21. MOVING CHARGES AND MAGNETISM

1. INTRODUCTION

In the previous chapters on electrostatics and current electricity, we have studied about the electric force and electric field. Another important property associated with moving charges is the magnetic force and the magnetic field. The current flowing in a conductor produces a magnetic field and any charge moving in this field will experience a magnetic force which will depend on the velocity (both magnitude and direction) as well as on some property of the field. We will study the properties and laws governing the magnetic field and magnetic force in detail in this chapter.

There are a wide variety of industrial and medical applications of magnetic fields and forces. Common example, is the use of electromagnet to lift heavy pieces of metal. Magnets are used in CD and DVD players, computer hard drives,loud speakers, headphones, TVs, and telephones. We are surrounded by magnets. Right from our doorbells to cars to security alarm systems and in our hospitals, magnets are being used everywhere.

2. LORENTZ FORCE: DEFINITION OF MAGNETIC FIELD B

If electric field and magnetic field occur simultaneously in a region then the force acting on a point charge q in the region will depend both on the position of the charge as well as on its velocity. The force $\vec{F}$ will have two components, viz. the electric force $\vec{F}_e$ and magnetic force $\vec{F}_m$. The force $\vec{F}_e$ does not depend on the motion of the charge, but only on its position, while $\vec{F}_m$ depends both on charge's velocity and position (see Fig. 21.1). The magnitude of $\vec{F}_e$ is qE and direction is along $\vec{E}$ (q is positive).

To know the direction and magnitude of $\vec{F}_m$ we introduced a vector $\vec{B}$ called magnetic flux density ormagnetic induction, which characterizes the magnetic field at a particular point. Experiments show that the force $\vec{F}_m$ isproportional to the magnitude of charge q, to the velocity $\vec{v}$ of the charge and the magnitude of density $\vec{B}$, this force being always perpendicular to vector $\vec{v}$ as well as vector $\vec{B}$. Also, if the charge moves along the direction of $\vec{B}$ at a point then the magnetic force on it is zero. We can summarize all these experimental results with the following vector equation:

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

That is, the force $\vec{F}_m$ on the point charge is equal to the

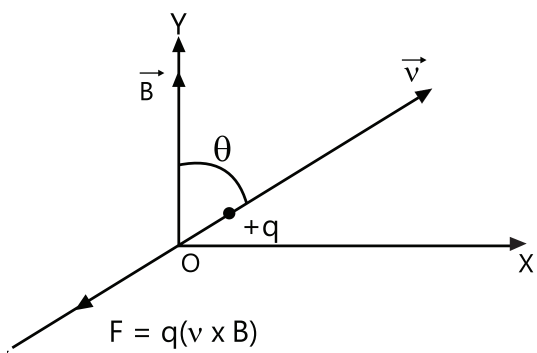

Figure 21.1: Magnetic Force on a Moving Charge

charge q times the cross product of its velocity $\vec{v}$ and the field $\vec{B}$ (all measured in the same reference frame). Using formula for the magnitude of cross product, we can write the magnitude of $\vec{F}_m$ as $F_m = |q| vB \sin\theta$ where θ is the angle between the velocity $\vec{v}$ and magnetic field $\vec{B}$.

If angle θ is 90°, then the above relation for magnetic force can be used to define the magnitude of magnetic flux density B as,

$$B = \frac{F_m}{|q| v_\perp}$$

where $v_\perp$ is the velocity component perpendicular to vector $\vec{B}$.

Thus, the total electromagnetic force acting on charge q is given as, $\vec{F} = \vec{F}_e + \vec{F}_m$

or $\quad \vec{F} = q\vec{E} + q[\vec{v} \times \vec{B}]$

This is called Lorentz force.

The unit of B is Tesla abbreviated as T. If q=1 C, v=1 ms^{-1}, sin θ=1 for θ = 90°, and F_m = 1 N,then B=1 T = 1 Weber-m^{-2}. Thus 1 Tesla is defined as the unit of magnetic field strength in S.I units which when acting on 1 C of charge moving with a velocity of 1 ms^{-1} at right angles to the magnetic field exerts a force of 1 N in a direction perpendicular to that of field and velocity vectors. C.G.S. units of magnetic field strength or magnetic induction is 1 gauss or 1 oersted. 1 gauss = 1 oersted= 10^{-4}T.

Illustration 1: A 2 MeV proton is moving perpendicular to uniform magnetic field of 2.5 T. What is the magnetic force on the proton? (Mass of proton = 1.6 x 10^{-27} kg) **(JEE MAIN)**

Sol: Kinetic energy of proton is $K.E. = \frac{m_p v^2}{2}$. 1 MeV=1.6 x 10^{-13}J.

K.E = 2 MeV = 2 x 1.6 x 10^{-13}J or $\frac{1}{2}mv^2$= 3.2 x 10^{-13}J

$$\therefore V = \sqrt{\frac{2x3.2x10^{-13}}{m}} = \sqrt{\frac{2x3.2x10^{-13}}{1.6x10^{-27}}} = 2 \times 10^7 \text{ m s}^{-1}$$

Now, magnetic force on proton, F= ev B = 1.6 x 10^{-19} x 2 x 10^7 x 2.5 = 8.0 x 10^{-12} N

Illustration 2: A charged particle is projected in a magnetic field $\vec{B} = (3\hat{i} + 4\hat{j})x10^{-2}$ T

The acceleration of the particle is found to be, $\vec{a} = (x\hat{i} + 2\hat{j})\ ms^{-2}$ Find the value of x. **(JEE MAIN)**

Sol: Magnetic force on a moving charge is perpendicular to the magnetic field. Therefore the dot product of force and magnetic field vector is zero.

As we have read $\vec{F}_m \perp \vec{B}$ i.e., the acceleration $\vec{a} \perp \vec{B}$ or $\vec{a} \cdot \vec{B} = 0$

or $(x\hat{i} + 2\hat{j}) \cdot (3\hat{i} + 4\hat{j})x10^{-2} = 0$; $(3x+8)x10^2=0$ $\quad \therefore\ x = -\frac{8}{3} ms^{-2}$

3. RELATION BETWEEN ELECTRIC AND MAGNETIC FIELD

Suppose in a particular inertial reference frame K, the electric field is zero and the magnetic field has a non-zero finite value. A point charge is moving with some velocity $\vec{v}$ in the frame K and thus experiences a magnetic force, and its velocity changes. Now suppose we have a frame K' translating with respect to frame K withconstantvelocity $\vec{v}$. In the frame K',the point charge is initially at rest, and so the magnetic force on it will be zero. Butas its velocity changes in the K frame, its velocity changes in the K' frame as well, i.e. it experiences a force in K' frame as well.

This initial force on it is the force $\vec{F}_e$ due to electric field in the K′ frame.Thus the magnetic field in K frame appears as a combination of electric field and magnetic field in K′ frame. The electric and magnetic fields are thus inter-dependent. We introduce a single physical entity called electromagnetic field. Whether the electromagnetic field will appear as electric field or magnetic field depends on the frame of reference. If we confine to a particular reference frame, we can treat electric fieldand magnetic field as separate entities. A field which is constant in one reference frame in the general case is found to vary in another reference frame.

4. MAGNETIC FIELD LINES

Magnetic field lines are used to represent the magnetic field in a region. The rules to construct the magnetic field lines are:-

(a) The direction of tangent to a magnetic field line at a point gives the direction of magnetic flux density vector $\vec{B}$ at that point.

(b) The density of the magnetic field lines at a point isproportional to the magnitude of vector $\vec{B}$ at that point. At points where the field lines are closer together, the magnetic field is stronger.

MASTERJEE CONCEPTS

- In case of a bar magnet, the density of magnetic field lines is high at points near the poles, and the density at pointsnear the center of the magnet is low.
- If we place a magnetic compass at any point in the earth's magnetic field, it will align itself in the direction of the magnetic field lines.

Vaibhav Krishnan (JEE 2009 AIR 22)

MASTERJEE CONCEPTS

- Common misconception about magnetic field lines is that it is the path followed by a magnetic north pole in a magnetic field.
- This is not correct. It is the instantaneous direction of the magnetic force acting on the magnetic north pole in the magnetic field.

Vaibhav Gupta (JEE 2009 AIR 54)

5. EARTH'S MAGNETIC FIELD

Magnetic field is present everywhere near the earth's surface. The line of earth's magnetic field lies in a vertical plane coinciding with the magnetic north-south direction at that place i.e. the plane passing through the geomagnetic poles. This plane is called the Magnetic Meridian. This plane is slightly inclined to the plane passing through the geographic poles called the geographic meridian.The angle between the magnetic meridian and the geographic meridian at a point is called the declination at that point. The earth's magnetic poles are opposite to the geographic poles i.e. at earth's North Pole, its magnetic south pole is situated and vice versa.

In the magnetic meridian plane, the magnetic field vector of the earth at any point, is generally inclined to the horizontalat that pointby an angle called the magneticdip at that point. If magnetic field of the earth at that point is B and the dip is θ,

B_v = the vertical component of B in the magnetic meridian plane = B sinθ

B_H = the horizontal component of B in the magnetic meridian plane = B cosθ.

$$\frac{B_V}{B_H} = \tan\theta$$

6. MOTION OF CHARGED PARTICLE IN ELECTRIC AND MAGNETIC FIELD

6.1 Trajectory of a Charged Particle Moving in Uniform Electric Field

Let a positively charged particle having charge +q and mass m enter at origin O with velocity v along X-direction in the region where electric field Eis along the Y-direction (see Fig. 21.2).

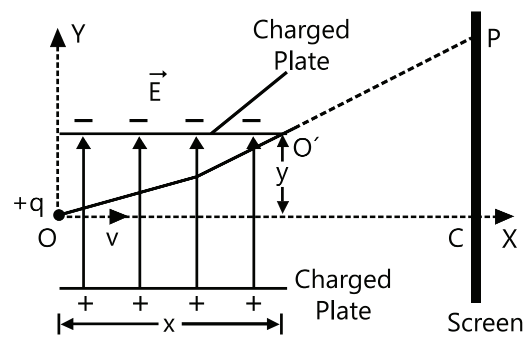

Figure 21.2: Charged particle moving in electric field

Force acting on the charge +q due to electric field E is given by

$$\vec{F} = q\vec{E}$$

Acceleration of the charged particle is $\vec{a} = \frac{\vec{F}}{m}$ or $\vec{a} = \frac{q\vec{E}}{m}$ ...(i)

The charged particle will accelerate in the direction of $\vec{E}$ and get deflected from its straight line path.

During its motion in the region of electric field, along x-axis we have u_x=v and a_x= 0 and x = vt

or $t = \frac{x}{v}$ ...(ii)

Along y axis we have, $u_y = 0$, $a_y = \frac{qE}{m}$ (∴ Initially the particle was moving along x-direction)

$$y = \frac{1}{2}a_y t^2$$

∴ $$y = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

Using Eq, (ii), we get $y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{v}\right)^2$ or $y = \frac{qEx^2}{2mv^2} = Kx^2$ ...(iii)

where $K = \frac{qE}{2mv^2}$ is a constant.Thus the charged particle moves along a parabolic trajectory.

6.2 Trajectory of a Charged Particle Movingin Uniform Magnetic Field

(a) Magnetic force acting on a charged particle moving with velocity $\vec{v}$ parallel (θ=0) or antiparallel (θ=180°) to $\vec{B}$, will be zero. Thus the trajectory of the particle is a straight line.

(b) If velocity $\vec{v}$ of the particle is perpendicular to $\vec{B}$ i.e. θ =90°, then magnetic force is F= qvB and the direction of this force is always perpendicular to v. The charged particle moves in a circular trajectory (see Fig. 21.3).

(c) If velocity $\vec{v}$ of the charged particle makes an angle θ with $\vec{B}$, the particle moves in a helical path. The component v sinθ which is perpendicular to $\vec{B}$ drives the charged particle along a circular path whereas the component v cosθ, which is parallel or antiparallel to $\vec{B}$, remains unchanged as there is no magnetic force along the direction of $\vec{B}$. Thus the charged particle moves along a helical path (see Fig. 21.4).

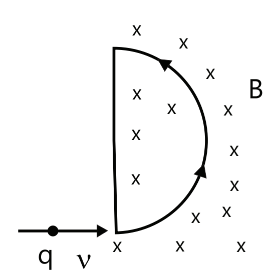

Figure 21.3: Charged particle moving in uniform magnetic field in electric field

(d) The magnetic force on the component of velocity perpendicular to the magnetic field provides the centripetal force to the charged particle to follow a circular trajectory of radius r.

$$qv_{\perp}B = \frac{mv_{\perp}^2}{r}$$

or $r = \frac{mv_{\perp}}{qB}$

Angular velocity, $\omega = \frac{v_{\perp}}{r} = \frac{qB}{m}$

Frequency $f = \frac{qB}{2\pi m}$

Time period $T = \frac{2\pi m}{qB}$

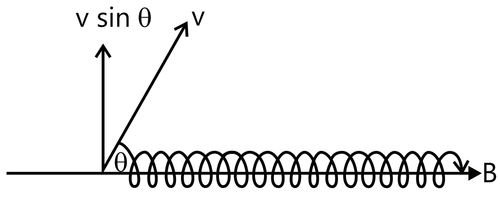

Figure 21.4: Charged particle moving in helical path in uniform magnetic field

Time period T is independent of v.

7.DISCOVERY OF ELECTRON

The Fig. 21.5 shows the simplified version of Thomson's' experiment. An electric field $\vec{E}$ is established in the region between the deflecting plates by connecting a battery across their terminals. The magnetic field $\vec{B}$ in the region between the deflecting plates is directed into the plane of the figure.

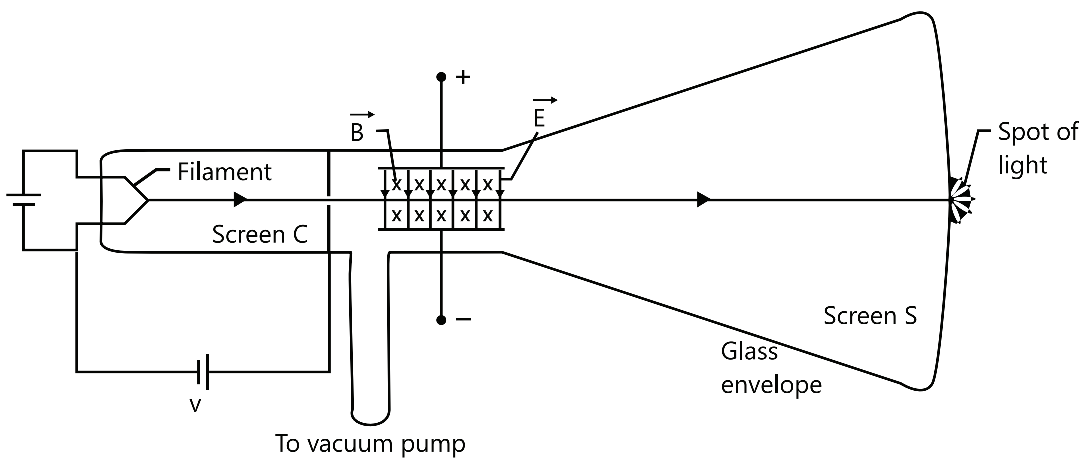

Figure 21.5: Thomson's experimental set up

Charged particles (electrons) are emitted by a hot filament at the rear of the evacuated cathoderay tube and are accelerated by an applied potential difference V. After they pass through a slit in screen C, they form a narrow beam. They then pass through the region between the deflecting plates, headed towards the center of fluorescent screen S, where they produce a spot of light. The crossed-fields $\vec{E}$ and $\vec{B}$ in the region between the deflecting plates can deflect them from the center of the screen. By controlling the magnitude and directions of the fields, $\vec{E}$ and $\vec{B}$ the deflection of the charged particles can be controlled.

When both the fields $\vec{E}$ and $\vec{B}$ are turned-off the beam of charged particles reaches the screen un-deflected.

When field $\vec{E}$ is turned-on the beam of charged particles is deflected.

Keeping the field $\vec{E}$ unchanged, field $\vec{B}$ is also turned-on.The magnitude of $\vec{B}$ is adjusted such that the deflection of the charged particles becomes zero. In this situation the electric force on the charged particles is balanced by the magnetic force.

$$q\vec{E} = -q\,\vec{v}\times\vec{B}$$

or $$\vec{E} = -\vec{v}\times\vec{B}$$

The ratio of magnitudes of $\vec{E}$ and $\vec{B}$ in this situation gives the speed of the charged particles.

$$v = \frac{E}{B}$$

When only field $\vec{E}$ is turned-on, the displacementof the charged particlesin the y-direction, when they reach the end of the plates, as derived in article 6.1 is

$$y = \frac{|q|EL^2}{2mv^2}$$

where v is the particle's speed along x-direction, mits mass, qits charge, and L is the length of the plates. The direction of deflection of charged particles show that the particles are negatively charged.

Substituting the value of v in terms of E and B we get,

$$y = \frac{|q|B^2L^2}{2mE}$$

or $$\frac{m}{|q|} = \frac{B^2L^2}{2yE}$$

Thus in this way the mass to charge ratio of electrons was discovered.

MASTERJEE CONCEPTS

Charged particle motion as a points on wheel

- 1. Suppose electric and magnetic field are perpendicular to each other and a charged particle is projected perpendicular to magnetic field, its motion can be assumed as that of the motion of a particle on a wheel
- 2. The point could be inside, on or outside the wheel depending on the problem
- 3. Suppose in this field it is projected in any other way (expect along the magnetic field) its horizontal motion is still like that of a point on a wheel, while vertical motion will be uniform velocity motion
- 4. To such problem, just resolve the particle velocity in to along the magnetic field and perpendicular to it
- 5. If electric field is not perpendicular, resolve it also into along and perpendicular to magnetic field and solve accordingly.

Nitin Chandrol (JEE 2012 AIR 134)

8. HALL EFFECT

The Hall Effect is the production of a voltage difference (the Hall voltage) across acurrent carrying conductor, lying in a magnetic field perpendicular to the current. The hall voltage is produced in the direction transverse to the electric current in the conductor. It was discovered by Edwin Hall in 1879.Hall Effect allows us to find out whether the charge carries in a conductor are positively or negatively charged and the number of charge carries per unit volume of the conductor.

External magnetic field $\vec{B}$, points into the plane of a copper strip of width d, carrying a current I as shown in Fig. 21.6.The magnetic force $\vec{F}_m$ will act on each drifting electron, towards the right edge of the strip. As the electrons accumulate on the right edge, positive charges are induced on the left edge and an electric field $\vec{E}$ is produced within the strip, directed from left to right.This field exerts an electric force $\vec{F}_e$ on each electron, towards the left edge of the strip.The hall potential difference V across the width of the strip,due to the electric field $\vec{E}$ isV=Ed.

When the electric and magnetic forces balance each other, $eE=ev_dB$ or $E = v_d B$

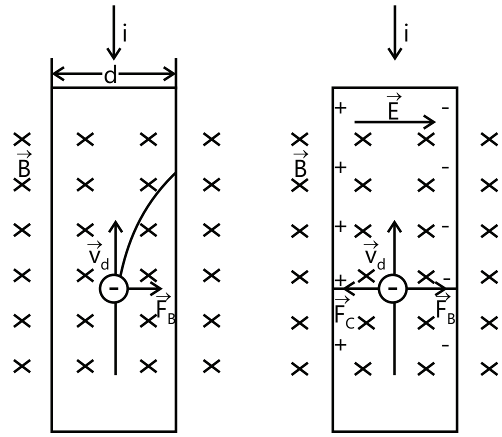

Figure 21.6: Hall Effect in conductor

Thedrift speed v_d is given as $v_d = \frac{J}{ne} = \frac{I}{neA}$

So we obtain $n = \frac{BI}{V\ell e}$ where $\ell\,(= \frac{\text{Cross} - \text{section Area}}{\text{Width}})$ is the thickness of the strip.

Illustration 3:Copper has 8.0×10^{28} conduction electrons per metre^3. A copper wire of length 1 m and cross-sectional area 8.0×10^{-6} m^2 carrying a current and lying at right angle to magnetic field of strength 5×10^{-3} T experiences a force of 8.0×10^{-2}N. Calculate the drift velocity of free electrons in the wire. **(JEE ADVANCED)**

Sol: If v is the drift speed of electrons then the magnetic force on the wire is

$$F = qvB\sin\theta = qvB\sin 90^{o} = qvB$$

where q is the total charge of electrons in the wire.

$n=8.0 \times 10^{28}$ m^{-3}

l= 1 m; $A=8.0 \times 10^{-6}$m^2

Charge on each electron, $e=1.6 \times 10^{-19}$ C

Number of electrons in the copper wire = n x volume of wire = n(A l)

Total charge in the wire, $q=n((Al)e$ or $q=8.0 \times 10^{28} \times 8.0 \times 10^{-6} \times 1 \times 1.6 \times 10^{-19}=1.024 \times 10^5$C

Using $F=qvB\sin\theta$, we have, $\nu = \frac{F}{qB\sin\theta} = \frac{8.0\text{x}10^{-2}}{1.024\text{x}10^{5}\text{x}5\text{x}10^{-3}\text{x}\sin 90^{o}}$ = 1.563×10^{-4}m s^{-1}

9. MAGNETIC FORCE ON A CURRENT CARRYING WIRE

Suppose in a conductor number of free electrons per unit volume is n, then in an infinitesimal volume dV in the conductor, the total charge of free electrons will be

$$dq = ne\,dV$$

If the magnetic field at the location of the elementary volume is $\vec{B}$, and the drift velocity of free electrons is $\vec{v}_d$ then the magnetic force on the elementary volume will be

$$d\vec{F} = ne[\vec{v}_d \times \vec{B}]dV$$

Now we know that the current density is given as

$\vec{j} = ne\vec{v}_d$

So $\quad d\vec{F} = [\vec{j} \times \vec{B}]dV$

Introducing the vector $d\vec{\ell}$ in the direction of current we can write, $\vec{j}dV = \vec{j}\Delta S d\ell = I\, d\vec{\ell}$. Here ΔS is the area of cross-section and $d\ell$ the length of the elementary volume dV.

So $\quad d\vec{F} = I\,[d\vec{\ell} \times \vec{B}]$

The total magnetic force on the conductor is $\vec{F} = I\int [d\vec{\ell} \times \vec{B}]$

For a thin straight wire of length L, if the field $\vec{B}$ is constant throughout the length of the wire and perpendicular to it, we can write

$F = I\,L\,B$

In vector form we can write, $\vec{F} = I\,\vec{L} \times \vec{B}$, where $\vec{L}$ is a length vector that has magnitude L and is directed along the wire segment in the direction of the (conventional) current.

Few important points regarding the force on current carrying conductor in magnetic field are given below:

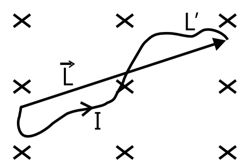

Figure 21.7: Current carrying conductor in uniform magnetic fieldr

(a) In a uniform magnetic field the force, dF= IBdℓ sinθ, does not depend on the position vector $\vec{r}$ of the current element.Thus this force is non-central. (Acentral force is a function of position vector $\vec{r}$, $\vec{F} = f(\vec{r})$)

(b) Theforce $d\vec{F}$ is always perpendicular to the plane containing $\vec{B}$ and $d\vec{\ell}$.Vectors $\vec{B}$ and $d\vec{\ell}$ may or may not be perpendicular to each other.

(c) As explained above, the total magnetic force on the conductor is

$\vec{F} = I\int [d\vec{\ell} \times \vec{B}]$

For uniform magnetic field, $\vec{B}$ can be taken out from the integral.

$\vec{F} = I\left[\int d\vec{\ell}\right] \times \vec{B}$

According to the law of vector addition $\int d\vec{\ell}$ is equal to the length vector $\vec{L}$ from initial to final point of the conductor as shown in Fig. 21.7. For a conductor of any arbitrary shape the magnitude of vector $\vec{L}$ is different from the actual length L′ of the conductor.

$\therefore \quad \vec{F} = I\vec{L} \times \vec{B}$

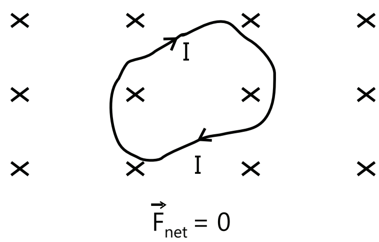

Figure 21.8: Current carrying loop in uniform magnetic field

(d) For a current carrying closed loop of any arbitrary shape placed in a uniform magnetic field (see Fig. 21.8),

$\vec{F} = I\left[\oint d\vec{\ell}\right] \times \vec{B} = 0$

Here as we add all the elementary vectors $d\vec{\ell}$ around the closed loop, the vector sum is zero because the final point is same as the initial point.

$\therefore \quad \oint d\vec{\ell} = 0$

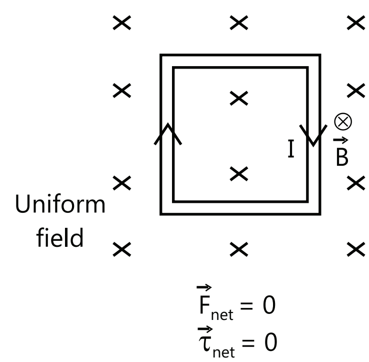

Figure 21.9: Area vector of closed loop is in direction of uniform magnetic field

Thus the net magnetic force on a current loop in a uniform magnetic field is always zero.

However, different parts of the loop do experience different net forces, although the vector some of all these forces comes out to be zero.

So the loop may experience some infinitesimal contraction or expansion, thus may be under tension.

Although the resultant of magnetic forces acting on the loop is zero, the resultant torque due the magnetic forces may not be zero.

Thus the torque on a loop in a uniform magnetic field is not always zero.

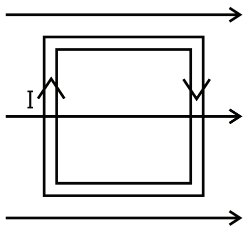

Figure 21.10: Area vector of closed loop is perpendicular to uniform magnetic field magnetic field

(e) When a current carrying closed loop is placed in a non-uniform magnetic field, in the general case it will experience non-zero net force as well as net torque.

Even a conductor of arbitrary shape not forming a loop, will experience a torque in a non-uniform field.

If the conductor is free to move, it will execute combined translational and rotational motion.

(f) When a current carrying conductor or closed loop translates or rotates in a magnetic field, the kinetic energy gained by it is, not due to the work done by magnetic forcesbut, at the expense of the energy supplied by the electric source which is maintaining current in the conductor/loop.

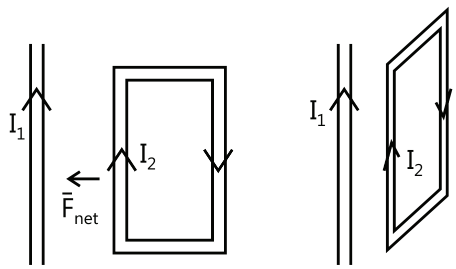

Figure 21.11: Closed loop in non-uniform magnetic field

The net work done by magnetic forces acting on a current carrying conductor is zero.

Though it may appear that,

$$W = \int \vec{F}.d\vec{r} = \int [I \int (d\vec{\ell} \times \vec{B})].d\vec{r} = \Delta K$$

but actually the kinetic energy is supplied by the electric source.

Illustration 4: A wire 12 cm long and carrying a current of 2 A is placed perpendicular to a uniform magnetic field. If a force of 0.8 N acts on it, calculate the value of the magnetic induction. **(JEE MAIN)**

Sol: This problem can be solved using formula $F = BI\ell\sin\theta$ for force on current carrying wire in uniform magnetic field.

ℓ = 12 cm = 12 x 10^{-2} m ; I = 2 A ; F = 0.8 N; $\theta = 90^o$

Using, F= BIlsin θ, we get $B = \frac{F}{I\ell\sin\theta} = \frac{0.8}{2x12x10^{-12}x\sin 90^o} = 3.3T$

9.1 Fleming's Left Hand Rule

If the thumb and the first two fingers of the left hand are stretched mutually perpendicular to each other and if the first finger points in the direction of the magnetic field and the second middle finger points in the direction of the current in the conductor, then the direction of thumb gives the direction of force on the conductor.

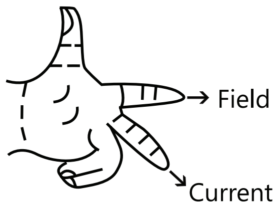

Figure 21.12: Fleming's Left hand Rule

10. TORQUE ON A CURRENT LOOP

Let us consider a square loop PQRS having side ℓ and area $A=l^2$ (See Figure). Let us introduce a unit vector $\hat{n}$ normal to the plane of the loop whose direction is related to the direction of current in the loop by the right-hand screw rule. Area of the loop can be written in vector form as $\vec{A} = \ell^2 \hat{n}$.

If current I in the loop is anti-clockwise then the vector $\hat{n}$ will be directed along the perpendicular to the plane of the paper towards the reader as shown in the Fig. 21.13. Suppose the loop is placed in a uniform magnetic field $\vec{B}$ directed along the perpendicular to the plane of the paper towards the reader, i.e. along the vector $\hat{n}$.In this situation, the magnitude of magnetic force on each of the branches of the loop will be IℓB, i.e. $|\vec{F}_1| = |\vec{F}_2| = |\vec{F}_3| = |\vec{F}_4| = I\ell B$. The direction of force on each branch can be found by Fleming's left hand rule. We can easily see that $\vec{F}_1 = -\vec{F}_3$ and $\vec{F}_1$ and $\vec{F}_3$ have same line of action. Similarly $\vec{F}_2 = -\vec{F}_4$ and $\vec{F}_2$ and $\vec{F}_4$ have same line of action.So, the net force as well as the net torque on the loop PQRS is zero.

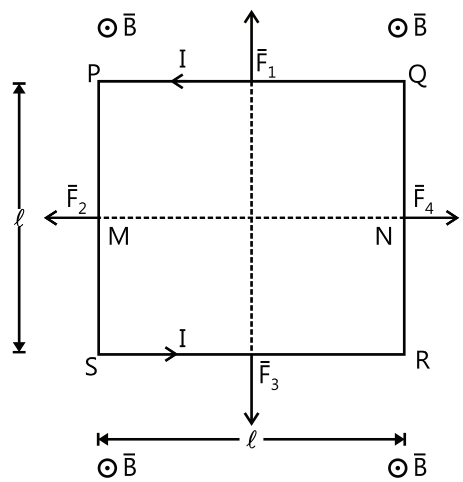

Figure 21.13: Zero torque on closed loop in uniform magnetic field

Now suppose the loop is rotated through an angle θ about the lineMN as shown in Fig. 21.14).So the anglebetween vector $\hat{n}$ and $\vec{B}$ will beθ. In this situation each of the sides Q'R' and S'P' makes an angle 90°-θ with the magnetic field $\vec{B}$ so that $|\vec{F}_2| = |\vec{F}_4| = I\ell B\cos\theta$ and again we have $\vec{F}_2 = -\vec{F}_4$ and $\vec{F}_2$ and $\vec{F}_4$ have same line of action. The side PQ shifts to P'Q' and RS shifts to R'S' such that PQ || P'Q' and RS || R'S' so that $|\vec{F}_1| = |\vec{F}_3| = I\ell B$ and again we have $\vec{F}_1 = -\vec{F}_3$, but the lines of action of $\vec{F}_1$ and $\vec{F}_3$ are displaced from each other by a distance of l$\sin\theta$. This forms a force couple, and the torque due to it will have magnitude

$$\tau = (I\,\ell\,B)\,\ell\sin\theta = I\,\ell^2 B\sin\theta = I\,A\,B\sin\theta$$

This torque is directed along the line MN.

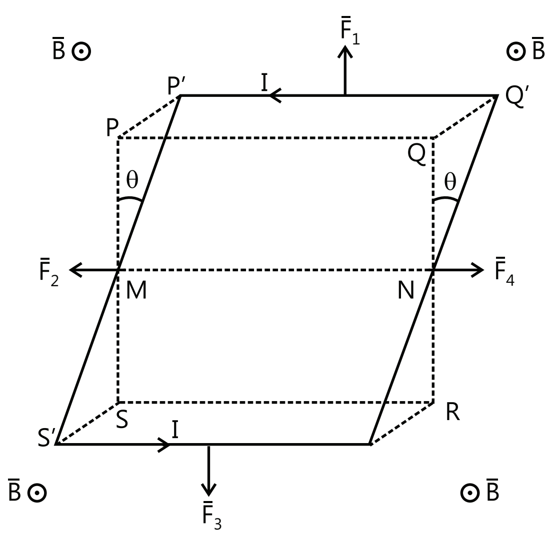

Figure 21.14: 14 Non-zero torque on closed loop in uniform magnetic field

In vector form we can write $\vec{\tau} = I\vec{A}\times\vec{B}$

Defining magnetic dipole moment of the loop as $\vec{M} = I\vec{A} = IA\hat{n}$, we can write torque as $\vec{\tau} = \vec{M}\times\vec{B}$

If the number of turns in the loop is N then we have, $\vec{M} = NI\vec{A} = NIA\hat{n}$

Note that although this formula has been derived for a square loop, it comes out to be true for any shape of the loop.

Illustration 5: A vertical circular coil of radius 0.1 m has moment of inertia as 1×10^{-1}kg m^2.It is free to rotate along y-axis coinciding with its diameter. Initially axis of the coil and direction of magnetic field of 1 T are along x-axis. The coil takes a quarter rotation. Find **(JEE ADVANCED)**

(i) Magnetic field strength at the center of the coil. Current of 3.19 A flows through this coil having 200 turns.

(ii) Magnetic moments of the coil.

(iii) Torque at the initial and final positions of the coil.

(iv) Angular speed at the final position.

Sol: The torque on coil is $\vec{\tau} = -\vec{M}\times\vec{B}$ where $\vec{M}$ the magnetic moment of coil is. As torque $\tau = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}\omega$, integrating equation of torque we get the angular velocity.

(i) Using $B = \frac{\mu_0 NI}{2R}$, we have $B = \frac{(4\pi x10^{-7})(200)(3.19)}{2x0.1} = 4x10^{-6}T$

(ii) Magnetic moment, m = NIA=NI (πR^2)= 200 x 3.19 x πx$(0.1)^2$ = 20Am^2

(iii) Torque$\tau = N_1 AB\sin\theta = m\sin\theta$; initially $\theta = 0$ so $\sin\theta = 0$ and $\tau = 0$

Finally, $\theta = 90^o$ so $\sin\theta = \sin 90^o = 1$ i.e., $\tau = mB$; i.e.$\tau = 20x4x10^{-6}x1 = 8x10^{-5}$Nm

(iv) $\Gamma = I\frac{d\omega}{dt}$ and $\Gamma = mB\sin\theta$; $I\frac{d\omega}{dt} = mB\sin\theta$, But $\frac{d\omega}{dt} = \frac{d\omega}{d\theta}x\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega$ Then, $I\omega d\omega = (mB\sin\theta)d\theta$

Integrating, we get $I\int_0^{\omega}\omega d\omega = mB\int_0^{\pi/2}\sin\theta d\theta$ i.e, $\frac{I\omega^2}{2} = -mB\cos\theta \Big|_0^{90} = mB$

i.e. $\omega = \left(\frac{2mB}{I}\right)^{1/2} = \left[\frac{2x8x10^{-5}}{0.1}\right]^{1/2}$ =4 x 10^{-2} rad s^{-1}

Note:

(a) Never use Fleming left-hand rule or right hand rule while solving questions. It becomes cumbersome to remember them precisely. Instead always find the direction of force by identifying the directions of motion and the field and then take the cross-product.

(b) Also, torque can be directly calculated by formula $\vec{M}\times\vec{B}$, where M is the magnetic dipolemoment as discussed below.

11. MAGNETIC DIPOLE MOMENT

Every current carrying loop behave like a magnetic dipole. It has two poles, north (N) and south (S) similar to a bar magnet. (see Fig. 21.15) Magnetic field lines are closed pathsdirected from the North Pole to the South Pole in the region outside the magnetic dipole and from South Pole to North Pole inside the magnetic dipole.

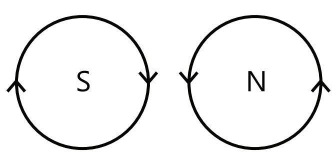

Figure 21.15: North and South Pole of current coil

Each loop has magnetic dipole moment defined as $\vec{M} = NI\vec{A}$, where N is the number of turns in the loop, I is the current in the loop andA is the area of cross-section of the loop.

For the direction of $\vec{M}$ any one of following methods can be used:

(a) The direction of $\vec{M}$ is from South Pole to North Poles we traverse inside the magnetic dipole. For a current loopthe North and the South Pole can be identified by the sense of current. The side fromwhere the current seems to flow clockwise is the South Pole and the opposite side from where it seems to flow anticlockwise is theNorth Pole.

(b) Vector $\vec{M}$ is along the normal to the plane of the loop. The direction of $\vec{M}$ is related to the direction of current in the loop by the right hand screw rule. Curl the fingers of the right hand around the perimeter of the loop in the direction of current as shown in Fig.21.16. Then thumb extendedperpendicular to the plane of the loop, points in the direction of $\overrightarrow{M}$.

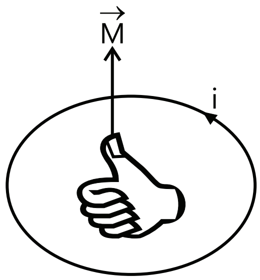

Figure 21.16: Right hand screw rule

The potential energy U of a magnetic dipole placed in a uniform magnetic field is

$$U = -MB\cos\theta$$

or $$U = -\vec{M}.\vec{B}$$

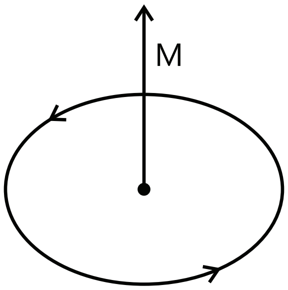

Figure 21.17: Direction of magnetic moment

For a bar magnet we define the magnetic dipole moment as

$$\vec{M} = m\vec{\ell}$$

Here m is the pole strength of the bar magnet and vector $\vec{\ell}$ is directed from South Pole to North Pole.

The unit of magnetic dipole moment is A-m^2.

The magnetic field at a large distance x on the magnetic axis of a bar magnet having magnetic dipole moment $\vec{M}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{2\vec{M}}{x^3}\right)$$

The magnetic field at a large distance x on the perpendicular bisector of a bar magnet having magnetic dipole moment $\vec{M}$ is

$$\vec{B} = -\frac{\mu_0}{4\pi}\left(\frac{\vec{M}}{x^3}\right)$$

Illustration 6: A square loop OABCO of side ℓ carries a current I. It is placed as shown in Fig. 21.18. Find magnetic moment of the loop. **(JEE MAIN)**

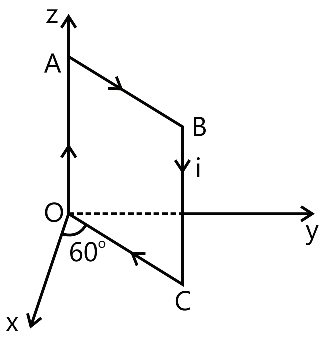

Figure 21.18

Sol: The magnetic moment of the loop is $M = IA$ for single turn. The direction of $\overrightarrow{M}$ is related to the direction of current in the loop by the right hand screw rule.

As discussed earlier, magnetic moments of the loop can be written as,

$$\vec{M} = I\left(\overrightarrow{BC} x \overrightarrow{CO}\right)$$

Here, $\overrightarrow{BC} = \ell\hat{k}$ $\qquad \overrightarrow{CO} = -\ell\cos 60^\circ\,\hat{i} - \ell\sin 60^\circ\,j = -\frac{\ell}{2}\hat{i} - \frac{\ell\sqrt{3}}{2}j$

$$\therefore\ \vec{M} = I\left[(-\ell\hat{k})x\left(-\frac{\ell}{2}\hat{i} - \frac{\ell\sqrt{3}}{2}\hat{j}\right)\right] \text{ or } \vec{M} = \frac{I\ell^2}{2}(\hat{j} - \sqrt{3}\,\hat{i})$$

Illustration 7: Find the magnitude of magnetic moment of the current carrying loop ABCDEFA. Each side of the loop is 10 cm long and current in the loop is i=20 A. **(JEE ADVANCED)**

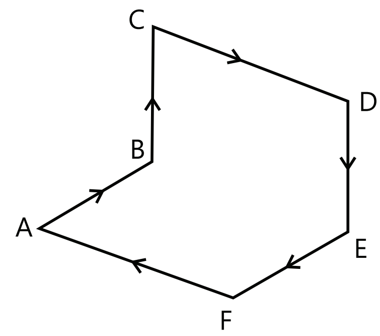

Figure 21.19

Sol: The magnetic moment of the loop is $M = IA$ for single turn. If a loop is divided into different parts, the magnetic moment of entire loop is vector sum of the magnetic moments of its individual parts.

By assuming two equal and opposite currents in BE, two current carrying loops (ABEFA and BCDEB) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other. Hence,

$$M_{net} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

Where M=iA-(2.0)(0.1)(0.1)=0.02 A-m^2 $\quad M_{net} = (\sqrt{2})(0.02)\ A - m^2$

=0.028 A-m^2

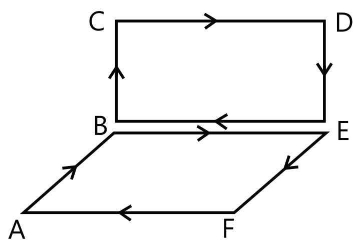

Figure 21.20

12. BIOT-SAVART LAW

Biot-Savart law is gives the strength of the magnetic field at any point due to a current element. If infinitesimalcurrent element of length $d\vec{\ell}$ carries a current I, the magnetic field or magnetic induction $d\vec{B}$ at any point P is given by Biot-Savart law as

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\cdot\frac{I d\vec{\ell}\times\vec{r}}{r^3}$$

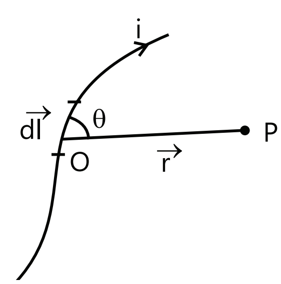

Figure 21.21 : Magnetic field due to current element dℓ

Here $\vec{r}$ is the position vector from the center of the element of length $d\vec{\ell}$ to the point of observation P. The direction of $d\vec{\ell}$ is along the direction of current I through it. If θ is the angle which $\vec{r}$ makes with the length $d\vec{\ell}$ of the conductor, the magnitude of magnetic induction is given by

$$\left|d\vec{B}\right| = \frac{\mu_o}{4\pi}\frac{I d\ell(r\sin\theta)}{r^3}$$

$$\left|d\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{I d\ell(\sin\theta)}{r^2}$$

Here μ_0 is the permeability of free space and $\frac{\mu_0}{4\pi} = 10^{-7}$ Tesla-meter/ampere.

The direction of $d\vec{B}$ is perpendicular to the plane containing current element $d\vec{\ell}$ and radius vector $\vec{r}$ which joins $d\vec{\ell}$ to P.

The total magnetic induction due to the conductor is given by, $\vec{B} = \int d\vec{B}$.

The magnetic intensity H at any point in the magnetic field is related to the magnetic induction as $H = \frac{B}{\mu}$ or $B = \mu H$ where μ is permeability of the medium. The unit of magnetic intensity H is $A\text{-}m^{-1}$

Maxwell's Cork Screw Rule: If a right handed cork screw is rotated so that its tip moves in the direction of flow of current through the conductor, then the direction of rotation of the head of the screw gives the direction of magnetic field lines around the conductor.

Right Hand Rule: If we hold the conductor in the right hand such that the thumb is stretched in the direction of current, the direction in which the fingers curl gives the direction on the magnetic field.

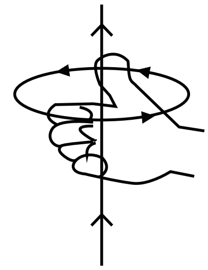

Figure 21.22: Right hand thumb rule

12.1 Application of Biot-Savart Law

Biot-Savart law is used to find the magnetic field due to current carrying conductors.

12.1.1 Magnetic Induction Due to Infinitely Long Straight Current Carrying Conductor

Suppose the current I flows through a long straightcurrent carrying conductor. We intend to find the magnetic field at point P at perpendicular distance r from the conductor. As shown in Fig. 21.23. the magnitude of field $d\vec{B}$ at P due toan infinitesimal element of length $d\ell$, is given by Biot-Savart law as:

$$\left|d\vec{B}\right| = dB = \frac{\mu_0 I d\ell \sin(90+\alpha)}{4\pi x^2}$$

where x is the distance between the current element and point P. The field $d\vec{B}$ is directed into the plane of the figure and perpendicular to it.

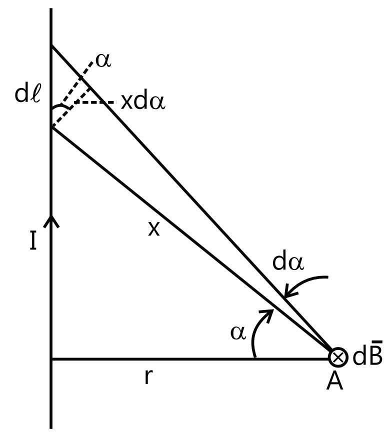

Figure 21.23: Magnetic field due to infinitely long straight wire

Now from Fig. 21.23. it is clear that, $d\ell \cos\alpha = x\, d\alpha$ and $x = \frac{r}{\cos\alpha}$, so we can write,

$$dB = \frac{\mu_0 I}{4\pi}\frac{\cos\alpha\, d\alpha}{r} \qquad \text{......(i)}$$

The conductor is infinitely long,so as the angle α varies from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, the infinitesimal element covers the infinite length of the conductor, and for allinfinitesimal elements making-up the conductor the field $d\vec{B}$ is directed into the plane of the figure. Thus we can add the magnitudes of $d\vec{B}$ due to all the infinitesimal elements to get the magnitude of total field as,

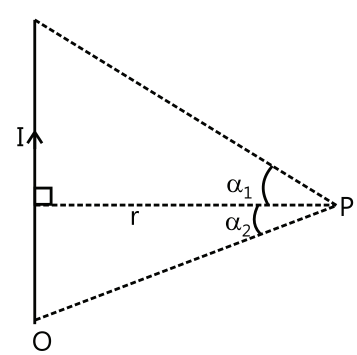

Figure 21.24: Magnetic field due to finite straight wire

$$B=\frac{\mu_o I}{4\pi r}\int_{-\pi/2}^{\pi/2}\cos\alpha\, d\alpha=\frac{\mu_o I}{2\pi r}$$

12.1.2 A Straight Conductor of Finite Length

If a conductor of finite length subtends an angle $\alpha_1$on one side and $\alpha_2$on the other side ofperpendicular from point P as shown in Fig. 21.24 then we can write,

$$B=\frac{\mu_o I}{4\pi r}\int_{-\alpha_2}^{\alpha_1}\cos\alpha\, d\alpha=\frac{\mu_o I}{4\pi r}\left|\sin\alpha\right|_{-\alpha_2}^{\alpha_1}=\frac{\mu_o I}{4\pi r}[\sin\alpha_1+\sin\alpha_2] \qquad \text{....(ii)}$$

12.1.3 At the End of a Straight Conductor of Infinite Length

In this case, the angle α varies from 0 to $\frac{\pi}{2}$, and we can write

$$B=\frac{\mu_o I}{4\pi r}\int_{0}^{\pi/2}\cos\alpha\, d\alpha=\frac{\mu_o I}{4\pi r}$$

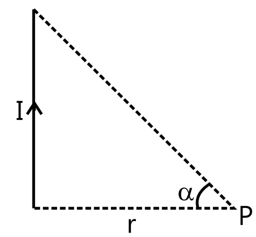

Figure 21.25: Magnetic field at end of straight wire of finite length

12.1.4 At The End of a Straight Conductor of Finite Length

In this case, (see Fig. 21.25) the angle α varies from 0 to α, and we can write

$$B=\frac{\mu_0 I}{4\pi r}\int_{0}^{\alpha}\cos\alpha\, d\alpha=\frac{\mu_o I\sin\alpha}{4\pi r}$$

12.1.5 At a Point Along the Length of the Straight Conductor Near Its End

In this case (see Fig. 21.26) $\alpha_1=\frac{\pi}{2}$ and $\alpha_2=-\frac{\pi}{2}$, and thus equation (ii)gives B=0. Actually in this case the value of α does not vary at all i.e. it is constant (at all points of the wire we have $\alpha=\frac{\pi}{2}$), thus $d\alpha = 0$ and thus equation (i) gives $dB = 0$.

Figure 21.26: Magnetic field along length of straight wire

Illustration 8: Calculate the magnetic field at the center of a coil in the form of a square of side 4 cm carrying a current of 5A. **(JEE MAIN)**

Sol: Square loop can be considered as four wires each of length ℓ. Magnetic field due to any one wire, at a the center is calculated as $B_1=\frac{\mu_0}{4\pi}\frac{1}{x}\left[\sin\theta_1+\sin\theta_2\right]$

A square coil carrying current is equivalent to four conductors of finite length.

Step 1

Magnetic field at O due to conductor BC is

$$B_1=\frac{\mu_0}{4\pi}\frac{1}{x}\left[\sin\theta_1+\sin\theta_2\right]$$

Here $\theta_1=\theta_2=45^0$; I=5A,x=2 cm=2x10^{-2}m

$$\therefore \quad B_1=\frac{10^{-7}x5}{2x10^{-2}}\left[\sin45^o+\sin45^o\right]=\frac{10^{-7}x5x\sqrt{2}}{2x10^{-2}}=3.54x10^{-5}T$$

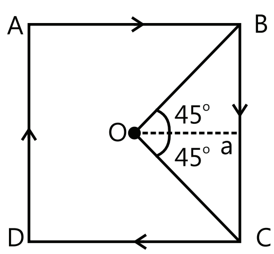

Figure 21.27

By symmetry, magnetic field intensity at O due to each arm will be same.Moreover, the direction of magnetic field at O due to each arm of the square is same

Step 2

∴ Net magnetic field at O due to current carrying square,

$B = 4B_1 \quad = 4 \times 3.54 \times 10^{-5}$ T or $\quad B = 1.42 \times 10^{-4}$ T

12.1.6 Magnetic Field on the Axis of a Current Carrying Circular Arc

If a current I is flowing in a circular arc of radius R lying in the y-z plane with center at origin O and subtending an angle φ at O, then the magneticfield $d\vec{B}$ at a point Pon x-axiswith coordinates (x, 0, 0) due to a small elementary arc of length $|d\vec{\ell}| = R\,d\theta$ at a distance r from P is given by Biot-Savart Law as:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell}\times\vec{r}}{r^3} \quad \text{... (i)}$$

where $\vec{r}$ is a vector from midpoint of $d\vec{\ell}$ to P.

As shown in Fig. 21.28 the coordinates of $d\vec{\ell}$ are (0, R cosθ, R sinθ), where θ is the angle between the radius of the arc through $d\vec{\ell}$ and the y-axis.

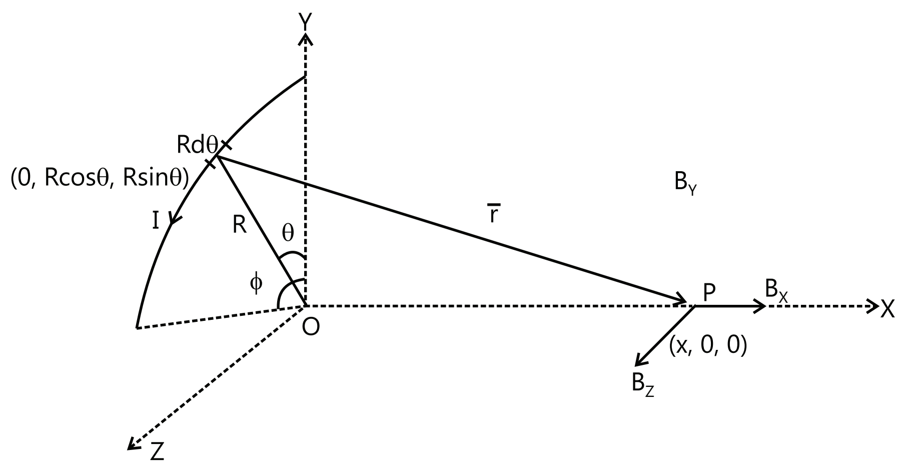

Figure 21.28: Magnetic field at a point on the axis of current carrying arc

So we can write $\vec{r} = x\hat{i} - R\cos\theta\,\hat{j} - R\sin\theta\,\hat{k}$ ... (ii)

Magnitude $r = \sqrt{x^2 + R^2}$ ... (iii)

Let us express $d\vec{\ell}$ in Cartesian coordinates system as shown in Fig. 21.29.

$d\vec{\ell} = -R\sin\theta\,d\theta\,\hat{j} + R\cos\theta\,d\theta\,\hat{k}$ ... (iv)

Put (ii), (iii) and (iv) in (i) to get

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I(-R\sin\theta\,d\theta\,\hat{j} + R\cos\theta\,d\theta\,\hat{k})\times(x\hat{i} - R\cos\theta\,\hat{j} - R\sin\theta\,\hat{k})}{(\sqrt{x^2+R^2})^3}$$

$$\Rightarrow \quad d\vec{B} = \frac{\mu_0 I}{4\pi(x^2+R^2)^{3/2}}(R^2\,d\theta\,\hat{i} + x\,R\cos\theta\,d\theta\,\hat{j} + xR\sin\theta\,d\theta\,\hat{k})$$

Resultant magnetic field at P is

$$\vec{B} = \frac{\mu_0 I}{4\pi(x^2+R^2)^{3/2}}\left(R^2\int_0^{\phi} d\theta\,\hat{i} + xR\int_0^{\phi}\cos\theta\,d\theta\,\hat{j} + xR\int_0^{\phi}\sin\theta\,d\theta\,\hat{k}\right)$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{4\pi(x^2+R^2)^{3/2}}[R^2\phi\,\hat{i} + xR\sin\phi\,\hat{j} + xR(1-\cos\phi)\hat{k}]$$

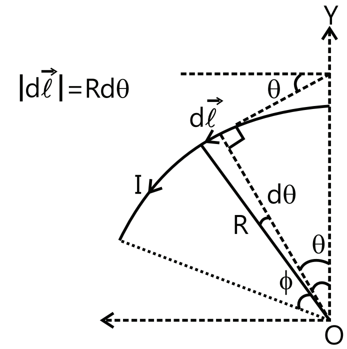

Figure 21.29: Vector is in the YZ plane

Thus $\vec{B}$ can be resolved into components parallel to the x, y and the z axes.

$$B_x = \frac{\mu_0 I R^2 \phi}{4\pi(x^2+R^2)^{3/2}}$$

$$B_y = \frac{\mu_0 I R x \sin\phi}{4\pi(x^2+R^2)^{3/2}}$$

$$B_z = \frac{\mu_0 I R x(1-\cos\phi)}{4\pi(x^2+R^2)^{3/2}}$$

The field at center of the arc: At center x = 0, so

$$B_x = \frac{\mu_0 I \phi}{4\pi R}$$

$$B_y = 0$$

$$B_z = 0$$

Thus at the center the field is normal to the plane of the arc.

For a semicircular loop, the angle subtended at the center is $\phi = \pi$, so $B = \frac{\mu_0 I}{4r}$

12.1.7 Magnetic Field on the Axis of a Current Carrying Circular Loop

The field $\vec{B}$ on the axis of a current carrying circular loop (see Fig. 21.30) can be obtained from the expression of $\vec{B}$ for a current carrying circular arc derived in the previous article by substituting the value of angle φ subtended at the center as 2π.

$$\therefore \quad \vec{B} = \frac{\mu_0 I}{4\pi(x^2+R^2)^{3/2}}[R^2(2\pi)\hat{i} + xR\sin 2\pi\,\hat{j} + xR(1-\cos 2\pi)\hat{k}]$$

$$\therefore \quad \vec{B} = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}\hat{i}$$

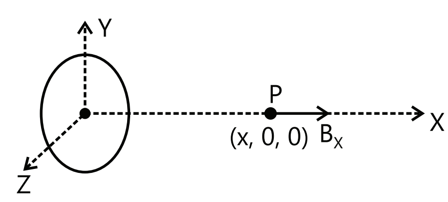

Figure 21.30: Magnetic field at a point on the axis of circular loop

Thus field $\vec{B}$ is directed along the axis of the circular loop.

For a coil havingN circular turns, $B = \frac{\mu_0 N I R^2}{2(R^2+x^2)^{3/2}}$

The field at center of the coil:

At center x = 0, so $B_0 = \frac{\mu_0 N I R^2}{2R^3}$

$$\therefore \quad B_0 = \frac{\mu_0 N I}{2R}$$

The direction of B at the center of circular current carrying arc or closed circular loop can be found as follows:

If we curl the fingers of the right hand in the direction of the current in the arc/loop, then the stretched thumb points in the direction of the field at the center.

If the point P is at a very large distance from the coil,then $x^2 >> R^2$, $B = \frac{\mu_0 NIR^2}{2x^3}$

If A is area of one turn of the coil, $A=\pi R^2$ $B = \frac{\mu_0 NIA}{2\pi x^3}$

Illustration 9: A straight wire carrying a current of 12 A is bent into a semi-circular are of radius 2.0 cm as shown in Fig. 21.31.(i) What is the direction and magnitude of magnetic field (B) at the center of the arc? **(JEE ADVANCED)**

(ii) Would the answer change if wire is bent in the opposite way?

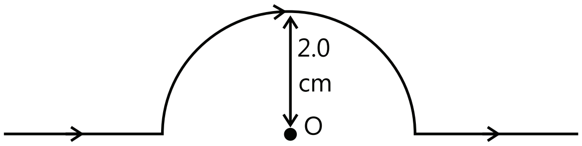

Figure 21.31

Sol: For given arrangement of wire, the magnetic field at the center due to the straight sections will be zero. The magnetic field at center will be due to the semicircular wire. Direction of field depends on direction of current and determined by right hand thumb rule.

(i) The wire is divided into three sections: (a) the straight section to be left (b) the straight section to the right and (c) circular arc.

Step 1. Magnetic field due to a current carrying element at a point is given by $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$

In the given case, angle between $\vec{dl}$ and r for the straight section is 0°or π .So sin 0 =sin π =0

Hence magnetic field at the center (O) of the arc due to straight sections is ZERO

Step 2. Magnetic field at the center due to current carrying semi-circular section is

$$B = \frac{1}{2} x \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0}{4\pi}\frac{\pi I}{r} = \frac{10^{-7} x3.142x12}{2x10^{-12}} = 1.89x\ 10^{-4}\ T$$

The magnetic field is directed into the plane of the paper.

(ii) Direction of the field will be opposite to the found out in (i).

Illustration 10: A current path shaped as shown in Fig. 21.32 produces a magnetic field at P, the center of the arc. If the arc subtends an angle of 30° and the radius of the arc is 0.6m, what are the magnitude and direction of the field produced at P if the current is 3.0 A **(JEE ADVANCED)**

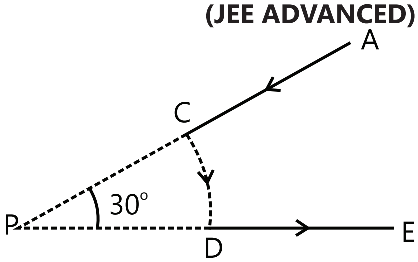

Figure 21.32

Sol: Magnetic field at the center P of arc CD is $B = \frac{\mu_0 I\phi}{4\pi R}$, and due to straight wires AC and DE is zero.

The magnetic field at P due to the straight segment AC and DE is zero, because $\vec{d\ell}$ is parallel to $\vec{r}$ along these paths, this means that $\vec{d\ell} \times \vec{r} = 0$. Each length element $d\vec{\ell}$ along path CD is at the same distance from P, hence B at P is due to segment CD which is given by

$$B = \frac{\mu_0 I}{4\pi r}\phi = \frac{\mu_0 I}{4\pi r} \times \frac{\pi}{6} = \frac{\mu_0 I}{24r}$$

13. FORCE BETWEEN PARALLEL CURRENTS

Consider two long wires kept parallel to each other such that the separation d between them is quite small as compared to their lengths. Suppose currents I_1 and I_2 flow through the wires in the same direction (see Fig. 21.33). Consider a small element $d\ell$ of the wire carrying current I_2. The magnetic field at $d\ell$ due to the wire carrying current

I_1 is $\vec{B} = \frac{\mu_0 I_1}{2\pi d}(-\hat{k})$ ...(i)

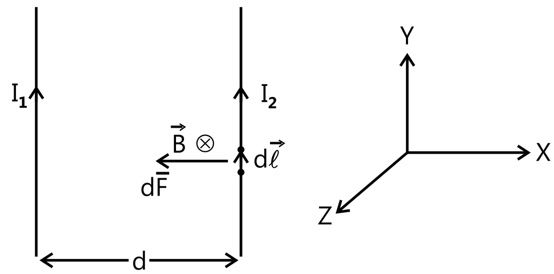

Figure 21.33: Force between parallel currents

($\vec{B}$ is normal to and directed into the plane of the figure)

The magnetic force on this element is $d\vec{F} = I_2 d\vec{\ell} \times \vec{B} = I_2 d\ell(\hat{j}) \times B(-\hat{k})$

or, $d\vec{F} = I_2 d\ell B(-\hat{i}) = \frac{\mu_0 I_1 I_2}{2\pi d} d\ell(-\hat{i})$ (directed towards the wire carrying current I_1)

Thus the wire carrying current I_2 is attracted towards the wire carrying current I_1. By Newton's third law the force acting on wire carrying current I_1 will also be attractive. Thus the two wires are attracted towards each other.

The force per unit length on each of the wires due to the other wire will be,

$$\frac{dF}{d\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Parallel currents attract each other, and antiparallel currents repel each other.

Note: Memorizing various formula of magnetic field due to ring and wire carrying current would easily help in calculating magnetic field due to complicated wire systems. Also, be careful about the direction of field in every problem you solve.

Illustration 11: A current of 10A flows through each two parallel long wires. The wires are 5 cm apart. Calculate the force acting per unit length of each wire. Use the standard values of constants required. **(JEE MAIN)**

Sol: Field of one wire exerts force on other wire and the force per unit length of wire is $\frac{F}{\ell} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}$.

Force acting per unit length of long conductor due to another long conductor parallel to it and carrying same current.

$\frac{dF}{d\ell} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}$; $I_1=I_2=10$A, r=5 cm $=5\times10^{-2}$m, $\frac{\mu_0}{4\pi} = 10^{-7}$TmA^{-1}; $\frac{dF}{d\ell} = \frac{10^{-7}\text{x}2\text{x}10\text{x}10}{5\text{x}10^{-2}} = 4 \text{ x } 10^{-4}$ N m^{-1}

Illustration 12: The wires which connect the battery of an automobile to its starting motor carry a current of 30A (for a short time). What is the force per unit length between the wires, if they are 70 cm long and 1.5 cm apart? Is the force attractive or repulsive? **(JEE ADVANCED)**

Sol: Field of one wire exerts force on other wire and the force per unit length of wire is $\frac{F}{\ell} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}$.

Force depends on direction of current. Parallel currents attract while anti-parallel currents repel.

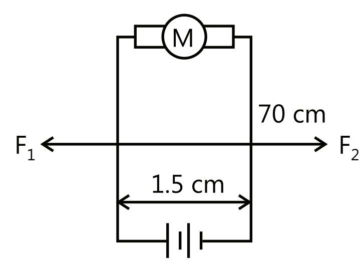

Figure 21.34

$\frac{dF}{d\ell} = \frac{\mu_0}{4\pi}\left(\frac{2I_1I_2}{d}\right)$; $I_1=I_2=300$A; r=1.5cm=1.5x 10^{-2}m

$\therefore \quad \frac{dF}{d\ell} = \frac{10^{-7} \times 2 \times 300 \times 300}{1.5 \times 10^{-2}} \quad = 1.2 Nm^{-1}$

Since current in both the wires flows in opposite direction, so the force is repulsive.

14. AMPERE'S LAW

This law is also called the 'Theorem on Circulation of Vector B'.

According to this law the line integral or circulation of magnetic field vector $\vec{B}$ around a closed path is equal to μ_0 times the algebraic sum of the currents enclosed by the closed path.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

The closed path is also called Amperian loop.

I_{enc} is the algebraic sum of all the currents passing through the area enclosed by the closed path. Current is assumed positive if it is along the direction associated with the direction of the circumvention of the closed path through the right-hand screw rule.If we curl the fingers of the right hand around the closed path, in the direction of circumvention, the stretched thumb gives the positive direction of current. The current in the opposite direction is negative.

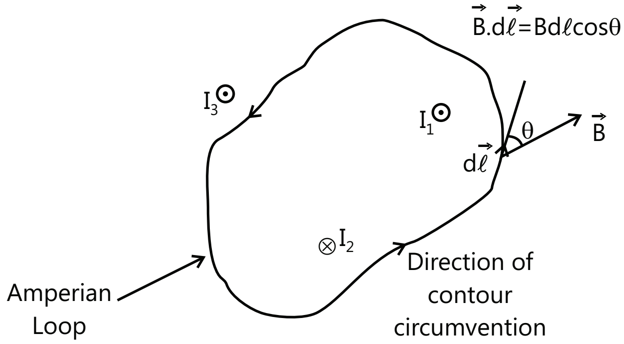

Figure 21.35: Current enclosed by amperian loop

For example in the Fig. 21.35 shown, the current directed out of the plane of the figure is positive, so we have $I_{enc} = I_1 - I_2$; $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left(I_1 - I_2\right)$

14.1 Limitations of Ampere's Circuital Law

Ampere's law is an important tool in calculating the magnetic field due to a current distribution. However this usefulness is limited to only a few cases where the magnetic field is having a symmetrical distribution in space. The Amperian loop is chosen in such a way that the magnetic field has a constant value along the loopand is directed tangentially at all points of the loop.If such a choice of a loop is not possible, then Ampere's law cannot be used to find out the magnetic field. For example this law can't be used to find the magnetic field at the center of a current carrying loop.

Note: Ampere's circuital law holds good for a closed path of any size and shape around a current carrying conductor.

14.2 Applications of Ampere's Law

14.2.1 Magnetic field due to current carrying circular wire of infinite length

Let R be the radius of the infinite circular wire carrying current I. The magnetic field lines are concentric circles with their centers on the axis of the wire.

(a) Magnetic field intensity at a point outside the wire

We intend to find magnetic field at a distance r> R from the axis of the wire. We choose a circular path of radius r and center at the axis of the wire as the Amperian loop. $\vec{B}$ will be constant and tangential at all points of this loop. Using Ampere's law,

$\oint \vec{B}.d\vec{\ell} = \mu_0 I$ or $\oint B\,d\ell \cos 0^0 = \mu_0 I$

or $B\oint d\ell = B(2\pi r) = \mu_0 I$

$\therefore \quad B = \frac{\mu_0 I}{2\pi r}$ ...(i)

Thus, the magnetic field intensity at a point outside the wire varies inversely as the distance of the point from the axis of the wire.

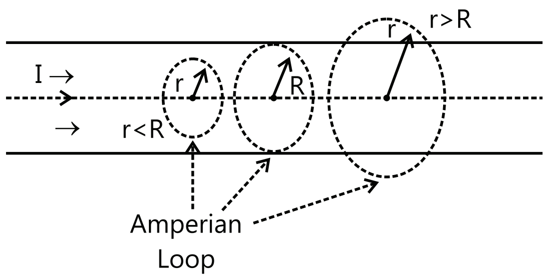

Figure 21.36: Circular cross-section of infinitely long straight wire

That is, $B \propto \frac{1}{r}$

At the surface of the wire, r = R, so $B = \frac{\mu_0 I}{2\pi R}$ ...(ii)

(b) Magnetic field intensity at a point inside the wire

We intend to find magnetic field at a distance r< R from the axis of the wire. We choose a circular path of radius r and center at the axis of the wire as the Amperian loop. $\vec{B}$ will be constant and tangential at all points of this loop. Using Ampere's law,

$\oint \vec{B}.d\vec{\ell} = \mu_0 I_{enc}$ or $\oint B\,d\ell \cos 0^0 = \mu_0 I_{enc}$

or $B\int d\ell = B(2\pi r) = \mu_0 I_{enc}$

If the current is uniformly distributed throughout the cross – section of the wire, then we have

$I_{enc} = \frac{I}{\pi R^2}(\pi r^2) = \frac{Ir^2}{R^2}$

$\therefore \quad B(2\pi r) = \mu_0 \frac{Ir^2}{R^2}$

$\therefore \quad B = \frac{\mu_0}{2\pi}\frac{Ir}{R^2}$

Thus, $B \propto r$

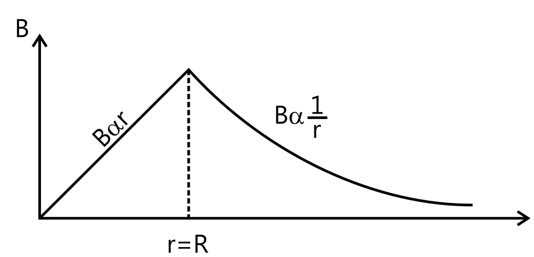

Figure 21.37: Variation of field with radial distance r

The variation of B with distance r from the axis of the wire is shown in Fig.21.37.

Illustration 13: Figure 21.38 shows the cross section of a long conducting cylinder with inner radius a=2.0 cm and outer radius b=4.0 cm.The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $j = cr^2$, with $c=3.0 \times 10^6$ A/m^{4}and r in meters. What is the magnetic field $\vec{B}$ at appoint that is 3.0 cm from the central axis of the cylindrical? **(JEE ADVANCED)**

Sol: The magnetic field in this case is symmetric. The field lines are concentric circles. We choose a circular amperian loop coaxial with the cylinder. First find the current enclosed for region a<x<r where r = 3 cm. Then use $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$ to find $\vec{B}$.

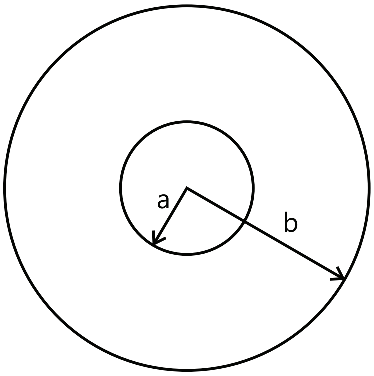

Figure 21.38

We write the integral as $i_{enc} = \int JdA = \int_a^r cr^2(2\pi r\,dr)$

$$= 2\pi c\int_a^r r^3\,dr = 2\pi c\left[\frac{r^4}{4}\right]_a^r = \frac{\pi c(r^4 - a^4)}{2}$$

The direction of integration indicated in Fig. 21.38 is (arbitrarily) clockwise. Applying the right-hand rule for Ampere's law to that loop, we find that we should take i_{enc} as negative because the current is directed out of the page but our thumb is directed into the page.

We next evaluate the left side of Ampere's law exactly as we did in figure.

Then Ampere's law, $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$,

Gives us $B(2\pi r) = -\frac{\mu_0 \pi c}{2}(r^4 - a^4)$

Solving for B and substituting known data yield $B = -\frac{\mu_0 \pi c}{4\pi r}(r^4 - a^4)$

$$= -\frac{\left(4\pi x10^{-7}\ T\cdot m/A\right)\left(3.0x10^{6}\ A/m^4\right)}{4(0.030m)\pi} x \left[(0.030\ m)^4 - (0.020m)^4\right] = -2.0 \text{ x } 10^{-5}\,T$$

Thus, the magnetic field $\vec{B}$ at a point 3.0 cm from the central axis has magnitude B=2.0 x 10^{-5} T and forms magnetic field lines that are directed opposite our direction of integration, hence counterclockwise in figure.

14.2.2 Magnetic Field Inside a Solenoid

A solenoid is an insulated wire wound closely into multiple turnsto form a helix. The length of the solenoid is assumed to be much larger than its diameter. At points very close to a turn, the magnetic field lines are almost concentric circles.The fields due to adjacent turns at points near the axis add-up while fields at points away from the axis cancel each other. If the solenoid is very tightly wound and its length is quite large, then the field inside it is uniform and parallel to its axis, while field outside it will be zero.

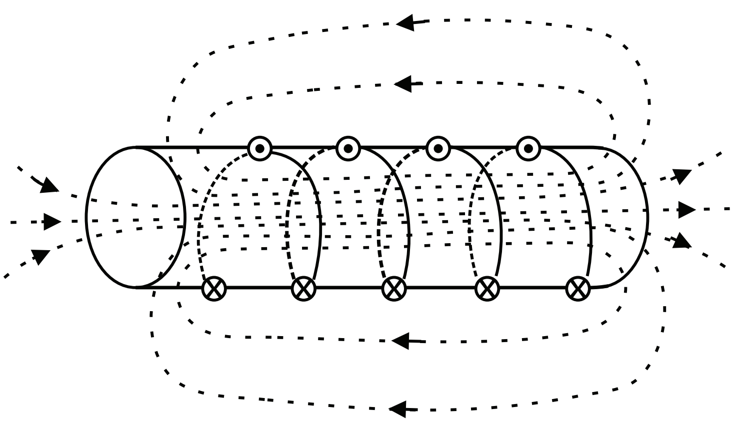

Figure 21.39: Magnetic field lines inside solenoid

We can apply Ampere's law to find the magnetic field inside the solenoid. We choose a rectangular Amperian loop abcd partly inside the solenoid and partly outside it as shown in Fig. 21.40, its length lbeing parallel to the solenoid's axis.

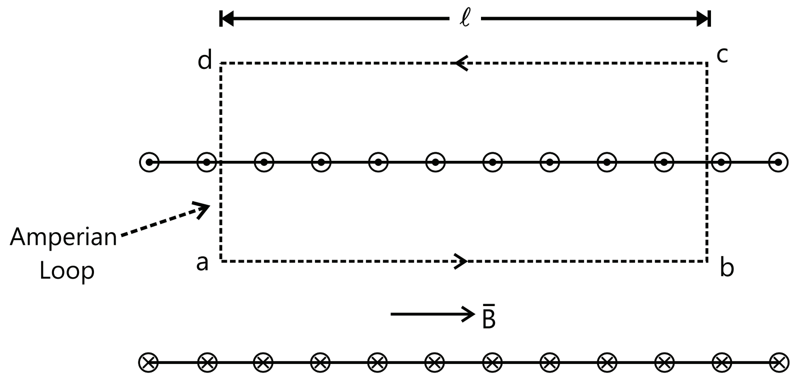

Figure 21.40: Rectangular amperian loop

There are four sides of the rectangle. We write $\oint \vec{B}.d\vec{\ell}$ as the sum of four integrals, one for each side:

$$\oint \vec{B}.d\vec{\ell} = \int_a^b \vec{B}.d\vec{\ell} + \int_b^c \vec{B}.d\vec{\ell} + \int_c^d \vec{B}.d\vec{\ell} + \int_d^a \vec{B}.d\vec{\ell}$$

The sides bc and da do not contribute to the line integral as the magnetic field is perpendicular to these sides at points inside the solenoid and at points outside the solenoid the magnetic field is zero. The side cd is completely outside the solenoid and hence the magnetic field is zero at all its points. So the only side that contributes to the line integral is ab.

Thus, we get $\oint \vec{B}.d\vec{\ell} = B\ell = \mu_0 n\ell I$

Here I is the current through each turn of the solenoid and n is the number of turns per unit length of the solenoid. The net current enclosed by the rectangle is $n\ \ell\ I$.

$\therefore \quad B = \mu_0 n I$

MASTERJEE CONCEPTS

(a) Magnetic field inside a solenoid and coil

(i) Magnetic field is considered uniform throughout the solenoid, while it is not true for coil

(ii) This is because, solenoid is long, while coil is thin.

(iii) Thus, magnetic field lines look very symmetric inside a solenoid, and of nearly equal length, while in a coil, the path are very different, and by Ampere's law, their magnitude is different

(b) Magnetic field on the axis at the end of a long solenoid

(i) Think of an infinite solenoid, if you could take the midpoint at the axis of this solenoid then the magnetic field strength at that point from each side would be $B = \frac{\mu_0 nI}{2}$ the situation you describe is like taking half of this infinite solenoid (as L>>d) and so $B = \frac{\mu_0 nI}{2}$

(μ_0 =permeability of free space, n= number of coils in the solenoid, I= current)

Anurag Saraf (JEE 2011 AIR 226)

Illustration 14:A closely wound solenoid 80 cm long has 5 layers of winding of 400 turns each. The diameter of the solenoid is 1.8 cm. if the current carries is 8.0 A, find the magnitude of B inside the solenoid near its center.

(JEE MAIN)

Sol: For solenoid of length ℓ the field at a point inside it is $B = \frac{\mu_0 NI}{\ell}$ where N is the number of turns in solenoid.

Magnetic field induction at a point inside the solenoid is

$$B = \frac{\mu_0 NI}{\ell} = \frac{4\pi 10^{-7} x(400x5)x8}{(80x10^{-2})} = 8\pi x10^{-3}T \approx 2.5\ x10^{-2}\ T$$

Illustration 15: A solenoid is 2 m long and 3 cm in diameter. Ithas 5 layers of winding of 1000 turns each and carries a current of 5A. What is the magnetic field at its center? **(JEE MAIN)**

Sol: For solenoid of length ℓ the field at a point inside it is $B = \frac{\mu_0 NI}{\ell}$ where N is the number of turns in solenoid.

Magnetic field at the center of a solenoid is given by,

$$B = \frac{\mu_0 NI}{l} = (4\pi x10^{-7})\left(\frac{5x1000}{2}\right)x5 = 1.57\ x\ 10^{-2}\ T$$

14.2.3 Magnetic field Inside a Toroid

Toroid is a circular solenoid. An insulated conducting wire is tightly wound on a ring (or torus) made ofnon-conducting material to form a toroid. The magnetic field inside a toroid can be obtained by using Ampere's law. We choose a circularAmperian loop of radius rinside the toroid concentric with it.

$$\oint \vec{B}\cdot d\vec{\ell} = \oint B\,d\ell = B\oint d\ell = B(2\pi r) = \mu_0 I_{enc}$$

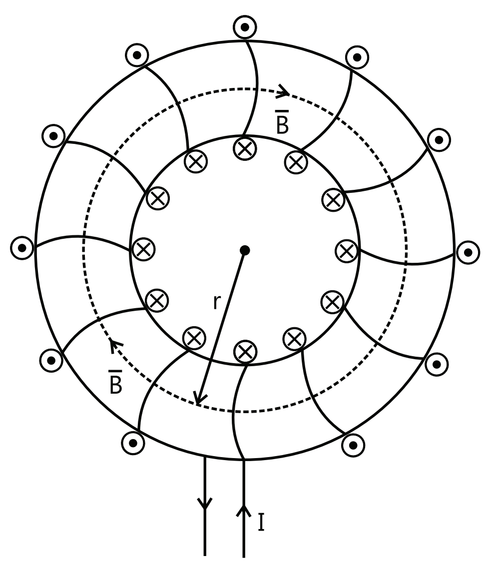

Figure 21.41: Magnetic field inside Toroid

If each turn of the toroid carries current I and the total number of turns in the toroid is N, then current enclosed by the Amperian loop is NI.

So $2\pi r\ B = \mu_0 N\ I$ or, $B = \frac{\mu_0 NI}{2\pi r}$

Illustration 16: A toroid of 4000 turns has outer radius of 26 cm and inner radius of 25 cm. If the current in the wire is 10A, calculate the magnetic field of the toroid also in the inner air space of the toroid. **(JEE ADVANCED)**

Sol: For toroid the field at a pointinside it at radial distance r from its center is $B=\frac{\mu_0 NI}{2\pi r}$ where N is the number of turns in toroid.

Radius of toroid $r=\frac{25+26}{2}=25.5\,cm=25.510^{-2}m$

Length of toroid $l=2\pi r=2\pi \times (25.5 \times 10^{-2} = 51 \times 10^{-2}\pi$ m

$\therefore$ Number of turns /unit length, $n=\frac{4000}{51 x 10^{-2}\pi}$

Field in a toroid is given by

$B=\mu_0 nI=4\pi x 10^{-2}\left(\frac{4000}{51 x 10^{-2}\pi}\right)x10$;= 3.14 x 10^2T

Field in the air space bounded by the toroid is zero because the field exists inside the envelope of the winding of the toroid.

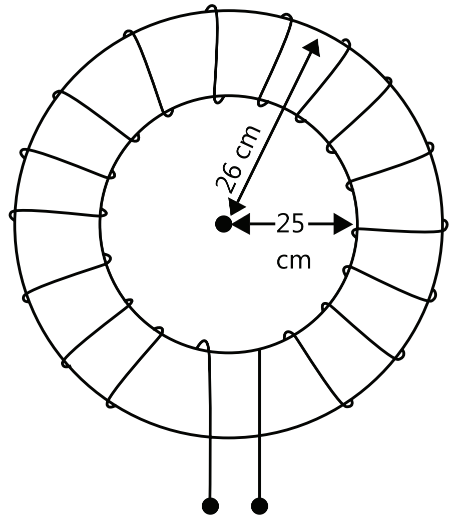

Figure 21.42

15. MOVING COIL GALVANOMETER

Moving Coil Galvanometer is a device used to detect/measure small electric current flowing in an electric circuit.

Principle: When a current carrying loop or coil is placed in the uniform magnetic field, it experiences a torque and thus starts rotating.

Construction: A moving coil galvanometer is shown in Fig. 21.43. It consists of a coil made of insulated copper wire wound on a soft-iron cylinder. The coil is suspended by a spiral spring between two cylindrical shaped poles of a permanent magnet.

The spring exerts a very small restoring torque on the coil.

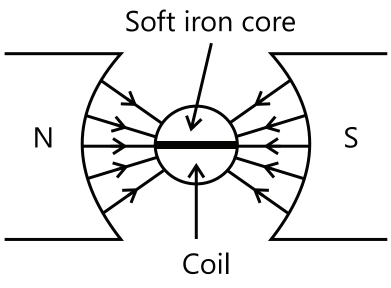

Figure 21.43: Moving coil galvanometer

Theory

Let B = Magnetic field

I = Current flowing through the coil

ℓ = Length of coil

b = Breadth of the coil

(ℓxb) = A= Area of the coil

N = Number of turns in the coil

When current flows through the coil, it experiences a torque, which is given by

τ=NIAB sinθ

where, θ is the angle between the normal to the plane of the coil and the direction of the magnetic field. Initially, $\theta = 90^o$, so τ=NIAB ...(i)

This torque is called deflecting torque.As the coil gets deflected, the spring is twisted and a restoring torque is developed in it which is proportional to the angle of deflection ϕ

$\tau_{res}=k\phi$ ...(ii)

Here k is a constant for a particular spring.

For equilibrium of the coil,

Deflecting torque= Restoring torque

i.e. $NIAB = k\phi$ ...(iii)

or $I = \dfrac{k\phi}{NAB}$ ...(iv)

or $I = G\phi$ ...(v)

where $G = \dfrac{k}{NAB}$ is Galvanometer constant

$\therefore$ $I \propto \phi$ ...(vi)

Thus, the current flowing through the coil is directly proportional to the deflection of the coil. Hence we can determine the current in the coil by measuring its deflection.

Use of a radial magnetic field in the moving coil galvanometer

A radial magnetic field, produced by cylindrical poles of permanent magnet is always parallel to the plane of the coil of the galvanometer. Thus the angle between the normal to the coil and the magnetic field is always 90°. Thus torque on the coil is $\tau = NIAB = k\phi$ or $I \propto \phi$. Thus, when radial magnetic field is used, the current in the coil is always proportional to the deflection. Hence, a linear scale can be used to determine the currentin the coil.

Use of Galvanometer

(a) It is used to detect electric current in a circuit e.g., Wheatstone Bridge.

(b) It is convertedinto an ammeter by putting a small resistance parallel toit.

(c) It is converted into a voltmeter by putting a high resistance in series with it.

(d) It is used as an ohmmeter.

Sensitivity of a Galvanometer

A galvanometer is said to be sensitive if a small current flowing through its coil produces a large deflection in it.

(a) Current Sensitivity

The current sensitivity of a galvanometer is the deflection produced in the galvanometer per unit current flowing through it.

i.e. Current sensitivity $= \dfrac{\phi}{I} = \dfrac{NAB}{k}$

Current sensitivity of galvanometer can be increased either by

(i) Increasing the magnetic field B by using a strong permanent horse-shoe shaped magnet.

(ii) Increasing the number of turns N.

(iii) Increasing the area of the coil A. (but this will make the galvanometer bulky and ultimately less sensitive)

(iv) Using a spring having small value of restoring torque constant k.

(b) Voltage Sensitivity

Voltagesensitivity is the deflection produced in the galvanometer per unit voltage applied to it.

Voltage sensitivity $= \dfrac{\phi}{V} = \dfrac{\phi}{IR}$ i.e., voltage sensitivity $= \dfrac{NBA}{kR}$ (R= resistance of the coil)

Voltage sensitivity can be increased by

(i) Increasing N

(ii) Increasing B

(iii) Increasing A

(iv) Decreasing k and

(v) Decreasing R.

Advantage of a moving coil galvanometer

(a) A minutely small current in the electric circuit can be detected using an extremely sensitively galvanometer.

(b) A linear scale can be used to read the current, since deflection of the coil is directly proportional to the current.

(c) The external magnetic fields (e.g. horizontal component of earth's magnetic field) cannot effect the deflection of the coil of the galvanometer,because the magnetic field of the permanent magnet is very strong. Thus the galvanometer can be placed in any location.

(d) A dead beat type galvanometer is used.(The coil of a dead beat type galvanometer comes to rest quickly after deflecting to its equilibrium position, i.e it does not oscillate)

16. CYCLOTRON

Cyclotron is a device used to accelerate positively charged particles (like protons,α particles, deuteron, ions etc.) to acquire enough energy to carry out nuclear disintegrations.

Principle: It works on the following principle: A positively charged particle is made to accelerate through an electric field and using a strong magnetic field it is circled back to the region of the electric field, to accelerate it again and again to acquire sufficiently large amount of energy.

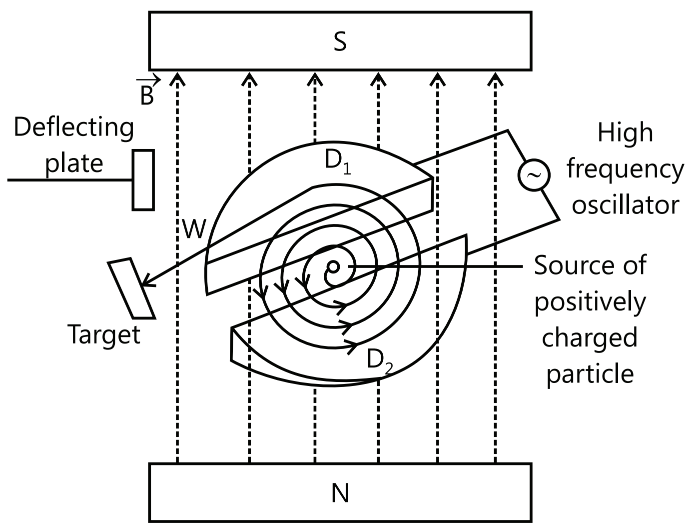

Figure 21.44: Cyclotron

Construction and Working: It consists of two hollow D-shaped metallic chambers D_1 and D_2 called dees. These dees are separated by a small gap where a source of positively charged particles is placed. Dees are connected to high frequency oscillator, which provides high frequency electric field across the gap of the dees which accelerates the particles. The magnetic field inside the dees is perpendicular to the plane of motion of particles and drives theminto a circular path. Suppose the particles start from rest and are accelerated towards chamber D_2. After completing a semicircle, when the particles reach the gap of the dees again, thereversal of the polarity of electric field ensures that the particlesareagain accelerated towards the other chamber D_1 by the electric field. Radius of the circular path increases with increase in speed, thusthe particles follow a spiral path (see Fig. 21.44)

Theory: The magnetic force on the positively charged particle provides the centripetal force to move in a circle of radius r.

$$\therefore \quad qvB = \frac{mv^2}{r} \text{ or } \quad r = \frac{mv}{qB} \qquad \text{... (i)}$$

Time taken by the particle to complete the semi-circle inside the dee,

$$t=\frac{\text{distance}}{\text{speed}}=\frac{\pi r}{v} \text{ or } t=\frac{\pi}{v}\times\frac{mv}{qB} \quad \text{or} \quad t=\frac{\pi m}{qB} \qquad \text{... (ii)}$$

This shows that time taken by the positively charged particle to complete any semi-circle (irrespective of its radius) is same

(a) Time Period: Let T be the period of the high frequency electric field, then the polarities of dees will change after time $\frac{T}{2}$.

The particle will be accelerated if time taken by it to describe the semi-circle is equal to $\frac{T}{2}$.

i.e. $$\frac{T}{2}=t=\frac{\pi m}{qB} \text{ or } T=\frac{2\pi m}{qB} \qquad \text{... (iii)}$$

(b) Cyclotron frequency: $f_c=\frac{1}{T}=\frac{qB}{2\pi m}$ ... (iv)

$\therefore$ Cyclotron angular frequency $\omega=2\pi f_c=\frac{qB}{m}$ ... (v)

(c) Energy gained: Energy gained by the positively charged particle in the cyclotron is given by $E=\frac{1}{2}mv^2$

From eqn.(i), we have $v=\frac{qBr}{m}$, then $E=\frac{1}{2}m \times \left(\frac{qBr}{m}\right)^2$ or $E=\frac{q^2B^2r^2}{2m}$ ... (vi)

Maximum energy gained by the positively charged particle will depend on the maximum value of radius of its path, i.e the radius of the dees.

$$E_{max}=\left(\frac{q^2B^2}{2m}\right)r_{max}^2 \qquad \text{... (vii)}$$

(d) Limitations of Cyclotron: Cyclotron cannot accelerate uncharged particles like neutron.

(e) Cyclotron cannot accelerate electrons because they have very small mass. Electrons start moving at a very high speed when they gain small energy in the cyclotron. The frequency of oscillating electric field required to keep them in phase with the electric field is very high, which is not feasible.

(f) The positively charged particle having large mass (i.e. ions) cannotbe accelerated after a certain speed in the cyclotron. When the speed of ion becomes comparable to the speed of light,the mass of ion increases as per the relation

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},$$

where m= mass of ion at velocity v, m_0=mass of ion at rest,cis speed of light (3×10^8 ms^{-1})

Time taken by the ion to describe semi-circular path increases as mass increases.So as the mass increases, the ion does not reach the gap between the two dees exactly at the instant the polarity is reversed and,it is not be accelerated further.

Uses of a Cyclotron

(a) It is used to produce radioactive material for medical purposes.

(b) It is used to synthesize fresh substances.

(c) It is used to improve the quality of solids by adding ions.

(d) It is used to bombard the atomic nuclei with highly accelerated particles to study the nuclear reactions.

Note: Sections after this are not in the syllabus of JEE ADVANCED but they are important for understanding the concepts completely.

Illustration 17:A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its dees is 60 cm. What is the kinetic energy (in MeV) of the proton beam produced by the acceleration? **(JEE MAIN)**

$\left(e = 1.60 \text{ x } 10^{-19} C, m_0 = 1.67 \text{ x } 10^{-27} \text{ kg, } 1\text{MeV} = 1.6 \text{ x } 10^{-13} J\right)$

Sol: The frequency of cyclotron is $f = \frac{Bq}{2\pi m}$ where q is the charge and m is the mass of the charged particle to be accelerated inside the cyclotron. The kinetic energy of the particle is $\left(\frac{mv^2}{2e}\right)$ in eV .

Cyclotron's oscillator frequency should be same as the proton's revolution frequency (in circular path)

$\therefore \quad f = \frac{Bq}{2\pi m}$ or

$B = \frac{2\pi mf}{q}$

Substituting the values in SI units, we have $B = \frac{(2)(22/7)(1.67 \text{x} 10^{-27})(10 \text{x} 10^6)}{1.6 \text{x} 10^{-19}}$ =0.67 T

The emerging beam of proton moves with the velocity

$v = \omega r = 2\pi f r = 2 \times \pi \times 10^7 \times 0.60 = 3.77 \times 10^7 \text{ ms}^{-1}$

Thus the kinetic energy (in MeV) is $\left(\frac{mv^2}{2e}\right) = \frac{1.67 \times 10^{-27} \times \left(3.77 \times 10^7\right)^2}{2 \times 1.6 \times 10^{-19}}$ eV = 7.42 MeV

17. MAGNETIC POLES AND BAR MAGNET

Two isolated charges of opposite signs are placed near each other, to form an electric dipole characterized by an electric dipole moment $\vec{p}$.On the other hand in magnetism an isolated 'magnetic charge' does not exist. The simplest magnetic structure is the magnetic dipole, characterized by a magnetic dipole moment $\vec{M}$.A current loop, a bar magnet and a solenoid of finite length are examples of magnetic dipoles.

When a magnetic dipole is placed in an external magnetic field $\vec{B}$, a torque act on it, given by $\vec{\tau} = \vec{M} \text{x} \vec{B}$

The magnetic field $\vec{B}$ due to a magnetic dipole at a point along its magnetic axis at (large) distance r from its center, is $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{r^3}$

A bar magnet has two poles (North and South) separated by a small distance. However, we cannot separate these poles apart. If a magnet is broken, the fragments prove to be dipoles and not isolated poles. If we break up a magnet into the electrons and nuclei that make up its atom, it will be found that even these elementary particle a re magnetic dipoles.

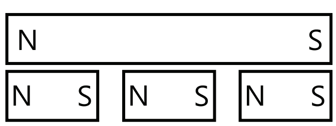

Figure 21.45: Poles of bar magnet

The poles of the bar magnet are modeled as follows:

(a) There are two types of magnetic charges; positive magnetic charge or North Pole and negative magnetic charge or South Pole. Every Pole has a strength m. The unit of Pole strength is A-m.

(b) A magnetic charge placed in a magnetic field experiences a force, $\vec{F} = m\vec{B}$. The force on positive magnetic charge is along the field and force on a negative magnetic charge is opposite to the field.

(c) A magnetic dipole is formed when a negative magnetic charge –m and a positive magnetic charge +m are placed at a small separation d. The magnetic dipole moment is, M=md. The direction of $\vec{M}$ is from –m to +m.

Geometrical Length and magnetic Length

In bar magnet, the poles are located at points which are slightly inside the two ends. The distance between the locations of the poles is called the magnetic length of the magnet. The distance between the ends is called the geometrical length of the magnet.

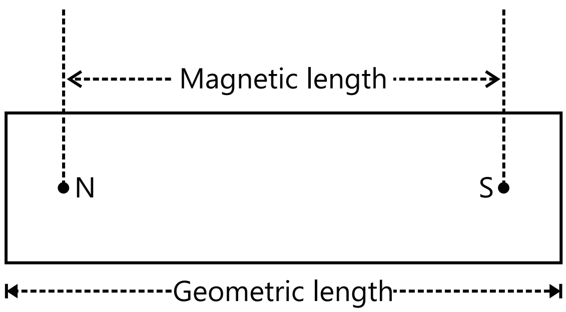

Figure 21.46: Geometric and Magnetic length of a bar magnet

Illustration18: Calculate the magnetic induction at a point 1 Å away from a proton, measured along its axis of spin. The magnetic moment of the proton is 1.4×10^{-26} A-m^2. **(JEE MAIN)**

Sol: On the axis of a magnetic dipole, magnetic induction is given by. $B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$

Substituting the values, we get $B = \frac{(10^{-7})(2)(1.4\text{x}10^{-26})}{(10^{-10})^3}$ = 2.8×10^{-3}T= 2.8 mT

18. MAGNETIC SUSCEPTIBILITY

For paramagnetic and diamagnetic materialsthe intensity of magnetization is directly proportional to the magnetic field intensity.

$$\vec{I} = \chi_m \vec{H}$$

The proportionality constant χ_m is called the magnetic susceptibility of the material. I and H have the dimensions of A-m^{-1} and the susceptibility χ_m is a dimensionless constant. For vacuum χ_m =0. For paramagnetic materials χ_m > 0, and for diamagnetic materials χ_m < 0 are diamagnetic.

19. CURIES'S LAW

When the temperature increase, due to thermal agitation the magnetization I decreases for a given magnetic intensity H,which means χ_m decreases as T increases. According to Curie's law, the susceptibility of a paramagnetic substance is inversely proportional to the absolute temperature: $\chi_m = \frac{c}{T}$ where c is a constant called the curie constant.

The magnetization of ferromagnetic material also decreases with increase in temperature, and on reaching a certain temperature,the ferromagnetic properties of the material disappear. This temperature is called Curie point (T_c). At temperatures above T_c ferromagnetic turns into a paramagnetic and its susceptibility varies with temperature as,

$$\chi_m = \frac{C'}{T - T_c}$$

where C′ is a constant.

20. PROPERTIES OF PARA-, DIA- AND FERRO-MAGNETISM

(a) **Paramagnetic Substances:** Example of such substances are platinum, aluminium, chromium, manganese, $CuSO_4$ solution, etc. They have the following properties:

(i) The substances, when placed in magnetic field, acquire a feeble magnetisation in the same sense as the applied field. Thus, the magnetic inductance inside the substance is slightly greater than outside to it.

(ii) In a uniform magnetic field, these substances rotate until their longest axes are parallel to the field.

(iii) These substances are attracted towards regions of stronger magnetic field when placed in a non-uniform magnetic field.

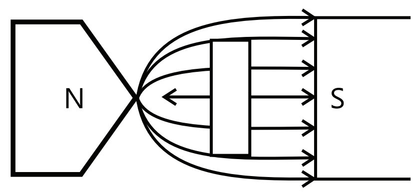

Figure 21.47: Paramagnetic material in strong magnetic field

(iv) Figure 21.47 shows a strong electromagnet in which one of the pole pieces is sharply pointed, while the other is flat. Magnetic field is much stronger near the pointed pole than near flat pole. If a small piece of paramagnetic material is suspended in this region, a force can be observed in the direction of arrow.

(v) If a paramagnetic liquid is filled in a narrow U-tube and one limb is placed in between the pole pieces of an electromagnet such that the level of the liquid is in line with the field, then the liquid will rise in the limb as the field is switched on.

(vi) For paramagnetic substances, the relative permeability μ_r is slightly greater than one.

(vii) At a given temperature the magnetic susceptibility χ_m does not change with the magnetizing field. However it varies inversely as the absolute temperature. As temperature increases χ_m decreases. At some higher temperature χ_m becomes negative and the substance become diamagnetic.

(b) **Diamagnetic Substances:** Examples of such substances are bismuth, antimony, gold, quartz, water, alcohol, etc. They have the following properties:

(i) These substances, when placed in a magnetic field, acquire feeble magnetization in a direction opposite to that of the applied field. Thus, the lines of induction inside the substance are smaller than those outside to it.

(ii) In a uniform field, these substances rotate until their longest axes are normal to the field.

(iii) In a non-uniform field, these substances move from stronger to weaker parts of the field.

(iv) If a diamagnetic liquid is filled in a narrow U-tube, and one limb is placed in between the pole of an electromagnet, the level depresses when the field is switched on.

(v) The relative permeability μ_r is slightly less than 1.

(vi) The susceptibility χ_m of such substances is always negative. It is constant and does not vary with field or the temperature.

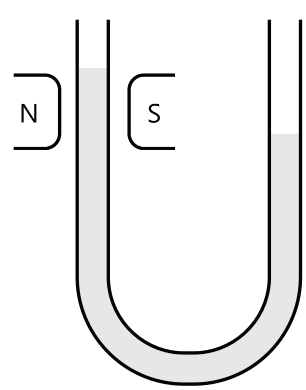

Figure 21.48: Liquid column of paramagnetic substance in strong magnetic field

(c) Ferromagnetic Substances: Examples of such substances are iron, nickel, steel, cobalt and their alloys. These substances resemble to a higher degree the paramagnetic substances with regards to their behaviour. They have the following additional properties:

(i) These substances are strongly magnetized by even a weak magnetic field.

(ii) The relative permeability is very large and is of the order of hundreds and thousands.

(iii) The susceptibility is positive and very large.

(iv) Susceptibility remains constant for very small values of $\vec{H}$, increases for larger values of $\vec{H}$ and then decreases for very large values of $\vec{H}$.

(v) Susceptibility decreases steadily with the rise of the temperature. Above a certain temperature, known as Curie temperature, the ferromagnetic substances become paramagnetic. For iron, it is 1000°C, 770°C for steel, 360°C for nickel, and 1150°C for cobalt.

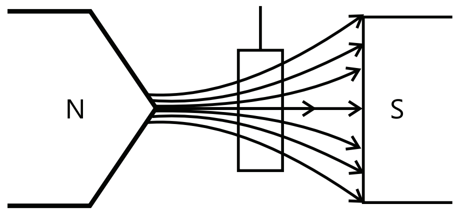

Figure 21.49: Diamagnetic substance in magnetic field

21. HYSTERESIS

Hysteresis is the dependence of the magnetic flux density B in a ferromagnetic material not only on its current magnetizing field H, but also on its history of magnetization or residual magnetization.

When a ferromagnetic material is magnetized in one direction, and then the applied magnetizing field is removed, then its magnetization will not be reduced to zero. It must be driven back to zero by a field in the opposite direction. If an alternating magnetic field intensity is applied to the material, its magnetization will trace out a loop called a hysteresis loop.

The phenomena in which magnetic flux density (B) lags behind the magnetizing field (H) in a ferromagnetic material during cycles of magnetization is called as hysteresis.

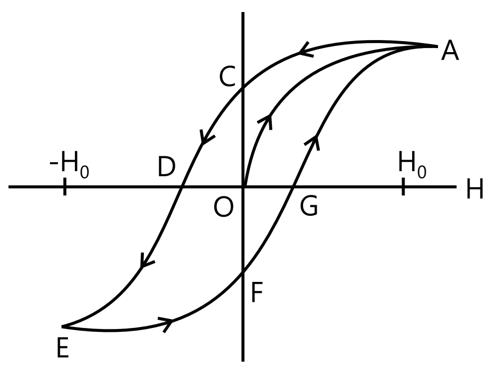

Figure 21.50: Hysteresis loop of I vs H

PROBLEM-SOLVING TACTICS

(a) General advice for this section involves learning of formulae and avoiding silly mistakes. Also it would be better to go by the usual algorithm of noting down known and unknown quantities and linking them.

(b) Much of manipulation and mathematical complexity is involved here which can't be avoided.

FORMULAE SHEET

(a) Magnetic Force on a charge moving with velocity $\vec{v}$ in magnetic field $\vec{B}$ is $\vec{F}_m = q\vec{v} \times \vec{B}$. Magnitude is $F_m = qvB\sin\theta$.

(b) Charged particle moving in uniform magnetic field

(i) Angular velocity $\omega = 2\pi f = \dfrac{|q|B}{m}$

(ii) Time period $T = \dfrac{2\pi m}{|q|B}$

(iii) Radius $r = \dfrac{mv}{qB} = \dfrac{m}{qB}\sqrt{\dfrac{2qV}{m}} = \dfrac{1}{B}\sqrt{\dfrac{2mV}{q}}$

(c) Helical Paths: Radius $r = \dfrac{mv_\perp}{qB}$ Pitch: $p = v_\perp T = v_\perp \dfrac{2\pi m}{|q|B}$

(d) The cyclotron $|q|B = 2\pi m f_{osc}$

(e) Crossed Fields: Lorentz Force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

(f) Trajectory of a charged particle in electric field $y = \dfrac{|q|Ex^2}{2mv^2}$

(g) Magnetic force on current element $d\vec{F} = Id\vec{\ell} \times \vec{B}$

(h) Magnetic force on a conductor in uniform field $\vec{F} = I\vec{L} \times \vec{B}$

(i) Magnetic dipole moment of a current coil having N turns $\vec{p}_m = NIA\hat{n}$

(j) Torque on a current coil $\vec{\tau} = \vec{p}_m \times \vec{B}$

(k) Potential energy of current coil $U = -\vec{p}_m . \vec{B}$

(l) Biot-Savart Law $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{\ell} \times \vec{r}}{r^3}$, $dB = \dfrac{\mu_0}{4\pi}\dfrac{Id\ell\sin\theta}{r^2}$

(m) Magnetic field at center of an arc subtending angle θ, $B = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{I\theta}{R}$

(n) Magnetic field at a point on the axis of a N turn coil $B = \dfrac{\mu_0}{2}\dfrac{NIR^2}{\left(z^2 + R^2\right)^{3/2}}$

(o) Magnetic field at center of N turn coil $B = \dfrac{\mu_0}{2}\dfrac{NI}{R}$

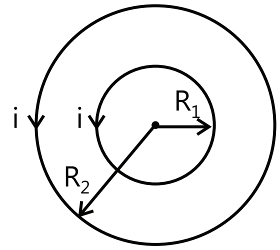

Figure 21.51

(p) Concentric coils with equal turns

(i) Similar currents flowing in the same direction

Net magnetic field, $$B = \frac{\mu_0}{2}\frac{NI}{R_1} + \frac{\mu_0}{2}\frac{NI}{R_2} = \frac{\mu_0}{2}NI\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(ii) Similar currents flowing in the opposite direction

Net magnetic field, $$B = \frac{\mu_0}{2}\frac{NI}{R_1} - \frac{\mu_0}{2}\frac{NI}{R_2} = \frac{\mu_0}{2}NI\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

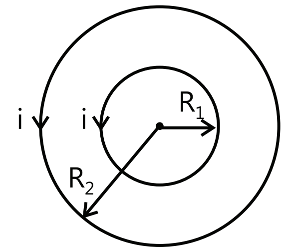

Figure 21.52

(q) Mutually perpendicular coils

Net Magnetic field, $$B = \sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{2\pi I}{R}$$

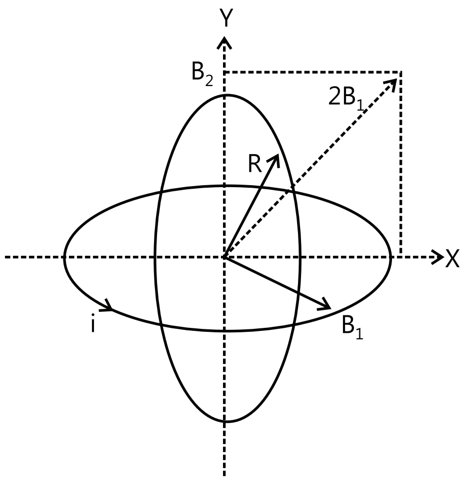

Figure 21.53

(r) Dispatched coils

Net Magnetic Field, $$B = \sqrt{2}\frac{\mu_0}{2}\frac{IR^2}{(R^2+x^3)^{3/2}} = \frac{\mu_0 IR^2}{\sqrt{2}(x^2+R^2)^{3/2}}$$

(s) Infinite straight wire $B = \frac{\mu_0 I}{2\pi R}$

(t) Semi-infinite straight wire $B = \frac{\mu_0 I}{4\pi R}$

(u) Force per unit length between two parallel currents separated by distance d, $\frac{dF}{d\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$

(v) Ampere's law $\oint \vec{B}.d\vec{\ell} = \mu_0 I_{enc}$

(w) Field inside infinite straight wire of circular cross-section $B = \frac{\mu_0 I}{2\pi R^2}r$

(x) Magnetic Field inside long solenoid having n turns per unit length $B = \mu_0 nI$

(y) Magnetic Field inside toroid having N turns $B = \frac{\mu_0 NI}{2\pi r}$

(z) Magnetic field due to bar magnet at end-on position $B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$

(aa) Magnetic field due to bar magnet at broadside-on position $B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$

(ab) Moving Coil Galvanometer $I = \frac{k\phi}{NAB}$

(ac) Magnetic field Intensity H, in vacuum is, $H = \frac{B}{\mu_0}$

(ad) Magnetic field Intensity H, in a medium is, $H = \frac{B}{\mu_r\mu_0}$

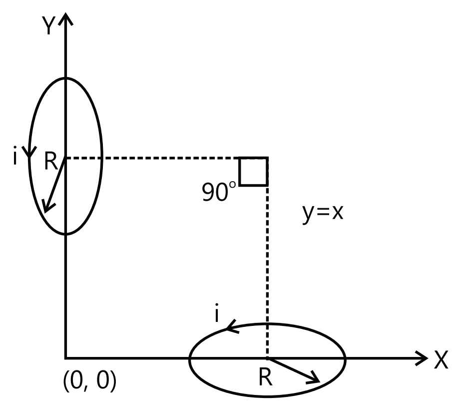

Figure 21.54