

Trigonometric Ratios, Identities and Equations

Solved Examples

JEE Main/Boards

Example 1: Solve: $2\cos 2\theta + \sqrt{2\sin \theta} = 2$

Sol: Solve this example by using $\sin^2 \theta = \frac{1}{2}[1 - \cos(2\theta)]$.

$$\sqrt{2\sin \theta} = 2(1 - \cos 2\theta) = 4\sin^2 \theta$$

$$\therefore 2 \sin \theta = 16 \sin^4 \theta : \sin \theta \geq 4$$

$$\sin \theta = 0 \text{ or } \sin^3 \theta = \frac{1}{8} \therefore \sin \theta = 0 \text{ or}$$

$$\frac{1}{2}, \theta = m\pi : m \in \mathbb{I} \text{ or } \theta = n\pi + (-1)^n \frac{\pi}{6} : n \in \mathbb{I}$$

Example 2: Solve: $8\tan^2 \frac{x}{2} = 1 + \sec x$

Sol: As we know that

$\tan^2 \frac{x}{2} = \frac{1 - \cos x}{1 + \cos x}$, substitute this to solve above example.

$$8\tan^2 \frac{x}{2} = 1 + \sec x \quad \dots (i)$$

$$\Rightarrow 8 \left(\frac{1 - \cos x}{1 + \cos x} \right) = \frac{\cos x + 1}{\cos x} : \cos x \neq 0, -1$$

$$\text{or } (8 - 8 \cos x)\cos x = (\cos x + 1)^2$$

$$\text{or } 8 \cos x - 8 \cos^2 x = \cos^2 x + 2\cos x + 1$$

$$\text{or } 9\cos^2 x - 6 \cos x + 1 = 0$$

$$\text{or } (3\cos x - 1)^2 = 0$$

$$\text{or } \cos x = \frac{1}{3} = \cos \beta, (\text{say}), \beta = \cos^{-1} \left(\frac{1}{3} \right)$$

$$\therefore x = 2n\pi \pm \beta : n \in \mathbb{I}$$

Example 3: Solve: $\sin x + \cos x - 2\sqrt{2}\sin x \cos x = 0$

Sol: We can write given equation as $\sin x + \cos x = \sqrt{2}\sin 2x$, multiplying and dividing L.H.S. by $\sqrt{2}$, we will get the result.

$$\text{or } \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x \right) = \sqrt{2} \sin 2x$$

$$\text{or } \sin \left(x + \frac{\pi}{4} \right) = \sin 2x$$

$$\Rightarrow 2x = n\pi + (-1)^n \left(x + \frac{\pi}{4} \right) : n \in I$$

Example 4: Find the general value of θ which satisfies both the equations $\cos \theta = -\frac{\sqrt{3}}{2}$ and $\sin \theta = \frac{1}{2}$.

Sol: Use the method for simultaneous equations.

$$\cos \theta = -\frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{5\pi}{6}, \frac{7\pi}{6} \dots$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6} \dots$$

Hence, the general solution is given by

$$\theta = 2n\pi + \frac{5\pi}{6}, n \in I$$

Example 5: Show that the equation

$$2\cos^2 \left(\frac{x}{2} \right) \sin^2 x = x^2 + x^{-2} \text{ for } 0 < x \leq \frac{\pi}{2}$$

has no real solution.

Sol: Here $2\cos^2 \frac{x}{2} \sin^2 x = x^2 + \frac{1}{x^2}$ holds only if $x^2 = 1$, hence by substituting $x = 1$ in above equation we can conclude that given equation has real solution or not.

$$x^2 + \frac{1}{x^2} \geq 2 \forall x \text{ with equality for}$$

$$x^2 = 1 \text{ alone. Since } 2\cos^2 \frac{x}{2} \sin^2 x \leq 2,$$

$$\therefore 2\cos^2 \frac{x}{2} \sin^2 x = x^2 + \frac{1}{x^2} \text{ holds only}$$

$$\text{If } x^2 = 1 \therefore x = 1 \text{ and } \cos \frac{x}{2} \sin x = \pm 1$$

$$\text{i.e. } \cos \left(\frac{1}{2} \right) \sin 1 = \pm 1, \text{ which is not true.}$$

Hence, the given equation has no solution.

Example 6: Determine 'a' for which the equation $a^2 - 2a + \sec^2(\pi(a+x)) = 0$ has solutions and find the solutions.

Sol: By using algebra and tangent of angle we can get the result.

$$a^2 - 2a + 1 + \tan^2(\pi(a+x)) = 0$$

$$\text{or } (a-1)^2 + [\tan \pi(a+x)]^2 = 0$$

$$\Rightarrow a-1 = 0 \text{ and } \tan \pi(a+x) = 0$$

$$\Rightarrow \tan(1+x)\pi = 0 \Rightarrow (1+x)\pi = n\pi : n \in I$$

$$\therefore x = n-1 : n \in I : a=1$$

Example 7: Solve the equation $\cos^7 x + \sin^4 x = 1$

Sol: Here $\cos^7 x \leq \cos^2 x$ and $\sin^4 x \leq \sin^2 x$, hence by solving this we will get the result.

$$\cos^7 x \leq \cos^2 x \text{ and } \sin^4 x \leq \sin^2 x$$

$$\therefore 1 = \cos^7 x + \sin^4 x \leq \cos^2 x + \sin^2 x = 1$$

$$\therefore \cos^7 x = \cos^2 x \text{ and } \sin^4 x = \sin^2 x$$

$$\cos^7 x = \cos^2 x \Rightarrow \cos^2 x (\cos^5 x - 1) = 0$$

$$\therefore \cos x = 0 \text{ or } \cos x = 1$$

$$\therefore x = (2n+1) \frac{\pi}{2}; n \in I$$

$$\text{or } x = 2m\pi; m \in I \quad \dots(i)$$

$$\sin^2 x = \sin^4 x \Rightarrow \sin^2 x \cos^2 x = 0$$

$$\Rightarrow \cos x = 0 \text{ or } \sin x = 0 \quad \dots(ii)$$

Since (i) satisfies the system (ii),

\therefore Solution set is given by (i)

Example 8: Solve for x and y:

$$12\sin x + 5\cos x = 2y^2 - 8y + 21$$

Sol: Multiply and divide L.H.S. by 13 and solve to get the result.

$$12\sin x + 5\cos x = 2y^2 - 8y + 21$$

$$\therefore \sqrt{12^2 + 5^2} \left(\frac{12}{13} \sin x + \frac{5}{13} \cos x \right) = 2(y^2 - 4y + 4) + 13$$

$$\text{or } 13\cos(x-\alpha) = 2(y-2)^2 + 13$$

$$\therefore \cos \alpha = \frac{5}{13} \text{ and } \sin \alpha = \frac{12}{13}$$

$$\text{Thus, } \cos(x-\alpha) = 1 \text{ and } y = 2 \text{ or}$$

$$x - \alpha = 2n\pi : n \in I \text{ and } y = 2$$

$$\therefore x = 2n\pi + \alpha : n \in I \text{ and } y = 2$$

JEE Advanced/Boards

Example 1: Solve for x, y : $\cos^3 y + 3\cos y \sin^2 y = 14$
 $x\sin^3 y + 3x\cos^2 y \sin y = 13$

Sol: Divide equation 1 by equation 2 and then by applying componendo and dividendo we can solve this problem.

We note that, " $x = 0; \sin y = 0$ or $\cos y = 0$ " do not yield a solution to given system.

$$\frac{\cos^3 y + 3\cos y \sin^2 y}{\sin^3 y + 3\cos^2 y \sin y} = \frac{14}{13}$$

By componendo and dividendo,

$$\begin{aligned} & \frac{\cos^3 y + 3\cos y \sin^2 y + 3\cos^2 y \sin y + \sin^3 y}{\cos^3 y + 3\cos y \sin^2 y - 3\cos^2 y \sin y - \sin^3 y} \\ &= \frac{14+13}{14-13} \text{ or } \left(\frac{\cos y + \sin y}{\cos y - \sin y} \right)^3 = 27 = (3)^3 \\ & \text{or } \frac{\cos y + \sin y}{\cos y - \sin y} = 3 \text{ or } \frac{1 + \tan y}{1 - \tan y} = \frac{3}{1} \end{aligned}$$

$$\tan y = \frac{1}{2} = \tan \alpha; y = n\pi + \alpha : n \in \mathbb{I}$$

Since $\sin y$ and $\cos y$ have signs, we have the following cases:

$$(i) \sin y = \frac{1}{\sqrt{5}} \text{ and } \cos y = \frac{2}{\sqrt{5}}; \text{ then}$$

$$x \left[\frac{8}{5\sqrt{5}} + 3 \frac{2}{\sqrt{5}} \cdot \frac{1}{5} \right] = 14 \Rightarrow x = 5\sqrt{5}$$

$$(ii) \sin y = -\frac{1}{\sqrt{5}} \text{ and } \cos y = -\frac{2}{\sqrt{5}}; \text{ then}$$

$$x \left[\frac{-8}{5\sqrt{5}} + 3 \left(\frac{-2}{\sqrt{5}} \right) \frac{1}{5} \right] = 14 \Rightarrow x = -5\sqrt{5}$$

Example 2: Solve: $\sin^4 x + \cos^4 x = \frac{7}{2} \sin x \cos x$

Sol: By substituting $2\sin x \cos x = t$ and solving we will be get the result.

$$\sin^4 x + \cos^4 x = \frac{7}{4} \sin 2x; \therefore \sin 2x > 0$$

$$(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x = \frac{7}{4} \sin 2x$$

$$\therefore 1 - \frac{t^2}{2} = \frac{7t}{4} \text{ or } 2t^2 + 7t - 4 = 0$$

$$\text{or } (2t-1)(t+4) = 0 \therefore \sin 2x = t = \frac{1}{2} = \sin \frac{\pi}{6}$$

$\Rightarrow 2x = n\pi + (-1)^n \frac{\pi}{6}; n \in \mathbb{I}$ General solution is

$$x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}, n \in \mathbb{I}$$

Example 3: Solve: $3\tan(\theta - 15^\circ) = \tan(\theta + 15^\circ)$

Sol: We can write given equation as $\frac{\tan(\theta + 15^\circ)}{\tan(\theta - 15^\circ)} = \frac{3}{1}$,

hence by applying componendo and dividendo we will get the result.

$$\text{Given, } 3\tan(\theta - 15^\circ) = \tan(\theta + 15^\circ)$$

$$\text{or } \frac{\tan(\theta + 15^\circ)}{\tan(\theta - 15^\circ)} = \frac{3}{1}$$

$$\text{or } \frac{\tan(\theta + 15^\circ) + \tan(\theta - 15^\circ)}{\tan(\theta + 15^\circ) - \tan(\theta - 15^\circ)} = \frac{4}{2}$$

$$\text{or } \frac{\sin(\theta + 15^\circ + \theta - 15^\circ)}{\sin(\theta + 15^\circ - \theta + 15^\circ)} = 2 \text{ or } \sin 2\theta = 1 = \sin \frac{\pi}{2}$$

$$\Rightarrow 2\theta = n\pi + (-1)^n \frac{\pi}{2}; n \in \mathbb{I} \therefore \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{4}; n \in \mathbb{I}$$

Example 4: Find value of θ for $\sin 2\theta = \cos 3\theta$, where $0 \leq \theta \leq 2\pi$; Use the above equation to find the value of $\sin 18^\circ$.

Sol: Here as we know $\sin \theta = \cos \left(\frac{\pi}{2} - \theta \right)$, hence we can write given equation as $\cos 3\theta = \cos \{(\pi/2) - 2\theta\}$.

Therefore by comparing their angle we will get the result.

The given equation is $\sin 2\theta = \cos 3\theta$ or, $\cos 3\theta = \sin 2\theta$ or, $\cos 3\theta = \cos \{(\pi/2) - 2\theta\}$

$$\text{or, } 3\theta = 2n\pi \pm \{(\pi/2) - 2\theta\} \quad \text{where } n \in \mathbb{I}$$

$$\text{Taking + sign, } 3\theta = 2n\pi + \{(\pi/2) - 2\theta\}$$

$$\text{or } 5\theta = (4n+1)(\pi/2)$$

$$\text{or, } \theta = (4n+1)(\pi/10), \text{ where } n \in \mathbb{I} \quad \dots (i)$$

$$\text{Again taking - sign, } 3\theta - 2n\pi = \{(\pi/2) - 2\theta\}$$

$$\text{or } \theta = (4n+1)(\pi/2) \quad \dots (ii)$$

Putting $n = 0, 1, 2, 3, \dots$ in (i) the values of θ in the interval $0 \leq \theta \leq 2\pi$ are given by

$$\theta = \pi/10, 5\pi/10, 9\pi/10, 13\pi/10, 17\pi/10 \text{ or } 18^\circ, 90^\circ, 162^\circ, 234^\circ, 346^\circ.$$

Again putting $n = 0, \pm 1, \pm 2, \dots$, in (ii) the value of θ in the interval $0 \leq \theta \leq 2\pi$ is $3\pi/2$ i.e. 270° only.

Hence the required values of θ in $0 \leq \theta \leq 2\pi$ are $18^\circ, 90^\circ, 162^\circ, 234^\circ, 270^\circ, 306^\circ$.

Example 5: Solve the equation:

$$\begin{aligned} & \cos(\pi 3^x) - 2\cos^2(\pi 3^x) + 2\cos(4\pi 3^x) - \cos(7\pi 3^x) \\ &= \sin(\pi 3^x) + 2\sin^2(\pi 3^x) - 2\sin(4\pi 3^x) \\ &+ 2\sin(\pi 3^{x+1}) - \sin(7\pi 3^x) \end{aligned}$$

Sol: Substitute $\pi 3^x = y$, and then by using sum to product formula we can solve this equation.

Denote $\pi 3^x$ by y to get

$$\begin{aligned} & \cos y - 2\cos^2 y + 2\cos 4y - \cos 7y \\ &= \sin y + 2\sin^2 y - 2\sin 4y + 2\sin 3y - \sin 7y \end{aligned} \quad \dots(i)$$

Transposing all terms to the left side,

$$\text{we have, } (\cos y - \cos 7y) + (\sin 7y - \sin y)$$

$$+ 2(\cos 4y + \sin 4y) - 2(\sin 3y + 1) = 0$$

$$\text{or, } 2\sin 4y \sin 3y + 2\cos 4y \sin 3y$$

$$+ 2(\cos 4y + \sin 4y) - 2(\sin 3y + 1) = 0$$

[Use C & D formulae]

$$\text{or, } 2\sin 3y(\sin 4y + \cos 4y)$$

$$+ 2(\cos 4y + \sin 4y) - 2(\sin 3y + 1) = 0$$

$$\text{or, } (\sin 3y + 1)(\sin 4y + \cos 4y - 1) = 0$$

This enables us to write down three groups of solutions:

$$y_1 = -\frac{\pi}{6} + \frac{2k\pi}{3}, \quad y_2 = \frac{n\pi}{2}, \quad y_3 = \frac{\pi}{8} + \frac{m\pi}{2}$$

where k, n and m are arbitrary integers. Recalling that $y = \pi 3^x$, we obtain an infinity of equations for determining the roots of the original equations:

$$\begin{aligned} 3^x &= -\frac{1}{6} + \frac{2k}{3}, \quad k = 0, \pm 1, \pm 2, \dots \quad 3^x = \frac{n}{2}, \quad n = 0, \pm 2, \\ &\dots = \frac{1}{8} + \frac{m}{2}, \quad m = 0, \pm 1, \pm 2, \dots \end{aligned}$$

The equation $3^x = a$ has a (unique) root only for positive a and it is given by the formula $x = \log_3 a$. Therefore, the equation (i) has a solution

only for those (integral) values of k, n, m for which the corresponding right members of the relations (i) are positive.

It is easy to see that of the first equation of (i) that is positive for integer $k > 0$, the right side of second equation of (i) is positive for integral $n > 0$; and the right side of the third equation of (i) is positive for $m \geq 0$. Thus, we have to solve (i) only for the indicated values of k, m, n . The resulting values of x are then the roots of the original equation:

$$x = \log_3 \left(-\frac{1}{6} + \frac{2k}{3} \right); \quad k = 1, 2, \dots$$

$$x = \log_3 \left(\frac{n}{2} \right), \quad n = 1, 2, \dots = \log_3 \left(\frac{1}{8} + \frac{1}{m} \right); \quad m = 0, 1, 2$$

Example 6: Solve the equation:

$$\sqrt{17\sec^2 x + 16 \left(\frac{1}{2} \tan x \sec x - 1 \right)} = 2\tan x (1 + 4\sin x)$$

Sol: Solve it like algebra by using product and Pythagorean identity.

The given equation is

$$\begin{aligned} & \sqrt{17\sec^2 x + 16 \left(\frac{1}{2} \tan x \sec x - 1 \right)} \\ &= 2\tan x (1 + 4\sin x) \end{aligned} \quad \dots(i)$$

$$\Rightarrow \sqrt{17\sec^2 x + 8\tan x \sec x - 16} = 2\tan x (1 + 4\sin x)$$

$$\Rightarrow \sqrt{17\sec^2 x + 8\tan x \sec x - 16} = 2\tan x (1 + 4\sin x)$$

$$\begin{aligned} & \Rightarrow \sqrt{17\sec^2 x + 8\tan x \sec x - 16(\sec^2 x - \tan^2 x)} \\ &= 2\tan x (1 + 4\sin x) \end{aligned}$$

$$\Rightarrow \sqrt{16\tan^2 x + 8\tan x \sec x + \sec^2 x}$$

$$= 2\tan x (1 + 4\sin x)$$

$$\sqrt{(4\tan x)^2 + 2 \times 4\tan x \sec x + \sec^2 x}$$

$$= 2\tan x (1 + 4\sin x)$$

$$4\tan x + \sec x = 2\tan x (1 + 4\sin x)$$

$$\Rightarrow 8\sin x \cdot \tan x - 2\tan x - \sec x = 0$$

$$\Rightarrow 8\frac{\sin^2 x}{\cos x} - 2\frac{\sin x}{\cos x} - \frac{1}{\cos x} = 0$$

$$\Rightarrow 8\sin^2 x - 2\sin x - 1 = 0$$

$$\therefore \sin x = \frac{1}{2} \text{ and } \sin x = -\frac{1}{4}$$

$$\sin x = \sin \pi/6 \Rightarrow x = \sin^{-1}\left(-\frac{1}{4}\right)$$

\therefore Solution of (i) is given by

$$x = n\pi \left(-1\right)^n \frac{\pi}{6} \text{ and } x = n\pi + \left(-1\right)^{n+1} \sin^{-1}\left(\frac{1}{4}\right),$$

where $n \in \mathbb{I}$

Example 7: Solve the equation:

$$\sin^{10} x + \cos^{10} x = \frac{29}{16} \cos^4 2x$$

Sol: we can represent given equation as

$$\left(\frac{2\sin^2 x}{2}\right)^5 + \left(\frac{2\cos^2 x}{2}\right)^5 = \frac{29}{16} \cos^4 2x, \text{ then use half angle formula to solve this problem.}$$

$$\text{Given, } \sin^{10} x + \cos^{10} x = \frac{29}{16} \cos^4 2x$$

$$\left(\frac{2\sin^2 x}{2}\right)^5 + \left(\frac{2\cos^2 x}{2}\right)^5 = \frac{29}{16} \cos^4 2x$$

$$\left(\frac{1-\cos 2x}{2}\right)^5 + \left(\frac{1+\cos 2x}{2}\right)^5 = \frac{29}{16} \cos^4 2x$$

$$\frac{(1-\cos 2x)^2 \cdot (1-\cos 2x)^3 + (1+\cos 2x)^2 \cdot (1+\cos 2x)^3}{32}$$

$$= \frac{29}{16} \cos^4 2x$$

$$\Rightarrow 10\cos^4 2x + 20\cos^2 2x + 2 = 58\cos^4 2x$$

$$\Rightarrow 48\cos^4 2x - 20\cos^2 2x - 2 = 0$$

$$\Rightarrow 24\cos^4 2x - 10\cos^2 2x - 1 = 0$$

$$\Rightarrow (2\cos^2 2x - 1)(12\cos^2 2x + 1) = 0$$

$$\therefore 2\cos^2 2x - 1 = 0 \quad [12\cos^2 2x + 1 \neq 0]$$

$$\Rightarrow \cos 4x = 0$$

$$\Rightarrow 4x = n\pi + \frac{\pi}{2}$$

$$\Rightarrow x = \frac{n\pi}{4} + \frac{\pi}{8}, n \in \mathbb{I}$$

Example 8: Consider the system of linear equations in x, y and z :

$$(\sin 3\theta)x - y + z = 0 \quad \dots (i)$$

$$(\cos 2\theta)x + 4y + 3z = 0 \quad \dots (ii)$$

$$\frac{\pi}{4} 2x + 7y + 7z = 0 \quad \dots (iii)$$

Find the value of θ for which the system has a non-trivial solution.

Sol: Here we can write given linear equation in matrix form, and as we know for the system having non-trivial solution $|A|$ must be 0.

We can write the given linear equation in the form of $AX=O$

$$A = \begin{bmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

As the system has a non-trivial solution,

$|A|$ must be 0

$$\therefore \begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

$$\Rightarrow 7\sin 3\theta + 14\cos 2\theta - 14 = 0$$

$$\Rightarrow \sin 3\theta + 2\cos 2\theta - 2 = 0,$$

$$(3\sin \theta - 4\sin^3 \theta) + 2(1 - 2\sin^2 \theta) - 2 = 0 \text{ or}$$

$$4\sin^3 \theta + 4\sin^2 \theta - 3\sin \theta = 0$$

$$\sin \theta (4\sin^2 \theta + 4\sin \theta - 3) = 0$$

$$\sin \theta (2\sin \theta - 1)(2\sin \theta + 3) = 0$$

$$\therefore \sin \theta = 0 \text{ or } \sin \theta = 1/2 \text{ or } \sin \theta = -3/2$$

But $\sin \theta = 0$ $\sin \theta = \frac{1}{2}$ is possible

And $\sin \theta = \frac{-3}{2}$ is not possible.

Now, $\sin \theta = 0 \therefore \theta = n\pi ; n \in \mathbb{I}$

and $\sin \theta = 1/2 = \sin(\pi/6)$,

$$\therefore \theta = n\pi + (-1)^n \pi/6; n \in \mathbb{I}$$

Therefore the required values of θ are

$$\theta = n\pi \text{ and } n\pi + (-1)^n (\pi/6), \text{ where } n \in \mathbb{I}$$

Example 9: Find the value of x ,

$$\frac{1}{2^{\cos^2 x}} \sqrt{y^2 - y + \frac{1}{2}} \leq 1$$

Sol: Re-write the expression inside the square root and then by using algebra we can find out the value of x .

Given

$$\frac{1}{2^{\cos^2 x}} \sqrt{y^2 - y + \frac{1}{2}} \leq 1 \quad \dots(i)$$

$$\frac{1}{2^{\cos^2 x}} \sqrt{y^2 - y + \frac{1}{4} + \frac{1}{4}} \leq 1$$

$$\frac{1}{2^{\cos^2 x}} \sqrt{\left(y - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \leq 1$$

Minimum value of $2^{\frac{1}{\cos^2 x}} = 2$

$$\text{Minimum value of } \sqrt{\left(y - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2}$$

\Rightarrow Minimum value of (i) is $2 \times \frac{1}{2} = 1$.

$$\therefore \frac{1}{2^{\cos^2 x}} \sqrt{\left(y - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1 \Rightarrow \cos^2 x = 1 \text{ and } y = \frac{1}{2}$$

$$\Rightarrow \cos^2 2n\pi = 1$$

$$\therefore x = 2n\pi$$

JEE Main/Boards

Exercise 1

Q.1 Solve the following trigonometric equations:

$$(i) \sin 2\theta = \frac{1}{2} \quad (ii) \cos 5\theta = -\frac{1}{2}$$

Q.2 Solve $7\cos^2 \theta + 3\sin^2 \theta = 4$

Q.3 Solve: $\tan x + \tan 2x + \sqrt{3} \tan x \tan 2x = \sqrt{3}$

Q.4 Solve: $3\tan(\theta - 15^\circ) = \tan(\theta + 15^\circ)$

Q.5 Solve the equations

$$\sin(x-y) = \frac{\sqrt{3}}{2} \text{ and } \cos(x+y) = \frac{1}{2}$$

Q.6 Solve the equation $\sin x = \tan x$

Q.7 Solve the equation $2\tan\theta - \cot\theta + 1 = 0$

Q.8 Solve the equations $\sin mx + \sin nx = 0$

Q.9 Solve the equation $\sec^2 2x = 1 - \tan 2x$

Q.10 Solve the equation:

$$4\sin x \cos x + 2\sin x + 2\cos x + 1 = 0$$

Q.11 Solve the equation: $\tan x + \sec x = 2\cos x$

Q.12 Solve: $2\sin^2 x - 5\sin x \cos x - 8\cos^2 x = -2$

Q.13 Solve: $4\sin x \sin 2x \sin 4x = \sin 3x$

Q.14 Solve the equation

$$(1 - \tan\theta)(1 + \sin 2\theta) = 1 + \tan\theta$$

Q.15 Solve $\sin x + \sqrt{3} \cos x = \sqrt{2}$

Q.16 Find the general solution of the following trigonometric equations:

$$(i) \tan 3\theta = -1 \quad (ii) \cos 5x = -\frac{1}{\sqrt{2}}$$

Q.17 Solve the following trigonometric equations:

$$(i) 3\cos^2 \theta + 7\sin^2 \theta = 4$$

$$(ii) \tan x + \tan 2x + \tan 3x = \tan 2x \tan 3x$$

$$(iii) \tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 4$$

Q.18 Solve the equation $\tan x + \cot x = 2$

Q.19 Find the general solution of the trigonometric equation: $\tan^3 x - 3\tan x = 0$

Q.20 Solve the following trigonometric equations:

(i) $\cos x + \sin x = 1$ (ii) $\sec x - \tan x = \sqrt{3}$

(iii) $\sin x + \cos x = \frac{1}{\sqrt{2}}$ (iv) $\cos x + \sqrt{3} \sin x = 1$

Q.21 Find the degree measures corresponding to the following radian measures.

(i) $\left(\frac{\pi}{6}\right)^c$ (ii) $\left(\frac{4\pi}{5}\right)^c$ (iii) $(1.2)^c$

Q.22 The angles in a triangle are in A.P. and the ratio of the smallest angle in degree to the greatest angle in radians is $60 : \pi$. Find the angles of the triangle in degrees and radians.

Q.23 Assuming the distance of the earth from the moon to be 38400 km and the angle subtended by the moon at the eye of a person on the earth to be $31'$, find the diameter of the moon.

Q.24 Assuming that a person of normal sight can read print at such a distance that the letters subtend an angle of $5'$ at his eye, find the height of the letters that he can read at a distance of 12 meters.

Q.25 Solve the equation $4\cos^2 x \sin x - 2\sin^2 x = 3\sin x$

Q.26 Solve the equation:

$$5\cos 2\theta + 2\cos^2 \frac{\theta}{2} + 1 = 0, -\pi < \theta < \pi$$

Q.27 Solve the equation: $4\sin^4 x + \cos^4 x = 1$

Q.28 Solve the equation: $\tan 2\theta \tan \theta = 1$

Q.29 Show that the equation: $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution.

Q.30 Does the equation $\sin^4 \theta - 2\sin^2 \theta - 1 = 0$ has a solution?

Exercise 2

Single Correct Choice Type

Q.1 If in a triangle ABC, $b\cos^2 \frac{A}{2} + a\cos^2 \frac{B}{2} = \frac{3}{2}c$, then a, b, c are:

- (A) In A.P. (B) In G.P. (C) In H.P. (D) None

Q.2 Given $a^2 + 2a + \operatorname{cosec}^2 \left(\frac{\pi}{2}(a+x) \right) = 0$

then, which of the following holds good?

- (A) $a = 1; \frac{x}{2} \in I$
 (B) $a = -1; \frac{x}{2} \in I$
 (C) $a \in R; x \in \phi$
 (D) a, x are finite but not possible to find

Q.3 In any triangle ABC, $(a+b)^2 \sin^2 \frac{C}{2} + (a-b)^2 \cos^2 \frac{C}{2} =$

- (A) $c(a+b)$ (B) $b(c+a)$ (C) $a(b+c)$ (D) c^2

Q.4 If in a ΔABC , $\sin^3 A + \sin^3 B + \sin^3 C$

$= 3\sin A \cdot \sin B \cdot \sin C$ then

- (A) ΔABC may be a scalene triangle
 (B) ΔABC is a right triangle
 (C) ΔABC is an obtuse angled triangle
 (D) ΔABC is an equilateral triangle

Q.5 $\sin 3\theta = 4\sin \theta \sin 2\theta \sin 4\theta$ in $0 \leq \theta \leq \pi$ has:

- (A) 2 real solutions (B) 4 real solutions
 (C) 6 real solutions (D) 8 real solutions

Q.6 With usual notations, in a triangle ABC, $a\cos(B-C) + b\cos(C-A) + c\cos(A-B)$ is equal to

- (A) $\frac{abc}{R^2}$ (B) $\frac{abc}{4R^2}$ (C) $\frac{4abc}{R^2}$ (D) $\frac{abc}{2R^2}$

Q.7 If $\cos \alpha = \frac{2\cos \beta - 1}{2 - \cos \beta}$ then $\tan \frac{\alpha}{2} \cot \frac{\beta}{2}$ has the value equal to, where ($0 < \alpha < \pi$ and $0 < \beta < \pi$)

- (A) 2 (B) $\sqrt{2}$ (C) 3 (D) $\sqrt{3}$

Q.8 If $x\sin\theta = y\sin\left(\theta + \frac{2\pi}{3}\right) = z\sin\left(\theta + \frac{4\pi}{3}\right)$ then

- (A) $x+y+z=0$ (B) $xy+yz+zx=0$
 (C) $xyz+x+y+z=1$ (D) None of these

Q.9 If $a\cos^2\alpha + 3a\cos\alpha\sin^2\alpha = m$

and $a\sin^3\alpha + 3a\cos^2\alpha\sin\alpha = n$. Then

$(m+n)^{2/3} + (m-n)^{2/3}$ is equal to:

- (A) $2a^2$ (B) $2a^{1/3}$ (C) $2a^{2/3}$ (D) $2a^3$

Q.10 The number of solutions of

$\tan(5\pi\cos\theta) = \cot(5\pi\sin\theta)$ for θ in $(0, 2\pi)$ is

- (A) 28 (B) 14 (C) 4 (D) 2

Q.11 In a $\triangle ABC$ if $B + C = 3A$ then $\cot\frac{B}{2} \cdot \cot\frac{C}{2}$ has the value equal to

- (A) 4 (B) 3 (C) 2 (D) 1

Q.12 The set of value of 'a' for which the equation, $\cos 2x + a\sin x = 2a - 7$ possess a solution is-

- (A) $(-\infty, 2)$ (B) $[2, 6]$ (C) $(6, \infty)$ (D) $(-\infty, \infty)$

Q.13 In $\triangle ABC$, the minimum value of $\frac{\sum \cot^2 \frac{A}{2} \cdot \cot^2 \frac{B}{2}}{\prod \cot^2 \frac{A}{2}}$ is

- (A) 1 (B) 2
 (C) 3 (D) Non-existent

Q.14 The general solution of

$\sin x + \sin 5x = \sin 2x + \sin 4x$ is:

- (A) $2n\pi$ (B) $n\pi$
 (C) $n\pi/3$ (D) $2n\pi/3$ Where $n \in I$

Q.15 Number of values of $\theta \in [0, 2\pi]$ satisfying the equation $\cot x - \cos x = 1 - \cot x \cdot \cos x$

- (A) 1 (B) 2 (C) 3 (D) 4

Q.16 The exact value of

$\cos^2 73^\circ + \cos^2 47^\circ + (\cos 73^\circ \cdot \cos 47^\circ)$ is

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

Q.17 The maximum value of

$$(7\cos\theta + 24\sin\theta) \times (7\sin\theta - 24\cos\theta)$$

for every $\theta \in R$.

- (A) 25 (B) 625 (C) $\frac{625}{2}$ (D) $\frac{625}{4}$

Q.18 If $x = \frac{n\pi}{2}$, satisfies the equation

$$\sin\frac{x}{2} - \cos\frac{x}{2} = 1 - \sin x \text{ & the inequality}$$

$$\left| \frac{x}{2} - \frac{\pi}{2} \right| \leq \frac{3\pi}{4}, \text{ then :}$$

- (A) $n = -1, 0, 3, 5$ (B) $n = 1, 2, 4, 5$
 (C) $n = 0, 2, 4$ (D) $n = -1, 1, 3, 5$

Q.19 The number of all possible triplets (a_1, a_2, a_3)

such that $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0$ for all x is

- (A) 0 (B) 1 (C) 3 (D) Infinite

Q.20 If A and B are complimentary angles, then:

$$(A) \left(1 + \tan\frac{A}{2}\right) \left(1 + \tan\frac{B}{2}\right) = 2$$

$$(B) \left(1 + \cot\frac{A}{2}\right) \left(1 + \cot\frac{B}{2}\right) = 2$$

$$(C) \left(1 + \sec\frac{A}{2}\right) \left(1 + \cosec\frac{B}{2}\right) = 2$$

- (D) A set containing two values

Previous Years' Questions

Q.1 The equation $2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + x^{-2}$,

$0 < x \leq \frac{\pi}{2}$ has

(1980)

- (A) No real solution

- (B) One real solution

- (C) More than one real solution

- (D) None of above

Q.2 The smallest positive root of the equation, $\tan x - x = 0$ lies in

(1987)

- (A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{2}, \pi\right)$ (C) $\left(\pi, \frac{3\pi}{2}\right)$ (D) $\left(\frac{3\pi}{2}, 2\pi\right)$

Q.3 The number of solution of the equation $\sin(e^x) = 5^x + 5^{-x}$ is (1991)
 (A) 0 (B) 1 (C) 2 (D) Infinitely many

Q.4 The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution, is (2002)
 (A) 4 (B) 8 (C) 10 (D) 12

Q.5 The set of values of θ satisfying the in equation $2\sin^2 \theta - 5\sin \theta + 2 > 0$, where $0 < \theta < 2\pi$, is (2006)

- (A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$
 (C) $\left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, 2\pi\right]$ (D) None of these

Q.6 The number of solutions of the pair of equations $2\sin^2 \theta - \cos 2\theta = 0$ and $2\cos^2 \theta - 3\cos \theta = 0$ in the interval $[0, 2\pi]$ is (2007)

- (A) 0 (B) 1 (C) 2 (D) 4

Q.7 Let $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$ and $Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$ be two sets. Then (2011)

- (A) $P \subset Q$ and $Q - P \neq \emptyset$
 (B) $Q \not\subset P$
 (C) $P \not\subset Q$
 (D) $P = Q$

Q.8 If $\alpha + \beta + \gamma = 2\pi$, then (1979)

- (A) $\tan \frac{\alpha}{2} + \tan \frac{\beta}{2} + \tan \frac{\gamma}{2} = \tan \frac{\alpha}{2} \tan \frac{\beta}{2} \tan \frac{\gamma}{2}$
 (B) $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} + \tan \frac{\beta}{2} \tan \frac{\gamma}{2} + \tan \frac{\gamma}{2} \tan \frac{\alpha}{2} = 1$
 (C) $\tan \frac{\alpha}{2} + \tan \frac{\beta}{2} + \tan \frac{\gamma}{2} = -\tan \frac{\alpha}{2} \tan \frac{\beta}{2} \tan \frac{\gamma}{2}$
 (D) None of these

Q.9 Given $A = \sin^2 \theta + \cos^4 \theta$, then for all real values of θ (1980)

- (A) $1 \leq A \leq 2$ (B) $\frac{3}{4} \leq A \leq 1$
 (C) $\frac{13}{16} \leq A \leq 1$ (D) $\frac{3}{4} \leq A \leq \frac{13}{16}$

Q.10 The expression (1986)

$$3 \left[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4 (3\pi + \alpha) \right] - 2 \left[\sin^6 \left(\frac{\pi}{2} + \alpha \right) + \sin^6 (5\pi - \alpha) \right]$$

is equal to

- (A) 0 (B) 1
 (C) 3 (D) $\sin 4\alpha + \cos 6\alpha$

Q.11 If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$, then $\tan \alpha$, equals (2001)

- (A) $2(\tan \beta + \tan \gamma)$ (B) $\tan \beta + \tan \gamma$
 (C) $\tan \beta + 2 \tan \gamma$ (D) $2 \tan \beta + \tan \gamma$

Q.12 Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan \theta)^{\tan \theta}$, $t_2 = (\tan \theta)^{\cot \theta}$
 $t_3 = (\cos \theta)^{\tan \theta}$ and $t_4 = (\cot \theta)^{\cot \theta}$, then (2006)

- (A) $t_1 > t_2 > t_3 > t_4$ (B) $t_4 > t_3 > t_1 > t_2$
 (C) $t_3 > t_1 > t_2 > t_4$ (D) $t_2 > t_3 > t_1 > t_4$

Q.13 The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as (2013)

- (A) $\sec A \cosec A + 1$ (B) $\tan A + \cot A$
 (C) $\sec A + \cosec A$ (D) $\sin A \cos A + 1$

Q.14 If $0 \leq x < 2\pi$, then the number of real values of x , which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is: (2016)

- (A) 5 (B) 7 (C) 9 (D) 3

JEE Advanced/Boards

Exercise 1

Q.1 Solve the equation: $\sin 5x = 16 \sin^5 x$

Q.2 Find all the solutions, of $4\cos^2 x \sin x - 2\sin^2 x = 3\sin x$

Q.3 Find the number of solutions of the equation

$$1 + \cos x + \cos 2x + \sin x + \sin 2x + \sin 3x = 0.$$

Which satisfy the condition $\frac{\pi}{2} < \left| 3x - \frac{\pi}{2} \right| \leq \pi$.

Q.4 Solve for x , $(-\pi \leq x \leq \pi)$ the equation; $2(\cos x + \cos 2x) + \sin 2x(1 + 2\cos x) = 2\sin x$

Q.5 Find the general solution of the following equation:

$$2(\sin x - \cos 2x) - \sin 2x(1 + 2\sin x) + 2\cos x = 0$$

Q.6 Find the values of x , between 0 & 2π . Satisfying the

$$\cos 3x + \cos 2x = \sin \frac{3x}{2} + \sin \frac{x}{2}.$$

Q.7 Solve: $\tan^2 2x + \cot^2 2x + 2\tan 2x + 2\cot 2x = 6$

Q.8 Solve the equation: $1 + 2 \operatorname{cosec} x = -\frac{\sec^2 x}{2}$ and

$$\text{the inequality } \frac{2\cos 7x}{\cos 3 + \sin 3} > 2^{\cos 2x}$$

Q.9 Solve $\sin\left(\frac{\sqrt{x}}{2}\right) + \cos\left(\frac{\sqrt{x}}{2}\right) = \sqrt{2} \sin \sqrt{x}$

Q.10 Find all values of 'a' for which every root of the equation, $a \cos 2x + |\text{a}| \cos 4x + \cos 6x = 1$ is also a

root of the equation, $\sin x \cos 2x = \sin 2x \cos 3x - \frac{1}{2} \sin 5x$, and conversely, every root of the second equation is also a root of the first equation.

Q.11 Solve for x , the equation $\sqrt{13 - 18 \tan x} = 6 \tan x - 3$, where $-2\pi < x < 2\pi$.

Q.12 Determine the smallest positive value of x which satisfy the equation $\sqrt{1 + \sin 2x} - \sqrt{2} \cos 3x = 0$

$$\mathbf{Q.13} \quad 2\sin\left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8\sin 2x \cdot \cos^2 2x}$$

Q.14 Find the number of principal solutions of the equation.

$$\sin x - \sin 3x + \sin 5x = \cos x - \cos 3x + \cos 5x$$

Q.15 Find the general solution of the trigonometric equation

$$3^{\left(\frac{1}{2} + \log_3(\cos x + \sin x)\right)} - 2^{\log_2(\cos x - \sin x)} = \sqrt{2}$$

Q.16 Find all values of θ between 0° & 180° satisfying the equation; $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$

Q.17 Find the solution set of the equation,

$$\log_{\frac{-x^2-6x}{10}}(\sin 3x + \sin x) = \log_{\frac{-x^2-6x}{10}}(\sin 2x)$$

Q.18 Find the value of θ , which satisfy

$$3 - 2\cos \theta - 4\sin \theta - \cos 2\theta + \sin 2\theta = 0.$$

Q.19 Find the sum of the roots of the equation $\cos 4x + 6 = 7\cos 2x$ on the interval $[0, 314]$.

Q.20 Find the least positive angle measured in degrees satisfying the equation

$$\sin^3 x + \sin^3 2x + \sin^3 3x = (\sin x + \sin 2x + \sin 3x)^3$$

Q.21 Find the number of solution of the equation

$$\sin(\pi - 6x) + \sqrt{3} \sin\left(\frac{\pi}{2} + 6x\right) = \sqrt{3} \text{ in } [0, 2\pi]$$

Q.22 Find the general values of θ for which the quadratic function

$$(\sin \theta)x^2 + (2\cos \theta)x + \frac{\cos \theta + \sin \theta}{2}$$

is the square of a linear function.

Q.23 Prove that the equations

$$(a) \sin x \cdot \sin 2x \cdot \sin 3x = 1$$

$$(b) \sin x \cdot \cos 4x \cdot \sin 5x = -1/2$$

(c) $\sin x \cos x \cos 2x + 1/2 = 0$

(d) $4 \sin 2x + \cos x = 5$

(e) $\sin 3x - \cos x = 2$

Have no solution

Q.24 Let $f(x) = \sin^6 x + \cos^6 x + k(\sin^4 x + \cos^4 x)$ for some real number k . Determine

(a) All real number k for which $f(x)$ is constant for all values of x .

(b) All real numbers k for which there exists a real number ' c ' such that $f(c) = 0$

(c) If $k = -0.7$, determine all solutions to the equation $f(x) = 0$

Q.25 If α and β are the roots of the equation, $a \cos \theta + b \sin \theta = c$ then match the entries of column I with the entries of column II.

Q.27

Column I	Column II
(A) The general solution of the equation $\sin^2 x + \cos^2 3x = 1$ is equal to	(p) $n\pi$ (where $n \in \mathbb{I}$)
(B) The general solution of the equation $e^{\cot^2 \theta} + \sin^2 \theta - 2 \cos^2 2\theta + 4$ $= 4 \sin \theta$, is	(q) $\frac{n\pi}{4}$
(C) For all real values of a , the general solution of the equation $a^2 \sin x - a \sin 2x + \sin x = 0$, is equal to	(r) $n\pi + \frac{\pi}{4}$
(D) The general solution of the equation $\sqrt[3]{2 \tan \theta - 1} + \sqrt[3]{\tan \theta - 1} = 1$, is	(s) $(4n+1)\frac{\pi}{2}$

Column I	Column II
(A) $\sin \alpha + \sin \beta$	(p) $\frac{2b}{a+c}$
(B) $\sin \alpha \cdot \sin \beta$	(q) $\frac{c-a}{c+a}$
(C) $\tan \frac{\alpha}{2} + \tan \frac{\beta}{2}$	(r) $\frac{2bc}{a^2+b^2}$
(D) $\tan \frac{\alpha}{2} \cdot \tan \frac{\beta}{2}$	(s) $\frac{c^2-a^2}{a^2+b^2}$

Q.26 Solve the equations for 'x' given in column I and match the entries of column II.

Column I	Column II
(A) $\cos 3x \cdot \cos^3 x + \sin 3x \cdot \sin^3 x = 0$	(p) $n\pi \pm \frac{\pi}{3}$
(B) $\sin 3\alpha = 4 \sin \alpha \sin(x+\alpha) \sin(x-\alpha)$ Where α is a constant $\neq n\pi$	(q) $n\pi + \frac{\pi}{4}, n \in \mathbb{I}$
(C) $ 2 \tan x - 1 + 2 \cot x - 1 = 2$	(r) $\frac{n\pi}{4} + \frac{\pi}{8}, n \in \mathbb{I}$
(D) $\sin^{10} x + \cos^{10} x = \frac{29}{16} \cos^4 2x$	(s) $\frac{n\pi}{2} \pm \frac{\pi}{4}$

Exercise 2

Single Correct Choice Type

Q.1 If in a ΔABC , $\cos A \cos B + \sin A \sin B \sin 2C = 1$ then, the statement which is correct?

(A) ΔABC is isosceles and right angled

(B) ΔABC is acute angled

(C) ΔABC is not right angled

(D) Least angle of the triangle is $\frac{\pi}{3}$

Q.2 The set of values of x satisfying the equation,

$$2^{\tan\left(\frac{x-\pi}{4}\right)} - 2(0.25)^{\frac{\sin^2\left(x-\frac{\pi}{4}\right)}{\cos 2x}} + 1 = 0$$

(A) An empty set

(B) A singleton

(C) A set containing two elements

(D) An infinite set

Q.3 The number of solutions of the equation,

$$\sum_{r=1}^5 \cos(rx) = 0$$

lying in $(0, \pi)$ is:

- (A) 2 (B) 3 (C) 5 (D) More than 5

Q.4 The value of $\cot 7\frac{1}{2}^\circ + \tan 67\frac{1}{2}^\circ - \cot 67\frac{1}{2}^\circ - \tan 7\frac{1}{2}^\circ$ is

- (A) A rational number (B) Irrational number
 (C) $2(3+2\sqrt{3})$ (D) $2(3-\sqrt{3})$

Q.5 If $A = 580^\circ$ then which one of the following is true?

- (A) $2\sin\left(\frac{A}{2}\right) = \sqrt{1+\sin A} - \sqrt{1-\sin A}$
 (B) $2\sin\left(\frac{A}{2}\right) = -\sqrt{1+\sin A} + \sqrt{1-\sin A}$
 (C) $2\sin\left(\frac{A}{2}\right) = -\sqrt{1+\sin A} - \sqrt{1-\sin A}$
 (D) $2\sin\left(\frac{A}{2}\right) = \sqrt{1+\sin A} + \sqrt{1-\sin A}$

Q.6 If $\tan \alpha = \frac{x^2 - x}{x^2 - x + 1}$ and

$\tan \beta = \frac{1}{2x^2 - 2x + 1}$ ($x \neq 0, 1$), where

$0 < \alpha, \beta < \frac{\pi}{2}$, then $\tan(\alpha + \beta)$ has the value equal to:

- (A) 1 (B) -1 (C) 2 (D) $\frac{3}{4}$

Q.7 Minimum value of $8\cos^2 x + 18\sec^2 x \forall x \in \mathbb{R}$ wherever it is defined, is:

- (A) 24 (B) 25 (C) 26 (D) 18

Q.8 If ϕ is eliminated from the equations $x=a \cos(\phi-\alpha)$

and $y=b \cos(\phi-\beta)$ then $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos(\alpha-\beta)$ is equal to

- (A) $\cos^2(\alpha-\beta)$ (B) $\sin^2(\alpha-\beta)$
 (C) $\sec^2(\alpha-\beta)$ (D) $\operatorname{cosec}^2(\alpha-\beta)$

Q.9 The general solution of the trigonometric equation $\tan x + \tan 2x + \tan 3x = \tan x \cdot \tan 2x \cdot \tan 3x$ is

- (A) $x = n\pi$ (B) $n\pi \pm \frac{\pi}{3}$
 (C) $x = 2n\pi$ (D) $x = \frac{n\pi}{3}$ Where $n \in \mathbb{I}$

Q.10 Number of principal solutions of the equation $\tan 3x - \tan 2x - \tan x = 0$, is

- (A) 3 (B) 5 (C) 7 (D) More than 7

Q.11 The value of x that satisfies the relation

$$x = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots \infty$$

- (A) $2\cos 36^\circ$ (B) $2\cos 144^\circ$
 (C) $2\sin 18^\circ$ (D) None of these

Multiple Correct Choice Type

Q.12 An extreme value of $1 + 4\sin \theta + 3\cos \theta$ is:

- (A) -3 (B) -4 (C) 5 (D) 6

Q.13 It is known that $\sin \beta = \frac{4}{5}$ & $0 < \beta < \pi$ then the value of

$$\frac{\sqrt{3} \sin(\alpha + \beta) - \frac{2}{\cos \frac{\pi}{6}} \cos(\alpha + \beta)}{\sin \alpha}$$

is:

(A) Independent of α for all β in $(0, \pi/2)$

(B) $\frac{5}{\sqrt{3}}$ for $\tan \beta > 0$

(C) $\frac{\sqrt{3}(7 + 24 \cot \alpha)}{15}$ for $\tan \beta < 0$

(D) None of these

Q.14 If $\sin t + \cos t = \frac{1}{5}$ then $\tan \frac{1}{5}$ is equal to:

- (A) -1 (B) -1/3 (C) 2 (D) -1/6

Previous Years' Questions

Q.1 Show that the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no real solution. **(1982)**

Q.2 Find the values of $x (-\pi, \pi)$ which satisfy the equation $2^{1+|\cos x|+|\cos^2 x|+\dots} = 4$. **(1984)**

Q.3 If $\exp \left\{ (\sin^2 x + \sin^4 x + \sin^6 x + \dots + \infty) \log_e 2 \right\}$, satisfies the equation $x^2 - 9x + 8 = 0$, find the value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$. **(1991)**

Q.4 Determine the smallest positive value of x (in degree) for which $\tan(x + 100^\circ) = \tan(x + 50^\circ) \tan(x) \tan(x - 50^\circ)$ **(1993)**

Q.5 Find the smallest positive number p for which the equation $\cos(p \sin x) - \sin(p \cos x) = 0$ has a solution $x \in [0, 2\pi]$ **(1995)**

Q.6 Find all values of θ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ satisfying the equation

$$(1 - \tan \theta)(1 + \tan \theta) \sec^2 \theta + 2^{\tan^2 \theta} = 0 \quad \text{**(1996)**}$$

Q.7 The values of θ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation

$$\begin{vmatrix} 1 + \sin^2 \theta & \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & 1 + \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0 \quad \text{**(1988)**}$$

- (A) $\frac{7\pi}{24}$ (B) $\frac{5\pi}{24}$ (C) $\frac{11\pi}{24}$ (D) $\frac{\pi}{24}$

Q.8 For $0 < \phi < \pi/2$, if $x = \sum_{n=0}^{\infty} \cos^{2n} \phi$, $y = \sum_{n=0}^{\infty} \sin^{2n} \phi$, $z = \sum_{n=0}^{\infty} \cos^{2n} \phi \sin^{2n} \phi$, then **(1993)**

- (A) $xyz = xy + y$ (B) $xyz = xy + z$
 (C) $xyz = x + y + z$ (D) $xyz = yz + x$

Q.9 $\sec^2 \theta = \frac{4xy}{(x+y)^2}$ is true if and only if **(1996)**

- (A) $x = y \neq 0$ (B) $x = y, x \neq 0$
 (C) $x = y$ (D) $x \neq 0, y \neq 0$

Q.10 If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then **(2009)**

- (A) $\tan^2 x = \frac{2}{3}$ (B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
 (C) $\tan^2 x = \frac{1}{3}$ (D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

Q.11 For $0 < \theta < \frac{\pi}{2}$, the solution(s) of

$$\sum_{m=1}^6 \operatorname{cosec} \left(\theta + \frac{(m-1)\pi}{4} \right) \operatorname{cosec} \left(\theta + \frac{m\pi}{4} \right) = 4\sqrt{2} \quad \text{is/are} \quad \text{**(2009)**}$$

- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

Q.12 The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan \theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is **(2010)**

Q.13 The positive value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)} \text{ is} \quad \text{**(2011)**}$$

Q.14 The number of all possible values of θ , where $0 < \theta < \pi$, for which the system of equations

$$(y+z)\cos 3\theta = (xyz)\sin 3\theta \quad x \sin 3\theta = \frac{2 \cos 3\theta}{y} + \frac{2 \sin 3\theta}{z}$$

And $(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y \sin 3\theta$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is.... **(2010)**

Q.15 The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan \theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is **(2010)**

Q.16 The maximum value of the expression

$$\frac{1}{\sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta} \text{ is} \quad \text{**(2010)**}$$

Q.17 Let $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$

and $Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$ be two sets. Then **(2011)**

- (A) $P \subset Q$ and $Q - P \neq \emptyset$ (B) $Q \not\subset P$
 (C) $P \not\subset Q$ (D) $P = Q$

Q.18 The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)} \text{ is} \quad \text{**(2011)**}$$

Q.19 Let $\theta, \phi \in [0, 2\pi]$ be such that

$$2 \cos \theta (1 - \sin \phi) = \sin^2 \theta \left(\tan \frac{\theta}{2} + \cot \frac{\theta}{2} \right) \cos \phi - 1$$

$$\tan(2\pi - \theta) > 0 \text{ and } -1 < \sin \theta < -\frac{\sqrt{3}}{2}$$

Then ϕ cannot satisfy

(A) $0 < \phi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$

(C) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \phi < 2\pi$

Q.20 For $x \in (0, \pi)$, the equation $\sin x + 2 \sin 2x - \sin 3x = 3$ has

(2012)

(2014)

- (A) Infinitely many solutions
- (B) Three solutions
- (C) One solution
- (D) No solution

Q.21 The number of distinct solution of the equation

$$\frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

In the interval $[0, 2\pi]$ is

(2015)

Q.22 Let $S = \left\{ x \in (-\pi, \pi) : x \neq 0, \pm \frac{\pi}{2} \right\}$.

The sum of all distinct solutions of the equation $\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

(2016)

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

- Q.11 Q.17 Q.24
Q.29

Exercise 2

- Q.6 Q.9 Q.14
Q.17 Q.19

Previous Years' Questions

- Q.1 Q.4 Q.5
Q.8 Q.10 Q.13

JEE Advanced/Boards

Exercise 1

- Q.7 Q.12 Q.17
Q.21 Q.24 Q.26

Exercise 2

- Q.4 Q.7 Q.8
Q.11

Previous Years' Questions

- Q.4 Q.5 Q.6
Q.8 Q.10 Q.12

Answer Key

JEE Main/Boards

Exercise 1

Q.1 (i) $\frac{n\pi}{2} + (-1)^n \frac{\pi}{12}$, $n \in \mathbb{I}$ (ii) $\frac{2n\pi}{5} \pm \frac{2\pi}{15}$, $n \in \mathbb{I}$

Q.2 $2n\pi \pm \frac{\pi}{3}$, $2n\pi \pm \frac{2\pi}{3}$, $n \in \mathbb{I}$

Q.3 $\frac{n\pi}{3} + \frac{\pi}{9}$, $n \in \mathbb{I}$

Q.4 $\frac{n\pi}{2} + (-1)^n \frac{\pi}{4}$, $n \in \mathbb{I}$

Q.5 $x = \frac{1}{2} \left[n\pi + (-1)^n \frac{\pi}{3} + 2m\pi \pm \frac{\pi}{3} \right]$, $n, m \in \mathbb{I}$,

$$y = \frac{1}{2} \left[2m\pi \pm \frac{\pi}{3} - n\pi - (-1)^n \frac{\pi}{3} \right]$$

Q.6 $x = n\pi$, $n \in \mathbb{I}$

Q.7 $\tan \alpha = \frac{1}{2}$

Q.8 $x = \frac{2a\pi}{m+n}$ or $x = \frac{(2b+1)\pi}{m-n}$, $a, b \in \mathbb{I}$

Q.9 $x = \frac{n\pi}{2}, \frac{m\pi}{2} - \frac{\pi}{8}$, $n \in \mathbb{I}$

Q.10 $x = n\pi - (-1)^n \frac{\pi}{6}$, $2n\pi \pm \frac{2\pi}{3}$, $n \in \mathbb{I}$

Q.11 $x = n\pi + (-1)^n \frac{\pi}{6}$ or $(2n+1)\frac{\pi}{2}$, $n \in \mathbb{I}$

Q.12 $x = n\pi + \tan^{-1}(-\frac{3}{4})$ or $x = n\pi + \tan^{-1}(2)$

Q.13 $n\pi$ or $\frac{n\pi}{3} \pm \frac{\pi}{9}$, where $n \in \mathbb{I}$

Q.14 $n\pi, n\pi - \frac{\pi}{4}$, where $n \in \mathbb{I}$

Q.15 $2n\pi + \frac{5\pi}{12}, 2n\pi - \frac{\pi}{12}$, $n \in \mathbb{I}$

Q.16 (i) $\frac{n\pi}{3} - \frac{\pi}{12}$, $n \in \mathbb{I}$ (ii) $\frac{2n\pi}{5} \pm \frac{3\pi}{20}$, $n \in \mathbb{I}$

Q.17 (i) $n\pi + (-1)^n \left(\pm \frac{\pi}{6} \right)$ (ii) $x = \frac{n\pi}{3}$, $n \in \mathbb{I}$
 (iii) $n\pi, n\pi \pm \frac{n\pi}{3}$, $n \in \mathbb{I}$

Q.18 $n\pi + \frac{\pi}{4}$, $n \in \mathbb{I}$

Q.19 $n\pi, n\pi \pm \frac{\pi}{3}$, $n \in \mathbb{I}$

Q.20 (i) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$, $n \in \mathbb{I}$ i.e.

$2n\pi$ or $2n\pi + \frac{\pi}{2}$, $n \in \mathbb{I}$ (ii) $2n\pi \pm \frac{\pi}{3} + \frac{\pi}{6}$, $n \in \mathbb{I}$

(iii) $2n\pi + \frac{7\pi}{12}$, $n \in \mathbb{I}$ (iv) $2n\pi, 2n\pi + \frac{2\pi}{3}$, $n \in \mathbb{I}$

Q.21 (i) 30° (ii) 144° (iii) $68^\circ 43' 37.8''$

Q.22 $30^\circ, 60^\circ, 90^\circ$ and $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$

Q.23 3466.36 km

Q.24 17.4 cm

Q.25 $x = n\pi, n\pi + (-1)^{n+1} \frac{3\pi}{10}, n\pi + (-1)^n \frac{\pi}{10}$

where $n = 0, \pm 1, \pm 2, \dots$

Q.26 $\theta = \frac{\pi}{3}, \frac{-\pi}{3}, \pi - \cos^{-1} \left(\frac{3}{5} \right)$

Q.27 $x = n\pi, x = n\pi \pm \alpha$ where $\sin \alpha = \frac{\sqrt{2}}{5}$

Q.28 $\theta = n\pi \pm \frac{\pi}{6}$

Q.30 NO real Solution

Exercise 2**Single Correct Choice Type****Q.1 A****Q.2 B****Q.3 D****Q.4 D****Q.5 D****Q.6 A****Q.7 D****Q.8 B****Q.9 C****Q.10 A****Q.11 C****Q.12 B****Q.13 A****Q.14 C****Q.15 B****Q.16 C****Q.17 C****Q.18 B****Q.19 D****Q.20 A****Previous Years' Questions****Q.1 A****Q.2 C****Q.3 A****Q.4 B****Q.5 A****Q.6 C****Q.7 D****Q.8 A****Q.9 B****Q.10 B****Q.11 C****Q.12 B****Q.13 A****Q.14 B****JEE Advanced/Boards****Exercise 1****Q.1** $x = n\pi$ or $x = n\pi \pm \frac{\pi}{6}$ **Q.2** $n\pi; n\pi + (-1)^n \frac{\pi}{10}$ or $n\pi + (-1)^n \left(\frac{3\pi}{10}\right)$ **Q.3** 2**Q.4** $\frac{\pm\pi}{3}, \frac{-\pi}{2}, \pm\pi$ **Q.5** $x = 2n\pi$ or $x = n\pi + (-1)^n \left(-\frac{\pi}{2}\right)$ or $x = n\pi + (-1)^n \frac{\pi}{6}$ **Q.6** $\frac{\pi}{7}, \frac{5\pi}{7}, \pi, \frac{9\pi}{7}, \frac{13\pi}{7}$ **Q.7** $x = \frac{n\pi}{2} + \frac{\pi}{8}, \frac{n\pi}{2} - \frac{\pi}{24}, \frac{n\pi}{2} - \frac{5\pi}{24}$ **Q.8** $x = 2n\pi - \frac{\pi}{2}$ **Q.9** $x = \left(4n\pi - \frac{\pi}{2}\right)^2$ or $x = \left(\frac{4m\pi}{3} + \frac{\pi}{2}\right)^2$ where $m, n \in \mathbb{W}$ **Q.10** $x = 0$ or $a < -1$ **Q.11** $\alpha - 2\pi; \alpha - \pi, \alpha + \pi$, where $\tan \alpha = \frac{2}{3}$ **Q.12** $x = \pi/16$ **Q.13** $x = n\pi + \frac{\pi}{12}; n \in \mathbb{I}$ **Q.14** 10 solutions**Q.15** $x = 2n\pi + \frac{\pi}{12}$ **Q.16** $30^\circ, 45^\circ, 90^\circ, 135^\circ, 150^\circ$ **Q.17** No Solution**Q.18** $\theta = 2n\pi$ or $2n\pi + \frac{\pi}{2}; n \in \mathbb{I}$ **Q.19** 4950π **Q.20** 72° **Q.21** 13**Q.23** $2n\pi + \frac{\pi}{4}$ or $(2n+1)\pi - \tan^{-1} 2; n \in \mathbb{I}$ **Q.24** (a) $-\frac{3}{2}$; (b) $k \in \left[-1, -\frac{1}{2}\right]$; (c) $x = \frac{n\pi}{2} \pm \frac{\pi}{6}$ **Q.25** A \rightarrow r; B \rightarrow s; C \rightarrow p; D \rightarrow q**Q.26** A \rightarrow s; B \rightarrow p; C \rightarrow q; D \rightarrow r**Q.27** A \rightarrow q; B \rightarrow s; C \rightarrow p; D \rightarrow r

Exercise 2**Single Correct Choice Type****Q.1 A****Q.2 A****Q.3 A****Q.4 B****Q.5 C****Q.6 A****Q.7 C****Q.8 B****Q.9 D****Q.10 C****Q.11 C****Multiple Correct Choice Type****Q.12 B, D****Q.13 A, B, C****Q.14 B, C****Previous Years' Questions****Q.2** $\left\{ \pm \frac{\pi}{3}, \pm \frac{2\pi}{3} \right\}$ **Q.3** $\frac{\sqrt{3}-1}{2}$ **Q.4** 30° **Q.5** $\frac{\pi}{2\sqrt{2}}$ **Q.6** $\theta = \pm \frac{\pi}{3}$ **Q.7** A C**Q.8** B, C**Q.9** A, B**Q.10** A, B**Q.11** D**Q.12** 3**Q.13** 7**Q.14** 3**Q.15** 3**Q.16** 2**Q.17** D**Q.18** 7**Q.19** A, C, D**Q.20** D**Q.21** 8**Q.22** C**Solutions****JEE Main/Boards****Exercise 1**

$$\text{Sol 1: (i)} \sin 2\theta = \frac{1}{2}$$

General solution of $\sin x = \frac{1}{2}$ is

$$x = n\pi + (-1)^n \frac{\pi}{6}, n \in \mathbb{I}$$

$$\therefore 2\theta = n\pi + (-1)^n \frac{\pi}{6}$$

$$\Rightarrow \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}, n \in \mathbb{I}$$

$$\text{(ii)} \cos 5\theta = -\frac{1}{2}$$

General solution of $\cos x = -\frac{1}{2}$ is

$$x = 2n\pi \pm \frac{2\pi}{3}, n \in \mathbb{I}$$

$$\therefore 5\theta = 2n\pi \pm \frac{2\pi}{3}, n \in \mathbb{I}$$

$$\Rightarrow \theta = \frac{2n\pi}{5} \pm \frac{2\pi}{15}, n \in \mathbb{I}$$

$$\text{Sol 2: } 7\cos^2\theta + 3\sin^2\theta = 4$$

$$\text{Since } \sin^2\theta + \cos^2\theta = 1$$

$$\therefore 4\cos^2\theta + 3(\cos^2\theta + \sin^2\theta) = 4$$

$$\Rightarrow 4\cos^2\theta + 3 = 4$$

$$\therefore \cos^2\theta = \frac{1}{4} \text{ or } \cos\theta = \frac{1}{2}, -\frac{1}{2}$$

$$\therefore \theta = 2n\pi \pm \frac{\pi}{3}, 2n\pi \pm \frac{2\pi}{3}, n \in \mathbb{I}$$

Sol 3: $\tan x + \tan 2x + \sqrt{3} \tan x \tan 2x = \sqrt{3}$
 $\Rightarrow \tan x + \tan 2x = \sqrt{3} (1 - \tan x \tan 2x)$

$$\therefore \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = \sqrt{3}$$

$$\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = \tan(A + B)$$

Applying the above formula

$$\tan(x + 2x) = \sqrt{3} \Rightarrow \tan 3x = \sqrt{3}$$

General solution of $\tan \theta = \sqrt{3}$ is

$$\theta = n\pi + \frac{\pi}{3}, n \in \mathbb{I}$$

$$\therefore 3x = n\pi + \frac{\pi}{3} \Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{9}$$

Sol 4: $3\tan(\theta - 15^\circ) = \tan(\theta + 15^\circ)$

We can write it as

$$\frac{\tan(\theta + 15^\circ)}{\tan(\theta - 15^\circ)} = \frac{\tan\left(\theta + \frac{\pi}{12}\right)}{\tan\left(\theta - \frac{\pi}{12}\right)} = 3$$

Applying $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$$\Rightarrow \frac{\sin\left(\theta + \frac{\pi}{12}\right)\cos\left(\theta - \frac{\pi}{12}\right)}{\sin\left(\theta - \frac{\pi}{12}\right)\cos\left(\theta + \frac{\pi}{12}\right)} = 3$$

Using $\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$

Above expression can be written as

$$\frac{\frac{1}{2} \left[\sin\left(\theta + \frac{\pi}{12} + \theta - \frac{\pi}{12}\right) + \sin\left(\theta + \frac{\pi}{12} - \theta + \frac{\pi}{12}\right) \right]}{\frac{1}{2} \left[\sin\left(\theta + \frac{\pi}{12} + \theta - \frac{\pi}{12}\right) + \sin\left(\theta - \frac{\pi}{12} - \theta + \frac{\pi}{12}\right) \right]} = 3$$

$$\Rightarrow \frac{\sin 2\theta + \sin \frac{\pi}{6}}{\sin 2\theta - \sin \frac{\pi}{6}} = 3$$

$$\Rightarrow 2\sin 2\theta = 4\sin \frac{\pi}{6} \Rightarrow \sin 2\theta = 2 \times \sin \frac{\pi}{6} = 1$$

$$\therefore \sin 2\theta = 1$$

General solution of $\sin x = 1$

$$\text{is } x = n\pi + (-1)^n \frac{\pi}{2}, n \in \mathbb{I}$$

$$\therefore \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{4}, n \in \mathbb{I}$$

Sol 5: $\sin(x - y) = \frac{\sqrt{3}}{2}, \cos(x + y) = \frac{1}{2}$

$$\therefore x - y = n\pi + (-1)^n \frac{\pi}{3} \quad n \in \mathbb{I} \quad \dots (i)$$

$$\text{and } x + y = 2m\pi \pm \frac{\pi}{3} \quad n \in \mathbb{I} \quad \dots (ii)$$

$$\therefore 2x = n\pi + (-1)^n \frac{\pi}{3} + 2m\pi \pm \frac{\pi}{3}$$

[Adding (i) and (ii)]

$$\text{and } x = \frac{1}{2} \left[n\pi + (-1)^n \frac{\pi}{3} + 2m\pi \pm \frac{\pi}{3} \right]$$

Similarly,

$$y = \frac{1}{2} \left[2n\pi \pm \frac{\pi}{3} - \left(n\pi + (-1)^n \frac{\pi}{3} \right) \right], m, n \in \mathbb{I}$$

[Subtracting (i) from (ii)]

Sol 6: $\sin x = \tan x$

$$\because \tan x = \frac{\sin x}{\cos x} \Rightarrow \sin x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \sin x \cos x = \sin x \Rightarrow \sin x (\cos x - 1) = 0$$

$$\therefore \sin x = 0 \text{ or } \cos x = 1 \text{ for } \sin x = 0$$

$$x = n\pi, n \in \mathbb{I}$$

$$\text{and for } \cos x = 1, x = 2m\pi, m \in \mathbb{I}$$

As the equation is valid for $\sin x = 0$ or $\cos x = 1$, is the solution will be union of both.

$$\therefore x = n\pi, n \in \mathbb{I}$$

Sol 7: $2\tan \theta - \cot \theta + 1 = 0$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\therefore 2\tan \theta - \frac{1}{\tan \theta} + 1 = 0$$

$$\Rightarrow \frac{2\tan^2 \theta + \tan \theta - 1}{\tan \theta} = 0$$

$$\Rightarrow \frac{(2\tan \theta - 1)(\tan \theta + 1)}{\tan \theta} = 0$$

$$\Rightarrow \tan \theta \neq 0 \text{ and } \tan \theta = \frac{1}{2} \text{ or } \tan \theta = -1$$

$$\therefore \text{From } \tan \theta \neq 0, \theta \neq n\pi \text{ and}$$

$$\tan \theta = -1, \theta = n\pi - \frac{\pi}{4}, n \in \mathbb{I}$$

\therefore Solution of equation is $\tan\theta = \frac{1}{2}, -1$

$$\text{i.e. } \theta = n\pi + \tan^{-1}\frac{1}{2}, n\pi - \frac{\pi}{4} n \in I$$

Sol 8: $\sin mx + \sin nx = 0$

$$\Theta \sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

$$\therefore 2 \sin\left(\frac{(m+n)x}{2}\right) \cos\left(\frac{(m-n)x}{2}\right) = 0$$

$$\therefore \sin(m+n)\frac{x}{2} = 0 \text{ or } \cos\left(\frac{(m-n)x}{2}\right) = 0$$

$$\Rightarrow (m+n)\frac{x}{2} = a\pi \text{ or } (m-n)\frac{x}{2} = (2b+1)\frac{\pi}{2} \quad a, b \in I$$

$$\therefore x = \frac{2a}{m+n}\pi \text{ or } \frac{(2b+1)}{m-n}\pi \quad a, b \in I$$

Sol 9: $\sec^2 2x = 1 - \tan 2x$

$$[1 + \tan^2\theta = \sec^2\theta]$$

$$\therefore 1 + \tan^2 2x = 1 - \tan 2x$$

$$\Rightarrow \tan^2 2x + \tan 2x = 0$$

$$\Rightarrow \tan 2x(1 + \tan 2x) = 0$$

$$\tan 2x = 0 \text{ or } \tan 2x = -1$$

$$2x = n\pi \text{ or } 2x = m\pi - \frac{\pi}{4} \quad n, m \in I$$

$$\therefore x = \frac{n\pi}{2} \text{ or } \frac{m\pi}{2} - \frac{\pi}{8} \quad n, m \in I$$

Sol 10: $4\sin x \cos x + 2\sin x + 2\cos x + 1 = 0$

$$\Rightarrow (2\sin x + 1) \times (2\cos x + 1) = 0$$

$$\Rightarrow \sin x = -\frac{1}{2} \text{ or } \cos x = -\frac{1}{2}$$

$$\therefore x = n\pi - (-1)^n \frac{\pi}{6} \quad (\text{when } \sin x = -\frac{1}{2})$$

$$\text{or } x = 2n\pi \pm \frac{2\pi}{3} \quad (\text{when } \cos x = -\frac{1}{2})$$

Sol 11: $\tan x + \sec x = 2\cos x$

$$\tan x = \frac{\sin x}{\cos x}, \sec x = \frac{1}{\cos x}$$

$$\therefore \frac{1+\sin x}{\cos x} = 2\cos x$$

$$\Rightarrow 1 + \sin x = 2\cos^2 x$$

$$[\because \sin^2 x + \cos^2 x = 1]$$

$$\therefore 1 + \sin x = 2(1 - \sin^2 x) = 2(1 - \sin x)(1 + \sin x)$$

$$\Rightarrow [2(1 - \sin x) - 1](1 + \sin x) = 0$$

$$\Rightarrow [(1 - 2\sin x)(1 + \sin x)] = 0$$

$$\therefore \sin x = \frac{1}{2} \text{ or } -1$$

$$\text{For } \sin x = \frac{1}{2} \quad x = n\pi + (-1)^n \frac{\pi}{6} \quad n \in I$$

$$\text{For } \sin x = -1 \quad x = (2n+1) \frac{\pi}{2} \quad n \in I$$

$$\therefore x = n\pi + (-1)^n \frac{\pi}{6} \text{ or } (2n+1) \frac{\pi}{2} \quad n \in I$$

Sol 12: $2\sin^2 x - 5\sin x \cos x - 8\cos^2 x + 2 = 0$

We can write 2 as $2(\cos^2 x + \sin^2 x)$

$$\therefore \cos^2 x + \sin^2 x = 1$$

$$\therefore 2\sin^2 x - 5\sin x \cos x - 8\cos^2 x + 2\cos^2 x + 2\sin^2 x = 0$$

$$\Rightarrow 4\sin^2 x - 5\sin x \cos x - 6\cos^2 x = 0$$

$$4\sin^2 x - 8\sin x \cos x + 3\sin x \cos x - 6\cos^2 x = 0$$

$$4\sin x(\sin x - 2\cos x) + 3\cos x(\sin x - 2\cos x) = 0$$

$$\therefore (4\sin x + 3\cos x)(\sin x - 2\cos x) = 0$$

$$\Rightarrow \sin x = -\frac{3}{4}\cos x \text{ or } \sin x = 2\cos x$$

$$\text{or } \tan x = -\frac{3}{4} \text{ or } \tan x = 2$$

$$x = n\pi + \tan^{-1}\left(-\frac{3}{4}\right) \text{ or } x = n\pi + \tan^{-1}(2)$$

Sol 13: $4\sin x \sin 2x \sin 4x = \sin 3x$

$$2\sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$2\sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$\therefore 2\sin x[2\sin 2x \sin 4x] = 2\sin x[\cos 2x - \cos 6x]$$

$$\Rightarrow 2\sin x \cos 2x - 2\sin x \cos 6x = \sin 3x$$

$$\Rightarrow \sin 3x + \sin(-x) - [\sin 7x + \sin(-5x)] = \sin 3x$$

$$\Rightarrow -\sin x = \sin 7x - \sin 5x$$

$$-\sin x = 2\cos 6x \sin x$$

$$\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

$$\Rightarrow \sin x(2\cos 6x + 1) = 0$$

$$\sin x = 0 \text{ or } \cos 6x = -\frac{1}{2}$$

$$x = n\pi \text{ or } 6x = 2n\pi \pm \frac{2\pi}{3} \Rightarrow x = n\pi, \frac{n\pi}{3} \pm \frac{\pi}{9}$$

Sol 14: $(1 - \tan\theta)(1 + \sin 2\theta) = (\tan\theta + 1)$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\therefore \left(\frac{\cos\theta - \sin\theta}{\cos\theta} \right)(1 + \sin 2\theta) = \left(\frac{\cos\theta + \sin\theta}{\cos\theta} \right)$$

$$\Rightarrow (1 + \sin 2\theta) = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$\therefore (1 + \sin 2\theta) = \frac{\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$(1 + \sin 2\theta) = \frac{1 + \sin 2\theta}{\cos 2\theta}$$

$$\Rightarrow \cos 2\theta (1 + \sin 2\theta) - (1 + \sin 2\theta) = 0$$

$$(\cos 2\theta - 1)(\sin 2\theta + 1) = 0$$

$$\sin 2\theta = -1 \text{ or } \cos 2\theta = 1$$

$$\therefore 2\theta = 2n\pi - \frac{\pi}{2} \text{ or } 2\theta = 2n\pi, n \in \mathbb{I}$$

$$\therefore \theta = n\pi - \frac{\pi}{4} \text{ or } n\pi, n \in \mathbb{I}$$

Sol 15: $\sin x + \sqrt{3} \cos x = \sqrt{2}$

$$\Rightarrow \frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \sin \frac{\pi}{6} \sin x + \cos \frac{\pi}{6} \cos x = \cos \frac{\pi}{4}$$

$$\Rightarrow \cos \left(x - \frac{\pi}{6} \right) = \cos \frac{\pi}{4}$$

$$x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4}$$

$$\therefore x = 2n\pi + \frac{5\pi}{12}, 2n\pi - \frac{\pi}{12}, n \in \mathbb{I}$$

Sol 16: (i) $\tan 3\theta = -1$

General solution of $\tan x = -1$ is

$$x = n\pi - \frac{\pi}{4}, n \in \mathbb{I}$$

$$\therefore 3\theta = n\pi - \frac{\pi}{4} \Rightarrow \theta = \frac{n\pi}{3} - \frac{\pi}{12}, n \in \mathbb{I}$$

$$\text{(ii)} \cos 5x = -\frac{1}{\sqrt{2}}$$

General solution for $\cos \theta = -\frac{1}{\sqrt{2}}$ is

$$\theta = 2n\pi \pm \frac{3\pi}{4}, n \in \mathbb{I}$$

$$\therefore 5x = 2n\pi \pm \frac{3\pi}{4} \Rightarrow x = \frac{2n\pi}{5} \pm \frac{3\pi}{20}, n \in \mathbb{I}$$

Sol 17: (i) $3\cos^2\theta + 7\sin^2\theta = 4$

$$3(\cos^2\theta + \sin^2\theta) + 4\sin^2\theta = 4$$

$$\Rightarrow 4\sin^2\theta = 4 - 3 = 1$$

$$\therefore \sin\theta = \frac{1}{2}, -\frac{1}{2} \Rightarrow \theta = n\pi + (-1)^n \left(\pm \frac{\pi}{6} \right)$$

$$\text{(ii)} \tan x + \tan 2x + \tan 3x = \tan x \tan 2x \tan 3x$$

$$\Rightarrow \tan x + \tan 2x + \tan 3x (1 - \tan x \tan 2x) = 0$$

$$\Rightarrow \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = -\tan 3x$$

$$\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = \tan(A + B)$$

$$\therefore \tan(x + 2x) = -\tan 3x$$

$$2\tan 3x = 0$$

$$\text{i.e. } \tan 3x = 0$$

$$\therefore 3x = n\pi, n \in \mathbb{I} \text{ or } x = \frac{n\pi}{3}, n \in \mathbb{I}$$

$$\text{(iii)} \tan \left(\frac{\pi}{4} + \theta \right) + \tan \left(\frac{\pi}{4} - \theta \right) = 4$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\therefore \tan \left(\frac{\pi}{4} + \theta \right) = \frac{1 + \tan \theta}{1 - \tan \theta}$$

$$\text{and } \tan \left(\frac{\pi}{4} - \theta \right) = \frac{1 - \tan \theta}{1 + \tan \theta}$$

$$\therefore \frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 - \tan \theta}{1 + \tan \theta} = 4$$

$$\Rightarrow \frac{(1 + \tan \theta)^2 + (1 - \tan \theta)^2}{(1 - \tan^2 \theta)} = 4$$

$$\therefore (1 + \tan^2 \theta) \times 2 = 4(1 - \tan^2 \theta) \Rightarrow 3\tan^2 \theta = 1$$

$$\tan \theta = \pm \frac{1}{\sqrt{3}} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}, n \in \mathbb{I}$$

Sol 18: $\tan x + \cot x = 2$

$$\cot x = \frac{1}{\tan x} \therefore \tan x + \frac{1}{\tan x} = 2$$

$$\tan^2 x - 2\tan x + 1 = 0$$

$$\Rightarrow (\tan x - 1)^2 = 0 \Rightarrow \tan x = 1$$

$$\therefore x = n\pi + \frac{\pi}{4}, n \in \mathbb{I}$$

Sol 19: $\tan^3 x - 3\tan x = 0$,

$$\tan x (\tan^2 x - 3) = 0$$

$$\tan x = 0 \text{ or } \tan x = \pm \sqrt{3}$$

$$x = n\pi \text{ or } x = n\pi \pm \frac{\pi}{3}, n \in \mathbb{I}$$

Sol 20: (i) $\cos x + \sin x = 1$

Multiply whole equation by $\frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = \frac{1}{\sqrt{2}}$$

$$\cos \frac{\pi}{4} \cos x + \sin \frac{\pi}{4} \sin x = \cos \left(x - \frac{\pi}{4} \right) = \frac{1}{\sqrt{2}}$$

$$\therefore x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$\therefore x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4} = 2n\pi \text{ or } 2n\pi + \frac{\pi}{2}, n \in \mathbb{I}$$

$$(ii) \sec x - \tan x = \sqrt{3}$$

$$\Rightarrow \frac{1}{\sqrt{3}} \sec x - \frac{\tan x}{\sqrt{3}} = 1 \Rightarrow \frac{1 - \sin x}{\sqrt{3} \cos x} = 1$$

$$\Rightarrow \sqrt{3} \cos x + \sin x = 1$$

Divide the equation by 2

$$\Rightarrow \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{1}{2}$$

$$\cos \left(x - \frac{\pi}{6} \right) = \cos \left(\frac{\pi}{3} \right)$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{6}, n \in \mathbb{I}$$

$$x = 2n\pi - \frac{\pi}{6} \text{ or } 2n\pi + \frac{\pi}{2}, n \in \mathbb{I}$$

$$(iii) \sin x + \cos x = \frac{1}{\sqrt{2}}$$

Multiply the equation by $\frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{2}$$

$$\Rightarrow \cos \left(x - \frac{\pi}{4} \right) = \cos \left(\frac{\pi}{3} \right)$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{4}$$

$$2n\pi + \frac{7\pi}{12} \text{ or } 2n\pi - \frac{\pi}{12}$$

$$(iv) \cos x + \sqrt{3} \sin x = 1$$

Divide the equation by 2

$$\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x = \frac{1}{2}$$

$$\Rightarrow \cos \left(x - \frac{\pi}{3} \right) = \cos \left(\frac{\pi}{3} \right)$$

$$\therefore x = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{3}$$

$$\text{i.e. } x = 2n\pi \text{ or } 2n\pi + \frac{2\pi}{3}, n \in \mathbb{I}$$

$$\text{Sol 21: (i)} \left(\frac{\pi}{6} \right)^c$$

$$180^\circ = \pi \text{ radian}$$

$$\therefore 1 \text{ radian} \rightarrow \frac{180^\circ}{\pi}$$

$$\frac{\pi}{6} \rightarrow \frac{180}{\pi} \times \frac{\pi}{6} = 30^\circ$$

$$\text{(ii)} \left(\frac{4\pi}{5} \right)^c$$

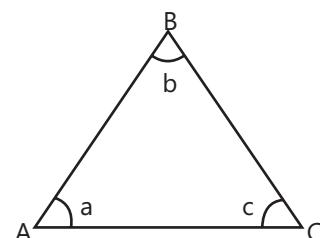
$$\frac{4\pi}{5} \text{ radian} = \frac{180}{\pi} \times \frac{4\pi}{5} = 144^\circ$$

$$\text{(iii)} (1.2)^c$$

$$1.2 \text{ radian} = \frac{180}{\pi} \times 1.2 = 68^\circ 43' 37.8''$$

$$\text{Note: } 1^\circ = 60', 1' = 60''$$

Sol 22: $a + b + c = 180^\circ$



Given angles are in A.P.

Let common difference = d

$$\therefore b = a + d, c = a + 2d$$

$$\Rightarrow a + (a + d) + (a + 2d) = 3(a + d) = 180^\circ$$

$$a + d = 60^\circ$$

$$\text{Also given } \frac{a}{c} = \frac{a}{a+2d} = \frac{60^\circ}{\pi} = \frac{\frac{\pi}{3}}{\pi} = \frac{1}{3}$$

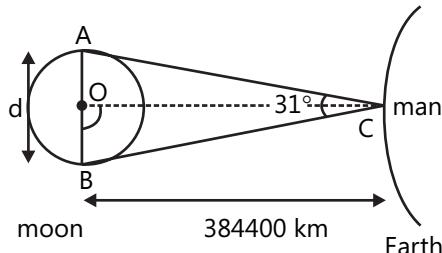
$$\Rightarrow 3a = a + 2d \Rightarrow a = d$$

From (i) and (ii)

$$a = d = 30^\circ$$

$$\therefore a = 30^\circ, b = 60^\circ, c = 90^\circ \text{ Ans.}$$

Sol 23:



Line OC divides AB into two equal parts

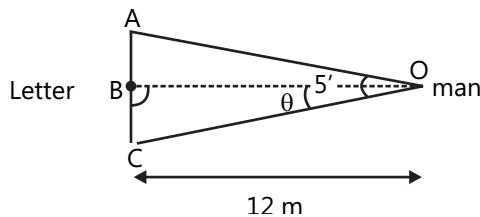
In $\triangle OBC$

$$\begin{aligned} \tan\left(\frac{31'}{2}\right) &= \frac{OB}{OC} \therefore OB = 384400 \times \tan(15.5^\circ) \\ &= 384400 \times \tan \frac{15.5}{60} = 173318 \text{ Km} \end{aligned}$$

$$AB = 2(OB) = 346636 \text{ Km}$$

$$\therefore \text{Diameter of moon} = 346636 \text{ Km.}$$

Sol 24:



Assuming letter to be symmetrically placed

$$\tan \theta = \frac{BC}{OB}$$

$$\tan(2.5') = \frac{BC}{12}$$

$$\therefore BC = 12 \tan(2.5') = 0.0873 \text{ m}$$

$$\therefore \text{Total length of letter} = 2BC = 0.174 \text{ m} = 17.4 \text{ m.}$$

Sol 25: $4\cos^2x \sin x - 2\sin^2x = 3\sin x$

One of the obvious solution is $\sin x = 0$

$$\dots \text{(i)} \quad \text{i.e. } x = n\pi$$

$$\text{Now if } x \neq n\pi \text{ i.e. } \sin x \neq 0 \Rightarrow 4\cos^2x = 3 + 2\sin x$$

$$\therefore 4 - 4\sin^2x = 3 + 2\sin x \Rightarrow 4\sin^2x + 2\sin x - 1 = 0$$

$$\sin x = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-1)}}{2 \times 4} = \frac{-2 \pm \sqrt{20}}{8}$$

$$\Rightarrow \sin x = \frac{-1 \pm \sqrt{5}}{4}$$

\therefore Possible solution is

$$\sin x = \frac{\sqrt{5}-1}{4} = \sin\left(\frac{\pi}{10}\right) \Rightarrow x = n\pi + (-1)^n \frac{\pi}{10}$$

$$\text{and } \sin x = \frac{-1-\sqrt{5}}{4} = \sin\left(-\frac{3\pi}{10}\right) = \sin\left(\pi + \frac{3\pi}{10}\right)$$

$$\therefore x = n\pi + (-1)^n \left(-\frac{3\pi}{10}\right) = n\pi + (-1)^{n+1} \frac{3\pi}{10}$$

$$\therefore x = n\pi, n\pi + (-1)^n \frac{\pi}{10}, n\pi + (-1)^{n+1} \frac{3\pi}{10}$$

$$\text{Sol 26: } 5\cos 2\theta + 2\cos^2 \frac{\theta}{2} + 1 = 0 \quad -\pi < \theta < n$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\therefore 2\cos^2 \frac{\theta}{2} = \cos \theta + 1$$

$$\text{and } \cos 2\theta = 2\cos^2 \theta - 1$$

$$\text{Putting both these in given equation}$$

$$\therefore 5(2\cos^2 \theta - 1) + \cos \theta + 1 + 1 = 0$$

$$10\cos^2 \theta + \cos \theta - 3 = 0$$

$$10\cos^2 \theta - 5\cos \theta + 6\cos \theta - 3 = 0$$

$$(5\cos \theta + 3)(2\cos \theta - 1) = 0$$

$$\Rightarrow \cos \theta = -\frac{3}{5} \text{ or } \cos \theta = \frac{1}{2}$$

$$\therefore \theta = \cos^{-1}\left(-\frac{3}{5}\right) \text{ or } \theta = \pm \frac{\pi}{3} = \pi - \cos^{-1}\frac{3}{5}$$

$$\therefore \theta = \frac{\pi}{3}, -\frac{\pi}{3}, \pi - \cos^{-1}\frac{3}{5} \quad (\text{As } -\pi < \theta < \pi)$$

Sol 27: $4\sin^4 x + \cos^4 x = 1$

$$4\sin^4 x = 1 - \cos^4 x = (1 - \cos^2 x)(1 + \cos^2 x)$$

$$4\sin^4 x = \sin^2 x(1 + \cos^2 x)$$

One of obvious solution is $\sin x = 0$ i.e. $x = n\pi$

If $\sin x \neq 0$

$$\Rightarrow 4\sin^2x = 1 + \cos^2x = 2 - \sin^2x$$

$$\Rightarrow \sin^2x = \frac{2}{5}$$

$$\sin x = \pm \sqrt{\frac{2}{5}}$$

$$\therefore x = n\pi \pm \sin^{-1} \sqrt{\frac{2}{5}}$$

$$\therefore x = n\pi, n\pi \pm \alpha, \sin \alpha = \sqrt{\frac{2}{5}}, n \in I$$

Sol 28: $\tan 2\theta \tan \theta = 1$

$$\tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta},$$

Substitute this in given equation, we get

$$\therefore \frac{2\tan^2 \theta}{1 - \tan^2 \theta} = 1 \Rightarrow 3\tan^2 \theta = 1$$

$$\tan \theta = \pm \frac{1}{\sqrt{3}} \quad \therefore \theta = n\pi \pm \frac{\pi}{6}$$

Sol 29: $e^{\sin x} - e^{-\sin x} - 4 = 0$

$$e^{\sin x} - \frac{1}{e^{\sin x}} = 4$$

Max. Value of L.H.S. can be attained only when $e^{\sin x}$ is max and $\frac{1}{e^{\sin x}}$ is min.

As max. Value of $\sin x$ is 1

$$\therefore e^{\sin x} \leq e^1 \text{ and } \frac{1}{e^{\sin x}} \geq \frac{1}{e}$$

$$\therefore \text{Max. Value of LHS} = e - \frac{1}{e} \approx 2.35$$

So there is no real value of x for which LHS = 4

Sol 30: $\sin^4 \theta - 2\sin^2 \theta - 1 = 0$

$$\text{Let } \sin^2 \theta = t \Rightarrow t^2 - 2t - 1 = 0$$

$$\therefore t = \frac{2 \pm \sqrt{(-2)^2 + 4 \times 1}}{2} = \frac{2 \pm \sqrt{8}}{2} = 1 \pm 2\sqrt{2}$$

$$\therefore \sin^2 \theta = 1 + \sqrt{2} \text{ or } 1 - \sqrt{2}$$

Since $-1 < \sin \theta < 1$ and $0 < \sin^2 \theta < 1$

\therefore No real solution.

Exercise 2

Single Correct Choice Type

$$\text{Sol 1: (A)} b \cos^2 \frac{A}{2} + a \cos^2 \frac{B}{2} = \frac{3}{2}c$$

$$b \frac{(\cos A + 1)}{2} + a \frac{(\cos B + 1)}{2} = \frac{3}{2}c$$

$$b \cos A + b + a \cos B + a = 3c$$

$$\Rightarrow a + b + (a \cos B + b \cos A) = 3c$$

$$\Rightarrow a \cos B + b \cos A = c$$

$$\therefore a + b = 2c \Rightarrow a, b, c \text{ are in A.P.}$$

$$\text{Sol 2: (B)} a^2 + 2a + \operatorname{cosec}^2 \left(\frac{\pi}{2}(a+x) \right) = 0$$

$$[\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta]$$

$$\Rightarrow a^2 + 2a + 1 + \cot^2 \left(\frac{\pi}{2}(a+x) \right) = 0$$

$$\Rightarrow (a+1)^2 + \cot^2 \left(\frac{\pi}{2}(a+x) \right) = 0$$

For the above equation to be valid

$$a+1=0 \text{ and } \cot \left(\frac{\pi}{2}(a+x) \right) = 0$$

$$\Rightarrow a = -1 \text{ and } \frac{\pi}{2}(a+x) = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow a+x = 2n+1 \Rightarrow x = 2n+2$$

$$\therefore a = -1 \text{ and } \frac{x}{2} \in I$$

$$\text{Sol 3: (D)} (a+b)^2 \sin^2 \frac{C}{2} + (a-b)^2 \cos^2 \frac{C}{2}$$

$$= (a^2 + b^2 + 2ab) \sin^2 \frac{C}{2} + (a^2 + b^2 - 2ab) \cos^2 \frac{C}{2}$$

$$= (a^2 + b^2) \frac{\pi}{3} + 2ab \left(\sin^2 \frac{C}{2} - \cos^2 \frac{C}{2} \right)$$

$$= a^2 + b^2 + 2ab(-\cos C)$$

$$= a^2 + b^2 - 2ab \frac{a^2 + b^2 - c^2}{2ab} = c^2$$

$$\text{Sol 4: (D)} \sin^3 A + \sin^3 B + \sin^3 C = 3 \sin A \sin B \sin C$$

$$\therefore \text{In triangle } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\text{i.e. } \sin A \propto a, \sin B \propto b, \sin C \propto c$$

$$\therefore a^3 + b^3 + c^3 = 3abc$$

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)=0$$

$$\therefore a^2 + b^2 + c^2 - ab - bc - ca = 0$$

$$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0 \Rightarrow a = b = c$$

∴ Triangle should be equilateral

Sol 5: (D) $\sin 3\theta = 4 \sin \theta \sin 2\theta \sin 4\theta \quad \theta \in [0, \pi]$

$$\sin 3\theta = 2 \sin \theta [\cos 2\theta - \cos 6\theta]$$

$$\sin 3\theta = \sin 3\theta + \sin(-\theta) - [\sin 7\theta + \sin(-5\theta)]$$

$$\Rightarrow \sin 7\theta - \sin 5\theta = -\sin \theta$$

$$\therefore -\sin \theta = 2 \cos 6\theta \sin \theta$$

$$\sin \theta [2 \cos 6\theta + 1] = 0 \Rightarrow \sin \theta = 0 \text{ or } \cos 6\theta = -\frac{1}{2}$$

$$\theta = n\pi \text{ or } 6\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = \frac{n\pi}{3} \pm \frac{\pi}{9}$$

$$\therefore \theta = 0, \pi, \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{2\pi}{9}, \frac{5\pi}{9}, \frac{8\pi}{9}$$

Sol 6: (A) $a \cos(B-C) + b \cos(C-A) + c \cos(A-B)$

$$= a(\cos B \cos C + \sin B \sin C) + b(\cos C \cos A + \sin C \sin A) + c(\cos A \cos B + \sin A \sin B)$$

$$= \cos C[a \cos B + b \cos A] + c \cos A \cos B + a \sin B \sin C + b \sin C \sin A + c \sin A \sin B$$

$$a = 2R \sin A, b = 2R \sin B, c = 2R \sin C$$

$$\text{and } a \cos B + b \cos A = c$$

$$= c \cos C + c \cos A \cos B + \frac{abc}{(2R)^2} + \frac{abc}{(2R)^2} + \frac{abc}{(2R)^2}$$

$$= c[\cos(\pi - (A+B)) + \cos A \cos B] + \frac{3abc}{4R^2}$$

$$= c[-\cos A \cos B + \sin A \sin B + \cos A \cos B] + \frac{3abc}{4R^2}$$

$$= c \sin A \sin B + \frac{3abc}{4R^2} = \frac{abc}{4R^2} + \frac{3abc}{4R^2} = \frac{abc}{R^2}$$

Sol 7: (D) $\cos \alpha = \frac{2 \cos \beta - 1}{2 - \cos \beta}$

$$2 \left(\frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right) - 1$$

$$\Rightarrow -\frac{1 + \tan^2 \frac{\beta}{2}}{2 - \left(\frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right)}$$

$$\Rightarrow \frac{2 \left(1 - \tan^2 \frac{\beta}{2} \right) - \left(1 - \tan^2 \frac{\beta}{2} \right)}{2 \left(1 + \tan^2 \frac{\beta}{2} \right) - \left(1 + \tan^2 \frac{\beta}{2} \right)}$$

$$\Rightarrow \frac{2 - 2 \tan^2 \frac{\beta}{2} - 1 - \tan^2 \frac{\beta}{2}}{2 + 2 \tan^2 \frac{\beta}{2} - 1 + 2 \tan^2 \frac{\beta}{2}}$$

$$\Rightarrow \frac{1 - 3 \tan^2 \frac{\beta}{2}}{1 + 3 \tan^2 \frac{\beta}{2}} = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$\Rightarrow \left(1 - 3 \tan^2 \frac{\beta}{2} \right) \left(1 + \tan^2 \frac{\beta}{2} \right) = \left(1 - \tan^2 \frac{\beta}{2} \right) \left(1 + 3 \tan^2 \frac{\beta}{2} \right)$$

$$\Rightarrow 1 - 3 \tan^2 \frac{\beta}{2} + \tan^2 \frac{\alpha}{2} - 3 \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}$$

$$= 1 + 3 \tan^2 \frac{\beta}{2} - \tan^2 \frac{\alpha}{2} = -3 \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}$$

$$\Rightarrow 2 \tan^2 \frac{\alpha}{2} = 6 \tan^2 \frac{\beta}{2}$$

$$\Rightarrow \tan^2 \frac{\alpha}{2} = 3 \tan^2 \frac{\beta}{2}$$

$$\Rightarrow \tan^2 \frac{\alpha}{2} \cdot \cot \frac{\beta}{2} = \sqrt{3}$$

Sol 8: (B) $x \sin \theta = y \sin \left(\theta + \frac{2\pi}{3} \right) = z \sin \left(\theta + \frac{4\pi}{3} \right)$

$$\Rightarrow \frac{x}{y} = \frac{\sin \theta \left(-\frac{1}{2} \right) + \cos \theta \left(+\frac{\sqrt{3}}{2} \right)}{\sin \theta} = -\frac{1}{2} + \frac{\sqrt{3}}{2} \cot \theta$$

$$\frac{x}{z} = \frac{\sin \theta \left(-\frac{1}{2} \right) + \cos \theta \left(-\frac{\sqrt{3}}{2} \right)}{\sin \theta} = -\frac{1}{2} - \frac{\sqrt{3}}{2} \cot \theta$$

$$\therefore \frac{x}{y} + \frac{x}{z} = -1 \Rightarrow xz + xy + yz = 0$$

Sol 9: (C) $m = \cos^3 \alpha + 3 \cos \alpha \sin^2 \alpha$

$$n = \sin^3 \alpha + 3 \cos^2 \alpha \sin \alpha$$

$$(m+n) = a(\sin^3 \alpha + \cos^3 \alpha + 3\cos \alpha \sin \alpha (\cos \alpha + \sin \alpha)) \\ = a(\sin \alpha + \cos \alpha)^3$$

$$(m-n) = a(\cos^3 \alpha - \sin^3 \alpha + 3\cos \alpha \sin \alpha (\sin \alpha - \cos \alpha)) \\ = a(\cos \alpha - \sin \alpha)^3 \\ \therefore (m+n)^{2/3} + (m-n)^{2/3}$$

$$= a^{2/3}(\sin \alpha + \cos \alpha)^2 + a^{2/3}(\cos \alpha - \sin \alpha)^2 = 2a^{2/3}$$

Sol 10: (A) $\tan(5\pi \cos \theta) = \cot(5\pi \sin \theta)$

$$\frac{\sin(5\pi \cos \theta)}{\cos(5\pi \cos \theta)} = \frac{\cos(5\pi \sin \theta)}{\sin(5\pi \sin \theta)}$$

$$\Rightarrow \cos(5\pi \cos \theta) \cos(5\pi \sin \theta) - \sin(5\pi \cos \theta) \sin(5\pi \sin \theta) = 0$$

$$\Rightarrow \cos(5\pi \cos \theta + 5\pi \sin \theta) = 0$$

$$\Rightarrow 5\pi(\cos \theta + \sin \theta) = 2n\pi \pm \frac{-2 \pm \sqrt{4+4 \times 4}}{2 \times 4} = -\frac{1 \pm \sqrt{5}}{4}$$

$$\Rightarrow \cos \theta + \sin \theta = \left(\frac{2n}{5} \pm \frac{1}{10} \right)$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta = \frac{1}{\sqrt{2}} \left(\frac{2n}{5} \pm \frac{1}{10} \right)$$

$$\sin \left(\theta + \frac{\pi}{4} \right) = \frac{1}{\sqrt{2}} \left(\frac{2n}{5} \pm \frac{1}{10} \right) n \in I$$

For this relation to satisfy

$$\frac{1}{\sqrt{2}} \left(\frac{2n}{5} \pm \frac{1}{10} \right) \in [-1, 1]$$

$$\therefore -\sqrt{2} \leq \frac{2n}{5} + \frac{1}{10} \leq \sqrt{2}$$

$$\text{and } -\sqrt{2} \leq \frac{2n}{5} - \frac{1}{10} \leq \sqrt{2}$$

From following both condition

$$\left(-\sqrt{2} - \frac{1}{10} \right) \frac{5}{2} \leq n \leq \left(\sqrt{2} - \frac{1}{10} \right) \frac{5}{2}$$

$$\text{and } \left(\frac{1}{10} - \sqrt{2} \right) \frac{5}{2} \leq n \leq \left(\sqrt{2} + \frac{1}{10} \right) \frac{5}{2}$$

Considering values of n and $\theta \in [0, 2\pi]$ total of 28 values are possible.

$$\text{Sol 11: (C)} \quad \tan \frac{B}{2} \tan \frac{C}{2} = \frac{s-a}{s}$$

$$2s = a + b + c$$

$$\text{Given } b + c = 3a$$

$$\therefore \cot \frac{B}{2} \cot \frac{C}{2} = \frac{s}{s-a}$$

$$= \frac{\frac{(a+b+c)}{2}}{\frac{(a+b+c)-a}{2}} \text{ or } \frac{\frac{(a+3a)}{2}}{\frac{(a+3a)-a}{2}} = \frac{2a}{2a-a}$$

$$\therefore \cot \frac{B}{2} \cot \frac{C}{2} = 2$$

Sol 12: (B) $\cos 2x + a \sin x = 2a - 7$

$$1 - 2\sin^2 x + a \sin x = 2a - 7$$

$$2\sin^2 x - a \sin x + 2a - 8 = 0$$

$$\sin x = \frac{a \pm \sqrt{a^2 - 4(2a-8) \times 2}}{2 \times 2}$$

$$\therefore \sin x = \frac{a \pm \sqrt{(a-8)^2}}{4} = \frac{a \pm |a-8|}{4}$$

For $a > 8$

$$\sin x = \frac{a \pm |a-8|}{4} \Rightarrow \sin x = \frac{2a-8}{4} \text{ or } 2$$

Since $\sin x \leq 1$ no value of $a > 8$ satisfies the equation

For $a < 8$

$$\sin x = \frac{a \pm (-a+8)}{4}$$

$$\sin x = 2, \frac{2a-8}{4}$$

$$\therefore -1 \leq \frac{2a-8}{4} \leq 1$$

$$2 \leq a \leq 6$$

\therefore The solution is $a \in [2, 6]$

$$\text{Sol 13: (A)} \quad \frac{\sum \cot^2 \frac{A}{2} \cot^2 \frac{B}{2}}{\prod \cot^2 \frac{A}{2}} = \frac{1}{\cot^2 \frac{C}{2}} + \frac{1}{\cot^2 \frac{B}{2}} + \frac{1}{\cot^2 \frac{A}{2}}$$

$$= \tan^2 \frac{A}{2} + \tan^2 \frac{B}{2} + \tan^2 \frac{C}{2}$$

$$= \left(\tan \frac{A}{2} + \tan \frac{B}{2} + \tan \frac{C}{2} \right)^2 - 2 \sum \tan \frac{A}{2} + \tan \frac{B}{2}$$

$$\text{For triangle } \sum \tan \frac{A}{2} + \tan \frac{B}{2} = 1$$

$$= \left(\tan \frac{A}{2} + \tan \frac{B}{2} + \tan \frac{C}{2} \right)^2 - 2$$

\therefore For LHS to be minimum $\left(\sum \tan \frac{A}{2}\right)^2$ should take its minimum value which is only possible for $A = B = C = \frac{\pi}{3}$

$$\therefore \left(\tan \frac{A}{2} + \tan \frac{B}{2} + \tan \frac{C}{2}\right)^2 = \left(\frac{1}{\sqrt{3}} \times 3\right)^2 = 3$$

\therefore Min. Value of given function = $3 - 2 = 1$

Sol 14: (C) $\sin x + \sin 5x = \sin 2x + \sin 4x$

$$2\sin 3x \cos 2x = 2\sin 3x \cos x$$

$$2\sin 3x(\cos 2x - \cos x) = 0$$

$\therefore 3x = n\pi$ or $\cos 2x = \cos x$

$$x = \frac{n\pi}{3} \text{ or } 2x = 2n\pi \pm x \Rightarrow x = \frac{2n\pi}{3}, 2np$$

$$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \dots$$

\therefore General solution is $\frac{n\pi}{3}$

Sol 15: (B) $\cot x - \cos x = 1 - \cot x \cos x$

$$\cos x \left(\frac{1 - \sin x}{\sin x} \right) = \frac{\sin x - \cos^2 x}{\sin x}$$

For $\sin x \neq 0$ i.e. $x \neq np$

$$\cos x - \cos x \sin x = \sin x - \cos^2 x$$

$$(\cos x - \sin x) + \cos x(\cos x - \sin x) = 0$$

$$\Rightarrow (1 + \cos x)(\cos x - \sin x) = 0$$

$$\cos x = -1 \text{ or } \cos x = \sin x$$

$$\therefore x = \pi, \frac{x}{4}, \frac{3\pi}{4}$$

On $\neq \pi$ as $\sin x \neq 0$

$$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}$$

\therefore Two solutions

Sol 16: (C) $\cos^2 73 + \cos^2 47 + (\cos 73 \cos 47)$

$$= (\cos 73 + \cos 47)^2 - \cos 47 \cos 73$$

$$\Rightarrow \left(2 \cos \frac{120}{2} \cos \frac{26}{2}\right)^2 - \left(\frac{\cos 120 + \cos 26}{2}\right)$$

$$= \cos^2 13 - \frac{1}{2} \left(-\frac{1}{2} + 2 \cos^2 13 - 1\right)$$

$$= \cos^2 13 + \frac{1}{4} - \cos^2 13 + \frac{1}{2} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

Sol 17: (C) $(7\cos\theta + 24\sin\theta) \times (7\sin\theta - 24\cos\theta)$

$$= 25 \left(\frac{7\cos\theta + 24\sin\theta}{25} \right) \times 25 \left(\frac{7\sin\theta - 24\cos\theta}{25} \right)$$

$$= -625 \sin(\alpha + \theta) \cos(\alpha + \theta)$$

$$\text{Where } \sin\alpha = \frac{7}{25}, \cos\alpha = \frac{24}{25}$$

$$= -\frac{625}{2} \sin 2(\alpha + \theta)$$

\therefore Maximum value occurs when

$$\sin 2(\alpha + \theta) = -1 \therefore \text{Maximum value} = \frac{625}{2}$$

Sol 18: (B) $\sin \frac{x}{2} - \cos \frac{x}{2} = 1 - \sin x$

$$\text{We know that } 1 - \sin x = \left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)^2$$

$$\Rightarrow \sin \frac{x}{2} - \cos \frac{x}{2} = \left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)^2$$

$$\Rightarrow \sin \frac{x}{2} = \cos \frac{x}{2} \text{ or } \sin \left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{x}{2} = m\pi + \frac{\pi}{4} \text{ or } \frac{x}{2} - \frac{\pi}{4} = m\pi + (-1)^m \frac{\pi}{4}$$

$$\Rightarrow x = (4m+1)\frac{\pi}{2} \text{ or } x = (4m+1)\frac{\pi}{2} + (-1)^m \frac{\pi}{2}$$

$$\text{Also } \left| \frac{x}{2} - \frac{\pi}{2} \right| \leq \frac{3\pi}{4}$$

$$\Rightarrow -\frac{3\pi}{4} \leq \frac{x}{2} - \frac{\pi}{2} \leq \frac{3\pi}{4} \Rightarrow -\frac{3\pi}{4} + \frac{\pi}{2} \leq \frac{x}{2} \leq \frac{3\pi}{4} + \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4} \leq \frac{x}{2} \leq \frac{5\pi}{4} \Rightarrow -\frac{\pi}{2} \leq x \leq \frac{5\pi}{2}$$

$$\therefore x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{4\pi}{2}, \frac{5\pi}{2}$$

i.e. $n = 1, 2, 4, 5$

Sol 19: (D) $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0$

$$a_1 + a_2(1 - 2\sin^2 x) + a_3 \sin^2 x = 0$$

$$\Rightarrow a_1 + a_2 + (a_3 - a_2) \sin^2 x = 0$$

For all x of this function has to be zero

$$\text{then } a_1 + a_2 = 0, a_3 - 2a_2 = 0$$

As there are three Variables and two equations so infinite solution are possible.

Sol 20: (A) Given $A + B = \frac{\pi}{2}$

$$\begin{aligned} \left(1 + \tan \frac{A}{2}\right) \left(1 + \tan \frac{B}{2}\right) &= \left(1 + \tan \frac{A}{2}\right) \left(1 + \tan \frac{\frac{\pi}{2} - A}{2}\right) \\ &= \left(1 + \tan \frac{A}{2}\right) \left(1 + \frac{1 - \tan \frac{A}{2}}{1 + \tan \frac{A}{2}}\right) = \left(1 + \tan \frac{A}{2}\right) \frac{(2)}{1 + \tan \frac{A}{2}} = 2 \end{aligned}$$

Previous Years' Questions

Sol 1: (A) Given equation is

$$2\cos^2\left(\frac{x}{2}\right)\sin^2x = x^2 + x^{-2}, x \leq \frac{\pi}{9}$$

$$\text{LHS} = 2\cos^2\left(\frac{x}{2}\right)\sin^2x < 2 \text{ and RHS} = x^2 + \frac{1}{x^2} \geq 2$$

\therefore The equation has no real solution.

Sol 2: (C) Let $f(x) = \tan x - x$

$$\text{We know, for } 0 < x < \frac{\pi}{2}$$

$$\Rightarrow \tan x > x$$

$\therefore f(x) = \tan x - x$ has no root in $(0, \pi/2)$

For $\pi/2 < x < \pi$, $\tan x$ is negative

$$\therefore f(x) = \tan x - x < 0$$

$$\text{So, } f(x) = 0 \text{ has no root in } \left(\frac{\pi}{2}, \pi\right)$$

For $\frac{3\pi}{2} < x < 2\pi$, $\tan x$ is negative

$$\therefore f(x) = \tan x - x < 0$$

$$\text{So, } f(x) = 0 \text{ has no root in } \left(\frac{3\pi}{2}, 2\pi\right)$$

$$\text{We have, } f(\pi) = 0 - \pi < 0 \text{ and } f\left(\frac{3\pi}{2}\right) = \tan\frac{3\pi}{2} - \frac{3\pi}{2} > 0$$

$$\therefore f(x) = 0 \text{ has at least one root between } \pi \text{ and } \frac{3\pi}{2}$$

Sol 3: (A) Given equation is $\sin(e^x) = 5^x + 5^{-x}$ is

$$\text{LHS} = \sin(e^x) \leq 1, \text{ for all } x \in \mathbb{R} \text{ and RHS} = 5^x + 5^{-x} \geq 2$$

$\therefore \sin(e^x) = 5^x + 5^{-x}$ has no solution.

Sol 4: (B) We know

$$-\sqrt{a^2 + b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2 + b^2}$$

$$\therefore -\sqrt{74} \leq 7\cos x + 5\sin x \leq \sqrt{74}$$

$$\text{i.e. } -\sqrt{74} \leq 2k + 1 \leq \sqrt{74}$$

Since, k is integer, $-9 < 2k + 1 < 9$

$$\Rightarrow -10 < 2k < 8 \quad \Rightarrow -5 < k < 4$$

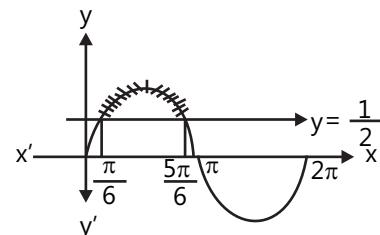
\Rightarrow Number of possible integer values of k is equal to 8.

Sol 5: (A) Since, $2\sin^2\theta - 5\sin\theta + 2 > 0$

$$\Rightarrow (2\sin\theta - 1)(\sin\theta - 2) > 0$$

[Where, $(\sin\theta - 2) < 0$ for all $\theta \in \mathbb{R}$]

$$\Rightarrow (2\sin\theta - 1) < 0$$



$$\Rightarrow \sin\theta < \frac{1}{2}$$

$$\therefore \text{From the graph, } \theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$$

$$\text{Sol 6: (C)} 2\sin^2\theta - \cos 2\theta = 0 \Rightarrow \sin^2\theta = \frac{1}{4}$$

$$\text{Also, } 2\cos^2\theta = 3\sin\theta \Rightarrow \sin\theta = \frac{1}{2}$$

\Rightarrow Two solutions exist in the interval $[0, 2\pi]$.

$$\text{Sol 7: (D)} P = \left\{ \theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta \right\}$$

$$\Rightarrow \cos\theta(\sqrt{2} + 1) = \sin\theta$$

$$\Rightarrow \tan\theta = \sqrt{2} + 1 \quad \dots\dots (i)$$

$$Q = \left\{ \theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta \right\} \Rightarrow \sin\theta(\sqrt{2} - 1) = \cos\theta$$

$$\tan\theta = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = (\sqrt{2}+1) \quad \dots\dots (ii)$$

$$\therefore P = Q$$

Sol 8: (A) $\alpha + \beta + \gamma = 2\pi$

$$\tan(\alpha + \beta + \gamma) = 0$$

$$\Rightarrow \frac{\tan(\alpha + \beta) + \tan \gamma}{1 - \tan(\alpha + \beta) \tan \gamma} = 0$$

$$\Rightarrow \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan \gamma = 0$$

$$\Rightarrow \tan \alpha + \tan \beta + \tan \gamma = \tan \alpha \tan \beta \tan \gamma$$

Sol 9: (B) $A \sin^2 \theta + \cos^4 \theta$

$$= \sin^2 \theta + (1 - \sin^2 \theta)^2$$

$$= \sin^2 \theta + \sin^4 \theta + 1 - 2 \sin^2 \theta$$

$$= 1 + \sin^4 \theta - \sin^2 \theta$$

$$= \left(\sin^2 \theta - \frac{1}{2} \right)^2 + 1 - \frac{1}{4}$$

$$= \left(\sin^2 \theta - \frac{1}{2} \right)^2 + \frac{3}{4}$$

$$\text{For minimum, } \sin \theta = \frac{1}{2}$$

$$A = \frac{3}{4}$$

$$\text{For maximum, } \sin \theta = 0$$

$$A = 1$$

$$\Rightarrow \frac{3}{4} \leq A \leq 1$$

$$\begin{aligned} \text{Sol 10: (B)} \quad & 3 \left[\sin^4 \left(\frac{3\pi}{2} - \alpha \right) + \sin^4 (3\pi + \alpha) \right] \\ & - 2 \left[\sin^6 \left(\frac{\pi}{2} + 2 \right) + \sin^6 (5\pi - 2) \right] \end{aligned}$$

$$= 3 \left[\cos^2 \alpha + \sin^4 \alpha \right] - 2 \left[\cos^6 \alpha + \sin^6 \alpha \right]$$

$$= 3 \cos^4 \alpha + 3 \sin^4 \alpha - 2 \cos^4$$

$$\alpha (1 - \sin^2 \alpha) - 2 \sin^4 \alpha (1 - \cos^2 \alpha)$$

$$= \sin^4 \alpha + \cos^4 \alpha + 2 \cos^4 \alpha \sin^2 \alpha + 2 \sin^4 \alpha \cos^2 \alpha$$

$$= (\sin^2 \alpha + \cos^2 \alpha)^2 - 2 \sin^2 \alpha \cos^2 \alpha$$

$$+ 2 \cos^2 \alpha \sin^2 \alpha (\cos^2 \alpha + \sin^2 \alpha)$$

$$\begin{aligned} &= 1 - 2 \sin^2 \alpha \cos^2 \alpha + 2 \cos^2 \alpha \sin^2 \alpha \\ &= 1 \end{aligned}$$

$$\text{Sol 11: (C)} \quad \alpha + \beta = \frac{\pi}{2} \Rightarrow \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = 0$$

$$\gamma = \alpha - \beta$$

$$\Rightarrow \tan \alpha \tan \beta = 1$$

$$\gamma = \alpha - \beta$$

$$\tan \gamma = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\Rightarrow \tan \gamma = \frac{\tan \alpha - \tan \beta}{2}$$

$$\Rightarrow 2 \tan \gamma = \tan \alpha - \tan \beta$$

$$\Rightarrow \tan \alpha = 2 \tan \gamma + \tan \beta$$

$$\text{Sol 12: (B)} \quad \theta \in \left(0, \frac{\pi}{4} \right)$$

$$\Rightarrow \tan \theta \in (1, 0)$$

$$\Rightarrow \cos \theta \in \left(\frac{1}{\sqrt{2}}, 1 \right)$$

$$\Rightarrow \cot \theta \in (1, 0)$$

$$\text{As } \theta \in \left(0, \frac{\pi}{4} \right)$$

$$\cot \theta > \tan \theta$$

$$\Rightarrow (\cot \theta)^{\cot \theta} > (\tan \theta)^{\cot \theta}$$

$$\Rightarrow t_4 > t_2$$

$$\text{Sol 13: (A)} \quad \frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$$

$$= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{1 / \tan A}{1 - \tan A}$$

$$= \frac{\tan^2 A}{1 - \tan A} + \frac{1}{\tan A (1 - \tan A)}$$

$$= \frac{-\tan^3 A + 1}{(1 - \tan A) \tan A}$$

$$= \frac{(1 - \tan A)(1 + \tan^2 A + \tan A)}{\tan A (1 - \tan A)}$$

$$= \frac{\sec^2 A + \tan A}{\tan A}$$

$$= \frac{\sec^2 A}{\tan A} + 1$$

$$= \sec A \operatorname{cosec} A + 1$$

Sol 14: (B) $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$$\Rightarrow \cos x + \cos 3x + \cos 2x + \cos 4x = 0$$

$$\Rightarrow 2\cos 2x \cos x + 2\cos 3x \cos x = 0$$

$$\Rightarrow 2\cos x(\cos 2x + \cos 3x) = 0$$

$$\Rightarrow 2\cos x \left[2\cos \frac{5x}{2} \cos \frac{x}{2} \right] = 0$$

$$\Rightarrow \cos \frac{x}{2} = 0 \text{ or } \cos x = 0 \text{ or } \cos \frac{5x}{2} = 0$$

$$\Rightarrow \frac{x}{2} = (2p+1)\frac{\pi}{2} \text{ or } x = (2q+1)\frac{\pi}{2} \text{ or } \frac{5x}{2} = (2r+1)\frac{\pi}{2}$$

$$\Rightarrow x = (2p+1)\pi \text{ or } x = (2q+1)\frac{\pi}{2} \text{ or } x = (2r+1)\frac{\pi}{5}$$

$$\Rightarrow x = \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

Total 7 solutions

JEE Advanced/Boards

Exercise 1

Sol 1: $\sin 5x = 16\sin^5 x$

$$\sin(3x + 2x) = \sin 3x \cos 2x + \cos 3x \sin 2x$$

$$= (3\sin x - 4\sin^3 x)(1 - 2\sin^2 x)$$

$$+ (-3\cos x + 4\cos^3 x)2\sin x \cos x$$

$$= \sin x[(3 - 4\sin^2 x)(1 - 2\sin^2 x)]$$

$$+ 2(3 - 4(1 - \sin^2 x))(1 - \sin^2 x) = 16\sin^5 x$$

One of the obvious solution is

$$\sin x = 0 \Rightarrow x = n\pi$$

If $\sin x \neq 0$

$$\Rightarrow 3 - 10\sin^2 x + 8\sin^4 x$$

$$+ 2(-1 + 4\sin^2 x)(1 - \sin^2 x) = 16\sin^4 x$$

$$\Rightarrow 3 - 10\sin^2 x + 8\sin^4 x + 10\sin^2 x - 8\sin^4 x - 2 = 16\sin^4 x$$

$$\Rightarrow 1 = 16\sin^4 x$$

$$\Rightarrow \sin^4 x = \frac{2\pi}{3} \Rightarrow \sin x = \pm \frac{\pi}{3}$$

$$\therefore x = n\pi \pm \frac{\pi}{6}, n\pi$$

Sol 2: $4\cos^2 x \sin x - 2\sin^2 x = 3\sin x$

One above solution is $\sin x = 0$ i.e. $x = n\pi$

If $\sin x \neq 0$

$$4\cos^2 x = 3 + 2\sin x$$

$$4(1 - \sin^2 x) = 3 + 2\sin x$$

$$4\sin^2 x + 2\sin x - 1 = 0$$

$$\sin x = \frac{-2 \pm \sqrt{4 + 4 \times 4}}{2 \times 4} = \frac{1 \pm \sqrt{5}}{4}$$

$$x = n\pi + (-1)^{n+1} \frac{3\pi}{10} \text{ or } n\pi + (-1)^n \frac{\pi}{10}$$

Sol 3: $1 + \cos x + \cos 2x + \sin x + \sin 2x + \sin 3x = 0$

$$\text{Given condition } \frac{\pi}{2} < \left| 3x - \frac{\pi}{2} \right| \leq \pi$$

$$1 + \cos x + 2\cos^2 x - 1 + (\sin x + \sin 3x) + \sin 2x = 0$$

$$\cos x(2\cos x + 1) + 2\sin 2x \cos x + \sin 2x = 0$$

$$(\cos x + \sin 2x)(2\cos x + 1) = 0$$

$$(2\sin x + 1)(2\cos x + 1) \cos x = 0$$

$$\sin x = -\frac{1}{2} \text{ or } \cos x = -\frac{1}{2} \text{ or } \cos x = 0$$

According to given condition

$$\frac{\pi}{2} < \left| 3x - \frac{\pi}{2} \right| \leq \pi$$

$$\Rightarrow \frac{\pi}{2} < 3x - \frac{\pi}{2} \leq \pi \text{ and } -\pi \leq 3x - \frac{\pi}{2} < -\frac{\pi}{2}$$

$$\Rightarrow \pi < 3x \leq \frac{3\pi}{2} \text{ and } -\frac{\pi}{2} \leq 3x < 0$$

$$\therefore \text{common condition is } x \in \left[-\frac{\pi}{6}, 0 \right) \cup \left[\frac{\pi}{3}, \frac{\pi}{2} \right)$$

Only two solutions are possible

$$\sin x = -\frac{1}{2} \text{ and } \cos x = 0$$

Sol 4: $2(\cos x + \cos 2x) + \sin 2x(1 + 2\cos x) = 2\sin x$

$$2(\cos x + 2\cos^2 x - 1) + \sin 2x(1 + 2\cos x) = 2\sin x$$

$$2\cos x(2\cos x + 1) - 2 + \sin 2x(1 + 2\cos x) = 2\sin x$$

$$2(\cos x + \sin 2x)(2\cos x + 1) - 2(1 + \sin x) = 0$$

$$2\cos x(1 + \sin x)(2\cos x + 1) - 2(1 + \sin x) = 0$$

$$(1 + \sin x)[\cos x(2\cos x + 1) - 1] = 0$$

$$\Rightarrow \sin x + 1 = 0 \text{ or } 2\cos^2 x + \cos x - 1 = 0$$

$$\text{i.e. } (2\cos x - 1)(\cos x + 1) = 0$$

$$\therefore \sin x = -1 \text{ or } \cos x = \frac{1}{2} \text{ or } \cos x = -1$$

$$x = (4n - 1)\frac{\pi}{2}$$

$$x = 2n\pi \pm \frac{\pi}{3} \text{ or } x = (2n + 1)\pi$$

$$\because x \in [-\pi, \pi]$$

$$\therefore x = -\frac{\pi}{2}, -\frac{\pi}{3}, \frac{\pi}{3}; -\pi, +\pi$$

$$\text{Sol 5: } 2(\sin x - \cos 2x) - \sin 2x(1 + 2\sin x) + 2\cos x = 0$$

$$2(\sin x + 2\sin^2 x) - 2 - \sin 2x(1 + 2\sin x) + 2\cos x = 0$$

$$(2\sin x - \sin 2x)(1 + 2\sin x) - 2(1 - \cos x) = 0$$

$$2\sin x(1 - \cos x)(1 + 2\sin x) - 2(1 - \cos x) = 0$$

$$2(1 - \cos x)[\sin x(1 + 2\sin x) - 1] = 0$$

$$\Rightarrow \cos x = 1 \text{ or } 2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0 \Rightarrow \sin x = \frac{1}{2} \text{ or } -1$$

$$x = 2n\pi \text{ or } x = n\pi + (-1)^n \frac{\pi}{6} \text{ or } x = (4n - 1)\frac{\pi}{2}$$

$$\Rightarrow x = 2n\pi, n\pi + (-1)^{n+1} \frac{\pi}{2}, n\pi + (-1)^n \frac{\pi}{6}$$

$$\text{Sol 6: } \cos 3x + \cos 2x = \sin \frac{3x}{2} + \sin \frac{x}{2}$$

$$\Rightarrow 2\cos \frac{5x}{2} \cos \frac{x}{2} = 2\sin x \cos \frac{x}{2}$$

$$\Rightarrow \cos \frac{x}{2} \left(\cos \frac{5x}{2} - \sin x \right) = 0$$

$$\cos \frac{x}{2} = 0 \text{ or } \cos \frac{5x}{2} = \sin x = \cos \left(\frac{\pi}{2} - x \right)$$

$$\Rightarrow \frac{x}{2} = 2n\pi \pm \frac{\pi}{2} \text{ or } \frac{5x}{2} = 2n\pi \pm \left(\frac{\pi}{2} - x \right)$$

$$\Rightarrow x = 4n\pi \pm \pi \text{ or } x = 4n\frac{\pi}{7} + \frac{\pi}{7} \text{ or } 4n\frac{\pi}{3} - \frac{\pi}{3} \text{ } n \in I$$

$$\therefore x \in (0, 2\pi)$$

$$\therefore x = \pi, \frac{\pi}{7}, \frac{5\pi}{7}, \frac{9\pi}{7}, \frac{13\pi}{7}$$

$$\text{Sol 7: } \tan^2 2x + \cot^2 2x + 2\tan 2x + 2\cot 2x = 6$$

$$\text{Let } \tan 2x = t$$

$$\Rightarrow t^2 + \frac{1}{t^2} + 2t + \frac{2}{t} = 6$$

$$\Rightarrow t^4 + 1 + 2(t^2 + 1)t = 6t^2$$

$$\Rightarrow t^4 + 2t^3 - 6t^2 + 2t + 1 = 0$$

$$\Rightarrow (t - 1)(t - 1)(t^2 + 4t + 1) = 0$$

$$\Rightarrow \tan 2x = 1 \text{ or } \frac{-4 \pm \sqrt{16 - 4}}{2} = -2 \pm \sqrt{3}$$

$$\therefore 2x = n\pi + \frac{\pi}{4} \text{ or } 2x = n\pi - \frac{\pi}{12} \text{ or } n\pi - \frac{5\pi}{12}$$

$$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}, \frac{n\pi}{2} - \frac{\pi}{12}, \frac{n\pi}{2} - \frac{5\pi}{12}$$

$$\text{Sol 8: } 1 + 2\operatorname{cosec} x = \frac{-\sec^2 \frac{x}{2}}{2}$$

$$1 + \frac{2}{\sin x} = -\frac{1}{2} \times \frac{1}{\cos^2 \frac{x}{2}}$$

$$\Rightarrow \frac{2 + \sin x}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = -\frac{1}{2 \cos^2 \frac{x}{2}}$$

$$\cos \frac{x}{2} \neq 0 \Rightarrow \frac{x}{2} \neq 2n\pi \pm \frac{\pi}{2} \Rightarrow x \neq 4n\pi \pm p$$

$$\Rightarrow 2 + \sin x = -\tan \frac{x}{2}$$

$$\Rightarrow 2 + \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = -\tan \frac{x}{2} \text{ (Let } \tan \frac{x}{2} = t)$$

$$\Rightarrow 2 + \frac{2t}{1+t^2} = -t \Rightarrow 2 + 2t^2 + 2t = -t - t^3$$

$$\Rightarrow t^3 + 2t^2 + 3t + 2 = 0$$

$$\Rightarrow (t + 1)(t^2 + t + 2) = 0$$

$$\Rightarrow t = -1 \text{ or } t^2 + t + 2 = 0$$

$$\therefore \tan \frac{x}{2} = -1$$

$$\frac{x}{2} = n\pi - \frac{\pi}{4} \Rightarrow x = 2n\pi - \frac{\pi}{2}$$

$$\text{Sol 9: } \sin \left(\frac{\sqrt{x}}{2} \right) + \cos \left(\frac{\sqrt{x}}{2} \right) = \sqrt{2} \sin \sqrt{x}$$

$$\frac{1}{\sqrt{2}} \sin \left(\frac{\sqrt{x}}{2} \right) + \frac{1}{\sqrt{2}} \cos \left(\frac{\sqrt{x}}{2} \right) = \cos \left(\frac{\pi}{2} - \sqrt{x} \right)$$

$$\Rightarrow \cos\left(\frac{\sqrt{x}}{2} - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2} - \sqrt{x}\right)$$

$$\therefore \frac{\sqrt{x}}{2} - \frac{\pi}{4} = 2n\pi \pm \left(\frac{\pi}{2} - \sqrt{x}\right)$$

$$\Rightarrow \sqrt{x} = \frac{4n\pi}{3} + \frac{\pi}{2}, -4n\pi + \frac{\pi}{2}$$

$$\therefore x = \left(\frac{4n\pi}{3} + \frac{\pi}{2}\right)^2, \left(4n\pi - \frac{\pi}{2}\right)^2, n \in I$$

Sol 10: As the roots are same both equations should be same.

Let us solve the second equation

$$\sin x \cos 2x = \sin 2x \cos 3x - \frac{1}{2} \sin 5x$$

$$\Rightarrow 2\sin x \cos 2x = 2\sin 2x \cos 3x - \sin 5x$$

$$\Rightarrow \sin 3x - \sin x = \sin 5x - \sin x - \sin 5x$$

$$\Rightarrow \sin 3x = 0 \therefore x = n\pi$$

Put $x = n\pi$ is given equation

$$a \cos 2x + |a| \cos 4x + \cos 6x = 1$$

$$a \cos 2n\pi + |a| \cos 4n\pi + \cos 6n\pi = 1$$

$$\Rightarrow a + |a| + 1 = 1$$

$$\therefore a \leq 0$$

$$\text{Sol 11: } \sqrt{13 - 18 \tan x} = 6\tan x - 3, -2\pi < x < 2\pi$$

$$\Rightarrow 13 - 18\tan x = (6\tan x - 3)^2$$

$$13 - 18\tan x = 36\tan^2 x + 9 - 36\tan x$$

$$18\tan^2 x - 9\tan x - 2 = 0$$

$$\Rightarrow (6\tan x + 1)(3\tan x - 2) = 0$$

$$\text{Also } 6\tan x - 3 > 0 \Rightarrow \tan x > \frac{1}{2}$$

$$\therefore \tan x = \frac{2}{3}$$

$$x = a - 2\pi, a - \pi, a, a + \pi, \text{ where } a = \tan^{-1} \frac{2}{3}$$

$$\text{Sol 12: } \left(\sqrt{1 + \sin 2x} - \sqrt{2} \cos 3x = 0\right)$$

$$1 + \sin 2x = 2\cos^2 3x = 2(1 - \sin^2 3x)$$

$$\Rightarrow 2\sin^2 3x + \sin 2x = 1$$

$$\Rightarrow 1 - \cos 6x + \sin 2x = 1$$

$$\Rightarrow \sin 2x = \cos 6x$$

$$\Rightarrow \cos\left(\frac{\pi}{2} - 2x\right) = \cos 6x$$

$$\Rightarrow \frac{\pi}{2} - 2x = 2n\pi \pm 6x$$

$$\Rightarrow x = \frac{n\pi}{4} + \frac{\pi}{16}, \frac{n\pi}{2} - \frac{\pi}{8}$$

$$\therefore \text{Smallest positive value} = \frac{\pi}{16}$$

$$\text{Sol 13: } 2\sin\left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8 \sin 2x \cos^2 2x}$$

$$\Rightarrow 2\left(\frac{1}{\sqrt{2}}\sin 3x + \frac{1}{\sqrt{2}}\cos 3x\right)$$

$$= \sqrt{1 + 4(2\sin 2x \cos 2x)\cos 2x}$$

$$\Rightarrow [\sqrt{2}(\sin 3x + \cos 3x)]^2 = 1 + 4\sin 4x \cos 2x$$

$$\Rightarrow 2(\sin^2 3x + \cos^2 3x + 2\sin 3x \cos 3x)$$

$$= 1 + 4\sin 4x \cos 2x$$

$$\Rightarrow 2(1 + 2\sin 3x \cos 3x) = 1 + 4\sin 4x \cos 2x$$

$$\Rightarrow 1 = 2(2\sin 4x \cos 2x) - 2(2\sin 3x \cos 3x)$$

$$= 2(\sin 6x + \sin 2x) - 2\sin 6x$$

$$\Rightarrow \sin 2x = \frac{1}{2}$$

$$\therefore \left(2x - \frac{\pi}{2}\right) = 2n\pi \pm \frac{\pi}{3}$$

$$x = n\pi + \frac{5\pi}{12}, n\pi + \frac{\pi}{12}, n \in I$$

$$\text{If } x = \frac{5\pi}{12} \Rightarrow \sin\left(3x + \frac{\pi}{4}\right) = -1$$

Which is not possible

$$\therefore x = n\pi + \frac{\pi}{12}$$

$$\text{Sol 14: } \sin x - \sin 3x + \sin 5x = \cos x - \cos 3x + \cos 5x$$

$$\frac{1}{\sqrt{2}}(\sin x - \cos x) - \frac{1}{\sqrt{2}}(\sin 3x - \cos 3x)$$

$$+ \frac{1}{\sqrt{2}}(\sin 5x - \cos 5x) = 0$$

$$\sin\left(x - \frac{\pi}{4}\right) - \sin\left(3x - \frac{\pi}{4}\right) + \sin\left(5x - \frac{\pi}{4}\right) = 0$$

$$\Rightarrow 2\sin\left(\frac{6x - \pi}{2}\right)\cos\left(\frac{x - 5x}{2}\right) - \sin\left(3x - \frac{\pi}{4}\right) = 0$$

$$\Rightarrow \sin\left(3x - \frac{\pi}{4}\right)[2\cos 2x - 1] = 0$$

$$\therefore \sin\left(3x - \frac{\pi}{4}\right) = 0 \text{ or } \cos 2x = \frac{1}{2}$$

$$\Rightarrow 3x = n\pi + \frac{\pi}{4} \text{ or } 2x = 2n\pi \pm \frac{\pi}{3}$$

$$\therefore x = \frac{n\pi}{3} + \frac{\pi}{12}, n\pi \pm \frac{\pi}{6}$$

$$\therefore \text{Principle solution are } \frac{\pi}{12}, \frac{\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12},$$

$$\frac{17\pi}{12}, \frac{21\pi}{12}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

i.e. 10 solutions.

$$\text{Sol 15: } 3^{\left(\frac{1}{2} + \log_3(\cos x + \sin x)\right)} - 2^{\log_2(\cos x - \sin x)} = \sqrt{2}$$

$$3^{\frac{1}{2} \log_3(\cos x + \sin x)} - 2^{\log_2(\cos x - \sin x)} = \sqrt{2}$$

$$\sqrt{3}(\cos x + \sin x) - (\cos x - \sin x) = \sqrt{2}$$

$$\Rightarrow \frac{\sqrt{3}-1}{2\sqrt{2}}\cos x + \frac{\sqrt{3}+1}{2\sqrt{2}}\sin x = \frac{1}{2}$$

$$\cos\left(x - \frac{5\pi}{12}\right) = 2n\pi \pm \frac{\pi}{3}$$

$\cos x - \sin x$ and $\cos x + \sin x > 0$

$$\therefore x = 2n\pi + \frac{\pi}{12}, 2n\pi + \frac{3\pi}{4}$$

\therefore For $x = 2n\pi + \frac{3\pi}{4}$, $\cos x + \sin x < 0$, which is not a solution.

$$\therefore x = 2n\pi + \frac{\pi}{12}$$

$$\text{Sol 16: } \cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$$

$$2\cos 4\theta \cos 2\theta + \cos 4\theta + 1 = 0$$

$$2\cos 4\theta \cos 2\theta + 2\cos^2 2\theta = 0$$

$$2\cos 2\theta (\cos 4\theta + \cos 2\theta)$$

$$2\cos 2\theta \times 2\cos 3\theta \cos \theta = 0$$

$$\Rightarrow \cos \theta = 0 \text{ or } \cos 2\theta = 0 \text{ or } \cos 3\theta = 0$$

$$\theta = 2n\pi \pm \frac{\pi}{2} \text{ or } \theta = n\pi + \frac{\pi}{4} \text{ or } \theta = \frac{2n\pi}{3} \pm \frac{\pi}{6}$$

$$\therefore \theta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{4}$$

$$= 30^\circ, 45^\circ, 90^\circ, 150^\circ, 135^\circ$$

$$\text{Sol 17: } \log_{\frac{-x^2-6x}{10}}(\sin 3x + \sin x) = \log_{\frac{-x^2-6x}{10}}(\sin 2x)$$

$$\Rightarrow -x^2 - 6x > 0 \text{ and } -x^2 - 6x \neq 10$$

$$\Rightarrow x(x+6) < 0 \text{ and } x^2 + 6x + 10 \neq 0$$

$$\therefore x \in (-6, 0)$$

But if x is negative then $\sin 2x$ will be negative.

Hence no solution is possible.

$$\text{Sol 18: } 3 - 2\cos \theta - 4\sin \theta - \cos 2\theta + \sin 2\theta = 0$$

$$3 - 2\cos \theta - 4\sin \theta - (1 - 2\sin^2 \theta) + 2\sin \theta \cos \theta = 0$$

$$\Rightarrow 2\cos \theta (\sin \theta - 1) + 2\sin \theta (\sin \theta - 1) - 2(\sin \theta - 1) = 0$$

$$\Rightarrow (\sin \theta - 1)(2\cos \theta + 2\sin \theta - 2) = 0$$

$$\therefore \sin \theta = 1 \text{ or } \sin \theta + \cos \theta = 1$$

$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{2} \text{ or } \cos\left(\theta - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$$

$$\therefore \theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$$

$$\therefore \theta = 2n\pi, 2n\pi + \frac{\pi}{2}$$

$$\text{Sol 19: } \cos 4x + 6 = 7\cos 2x$$

$$\cos 4x - \cos 2x = 6(\cos 2x - 1)$$

$$\Rightarrow 2\sin 3x \sin(-x) = 6(-2\sin^2 x)$$

$$\Rightarrow \sin 3x \sin x = 6\sin^2 x$$

$$\sin 3x \sin x = 6\sin^2 x$$

$$\sin x(\sin 3x - 6\sin x) = 0$$

$$\sin x(3\sin x - 4\sin^3 x - 6\sin x) = 0$$

$$\Rightarrow \sin^2 x(-3 - 4\sin^2 x) = 0$$

$$\Rightarrow \sin x = 0 \text{ or } \sin^2 x = \frac{-3}{4}$$

Which is not possible

$$\therefore x = n\pi$$

\therefore Possible solutions are

$$0, \pi, 2\pi, 3\pi, \dots, 99\pi$$

$$\therefore 100\pi = 314.159 > 314$$

$$\therefore \text{Sum} = \frac{99 \times 100\pi}{2} = 4950\pi$$

$$\text{Sol 20: } \sin^3 x + \sin^3 2x + \sin^3 3x$$

$$= (\sin x + \sin 2x + \sin 3x)^3$$

$$a^3 + b^3 + c^3 = (a + b + c)^3 - 3(a + b)$$

$$(b + c)(c + a)$$

$$\Rightarrow (\sin x + \sin 2x)(\sin 2x + \sin 3x)(\sin 3x + \sin x) = 0$$

$$\left(2\sin \frac{3x}{2} \cos \frac{x}{2}\right) \left(2\sin \frac{5x}{2} \cos \frac{x}{2}\right) (2\sin 2x \cos x) = 0$$

$$\Rightarrow \sin \frac{3x}{2} = 0 \text{ or } \sin \frac{5x}{2} = 0 \text{ or } \sin 2x = 0 \text{ or}$$

$$\cos \frac{x}{2} = 0 \text{ or } \cos x = 0$$

$$\therefore x = \frac{2n\pi}{3}, \frac{2n\pi}{5}, n\pi, 4n\pi \pm \pi, 2n\pi \pm \frac{\pi}{2}$$

$$\therefore \text{Least positive angle would be } \frac{2\pi}{5} = 72^\circ.$$

Sol 21: $\sin(\pi - 6x) + \sqrt{3} \sin \left(\frac{\pi}{2} + 6x \right) = \sqrt{3}$ in $[0, 2\pi]$

$$\frac{1}{2} \sin 6x + \frac{\sqrt{3}}{2} \cos 6x = \frac{\sqrt{3}}{2}$$

$$\cos \left(6x - \frac{\pi}{6} \right) = \cos \left(\frac{\pi}{6} \right)$$

$$6x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{6} \Rightarrow x = \frac{n\pi}{3}, \frac{n\pi}{3} + \frac{\pi}{18}$$

$$\therefore x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi, \frac{\pi}{18},$$

$$\frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$$

\therefore Total no. of solutions = 13

Sol 22: $(\sin \theta)x^2 + (2\cos \theta)x + \frac{\cos \theta + \sin \theta}{2}$

For perfect square of linear equation $D = 0$

$$\Rightarrow b^2 - 4ac = 0$$

$$4\cos^2 \theta - 4 \left(\frac{\cos \theta + \sin \theta}{2} \right) \sin \theta = 0$$

$$2\cos^2 \theta = (\cos \theta + \sin \theta) \sin \theta$$

$$\Rightarrow 2\cot \theta = 1 + \tan \theta$$

$$\Rightarrow \tan^2 \theta + \tan \theta - 2 = 0$$

$$(\tan \theta + 2)(\tan \theta - 1) = 0$$

$$\tan \theta = -2, 1$$

$$n\pi + \frac{\pi}{4}, n\pi - \tan^{-1} 2 \quad n \in \mathbb{I}$$

Sol 23: (a) $\sin x \sin 2x \sin 3x = 1$

$$\frac{1}{\tan x} (\cos(-2x) - \cos 4x) \sin 2x = 1$$

$$2\cos 2x \sin 2x - 2\cos 4x \sin 2x = 4$$

$$\Rightarrow \sin 4x + \sin 2x - \sin 6x = 4$$

As maximum value of $\sin \theta = 1$ so if $\sin 4x, \sin 2x$ takes maximum value 1 and $\sin 6x$ takes the value -1 even then LHS will be less than RHS so no solution possible.

(b) $\sin x \cos 4x \sin 5x = -\frac{1}{2}$

$$(\cos 4x - \cos 6x) \cos 4x = -1$$

$$2\cos^2 4x - 2\cos 6x \cos 4x = -2$$

$$\Rightarrow 1 + \cos 8x - \cos 10x - \cos 2x = -2$$

$$\Rightarrow \cos 8x - \cos 10x - \cos 2x = -3$$

For LHS = RHS

$$\cos 10x = \cos 2x = 1 \text{ and } \cos 8x = 1$$

$$\therefore 8x = 2n\pi \text{ and } 10x = (2m + 1)\pi \text{ and}$$

$$2x = (2m + 1)p$$

$$\Rightarrow x = \frac{n\pi}{4}, x = (2m + 1) \frac{\pi}{10}, x = (2m + 1) \frac{\pi}{2}$$

$$\frac{n\pi}{4} = (2m + 1) \frac{\pi}{10} \text{ and } \frac{n\pi}{4} = (2m + 1) \frac{\pi}{2}$$

There is no integer value of n and m for which above results hold. So no solution.

(c) $2\sin x \cos x \cos 2x = -1$

$$2\sin x [\cos 3x + \cos x] = -2$$

$$\Rightarrow \sin 4x + \sin(-2x) + \sin 2x = -2$$

$$\Rightarrow \sin 4x = -2$$

\therefore No solution

(d) $4\sin 2x + \cos x = 5$

For this result to hold

$$\sin 2x = 1 \text{ and } \cos x = 1$$

$$\therefore 2x = n\pi + (-1)^n \frac{\pi}{2} \text{ and } x = 2m\pi$$

$$\therefore x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{2} \text{ and } x = 2m\pi$$

There exist no m, n for which the above relation is valid. No solution

(e) $\sin 3x - \cos x = 2$

For this

$$\sin 3x = 1 \text{ and } \cos x = -1$$

$$3x = n\pi + (-1)^n \frac{\pi}{2} \text{ and } x = (2m + 1)\pi$$

$$x = \frac{n\pi}{3} + (-1)^n \frac{\pi}{6} \text{ and } x = (2m + 1)\pi$$

Both the values cannot be same for any integral value of m and n. So no. solution

Sol 24: (a) $f(x) = \sin^6x + \cos^6x + k(\sin^4x + \cos^4x)$

$$\begin{aligned} &= (\sin^2x + \cos^2x)^3 - 3\sin^4x \cos^2x \\ &\quad - 3\sin^2x \cos^4x + k(1 - 2\sin^2x \cos^2x) \\ &= 1 + k - 2k\sin^2x \cos^2x - 3\sin^2x \cos^2x \\ &= (1 + k) - \frac{(2k+3)}{4} \sin^22x \end{aligned}$$

For $f(x) = \text{constant}$

$$\frac{2k+3}{4} = 0 \Rightarrow k = -\frac{3}{2}$$

$$(b) (1 + k) - \frac{(2k+3)}{4} \sin^22x = 0$$

$$\Rightarrow \sin^22x = \frac{(1+k)^4}{(2k+3)}$$

$$\therefore 0 \leq \frac{4(1+k)}{2k+3} \leq 1 \Rightarrow k \in \left[-1, \frac{-1}{2}\right]$$

(c) If $k = -0.7$

$$\Rightarrow (1-0.7) - \frac{2(-0.7)+3}{4} \sin^22x = 0$$

$$\Rightarrow \sin^22x = \frac{0.3 \times 4}{1.6} = \frac{3}{4}$$

$$\therefore \sin2x = \pm \frac{\sqrt{3}}{2}$$

$$\Rightarrow 2x = n\pi \pm \frac{\pi}{3} \Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{6}$$

Sol 25: a & b

$$a \cos \theta + b \sin \theta = c$$

$$\Rightarrow a \cos \theta = c - b \sin \theta$$

$$\Rightarrow a^2 \cos^2 \theta = c^2 - b^2 \sin^2 \theta - 2bc \sin \theta$$

$$\Rightarrow a^2 (1 - \sin^2 \theta) = c^2 + b^2 \sin^2 \theta - 2bc \sin \theta$$

$$\Rightarrow (a^2 + b^2) \sin^2 \theta - 2bc \sin \theta + c^2 - a^2 = 0$$

If α & β are roots, then

$$\sin \alpha + \sin \beta = \frac{2bc}{a^2 + b^2}$$

$$\sin \alpha \cdot \sin \beta = \frac{c^2 - a^2}{a^2 + b^2}$$

$$(c) \tan \frac{\alpha}{2} + \tan \frac{\beta}{2}$$

$$\sin \theta = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}},$$

$$\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$\text{Let } \tan \frac{\theta}{2} = t$$

$$\therefore a \cos \theta + b \sin \theta = c$$

$$a(1 - t^2) + b(2t) = c(1 + t^2)$$

$$= (a + c)t^2 - 2bt + c - a = 0$$

$$\therefore \tan \frac{\alpha}{2} + \tan \frac{\beta}{2} = \frac{2b}{a+c} \text{ (P) (sum of roots)}$$

$$\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{c-a}{c+a}$$

(Q) (Product of roots)

Sol 26:

$$(A) \cos 3x \cos^3 x + \sin 3x \sin^3 x = 0$$

$$\Rightarrow (4\cos^3 x - 3\cos x) \cos^3 x + (3\sin x - 4\sin^3 x) \sin^3 x = 0$$

$$\Rightarrow 4\cos^6 x - 4\sin^6 x + 3\sin^4 x - 3\cos^4 x = 0$$

$$\Rightarrow 4[(\cos^2 x)^3 - (\sin^2 x)^3]$$

$$+ 3(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = 0$$

$$\Rightarrow 4(\cos^2 x - \sin^2 x)(1 + \cos^2 x \sin^2 x) - 3\cos 2x = 0$$

$$\Rightarrow 4\cos 2x(1 + \cos^2 x \sin^2 x) = 3\cos 2x$$

$$\Rightarrow \cos 2x + 4\cos 2x(\sin 2x)^2 = 0$$

$$\Rightarrow \cos 2x(1 + \sin^2 2x) = 0$$

$$\sin^2 2x = -1$$

Not possible (No real solution)

$$\cos 2x = 0 \Rightarrow 2x = 2n\pi \pm \frac{\pi}{2} \Rightarrow x = \frac{n\pi}{2} \pm \frac{\pi}{4}$$

$$(B) \sin 3\alpha = 4\sin \alpha \sin(x + \alpha) \sin(x - \alpha)$$

$$\sin 3\alpha = 2\sin \alpha [\cos 2\alpha - \cos 2x]$$

$$\sin 3\alpha = \sin 3\alpha + \sin(-\alpha) - 2\sin \alpha \cos 2x$$

$$\Rightarrow \sin \alpha (1 + 2\cos 2x) = 0$$

$$\cos 2x = -\frac{1}{2}$$

$$2x = 2n\pi \pm \frac{2\pi}{3}$$

$$x = n\pi \pm \frac{\pi}{3}$$

$$(C) |2\tan x - 1| + |2\cot x - 1| = 2$$

For $\tan x > 2$

$$2\tan x - 1 + 1 - 2\cot x = 2$$

$$\tan x - \frac{1}{\tan x} = 1$$

$$\tan^2 x - \tan x - 1 = 0$$

$$\tan x = \frac{1 \pm \sqrt{5}}{2} < 2$$

∴ No. solution.

$$\text{for } \tan x \in \left[\frac{1}{2}, 2 \right]$$

$$2\tan x - 1 + 2\cot x - 1 = 2$$

$$2\tan x + \frac{2}{\tan x} = 4$$

$$\tan^2 x - 2\tan x + 1 = 0$$

$$\tan x = 1 \in \left[\frac{1}{2}, 2 \right]$$

$$\therefore x = n\pi + \frac{\pi}{4} \text{ is one solution}$$

$$\text{For } x < \frac{1}{2}$$

$$1 - 2\tan x + \frac{2}{\tan x} - 1 = 2$$

$$\tan^2 x + \tan x - 1 = 0$$

$$\tan x = \frac{-1 \pm \sqrt{5}}{2}$$

$$\tan x = \frac{-1 - \sqrt{5}}{2} \text{ is acceptable}$$

∴ From option (Q)

$$(D) \sin^{10} x + \cos^{10} x = \frac{29}{16} \cos^4 2x$$

$$\left(\frac{1 - \cos 2x}{2} \right)^5 + \left(\frac{1 + \cos 2x}{2} \right)^5 = \frac{29}{16} \cos^4 2x$$

Let $\cos 2x = t$

$$\left(\frac{1-t}{2} \right)^5 + \left(\frac{1+t}{2} \right)^5 = \frac{29}{16} t^4$$

$$24t^4 - 10t^2 - 1 = 0 \text{ or } (2t^2 - 1)(12t^2 + 1) = 0$$

$$\Rightarrow t = \pm \frac{1}{2}$$

$$\therefore \cos^2 x = \frac{1}{2}$$

$$\Rightarrow 2\cos^2 2x - 1 = 0$$

$$\Rightarrow \cos 4x = 0$$

$$4x = n\pi + \frac{\pi}{2} \text{ or } x = \frac{n\pi}{4} + \frac{\pi}{8}, n \in I$$

$$\text{Sol 27: (A) } \sin^2 x + \cos^2 3x = 1$$

$$\cos^2 3x = \cos^2 x$$

$$\Rightarrow \cos 3x = \pm \cos x$$

$$\therefore 3x = 2n\pi \pm x \text{ or } 3x = 2n\pi \pm (\pi - x)$$

$$x = n\pi, \frac{n\pi}{2} \text{ or } x = \frac{n\pi}{2} + \frac{\pi}{4}, n\pi - \frac{\pi}{2}$$

From options one can say that all the options which satisfying the equation are p, q, r, s

$$(B) e^{\cot^2 \theta} + \sin^2 \theta - 2\cos^2 \theta + 4 = 4\sin \theta$$

$e^{\cot^2 \theta}$ is not possible at $\theta = n\pi, n \in I$

$$\text{So at } \theta = \frac{\pi}{4}$$

$$\Rightarrow e^1 + \frac{1}{2} - 2(0) + 4, \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

L. H. S. ≠ R. H. S.

$$\text{at } \theta = \frac{\pi}{2}$$

$$\Rightarrow e^0 + 1^2 - 2(1)^2 + 4 = 4(1)$$

$$1 + 1 - x + 4 = 4$$

L. H. S. = R. H. S.

$$\text{So Ans (s) } (4n+1) \frac{\pi}{2}$$

$$(C) a^2 \sin x - a \sin 2x + \sin x = 0$$

$$(a^2 + 1)\sin x = 2a \sin x \cos x$$

$$\sin x[(a^2 + 1) - 2a \cos x] = 0$$

$$\sin x = 0, \cos x = \frac{a^2 + 1}{2a} \geq 1$$

From options Ans. (p)

$$(D) \sqrt[3]{2 \tan \theta - 1} + \sqrt[3]{\tan \theta - 1} = 1$$

From all the given options we can directly reject P, Q, S as they are not satisfying the given equation and $\frac{\pi}{2}$ is not in domain of $\tan \theta$.

Exercise 2

Single Correct Choice Type

Sol 1: (A) If $B = C$ then $\angle A = 90^\circ$ $\angle B = 45^\circ$ $\angle C = 45^\circ$

$$\text{Sol 2: (A)} \quad 2^{\tan\left(x - \frac{\pi}{4}\right)} - 2(0.25)^{\sin^2\left(x - \frac{\pi}{4}\right)} + 1 = 0$$

$$\frac{\sin^2\left(x - \frac{\pi}{4}\right)}{\cos 2x} = \frac{1}{2} \left(1 - \cos 2\left(x - \frac{\pi}{4}\right)\right)$$

$$= \frac{1}{2} \frac{(1 - \sin 2x)}{\cos 2x} = \frac{1}{2} \tan\left(\frac{\pi}{4} - x\right)$$

$$\therefore \text{Take } \tan\left(x - \frac{\pi}{4}\right) = t$$

Then expression would be

$$2^t - 2\left(\frac{1}{4}\right)^{t/2} + 1 = 0$$

$$\Rightarrow 2^t - 2\left(\frac{1}{2}\right)^t + 1 = 0$$

$$(2^t)^2 + (2^t) - 2 = 0$$

$$(2^t + 2)(2^t - 1) = 0$$

$$\Rightarrow 2^t = -2 \text{ (Not possible)} \text{ or } 2^t = 1$$

$$\Rightarrow t = 0$$

$$\therefore \tan\left(x - \frac{\pi}{4}\right) = 0; \quad x = n\pi + \frac{\pi}{4}$$

But in equation $\frac{1}{\cos 2x}$ does not exist at

$x = n\pi + \frac{\pi}{4}$, therefore no value of x exists.

$$\text{Sol 3: (A)} \quad \sum_{r=1}^5 \cos r x = 0$$

$$\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x = 0$$

$$\Rightarrow \cos x + \cos 5x + \cos 2x + \cos 4x + \cos 3x = 0$$

$$\Rightarrow 2 \cos 3x \cos 2x + 2 \cos 3x \cos x + \cos 3x = 0$$

$$\Rightarrow \cos 3x [2 \cos 2x + 2 \cos x + 1] = 0$$

$$\Rightarrow \cos 3x \left[2 \times 2 \cos \frac{3x}{2} \cos \frac{x}{2} + 1 \right] = 0$$

$$\Rightarrow \cos 3x \left[4 \cos \frac{3x}{2} \cos \frac{x}{2} + 1 \right] = 0$$

$$\Rightarrow \cos 3x = 0 = \cos(2n+1)\frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}$$

⇒ Two solutions

$$\text{Sol 4: (B)} \quad \cot 7\frac{1}{2}^\circ + \tan 67\frac{1}{2}^\circ - \cot 67\frac{1}{2}^\circ - \tan 7\frac{1}{2}^\circ$$

$$= \cot \frac{A}{2} + \tan \frac{B}{2} - \cot \frac{B}{2} - \tan \frac{A}{2}$$

$$A = 15^\circ, B = 135^\circ$$

$$= \frac{1 - \tan^2 \frac{A}{2}}{\tan \frac{A}{2}} + \frac{\tan^2 \frac{B}{2} - 1}{\tan \frac{B}{2}}$$

$$= 2 \cot A - 2 \cot B = 2(\cot 15^\circ - \cot 135^\circ)$$

$$= 2(2 + \sqrt{3} + 1) = 2(3 + \sqrt{3})$$

Which is an irrational number.

$$\text{Sol 5: (C)} \quad A = 580^\circ = 3\pi + \frac{2\pi}{9}$$

$$\sqrt{1 + \sin A} = \sqrt{\sin^2 \frac{A}{2} + \cos^2 \frac{A}{2} + 2 \sin \frac{A}{2} \cos \frac{A}{2}}$$

$$= \sqrt{\left(\sin \frac{A}{2} + \cos \frac{A}{2}\right)^2} = \left|\sin \frac{A}{2} + \cos \frac{A}{2}\right|$$

$$\therefore \text{for } A = 3\pi + \frac{2\pi}{9}$$

$$\sqrt{1 + \sin A} = -\left(\sin \frac{A}{2} + \cos \frac{A}{2}\right)$$

$$\sqrt{1 - \sin A} = \sqrt{\sin^2 \frac{A}{2} + \cos^2 \frac{A}{2} - 2 \sin \frac{A}{2} \cos \frac{A}{2}}$$

$$= \left|\sin \frac{A}{2} - \cos \frac{A}{2}\right|$$

$$\text{For } A = 3\pi + \frac{2\pi}{9}$$

$$\sqrt{1 - \sin A} = -\left(\sin \frac{A}{2} - \cos \frac{A}{2}\right)$$

$$\therefore 2 \sin \frac{A}{2} = -\sqrt{1 + \sin A} - \sqrt{1 - \sin A}$$

Sol 6: (A) $\tan\alpha = \frac{x^2 - x}{x^2 - x + 1}$ and $\tan\beta = \frac{1}{2x^2 - 2x + 1}$

$$\Rightarrow 2(x^2 - x) + 1 = \frac{1}{\tan\beta}$$

$$x^2 - x = \left(\frac{1 - \tan\beta}{\tan\beta}\right)\frac{1}{2}$$

$$\therefore \tan\alpha = \frac{\frac{1}{2}\left(\frac{1 - \tan\beta}{\tan\beta}\right)}{\frac{1}{2}\left(\frac{1 - \tan\beta}{\tan\beta}\right) + 1} = \frac{\frac{1}{2}\left(\frac{1 - \tan\beta}{\tan\beta}\right)}{\frac{1}{2}\left(\frac{1 + \tan\beta}{\tan\beta}\right)}$$

$$\Rightarrow \tan\alpha = \frac{1 - \tan\beta}{1 + \tan\beta}$$

$$\Rightarrow \tan\alpha + \tan\alpha \tan\beta = 1 - \tan\beta$$

$$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} = \tan(\alpha + \beta) = 1$$

Sol 7: (C) $8\cos^2x + 18\sec^2x$

$$f(X) = 8X^2 + \frac{18}{X^2} \text{ let } \cos^2x = X$$

$$f'(X) = 16X - \frac{36}{X^3} = 0$$

$$X = \left(\frac{36}{16}\right)^{1/4} = (2 \cdot 25)^{1/4} > 1$$

∴ Min. Value of this function will be

$$\text{When } \cos x = 1$$

$$\therefore \text{Min. Value} = 8 + 18 = 26$$

Sol 8: (B) $x = a \cos(\phi - \alpha)$

$$y = b \cos(\phi - \beta)$$

$$\Rightarrow \cos^{-1}\left(\frac{x}{a}\right) = \phi - \alpha \quad \dots\dots(i)$$

$$\Rightarrow \cos^{-1}\left(\frac{y}{b}\right) = \phi - \beta \quad \dots\dots(ii)$$

(ii) - (i)

$$\Rightarrow \alpha - \beta = \cos^{-1}\left(\frac{x}{a}\right) - \cos^{-1}\left(\frac{y}{b}\right)$$

$$\Rightarrow \alpha - \beta = \cos^{-1}\left[\frac{xy}{ab} + \sqrt{1 - \frac{x^2}{a^2}} \sqrt{1 - \frac{y^2}{b^2}}\right]$$

$$\Rightarrow \cos(\alpha - \beta) = \frac{xy}{ab} + \sqrt{1 - \frac{x^2}{a^2}} \cdot \sqrt{1 - \frac{y^2}{b^2}}$$

$$\Rightarrow \cos(\alpha - \beta) = \frac{xy}{ab} + \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2 y^2}{a^2 b^2}}$$

$$\Rightarrow \left(\cos(\alpha - \beta) - \frac{xy}{ab}\right)^2 = \left(\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2 y^2}{a^2 b^2}}\right)^2$$

$$\Rightarrow \cos^2(\alpha - \beta) + \frac{x^2 y^2}{a^2 b^2} - 2 \cos(\alpha - \beta) \times \frac{xy}{ab}$$

$$= 1 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) + \frac{x^2 y^2}{a^2 b^2}$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos(\alpha - \beta) = 1 - \cos^2(\alpha - \beta)$$

$$= \sin^2(\alpha - \beta)$$

Sol 9: (D) $\tan x + \tan 2x + \tan 3x = \tan x \cdot \tan 2x \cdot \tan 3x$

$$\frac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = -\tan 3x$$

$$\Rightarrow \tan 3x = -\tan 3x \Rightarrow \tan 3x = 0$$

$$\Rightarrow 3x = n\pi$$

$$\Rightarrow x = \frac{n\pi}{3}$$

Sol 10: (C) $\tan 3x - \tan 2x - \tan x = 0$

$$\tan(3x - 2x)[1 + \tan 3x \tan 2x] - \tan x = 0$$

$$\Rightarrow \tan x \tan 2x \tan 3x = 0$$

$$\therefore x = n\pi, \frac{n\pi}{2}, \frac{n\pi}{3}$$

$$\therefore x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$$

i.e. 7 solutions

$$\therefore x \neq \frac{\pi}{2}$$

As at $x = \frac{\pi}{2}$ $\tan x$, is not defined

Sol 11: (C) $x = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots \dots \infty$

$$x = 1 - x(1 - x + x^2 - x^3 + x^4 \dots \dots \infty)$$

$$x = 1 - x \cdot x$$

$$\Rightarrow x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{5}}{2} = 2\sin 18^\circ$$

Multiple Correct Choice Type

Sol 12: (B, D) $1 + 4\sin\theta + 3\cos\theta = 1 + 5\left(\frac{4}{5}\sin\theta + \frac{3}{5}\cos\theta\right)$

$$= 1 + 5\sin(\theta + \alpha)\cos\alpha = \frac{4}{5}$$

∴ Maximum value is $1 + 5 = 6$

When $\sin(\theta + \alpha) = 1$

Minimum value is $1 - 5 = -4$

When $\sin(\theta + \alpha) = -1$

Sol 13: (A, B, C) $\frac{\sqrt{3}\sin(\alpha + \beta) - \frac{2}{\cos\frac{\pi}{6}}\cos(\alpha + \beta)}{\sin\alpha}$

$$= \left(\frac{\sqrt{3} \times \frac{\sqrt{3}}{2} \sin(\alpha + \beta) - 2 \cos(\alpha + \beta)}{\sin\alpha} \right) \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}} \left[\frac{\frac{3}{2}(\sin\alpha\cos\beta + \cos\alpha\sin\beta) - 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)}{\sin\alpha} \right]$$

$$= \frac{2}{\sqrt{3}} \left[\frac{3}{2}\cos\beta + \frac{3}{2}\cot\alpha\sin\beta - 2\cot\alpha\cos\beta + 2\sin\beta \right]$$

If $\sin\beta = \frac{4}{5}$ and $\cos\beta = \frac{3}{5}$

i.e. $\tan\beta > 0$ i.e. $\beta \in \left(0, \frac{\pi}{2}\right)$

R. H. S.

$$= \frac{2}{\sqrt{3}} \left[\frac{3}{2} \times \frac{3}{5} + \frac{3}{2} \cot\alpha \times \frac{4}{5} - 2\cot\alpha \times \frac{3}{5} + 2 \times \frac{4}{5} \right]$$

$$= \frac{2}{\sqrt{3}} \left[\frac{9}{10} + \frac{8}{5} \right] = \frac{5}{\sqrt{3}}$$

If $\cos\beta = -\frac{3}{5}$ i.e. $\tan\beta < 0$

⇒ R. H. S.

$$= \frac{2}{\sqrt{3}} \left[\frac{3}{2} \left(-\frac{3}{5} \right) + \frac{3}{2} \cot\alpha \times \frac{4}{5} + 2\cot\alpha \times \frac{3}{5} + 2 \times \frac{4}{5} \right]$$

$$= \frac{2}{\sqrt{3}} \left[\frac{12}{5} \cot\alpha + \frac{7}{10} \right]$$

$$= \frac{[24\cot\alpha + 7]}{15} \sqrt{3}$$

Sol 14: (B, C) $\sin t + \cos t = \frac{1}{5}$

$$\Rightarrow \frac{2\tan\frac{t}{2}}{1+\tan^2\frac{t}{2}} + \frac{1-\tan^2\frac{t}{2}}{1+\tan^2\frac{t}{2}} = \frac{1}{5}$$

Let $\tan\frac{t}{2} = a$

$$5(2a + 1 - a^2) = 1 + a^2$$

$$\Rightarrow 6a^2 - 10a - 4 = 0 \Rightarrow 3a^2 - 5a - 2 = 0$$

$$\Rightarrow (3a + 1)(a - 2) = 0$$

$$\therefore a = -\frac{1}{3}, 2$$

$$\tan\frac{t}{2} = -\frac{1}{3}, 2$$

Previous Years' Questions

Sol 1: Given, $e^{\sin x} - \frac{1}{e^{\sin x}} = 4$

$$\Rightarrow (e^{\sin x})^2 - 4(e^{\sin x}) - 1 = 0$$

$$\Rightarrow e^{\sin x} = \frac{4 \pm \sqrt{16+4}}{2} = 2 \pm \sqrt{5}$$

But since, $e \sim 2.72$ and we know, $0 < e^{\sin x} < e$

$\therefore e^{\sin x} = 2 \pm \sqrt{5}$ is not possible.

Hence, no solution.

Sol 2: Given, $2^{1+|\cos x|+|\cos^2 x|+|\cos^3 x|+\dots} = 2^2$

$$\Rightarrow 2^{\frac{1}{1-|\cos x|}} = 2^2 \quad \Rightarrow \frac{1}{1-|\cos x|} = 2$$

$$\Rightarrow \cos x = \pm \frac{1}{2}$$

($\because x \in (-\pi, \pi)$)

Thus, the solution set is

$$\left\{ +\frac{\pi}{3}, \pm \frac{2\pi}{3} \right\}$$

Sol 3: Exp $\left\{ \left(\sin^2 x + \sin^4 x + \sin^6 x + \dots + \infty \right) \log_e 2 \right\}$

$$= e^{\frac{\sin^2 x}{1 - \sin^2 x} \log_e 2} = e^{\log_e 2 \frac{\sin^2 x}{\cos^2 x}}$$

$$\Rightarrow 2^{\tan^2 x} \text{ satisfy } x^2 - 9x + 8 = 0$$

$$\Rightarrow x = 1, 8$$

$$\therefore 2^{\tan^2 x} = 1 \text{ and } 2^{\tan^2 x} = 8$$

$$\Rightarrow \tan^2 x = 0 \text{ and } \tan^2 x = 3$$

$$\Rightarrow x = n\pi \text{ and } \tan^2 x = \left(\tan \frac{\pi}{3} \right)^2$$

$$\Rightarrow x = n\pi \text{ and } x = n\pi \pm \frac{\pi}{3}$$

$$\text{Neglecting } x = n\pi \text{ as } 0 < x < \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{3} \in \left(0, \frac{\pi}{2} \right)$$

$$\therefore \frac{\cos x}{\cos x + \sin x} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2}} = \frac{1}{1 + \sqrt{3}} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$\Rightarrow \frac{\cos x}{\cos x + \sin x} = \frac{\sqrt{3} - 1}{2}$$

Sol 4: $\tan(x + 100^\circ) = \tan(x + 50^\circ) \tan x \tan(x - 50^\circ)$

$$\Rightarrow \frac{\tan(x + 100^\circ)}{\tan x} = \tan(x + 50^\circ) \tan(x - 50^\circ)$$

$$\Rightarrow \frac{\sin(x + 100^\circ)}{\cos(x + 100^\circ)} \cdot \frac{\cos x}{\sin x} = \frac{\sin(x + 50^\circ) \sin(x - 50^\circ)}{\cos(x + 50^\circ) \cos(x - 50^\circ)}$$

$$\Rightarrow [\sin(2x + 100^\circ) + \sin 100^\circ][\cos 100^\circ + \cos 2x]$$

$$= [\cos 100^\circ - \cos 2x] \times [\sin(2x + 100^\circ) - \sin 100^\circ]$$

$$\Rightarrow \sin(2x + 100^\circ) \cdot \cos 100^\circ$$

$$+ \sin(2x + 100^\circ) \cdot \cos 2x + \sin 100^\circ \cos 100^\circ$$

$$+ \sin 100^\circ \cos 2x$$

$$= \cos 100^\circ \sin(2x + 100^\circ)$$

$$- \cos 100^\circ \sin 100^\circ - \cos 2x \sin$$

$$(2x + 100^\circ) + \cos 2x \sin 100^\circ$$

$$\Rightarrow 2 \sin(2x + 100^\circ) \cos 2x + 2 \sin 100^\circ \cos 100^\circ = 0$$

$$\Rightarrow \sin(4x + 100^\circ) + \sin 100^\circ + \sin 200^\circ = 0$$

$$\Rightarrow \sin(4x + 100^\circ) + 2 \sin 150^\circ \cos 50^\circ = 0$$

$$\Rightarrow \sin(4x + 100^\circ) + 2 \cdot \frac{1}{2} \sin(90^\circ - 50^\circ) = 0$$

$$\Rightarrow \sin(4x + 100^\circ) + \sin 40^\circ = 0$$

$$\Rightarrow \sin(4x + 100^\circ) + \sin(-40^\circ) = 0$$

$$\Rightarrow 4x = n(180^\circ) + (-1)^n (-40^\circ) - 100^\circ$$

$$\Rightarrow x = \frac{1}{4} [n(180^\circ) + (-1)^n (-40^\circ) - 100^\circ]$$

The smallest positive value of x is obtained when n = 1.

$$\text{Therefore, } x = \frac{1}{4} (180^\circ + 40^\circ - 100^\circ)$$

$$\Rightarrow x = \frac{1}{4} (120^\circ) = 30^\circ$$

Sol 5: Given,

$$\cos(p \sin x) = \sin(p \cos x), \forall x \in [0, 2\pi]$$

$$\Rightarrow \cos(p \sin x) = \cos\left(\frac{\pi}{2} - p \cos x\right)$$

$$\Rightarrow p \sin x = 2n\pi \pm \left(\frac{\pi}{2} - p \cos x \right), n \in I$$

$$(\because \cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I)$$

$$\Rightarrow p \sin x + p \cos x = 2n\pi + \pi/2$$

$$\text{or } p \sin x - p \cos x = 2n\pi - \pi/2, n \in I$$

$$\Rightarrow p(\sin x + \cos x) = 2n\pi + \pi/2$$

$$\text{or } p(\sin x - \cos x) = 2n\pi - \pi/2, n \in I$$

$$\Rightarrow p\sqrt{2} \left(\cos \frac{\pi}{4} \sin x + \sin \frac{\pi}{4} \cos x \right) = 2n\pi - \frac{\pi}{2}$$

$$\text{or } p\sqrt{2} \left(\cos \frac{\pi}{4} \sin x - \sin \frac{\pi}{4} \cos x \right) = 2n\pi - \frac{\pi}{2}, n \in I$$

$$\Rightarrow p\sqrt{2} \left[\sin(x + \pi/4) \right] = \frac{(4n+1)\pi}{2}$$

$$\text{or } p\sqrt{2} \left[\sin(x - \pi/4) \right] = (4n-1)\frac{\pi}{2}, n \in I$$

$$\text{Now, } -1 \leq \sin(x \pm \pi/4) \leq 1$$

$$\Rightarrow -p\sqrt{2} \leq p\sqrt{2} \sin(x \pm \pi/4) \leq p\sqrt{2}$$

$$\Rightarrow -p\sqrt{2} \leq \frac{(4n+1)\pi}{2} \leq p\sqrt{2}, n \in I$$

Second inequality is always a subset of first, therefore, we have to consider only first.

It is sufficient to consider $n \leq 0$, because for $n > 0$, the solution will be same for $n \geq 0$.

$$\text{If } n \geq 0, -\sqrt{2}p \leq (4n+1)\pi/2$$

$$\Rightarrow (4n+1)\pi/2 \leq \sqrt{2}p$$

For p to be least, n should be least.

$$\Rightarrow n = 0$$

$$\Rightarrow \sqrt{2}p \geq \pi/2$$

$$\Rightarrow p \geq \frac{\pi}{2\sqrt{2}}$$

Therefore, least value of $p = \frac{\pi}{2\sqrt{2}}$

Sol 6: Given,

$$(1 - \tan^2 \theta)(1 + \tan^2 \theta) \sec^2 \theta + 2^{\tan^2 \theta} = 0$$

$$\Rightarrow (1 - \tan^2 \theta) \cdot (1 + \tan^2 \theta) + 2^{\tan^2 \theta} = 0$$

$$\Rightarrow 1 - \tan^4 \theta + 2^{\tan^2 \theta} = 0$$

Substitute $\tan^2 \theta = x$

$$\therefore 1 - x^2 + 2^x = 0$$

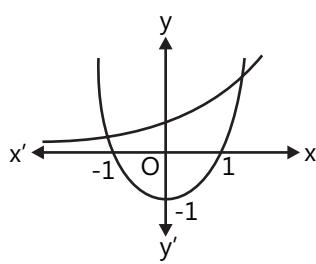
$$\Rightarrow x^2 - 1 = 2^x$$

Note: 2^x and $x^2 - 1$ are incompatible functions, therefore, we have to consider range of both functions.

Curves $y = x^2 - 1$

And $y = 2^x$

It is clear from the graph that two curves intersect at one point at $x = 3, y = 8$.



$$\text{Therefore, } \tan^2 \theta = 3$$

$$\Rightarrow \tan \theta = \pm \sqrt{3}$$

$$\Rightarrow \theta = \pm \frac{\pi}{3}$$

Sol 7: (A, C) Given,

$$\begin{vmatrix} 1 + \sin^2 \theta & \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & 1 + \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0$$

Applying

$R_3 \rightarrow R_3 - R_1$ and $R_2 \rightarrow R_2 - R_1$ we get

$$\begin{vmatrix} 1 + \sin^2 \theta & \cos^2 \theta & 4 \sin 4\theta \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 + C_2$

$$\Rightarrow \begin{vmatrix} 2 & \cos^2 \theta & 4 \sin 4\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 2 + 4 \sin 4\theta = 0$$

$$\Rightarrow \sin 4\theta = -\frac{1}{2}$$

$$\Rightarrow 4\theta = n\pi + (-1)^n \left(-\frac{\pi}{6}\right)$$

$$\Rightarrow 4\theta = \frac{n\pi}{4} + (-1)^{n+1} \left(\frac{\pi}{24}\right)$$

Clearly, $\theta = \frac{7\pi}{24}, \frac{11\pi}{24}$ are two values of θ lying between 0° and $\frac{\pi}{2}$

Sol 8: (B, C) For $0 < \phi < \pi/2$ we have

$$x = \sum_{n=0}^{\infty} \cos^{2n} \phi = 1 + \cos^2 \phi$$

$$+ \cos^4 \phi + \cos^6 \phi + \dots$$

It is clearly a GP with common ratio of $\cos^2 \phi$ which is ≤ 1 .

$$\text{Hence, } x = \frac{1}{1 - \cos^2 \phi} = \frac{1}{\sin^2 \phi}$$

$$\left[\because S_{\infty} = \frac{a}{1-r}, -1 < r < 1 \right]$$

$$\text{Similarly, } y = \frac{1}{\cos^2 \phi}$$

$$\text{And } z = \frac{1}{1 - \sin^2 \phi \cos^2 \phi}$$

$$\text{Now, } x+y = \frac{1}{\sin^2 \phi} + \frac{1}{\cos^2 \phi}$$

$$= \frac{\cos^2 \phi + \sin^2 \phi}{\cos^2 \phi \sin^2 \phi} = \frac{1}{\cos^2 \phi \sin^2 \phi}$$

$$\text{Again, } \frac{1}{x} = 1 - \sin^2 \phi \cos^2 \phi = 1 - \frac{1}{xy}$$

$$\Rightarrow \frac{1}{x} = \frac{xy-1}{xy} \Rightarrow xy = xyz - z$$

$$\Rightarrow xy + z = xyz$$

Therefore, (b) is the answer from eq. (i) (putting the value of xy)

$$\Rightarrow xyz = x + y + z$$

Sol 9: (A, B) We know that, $\sec^2 \theta \geq 1$

$$\Rightarrow \frac{4xy}{(x+y)} \geq 1$$

$$\Rightarrow 4xy \geq (x+y)^2$$

$$\Rightarrow (x+y)^2 - 4xy \leq 0$$

$$\Rightarrow (x-y)^2 \leq 0$$

$$\Rightarrow x-y = 0$$

$$\Rightarrow x = y$$

Therefore, $x + y = 2x$ (add x both sides)

But $x + y \neq 0$ since it lies in the denominator,

$$\Rightarrow 2x \neq 0$$

$$\Rightarrow x \neq 0$$

Hence, $x = y, x \neq 0$ is the answer.

Sol 10: (A, B) $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$

$$\Rightarrow \frac{3 \sin^4 x + 2 \cos^4 x}{6} = \frac{1}{5}$$

$$\Rightarrow \sin^4 x + 2(\sin^4 x + \cos^4 x) = \frac{6}{5}$$

$$\Rightarrow \sin^4 x + 2[1 - 2 \sin^2 x (\cos^2 x)] = \frac{6}{5}$$

$$\Rightarrow \sin^4 x + 2 - 4 \sin^2 x (1 - \sin^2 x) = \frac{6}{5}$$

$$\Rightarrow \sin^4 x + 2 - 4 \sin^2 x + 4 \sin^4 x = \frac{6}{5}$$

$$\Rightarrow 5 \sin^4 x - 4 \sin^2 x + \frac{4}{5} = 0$$

$$\Rightarrow \sin^2 x = \frac{4 \pm \sqrt{16 - 4 \times 5 \times \frac{4}{5}}}{10}$$

$$= \frac{4 \pm \sqrt{16 - 16}}{10}$$

$$\Rightarrow \sin^2 x = \frac{2}{5}$$

$$\Rightarrow \sin x = \pm \sqrt{\frac{2}{5}}$$

$$\sin^2 x = \frac{2}{5}$$

$$\Rightarrow \cos^2 x = \frac{3}{5}$$

$$\Rightarrow \sec^2 x = \frac{5}{3}$$

$$\Rightarrow \tan^2 x = \sec^2 x - 1$$

$$= \frac{5}{3} - 1 = \frac{2}{3}$$

$$\text{Now, } \frac{\sin^8 x}{8} + \frac{\cos^8 x}{27}$$

$$= \frac{\left(\frac{2}{5}\right)^4}{8} + \frac{\left(\frac{3}{5}\right)^4}{27} = \frac{2}{5^4} + \frac{3}{5^4} = \frac{1}{125}$$

Sol 11: (D)

$$\sum_{m=1}^6 \cosec \left[\theta + \frac{m\pi}{4} - \frac{\pi}{4} \right] \cosec \left(\theta + \frac{m\pi}{4} \right) = 4\sqrt{2}$$

$$\sum_{m=1}^6 \sqrt{2} \times \frac{\sin \left[\left(\theta + \frac{m\pi}{4} \right) - \left(\theta + \frac{m\pi}{4} - \frac{\pi}{4} \right) \right]}{\sin \left(\theta + \frac{m\pi}{4} \right) \sin \left(\theta + \frac{m\pi}{4} - \frac{\pi}{4} \right)} = 4\sqrt{2}$$

$$\sum_{m=1}^6 \frac{\sin \left(\theta + \frac{m\pi}{4} \right) \cos \left(\theta + \frac{m\pi}{4} - \frac{\pi}{4} \right) - \cos \left(\theta + \frac{m\pi}{4} \right) \sin \left(\theta + \frac{m\pi}{4} - \frac{\pi}{4} \right)}{\sin \left(\theta + \frac{m\pi}{4} \right) \sin \left(\theta + \frac{m\pi}{4} - \frac{\pi}{4} \right)}$$

$$\sum_{m=1}^6 \sqrt{2} \times \left[\cot \left(\theta + \frac{m\pi}{4} - \frac{\pi}{4} \right) - \cot \left(\theta + \frac{m\pi}{4} \right) \right] = 4\sqrt{2}$$

$$\sqrt{2} \left[\cot \theta - \cot \left(\theta + \frac{3\pi}{2} \right) \right] = 4\sqrt{2}$$

$$\Rightarrow \cot \theta - \cot \left(\frac{3\pi}{2} + \theta \right) = 4$$

$$\Rightarrow \cot \theta - \tan \theta = 4$$

$$\Rightarrow \tan^2 \theta - 4 \tan \theta + 1 = 0$$

$$\Rightarrow \tan \theta = \frac{4 \pm \sqrt{16 - 4}}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$\Rightarrow \tan \theta = \pm 2\sqrt{3}$$

$$\Rightarrow \theta = \frac{5\pi}{12}$$

Sol 12: Given, $\tan \theta = \cot 5\theta$

$$\Rightarrow \tan \theta = \tan \left(\frac{\pi}{2} - 5\theta \right)$$

$$\Rightarrow \frac{\pi}{2} - 5\theta = n\pi + \theta$$

$$\Rightarrow 6\theta = \frac{\pi}{2} - n\pi$$

$$\Rightarrow \theta = \frac{\pi}{12} - \frac{n\pi}{6}$$

$$\text{Also, } \cos 4\theta = \sin 2\theta = \cos \left(\frac{\pi}{2} - 2\theta \right)$$

$$\Rightarrow 4\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta \right)$$

Taking positive Δ sign

$$\Rightarrow 6\theta = 2n\pi + \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{12}$$

Taking negative sign

$$\Rightarrow 2\sin \frac{2\pi}{n} \cdot \cos \frac{2\pi}{n} = \sin \frac{3\pi}{n}$$

$$\Rightarrow \sin \frac{4\pi}{n} = \sin \frac{3\pi}{n}$$

$$\Rightarrow 2\theta = 2n\pi - \frac{\pi}{2}$$

$$\Rightarrow \theta = n\pi - \frac{\pi}{4}$$

Above values of θ suggest that there are only 3 common solutions.

Sol 13: Given, $n > 3 \in \text{Integer}$

$$\text{and } \frac{1}{\sin \left(\frac{\pi}{n} \right)} = \frac{1}{\sin \left(\frac{2\pi}{n} \right)} + \frac{1}{\sin \left(\frac{3\pi}{n} \right)}$$

$$\Rightarrow \frac{1}{\sin \frac{\pi}{n}} - \frac{1}{\sin \frac{3\pi}{n}} = \frac{1}{\sin \frac{2\pi}{n}}$$

$$\Rightarrow \frac{\sin \frac{3\pi}{n} - \sin \frac{\pi}{n}}{\sin \frac{\pi}{n} \cdot \sin \frac{3\pi}{n}} = \frac{1}{\sin \frac{2\pi}{n}}$$

$$\Rightarrow 2\cos \left(\frac{2\pi}{n} \right) \cdot \sin \frac{\pi}{n} = \frac{\sin \frac{\pi}{n} \cdot \sin \frac{3\pi}{n}}{\sin \frac{2\pi}{n}}$$

$$\Rightarrow \frac{4\pi}{n} = \pi - \frac{3\pi}{n} \quad \Rightarrow \frac{7\pi}{n} = \pi; \Rightarrow n = 7$$

$$\text{Sol 14: } (y+z) \cos 3\theta = xyz \sin 3\theta \quad \dots(i)$$

$$xyz \sin 3\theta = 2z \cos 3\theta + 2y \sin 3\theta \quad \dots(ii)$$

$$xyz \sin 3\theta = (y+2z) \cos 3\theta + y \sin 3\theta \quad \dots(iii)$$

From (i), (ii) and (iii), we get

$$(y+z) \cos 3\theta = 2z \cos 3\theta + 2y \sin 3\theta \\ = (y+2z) \cos 3\theta + y \sin 3\theta$$

$$\Rightarrow y(\cos 3\theta - 2\sin 3\theta) = z \cos 3\theta \text{ and}$$

$$y(\cos 3\theta - \sin 3\theta) = 0$$

$$\Rightarrow \cos 3\theta = \sin 3\theta$$

$$\Rightarrow \tan 3\theta = 1 = \tan \frac{\pi}{4}$$

$$\Rightarrow 3\theta = n\pi + \frac{\pi}{4}$$

$$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{12}$$

$$\Rightarrow \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$$

Total three solution in $(0, \pi)$

Sol 15: $\tan \theta = \cot 5\theta$

$$\tan \theta = \tan \left(\frac{\pi}{2} - 5\theta \right)$$

$$\Rightarrow \theta = n\pi + \frac{\pi}{2} - 5\theta$$

$$\Rightarrow 6\theta = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{n\pi}{6} + \frac{\pi}{12} \Rightarrow \theta = \pm \frac{\pi}{12}, \frac{\pm\pi}{4}, \frac{\pm 5\pi}{12}, \dots$$

$$\sin 2\theta = \cos 4\theta$$

$$\Rightarrow \cos 4\theta = \cos \left(\frac{\pi}{2} - 2\theta \right)$$

$$\Rightarrow 4\theta = 2m\pi \pm \left(\frac{\pi}{2} - 2\theta \right)$$

$$\Rightarrow \theta = \frac{m\pi}{3} + \frac{\pi}{12}, m\pi - \frac{\pi}{4}$$

$$\Rightarrow \theta = \frac{\pi}{12}, -\frac{\pi}{4}, \frac{5\pi}{12}$$

....(i)

In (i) and (ii) only (iii) solutions are common

$$-\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$$

....(ii)

Sol 16: Given

$$\frac{1}{\sin^2 \theta + 3\sin \theta \cos \theta + 5\cos^2 \theta}$$

Let

$$\begin{aligned} A &= \sin^2 \theta + 3\sin \theta \cos \theta + 5\cos^2 \theta \\ &= 1 + 4\cos^2 \theta + 3\sin \theta \cos \theta \end{aligned}$$

$$= 1 + 2(1 + \cos 2\theta) + \frac{3}{2} \sin 2\theta$$

$$= 3 + 2\cos 2\theta + \frac{3}{2} \sin 2\theta$$

$$\left[a \cos \theta + b \sin \theta \geq -\sqrt{a^2 + b^2} \right]$$

$$A_{\min} = 3 - \sqrt{2^2 + \left(\frac{3}{2}\right)^2}$$

$$= 3 - \sqrt{4 + \frac{9}{4}}$$

$$= 3 - \sqrt{\frac{25}{4}} = 3 - \frac{5}{2} = \frac{1}{2}$$

The maximum value of given expression

$$\frac{1}{\sin^2 \theta + 3\sin \theta \cos \theta + 5\cos^2 \theta} \text{ is } \frac{1}{1/2} = 2$$

Sol 17: (D) $P = \left\{ \theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta \right\}$

$$\Rightarrow \sin \theta = (\sqrt{2} + 1) \cos \theta$$

$$\Rightarrow \tan \theta = \sqrt{2} + 1$$

$Q = \left\{ \theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta \right\}$

$$\Rightarrow \cos \theta = (\sqrt{2} - 1) \sin \theta$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$$

$$\therefore P = Q$$

$$\text{Sol 18: } \frac{1}{\sin \frac{\pi}{n} \sin \frac{2\pi}{n} \sin \frac{3\pi}{n}} = \frac{1}{\sin \frac{\pi}{n}} + \frac{1}{\sin \frac{3\pi}{n}}$$

$$\Rightarrow \frac{\sin \frac{3\pi}{n} - \sin \frac{\pi}{n}}{\sin \frac{\pi}{n} \sin \frac{3\pi}{n}} = \frac{1}{\sin 2\frac{\pi}{n}}$$

$$\Rightarrow \frac{2 \cos \frac{\left(\frac{3\pi}{n} + \frac{\pi}{n}\right)}{2} \sin \frac{\left(\frac{3\pi}{n} - \frac{\pi}{n}\right)}{2}}{\sin \frac{\pi}{n} \sin \frac{3\pi}{n}} = \frac{1}{\sin \frac{2\pi}{n}}$$

$$\Rightarrow \frac{2 \cos \frac{2\pi}{n} \sin \frac{\pi}{n}}{\sin \frac{\pi}{n} \sin \frac{3\pi}{n}} = \frac{1}{\sin 2\frac{\pi}{n}}$$

$$= 2 \sin \frac{2\pi}{n} \cos \frac{2\pi}{n} = \sin \frac{3\pi}{n}$$

$$\Rightarrow \sin \frac{4\pi}{n} = \sin \frac{3\pi}{n}$$

$$\Rightarrow \sin \frac{4\pi}{n} - \sin \frac{3\pi}{n} = 0$$

$$\Rightarrow 2 \cos \frac{7\pi}{2n} \sin \frac{\pi}{2n} = 0$$

$$\Rightarrow \cos \frac{7\pi}{2n} = 0 \text{ or } \sin \frac{\pi}{2n} = 0$$

$$\Rightarrow n = \frac{7}{2m+1}$$

For integral values of $n > 3$

Sol 19: (A, C, D) $\theta \in [0, 2\pi]$

$$2 \cos \theta (1 - \sin \phi) = \sin^2 \theta \left(\tan \frac{\theta}{2} + \cot \frac{\theta}{2} \right) \cos \phi - 1$$

$$\Rightarrow 2 \cos \theta (1 - \sin \phi) = \frac{\sin^2 \theta \left(\tan^2 \frac{\theta}{2} + 1 \right)}{\tan \frac{\theta}{2}} \cos \phi - 1$$

$$\Rightarrow 2 \cos \theta (1 - \sin \phi) = \frac{\sin^2 \theta \cdot \sec^2 \frac{\theta}{2}}{\tan \frac{\theta}{2}} \cos \phi - 1$$

$$\Rightarrow 2 \cos \theta (1 - \sin \phi) = 2 \sin \theta (\cos \phi) - 1$$

$$\Rightarrow 1 + 2 \cos \theta = 2 \sin (\theta + \phi) \quad \dots \text{(i)}$$

$$\text{Now, given } \tan(2\pi - \theta) > 0, -1 < \sin \theta < -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \tan \theta < 0 \text{ and } \frac{3\pi}{2} < \theta < \frac{5\pi}{3}$$

$$\Rightarrow \frac{3\pi}{2} < \theta < \frac{5\pi}{3}$$

$$\text{From (i), as } \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3} \right)$$

$$\Rightarrow 1 < 2 \cos \theta + 1 < 2$$

$$\Rightarrow 1 < 2 \sin (\theta + \phi) < 2$$

$$\Rightarrow \frac{1}{2} < \sin (\theta + \phi) < 1 \Rightarrow \theta + \phi \in \left(\frac{\pi}{6}, \frac{5\pi}{6} \right), \theta + \phi \neq \frac{\pi}{2}$$

Or

$$\theta + \phi \in \left(\frac{13\pi}{6}, \frac{17\pi}{6} \right), \theta + \phi \neq \frac{5\pi}{2} \Rightarrow \frac{\pi}{6} - \theta < \phi < \frac{5\pi}{6} - \theta$$

$$\text{or } \frac{13\pi}{6} - \theta < \phi < \frac{17\pi}{6} - \theta$$

$$\text{As, } \theta \in \left(-\frac{3\pi}{2}, -\frac{2\pi}{3} \right) \text{ or } \left(\frac{2\pi}{3}, \frac{7\pi}{6} \right)$$

Sol 20: (D) $\sin x + 2 \sin 2x - \sin 3x = 3$

$$\Rightarrow \sin x - \sin 3x + 2 \sin 2x = 3$$

$$\Rightarrow 2 \cos 2x \sin(-x) + 2 \sin 2x = 3$$

$$\Rightarrow -2 \sin x [2 \cos^2 x - 1] + 2 \times 2 \sin x \cos x = 3$$

$$\Rightarrow -4 \sin x \left[\cos^2 x - \frac{1}{2} - \cos x \right] = 3$$

$$\Rightarrow -4 \sin x \left[\left(\cos x - \frac{1}{2} \right)^2 - \frac{3}{4} \right] = 3$$

$$\Rightarrow \left(\cos x - \frac{1}{2} \right)^2 - \frac{3}{4} = -\frac{3}{4 \sin x}$$

$$\Rightarrow \left(\cos x - \frac{1}{2} \right)^2 = \frac{3}{4} - \frac{3}{4 \sin x}$$

$$\Rightarrow \left(\cos x - \frac{1}{2} \right)^2 = \frac{3}{4} \left(1 - \frac{3}{\sin x} \right)$$

L.H.S. > 0 and R.H.S. < 0

∴ No solution

$$\text{Sol 21: } \frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

$$\Rightarrow \frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^4 x$$

$$[1 - \sin^2 x] + \sin^4 x [1 - \cos^2 x] = 2$$

$$\Rightarrow \frac{5}{4} \cos^2 2x + 2(\cos^4 x + \sin^4 x) - \sin^2 x \cos^2 x$$

$$(\sin^2 x + \cos^2 x) = 2$$

$$\Rightarrow \frac{5}{4} \cos^2 2x + 2[(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x]$$

$$-\sin^2 x \cos^2 x = 2$$

$$\Rightarrow \frac{5}{4} \cos^2 2x - 5 \sin^2 x \cos^2 x = 0$$

$$\Rightarrow \frac{5}{4} \cos^2 2x = \frac{5}{4} \sin^2 2x$$

$$\Rightarrow \tan^2 2x = 1$$

$$\Rightarrow \tan 2x = \pm = \tan \left(\pm \frac{\pi}{4} \right)$$

$$\Rightarrow 2x = n\pi \pm \frac{\pi}{4}$$

$$\Rightarrow x = \frac{n\pi}{2} \pm \frac{\pi}{8}$$

$$\Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \rightarrow$$

Total 8 solution.

$$\text{Sol 22: (C)} \quad \sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$$

$$\Rightarrow \frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x} + 2 \left[\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} \right] = 0$$

$$\Rightarrow \sqrt{3} \sin x + \cos x + 2 \left[\sin^2 x - \cos^2 x \right] = 0$$

$$\Rightarrow \sqrt{3} \sin x + \cos x = 2 \cos 2x$$

$$\Rightarrow \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = \cos 2x$$

$$\Rightarrow \sin \frac{\pi}{3} \sin x + \cos \frac{\pi}{3} \cos x = \cos 2x$$

$$\Rightarrow \cos \left(x - \frac{\pi}{3} \right) = \cos 2x$$

$$\Rightarrow x - \frac{\pi}{3} = 2n\pi \pm 2x$$

$$\Rightarrow x = - \left(2n\pi + \frac{\pi}{3} \right) \text{ or } x = \frac{2n\pi}{3} + \frac{\pi}{9}$$

$$\Rightarrow x = -\frac{\pi}{3}, \frac{\pi}{9}, -\frac{5\pi}{9}, \frac{7\pi}{9}$$

Sum = 0