PROBLEM-SOLVING TACTICS

- Making a habit of writing angle values in radians rather in degrees makes the calculation of inverse trigonometric functions easier.
- Try to remember graphs of inverse trigonometric functions. Sometimes it is easier to approximate answers using graphical methods.
- Always verify whether the results are in the range or domain of the respective function.
- In some cases, constructing a right angled triangle for the given inverse function and then solving using properties of triangle is much helpful.
- In case of identities in inverse circular functions, principal values should be taken. As such signs of x, y, etc., will determine the quadrant in which the angles will fall. In order to bring the angles of both sides in the same quadrant, one should make an adjustment by π.

FORMULAE SHEET

1.	If y = sin x, then x = $\sin^{-1}$ y, similarly for other inverse T-functions.

2. Domain and Range of Inverse T-functions:

Function	Domain(D)	Range (R)
$\sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\cos^{-1} x$	$-1 \le x \le 1$	$0 \le \theta \le \pi$
$\tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$\cot^{-1} x$	$-\infty < x < \infty$	$0 < \theta < \pi$
$\sec^{-1} x$	$x \le -1,\ x \ge 1$	$0 \le \theta \le \pi,\ \theta \ne \frac{\pi}{2}$
$\operatorname{cosec}^{-1} x$	$x \le -1,\ x \ge 1$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2},\ \theta \ne 0$

3. Properties of Inverse T-functions:

(i) $\sin^{-1}(\sin\theta) = \theta$ provided $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\cos^{-1}(\cos\theta) = \theta$ provided $0 \le \theta \le \pi$

$\tan^{-1}(\tan\theta) = \theta$ provided $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\cot^{-1}(\cot\theta) = \theta$ provided $0 < \theta < \pi$

	$\sec^{-1}(\sec\theta) = \theta$ provided $0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$ $\text{cosec}^{-1}(\text{cosec}\,\theta) = \theta$ provided $-\frac{\pi}{2} \le \theta < 0$ or $0 < \theta \le \frac{\pi}{2}$	
	(ii) $\sin(\sin^{-1}x) = x$ provided $-1 \le x \le 1$ $\cos(\cos^{-1}x) = x$ provided $-1 \le x \le 1$ $\tan(\tan^{-1}x) = x$ provided $-\infty < x < \infty$ $\cot(\cot^{-1}x) = x$ provided $-\infty < x < \infty$ $\sec(\sec^{-1}x) = x$ provided $-\infty < x \le -1$ or $1 \le x < \infty$ $\text{cosec}(\text{cosec}^{-1}x) = x$ provided $-\infty < x \le -1$ or $1 \le x < \infty$	
	(iii) $\sin^{-1}(-x) = -\sin^{-1}x$, $\cos^{-1}(-x) = \pi - \cos^{-1}x$ $\tan^{-1}(-x) = -\tan^{-1}x$ $\cot^{-1}(-x) = \pi - \cot^{-1}x$ $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$ $\sec^{-1}(-x) = \pi - \sec^{-1}x$	**(iv)** $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, \quad \forall\, x \in [-1, 1]$ $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, \quad \forall\, x \in R$ $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}, \quad \forall\, x \in (-\infty,-1] \cup [1,\infty)$

4.	Value of one inverse function in terms of another inverse function:
	(i) $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x}$ $= \sec^{-1}\frac{1}{\sqrt{1-x^2}} = \text{cosec}^{-1}\frac{1}{x},\ 0 \le x \le 1$ **(ii)** $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}}$ $= \sec^{-1}\frac{1}{x} = \text{cosec}^{-1}\frac{1}{\sqrt{1-x^2}},\ 0 \le x \le 1$ **(iii)** $\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \cot^{-1}\frac{1}{x} = \sec^{-1}\sqrt{1+x^2} = \text{cosec}^{-1}\frac{\sqrt{1+x^2}}{x},\ x \ge 0$ **(iv)** $\sin^{-1}\left(\frac{1}{x}\right) = \text{cosec}^{-1}x,\ \forall\ x \in (-\infty,1] \cup [1,\infty)$ **(v)** $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x,\ \forall\ x \in (-\infty,1] \cup [1,\infty)$ **(vi)** $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1}x & \text{for } x > 0 \\ -\pi + \cot^{-1}x & \text{for } x < 0 \end{cases}$

5.	Formulae for sum and difference of inverse trigonometric function:
	(i) $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$; if $x>0,\ y>0,\ xy<1$
	(ii) $\tan^{-1}x+\tan^{-1}y=\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right)$; if $x>0,\ y>0,\ xy>1$
	(iii) $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1+xy}\right)$; if $xy>-1$
	(iv) $\tan^{-1}x-\tan^{-1}y=\pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right)$; if $x>0,\ y<0,\ xy<-1$
	(v) $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$
	(vi) $\sin^{-1}x\pm\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}\pm y\sqrt{1-x^2}\right]$; If $x,y,\geq 0\ \&\ x^2+y^2\leq 1$
	(vii) $\sin^{-1}x\pm\sin^{-1}y=\pi-\sin^{-1}\left[x\sqrt{1-y^2}\pm y\sqrt{1-x^2}\right]$; If $x,y,\geq 0\ \&\ x^2+y^2>1$
	(viii) $\cos^{-1}x\pm\cos^{-1}y=\cos^{-1}\left[xy\mp\sqrt{1-x^2}\ \sqrt{1-y^2}\right]$; If $x,y,>0\ \&\ x^2+y^2\leq 1$
	(ix) $\cos^{-1}x\pm\cos^{-1}y=\pi-\cos^{-1}\left[xy\mp\sqrt{1-x^2}\ \sqrt{1-y^2}\right]$; If $x,y,>0\ \&\ x^2+y^2>1$

6.	Inverse trigonometric ratios of multiple angles
	(i) $2\sin^{-1}x=\sin^{-1}(2x\sqrt{1-x^2})$, if $-1\leq x\leq 1$
	(ii) $2\cos^{-1}x=\cos^{-1}(2x^2-1)$, if $-1\leq x\leq 1$
	(iii) $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
	(iv) $3\sin^{-1}x=\sin^{-1}(3x-4x^3)$
	(v) $3\cos^{-1}x=\cos^{-1}(4x^3-3x)$
	(vi) $3\tan^{-1}x=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$