I solution	II solution
$C = 50^o1'12"$ $A = 180^o - (B + C)$ $A = 180^o - (42^o47' + 50^o1'12") = 87^o11'48"$ To find a $\frac{a}{\sin A} = \frac{b}{\sin B}$ $a = \frac{b\sin A}{\sin B} = \frac{72.95 \times \sin 87^o11'48"}{\sin 42^o27'}$ $a = 107.95$	$C = 129^o58'48"$ $A = 180^o - (B + C)$ $= 180^o - (42^o47' + 129^o58'48") = 7^o14'12"$ To find a $\frac{a}{\sin A} = \frac{b}{\sin B}$ $a = \frac{b\sin A}{\sin B} = \frac{72.95 \times \sin 7^o14'12"}{\sin 42^o27'}$ $a = 13.62$

$\therefore$ Two solutions are

$C_1 = 50^o1'12"$ $A_1 = 87^o11'48"$ $a_1 = 107.95$ $C_2 = 129^o58'48"$ $A_2 = 7^o14'12"$ $a_2 = 13.62$

Geometrically, we draw the triangle with given data c,b and angle B.

(a) If $AN(= c\sin B) = b$ (exactly). The triangle is a right angled triangle.

(b) If $AN(= c\sin B) > b$, the triangle cannot be drawn.

(c) If $AN(= c\sin B) < b < c$, two triangles are possible.

(d) b>c, only one triangle is possible.

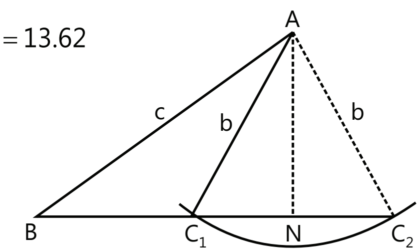

Figure 19.19

Illustration 27: In a triangle ABC, b=16cm, c=25cm, and $B = 33^o15'$. Find the angle C. **(JEE MAIN)**

Sol: Simply by using sine rule, we can find out the angle C.

We know that, $\frac{\sin C}{c} = \frac{\sin B}{b}$ [Here, b=16cm, c=25cm, B= $33^o15'$]

$\sin C = \frac{c}{b}\sin B = \frac{25\sin 33^o15'}{16} = 0.8567$; $C = \sin^{-1}(0.8567) = 58^o57'$; $C_1 = 58^o57'$; $C_2 = 180^o - 58^o57' = 121^o3'$

PROBLEM SOLVING TACTICS

In the application of sine rule, the following points are to be noted. We are given one side a and some other side x is to be found. Both these are in different triangles. We choose a common side y of these triangles. Then apply sine rule for a and y in one triangle and for x and y for the other triangle and eliminate y. Thus, we will get the unknown side x in terms of a.

In the adjoining figure, a is the known side of ΔABC and x is the unknown side of triangle ACD. The common side of these triangles is AC=y(say). Now, apply sine rule.

$\therefore \frac{a}{\sin\alpha} = \frac{y}{\sin\beta}$ ...(i) and $\frac{x}{\sin\theta} = \frac{y}{\sin\gamma}$ ... (ii)

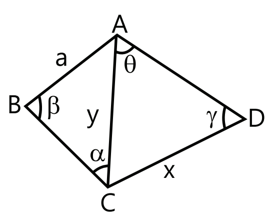

Figure 19.20

Dividing (ii) by (i) we get, $\frac{x\sin\alpha}{a\sin\theta} = \frac{\sin\beta}{\sin\gamma}$; $\therefore x = \frac{a\sin\beta\sin\theta}{\sin\alpha\sin\gamma}$

In case of generalized triangle problems, option verification is very useful using equilateral, isosceles or right angle triangle properties. So, it is advised to remember properties of these triangles.

FORMULAE SHEET

(a) In $\Delta ABC,\ \angle A+\angle B+\angle C=\pi$

(a) $\sin(B+C)=\sin(\pi-A)=\sin A$

(b) $\cos(C+A)=\cos(\pi-B)=-\cos B$

(c) $\sin\frac{A+B}{2}=\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)=\cos\frac{C}{2}$

(d) $\cos\frac{B+C}{2}=\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)=\sin\frac{A}{2}$

(b) **Sine rule:** In, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$ Where R = Circumradius and a, b, c are sides of triangle.

(c) **Cosine rule:** $\cos A=\frac{b^2+c^2-a^2}{2bc}$, $\cos B=\frac{a^2+c^2-b^2}{2ac}$, $\cos C=\frac{a^2+b^2-c^2}{2ab}$

(d) **Trigonometric ratios of half – angles:**

(a) $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$ where 2s = a + b + c; **(b)** $\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$; **(c)** $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

(e) **Area of a triangle :** $\Delta=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\frac{1}{2}ab\sin C$

(f) **Heron's formula :** $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\frac{a+b+c}{2}$.

(g) **Circumcircle Radius :** $R=\frac{a}{2\sin A}=\frac{b}{2\sin B}=\frac{c}{2\sin C}=\frac{abc}{4\Delta}$

(h) **Incircle Radius :** **(a)** $r=\frac{\Delta}{s}$; **(b)** $r=(s-a)\tan\left(\frac{A}{2}\right)$, $r=(s-b)\tan\left(\frac{B}{2}\right)$ and $r=(s-c)\tan\left(\frac{C}{2}\right)$

(i) **Radius of the Escribed Circle :**

(a) $r_1=\frac{\Delta}{s-a},\ r_2=\frac{\Delta}{s-b},\ r_3=\frac{\Delta}{s-c}$

(b) $r_1=s\tan\frac{A}{2},\ r_2=s\tan\frac{B}{2},\ r_3=s\tan\frac{C}{2}$

(c) $r_1=a\frac{\cos\frac{B}{2}\cos\frac{C}{2}}{\cos\frac{A}{2}},\ r_2=b\frac{\cos\frac{C}{2}\cos\frac{A}{2}}{\cos\frac{B}{2}},\ r_3=c\frac{\cos\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{C}{2}}$

(d) $r_1=4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2},\ r_2=4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2},\ r_3=4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

(j) **Length of Angle bisector and Median:**

$m_a=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}$ and $\beta_a=\frac{2bc\cos\frac{A}{2}}{b+c}\Rightarrow m_a$ - length of median, β_a - length of bisector.