PROBLEM SOLVING TACTICS

(a) Any formula that gives the value of $\sin\frac{A}{2}$ in terms of sin A shall also give the value of sin $\frac{n\pi+(-1)^n A}{2}$.

(b) Any formula that gives the value of $\cos\frac{A}{2}$ in terms of cos A shall also give the value of cos $\frac{2n\pi \pm A}{2}$.

(c) Any formula that gives the value of $\tan\frac{A}{2}$ in terms of tan A shall also give the value of tan $\frac{n\pi \pm A}{2}$.

(d) If α is the least positive value of θ which satisfies two given trigonometric equations, then the general value of θ will be $2n\pi+\alpha$. For example, $\sin\theta=\sin\alpha$ and $\cos\theta=\cos\alpha$, then, $\theta=2n\pi+\alpha, n\in I$

(i) $\sin(n\pi+\theta)=(-1)^n\sin\theta,\ n\in I$

(ii) $\cos(n\pi+\theta)=(-1)^n\cos\theta,\ n\in I$

(iii) $\sin(n\pi-\theta)=(-1)^{n-1}\sin\theta,\ n\in I$

FORMULAE SHEET

Tangent and cotangent Identities	$\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$
Product Identities	$\sin\theta \times \operatorname{cosec}\theta = 1$, $\cos\theta \times \sec\theta = 1$, $\tan\theta \times \cot\theta = 1$
Pythagorean Identities	$\sin^2\theta + \cos^2\theta = 1$, $\tan^2\theta + 1 = \sec^2\theta$, $1 + \cot^2\theta = \csc^2\theta$
Even/Odd Formulas	$\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$, $\tan(-\theta) = -\tan\theta$, $\cot(-\theta) = -\cot\theta$, $\sec(-\theta) = \sec\theta$, $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
Periodic Formulas (If n is an integer)	$\sin(2n\pi + \theta) = \sin\theta$, $\cos(2n\pi + \theta) = \cos\theta$, $\tan(n\pi + \theta) = \tan\theta$, $\cot(n\pi + \theta) = \cot\theta$, $\sec(2n\pi + \theta) = \sec\theta$, $\operatorname{cosec}(2n\pi + \theta) = \operatorname{cosec}\theta$
Double and Triple Angle Formulas	$\sin(2\theta) = 2\sin\theta\cos\theta$, $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$, $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$
Complementary angles	$\sin\left(\frac{\pi}{2} \pm \theta\right) = \cos\theta$, $\cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin\theta$, $\tan\left(\frac{\pi}{2} \pm \theta\right) = \mp\cot\theta$, $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$, $\sec\left(\frac{\pi}{2} - \theta\right) = \operatorname{cosec}\theta$, $\operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec\theta$
Half Angle	$\sin^2\theta = \frac{1}{2}\left[1 - \cos(2\theta)\right]$, $\cos^2\theta = \frac{1}{2}\left[1 + \cos(2\theta)\right]$, $\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$
Sum and Difference	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$, $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$, $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$,

Product to Sum	$\sin\alpha\sin\beta=\frac{1}{2}\left[\cos\left(\alpha-\beta\right)-\cos\left(\alpha+\beta\right)\right],$ $\sin\alpha\cos\beta=\frac{1}{2}\left[\sin\left(\alpha+\beta\right)+\sin\left(\alpha-\beta\right)\right],$ $\cos\alpha\cos\beta=\frac{1}{2}\left\lfloor\cos\left(\alpha-\beta\right)+\cos\left(\alpha+\beta\right)\right\rfloor,$ $\cos\alpha\sin\beta=\frac{1}{2}\left[\sin\left(\alpha+\beta\right)-\sin\left(\alpha-\beta\right)\right],$
Sum to Product	$\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right),$ $\sin\alpha-\sin\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$ $\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ $\cos\alpha-\cos\beta=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$