MasterJEE

IIT-JEE | Medical | Foundations

Time : 3 hrs. **M.M. : 360**

Answers & Solutions

for

JEE (MAIN)-2019

(Online CBT Mode)

(Physics, Chemistry and Mathematics)

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 **(four)** marks for each correct response.
4. *Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for a question in the answer sheet.*
5. There is only one correct response for each question.

PHYSICS

1. An amplitude modulated signal is given by V(t) = 10 [1 + 0.3cos (2.2 × $10^4 t$)] sin(5.5 × $10^5 t$). Here t is in seconds. The sideband frequencies (in kHz) are, [Given π =22/7]

(1) 1785 and 1715
(2) 178.5 and 171.5
(3) 89.25 and 85.75
(4) 892.5 and 857.5

Answer (3)

Sol. ω_U = (2.2 × 10^4 + 5.5 × 10^5) rad/s

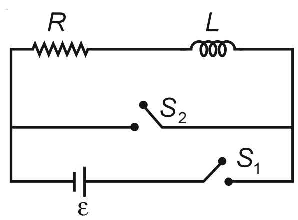
ω_L = (5.5 × 10^5 – 2.2 × 10^4) rad/s

ω_U = (2.2 + 55) ×10^4 = 57.2 × 10^4 rad/s

$$f_U = \frac{572}{2\pi}\text{ kHz} \simeq 91\text{ kHz}$$

$$f_L = \frac{528}{2\pi}\text{ kHz} \simeq 84\text{ kHz}$$

2. In the circuit shown,



the switch S_1 is closed at time t = 0 and the switch S_2 is kept open. At some later time(t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time 't' is given by

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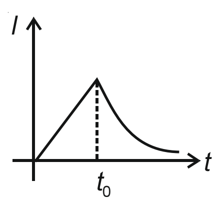

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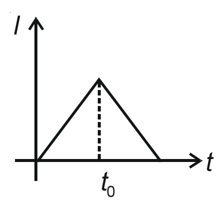

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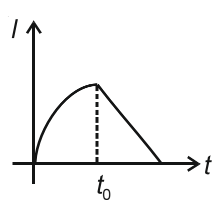

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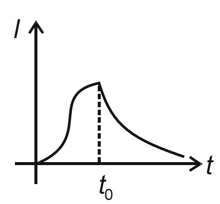
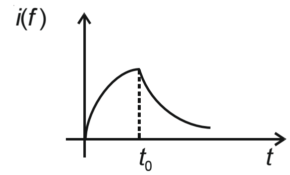


Answer (3)

Sol. $i(f) = \frac{V}{R}(1 - e^{-\frac{R}{L}t}),\quad t \le t_0$

$i(f) = \frac{V}{R}e^{-\frac{R}{L}(t-t_0)},\quad t > t_0$



* The closest to appropriate graph is in option 3.

3. The force of interaction between two atoms is given by $F = \alpha\beta\exp\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is

(1) $M^0L^2T^{-4}$
(2) M^2LT^{-4}
(3) MLT^{-2}
(4) $M^2L^2T^{-2}$

Answer (2)

Sol. $[x^2] = [\alpha KT]$

$[\alpha ML^2T^{-2}] = L^2$

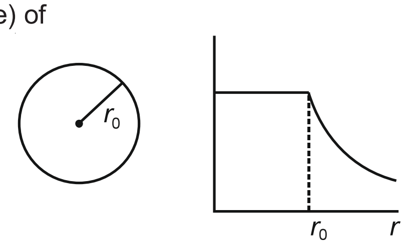
$[\alpha] = M^{-1}T^2$

$[\alpha{\cdot}\beta] = MLT^{-2}$

$M^{-1}T^{+2}[\beta] = MLT^{-2}$

$[\beta] = M^2LT^{-4}$

4. The given graph shows variation (with distance r from centre) of

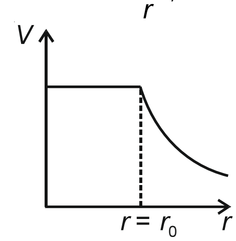


(1) Potential of a uniformly charged spherical shell
(2) Electric field of a uniformly charged sphere
(3) Electric field of uniformly charged spherical shell
(4) Potential of a uniformly charged sphere

Answer (1)

Sol. For spherical shell $V = \frac{K.q}{r_0},\ r \le r_0$

$= \frac{Kq}{r},\ r > r_0$



5. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

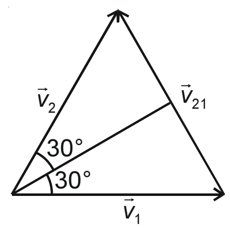
(1) 10 m/s (2) Zero

(3) $10\sqrt{3}$ m/s (4) $10\sqrt{2}$ m/s

Answer (1)

Sol. $\vec{v}_1 + \vec{v}_{21} = \vec{v}_2$

$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1$



$= |\vec{v}_{21}| = 2v\sin 30°$

$= 2v \times \frac{1}{2} = v$

6. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state, in terms of Bohr radius a_0, will be

(hc = 12500 eV-Å)

(1) $4a_0$ (2) $9a_0$

(3) $25a_0$ (4) $16a_0$

Answer (4)

Sol. $\Delta E = \frac{hc}{\lambda}$

$\Delta E = \frac{12500}{980} = 12.76$ eV

$E_n - E_1 = 12.76$

$E_n = E_1 + 12.76$

$= -13.6 + 12.76$

$E_n = -0.84 \text{ eV} = \frac{-13.6}{n^2}$ eV

$\Rightarrow n = 4$

$\Rightarrow r_n = 16a_0$

7. Two equal resistances when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be

(1) 60 W (2) 30 W

(3) 120 W (4) 240 W

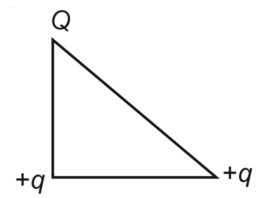
Answer (4)

Sol. When in series $P_0 = \frac{P_1P_2}{P_1 + P_2} = 60$ W

$P_0 = \frac{P}{2}$ $P = 120$ watt

When in parallel $P_0' = 2P = 2 \times 120 = 240$ W

8. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right-angle isosceles triangles as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is



(1) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

(2) $+q$

(3) $-2q$

(4) $\frac{-q}{1+\sqrt{2}}$

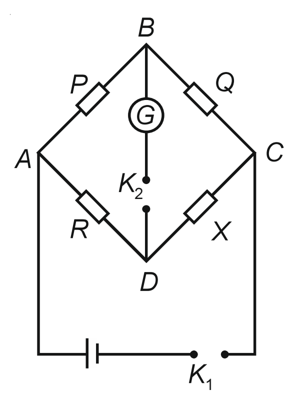
Answer (1)

Sol. $0 = U = \frac{kq^2}{a} + \frac{kQq}{a} + \frac{kQq}{\sqrt{2}a}$

$-q = Q\left(1 + \frac{1}{\sqrt{2}}\right)$

$Q = \frac{-q(\sqrt{2})}{\sqrt{2}+1}$

9. In a Wheatstone bridge (see fig.), Resistances P and Q are approximately equal. When $R = 400\ \Omega$, the bridge is balanced. On interchanging P and Q, the value of R, for balance, is 405 Ω. The value of X is close to



(1) 404.5 ohm (2) 401.5 ohm

(3) 402.5 ohm (4) 403.5 ohm

Answer (3)

Sol. $\frac{P}{R} = \frac{Q}{X}$

$\frac{P}{400} = \frac{Q}{X}$

$\frac{Q}{405} = \frac{P}{X} \quad \Rightarrow P = \frac{QX}{405}$

$\frac{QX}{400 \times 405} = \frac{Q}{X}$

$X = \sqrt{400 \times 405}$

$X = 402.5\ \Omega$

10. There are two long co-axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self inductance of the inner-coil is

(1) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$ (2) $\frac{n_2}{n_1}$

(3) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$ (4) $\frac{n_1}{n_2}$

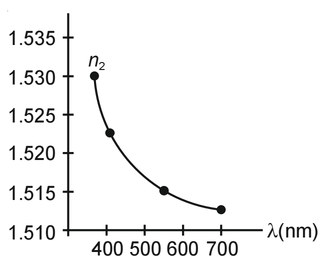
Answer (2)

Sol. $M = \mu_0 n_1 n_2 \pi r_1^2 l$

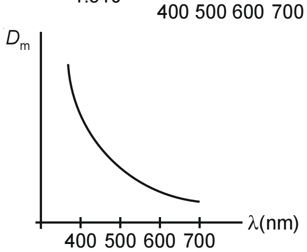
$L = \mu_0 n_1^2 \pi r_1^2 l$

$\frac{M}{L} = \frac{n_2}{n_1}$

11. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?



(1)



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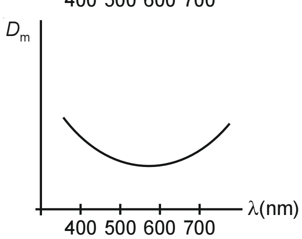

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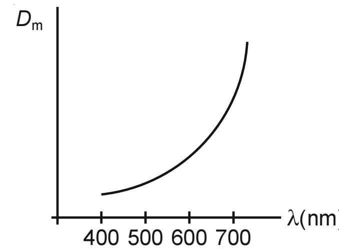

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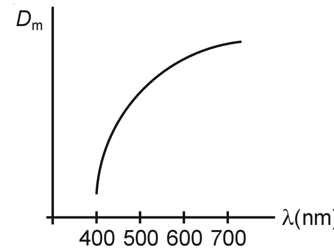
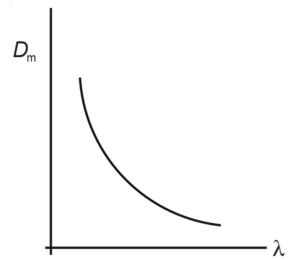


Answer (1)

Sol. $D_m = (\mu - 1)A$



12. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210 s will be

(1) 1 (2) 3

(3) 2 (4) $\frac{1}{9}$

Answer (Bonus)

Sol. $\frac{KE}{PE} = \frac{\frac{1}{2}kA^2 - \frac{1}{2}kA^2\sin^2\frac{\pi t}{90}}{\frac{1}{2}kA^2\sin^2\frac{\pi t}{90}} = \frac{1}{3}$

13. In an experiment, electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied. [Charge of the electron = 1.6×10^{-19} C, Mass of the electron = 9.1×10^{-31} kg]

(1) 7.5×10^{-3} m (2) 7.5 m

(3) 7.5×10^{-2} m (4) 7.5×10^{-4} m

Answer (2)

Sol. $r = \frac{mv}{Bq} = \frac{\sqrt{2mqV}}{Bq}$

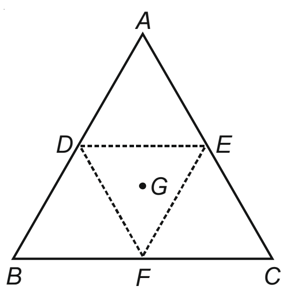
$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times V}}{B\sqrt{q}}$

$$=\frac{\sqrt{2\times 9.1\times 10^{-31}\times 500}}{100\times 10^{-3}\sqrt{1.6\times 10^{-19}}}$$

$$=\frac{1}{100\times 10^{-3}}\frac{\sqrt{2\times 9.1\times 500\times 10^{-12}}}{1.6}$$

$$=\frac{75.4\times 10^{-6}}{100\times 10^{-3}}=7.5\times 10^{-4}\text{ m}$$

14. An equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then



(1) $I=\frac{3}{4}I_0$ (2) $I=\frac{15}{16}I_0$

(3) $I=\frac{I_0}{4}$ (4) $I=\frac{9}{16}I_0$

Answer (2)

Sol. $I_0 = K\cdot ML^2$

$$I_1=K\cdot\frac{M}{4}\left(\frac{L}{2}\right)^2=K\cdot\frac{ML^2}{16}$$

$$I_2 = I_0 - I_1$$

$$I_2=\frac{15}{16}\text{ KML}^2$$

$$=\frac{15}{16}I_0$$

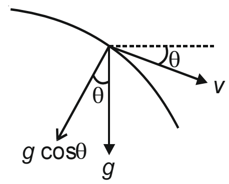
15. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1 s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is

(1) 5.1 m (2) 2.5 m

(3) 2.8 m (4) 10.3 m

Answer (3)

Sol. $T=\frac{2u\sin\theta}{g}$



$$=\frac{2\times 10}{10}\times\frac{\sqrt{3}}{2}$$

$$T=\sqrt{3}\ s$$

$$V_y=5\sqrt{3}-10=-1.34\text{ ms}^{-1}$$

$$V_x=10\times\frac{1}{2}=5\text{ ms}^{-1}$$

$$|\tan\theta|=\left|\left(-\frac{1.34}{5}\right)\right|$$

$$\theta = 15°$$

$$R=\frac{V^2}{g\cos\theta}=\frac{26.79}{10\times 0.97}=2.77\text{ m}$$

$$\simeq 2.8\text{ m}$$

16. Equation of travelling wave on a stretched string of linear density 5 g/m is y = 0.03 sin(450t – 9x) where distance and time are measured in SI units. The tension in the string is

(1) 10 N (2) 7.5 N

(3) 5 N (4) 12.5 N

Answer (4)

Sol. $Y=A\sin\omega\left(t-\frac{x}{v}\right)$

V = 50 m/s by comparison.

$$50=\sqrt{\frac{T}{\mu}}$$

$$T = 2500\times 5\times 10^{-3}$$

$$T = 12.5\text{ N}$$

17. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is

(1) 4 RT (2) 12 RT

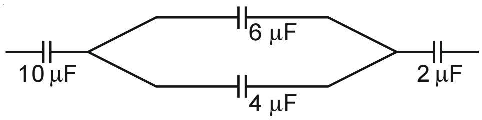
(3) 15 RT (4) 20 RT

Answer (3)

Sol. $U=3\times\frac{5}{2}RT+5\times\frac{3}{2}RT$

$\Rightarrow$ $U = 15\ RT$

18. In the figure shown below, the charge on the left plate of the 10 μF capacitor is –30 μC. The charge on the right plate of the 6 μF capacitor is



(1) +18 μC (2) –12 μC

(3) +12 μC (4) –18 μC

Answer (1)

Sol. Let charge be Q_1 & Q_2

$$\frac{Q_1}{6} = \frac{Q_2}{4}$$

$\Rightarrow$ $Q_1 + Q_2 = 30$

$\Rightarrow$ $Q_1 = 18\ \mu C$, $Q_2 = 12\ \mu C$

19. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be

(1) 0.92×10^{-3} m/s away from the lens

(2) 2.26×10^{-3} m/s away from the lens

(3) 1.16×10^{-3} m/s towards the lens

(4) 3.22×10^{-3} m/s towards the lens

Answer (3)

Sol. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$u = -20$ m, $f = 0.3$

$$\frac{1}{v} = \frac{1}{0.3} - \frac{1}{20}$$

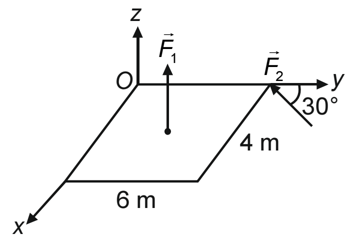
$$\frac{1}{v} = \frac{10}{3} - \frac{1}{20}$$

$$v = \frac{60}{197}\ \text{m}$$

$$v_{image} = \left(\frac{3}{197}\right)^3 \times 5$$

$= 1.16 \times 10^{-3}$ m/s toward the lens.

20. A slab is subjected to two forces $\vec{F_1}$ and $\vec{F_2}$ of same magnitude F as shown in the figure. Force $\vec{F_2}$ is in XY-plane while force F_1 acts along z-axis at the point $(2\vec{i} + 3\vec{j})$. The moment of these forces about point O will be



(1) $(3\hat{i} - 2\hat{j} + 3\hat{k})F$ (2) $(3\hat{i} + 2\hat{j} - 3\hat{k})F$

(3) $(3\hat{i} + 2\hat{j} + 3\hat{k})F$ (4) $(3\hat{i} - 2\hat{j} - 3\hat{k})F$

Answer (1)

Sol. $\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$

$$\vec{\tau}_1 = (2\hat{i} + 3\hat{j}) \times F\hat{k} = F(3\hat{i} - 2\hat{j})$$

$$\vec{\tau}_2 = 6\hat{j} \times F(-\sin 30^\circ \hat{i} - \cos 30^\circ \hat{j})$$

$$\vec{\tau}_2 = 3F\hat{k}$$

$$\tau = F(3\hat{i} - 2\hat{j} + 3\hat{k})$$

21. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by

(1) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$

(2) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

(3) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$

(4) $\left(\sqrt{n}, \sqrt{n}\right)$

Answer (2)

Sol. $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$V = \frac{1}{\sqrt{k\varepsilon_0 \mu_0}}$$

$$\frac{C}{V} = \sqrt{k} = n$$

$$\frac{1}{2}\varepsilon_0 E_0^2 C = \frac{1}{2}\varepsilon_0 k E^2 V$$

$$\frac{E_0}{E} = \sqrt{n}$$

Similarly,

$$\frac{B_0}{B} = \frac{1}{\sqrt{n}}$$

22. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be

(1) $\frac{3}{4}\rho v^2$ (2) $\frac{1}{4}\rho v^2$

(3) $\frac{1}{2}\rho v^2$ (4) ρv^2

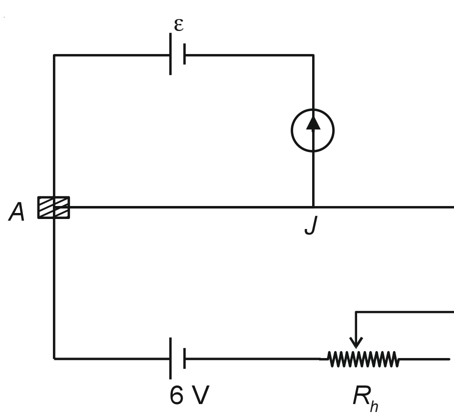
Answer (1)

Sol. Let area be A.

$$F = \frac{\rho A}{4} \times v^2 + \frac{\rho A}{4} \times 2v^2$$

$$\text{Pressure} = \frac{3\rho A v^2}{4A} = \frac{3\rho}{4} v^2$$

23. The resistance of the metre bridge AB in given figure is 4 Ω. With a cell of emf ε = 0.5 V and rheostat resistance R_h = 2 Ω the null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$ the same null point J is found for R_h = 6 Ω.The emf ε_2 is



(1) 0.6 V (2) 0.5 V

(3) 0.3 V (4) 0.4 V

Answer (3)

Sol. Case 1 :

$$E_1 = \frac{6 \times 4x}{4+2}$$

$$E_2 = \frac{6 \times 4x}{4+6}$$

6 × 0.5 = E_2

E_2 = 0.3 V

24. A body of mass 1 kg falls freely from a height of 100 m, on a platform of mass 3 kg which is mounted on a spring having spring constant k = 1.25 × 10^6 N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that g = 10 ms^{-2}, the value of x will be close to

(1) 80 cm (2) 8 cm

(3) 4 cm (4) 40 cm

Answer (Bonus)

Sol. Initial compression = $\frac{3 \times 10}{k}$, since spring constant is high. So initial compression is low.

Let v_1 be velocity after collision.

$4v_1 = v_0$

$v_0 = \sqrt{2g \times 100}$

$\frac{1}{2} \times 4 \times v_1^2 = \frac{1}{2}kx^2$

x = 2 cm

None of the option is correct.

25. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to

(1) 0.74 (2) 0.94

(3) 0.80 (4) 0.85

Answer (4)

Sol. $\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$

$$I = 4I_0 \cos^2(\pi/8)$$

$$\frac{I}{4I_0} = \cos^2(\pi/8) = 0.85$$

26. A satellite is revolving in a circular orbit at a height h from the earth surface, such that $h << R$ where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of eath is

(1) $\sqrt{2gR}$ (2) $\sqrt{gR}(\sqrt{2}-1)$

(3) $\sqrt{gR}$ (4) $\sqrt{\frac{gR}{2}}$

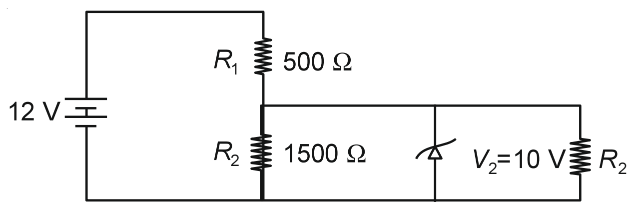
Answer (2)

Sol. $v_0 = \sqrt{2gR}$

$v_e = \sqrt{2gR}$

$\Delta v = \sqrt{gR}\,(\sqrt{2} - 1)$

27. In the given circuit the current through Zener Diode is close to



(1) 6.7 mA

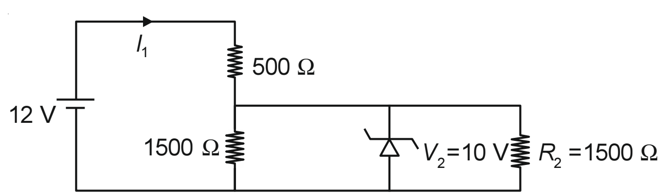
(2) 0.0 mA

(3) 4.0 mA

(4) 6.0 mA

Answer (2)

Sol.



$$(V_{R_2})_{max} = \frac{12 \times 750}{1250}$$

$(V_{R_2})_{max} < V_Z$

So, current through Zener diode is zero.

28. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x = constant, then x is

(1) $\frac{2}{5}$

(2) $\frac{2}{3}$

(3) $\frac{5}{3}$

(4) $\frac{3}{5}$

Answer (1)

Sol. For adiabatic

PV^γ = constant

TV^x = constant

$\Rightarrow$ PVV^x = constant

$x + 1 = \gamma$

$x = 2/5$

29. Ice at –20°C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount off ice added to the water was close to

(Specific heat of water = 4.2 J/g/°C

Specific heat of Ice = 2.1 J/g/°C

Heat of fusion of water at 0°C = 334 J/g)

(1) 100 g

(2) 40 g

(3) 50 g

(4) 60 g

Answer (2)

Sol. Heat lost by water = 50 × 40 = 2000 cal.

Let amount of ice be x g.

$$x \times \frac{1}{2} \times 20 + (x - 20) \times 80 = 2000$$

$90x = 3600$

$x = 40$ g

30. If the deBroglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to :

(Speed of light = 3×10^8 m/s

Planck's constant = 6.63×10^{-34} J-s

Mass of electron = 9.1×10^{-31} kg)

(1) 1.7×10^6 m/s

(2) 1.45×10^6 m/s

(3) 1.8×10^6 m/s

(4) 1.1×10^6 m/s

Answer (2)

Sol. $$\lambda_1 = \frac{10^{-3} \times 3 \times 10^8}{6 \times 10^{14}} = 0.5 \times 10^{-9}$$

$$\lambda_1 = 5 \times 10^{-10} \text{ m}$$

$$v = \frac{h}{m\lambda_1} = \frac{6.6 \times 10^{-34}}{5 \times 10^{-10} \times 9.1 \times 10^{-31}}$$

$= 1.45 \times 10^6$ m/s

CHEMISTRY

1. Match the ores (**column A**) with the metals (**column B**):

(Column A) Ores		(Column B) Metals
(I) Siderite	(a)	Zinc
(II) Kaolinite	(b)	Copper
(III) Malachite	(c)	Iron
(IV) Calamine	(d)	Aluminium

(1) (I) - (a); (II) - (b); (III) - (c); (IV) - (d)
(2) (I) - (c); (II) - (d); (III) - (a); (IV) - (b)
(3) (I) - (c); (II) - (d); (III) - (b); (IV) - (a)
(4) (I) - (b); (II) - (c); (III) - (d); (IV) - (a)

Answer (3)

Sol. Siderite = $FeCO_3$

Calamine = $ZnCO_3$

Malachite = $CuCO_3.Cu(OH)_2$

Kaolinite = $Al_2Si_2O_5(OH)_4$

2. The concentration of dissolved oxygen (DO) in cold water can go upto:

(1) 14 ppm (2) 16 ppm
(3) 10 ppm (4) 8 ppm

Answer (3)

Sol. DO in cold water can go upto 10 ppm

(Ref - NCERT)

3. The freezing point of a diluted milk sample is found to be –0.2°C, while it should have been –0.5°C for pure milk. How much water has been added to pure milk to make the diluted sample?

(1) 3 cups of water and 2 cups of pure milk
(2) 1 cup of water and 2 cups of pure milk
(3) 2 cups of water to 3 cups of pure milk
(4) 1 cup of water to 3 cups of pure milk

Answer (1)

Sol. Freezing point of diluted milk = – 0.2°C

$$\Delta T_f' = 0.2°C$$

Freezing point of pure milk = – 0.5°C

$$\Delta T_f = 0.5°C$$

$$\frac{\Delta T_f}{\Delta T_f'} = \frac{k_f}{k_f}\frac{m}{m'}$$

$$\Rightarrow \frac{0.5}{0.2} = \frac{w_1'}{w_1}$$

$$\Rightarrow \frac{w_1'}{w_1} = \frac{5}{2}$$

2 cups of pure milk mixed with 3 cups of water overall 5 cups of diluted milk.

4. The correct match between **item (I)** and **item (II)** is:

Item - I	Item - II
(A) Norethindrone	(P) Anti-biotic
(B) Ofloxacin	(Q) Anti-fertility
(C) Equanil	(R) Hypertension
	(S) Analgesics

(1) (A) → (R) ; (B) → (P) ; (C) → (R)
(2) (A) → (R) ; (B) → (P) ; (C) → (S)
(3) (A) → (Q) ; (B) → (P) ; (C) → (R)
(d) (A) → (Q) ; (B) → (R) ; (C) → (S)

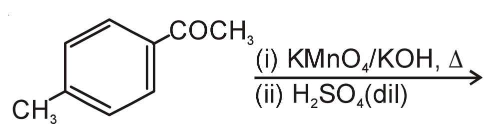
Answer (3)

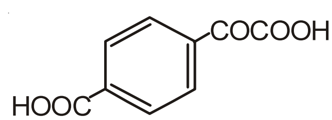
Sol. (A) Norethindrone - Antifertility (Q)

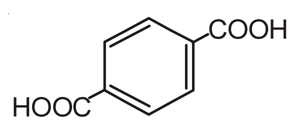
(B) Ofloxacin - Anti-biotics (P)

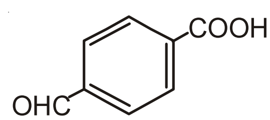
(C) Equanil - Tranquilizer (R)

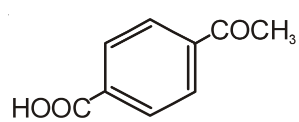
5. The major product of the following reaction is



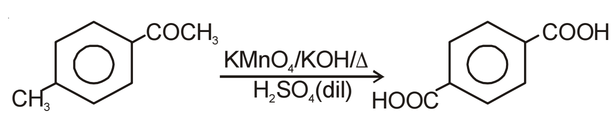
(1) 

(2) 

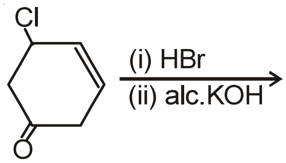
(3) 

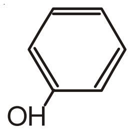
(4) 

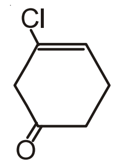
Answer (2)

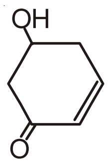
Sol. 

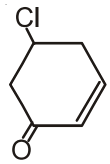
6. The major product of the following reaction is:



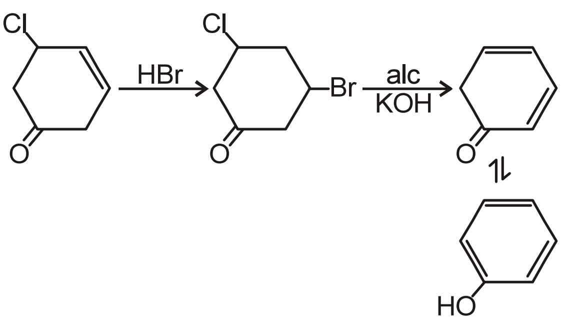
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

7. The chloride that CANNOT get hydrolysed is:

(1) $PbCl_4$ (2) CCl_4

(3) $SnCl_4$ (4) $SiCl_4$

Answer (2)

Sol. CCl_4 cannot be hydrolysed due to absence of d orbitals. Carbon cannot extend its coordination number beyond four.

8. If a reaction follows the Arrhenius equation, the plot lnk vs $\frac{1}{(RT)}$ gives straight line with a gradient (–y) unit. The energy required to activate the reactant is:

(1) yR unit (2) y/R unit

(3) –y unit (4) y unit

Answer (4)

Sol. $k = Ae^{-E_a/RT}$

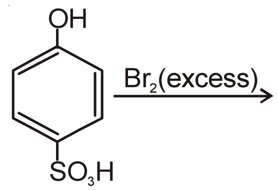
$$\ln k = \ln A - \frac{E_a}{RT}$$

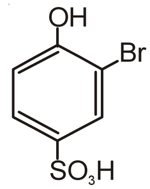
For lnk versus $\frac{1}{RT}$, slope = –y

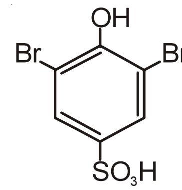
$-y = -E_a$

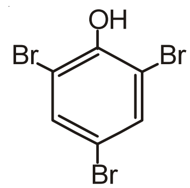
$\Rightarrow E_a = y$

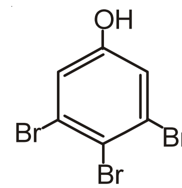
9. The major product of the following reaction is



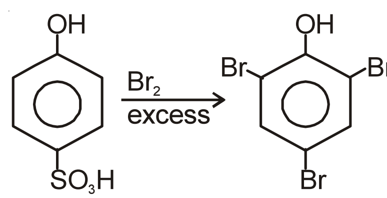
(1) 

(2) 

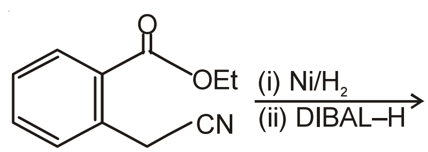
(3) 

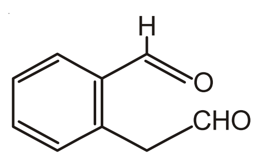
(4) 

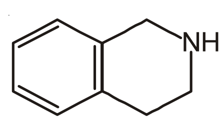
Answer (3)

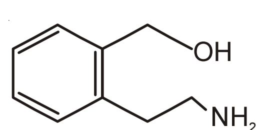
Sol. 

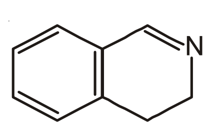
10. The major product of the following reaction is:



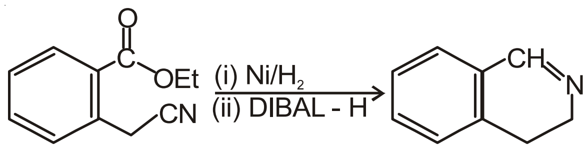
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. 

11. A solid having density of 9×10^3 kg m^{-3} forms face centred cubic crystals of edge length $200\sqrt{2}$ pm. What is the molar mass of the solid?

[Avogadro constant $\cong$ 6×10^{23} mol^{-1}, $\pi \cong 3$]

(1) 0.0305 kg mol^{-1} (2) 0.4320 kg mol^{-1}

(3) 0.0432 kg mol^{-1} (4) 0.0216 kg mol^{-1}

Answer (1)

Sol. $a = \dfrac{Z \times M}{N \times a^3}$

$$9 \times 10^3 = \frac{4 \times M}{\left(200 \times \sqrt{2} \times 10^{-12}\right)^3 6 \times 10^{23}}$$

M = 0.03 kg/mole

12. The correct match between items I and II is

Item-I (Mixture)	Item-II (Separation method)
(A) H_2O : Sugar	(P) Sublimation
(B) H_2O : Aniline	(Q) Recrystallization
(C) H_2O : Toluene	(R) Steam distillation
	(S) Differential extraction

(1) (A) $\rightarrow$ (R), (B) $\rightarrow$ (P), (C) $\rightarrow$ (S)

(2) (A) $\rightarrow$ (S), (B) $\rightarrow$ (R), (C) $\rightarrow$ (P)

(3) (A) $\rightarrow$ (Q), (B) $\rightarrow$ (R), (C) $\rightarrow$ (P)

(4) (A) $\rightarrow$ (Q), (B) $\rightarrow$ (R), (C) $\rightarrow$ (S)

Answer (4)

Sol. H_2O : Sugar – Recrystallisation.

Sugar is purified by this method.

H_2O : Aniline – Separation by steam distillation.

13. The correct order of the atomic radii of C, Cs, Al, and S is

(1) S < C < Al < Cs (2) C < S < Cs < Al

(3) S < C < Cs < Al (4) C < S < Al < Cs

Answer (4)

Sol. Carbon is smallest being 2nd period element and Cs belongs to 6th period so largest. On moving from left to right, size decreases so C < S < Al < Cs

14. For the cell $Zn(s)|Zn^{2+}(aq)||M^{x+}(aq)|$ M(s), different half cells and their standard electrode potentials are given below

$M^{x+}(aq)/M(s)$	$Au^{3+}(aq)/Au(s)$	$Ag^{+}(aq)/Ag(s)$	$Fe^{3+}(aq)/Fe^{2+}(aq)$	$Fe^{2+}(aq)/Fe(s)$
$E^\circ_{M^{x+}/M}/(V)$	1.40	0.80	0.77	– 0.44

If $E^\circ_{Zn^{2+}/Zn} = -0.76$ V, which cathode will give a maximum value of E°_{cell} per electron transferred?

(1) Fe^{2+}/Fe (2) Ag^{+}/Ag

(3) Fe^{3+}/Fe^{2+} (4) Au^{3+}/Au

Answer (4)

Sol. $E_{cell} = (E^\circ_{R.P})_{Cathode} - (E^\circ_{R.P})_{Anode}$

All electrodes act as cathode w.r.t. Zn so the ion which has highest reduction potential will give maximum value of E°_{cell} so Au^{3+}/Au produce highest E°_{cell}.

15. Consider the reaction

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

The equilibrium constant of the above reaction is K_P. If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by (Assume that $P_{NH_3} << P_{total}$ at equilibrium)

(1) $\dfrac{K_P^{1/2} P^2}{4}$ (2) $\dfrac{3^{3/2} K_P^{1/2} P^2}{4}$

(3) $\dfrac{K_P^{1/2} P^2}{16}$ (4) $\dfrac{3^{1/2} K_P^{1/2} P^2}{16}$

Answer (4)

Sol. $\underset{P_0 - 2x}{2NH_3(g)} \rightleftharpoons \underset{x}{N_2(g)} + \underset{3x}{3H_2(g)}$, $K = \dfrac{1}{K_P}$

$\therefore$ $P = P_0 + 2x$

and $x = \dfrac{P_0}{2} \Rightarrow 4x = P$

$\therefore$ $K = \dfrac{1}{K_P} = \dfrac{x(3x)^3}{P_{NH_3}{}^2}$

$\Rightarrow$ $P_{NH_3}{}^2 = 3^3 x^4 K_P$

$\Rightarrow$ $P_{NH_3} = 3^{\frac{3}{2}} x^2 K_P^{\frac{1}{2}}$

$= \dfrac{3^{\frac{3}{2}} \cdot P^2 \cdot K_P^{\frac{1}{2}}}{16}$

16. For the chemical reaction $X \rightleftharpoons Y$, the standard reaction Gibbs energy depends on temperature T (in K) as

$$\Delta_r G^\circ \text{ (in kJ mol}^{-1}) = 120 - \frac{3}{8}T$$

The major component of the reaction mixture at T is

(1) Y if T = 280 K (2) X if T = 315 K

(3) X if T = 300 K (4) X if T = 350 K

Answer (2)

Sol. If ΔG° is positive then $K_{eq} < 1$.

so, $\frac{[Y]}{[X]} < 1$

If ΔG° is negative then $K_{eq} > 1$

so, $\frac{[Y]}{[X]} > 1$

$$\Delta G^\circ = 120 - \frac{3}{8}T$$

At 315 K, ΔG° = 120 – 118.125 = positive

so, [X] > [Y].

17. An organic compound is estimated through Dumas method and was found to evolve 6 moles of CO_2, 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is

(1) $C_6H_8N_2$ (2) $C_{12}H_8N$

(3) C_6H_8N (4) $C_{12}H_8N_2$

Answer (1)

Sol. Mol of CO_2 = 6 so mol of C is = 6

Mol of H_2O = 4 so mol of H is = 8

Mol of N_2 = 1 so mol of N is = 2

Formula $C_6H_8N_2$

18. Match the metals (column I) with the coordination compound(s)/enzyme(s) (column II)

(Column I) Metals	**(Column II) Coordination compound(s)/ enzyme(s)**
(A) Co	(i) Wilkinson catalyst
(B) Zn	(ii) Chlorophyll
(C) Rh	(iii) Vitamin B_{12}
(D) Mg	(iv) Carbonic anhydrase

(1) (A) - (iv), (B) - (iii), (C) - (i), (D) - (ii)

(2) (A) - (i), (B) - (ii), (C) - (iii), (D) - (iv)

(3) (A) - (ii), (B) - (i), (C) - (iv), (D) - (iii)

(4) (A) - (iii), (B) - (iv), (C) - (i), (D) - (ii)

Answer (4)

Sol. Wilkinson catalyst is $[Rh(PPh)_3Cl]$

Chlorophyll contains Mg.

Vitamin B_{12} contains Co.

Carbonic anhydrase contains Zn.

19. Two blocks of the same metal having same mass and at temperature T_1 and T_2, respectively, are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS, for this process is

(1) $2C_P \ln\left(\frac{T_1+T_2}{4T_1T_2}\right)$

(2) $C_P \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$

(3) $2C_P \ln\left[\frac{T_1+T_2}{2T_1T_2}\right]$

(4) $2C_P \ln\left[\frac{(T_1+T_2)^{\frac{1}{2}}}{T_1T_2}\right]$

Answer (2)

Sol. Final temperature = $\frac{T_1+T_2}{2}$, let $T_2 > T_1$

$$\therefore \quad dS = \frac{dq}{T} = \frac{C_P dT}{T}$$

$$\therefore \quad \Delta S = C_P \ln\left(\frac{T_f}{T_i}\right)$$

$$\therefore \quad \Delta S_{total} = C_P \ln\left(\frac{T_1+T_2}{2T_1}\right) + C_P \ln\left(\frac{T_1+T_2}{2T_2}\right)$$

$$= C_P \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$$

20. The correct statements among (a) to (d) regarding H_2 as a fuel are

(a) It produces less pollutants than petrol.

(b) A cylinder of compressed dihydrogen weighs ~ 30 times more than a petrol tank producing the same amount of energy.

(c) Dihydrogen is stored in tanks of metal alloys like $NaNi_5$.

(d) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 kJ, respectively.

(1) (b) and (d) only (2) (a) and (c) only

(3) (b), (c) and (d) only (4) (a), (b) and (c) only

Answer (4)

Sol. The energy released by combustion of one gm dihydrogen is more than L.P.G.

21. The element that usually does NOT show variable oxidation states is

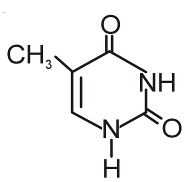
(1) Cu (2) Ti

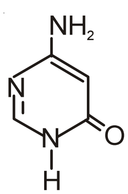
(3) V (4) Sc

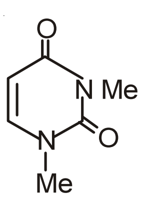
Answer (4)

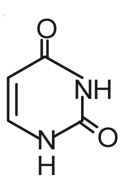
Sol. Sc shows fixed oxidation state of +3

22. Among the following compounds, which one is found in RNA?

(1) 

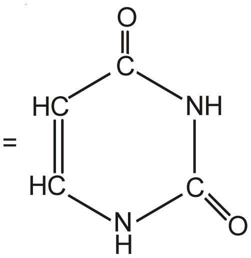
(2) 

(3) 

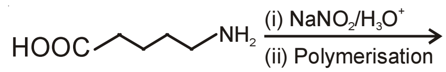
(4) 

Answer (4)

Sol. R.N.A contain Uracil

U = 

23. The polymer obtained from the following reactions is



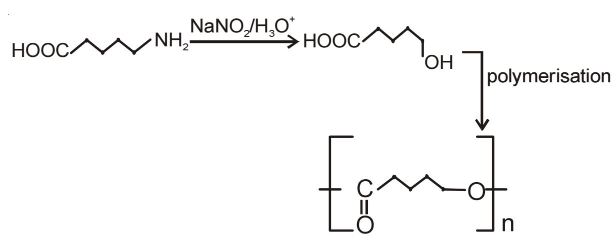
(1) $\left[O-(CH_2)_4-\overset{O}{\overset{\|}{C}} \right]_n$

(2) $\left[HN\overset{O}{\overset{\|}{C}}(CH_2)_4-\overset{O}{\overset{\|}{C}}-\overset{H}{N} \right]_n$

(3) $\left[O\overset{O}{\overset{\|}{C}}(CH_2)_4O \right]_n$

(4) $\left[\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{H}{N} \right]_n$

Answer (1)

Sol. 

24. NaH is an example of

(1) Metallic hydride

(2) Electron-rich hydride

(3) Molecular hydride

(4) Saline hydride

Answer (4)

Sol. NaH are saline hydride

25. The amphoteric hydroxide is

(1) $Mg(OH)_2$

(2) $Be(OH)_2$

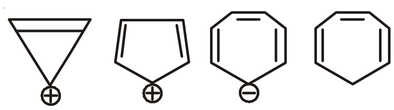
(3) $Sr(OH)_2$

(4) $Ca(OH)_2$

Answer (2)

Sol. $Be(OH)_2$ is amphoteric in nature.

26. Which compound(s) out of following is/are not aromatic?



(A) (B) (C) (D)

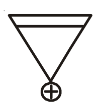
(1) (B), (C) and (D)

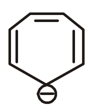
(2) (A) and (C)

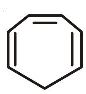
(3) (C) and (D)

(4) (B)

Answer (1)

Sol.  is aromatic as it has $2\pi e^-$ in complete conjugation

 and  are antiaromatic.

 is non aromatic

27. Peroxyacetyl nitrate (PAN), an eye irritant is produced by

(1) Classical smog

(2) Acid rain

(3) Organic waste

(4) Photochemical smog

Answer (4)

Sol. P.A.N is produced by Photochemical smog.

28. A 10 mg effervescent tablet containing sodium bicarbonate and oxalic acid releases 0.25 ml of CO_2 at T = 298.15 K and p = 1 bar. If molar volume of CO_2 is 25.0 L under such condition, what is the percentage of sodium bicarbonate in each tablet?

[Molar mass of $NaHCO_3$ = 84 g mol^{-1}]

(1) 33.6 (2) 8.4

(3) 0.84 (4) 16.8

Answer (2)

Sol. Moles of CO_2 evolved $= \dfrac{0.25}{25\times10^3} = 10^{-5}$

$\therefore$ moles of $NaHCO_3 = 10^{-5}$

$\therefore$ mass of $NaHCO_3 = 84 \times 10^{-5}$ g

$= 0.84 \times 10^{-3}$ g

$= 0.84$ mg

$\therefore$ % by weight $= \dfrac{0.84}{10}\times100$

$= 8.4$ %

29. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H-atom is suitable for this purpose?

[$R_H = 1 \times 10^5$ cm, $h = 6.6 \times 10^{-34}$ Js, $c = 3 \times 10^8$ ms^{-1}]

(1) Balmer, $\infty\rightarrow2$

(2) Lyman, $\infty\rightarrow1$

(3) Paschen, $5 \rightarrow3$

(4) Paschen, $\infty\rightarrow3$

Answer (4)

Sol. $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$n_1 = 3, n_2 = \infty$

$\dfrac{1}{\lambda} = R\left(\dfrac{1}{9}\right) \Rightarrow \lambda = \dfrac{9}{R} = \dfrac{9}{10^5} = 9\times10^{-5}$ cm

$= 900$ nm

30. An example of solid sol is.

(1) Butter

(2) Hair cream

(3) Paint

(4) Gem stones

Answer (4)

Sol. Gem stones are solid sol

1. Let $f(x)=\begin{cases}-1, & -2\le x<0\\ x^2-1, & 0\le x\le 2\end{cases}$ and

$g(x) = |f(x)|+f(|x|)$. Then, in the interval $(-2, 2)$, g is

(1) not differentiable at two points

(2) not differentiable at one point

(3) not continuous

(4) differentiable at all points

Answer (2)

Sol. $f(x) = \begin{cases}-1, & -2\le x<0\\ x^2-1, & 0\le x\le 2\end{cases}$

$f(|x|) = \begin{cases}-1, & -2\le |x|<0\\ |x|^2-1, & 0\le |x|\le 2\end{cases}$

$f(|x|) = x^2 - 1, -2 \le x \le 2$

$\Rightarrow$ $g(x) = \begin{cases}x^2, & -2\le x<0\\ (x^2-1)+|x^2-1|, & 0\le x\le 2\end{cases}$

$= \begin{cases}x^2, & -2\le x<0\\ 0, & 0\le x<1\\ 2(x^2-1), & 1\le x\le 2\end{cases}$

$g'(0^-) = 0, g'(0^+) = 0, g'(1^-) = 0, g'(1^+) = 4$

$\Rightarrow$ $g(x)$ is non-differentiable at $x = 1$

$\Rightarrow$ Option (2) is correct.

2. The plane containing the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points?

(1) (0, –2, 2) (2) (2, 2, 0)

(3) (–2, 2, 2) (4) (2, 0, –2)

Answer (4)

Sol. Let normal to the required plane is $\vec{n}$

$\Rightarrow$ $\vec{n}$ is perpendicular to both vector $2\hat{i}-\hat{j}+3\hat{k}$ and $2\hat{i}+3\hat{j}-\hat{k}$.

$\Rightarrow$ $\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 3\\ 2 & 3 & -1\end{vmatrix}=-8\hat{i}+8\hat{j}+8\hat{k}$

$\Rightarrow$ equation of the required plane is

$(x - 3)(-1) + (y + 2) \times 1 + (z - 1) \times 1 = 0$

$x - 3 - y - 2 - z + 1 = 0$

$x - y - z = 4$ passes through (2, 0, –2)

$\Rightarrow$ Option (4) is correct

3. Let $f : R \to R$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is

(1) $R-\left[-\frac{1}{2},\frac{1}{2}\right]$ (2) $\left[-\frac{1}{2},\frac{1}{2}\right]$

(3) (–1, 1) – {0} (4) R – [–1, 1]

Answer (2)

Sol. $f(x) = \frac{x}{1+x^2}, x\in R$

$y = \frac{x}{1+x^2}$

$yx^2 - x + y = 0$

$D \ge 0$

$1 \ge 4y^2$

$|y| \le \frac{1}{2}$

$-\frac{1}{2}\le y\le\frac{1}{2}$

$\Rightarrow$ Option (2) is correct.

4. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2}-d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}+d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals

(1) $\sqrt{2}$ (2) $\frac{\sqrt{5}}{2}$

(3) $\frac{2}{3}$ (4) 2

Answer (1)

Sol. Outcomes are $\left(\frac{1}{2}-d\right),\left(\frac{1}{2}-d\right)$, ..., 10 times, $\frac{1}{2},\frac{1}{2},\ldots,$ 10 times, $\frac{1}{2}+d,\frac{1}{2}+d,\ldots,$ 10 times

$$\text{Mean} = \frac{1}{30}\left(\frac{1}{2}\times 30\right) = \frac{1}{2}$$

$$\sigma^2 = \frac{1}{30}\Sigma x_i^2 - (\bar{x})^2$$

$$= \frac{1}{30}\left[\left(\frac{1}{2}-d\right)^2 \times 10 + \left(\frac{1}{2}\right)^2 \times 10 + \left(\frac{1}{2}+d\right)^2 \times 10\right] - \frac{1}{4}$$

$$\Rightarrow \frac{4}{3} = \frac{1}{30}\left[30 \times \frac{1}{4} + 20d^2\right] - \frac{1}{4}$$

$$\Rightarrow \frac{4}{3} = \frac{1}{4} + \frac{2}{3}d^2 - \frac{1}{4}$$

$\Rightarrow$ $d^2 = 2 \Rightarrow |d| = \sqrt{2}$

$\Rightarrow$ Option (1) is correct.

5. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is:

(1) $-14\hat{i} + 5\hat{j}$ (2) $-10\hat{i} - 5\hat{j}$

(3) $-14\hat{i} - 5\hat{j}$ (4) $-10\hat{i} + 5\hat{j}$

Answer (4)

Sol. For coplanar vectors,

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & (\lambda^2 - 1) \end{vmatrix} = 0$$

$\Rightarrow$ $\lambda^3 - \lambda - 16 + 2(8 - \lambda^2 + 1) + 4(4 - 2\lambda) = 0$

$\Rightarrow$ $\lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

i.e., $(\lambda - 2)(\lambda - 3)(\lambda + 3) = 0$

For $\lambda = 2$, $\vec{c} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$$\therefore \quad \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$$

For $\lambda = 3$ or -3, $\vec{a} \times \vec{c} = 0$ **(Rejected)**

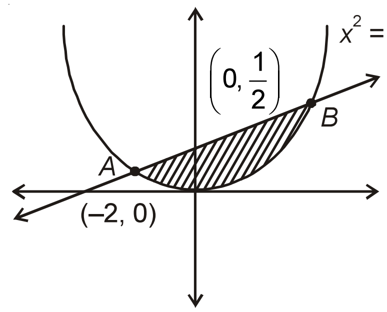
6. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

(1) $\frac{7}{8}$ (2) $\frac{5}{4}$

(3) $\frac{9}{8}$ (4) $\frac{3}{4}$

Answer (3)

Sol.



Let points of intersection of the curve and the line be A and B

$$x^2 = 4\left(\frac{x+2}{4}\right)$$

$x^2 - x - 2 = 0$

$x = 2, -1$

Points are (2, 1) and $\left(-1, \frac{1}{4}\right)$

$$\text{Area} = \int_{-1}^{2}\left[\left(\frac{x+2}{4}\right) - \left(\frac{x^2}{4}\right)\right]dx = \left[\frac{x^2}{8} + \frac{1}{2}x - \frac{x^3}{12}\right]_{-1}^{2}$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{3}\right) - \left(\frac{1}{8} - \frac{1}{2} + \frac{1}{12}\right) = \frac{9}{8}$$

7. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals

(1) 5^3 (2) 5^4

(3) $2(5^2)$ (4) $4(5^2)$

Answer (2)

Sol. Let $a_1 = a$, $a_2 = ar$, $a_3 = ar^2$ ... $a_{10} = ar^9$

where r = common ratio of given G.P.

As $\frac{a_3}{a_1} = 25$

$\Rightarrow$ $\frac{ar^2}{a} = 25$

$\Rightarrow$ $r = \pm 5$

Now, $\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = (\pm 5)^4 = 5^4$

8. If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

where a, b, c are non-zero real numbers, has more than one solution, then

(1) $b - c + a = 0$ (2) $b + c - a = 0$

(3) $a + b + c = 0$ (4) $b - c - a = 0$

Answer (4)

$_1 = \Delta_2 = \Delta_3 = 0$ for infinite solution

$$\Delta_1 = \begin{vmatrix} a & 2 & 3 \\ b & -1 & 5 \\ c & -3 & 2 \end{vmatrix} = a(13) + 2(5c - 2b) + 3(-3b + c)$$

$= 13a - 13b + 13c = 0$

i.e, $a - b + c = 0$

or $b - c - a = 0$

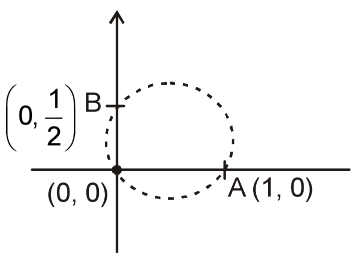
9. The straight line $x + 2y = 1$ meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is

(1) $\dfrac{\sqrt{5}}{4}$ (2) $\dfrac{\sqrt{5}}{2}$

(3) $4\sqrt{5}$ (4) $2\sqrt{5}$

Answer (2)

Sol.



Let equation of circle be $x^2 + y^2 + 2gx + 2fy = 0$

As length of intercept on x axis is $1 \quad 2\sqrt{g^2 \quad c}$

$\Rightarrow |g| = \dfrac{1}{2}$

length of intercept on y-axis $= \dfrac{1}{2} = 2\sqrt{f^2 - c}$

$\Rightarrow |f| = \dfrac{1}{4}$

Equation of circle that passes through given points is $x^2 + y^2 - x - \dfrac{y}{2} = 0$

Tangent at (0, 0) is $\dfrac{x}{2} + \dfrac{y}{4} = 0$

$\Rightarrow 2x + y = 0$

Sum of perpendicular distance $= \dfrac{\frac{1}{2} + 2}{\sqrt{5}} = \dfrac{\sqrt{5}}{2}$.

10. Let $[x]$ denote the greatest integer less than or equal to x. Then

$$\lim_{x \to 0} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$$

(1) equals 0 (2) equals $\pi + 1$

(3) equals π (4) does not exist

Answer (4)

Sol. $\displaystyle \lim_{x \to 0^+} \frac{\tan(\pi \sin^2 x) + (x - 0)^2}{x^2}$

$= \displaystyle \lim_{x \to 0^+} \left(\frac{\tan(\pi \sin^2 x)}{x^2} + 1 \right)$

$= 1 + \pi$

Also, $\displaystyle \lim_{x \to 0^-} \frac{\tan(\pi \sin^2 x) + (-x + \sin x)^2}{x^2}$

$\displaystyle \lim_{x \to 0^-} \frac{\tan(\pi \sin^2 x) + x^2 + \sin^2 x - 2x \sin x}{x^2}$

$= \pi + 1 + 1 - 2 = \pi$

As LHL $\neq$ RHL

Limit does not exist

11. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then $|p|$ is:

(1) $\dfrac{1}{\sqrt{3}}$ (2) $\dfrac{1}{\sqrt{6}}$

(3) $\dfrac{1}{\sqrt{5}}$ (4) $\dfrac{1}{\sqrt{2}}$

Answer (4)

Sol. $A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$

$\therefore \quad A \cdot A^T = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix} \times \begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{bmatrix}$

$= \begin{bmatrix} 4q^2 + r^2 & 2q^2 - r^2 & -2q^2 + r^2 \\ 2q^2 - r^2 & p^2 + q^2 + r^2 & p^2 - q^2 - r^2 \\ -2q^2 + r^2 & p^2 - q^2 - r^2 & p^2 + q^2 + r^2 \end{bmatrix}$

$\because \quad AA^T = I$

$\therefore \quad 4q^2 + r^2 = p^2 + q^2 + r^2 = 1$

and $2q^2 - r^2 = 0 = p^2 - q^2 - r^2$

$\therefore \quad p^2 = 3q^2$ and $r^2 = 2q^2$

$\therefore \quad p^2 = \dfrac{1}{2}, q^2 = \dfrac{1}{6}$ and $r^2 = \dfrac{1}{3}$

$\therefore \quad |p| = \dfrac{1}{\sqrt{2}}$.

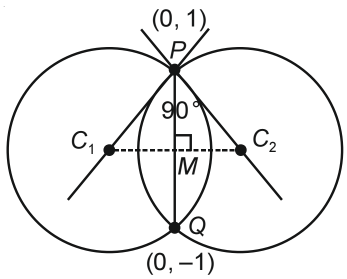
12. Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is:

(1) 1 (2) $\sqrt{2}$

(3) $2\sqrt{2}$ (4) 2

Answer (4)

Sol. $\because$ Two circles of equal radii intersect each other orthogonally. Then M is mid point of PQ.



and $PM = C_1M = C_2M$

$$PM = \frac{1}{2}\sqrt{(0-0)^2 + (1+1)^2} = 1$$

$\therefore$ Distance between centres = 1 + 1 = 2.

13. The value of r for which ${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + \ldots + {}^{20}C_0\,{}^{20}C_r$ is maximum, is:

(1) 10 (2) 20

(3) 15 (4) 11

Answer (2)

Sol. ${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + \ldots + {}^{20}C_0 \cdot {}^{20}C_r$

For maximum value of above expression r should be equal to 20.

as ${}^{20}C_{20} \cdot {}^{20}C_0 + {}^{20}C_{19} \cdot {}^{20}C_1 + \ldots + {}^{20}C_{20} \cdot {}^{20}C_0$

$$= \left({}^{20}C_0\right)^2 + \left({}^{20}C_1\right)^2 + \cdots + \left({}^{20}C_{20}\right)^2 = {}^{40}C_{20}.$$

Which is maximum

So $r = 20$

14. If $x \log_e(\log_e x) - x^2 + y^2 = 4$ ($y > 0$), then $\frac{dy}{dx}$ at $x = e$ is equal to:

(1) $\frac{(2e-1)}{2\sqrt{4+e^2}}$ (2) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(3) $\frac{(1+2e)}{\sqrt{4+e^2}}$ (4) $\frac{e}{\sqrt{4+e^2}}$

Answer (1)

Sol. $x \log_e(\log_e x) - x^2 + y^2 = 4$

Differentiate both sides w.r.t. x, we get

$$\log_e(\log_e x) + x \cdot \frac{1}{x \cdot \log_e x} - 2x + 2y\frac{dy}{dx} = 0$$

$$\log_e(\log_e x) + \frac{1}{\log_e x} - 2x + 2y\frac{dy}{dx} = 0 \quad \ldots(1)$$

When $x = e$, $y = \sqrt{4+e^2}$.

When $x = e$ in equation (1)

$$0 + 1 - 2e + 2\sqrt{4+e^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2e-1}{2\sqrt{4+e^2}}.$$

15. If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function $A(x)$, where C is a constant of integration, then $(A(x))^m$ equals:

(1) $\frac{-1}{3x^3}$ (2) $\frac{1}{27x^6}$

(3) $\frac{1}{9x^4}$ (4) $\frac{-1}{27x^9}$

Answer (4)

Sol. $A(x)\left(\sqrt{1-x^2}\right)^m + C = \int \frac{\sqrt{1-x^2}}{x^4}dx$

$$= \int \frac{\sqrt{\frac{1}{x^2}-1}}{x^3}dx$$

Let $\frac{1}{x^2} - 1 = t^2$

$$\Rightarrow -\frac{2}{x^3} = \frac{2t\,dt}{dx}$$

$$\frac{dx}{x^3} = -\frac{2t}{2}dt$$

$$A(x)\left(\sqrt{1-x^2}\right)^m + C = \int(-t^2)\,dt = -\frac{t^3}{3} + C$$

$$= -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}} + C$$

$$= -\frac{1}{3} \cdot \frac{1}{x^3} \cdot (1-x^2)^{\frac{3}{2}} + C$$

$$= \frac{-1}{3x^3}\left(\sqrt{1-x^2}\right)^3 + C$$

$\therefore$ $A(x) = -\frac{1}{3x^3}$

$\therefore$ $(A(x))^3 = \frac{-1}{27x^9}$

16. Two integers are selected at random from the set {1, 2, ..., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is:

(1) $\frac{3}{5}$ (2) $\frac{7}{10}$

(3) $\frac{1}{2}$ (4) $\frac{2}{5}$

Answer (4)

Sol. Probability that sum of selected two numbers is even

$$= P(E_1) = \frac{^6C_2 + {^5C_2}}{^{11}C_2}$$

Probability that sum is even and selected numbers are also even $P(E_2) = \frac{^5C_2}{^{11}C_2}$

$$\therefore \quad P\left(\frac{E_2}{E_1}\right) = \frac{^5C_2}{^6C_2 + {^5C_2}} = \frac{10}{15+10} = \frac{2}{5}.$$

17. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is:

(1) $4x + 2y + 1 = 0$ (2) $x + 2y + 4 = 0$

(3) $x - 2y + 4 = 0$ (4) $x + y + 1 = 0$

Answer (2)

Sol. Equation of a tangent to parabola $y^2 = 4x$ is :

$$y = mx + \frac{1}{m}$$

This line is a tangent to $xy = 2$

$$\therefore \quad x\left(mx + \frac{1}{m}\right) = 2$$

$$mx^2 + \frac{1}{m}x - 2 = 0$$

$$\therefore \quad D = \left(\frac{1}{m}\right)^2 - 4 \cdot m \cdot (-2) = 0$$

$$\frac{1}{m^2} + 8m = 0$$

$$1 + 8m^3 = 0$$

$$m^3 = -\frac{1}{8}$$

$$m = -\frac{1}{2}$$

$\therefore$ Equation of common tangent: $y = -\frac{1}{2}x - 2$

$$2y = -x - 4$$

$\therefore \quad x + 2y + 4 = 0$

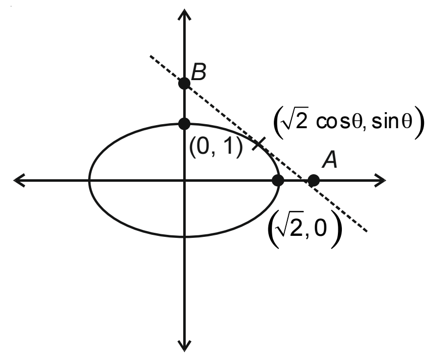
18. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve:

(1) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$ (2) $\frac{x^2}{2} + \frac{y^2}{4} = 1$

(3) $\frac{x^2}{4} + \frac{y^2}{2} = 1$ (4) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

Answer (4)

Sol. Equation of tangent is



$$\frac{\sqrt{2}\cos\theta\, x}{2} + y\sin\theta = 1$$

$$A\left(\frac{\sqrt{2}}{\cos\theta}, 0\right) \text{ and } B\left(0, \frac{1}{\sin\theta}\right)$$

Let mid point be (h, k)

$$\Rightarrow \quad h = \frac{1}{\sqrt{2}\cos\theta}, k = \frac{1}{2\sin\theta}$$

As $\cos^2\theta + \sin^2\theta = 1$

$$\therefore \quad \frac{1}{2h^2} + \frac{1}{4k^2} = 1$$

Locus is $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

19. A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is:

(1) 6 (2) $\sqrt{41}$

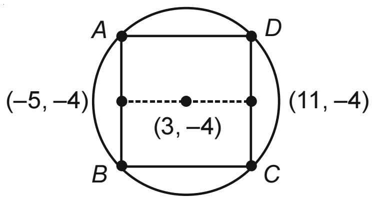
(3) 13 (4) $\sqrt{137}$

Answer (2)

Sol. $x^2 + y^2 - 6x + 8y - 103 = 0$

$C(3, -4)$, $r = 8\sqrt{2}$

$\Rightarrow$ Length of side of square = $\sqrt{2}r = 16$



$\Rightarrow$ $A(-5, 4)$, $B(-5, -12)$
$C(11, -12)$, $D(11, 4)$

$\Rightarrow$ Required distance = $OA = \sqrt{41}$

$\Rightarrow$ Option (2) is correct.

20. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is:

(1) $\frac{c}{\sqrt{3}}$ (2) $\frac{3}{2}y$

(3) $\frac{c}{3}$ (4) $\frac{y}{\sqrt{3}}$

Answer (1)

Sol. $a + b = x$

$ab = y$

$x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$

$\Rightarrow (a + b - c)(a + b + c) = ab$

$\Rightarrow 2(s - c)(2s) = ab$

$\Rightarrow 4s(s - c) = ab$

$\Rightarrow \frac{s(s-c)}{ab} = \frac{1}{4}$

$\Rightarrow \cos^2\frac{c}{2} = \frac{1}{2}$

$\Rightarrow \cos c = -\frac{1}{2} \Rightarrow c = 120°$

$\Rightarrow \Delta = \frac{1}{2}ab(\sin 120°) = \frac{\sqrt{3}}{4}ab \Rightarrow R = \frac{abc}{\sqrt{3}\,ab} = \frac{c}{\sqrt{3}}$

21. Let $\left(-2-\frac{1}{3}i\right)^3 = \frac{x+iy}{27}$ $(i = \sqrt{-1})$, where x and y are real numbers, then $y - x$ equals

(1) –85 (2) –91

(3) 85 (4) 91

Answer (4)

Sol. $-(6 + i)^3 = x + iy$

$-[216 - i + 18i(6 + i)] = x + iy$

$\Rightarrow -[216 - i + 108i - 18] = x + iy$

$\Rightarrow -216 + i - 108i + 18 = x + iy$

$\Rightarrow -198 - 107i = x + iy$

$\Rightarrow x = -198,\ y = -107$

$\Rightarrow y - x = -107 + 198 = 91$

22. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

(1) $p \vee r$ (2) $(p \wedge r) \rightarrow (p \vee r)$

(3) $(p \vee r) \rightarrow (p \wedge r)$ (4) $p \wedge r$

Answer (2)

Sol. q is false

$[(p \wedge q) \leftrightarrow r]$ is true

As $(p \wedge q)$ is false

[False $\leftrightarrow r$] is true

Hence r is false

Option (1): says $p \vee r$, As r is false

Hence $(p \vee r)$ can either be true or false

Option (2): says $(p \wedge r) \rightarrow (p \vee r)$

$(p \wedge r)$ is false

As $F \rightarrow T$ is true and

$F \rightarrow F$ is also true

Hence it is a tautology

Option (3): $(p \vee r) \rightarrow (p \wedge r)$

i.e. $(p \vee r) \rightarrow F$

It can either be true or false

Option (4): $(p \wedge r)$, As r is false

Hence $(p \wedge r)$ is false

23. If $y(x)$ is the solution of the differential equation $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}, x > 0$, where $y(1) = \frac{1}{2}e^{-2}$, then

(1) $y(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$

(2) $y(\log_e 2) = \frac{\log_e 2}{4}$

(3) $y(\log_e 2) = \log_e 4$

(4) $y(x)$ is decreasing in (0, 1)

Answer (1)

Sol. $\frac{dy}{dx} + \left(2 + \frac{1}{x}\right)y = e^{-2x}, x > 0$

$\text{IF} = e^{\int\left(2+\frac{1}{x}\right)dx}\ e^{2x+\ln x}$

$$y(x)\cdot e^{2x+\ln x} = \int e^{2x+\ln x}\cdot e^{-2x}dx + c$$

$$= \int x\,dx + c$$

$$y(x)\cdot e^{2x}\cdot x = \frac{x^2}{2} + c$$

$$y(1) = \frac{1}{2}e^{-2} \text{ gives } \frac{1}{2}e^{-2}\cdot e^{2}\cdot 1 = \frac{1}{2} + c \quad \Rightarrow \quad c = 0$$

$$\therefore \quad y(x) = \frac{x^2}{2}\cdot\frac{e^{-2x}}{x}$$

$$y(x) = \frac{x}{2}\cdot e^{-2x}$$

$$y'(x) = \frac{e^{-2x}}{2}(1-2x) < 0 \ \forall\ x \in \left(\frac{1}{2}, 1\right)$$

Hence, y(x) is decreasing in $\left(\frac{1}{2}, 1\right)$

24. The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$ are

(1) $2\sqrt{3}, 1, -1$ (2) $2, \sqrt{2}, -\sqrt{2}$

(3) 2, –1, 1 (4) $\sqrt{2}, 1, -1$

Answer (2, 4)

Sol. Let the d.r's of the normal be $\langle a, b, c\rangle$

Equation of the plane is

$a(x-0) + b(y+1) + c(z-0) = 0$

It passes through (0, 0, 1)

$\therefore \quad b + c = 0$

Also $\dfrac{0\cdot a + b - c}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad b\cdot c = \sqrt{a^2+b^2+c^2}$

And $b + c = 0$

Solving we get $b = \pm\dfrac{1}{\sqrt{2}}a.$

$\therefore$ The d.r's are $\sqrt{2}, 1, -1$

Or $2, \sqrt{2}, -\sqrt{2}$

Note: Options (2) and (4) are correct.

25. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R: x^2 + 30 \le 11x\}$ is

(1) 122 (2) –122

(3) 222 (4) –222

Answer (1)

Sol. $f(x) = 3x(x-3)^2 - 40$

Now $S = \{x \in R : x^2 + 30 \le 11x\}$

So $x^2 - 11x + 30 \le 0$

$x \in [5, 6]$

For given interval, $f(x)$ will have maximum value for $x = 6$

$f(6) = 3 \times 6 \times 3 \times 3 - 40 = 122$

26. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to

(1) $\frac{-1}{12}$ (2) $\frac{1}{12}$

(3) $\frac{5}{12}$ (4) $\frac{1}{4}$

Answer (2)

Sol. $f_k(x) = \dfrac{1}{k}\left(\sin^k x + \cos^k x\right)$

$$f_4(x) = \frac{1}{4}[\sin^4 x + \cos^4 x] = \frac{1}{4}\left[1 - \frac{(\sin 2x)^2}{2}\right]$$

$$f_6(x) = \frac{1}{6}[\sin^6 x + \cos^6 x] = \frac{1}{6}\left[1 - \frac{3}{4}(\sin 2x)^2\right]$$

$$\text{Now } f_4(x) - f_{(6)}(x) = \frac{1}{4} - \frac{1}{6} - \frac{(\sin 2x)^2}{8} + \frac{1}{8}(\sin 2x)^2$$

$$= \frac{1}{12}$$

27. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3} + \frac{3}{x}\right)^8$ equals 5670 is

(1) 4 (2) 8

(3) 0 (4) 6

Answer (3)

Sol. Middle term, $\left(\dfrac{n}{2}+1\right)^{\text{th}}$

$$T_{4+1} = {}^8C_4\left(\frac{x^3}{3}\right)^4\left(\frac{3}{x}\right)^4 = 5670$$

$$\Rightarrow \frac{8\times7\times6\times5}{4\times3\times2}\times x^8 = 5670$$

$x^8 = 81$

$x^8 - 81 = 0$

Now sum of all values of x = zero

28. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is

(1) $\frac{1}{3}$

(2) $\frac{2}{9}$

(3) $\frac{2}{3}$

(4) $\frac{4}{9}$

Answer (3)

Sol. Let any series

$a, ar, ar^2, \ldots \infty$

So $\frac{a}{1-r} = 3$ ...(i)

Now sum of cubes of its terms is $\frac{27}{19}$

So $a^3, a^3r^3, \ldots,$

$$\frac{a^3}{1-r^3} = \frac{27}{19}$$

$$\Rightarrow \frac{a}{1-r}\times\frac{a^2}{(1+r^2+r)} = \frac{27}{19}$$

$$\Rightarrow \frac{9(1+r^2-2r)\times3}{1+r^2+r} = \frac{27}{19}$$

$$\Rightarrow 6r^2 - 13r + 6 = 0$$

$\Rightarrow (3r-2)(2r-3) = 0$

$$\Rightarrow r = \frac{2}{3}, \frac{3}{2}$$

As $|r| < 1$

So $r = \frac{2}{3}$

29. The value of the integral $\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$ (where $[x]$ denotes the greatest integer less than or equal to x) is

(1) $\sin 4$

(2) $4 - \sin 4$

(3) 0

(4) 4

Answer (3)

Sol. Let $f(x) = \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}$

Now $f(-x) = \frac{\sin^2(-x)}{\left[\frac{-x}{\pi}\right]+\frac{1}{2}}$ $\quad \because [-x] = -1-[x]$

$$f(-x) = \frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}} = \frac{\sin^2 x}{-\frac{1}{2}-\left[\frac{x}{\pi}\right]} = -f(x)$$

So $f(x)$ is odd function

So $\int_{-2}^{2} f(x)\,dx = 0$

30. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

(1) –300

(2) 144

(3) –81

(4) 100

Answer (1)

Sol. $81x^2 + kx + 256 = 0 \genfrac{}{}{0pt}{}{\alpha}{\beta}$

Given $(\alpha)^{\frac{1}{3}} = \beta$

$\alpha = \beta^3$

So $(\alpha)(\beta) = \frac{256}{81}$

$$\Rightarrow \beta^4 = \left(\frac{4}{3}\right)^4 \Rightarrow \beta = \frac{4}{3}$$

Now $\alpha = \frac{64}{27}$

Now $\alpha+\beta = -\frac{k}{81} \Rightarrow \frac{4}{3}+\frac{64}{27} = -\frac{k}{81}$

$k = -300$

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