(j) Truth of the statement:

(i) If two persons A and B speak the truth with probabilities P_1& P_2 respectively and if they agree on a statement, then the probability that they are speaking the truth will be given by $\frac{P_1P_2}{P_1P_2+(1-P_1)(1-P_2)}$.

(ii) If A and B both assert that an event has occurred, the probability of occurrence of which is α, then the probability that the event has occurred $\frac{\alpha P_1P_2}{\alpha P_1P_2+(1-\alpha)(1-P_1)(1-P_2)}$ given that the probability of A & B speaking truth is $p_{1,}p_2$ respectively.

(iii) If in the second part, the probability that their lies coincide is β, then from the above case, the required probability will be $\frac{\alpha P_1P_2}{\alpha P_1P_2+(1-\alpha)(1-P_1)(1-P_2)\beta}$

Solved Examples

JEE Main/Boards

Example 1: If there are two events A and B such that P(A′) = 0.3 P(B) = 0.5 and P(A ∩ B) = 0.3, then P(B|A ∪ B′) is:

(A) 3/8 (B) 2/3 (C) 5/6 (D) 1/4

Sol: Use set theory and probability of complimentary events to calculate P(A ∪ B′)

We have P(A ∪ B′)

= P(A) + P(B′) – P(A ∩ B′)

= [1 – P(A′)] + [1 – P(B)] – [P(A) – P(A ∩ B)]

= (1 – 0.3) + (1 – 0.5) – (0.7 – 0.3) = 0.8

Now, P(B|A ∪ B′) = $\frac{P[B\cap(A\cup B')]}{P(A\cup B')}$

$=\frac{P[(B\cap A)\cup(B\cap B')]}{P(A\cup B')}=\frac{P(A\cap B)}{P(A\cup B')}=\frac{0.3}{0.8}=\frac{3}{8}$

Example 2: Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals:

(A) 1/2 (B) 7/15 (C) 2/15 (D) 1/3

Sol: Each black balls can be arranged in between any two white balls. Us e this idea to find the number of ways in which no two black balls are together.

The number of ways of placing 3 black balls at 10 places is $^{10}C_3$. The number of ways in which two black balls are not together is equal to the number of ways of choosing 3 places marked with X out of eight places

X W X W X W X W X W X W X W X

This can be done in $^8C_3$ ways. Thus, probability of the required event is $\frac{^8C_3}{^{10}C_3}=\frac{8\times7\times6}{10\times9\times8}=\frac{7}{15}$

Example 3: A group of 2n boys and 2n girls is randomly divided into two equal groups. The probability that catch contains the same number of boys and girls is:

(A) 1/2 (B) 1/n

(C) 1/2n (D) None of these

Sol: If one group is selected the second group automatically gets created. Hence, select n boys and n girls from the given group.

Total number of ways of choosing a group is $^{4n}C_{2n}$ The number of ways in which each group contains equal number of boys and girls is $(^{2n}C_n)$ $(^{2n}C_n)$

∴ Required probability = $\frac{(^{2n}C_n)^2}{^{4n}C_{2n}}$.

Example 4: Let A and B be two events such that P(A) = 0.3 and P(A ∪ B) = 0.8. If A and B are independent events, then P(B) is:

(A) 3/7 (B) 4/7 (C) 5/7 (D) 6/7

Sol: If say A and B are two independent events then P(A ∩ B) = P(A) × P(B)

We have $0.8 = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)\,P(B)$

[$\because$ A and B are independent]

$= 0.3 + P(B) - (0.3)\,P(B)$

$\Rightarrow 0.5 = (0.7)\,P(B) \Rightarrow P(B) = \frac{5}{7}$

Example 5: A natural number x is chosen at random from the first one hundred natural numbers.

The probability that $\frac{(1-20)(x-40)}{x-30} < 0$ is:

(A) 1/50 (B) 3/50 (C) 3/25 (D) 7/25

Sol: Find the range of values the variable x can take and then find the required probability.

Let $E = \frac{(x-20)(x-40)}{x-30} = \frac{(x-20)(x-30)(x-40)}{(x-30)^2}$

Sign of E is same as that of sign of1

$(x-20)(x-30)(x-40) = F$(say)

Note that $F < 0$ if and only if

$0 < x < 20$ or $30 < x < 40$

$\therefore$ $E < 0$ in $(0, 20) \cup (30, 40)$

Thus E is negative for x = 1, 2, ... , 19, 31, 32, ..., 39 that is E, < 0 for 28 natural numbers

$\therefore$ Required probability $= \frac{28}{100} = \frac{7}{25}$

Example 6: Let E and F be two independent events such that $P(E) < P(F)$. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happen is $\frac{1}{2}$. Then,

(A) $P(E) = 1/3$, $P(F) = \frac{1}{2}$ (B) $P(E) = 1/2$, $P(F) = \frac{2}{3}$

(C) $P(E) = 2/3$, $P(F) = \frac{3}{4}$ (D) $P(E) = 1/4$, $P(F) = \frac{1}{3}$

Sol: Use the concept of Probability for independent events.

We are given $P(E \cap F) = \frac{1}{12}$ and $P(E' \cap F') = \frac{1}{2}$

As E and F are independent, we get P(E) P(F)

$= \frac{1}{12}$ and $P(E')\,P(F') = \frac{1}{2}$

$\Rightarrow (1 - P(E)\,(1 - P(F)) = \frac{1}{2}$

$\Rightarrow 1 - (P(E) + P(F) - P(E)\,P(F)) = \frac{1}{2}$

$\Rightarrow P(E) + P(F) = 1 + \frac{1}{12} - \frac{1}{2} = \frac{7}{12}$

$\therefore$ Equation whose roots are P(E) and P(F) is

$x^2 - (P(E) + P(F))x + P(E)\,P(F) = 0$

or $x^2 - \frac{1}{12}x + \frac{1}{12}$

$\Rightarrow 12x^2 - 7x + 1 = 0$

$\Rightarrow (3x - 1)(4x - 1) = 0$

$\Rightarrow x = \frac{1}{3}, \frac{1}{4}$

As $P(E) < P(F)$, we take $P(E) = \frac{1}{4}$ and $P(F) = \frac{1}{3}$

Example 7: Fifteen coupons are numbered 1, 2, ... , 15 respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number on a selected coupon as 9 is:

(A) $\left(\frac{9}{15}\right)^6$ (B) $\left(\frac{8}{15}\right)^7$

(C) $\left(\frac{3}{5}\right)^7$ (D) None of these

Sol: Calculate the probability for getting highest number as 9 and 8. Subtract the two to get the desired probability.

Let p = the probability that a selected

coupon bears number ≤ 9.

$\Rightarrow p = \frac{9}{15} = \frac{3}{5}$ and

n = Number of coupons drawn with replacement

X = The number of coupons bearing number ≤ 9

Note that $X - B(n, p)$

Probability that the largest number on the

selected coupons does not exceed 9

= probability that all the coupons bear

number ≤ 9

$= P(X = 7) = {}^7C_7\,p^7 = \left(\frac{3}{5}\right)^7$

Similarly, probability that largest number on the selected coupon is ≤ 8 is $\left(\frac{8}{15}\right)^7$.

Hence, probability of the required event = $\left(\frac{3}{5}\right)^7 - \left(\frac{8}{15}\right)^7$.

Example 8: A four digit number (numbered from 0000 to 9999) is said to be lucky if the sum of its first two digits is equal to the sum of its last two digits. If a four digit number is picked up at random, then the probability that it is lucky is:

(A) 0.065 (B) 0.064 (C) 0.066 (D) 0.067

Sol: The sum of the first two digits can be any number from 0 to 18. Use the formula for the number of non-negative integral solutions of x+y=m to proceed further.

The total number of ways of choosing the ticket is 10000.

Let the four digits number on the ticket be $x_1 x_2 x_3 x_4$. Note that $0 \le x_1 + x_2 \le 18$ and $0 \le x_3 + x_4 \le 18$.

Also, the number of non-negative integral solutions of x + y = m (with $0 \le x, y \le 9$) is

m + 1 if $0 \le m \le 9$ and is 19 – m if 10 ≤ m £18.

Thus, the number of favorable ways

= 1 × 1 + 2 × 2 + ... + 10 × 10 + 9 × 9 + 8 × 8 + ... + 1 × 1

$= 2\left\{\frac{9\times10\times19}{6}\right\} + 100 = 670$

$\therefore$ Probability of required event = $\frac{670}{10000} = 0.067$

Example 9: Three numbers are chosen at random without replacement from {1, 2, 3, ...10).The probability that minimum of the chosen number is 3 or their maximum is 7, is:

(A) $\frac{11}{30}$ (B) $\frac{11}{40}$ (C) $\frac{1}{7}$ (D) $\frac{1}{8}$

Sol: Find the probability for getting 3 as the minimum and 7 as the maximum number among the three numbers selected. Then use the formula P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

Let A and B denote the following events

A: minimum of the chosen number is 3

B: maximum of the chosen number is 7

We have, P(A) = P(choosing 3 and two other numbers from 4 to 10)$=\frac{{}^7C_2}{{}^{10}C_3} = \frac{7\times6}{2}\times\frac{3\times2}{10\times9\times8} = \frac{7}{40}$

P(B) = P(Choosing 7 and two other numbers from 1 to 6)$=\frac{{}^6C_2}{{}^{10}C_3} = \frac{6\times5}{2}\times\frac{3\times2}{10\times9\times8} = \frac{1}{8}$

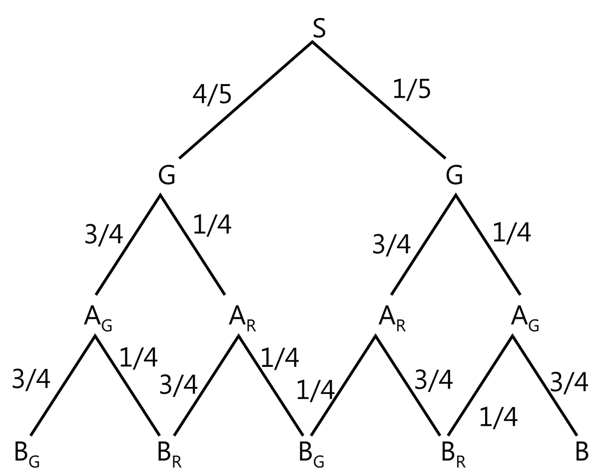
P(A ∩ B) = P (choosing 3 and 7 and one other number from 4 to 6)$=\frac{3}{{}^{10}C_3} = \frac{3\times3\times2}{10\times9\times8} = \frac{1}{40}$

Now, $P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{7}{40} + \frac{1}{8} - \frac{1}{40} = \frac{11}{40}$

Example 10: A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is:

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Sol: Draw a tree diagram for all the possibilities and calculate the probability for all the different cases.



Let G, E_1, E_2 and E denote the following events:

G: Original signal is green

E_1: A receives the signal correctly

E_2:B receives the signal correctly

E = B receives the green signal

We have,

$E = GE_1E_2 \cap GE'_1E'_2 \cap G'E_1E'_2 \cap G'E'_1E_2$

$\Rightarrow P(E) = P(GE_1E_2) + P(G'E'_1E'_2) + P(G'E_1E'_2) + P(G'E'_1E_2)$

$= \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) +$

$\left(\frac{1}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{5}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)$

$= \frac{36 + 4 + 3 + 3}{80} = \frac{23}{40}$

Also, $P(G \cap E) = P(GE_1E_2) + P(GE'_1E'_2) = \frac{40}{80} = \frac{1}{2}$

$\therefore P(G/E) = \frac{P(G \cap E)}{P(E)} = \frac{1/2}{23/40} = \frac{20}{23}$

JEE Advanced/Boards

Example 1: Let A, B, C, be three mutually independent events. Consider the two statement S_1 and S_2

S_1: A and $B \cup C$ are independent

S_2: A and $B \cap C$ are independent

Then,

(A) Both S_1 and S_2 are true (B) Only S_1 is true

(C) Only S_2 is true (D) Neither S_1 nor S_2 is true

Sol: Use the basic understanding of sets and probability of union and intersection of two sets to find the answer.

We are given that

$P(A \cap B) = P(A)\,P(B)$

$P(B \cap C) = P(B)\,P(C)$, $P(C \cap A) = P(C)\,P(A)$,

and $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$

We have

$P(A \cap (B \cap C)) = P(A \cap B \cap C)$

$= P(A)\,P(B)\,P(C) = P(A)\,P(B \cap C)$

$\Rightarrow$A and $B \cap C$ are independent. Therefore, S_2 is true.

Also $P[(A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$= P(A)\,P(B) + P(A)\,P(C) - P(A)\,P(B)\,P(C)$

$= P(A)\,[P(B) + P(C) - P(B)\,P(C)]$

$\therefore$A and $B \cup C$ are independent.

Example 2: A bag contains some white and some black balls, all combinations of balls being equally likely. The total number of balls in the bag is 10. If three balls are drawn at random without replacement and all of them are found to be black, the probability that the bag contains 1 white and 9 black balls is:

(A) $\frac{14}{55}$ (B) $\frac{12}{55}$ (C) $\frac{2}{11}$ (D) $\frac{8}{55}$

Sol: In this case, the number of black balls can be anything between 3 and 10. Apply Baye's theorem to find the required probability.

Let E_i denote the event that the bag contains i black and (10 – i) white balls (i =0, 1, 2, ..., 10). Let A denote the event that the three balls drawn at random from the bag are black. We have,

$P(E_i) = \frac{1}{11}$ (i = 0, 1, 2, ... , 10)

$P(A|E_i) = 0$ for i = 0, 1, 2

and $P(A|E_i) = \frac{^iC_3}{^{10}C_3}$ for $i \geq 3$

Now, by the total probability rule,

$$P(A) = \sum_{i=0}^{10} P(E_i)P(A \mid E_i)$$

$$= \frac{1}{11} \times \frac{1}{^{10}C_3}\,[^3C_3 + {^4C_3} + \dots + {^{10}C_3}]$$

But $^3C_3 + {^4C_3} + {^5C_3} + \dots + {^{10}C_3}$

$= {^4C_4} + {^4C_3} + {^5C_3} + \dots + {^{10}C_3}$

$= {^5C_4} + {^5C_3} + {^6C_3} + \dots + {^{10}C_3}$

$= {^6C_4} + {^6C_3} + \dots + {^{10}C_3} = \dots = {^{11}C_4}$

Thus, $P(A) = \frac{^{11}C_4}{11 \times {^{10}C_3}} = \frac{1}{4}$

By the Bayes' rule

$$P(E_9|A) = \frac{P(E_9)P(A \mid E_9)}{P(A)} = \frac{\frac{1}{11}\frac{(^9C_3)}{^{10}C_3}}{\frac{1}{4}} = \frac{14}{55}$$

Example 3:A pair of biased dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is:

(A) 2/5 (B) 3/5

(C) 4/5 (D) None of these

Sol: The possible outcomes could be 5, X5, XX5, XXX5, XXXX5 and so on, where X denotes a sum of neither 5 nor 7. Also it can be easily understood that this sequence goes on till infinity.

Let A denote the event that a sum of 5 occurs, B denote the event that a sum of 7 occurs and C the event that neither a sum of 5 nor a sum of 7 occurs. We have,

$P(A) = \frac{4}{36} = \frac{1}{9}$

$P(B) = \frac{6}{36} = \frac{1}{6}$

and $P(C) = \frac{26}{36} = \frac{13}{18}$

Thus,

P(A occurs before B) = P[A or $(C \cap A)$ or $(C \cap C \cap A)$ or ....]

$= P(A) + P(C \cap A) + P(C \cap C \cap A) + ...$

$= P(A) + P(C)\,P(A) + P(C)^2\,P(A) + ...$

$$= \frac{1}{9} + \left(\frac{13}{18}\right)\frac{1}{9} + \left(\frac{13}{18}\right)^2\frac{1}{9} + ...$$

$$= \frac{1/9}{1 - \frac{13}{18}} = \frac{2}{5} \text{ [sum of an infinite G.P.]}$$

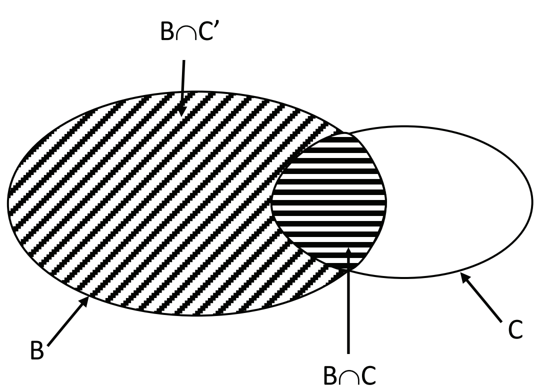
Example 4: If A, B and C are three events such that P(B) $= \frac{3}{4}$, $P(A \cap B \cap C') = \frac{1}{3}$ and $P(A' \cap B \cap C') = \frac{1}{3}$, then $P(B \cap C)$ is equal to:

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{15}$ (D) $\frac{1}{9}$

Sol: Apply the knowledge of set theory to write $B \cap C'$ in terms of $A \cap B \cap C'$ and $A' \cap B \cap C'$.

We have, $P(B \cap C') = P[(A \cup A') \cap (B \cap C')]$

$= P(A \cap B \cap C') + P(A' \cap B \cap C') = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$



Now, $P(B \cap C) = P(B) - P(B \cap C') = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

Example 5: A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operative at least two of its engine must function. Let X denote the event that ship is operational and let X_1, X_2 and X_3 respectively the events that the engines E_1, E_2 and E_3 are functioning. Let,

(A) $P(X'_1 / X) = \frac{3}{8}$

(B) $P(X/X_2) = \frac{7}{8}$

(C) P(Exactly two engines are functioning) $= \frac{7}{8}$

(D) $P(X/X_1) = 7/16$

Sol: The ship can be operational in four possible cases. Calculate the probability of the ship being operational and then proceed accordingly.

We have, $X = (X_1X_2X_3') \cup (X_1X_2'X_3) \cup (X_1'X_2X_3) \cup (X_1X_2X_3)$

and $X'_1 \cap X = X'_1 X_2 X_3$

Now, $P(X'_1 / X) = \frac{P(X'_1 \cap X)}{P(X)} = \frac{P(X'_1 X_2 X_3)}{P(X)}$

We have $P(X'_1 X_2 X_3) = P(X'_1)P(X_2)P(X_3)$

$\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{32}$

and $P(X) = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) +$

$\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{4}$

$\therefore P(X'_1 / X) = \frac{1}{8}$

Next, $X \cap X_2 = X - X_1X_2'X_3$

$P(X \cap X_2) = P(X) - P(X_1 x_2' X_3) = \frac{5}{32}$

$\therefore P(X / X_2) = \frac{P(X \cap X_2)}{P(X_2)} = \frac{5/32}{1/4} = \frac{5}{8}$

Example 6: A fair coin is tossed 100 times. The probability of getting tails 1, 3 ... ...49 times is:

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

Sol: Let p = probability of getting a tail in a single trial $= \frac{1}{2}$ and n = number of trials = 100

X = Number of trials in 100 trials

We have, $P(X = r) = {}^{100}C_r\, p^r\, q^{n-r}$

$$= {}^{100}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{100-r} = {}^{100}C_r \left(\frac{1}{2}\right)^{100}$$

Now,

$P(X = 1) + P(X = 3) + \ldots.. + P(X = 49) = {}^{100}C_1 \left(\frac{1}{2}\right)^{100} +$

$${}^{100}C_3 \left(\frac{1}{2}\right)^{100} + \ldots\ldots\ldots + {}^{100}C_{49} \left(\frac{1}{2}\right)^{100}$$

$$= ({}^{100}C_1 + {}^{100}C_3 + \ldots\ldots + {}^{100}C_{49}) \left(\frac{1}{2}\right)^{100}$$

But ${}^{100}C_1 + {}^{100}C_3 + \ldots\ldots. + {}^{100}C_{99} = 2^{99}$

Also, ${}^{100}C_{99} = {}^{100}C_1$,

${}^{100}C_{97} = {}^{100}C_3$, …….. ${}^{100}C_{51}$

${}^{100}C_{49}$

Thus, $2({}^{100}C_1 + {}^{100}C_3 + \ldots\ldots + {}^{100}C_{49}) = 2^{99}$

$\Rightarrow {}^{100}C_1 + {}^{100}C_3 + \ldots\ldots + {}^{100}C_{49} = 2^{98}$

$\therefore$ Probability of required even $\frac{2^{98}}{2^{100}} = \frac{1}{4}$

Example 7: If A, B and C are three events, then

(A) P (exactly two of A, B, C occur) $\le P(A \cap B) + P(B \cap C) + P(C \cap A)$

(B) $P(A \cup B \cup C) \le P(A) + P(B) + P(C)$

(C) P(exactly one of A, B, C occur $\le P(A) + P(B) + P(C) - P(B \cap C) - P(C \cap A) - P(A \cap B)$

(D) P (A and at least one of B, C occurs)

$\le P(A \cap B) + P(A \cap C)$

Sol: Apply Boole's Inequality.

We have, P(exactly two of A, B, C occur)

$= P(A \cup B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C) \le P(A \cap B) + P(B \cap C) + P(C \cap A)$

Also, $P(A \cup B \cup C) \le P(A \cup B) + P(C) \le P(A) + P(B) + P(C)$

Next P(exactly one of A, B, C occurs)

$= P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(C \cap A) + 3P(A \cap B \cap C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(C \cap A)$

$- [P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)]$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$P(C \cap A)$ – P (exactly two of A, B, C occur)

$\le P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$P(C \cap A)]$

Lastly P(A and at least one of B, C occur)

$= P[A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$\le P(A \cap B) + P(A \cap C)$

Example 8: For two events A and B, if P(A) = $P(A \mid B) = \frac{1}{4}$ and $P(B \mid A) = \frac{1}{2}$, then

(A) A and B are independent

(B) A and B are mutually exclusive

(C) $P(A' \mid B) = \frac{3}{4}$

(D) $P(B' \mid A') = \frac{1}{2}$

Sol: Use basic formulae.

We have, $P(A) = P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A)$ $P(B)$

Therefore, A and B are independent. Also

$$P(A \cap B) = P(A)\, P(B \mid A) = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{8}\right)^{1}0$$

$\therefore$ A and B are mutually exclusive.

As A and B are independent

$$P(A' \mid B) = P(A') = 1 - P(A) = 1 - \left(\frac{1}{4}\right) = \left(\frac{3}{4}\right)$$

Since A and B are independent.

$P(B) = P(B \mid A) = \frac{1}{2}$

$\Rightarrow P(B' \mid A') = P(B') = 1 - P(B) = \frac{1}{2}$

Example 9: Let X be a set containing n elements. If two subsets,A and B, of X are picked at random, the probability that A and B have the same number of elements is:

(A) $\frac{^{2n}C_n}{2^{2n}}$ (B) $\frac{1}{^{2n}C_n}$

(C) $\frac{1.3.5 \;\ldots\ldots\; (2n-1)}{2^n(n!)}$ (D) $\frac{3^n}{4^n}$

Sol: The number of ways of choosing a set of k elements is ^nC_k. The total number of subsets from a set of n elements is 2^n.

We know that the number of subsets of a set containing n elements is 2^n. Therefore, the number of ways of choosing A and B is $2^n . 2^n = 2^{2n}$. We also know that the number of subsets (of X) which contain exactly r elements is ^nC_r. Therefore, the number of ways of choosing A and B so that they have the same number of elements is

$(^nC_0)^2 + (^nC_1)^2 + (^nC_2)^2 + \ldots\ldots + (^nC_n)^2$

$= {}^{2n}C_n = \frac{1.2.3 \;\ldots\ldots\; (2n-1)(2n)}{n!n!}$

$= \frac{[1.3.5 \ldots\ldots (2n-1)][2.4.6 \ldots\ldots (2n)]}{n!n!}$

Thus, the probability of the required event is

$\frac{^{2n}C_n}{2^{2n}} = \frac{1.3.5 \ldots\ldots (2n-1)}{2^n(n!)}$

Example 10: Two numbers are selected at random from the number 1, 2, ..., n. Let p denote the probability that the difference between the first and second is not less than m (where 0 < m < n). If n = 25 and m = 10, find 5p.

Sol: Apply Total probability theorem.

Let the first number be x and the second be y. Let A denote the event that the difference between the first and second numbers is at least m. Let E_x denote the event that the first number chosen is x. We must have $x - y \geq m$ or $y \leq x - m$. Therefore $x > m$ and $y < n - m$.

Thus, $P(E_x) = 0$ for $0 < x \leq m$ and $P(E_x) = 1/n$ for $m < x \leq n$.

Also, $P(A \mid E_x) = (x - m)/(n - 1)$.

Therefore, $P(A) = \sum_{x=1}^{n} P(E_x)\, P(A \mid E_x)$

$= \sum_{x=m+1}^{n} P(E_x)\, P(A \mid E_x) = \sum_{x=m+1}^{n} \frac{1}{n}.\frac{x-m}{n-1}$

$\frac{1}{n(n-1)}[1 + 2 + \ldots\ldots + (n-m)] = \frac{(n-m)(n-m+1)}{2n(n-1)}$

Put n = 25 and m = 10

$\Rightarrow 5P = 5\,\frac{(25-10)(25-10+1)}{2.25(25-1)} = 1$

JEE Main/Boards

Exercise 1

Q.1 Given $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{4}$. Are the events independent?

Q.2 Given $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{6}$. Are the events A and B independent?

Q.3 A die is thrown twice. Find the probability of getting a number 6 on the first throw and number > 4 on the second.

Q.4 Given P(A) = 0.3, P(B)= 0.2. Find P(B/A) if A and B are mutually exclusive events.

Q.5 If P(A)=0.4, P(B)=p and $P(A \cup B)=0.7$. Find the value of p, if A and B are independent set of events.

Q.6 Does the following table represents a probability distribution? Give reasons.

X	–2	–1	0	1	2
P(X)	0.1	0.2	–0.2	0.4	0.5

Q.7 Given P(A) = 0.2, P(B) = 0.3 and $P(A \cap B) = 0.1$ Find P(A/B).

Q.8 The parameters n and p of a binomial distribution are 12 and 1/3 respectively, Find the standard deviation.

Q.9 Given P(A) = 0.4, P(B) = 0.7 and P(B/A) = 0.6. Find $P(A \cup B)$.

Q.10 A coin is tossed three times and the Random variable X represents "number of heads". What values X can take?

Q.11 Does the following table represents a probability distribution? Give reasons.

X	0	1	2
P(X)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

Q.12 Find the value of k, such that the following distribution represents a probability distribution

X	0	1	2	3	4
P(X)	k	0	3k	2k	4k

Q.13 Two cards are drawn successively, with replacement, from a deck of 52 cards. Find the probability of getting both spades.

Q.14 Find the mean of the distribution.

X	1	2	3
P(X)	0.4	0.3	0.3

Q.15 A coin is tossed 7 times, write the Probability distribution of getting r heads.

Q.16 In two successive throws of a pair of dice, find the probability of getting a total of 8 each time.

Q.17 Events E and F are given to be independent. Find P(F) if it is given that: P(E) = 0.60 and $P(E \cap F) = 0.35$.

Q.18 If A and B are two independent events such that $P(A \cup B)=0.7$ P(A)=0.4. Find P(B).

Q.19 Two cards are drawn from a pack of 52 cards at random and kept out. Then one card is drawn from the remaining 50 cards. Find the probability that it is an ace.

Q.20 Three cards are drawn with replacement from a well shuffled pack of cards. Find the probability that the cards are a king, a queen and a jack.

Q.21 A policeman fires four bullets on a dacoit. The probability that the dacoit will be killed by one bullet is 0.6. What is the probability that dacoit is still alive?

Q.22 A bag contains tickets numbered 1,2,3,..... , 50 of which five tickets x_1, x_2,......

x_5 are drawn at random and arranged in

ascending order of magnitude $x_1, < x_2 < x_3 <$

$x_4 < x_5$. What is the probability that $x_3 = 30$?

Q.23 A random variable X has the following probability distribution:

X	–2	–1	0	1	2	3
P(X)	0.1	k	0.2	2k	0.3	k

Find the value k(i) $P(X \le 1)$

(ii) $P(X \ge 0)$

Q.24 Two cards are drawn successfully with replacement from a well shuffled pack of 52cards.Find the probability distribution of number of aces.

Q.25 In a lottery, a person choose six different numbers at random from 1 to 20 and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game, (order of numbers is not important)?

Q.26 The probability of students A passing an examination is 3/5 and of student B is 4/5. Assuming that the two events "A passes", "B passes" as independent. Find the probability of:

(i) Both the students passing the examination (ii) only A passing the examination

(iii) Only one of them passing the examination (iv) none of them passing the examination.

Q.27 A box contains 12 items of which 3 are defective. A sample of 3 items is selected from the box. Let x denote the number of defective items in a sample, find the probability distribution of X.

Q.28 Two dice are thrown. Find the probability that the numbers appeared have a sum 8 if it is known that the second dice always exhibits 4.

Q.29 In an examination, an examinee either guesses or copies or knows the answer of multiple choice questions with four choices. The probability that he makes a guess is 1/3 and probability that he copies the answer is 1/6.The probability that his answer is correct, given that he copied it, is 1/8. Find the probability that he known the answer to the question, given that he correctly answered it.

Q.30 There are three bags which contains 2 white, 3black; 4 white, 1 black; 3 white, 7 black balls respectively A ball is drawn at random from one of the bags and is found to be black. Find the probability that is was drawn from the bag containing

(i) Maximum number of black balls.

(ii) Maximum number of white balls.

Q.31 Two cards are drawn successively with replacement from a pack of 52 cards. Find the probability distribution of the number of aces. Find its mean and standard deviation.

Q.32 The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05.Find the probability that out of 5 such bulbs.

(i) None

(ii) Not more than one

(iii) More than one will fuse after 150 days of use.

(iv) At least one

Q.33 In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is 5/6. What is the probability that he will knock down fewer than 2 hurdles?

Q.34 If on an average 1 ship in every 10 sinks, then find the chance that out of 5 ships at least 4 will arrive safely.

Q.35 About 70% of certain kind of seeds sold in the retail market germinate when planted under normal conditions. Suppose one packed contains 10 seeds. If these are planted, then what is the probability of 2 of these germinating?

Q.36 A man takes a step forward with probability0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps, he is just one step away from the starting point.

Q.37 A bag contains 10 balls, each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with digit 6?

Q.38 Six dice are thrown 729 times. How many times do you expect at least three dice to show five or six?

Q.39 A survey of200 families each having 4 children was conducted. In how many families do you expect 3 boys and 1 girl if boys and girls are equal probable?

Q.40 Past experience shows that 80% of Operations performed by a doctor are successful. If he performs 4 operations in a day, what is the probability that at least three operations will be successful?

Q.41 The probability that a student entering a collage will graduate is 0.6. Find the probability that out of a group of 6 students,

(i) None (ii) Atleast one

(iii) At most 3 will graduate.

Q.42 The probability of a bomb hitting a target is $\frac{1}{3}$. Two bombs are enough to destroy a bridge. If five bombs are dropped at the bridge, find the probability that the bridge is destroyed.

Q.43 In a binomial distribution, the sum of mean and variance is 42 and their product is 360.Findthe distribution.

Q.44 A bag contains 3 red and 7 black balls. Two balls are selected are selected at random without replacement. If the second selected is given to be red, what is the probability that the first selected is also red?

Q.45 Five dice are thrown simultaneously. If the occurrence of an odd number in a single die is considered a success, find the probability that there are odd number of successes.

Q.46 A die is thrown 10 times. If getting an even number is a success, find the probability of getting at least 9 successes.

Q.47 There are three urns A, B and C. Urn A contain4 red balls and 4 green balls. Urn B contains red ball and 5 green balls. Urn c contains 5 red balls and 2 green balls. One ball is drawn from each of the three urns. What is the probability out of these three drawn, two are green ball one is a red ball?

Q.48 A bag contains 4 red, 3 black and 3 white ball two balls are drawn from the bag. What is the probability that none of the balls drawn is white ball?

Q.49 A and B appear for an interview for two post, the probability of A's selection is 1/3 and the of B's selection is 2/5. Find the probability the only of them will be selected.

Q.50 A coin is tossed thrice and all eight out come are assumed equally likely Let the event E"the first throw results in head" and event F"the last throw results in tail". Find whether events E and F are independent.

Exercise 2

Single Correct Choice Type

Q.1 If cards are drawn at random from a pack of 52 playing cards without replacement then the probability that a particular card is drawn at the n^{th} draw is:

(A) 1/(53-n) (B) 1/52

(C) n/52 (D) n/(53 – n)

Q.2 4 persons are asked the same question by an interviewer. If each has independent probability 1/6 of answering the question correctly. The probability that at least one answers correctly is:

(A) 2/3 (B) $(1/6)^4$

(C) $1-(5/6)^4$ (D) $1-(1/6)^4$

Q.3 A person draws a card from a pack of 52 cards, replaces it & shuffles the pack. He continues doing this till he draws a spade. The probability that he will fail exactly the first two times is:

(A) 1/64 (B) 9/64 (C) 36/64 (D) 60/64

Q.4 A committee of 5 is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is:

(A) 1/2 (B) 5/9 (C) 4/9 (D)2/3

Q.5 For a biased die the probabilities for the different faces to turn up are given below:

Faces: 1 2 3 4 5 6

Prob.: 0.10 0.32 0.21 0.15 0.05 0.17

The die is tossed & you are told that either face one or face two has turned up. Then the probability that it is face one is:

(A) 1/6 (B)1/10 (C) 5/49 (D) 5/21

Q.6 For any 2 events A&B, the probabilities P(A), P(A+B), P(AB) & P(A) + P(B) when arranged in the increasing order of their magnitudes is:

(A) $P(AB) \le P(A) \le P(A+B) \le P(A)+P(B)$

(B) $P(A)+P(B) \le P(A+B) \le P(AB) \le P(A)$

(C) $P(A+B) \le P(AB) \le P(A)+P(B) \le P(A)$

(D) $P(AB) \le P(A) \le P(A)+P(B) \le P(A+B)$

Q.7 An integer x is chosen from the first 50 positive integers. The probability that, $x + \frac{100}{x} > 50$, is:

(A) $\frac{1}{25}$ (B) $\frac{2}{25}$ (C) $\frac{1}{10}$ (D) None of these

Q.8 The probability of India winning a test match against West Indies is 1/2. Assuming independent from match to match the probability that in a 5 match series, India's second win occurs at the 3rd test is:

(A) 1/4 (B) 1/8 (C) 1/2 (D) 2/3

Previous Years' Questions

Q.1 The probability that an event A happens in one trial of an experiment is 0.4. Three independent trials of the experiments are performed. The probability that the event A happens at least ones is ***(1980)***

(A) 0.936 (B) 0.784

(C) 0.904 (D) None

Q.2 If A and B are two independent events such that P(A)> 0,and P(B) $\neq$ 1, then $P(\bar{A}/\bar{B})$ is equal to ***(1982)***

(A) 1 – P(A/B) (B) $1-P(A/\bar{B})$

(C) $\frac{1-P(A\cup B)}{P(B)}$ (D) $\frac{P(\bar{A})}{P(\bar{B})}$

Q.3 The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A})+P(\bar{B})$ is: ***(1987)***

(A) 0.4 (B) 0.8 (C) 1.2 (D) 1.4

(Here $\bar{A}$ and $\bar{B}$ are complements of A and B, respectively).

Q.4 One hundred identical coins, each with probability p, of showing up heads are tossed once. If 0< p< 1 and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is **(1988)**

(A) $\frac{1}{2}$ (B) 49/101 (C) 50/101 (D) 51/101

Q.5 Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently, equals **(1998)**

(A) $\frac{1}{2}$ (B) $\frac{7}{15}$ (C) $\frac{2}{15}$ (D) $\frac{1}{3}$

Q.6 If E and F are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then: **(1998)**

(A) Occurrence of E $\Rightarrow$ occurrence of F

(B) Occurrence of F $\Rightarrow$ occurrence of E

(C) Non-occurrence of E $\Rightarrow$ non-occurrence of F

(D) None of the above implication holds

Q.7 If $P(B) = \frac{3}{4}$, $P(\bar{A} \cap \bar{B} \cap \bar{C}) = \frac{1}{3}$ and

$P(\bar{A} \cap \bar{B} \cap \bar{C}) = \frac{1}{3}$ then $P(B \cap C)$ is: **(2002)**

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{15}$ (D) $\frac{1}{9}$

Q.8 If three distinct numbers are chosen randomly from the first100 natural numbers, then the probability that all three of them are divisible by both 2 and 3, is **(2004)**

(A) $\frac{4}{55}$ (B) $\frac{4}{35}$ (C) $\frac{4}{33}$ (D) $\frac{4}{1155}$

Q.9 One Indian and four American men and their wives are to be seated randomly around a circular table. Then, the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is **(2007)**

(A) 1/2 (B) 1/3 (C)2/5 (D)1/5

Q.10 An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is **(2008)**

(A) 2,4 or 8 (B) 3, 6 or 9

(C) 4 or 8 (D)5 or 10

Q.11 Let (0 be a complex cube root of unity with $\omega \ne 1$. A fair die is thrown three times. If r_1, r_2 and $r_3$are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is: **(2010)**

(A) 1/18 (B) 1/9 (C) 2/9 (D) 1/36

Q.12 A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal is green is **(2010)**

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Q.13 It is given that the event A and B are such that $P(A) = \frac{1}{4}, P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$. Then P(B) is **(2008)**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$

Q.14 A die thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is**(2008)**

(A) $\frac{3}{5}$ (B) 0 (C) 1 (D) $\frac{2}{5}$

Q.15 The mean of the number a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b? **(2008)**

(A) a = 0, b = 7 (B) a = 5, b = 2

(C) a = 1, b = 6 (D) a = 3, b = 4

Q.16 In a stop there are five types of ice-creams available. A child buys six ice-creams

Statement-I: The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$

Statement-II: The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A/s and 4 B's in a row. **(2008)**

(A) Statement-I is false, statement-II is true.

(B) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I

(C) Statement-I is true, statement-II is true ; statement-II is not a correct explanation for statement-I.

(D) Statement-I is true, statement-II is false

Q.17 How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent? ***(2008)***

(A) $8 \cdot {}^6C_4 \cdot {}^7C_4$ (B) $6 \cdot 7 \cdot {}^8C_4$

(C) $6 \cdot 8 \cdot {}^7C_4$ (D) $7 \cdot {}^6C_4 \cdot {}^8C_4$

Q.18 If the mean deviation of number 1, 1 + d, 1 + 2d, ..., 1 + 100d from their mean is 255, then the d is equal to ***(2009)***

(A) 10.0 (B) 20.0 (C) 10.1 (D) 20.2

Q.19 From 6 different novels and 3 different dictionaries, 4 movies and 1 dictionary are to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then the number of such arrangements is ***(2009)***

(A) Less than 500

(B) At least 500 but less than 750

(C) At least 750 but less than 1000

(D) At least 1000

Q.20 One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals. ***(2009)***

(A) $\frac{1}{14}$ (B) $\frac{1}{7}$ (C) $\frac{5}{14}$ (D) $\frac{1}{50}$

Q.21 Statement-I: The variance of first n even natural numbers is $\frac{n^2-1}{4}$

Statement-II: The sum of first n natural number is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$ ***(2009)***

(A) Statement-I: is true, statement is true; statement-II is a correct explanation for statement-I

(B) Statement-I is true, statement-II is true; statement-II is not a correct explanation for Statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Q.22 Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20} ***(2010)***

Statement-I: The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement-II: If the four chosen numbers from an AP, then the set of all possible values of common difference is $\{\pm1, \pm2, \pm3, \pm4, \pm5\}$.

(A) Statement-I is true, statement-II is true; statement-II is not the correct explanation for statement-I

(B) Statement-I is true, statement-II is false

(C) Statement-I is false, statement-II is true

(D) Statement-I is true, statement-II is true; statement-II is the correct explanation for statement-I

Q.23 Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 \Rightarrow \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{20}C_j$. ***(2010)***

Statement-I: $S_3 = 55 \times 2^9$

Statement-II: $S_1 = 55 \times 2^8$ and $S_2 = 10 \times 2^8$

(A) Statement-I is true, statement-II is true; statement-II is not the correct explanation for statement-I

(B) Statement-I is true, statement-II is false

(C) Statement-I is false, statement-II is true

(D) Statement-I is true, statement-II is true, Statement-II is the correct explanation for statement-I

Q.24 There are two urns. urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is ***(2010)***

(A) 36 (B) 66 (C) 108 (D) 3

Q.25 An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colour is ***(2010)***

(A) $\frac{2}{7}$ (B) $\frac{1}{21}$ (C) $\frac{2}{23}$ (D) $\frac{1}{3}$

Q.26 For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is ***(2010)***

(A) $\frac{11}{2}$ (B) 6 (C) $\frac{13}{2}$ (D) $\frac{5}{2}$

Q.27 Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is **(2012)**

(A) 880 (B) 629 (C) 630 (D) 879

Q.28 Three numbers are chosen at random without replacement {1, 2, 3, ….., 8}. The probability that their minimum is 3, given that their maximum is 6, is ***(2012)***

(A) $\frac{3}{8}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

Q.29 All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given? ***(2013)***

(A) Mean (B) Median

(C) Mode (D) Variance

Q.30 Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are ***(2014)***

(A) Independent but not equally likely

(B) Independent and equally likely

(C) Mutually exclusive and independent

(D) Equally likely but not independent

Q.31 The variance of first 50 even natural numbers is ***(2014)***

(A) 437 (B) $\frac{437}{4}$ (C) $\frac{833}{4}$ (D) 833

Q.32 The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8 without repetition is: ***(2015)***

(A) 216 (B) 192 (C) 120 (D) 72

Q.33 If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is ***(2015)***

(A) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (B) $55\left(\frac{2}{3}\right)^{10}$

(C) $220\left(\frac{1}{3}\right)^{12}$ (D) $22\left(\frac{1}{3}\right)^{11}$

Q.34 The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observation valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is ***(2015)***

(A) 16.8 (B) 16.0 (C) 15.8 (D) 14.0

Q.35 If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true***(2016)***

(A) $3a^2 - 32a + 84 = 0$ (B) $3a^2 - 34a + 91 = 0$

(C) $3a^2 - 23a + 44 = 0$ (D) $3a^2 - 26a + 55 = 0$

Q.36 Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shown up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true? ***(2016)***

(A) E_2 and E_3 are independent

(B) E_1 and E_3 are independent

(C) E_1, E_2 and E_3 are independent

(D) E_1 and E_2 are independent

Q.37 If all the words (with or without meaning) having five letters, formed using the letter of the word SMALL and arranged as in dictionary, then the position of the work SMALL is ***(2016)***

(A) 59 (B) 52 (C) 58 (D) 46

JEE Advanced/Boards

Exercise 1

Q.1 There are 2 groups of subjects one of which consists of 5 science subjects and 3 engineering subjects and other consists of 3 science and 5 engineering subjects. An unbiased die is cast. If the number 3 or 5 turns up a subject is selected at random from first group, otherwise the subject is selected from 2^{nd}group. Find the probability that an engineering subject is selected.

Q.2 A pair of fair dice is tossed. Find the probability that the maximum of the two numbers is greater than 4.

Q.3 In a given race, the odds in favor of four horses A, B, C & D are 1: 3, 1: 4, 1: 5 and 1: 6 respectively. Assuming that a dead heat is impossible, find the chance that one of them wins the race.

Q.4 A covered basket of flowers has some lilies and roses. In search of rose, sweety and shweta alternately pick up a flower from the basket but puts it back if it is not a rose. Sweety is 3 times more likely to be the first one to pick a rose. If Sweety begin this 'rose hunt' and if there are 60 lilies in the basket, find the number of roses in the basket.

Q.5 A hotel packed breakfast for each of the three guests. Each breakfast should have consisted of three types of rolls, one each of nut, cheese and fruit rolls. The preparer wrapped each of the nine rolls and once warped, the rolls were indistinguishable from one another. She then randomly put three rolls in a bag for each of the guests. If the probability that each guest got one roll of each type is m where m and n are relatively prime integers, find the value of(m + n).

Q.6 Players A and B alternately toss a biased coin, with A going first. A wins if A tosses a Tail before B tosses a Head; otherwise B wins. If the probability of a head is p, find the value of p for which the game is fair to both players.

Q.7 The entries in a two-by-two determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ are integers that are chosen randomly and independently, and, for each entry, the probability that the entry is odd is p. If the probability that the value of the determinant is even is 1/2, then find the value of p.

Q.8 Let an ordinary fair dice is thrown for five times. If $P = \frac{a}{b}$ expressed in lowest form be the probability that the outcome of the fifth throw was already thrown, then find the value of (a + b).

Q.9 A bomber wants to destroy a bridge. Two bombs are sufficient to destroy it. If four bombs are dropped, what is the probability that it is destroyed, if the chance of a bomb hitting the target is 0.4

Q.10 The chance of one event happening is the square of the chance of a 2^{nd} event, but odds against the first are the cubes of the odds against the 2^{nd}. Find the chances of each, (assume that both events are neither sure nor impossible).

Q.11 A box contains 5 radio tubes of which 2 are defective. The tubes are tested one after the other until the 2 defective tubes are discovered. Find the probability that the process stopped on the (i) Second test; (ii) Third test. If the process stopped on the third test, (iii) find the probability that the first tube is non-defective.

Q.12 An aircraft gun can take a maximum of four shots at an enemy's plane moving away from it. The probability of hitting the plane at first, second, third & fourth shots are 0.4, 0.3, 0.2 & 0.1 respectively. What is the probability that the gun hits the plane?

Q.13 In a batch of 10 articles, 4 articles are defective. 6 articles are taken from the batch for inspection. If more than 2 articles in this batch are defective, the whole batch is rejected. Find the probability that the batch will be rejected.

Q.14 A game is played with a special fair cubic die which has one red side, two blue sides, and three green sides. The result is the colour of the top side after the die has been rolled. If the die is rolled repeatedly, the probability that the second blue result occurs on or before the tenth roll, can be expressed in the form $\frac{3^p - 2^q}{3^r}$ where p, q, r are positive integers, find the value of $p^2 + q^2 + r^2$.

Q.15 An author writes a good book with a probability of 1/2. If it is good it is published with a probability of 2/3. If it is not, it is published with a probability of 1/4. Find the probability that he will get atleast one book published if he writes two.

Q.16 Consider 4 independent trials in which an event A occurs with probability $\frac{1}{3}$. The event B will occur with probability if the event A occurs atleast twice, it can not occur if the event A does not occur and it occurs with a probability $\frac{1}{2}$ if the event A occurs once. If the probability P of the occurrence of event B can be expressed as $\frac{m}{n}$, find the least value of(m + n), where m, n $\in$ N.'

Q.17 A uniform unbiased die is constructed in the shape of a regular tetrahedron with faces numbered 1,2,3 and 4 and the score is taken from the face on which the die lands. If two such dice are thrown together, find the probability of scoring.

(i) Exactly 6 on each of 3 successive throws.

(ii) More than 4 on at least one of the three successive throws.

Q.18 Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that one of them is a red card & the other is a queen.

Q.19 A person flips 4 fair coins and discards those which turn up tails. He again flips the remaining coin and then discards those which turn up tails. Find the probability that he discards atleast 3 coins.

Q.20 Each of the 'n' passengers sitting in a bus may get down from it at the next stop with probability p. Moreover, at the next stop either no passenger or exactly one passenger boards the bus. The probability of no passenger boarding the bus at the next stop being p_o. Find the probability that when the bus continues on its way after the stop, there will again be 'n' passengers in the bus.

Q.21 A jar contains 2n thoroughly mixed balls, n white and n black balls. n persons each of whom draw 2 balls simultaneously from the bag without replacement.

(i) If the probability that each of the n person draw both balls of different colours is 8 35, then find the value of n.

(ii) If n = 4 then find the probability that each of the 4 persons draw balls of the same colour.

(iii) If n = 7 then the find the probability that each of the 7 persons draw balls of same colour.

Q.22 Eight players P_1, P_2, P_3, ... P_8 play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_j will win if i<j. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final.

Q.23 Let A & B be two events defined on a sample space. Given P (A) = 0.4 ; P(B) = 0.80and $P(\bar{A}/B)$ = 0.10. Then find;

(i) $P(\bar{A} \cup B)$ and $P\left[\left(\bar{A} \cap B\right) \cup \left(A \cap \bar{B}\right)\right]$

Q.24 Mr. A randomly picks 3 distinct numbers from the set {1, 2, 3, 4, 5, 6, 7, 8, 9} and arranges them in the descending order to form a three digit number. Mr. B randomly picks 3 distinct numbers from the set {1, 2, 3, 4, 5, 6, 7, 8} and also arranges them in descending order to form a 3 digit number.

(i) Find the probability that A and B has the

same three digit number.

(ii) Find the probability that Mr. A's number is larger than Mr. B's number.

Q.25 A pair of students is selected at random from a probability class. The probability that the pair selected will consist of one male and one female student is $\frac{10}{19}$. Find the maximum number of students the class can contain.

Exercise 2

Single Correct Choice Type

Q.1 Suppose, that it is 9 to 7 against a person A who is now 35 years of age living till he is 65 and 3 to 2 against a person B now 45 living till he is 75, then the chance that one at least of these persons will be alive 30 years hence is:

(A) 14/27 (B) 53/80

(C) l/2 (D) None of these

Q.2 An experiment results in four possible outcomes S_1, S_2, S_3 and S_4 with probabilities p_1, p_2, p_3&p_4 respectively. Which one of the following probability assignment is possible? [Assume S_1, S_2, S_3, S_4 are mutually exclusive]

(A) $p_1=0.25$, $p_2=0.35$, $p_3=0.10$, $p_4=0.05$

(B) $p_1=0.40$, $p_2=-0.20$, $p_3=0.60$, $p_4=0.20$

(C) $p_1= 0.30$, $p_2 = 0.60$, $p_3 = 0.10$, $p_4= 0.10$

(D) $p_1 = 0.20$, $p_2 = 0.30$, $p_3= 0.40$, $p_4 = 0.10$

Q.3 Let P denotes the probability that in a group of 4 persons all are born on different days of the week, then P must lie in the interval:

(A) $\frac{1}{3} < P < \frac{1}{2}$ (B) $\frac{1}{4} < P < \frac{1}{5}$

(C) $\frac{1}{6} < P < \frac{1}{3}$ (D) None of these

Q.4 The probability that 4^{th} power of a positive integer ends in the digit 6 is:

(A) 10% (B) 20% (C) 25% (D) 40%

Q.5 India plays 2 matches each with West-Indies & Australia. In any match the probabilities of India getting points 0,1 & 2 are 0.45, 0.05 & 0.50 respectively Assuming that the outcomes are independent, the probability of India getting atleast 7 points is:

(A) 0.8750 (B) 0.0875

(C) 0.0625 (D) 0.0250

Q.6 A women has 'n' keys, of which one will open her door. If she tries the keys randomly, discarding those that do not work (with out using the discarded key again), the probability that she will open the door with the last key is:

(A) $\frac{1}{n-1}$ (B) $\frac{1}{n}$ (C) $\frac{1}{(n-1)!}$ (D) $\frac{1}{2^n}$

Q.7 If A & B are two independent events, each with probability P, ($P \neq 0$) then $P(A/A \cup B)$ is:

(A) 1/P (B) 1/2

(C)2/P (D) 1/(2 – P)

Q.8 The probability of obtaining more tails than heads in 6 tosses of a fair coins is:

(A) 2/64 (B) 22/64

(C) 21/64 (D) None of these

Q.9 A gambler has one rupee in his pocket. He tosses an unbiased normal coin unless either he is ruined or unless the coin has been tossed for a maximum of five times. If for each head he wins a rupee and for each tail he loses a rupee, then the probability that the gambler is ruined is:

(A) 1/2 (B) 5/8 (C) 3/8 (D) 22/32

Q.10 If x be chosen randomly from the set of first 50 natural numbers, then the probability that x^x is perfect square of a natural number is-

(A) 12/25 (B)1/2 (C) 29/50 (D) 31/50

Q.11 A and B independently solve a problem. The chance that A and B will solve the problem correctly are P & 1/2 respectively. The chance that they will make the same mistake is $\frac{1}{100}$. If the probability that their answer is correct and they get the same answer which is $\frac{300}{301}$, then P is:

(A) 1/2 (B) 3/4 (C) 1/4 (D) None of these

Q.12 Two dice are thrown until a 6 appears on atleast one of them. Then the probability that for the first time, a 6 appears in the second throw is:

(A) 175/1296 (B) 275/1296

(C) 375/1296 (D) None of these

Q.13 Box A has 3 white & 2 red balls, box B has 2 white & 4 red balls. If two balls are selected at random (without replacement) from A & 2 more are selected at random from B, the probability that all the four balls are white is:

(A) 10% (B) 2% (C) 12% (D) 4%

Q.14 A & B are two independent events such that $P(\bar{A}) = 0.7$, $P(\bar{B}) = a$ & $P(A \cup B) = 0.8$, then, a =

(A) 5/7 (B) 2/7 (C) 1 (D) None

Q.15 A writes a letter to his friend B and gives it to his son to post it. The chance that his son will post the letter is 1/2 and the chance that a letter posted will reach it's destination is 5/6. If the letter was not received by B, the chance A'sson did not post the letter is-

(A) $\frac{5}{11}$ (B) $\frac{6}{11}$ (C) $\frac{2}{3}$ (D) $\frac{6}{7}$

Q.16 Two numbers are randomly selected from the set of first 20 natural numbers. Find the chance that their product is even given that their sum is odd-

(A) $\frac{9}{19}$ (B) $\frac{10}{19}$ (C) $\frac{29}{38}$ (D) None of these

Previous Years' Questions

Q.1 A box contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept beside the first. This process is repeated till all the balls are drawn from the box. Find the probability that the balls drawn are in the sequence of 2 black, 4 white and 3 red. ***(1979)***

Q.2 A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. Find the probability of the occurrence of A. ***(1984)***

Q.3 In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks the correct answers. The candidates decide to tick the answers at random, if he is allowed up to three chances to answer the questions, find the probability that he will get marks in the question. ***(1985)***

Q.4 Three players. A, B and C. toss a coin cyclically in that order(that is A, B, C, A, B, C, A, B......) till a head shows. Let p be the probability that the coin shows a head. Let α, β and γ be, respectively, the probabilities that A, B and C gets the first head. Prove that $\beta = (1-p)\alpha$. Determine α, β and γ(in terms of P) ***(1998)***

Q.5 An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6, is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6 only three numbers appear in this list? ***(2001)***

Q.6 A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which at least 4 balls are white. Find the probability that in the next two drawn exactly one white ball is drawn. (Leave the answer in ${}^{n}C_{r}$). ***(2004)***

Q.7 A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7},\frac{3}{7},\frac{2}{7}$ and $\frac{1}{7}$. respectively. Probability that he reaches offices late, if he takes car, scooter, bus or train is $\frac{2}{9}$, $\frac{1}{9}$, $\frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car? ***(2005)***

Paragraph 1 (Q.8 to Q.9): Read the following Paragraph and answer the questions.

There are n urns each containing (n + 1) balls such that the ith urn contains 7 white halls and (n + 1–i) red halls. Let u, be the event of selecting ith urn, i = 1,2,3, ..., n and W denotes the event of getting a white balls. ***(2006)***

Q.8 If $(u_i) \propto$ I, where i = 1, 2,3, ... , then $\lim_{n\to\infty} P(W)$ is equal to

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Q.9 If $P(u_i)=c$, where c is a constant, then $P(u_n/W)$ is equal to

(A) $\frac{2}{n+1}$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{2}$

Q.10 If E and F are independent events such that $0< P(E)<1$ and$0<P(F)<1$, then ***(1989)***

(A) E and F are mutually exclusive

(B) E and F^c (the complement of the event F) are independent

(C) E^C and F^C are independent

(D) $P(E/F)+P(E^C/F)= 1$

Q.11 Let E and F be two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happen is 1/2. Then, ***(1993)***

(A) P(E) = 1/3, P(F) = 1/4

(B) P(E)=1/2,P(F)= 1/6

(C) P(E)= 1/6, P(F) = 1/2

(D) P(E) = 1/4, P(F) = 1/3

Q.12 If $\bar{E}$ and $\bar{F}$ are the complementary events of E and F respectively and if $0 < P(F) < 1$, then ***(1998)***

(A) $P(E/F)+P(\bar{E}/F)= 1$

(B) $P(E/F) + P(E/\bar{F})= 1$

(C) $P(\bar{E}/F) + P(E/\bar{F})= 1$

(D) $P(E/\bar{F})+P(\bar{E}/\bar{F})= 1$

Q.13 Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then ***(2011)***

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$ (B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$ (D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

Q.14 One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is ***(2007)***

(A) 1/2 (B) 1/3 (C) 2/5 (D) 1/5

Q.15 Let $H_1, H_2, ...H_n$ be mutually exclusive and exhaustive event with $P(H_1) > 0$, I = 1, 2, ..., n Let E be any other event with $0 < P(E) < 1$. ***(2007)***

Statement-I: $P(H_i|E) > P(E \mid H_i).\ P(H_i)$ for I = 1, 2, ...,n

Beause

Statement-II: $\sum_{i=1}^{n} P(H_i) = 1$

(A) Statement-I is True, statement-II is true, statement-II is a correct explanation for statement-I

(B) Statement-I is True, statement-II is True, statement-II is NOT a correct explanation for statement-I

(C) Statement-I is True, statement-II is False

(D) Statement-I is False, statement-II is True.

Q.16 The letters of the word **COCHIN** are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the work **COCHIN** is ***(2007)***

(A) 360 (B) 192 (C) 96 (D) 48

Q.17 Let E^c denote the complement of an event E. Let E, F, g be pairwise independent events with P(G) > 0 and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c | G)$ equals ***(2007)***

(A) $P(E^c) + P(F^c)$ (B) $P(E^c) - P(F^c)$

(C) $P(E^c) - P(F)$ (D) $P(E) - P(F^c)$

Q.18 An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is ***(2008)***

(A) 2, 4 or 8 (B) 3, 6 or 9

(C) 4 or 8 (D) 5 or 10

Q.19 Consider all possible permutations of the letters of the word ENDEANOEL.

Match the statement/expressions in column I with the statement/expressions in column II. ***(2008)***

Column I	**Column II**
(A) The number of permutations containing the word ENDEA is	(p) 5!
(B) The number of permutations in which the letter E occurs in the first and the last positions is	(q) 2 × 5!
(C) The number of permutations in which none of the letters D, L, N occurs in the last five position is	(r) 7 × 5!
(D) The number of permutations in which the letter A, E, O occur only in odd positions is	(s) 21 × 5!

Q.20 The number of seven digit integers, with sum of the digit equal to 10 and formed by using the digits 1, 2 and 3 only, is ***(2009)***

(A) 55 (B) 66 (C) 77 (D) 88

Paragraph 2 (Q.21 to Q.23): A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required. ***(2009)***

Q.21 The probability that X = 3 equals

(A) $\frac{25}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{125}{216}$

Q.22 The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{25}{216}$

Q.23 The conditional probability that $X \geq 6$ given X > 3 equals

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

Q.24 A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is ***(2010)***

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Paragraph 3 (Q.25 to Q.26): Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 balls is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2. ***(2011)***

Q.25 The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

Q.26 Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

Q.27 The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is ***(2012)***

(A) 75 (B) 150 (C) 210 (D) 243

Q.28 A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is(are) true ? ***(2012)***

(A) $P\left[X_1^c \mid X\right] = \frac{3}{16}$

(B) P [Exactly two engines of the ship are functioning | X] $= \frac{7}{8}$

(C) $P[X \mid X_2] = \frac{5}{16}$

(D) $P[X \mid X_1] = \frac{7}{16}$

Q.29 Four fair dice D_1, D_2, D_3 and D_4 each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is ***(2012)***

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

Q.30 Let X and Y be two events such that $P(X \mid Y) = \frac{1}{2}$, $P(Y \mid X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is (are) correct ? ***(2012)***

(A) $P(X \cup Y) = \frac{2}{3}$

(B) X and Y are independent

(C) X and Y are not independent

(D) $P(X^c \cap Y) = \frac{1}{3}$

Q.31 Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by the at least one of them is ***(2013)***

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{3}{256}$ (D) $\frac{253}{256}$

Q.32 Of the three independent events E_1, E_2 and E_3 the probability that only E_1 occurs is α only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occurence of } E_1}{\text{Probability of occurrence of } E_3} =$ ________

Q.33 A pack contains n cards numbered from 1 to n. Two consecutive numbered card are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k – 20 = ***(2013)***

Q.34 If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is ***(2013)***

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$ (C) $\frac{558}{648}$ (D) $\frac{556}{648}$

Q.35 If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is ***(2013)***

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$ (C) $\frac{65}{181}$ (D) $\frac{55}{181}$

Q.36 Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is ***(2014)***

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Q.37 Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is ***(2014)***

(A) 264 (B) 265 (C) 53 (D) 67

Paragraph 4 (Q.38 to Q.39): Box 1 contains three cards bearing number 1, 2, 3 ; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, I = 1, 2, 3. ***(2014)***

Q.38 The probability that $x_1 + x_2 + x_3$ is odd, is

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

Q.39 The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

Q.40 The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96 is ***(2015)***

Q.41 Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is ***(2015)***

Paragraph 5 (Q.42 to Q.45): Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II ***(2015)***

Q.42 One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible of n_1, n_2, n_3 and n_4 is (are)

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$

(B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$

(D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

Q.43 A ball is drawn at random from box II. If the probability of drawing a red ball from box I, after this transfer, is 1/3, then the correct option(s) with the possible values of n_1 and n_2 is (are)

(A) $n_1 = 4, n_2 = 6$

(B) $n_1 = 2, n_2 = 3$

(C) $n_1 = 10, n_2 = 20$

(D) $n_1 = 3, n_2 = 6$

Q.44 A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

P (computer turns out to be defective given that it is produced in plant T_1)

= 10 P (computer turns out to be defective given that it is produced in plant T_2)

Where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective.

Then the probability that it is produced in plant T_2 is ***(2016)***

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Q.45 A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is ***(2016)***

(A) 380 (B) 320 (C) 260 (D) 95

Paragraph 6 (Q.46 to Q.47): Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and lo sin a game against t_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored teams T_1 and T_2 respectively, after two games ***(2016)***

Q.46 P (X > Y) is

(A) $\frac{1}{4}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{7}{12}$

Q.47 P (X = Y) is

(A) $\frac{11}{36}$ (B) $\frac{1}{3}$ (C) $\frac{13}{36}$ (D) $\frac{1}{2}$

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.6 Q.9 Q.14
Q.22 Q.25 Q.31
Q.36 Q.39 Q.42
Q.44

Exercise 2

Q.1 Q.3 Q.7
Q.9

Previous Years' Questions

Q.6 Q.7 Q.9
Q.11 Q.12

JEE Advanced/Boards

Exercise 1

Q.5 Q.12 Q.14
Q.20 Q.22 Q.25

Exercise 2

Q.3 Q.6 Q.9
Q.15 Q.16

Previous Years' Questions

Q.3 Q.4 Q.7
Q.11 Q.13

Answer Key

JEE Main/Boards

Exercise 1

Q.1 Yes **Q.2** Yes **Q.3** 1/18 **Q.4** 0 **Q.5** 0.5 **Q.6** No

Q.7 1/3 **Q.8** 1.63 **Q.9** 0.86 **Q.10** 0, 1, 2, 3 **Q.11** No **Q.12** 0.1

Q.13 1/16 **Q.14** 1.9 **Q.15** $P(r) = {}^7C_r\left(\frac{1}{2}\right)^7$, r = 0, 1, 2, ... 7 **Q.16** $\frac{25}{1296}$ **Q.17** $\frac{7}{12}$

Q.18 0.5 **Q.19** 1/13 **Q.20** 6/2197 **Q.21** $(0.4)^4$ **Q.22** $\frac{551}{15134}$

Q.23 k=0.1 (i) 0.6 (ii) 0.8

Q.24

X	0	1	2
P(X)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Q.25 $\frac{1}{{}^{20}C_6}$

Q.26. (i) $\frac{12}{25}$ (ii) $\frac{3}{25}$ (iii) $\frac{11}{25}$ (iv) $\frac{2}{25}$

Q.27

X	0	1	2	3
P(X)	$\frac{21}{55}$	$\frac{27}{55}$	$\frac{27}{220}$	$\frac{1}{220}$

Q.28 1/6

Q.29 $\frac{24}{29}$ **Q.30** (i) $\frac{7}{15}$ (ii) $\frac{2}{15}$ **Q.31** $\mu = \frac{2}{13}, \sigma = 0.38$

Q.32 (i) $\left(\frac{19}{20}\right)^5$ (ii) $\left(\frac{19}{20}\right)^4$ (iii) $1-\frac{6}{5}\left(\frac{19}{20}\right)^4$ (iv) $1-\left(\frac{19}{20}\right)^5$

Q.33 0.4845 **Q.34** 0.9185 **Q.35** 0.00145 **Q.36** $462(0.24)^5$ **Q.37** $\left(\frac{9}{10}\right)^4$

Q.38 233 **Q.39** 50 **Q.40** 0.8192 **Q.41** (i) $\left(\frac{2}{5}\right)^6$ (ii) $1-\left(\frac{2}{5}\right)^6$ (iii) $\frac{1424}{3125}$

Q.42 $\frac{131}{243}$ **Q.43** $\left(\frac{2}{5}+\frac{3}{5}\right)^{50}$ **Q.44** 2/9 **Q.45** 1/2 **Q.46** $\frac{11}{1024}$

Q.47 $\frac{41}{112}$ **Q.48** $\frac{7}{15}$ **Q.49** 7/15 **Q.50** Yes independent

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** C **Q.3** B **Q.4** D **Q.5** D **Q.6** A

Q.7 C **Q.8** A

Previous Years' Questions

Q.1 B	**Q.2** B	**Q.3** C	**Q.4** D	**Q.5** B	**Q.6** D
Q.7 A	**Q.8** D	**Q.9** C	**Q.10** D	**Q.11** C	**Q.12.** C
Q.13 B	**Q.14** C	**Q.15** D	**Q.16** A	**Q.17** D	**Q.18** C
Q.19 D	**Q.20** A	**Q.21** D	**Q.22** B	**Q.23** B	**Q.24** C
Q.25 A	**Q.26** A	**Q.27** D	**Q.28** B	**Q. 29** D	**Q.30** A
Q.31 D	**Q.32** D	**Q.33** A	**Q.34** D	**Q.35** A	**Q.36** C
Q.37 C					

JEE Advanced/Boards

Exercise 1

Q.1 $\frac{13}{24}$	**Q.2** 5/9	**Q.3** 319/420	**Q.4** 120	**Q.5** 79	**Q.6** $\frac{\sqrt{5}-1}{2}$
Q.7 $\frac{1}{\sqrt{2}}$	**Q.8** 1967	**Q.9** $\frac{328}{625}$	**Q.10** $\frac{1}{9},\frac{1}{3}$	**Q.11** (i) 1/10 (ii) 3/10 (iii) 2/3	
Q.12 0.6976	**Q.13** 19/42	**Q.14** 283	**Q.15** 407/576	**Q.16** 130	

Q.17 (i) $\frac{125}{16^3}$ (ii) $\frac{63}{64}$ **Q.18** 101/1326 **Q.19** $\frac{189}{256}$ **Q.20** $(1-p)^{n-1}.[p_0(1-p) + np(1-p_0)]$

Q.21 (i) 4 (ii) $\frac{3}{35}$ (iii) 0 **Q.22** 4/35 **Q.23** (i) 0.82 (ii) 0.76

Q.24 (i) $\frac{1}{84}$ (ii) $\frac{37}{56}$ **Q.25** 20

Exercise 2

Single Correct Choice Type

Q.1 B	**Q.2** D	**Q.3** A	**Q.4** D	**Q.5** B	**Q.6** B
Q.7 D	**Q.8** B	**Q.9** D	**Q.10** C	**Q.11** D	**Q.12** B
Q.13 B	**Q.14** B	**Q.15** D	**Q.16** D		

Previous Years' Questions

Q.1 $\frac{1}{1260}$ **Q.2** $\frac{1}{3}$ or $\frac{1}{2}$ **Q.3** 1/5 **Q.4** $\alpha = \frac{p}{1-(1-p)^3}$, $\beta = \frac{p(1-p)}{1-(1-p)^3}$, $\gamma = \frac{p-2p^2+p^3}{1-(1-p)^3}$

Q.5 $\frac{(3^n - 3.2^n + 3)\times {}^6C_3}{6^n}$ **Q.6** $\frac{{}^{12}C_2.{}^6C_4}{{}^{18}C_6}.\frac{{}^{10}C_1.{}^2C_1}{{}^{12}C_2} + \frac{{}^{12}C_2.{}^6C_5}{{}^{18}C_6}.\frac{{}^{11}C_1.{}^1C_1}{{}^{12}C_2}$ **Q.7** $\frac{1}{7}$

Q.8 B **Q.9** A **Q.10** B, C, D **Q.11** A, D **Q.12** A, D **Q.13** A, D

Q.14 C **Q.15** D **Q.16** C **Q.17** C **Q.18** D

Q.19 A $\to$ p; B $\to$ s; C $\to$ q; D $\to$ q **Q.20** C **Q.21** A **Q.22** B **Q.23** D

Q.24 C **Q.25** B **Q.26** D **Q.27** B **Q.28** B, D **Q.29** A

Q.30 A, B **Q.31** A **Q.32** 6 **Q.33** 5 **Q.34** A **Q.35** D

Q.36 A **Q.37** C **Q.38** B **Q.39** C **Q.40** 8 **Q.41** 5

Q.42 A, B **Q.43** C, D **Q.44** C **Q.45** A **Q.46** B **Q.47** C

Solutions

JEE Main/Boards

Exercise 1

Sol 1: P (A) = $\frac{1}{4}$, P(B) = $\frac{2}{3}$, P(A $\cup$ B) = $\frac{3}{4}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{3}{4} = \frac{1}{4} + \frac{2}{3} - P(A \cap B)$

$P(A \cap B) = \frac{1}{4} + \frac{2}{3} - \frac{3}{4} = \frac{3+8-9}{12} = \frac{+2}{12} = \frac{+1}{6}$

$P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{2}{3} = \frac{1}{6} = P(A \cap B)$

So the events are independent.

Sol 2: P (A) = $\frac{1}{2}$, P(B) = $\frac{1}{3}$

$P(A \cap B) = \frac{1}{6}$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} = P(A \cap B)$

So events A and B are independent.

Sol 3: A dice has 6 number on it 1, 2, 3, 4, 5, 6

In a thrown probability of getting a member 6 P (A)= $\frac{1}{6}$
Total number of possibility

4 has two possibility = 5 or 6 both are greater than 4.

So P(B) = Probability of getting 5 or 6

$\Rightarrow \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$P(A \cap B) = P(A)\,P(B) = \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18}$

Sol 4: P(A) = 0. 3P(B) = 0. 2

$P(B/A) = \frac{P(B \cap A)}{P(A)}$

$P(A \cap B) = 0$

Because it's given that A and B both are exclusive events

$\therefore P(B/A) = 0$

Sol 5: P(A) = 0. 4,P(B) = P

$P(A \cap B) = 0$

A and B are independents so

$P(A \cap B) = P(A) \cdot P(B) = 0.4P$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = 0.4 + P - 0.4P \Rightarrow 0.4 + 0.6P$

$0.7 - 0.4 = 0.3 = 0.6P$

$\Rightarrow P = \frac{0.3}{0.6} = \frac{1}{2} = 0.5$

Sol 6:

x	–2	–1	0	1	2
P(x)	0.1	0.2	–0.2	0.4	0.5

The table does not represents probability distribution as probability never can be negative.

Sol 7: P(A) = 0. 2,P(B) = 0. 3

$P(A \cap B)$ = 0. 1

$$P(A/B)=\frac{P(A\cap B)}{P(B)} = \frac{0.1}{0.3}=\frac{1}{3}$$

Sol 8: n = 12,P = 1/3

Standard deviation = $\sqrt{npq}$

$$q = 1-P = 1-\frac{1}{3}=\frac{2}{3} = \sqrt{\frac{12}{1}\times\frac{1}{3}\times\frac{2}{3}} = 1.\ 63$$

Sol 9: P (A) = 0. 4,P(B) = 0. 7

$$P(A/B)=\frac{P(B\cap A)}{P(A)} = 0.\ 6$$

$\Rightarrow P(B \cap A) = P(A)\ 0.\ 6 = (0.\ 4)\ (0.\ 6) = 0.\ 24$

$P(A \cup B)\ P(A) + P(B) - P(A \cap B)$

= 0. 4 + 0. 7 – 0. 24 = 1. 1 – 0. 24 = 0. 86

Sol 10: A coin is tossed 3 times

X = number of head

X = 0 (T, T, T), 1(T, H, T), 2(H, H, T), 3(H, H, H)

X can be 0, 1, 2, 3

Sol 11:

x	0	1	2
P(x)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

$$\Sigma P(x) = \frac{1}{3}+\frac{1}{3}+\frac{1}{6}=\frac{2}{3}+\frac{1}{6}=\frac{5}{6}\neq 1$$

So it's not probability distribution

Sol 12:

x	0	1	2	3	4
P(x)	k	0	3k	2k	4k

Σ P(x) = 1

K + 0 + 3k + 2k + 4k = 1

⇒ 10 k = 1

$$k = \frac{1}{10}=0.1$$

Sol 13: Total spades in a deck of 52 cards = 13

In a time probability of getting one spades = $\frac{13}{52}=\frac{1}{4}=P$

P(In two times and both time gets spades)

$$= P.\ P = \frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$$

Sol 14:

x	1	2	3
P(x)	0.4	0.3	0.3

Mean = Σ P(x); x

= 1(0. 4) +2 (0. 3) +3 (0. 3)

= 0. 4 + 0. 6 + 0. 9 = 1. 9

Sol 15: A coin is tossed 7 times

P (getting heads in one time) = $\frac{1}{2}$

If total number of heads = r

There is ${}^7C_r$ way to get r heads in 7 times probability of getting r heads

$${}^7C_r\left(\frac{1}{2}\right)^r,\ r = 0\ 1, 2, 3, 4, 5, 6, 7$$

r could be any integer between 0 and 7

Sol 16: P(getting 8 in one throw of a pair dice) = $\frac{5}{36}$

P (in two successive throws of a pair of dice getting 8 each time)

$$= \frac{5}{36}\times\frac{5}{36}=\frac{25}{1296}$$

Sol 17: E and F are independent

P(E) = 0. 60 and $P(E \cap F)$ = 0. 35

$\because\ P(E \cap F) = P(E) \cdot P(F)$

$$P(F) = \frac{P|E\cap F|}{P(E)} = \frac{0.35}{0.60}=\frac{7}{12}$$

Sol 18: $P(A \cup B)$ = 0. 7P(A) = 0. 4

A and B are independent 50

$P(A \cap B) = P(A) \cdot P(B)$

$\because\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$

⇒ 0. 7 = 0. 4 + P(B) – P(A) P(B)P(B) (1 – 0. 4) = 0. 7 – 0. 4 = 0. 3

$$P(B) = \frac{0.3}{0.6}=\frac{1}{2}$$

Sol 19: There is 3 possibility –

First two cards are not ace = $\frac{{}^{48}C_2}{{}^{52}C_2}$

One of them is ace = $\frac{48\times 4}{51\times 52}$

Both are ace = $\frac{4\times3}{51\times52}$

Respectively probability of getting ace in third draw

$\Rightarrow \frac{4}{50},\frac{3}{50},\frac{2}{50}$

$$P = \frac{{}^{48}C_2}{{}^{52}C_2}\times\frac{4}{50}+\frac{48\times4}{52\times51}\frac{3x^2}{50}+\frac{4\times3}{51\times52}\times\frac{2}{50}$$

$= \frac{425}{5525} = \frac{1}{13}$

Sol 20: There is 4 King, queen and 4 jack is a deck of 52 cards. For three drawn

P(One king, one queen, one jack) = $\frac{4}{52}\cdot\frac{4}{52}\cdot\frac{4}{52}\cdot 3!$

There is 3! types of way to get them = $\frac{6}{169\times13} = \frac{6}{2197}$

Sol 21: Total fires = 4

P = Probability of killed by one bullet = 0. 6 after 4 fire.

P(still alive) = $(1 – 0.6)^4 = (0.4)^4$

Sol 22 : Drawn randomly 5 cards

x_1, x_2, x_3, x_4, x_5 and $x_1 < x_2 < x_3 < x_4 < x_5$

$\left| \begin{matrix} 1,2,3\ldots \\ \ldots 49,50 \end{matrix} \right|$

P = Probability of x_3 = 30. So, x_1 and x_2 should be less than 30 for this number of total way to get x_1 and $x_2 = {}^{29}C_2$

And $x_4, x_5 > 30 \Rightarrow$ Total way $\Rightarrow {}^{20}C_2$

Total way for $x_1\ x_2\ x_3\ x_4\ x_5 = {}^{50}C_5$

$$P = \frac{{}^{29}C_2\ {}^{20}C_2}{{}^{50}C_5} = \frac{551}{15134}$$

Sol 23 :

x	–2	–1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	k

Σ P(x); = 1(always true)

0. 1 + K + 0. 2 + 2k + 0. 3 + k = 1

4k = 1 – 0. 1 – 0. 2 – 0. 34k = 0. 4

$\Rightarrow k = \frac{0.4}{4} = 0.1$

(i) $P(x \le 1)$

= P(x = –2) + P(x = –1) + P (x = 0) + P(x = 1)

= 0. 1 + k + 0. 2 + 2k = 3k + 0. 3 = 0. 6

(ii) $P(x \ge 0)$

= P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)

= 0. 2 + 2k + 0. 3 + k = 0. 5 + 3k

= 0. 5 + 3(0. 1) = 0. 5 + 0. 3 = 0. 8

Sol 24: Total aces in a pack of 52 cards = 4

Two cards are drawn with replacement assume x = number of getting aces in two drawn

So (i) x_1 = 0,no aces

$$P(x_1) = \left(\frac{52-4}{52}\right)\left(\frac{52-4}{52}\right) = \frac{48}{52}\cdot\frac{48}{52} = \frac{12^2}{13^2} = \frac{144}{169}$$

(ii) x_2 = 1 (one ace)

$$P(x_2) = 2\times\frac{4}{52}\times\frac{48}{52} = \frac{24}{169}$$

(iii) x_3 = 2 (both are ace)

$$P(x_3) = \frac{4}{52}\cdot\frac{4}{52} = \frac{1}{169}$$

x	0	1	2
P(x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Sol 25: Number of way to get 6 numbers from 1 to 20 = ${}^{20}C_6$

Number of way to get the fixed 6 numbers = 1

$P = \frac{1}{{}^{20}C_6}$

Sol 26: P(A) = probability of student A passing exam

P (A) = 3/5P(B) = 4/5

$\Rightarrow$ P(A) and P(B) are independent

So, $P(A \cap B) = P(A)\cdot P(B) = \frac{3}{5}\cdot\frac{4}{5} = \frac{12}{25}$

(i) Both the student passing exam

$P(A \cap B) = \frac{12}{25}$

(ii) Only A pass the exam

$= P(A) - P(A\cap B) = \frac{3}{5}-\frac{12}{25} = \frac{15-12}{25} = \frac{3}{25}$

(iii) Only one of them passing exam

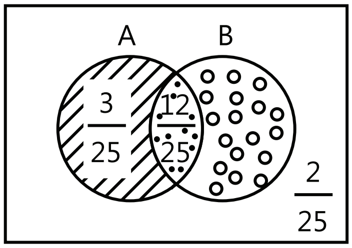
$= P(A) + P(B) - 2P(A \cap B)$

$= \frac{3}{5}+\frac{4}{5}-\frac{2.12}{25} = \frac{15+20-24}{25} = \frac{11}{25}$

(iv) None of them passing exam

$= 1 - P(A \cup B) = 1 + P(A \cap B) - P(A) - P(B)$

$$= 1 + \frac{12}{25} - \frac{3}{5} - \frac{4}{5} = \frac{25+12-15-20}{25} = \frac{2}{25}$$



Sol 27: 3 are defective out of 12 items

A sample of 3 items is selected from the box

Where x = number of defective items

(i) $x_0 = 0$, all are in will condition

$$\Rightarrow P(x_0) = \frac{{}^3C_1\,{}^9C_3}{{}^{12}C_3} = \frac{21}{55} \qquad \boxed{12 = 9 + 3\ \text{def}}$$

(ii) $x_1 = 1$, one defective

$$P(x_1) = \frac{{}^3C_1\,{}^9C_2}{{}^{12}C_3} = \frac{27}{55}$$

(iii) $x_2 = 2$, two defective

$$P(x_2) = \frac{{}^3C_2\,{}^9C_2}{{}^{12}C_3} = \frac{27}{220}$$

(iv) $x_3 = 3$, all are defective

$$P(x_3) = \frac{{}^3C_3}{{}^{12}C_3} = \frac{1}{220}$$

x	0	1	2	3
P(x)	$\frac{21}{55}$	$\frac{27}{55}$	$\frac{27}{220}$	$\frac{1}{220}$

Sol 28: Two dice are thrown, second dice always exhibits.

S = All possibility: (1, 4) (2, 4) (3, 4) (5, 4) (6, 4) (4, 4)

Sum of 8 $\rightarrow$ (4, 4)

$$\Rightarrow P = \frac{x(4,4)}{n(S)} = \frac{1}{6}$$

Sol 29 : C $\rightarrow$ Copies ans.

K $\rightarrow$ Known ans.

G $\rightarrow$ Guess ans.

R $\rightarrow$ Ans. is right

P(C) = 1/6P(G) = 1/3

$$\Rightarrow P(K) = 1 - \frac{1}{6} - \frac{1}{3} = \frac{1}{2}$$

$$P(R / C) = \frac{P(R \cap C)}{P(C)} = \frac{1}{8}$$

P(R) =P(CR) + P(KR) + P(GR) there is 4 choices for each question

$$\text{So, } P(GR) = \frac{1}{4} \cdot P(G) = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

$$\frac{P(KR)}{P(R)} = \frac{\frac{1}{2}}{\frac{1}{8} \times \frac{1}{6} + \frac{1}{12} + \frac{1}{2}} = \frac{24}{1+4+24} = \frac{24}{29}$$

Sol 30 :

I	II	III
2 white	4 white	3 white
3 black	1 black	7 black

B = Ball is black

P(B) = P(BI) + P(BII) + P(BIII)

$$= \frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{7}{10}$$

$$= \frac{3}{15} + \frac{1}{15} + \frac{7}{30} = \frac{6+2+7}{30} = \frac{15}{30} = \frac{1}{2}$$

(i) Bag II has max. number of black balls

$$P\left(\frac{III}{B}\right) = \frac{P(IIIB)}{P(B)} = \frac{\frac{7}{30}}{\frac{1}{2}} = \frac{7}{15}$$

(ii) Bag II has max. white balls

$$\text{So } P\left(\frac{II}{B}\right) = \frac{P(IIB)}{P(B)} = \frac{\frac{1}{15}}{1/2} = \frac{2}{15}$$

Sol 31: Two cards are drawn with replacement

X = number of getting aces from question number 29

x	0	1	2
P(x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

$$\text{mean} = \Sigma P(x)x_1 = 0\left(\frac{144}{169}\right) + 1\left(\frac{24}{169}\right) + 2\left(\frac{1}{169}\right)$$

$$= \frac{24+2}{169} = \frac{2}{13}$$

$$\sigma = SD = \sqrt{\Sigma P(x)(x_1)^2 - (\Sigma P(x) \cdot (x_1))^2}$$

$$= \sqrt{(0)^2\left(\frac{144}{169}\right)+1^2.\left(\frac{24}{169}\right)+2^2.\left(\frac{1}{169}\right)-\left(\frac{2}{13}\right)^2}$$

$$= \frac{1}{\sqrt{169}}\sqrt{24+2^2-2^2}$$

$$= \frac{\sqrt{24}}{13} = 0.377 = 0.38$$

SD = 0. 38, $\mu = \frac{2}{13}$

Sol 32 : Total choose bulb = 5

P = P (bulb will fuse after 150 days) = 0. 05

$q = 1 - P = 0.95 = \frac{19}{20}$, n = 5,

x = number of fuse

(i) P(none of 5 will fuse) = $(q)^5 = \left(\frac{19}{20}\right)^5$

(ii) P(not more than ore) = $P(x \le 1)$

$= P(x = 0) + P(x = 1) = (q)^5 + {}^5C_1\, q^4\, P^1$

$$= \left(\frac{19}{20}\right)^5 + \left(\frac{19}{20}\right)^4\left(\frac{1}{20}\right) = \left(\frac{19}{20}\right)^5\left[1+\frac{20}{19}\cdot\frac{1}{20}\right]$$

$$= \frac{20}{19}\times\left(\frac{19}{20}\right)^5 = \left(\frac{19}{20}\right)^4$$

(iii) $P(x > 1) = 1 - P(x = 0) - P(x = 1) = 1-\frac{6}{5}\left(\frac{19}{20}\right)^4$

(iv) $P(x \ge 1) = 1 - P(x = 0) = 1-\left(\frac{19}{20}\right)^5$

Sol 33 : x = number of hurdles; he down

Total hurdles = 10 = n

$P = 5/6 \Rightarrow q = 1 - P = 1/6$

$P(x < 2) = P(x = 0) + P(x = 1)$

$$= \left(\frac{5}{6}\right)^{10} + 10\times\left(\frac{5}{6}\right)^9\times\frac{1}{6} = 0.4845$$

Sol 34 : Total ship = 5

X = n (arrive safe ships)

$q = \frac{1}{10} \Rightarrow P = 1 - q = \frac{9}{10}$

$P(x \ge 4) = P(x = 4) + P(x = 5)$

$$= {}^5C_1\left(\frac{9}{10}\right)^4\frac{1}{10}+\left(\frac{9}{10}\right)^5 = 0.9185$$

Sol 35 : n = 10,P = 0. 7

x = number of seed germinate

$P(x = 2) = {}^{10}C_2\,(0.7)^2\,(1 - 0.7)^8$

$$= \frac{10\times 9}{2}\times(0.49)(0.3)^8 = 0.00145$$

Sol 36 : P(F) = P(step forward) = 0. 4

P(B) = P(step backward) = 0. 6

N = 11 (number of step forward)

$x_F + x_B = 11 |\, x_F - x_B | = 1$

$\Rightarrow |\, 11 - 2x_B \,| = 1$

$\Rightarrow x_B = \frac{11-1}{2} = 5$ or $2x_B - 11 = 1$

$\Rightarrow 2x_B = 1 + 11 = 12$

$x_B = 6\ x_F = 11 - 5$

or 11 – 6 = 6 or 5

$P(x_F = 6, 5) = P(x = 6) + P(x = 5)$

$= {}^{11}C_6\,(0.4)^6\,(0.6)^5 + {}^{11}C_5\,(0.4)^5\,(0.6)^6$

$= {}^{11}C_5\,[(0.4 + 0.6)\,(0.24)^5]$

$= {}^{11}C_5\,(0.24)^5 = 462\,(0.24)^5$

Sol 37 : 4 balls are drawn with replacement from the bag. Assume x = number ball marked with 6

$$P(x = 0) = \left(\frac{10-1}{10}\right)^4 = \left(\frac{9}{10}\right)^4$$

($\because$ There is only one ball which is marked 6 out of 10)

Sol 38 : 6 dice are thrown assume x_5 = number of dice which show 5 or 6

$P = \frac{2}{6} = \frac{1}{3}$, $q = \frac{4}{6} = \frac{2}{3}$

$P(x_5 \ge 3) = P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6)$

$$= {}^6C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right)^3 + {}^6C_4\left(\frac{2}{6}\right)^4\left(\frac{4}{6}\right)^2$$

$$+{}^6C_5\left(\frac{2}{6}\right)^5\left(\frac{4}{6}\right) + {}^6C_6\left(\frac{2}{6}\right)^6 = \frac{233}{729}$$

Expected number $\rightarrow 729 \times \frac{233}{729} = 233$

Sol 39 : 200 families, $P(B) = \frac{1}{2} = P(G)$ Total children = 4 for each family

$P(3B, 1G) = {}^4C_1 \times \left(\frac{1}{2}\right)^4 = \frac{4}{4\times 4} = \frac{1}{4}$

expectation $\rightarrow 200 \times \frac{1}{4} = 50$ families

Sol 40 : P = probability of success of a operation

P = 0. 8, n = 4x = number of successful operation

$P(x \geq 3) = P(x = 3) + P(x = 4)$

$= [{}^4C_3 (0.8)^3 (1–0.8) + {}^4C_4(0.8)^7]$

$= [4 \times 0.8^3 \cdot 0.2 + (0.8)^4]$

$= 0.8^4 \times 2 = 0.8192$

Sol 41 : P = Probability of graduate student

P = 0. 6

$\Rightarrow q = 1 – 0.6 = 0.4$

x = number of students will graduate

(i) $P(x = 0) = {}^6C_0 (0.4)^6 = \left(\frac{2}{5}\right)^6$

(ii) $P(x \geq 1) = 1 – P(x = 0) = 1-\left(\frac{2}{5}\right)^6$

(iii) $P(x \leq 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$

$= \left(\frac{2}{5}\right)^6 + {}^6C_1\left(\frac{2}{5}\right)^5\frac{3}{5} + {}^6C_2\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2 + {}^6C_3\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^3$

$= \frac{1}{5^6}[2^6 + 6 \cdot 3 \cdot 2^5 + 15 \cdot 2^4 \cdot 3^2 + 20 \cdot 2^3 \cdot 3^3]$

$= \frac{7120}{5^6} = \frac{1424}{3125}$

Sol 42 : P = P(bomb hitting a target)

$P = \frac{1}{3} \Rightarrow q = 1-\frac{1}{3} = \frac{2}{3}$

Two bombs are enough to destroy target

n = 5x = number of bombs that hit the target

so for destroy bridge

$P(x \Rightarrow 2) = 1 – P(x = 0) – P(x = 1)$

$= 1-\left[\left(\frac{2}{3}\right)^5 + {}^5C_1\left(\frac{2}{3}\right)^4 \times \frac{1}{3}\right] = 1-\frac{1}{3^5}[2^5 + 5.2^4]$

$= 1-\frac{112}{3^5} = \frac{243-112}{243} = \frac{131}{243}$

Sol 43: $(p + q)^n$

variance $\sigma^2 = npq$ mean $\mu = np$

Given $\rightarrow np + npq = 42$

$\Rightarrow np(1 + q) = 42$ ...(i)

and $(np)(npq) = 360$

$\Rightarrow (np)^2 q = 360$ ...(ii)

$(1)^2\ (2) \rightarrow \frac{(np)^2(1+q)^2}{(np)^2 q} = \frac{422}{360} = \frac{49}{10}$

$\Rightarrow 10(1 + q^2 + 2q) = 49q$

$\Rightarrow 10q^2 + 202 – 49q + 10 = 0$

$\Rightarrow 10q^2 – 29q + 10 = 0$

$\Rightarrow 10q^2 – 25q – 4q + 10 = 0$

$\Rightarrow 5q(2q – 5) – 2(2q – 5) = 0$

$\Rightarrow (2q – 5)(5q – 2) = 0$

$q = \frac{5}{2}$ or $\frac{2}{5}$, $q < 1$

So, $q = \frac{2}{5} \Rightarrow P = 1 – q = \frac{3}{5}$

and $(np)^2q = 360$

$n^2 = \frac{360}{215\times\left(\frac{3}{5}\right)^2} = \frac{360\times 5^3}{2\times 9}$

$n^2 = 10^2 \times 5^2 n = 10 \times 5 = 50 \Rightarrow \left(\frac{2}{5}+\frac{3}{5}\right)^{50}$

Sol 44 : R_1 = I[st] ball is red

R_2 = 2[nd] ball is red | 3 red, 7 black |

Total 2 balls are related without replacement $P\left(\frac{R_1}{R_2}\right) > ?$

$\Rightarrow \frac{P(R_1 \cap R_2)}{P(R_2)}$

$P(R_2)_2 = \frac{7}{10}\times\frac{3}{9}+\frac{3}{10}\times\frac{2}{9} = \frac{21+6}{90} = \frac{22}{90} = \frac{3}{10}$

$P(R_1 \cap R_2) = \frac{3}{10}\times\frac{2}{9} = \frac{6}{90} = \frac{1}{15}$

$P\left(\frac{R_1}{R_2}\right) = \frac{1/15}{3/10} = \frac{10}{3\times 15} = \frac{2}{9}$

Sol 45: Five dice are thrown

Success = odd number

There is number of odd number and even number are same which is 3 (1, 3,5 and 2, 4, 6)

So, P(success) = P(no success)

but P(success) + P (no success) = 1

∵ There is either odd or even number

P (success) = $\frac{1}{2}$

Sol 46: n = 10x = number of getting even number

$P = \frac{1}{2} \Rightarrow q = 1-\frac{1}{2} = \frac{1}{2}$

$P(x \geq 9) = P(x = 9) + P(x = 10)$

$$= {}^{10}C_9\left(\frac{1}{2}\right)^{10} + 1\left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{10}(1+10) = \frac{11}{2^{10}} = \frac{11}{1024}$$

Sol 47:

4 red, 4 green	3 red, 5 green	5 red, 2 green
A	B	C
↓	↓	↓
one ball	one ball	one ball

= 2 green balls and one is red

P(2G, 1R) = P[(GRG) + (GGR) + (RGG)]

$$P(2G, 1R) = \frac{4}{8}\left[\frac{5}{8}\cdot\frac{5}{7}+\frac{3}{8}\cdot\frac{2}{7}\right]+\frac{4}{8}\times\frac{5}{8}\cdot\frac{2}{7}$$

$$= \frac{1}{2}\left[\frac{25}{56}+\frac{6}{56}\right]+\frac{5}{56} = \frac{1}{56}\times\frac{1}{2}[25+6+10] = \frac{41}{112}$$

Sol 48: Two balls are drawn from $\left|\begin{array}{c}\text{4 red}\\ \text{3 black}\\ \text{3 white}\end{array}\right|$

P(none of the ball is white)= $\frac{{}^{7}C_2}{{}^{10}C_2} = \frac{7}{15}$

Sol 49: P(A) = probability of A selection = $\frac{1}{3}$

$P(B) = \frac{2}{5}$

Total post = 2

P(only one of them will selected) = $P(A) + P(B) - 2P(A \cap B)$

Since both events are independent

So, $P(A \cap B) = P(A)\,P(B) = \frac{1}{3}\cdot\frac{2}{5} = \frac{2}{15}$

$$\Rightarrow P = \frac{1}{3}+\frac{2}{5}-2\cdot\frac{2}{15} = \frac{5+6-4}{15} = \frac{7}{15}$$

Sol 50 : E = 1^{st} throw is head

F = Last throw is tail

$$P(E) = \frac{1}{2}\times\frac{2}{2}-\frac{2}{2} = \frac{1}{2}\quad P(F) = \frac{2}{2}\cdot\frac{2}{2}-\frac{1}{2} = \frac{1}{2}$$

P(E∩F) = P(1^{st} is head and last is tail)

$$= \frac{1}{2}\times\frac{2}{2}\times\frac{1}{2} = \frac{1}{4}$$

$P(E)\cdot P(F) = \frac{1}{4} = P(E \cap F)$

So, E and F are independent.

Exercise 2

Single Correct Choice Type

Sol 1: (B) P(special card at n^{th} drawn)

$$\Rightarrow \frac{51}{52}\cdot\frac{1}{51} = \frac{1}{52}$$

Sol 2: (C) n = 4 (r person)

$P = \frac{1}{6}$ (correct ans. by one)

x = number of correct ones

$$P(x \geq 1) = 1 - P(x = 0) = 1-\left(1-\frac{1}{6}\right)^4 = 1-\left(\frac{5}{6}\right)^4$$

Sol 3: (B) n = 2

x = get spade

Total spades in 52 cards = 13

P = P(he fails exactly first two times)

$$P = \left(\frac{52-13}{52}\right)\left(\frac{52-13}{52}\right)\cdot\frac{13}{52} = \frac{39}{52}\cdot\frac{39}{52}\cdot\frac{13}{52} = \frac{9}{64}$$

Sol 4: (D) 5 is to be chosen from 9 people there is a couple in group of 9

P = P(couple chosen)

q = P(couple don't chose)

$$P + q = \frac{1}{{}^9C_5}\left[{}^7C_5 + {}^7C_3 . \frac{2}{2}\right]$$

$$= \frac{1.2.3.4}{9.8.7.6}\left[\frac{7.6}{1.2} + \frac{7.6.5}{2.3}\right] = \frac{7.8.4}{9.8.7} = \frac{4}{9}$$

Sol 5: (D) P(n) = probability of shown in fall

$$P\left(\frac{1}{1 \cup 2}\right) = \frac{P(1 \cap (1 \cup 2)}{P(1 \cup 2)} = \frac{P(1)}{P(1) + P(2)}$$

$\therefore P(1 \cap 2) = 0$

$$= \frac{0.1}{0.1 + 0.32} = \frac{0.10}{0.42} = \frac{5}{21}$$

Sol 6: (A) P(A + B) = P(A) + P(B) – P(AB)

and P(A), P(B) > P(AB)

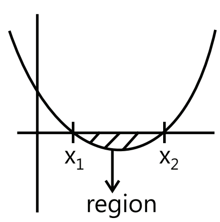
$\therefore$ P(A + B) > P(A), (AB)

and P(A), P(B), P(AB), P(A + B) ≥ 0

$\therefore$ option (A) is correct

P(AB) $\leq$ P(A) $\leq$ P(A + B) $\leq$ P(A) + P(B)

Sol 7: (C) $x \in \{1, 2, \ldots\ldots, 50\}$



$x + \frac{100}{x} > 50$; $\Rightarrow x^2 - 50x + 100 > 0$

$$\Rightarrow x = \frac{50 \pm \sqrt{50^2 - 4(100)}}{2}$$

$x = 25 \pm \frac{1}{2}\sqrt{2500 - 400}$; $x = 25 \pm \frac{10\sqrt{21}}{2}$

$x = 25 \pm 5\sqrt{2}$

x_1 = 2. 0871 and x_2 = 47. 912

In region integer $y \in \{3, 4, \ldots\ldots, 47\}$

So only for x = 1, 2, 48, 49, 50;

y is greater than 0 or $x + \frac{100}{x} > 50$

$$\Rightarrow P = \frac{n(1, 2, 48, 49, 50)}{50} = \frac{5}{50} = \frac{1}{10}$$

Sol 8: (A) $P = \frac{1}{2}$ events

P = probability of win test match

L = Lose match; W = win match

$P(L) = 1 - \frac{1}{2} = \frac{1}{2}$

There is 5 match series.

Total possibility = 2^5

P(India's second win occurs at the 3rd test)

P(LWW) + P(WLW)

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 2 = \frac{1}{4}$$

Previous Years' Questions

Sol 1: (B) Given that , P(A) = 0.4, $P(\bar{A})$ = 0.6

P(the event A happens at least one

= 1 – P(none of the event happens)

= 1 – (0.6) (0.6) (0.6) = 1 – 0.216 = 0.784

Sol 2: (B) Since, $P(A/\bar{B}) + P(\bar{A}/\bar{B}) = 1$

$\therefore P(\bar{A}/\bar{B}) = 1 - P(A/\bar{B})$

Sol 3: (C) Given, $P(A \cup B) = 0.6$, $P(A \cap B) = 0.2$

$\therefore P(\bar{A}) + P(\bar{B})$ = [1 – P(A)] + [1 – P(B)]

= 2 – [P(A) + P(B)] = 2 – [$P(A \cup B) + P(A \cap B)$]

= 2 – [0.6 + 0.2] = 1.2

Sol 4: (D) Let X be the number of coins shoeing heads. Let X be a binomial variate with parameter n = 100 and p.

Since, p(X = 50) = P(X = 51)

$$\Rightarrow {}^{100}C_{50}P^{50}(1-p)^{50} = {}^{100}C_{51}(p)^{51}(1-p)^{49}$$

$$\Rightarrow \frac{(100)!}{(50!)(50!)}\frac{(51!)\times(49!)}{100!} = \frac{p}{1-p}$$

$$\Rightarrow \frac{p}{1-p} = \frac{51}{50} \qquad \Rightarrow p = \frac{51}{101}$$

Sol 5: (B) The number of ways of placing 3 black balls without any restriction is ${}^{10}C_3$. Since, we have total 10 places of putting 10 balls in a row. Now the number of ways in which no two black balls put together is equal to the number of ways of choosing 3 places marked '.........' Out of eight places

–W–W–W–W–W–W–W–

The can be done in ${}^8C_3$ ways

$\therefore$ Required probability $= \dfrac{{}^8C_3}{{}^{10}C_3} = \dfrac{8\times7\times6}{10\times9\times8} = \dfrac{7}{15}$

Sol 6: (D) It is given that $P(E) \le P(F) \Rightarrow E \subseteq F$ ..... (i)

and $P(E \cap F) > 0 \Rightarrow E \subset F$ ..... (ii)

(a) Occurrence of E $\Rightarrow$ occurrence of

F [from Eq.; (i)]

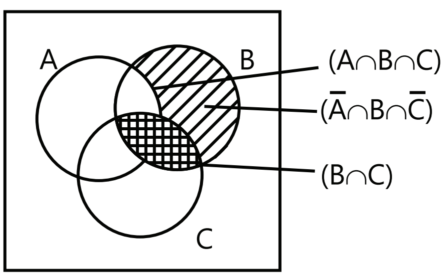
(b) Occurrence of F $\Rightarrow$ occurrence of E

[from Eq. (ii)]

(c) Non-occurrence of E $\Rightarrow$ non-occurrence of F

[from Eq. (i)]

Sol 7: (A) Given, $P(B) = \dfrac{3}{4}$ $(A \cap B \cap \overline{C}\ ) = \dfrac{1}{3}$ and



$P = (\overline{A} \cap \overline{B} \cap \overline{C}) = \dfrac{1}{3}$ Which can be shown in Venn diagram.

$\therefore P(B \cap C) = P(B) - \{P(A \cap B \cap \overline{C}\ ) + P(\overline{A} \cap B \cap \overline{C}\ )$

$= \dfrac{3}{4} - \left(\dfrac{1}{3} + \dfrac{1}{3}\right) = \dfrac{3}{4} - \dfrac{2}{3} = \dfrac{1}{12}$

Sol 8: (D) Since, three distinct number are to be selected from first 100 natural numbers.

$\Rightarrow n(S) = {}^{100}C_3$

$E_{\text{(fuvourable events)}}$ = All three of them are divisible by both 2 and 3.

$\Rightarrow$ divisible by 6 i.e., {6, 12, 18, ... , 96}

Thus, out of 16 we have to select 3.

$\therefore n(E) = {}^{16}C_3$

$\therefore$ Required probability $= \dfrac{{}^{16}C_3}{{}^{100}C_3} = \dfrac{4}{1155}$

Sol 9: (C) Let E = event when each American man is seated adjacent to his wife and

A = Event when Indian man is seated adjacent to his wife.

Now, $n(A \cap E) = (4!); \times (2!)^5$

Even when each American man is seated adjacent to his wife

Again, $n(E) = (5!) \times (2!)^4$

$\therefore P(A / E) = \dfrac{n(A \cap E)}{n(E)} = \dfrac{(4!)\times(2!)^5}{(5!)\times(2!)^4} = \dfrac{2}{5}$

Alternate Solution: Fixing four American couples and one Indian man in between any two couples; we have 5 different ways in which his wife can be seated, of which 2 cases are favourable.

$\therefore$ Required probability $= \dfrac{2}{5}$

Sol 10: (D) Since, $P(A) = \dfrac{2}{5}$

For independent events,

$P(A \cap B) = P(A)P(B) \Rightarrow P(A \cap B) \le \dfrac{2}{5}$

$\Rightarrow P(A \cap B) = \dfrac{1}{10}, \dfrac{2}{10}, \dfrac{3}{10}, \dfrac{4}{10}$

(Maximum 4 outcomes may be in $A \cap B$)

1. Now, $P(A \cap B) = \dfrac{1}{10}$

$\Rightarrow P(A).P(B) = \dfrac{1}{10}$

$\Rightarrow P(B) = \dfrac{1}{10} \times \dfrac{5}{2} = \dfrac{1}{4}$, not possible

2. Now, $P(A \cap B) = \dfrac{2}{10}$

$\Rightarrow \dfrac{2}{5} \times P(B) = \dfrac{2}{10}$

$\Rightarrow P(B) = \dfrac{5}{10}$,

Outcomes of B = 5

3. Now, $P(A \cap B) = \dfrac{3}{10}$

$\Rightarrow P(A)\ P(B) = \dfrac{3}{10} \Rightarrow \dfrac{2}{5} \times P(B) = \dfrac{3}{10}$

$P(B) = \dfrac{3}{4}$, not possible

4. Now, $P(A \cap B) = \dfrac{4}{10}$

$\Rightarrow$ P(A).P(B) = $\frac{4}{10}$

$\Rightarrow$ P(B) = 1, outcomes of B = 10

Sol 11 : (C) Sample space A dice is thrown thrice, n(s) = 6 × 6 × 6.

Favourable events $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$

i.e. (r_1, r_2, r_3) are ordered 3-triple which can take values.

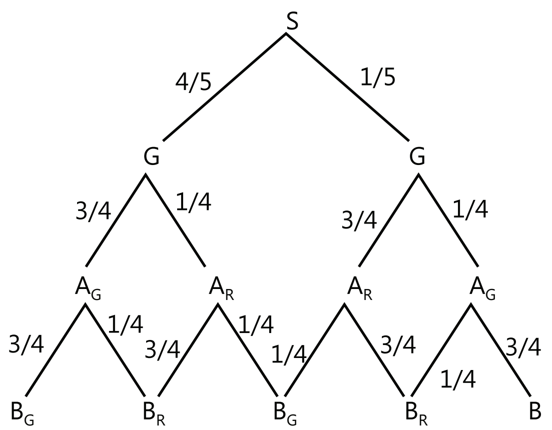
$$\left.\begin{matrix}(1,2,3), & (1,5,3), & (4,2,3), & (4,5,3) \\ (1,2,6), & (1,5,6), & (4,2,6), & (4,5,6)\end{matrix}\right\}$$

i.e. 8 orc and each can be arranged in 3! Ways = 6

$\therefore$ n(E) = 8 × 6

$\Rightarrow$ P(E) = $\frac{8\times 6}{6\times 6\times 6} = \frac{2}{9}$

Sol 12: (C) From the tree-diagram it follows that



P(B_G) = $\frac{46}{80}$

P(B_G|G) = $\frac{10}{16} = \frac{5}{8}$

$\therefore$ P($B_G \cap$ G) = $\frac{5}{8}\times\frac{4}{5} = \frac{1}{2}$

P(G/B_G) = $\frac{1/2}{P(B_G)} = \frac{1}{2}\times\frac{80}{46} = \frac{20}{23}$.

Sol 13: (B) $\frac{P(A\cap B)}{P(B)} = \frac{1}{2}, \frac{P(A\cap B)}{P(A)} = \frac{2}{3}$

Hence $\frac{P(A)}{P(B)} = \frac{3}{4}$. (But P (A) = 1/4)

$\Rightarrow P(B) = \frac{1}{3}$

Sol 14: (C) A = {4, 5, 6}, B = {1, 2, 3, 4}

Obviously P(A $\cup$ B) = 1

Sol 15: (D) Mean of a, b, 8, 5, 10 is 6

$\Rightarrow \quad \frac{a+b+8+5+10}{5} = 5$

$\Rightarrow \quad a+b=7$ ...(i)

$\therefore \quad \text{Variance} = \frac{\Sigma(x_1 - A)^2}{n}$

$= \frac{(a-6)^2 + (b-6)^2 + 4 + 1 + 16}{5} = 6.8$

$\Rightarrow \quad a^2 + b^2 = 25$

$a^2 + (7-a)^2 = 25$ (Using (i))

$\Rightarrow \quad a^2 - 7a + 12 = 0$

$\therefore \quad a = 3,3$ and $b = 3,4.$

Sol 16: (A) $x_1 + x_2 + x_3 + x_5 = 6$

${}^{5+6-1}C_{5-1} = {}^{10}C_4$

Sol 17: (D) Other than S, seven letters M, I, I, I, P, P, I can be arranged in $\frac{7!}{2!4!} = 7 . 5 . 3.$

Now four S can be placed in 8 spaces in ${}^8C_4$ ways.

Desired number of ways $= 7 . 5\ 3 .\ {}^8C_4\ 7 .\ {}^6C_4 .\ {}^8C_4$

Sol 18: (C) Mean $(\bar{x}) = \frac{\text{sum of quantities}}{n} = \frac{\frac{n}{2}(a+1)}{n}$

$= \frac{1}{2}[1+1+100d] = 1+50d$

M.D. $= \frac{1}{n}\Sigma|x_1 - \bar{x}| \Rightarrow 255$

$= \frac{1}{101}[50d + 49d + 48d + ... + d + 0 + d + ... + 50d]$

$= \frac{2d}{101}\left[\frac{50\times 51}{2}\right]$

$\Rightarrow \quad d = \frac{255\times 101}{50\times 51} = 10.1$

Sol 19: (D) 4 novels can be selected from 6 novels in ${}^6C_4$ ways. 1 dictionary can be selected and 3 dictionaries in ${}^3C_1$ ways. As the dictionary selected is fixed in the middle, the remaining 4 novels can be arranged in 4! Ways.

$\therefore$ The required number of ways of arrangement $= {}^6C_4 \times {}^3C_1 \times 4! = 1080$

Sol 20: (A) S = {00, 01, 02, ..., 49}

Let A be the even that sum of the digits on the selected ticket is 8 then

A = {08, 17, 26, 35, 44}

Let B be the event that the product of the digits is zero

B = {00, 01, 02, 03, ..., 09, 10, 20, 30, 40}

$A \cap B = \{8\}$

Required probability $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{50}}{\frac{14}{50}} = \frac{1}{14}$

Sol 21: (D) Statement-II is true

Statement-I: Sum of n even natural numbers = n (n + 1)

Mean $(\overline{x}) = \frac{n(n+1)}{n} = n+1$

Variance

$$= \left[\frac{1}{n}\Sigma(x_1)^2\right] - (\overline{x}) = \frac{1}{2}[2^2 + 4^2 + ... + (2n)^2] - (4+1)^2$$

$$= \frac{1}{2}2^2[1^2 + 2^2 + ... + n^2] - (n+1)^2$$

$$= \frac{4}{n}\frac{n(n+1)(2n+1)}{6} - (n+1)^2$$

$$= \frac{(n+1)[2(2n+1) - 3(n+1)]}{3}$$

$$= \frac{(n+1)[4n+2-3n-3]}{3}$$

$$= \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$$

$\therefore$ Statement-I is false.

Sol 22: (B) $N(S) = {}^{20}C_4$

Statement-I:

Common difference is 1 ; total number of cases = 17

Common difference is 2 ; total number of cases = 14

Common difference is 3 ; total number of cases = 11

Common difference is 4 ; total number of cases = 8

Common difference is 5 ; total number of cases = 5

Common difference is 6 ; total number of cases = 2

$$\text{Prob.} = \frac{17+14+11+8+5+2}{{}^{20}C_4} = \frac{1}{85}$$

Sol 23: (B) $S_1 = \sum_{j=1}^{10} j(j-1)\frac{10!}{j(j-1)(j-2)!(10-j)!}$

$$= 90\sum_{j=2}^{10}\frac{8!}{(j-2)!(8-(j-2))!} = 90 \cdot 2^8$$

$$S_2 = \sum_{j=1}^{10}\frac{10!}{j(j-1)!(9-(j-1))!}$$

$$= 10\sum_{j=1}^{10}\frac{9!}{(j-1)!(9-(j-1))!} = 10 \cdot 2^9$$

$$S_3 = \sum_{j=1}^{10}[j(j-1) + j]\frac{10!}{j!(10-j)!}$$

$$= \sum_{j=1}^{10} j(j-1)\, {}^{10}C_j = \sum_{j=1}^{10} j\, {}^{10}C_j = 90 \cdot 2^8 + 10 \cdot 2^9$$

$$= 90 \cdot 2^8 + 20 \cdot 2^8 = 110 \cdot 2^8 = 55 \cdot 2^9$$

Sol 24: (C) The number of ways $= {}^3C_2 \times {}^9C_2$

$$= 3 \times \frac{9 \times 9}{2} = 3 \times 36 = 108$$

Sol 25: (A) $n(S) = {}^9C_3$

$n(E) = {}^3C_1 \times {}^4C_1 \times {}^2C_1$

$$= \frac{3 \times 4 \times 2}{{}^9C_3} = \frac{24 \times !}{9!} \times 6! = \frac{24 \times 6}{9 \times 8 \times 7} = \frac{2}{7}$$

Sol 26: (A) $\sigma_x^2 = 4$, $\sigma_y^2 = 5$, $\overline{x} = 2$, $\overline{y} = 4$

$$\frac{\Sigma x_i}{5} = 2 \qquad \Sigma x_i = 10; y_i = 20$$

$$\sigma_x^2 = \left(\frac{1}{2}\Sigma x_i^2\right) - (\overline{x})^2 = \frac{1}{5}(\Sigma y_i^2) - 16$$

$$\Sigma x_i^2 = 40, \ \Sigma y_i^2 = 105$$

$$\sigma_2^2 = \frac{1}{10}\left(\Sigma x_i^2 + \Sigma y_i^2\right) - \left(\frac{\overline{x}+\overline{y}}{2}\right)^2$$

$$= \frac{1}{10}(40+105) - 9 = \frac{145-90}{10} = \frac{55}{10} = \frac{11}{2}$$

Sol 27: (D) Number of ways of selecting one or more balls from 10 white, 9 green, and 7 black balls

$= (10+1)(9+1)(7+1)-1$

$= 11\times 10\times 8-1 = 879$.

Sol 28: (B) Let A be the event that maximum is 6.

B be event that minimum is 3

$P(A) = \frac{^5C_2}{^8C_3}$ (the numbers < 6 are 5)

$P(B) = \frac{^5C_2}{^8C_3}$ (the numbers > 3 are 5)

$P(A\cap B) = \frac{^2C_1}{^8C_3}$

Required probability is $P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)} = \frac{^2C_1}{^5C_2} = \frac{2}{10} = \frac{1}{5}$

Sol 29: (D) If initially all marks were x_1 then

$$\sigma_1^2 = \frac{\Sigma(x_1-\bar{x})^2}{N}$$

Now each is increased by 10

$$\sigma_2^2 = \frac{\Sigma[(x_1+10)-(\bar{x}+10)]^2}{N} = \sigma_1^2$$

So, variance will not change whereas mean, median and mode will increase by 10.

Sol 30: (A) $P(\overline{A\cup B}) = \frac{1}{2} \Rightarrow P(A\cup B) = 1-\frac{1}{6} = \frac{5}{6}$

$P(\bar{A}) = \frac{1}{4} \Rightarrow P(A) = 1-\frac{1}{4} = \frac{3}{4}$

$\because\quad P(A\cup B) = P(A)+P(B)-P(A\cap B)$

$\frac{5}{6} = \frac{3}{4}+P(B)-\frac{1}{4}$

$P(B) = \frac{1}{3}$

$\because\quad P(A)\neq P(B)$ so they are not equally likely.

Also $P(A)\times P(B) = \frac{3}{4}\times\frac{1}{3} = \frac{1}{4} = P(A\cap B)$

$\because\quad P(A\cap B) = P(A)\cdot P(B)$ so A and B are independent.

Sol 31: (D) Variance $= \frac{\sum x_i^2}{N} - (\bar{x})^2$

$$\Rightarrow \quad \sigma^2 = \frac{2^2+4^2+.....+100^2}{50} - \left(\frac{2+4+....+100}{50}\right)^2$$

$$= \frac{4(1^2+2^2+3^2+...+50^2)}{50} - (51)^2$$

$$= 4\left(\frac{50\times 51\times 101}{50\times 6}\right) - (51)^2 = 3434 - 2601$$

$$\Rightarrow \quad \sigma^2 = 833$$

Sol 32: (D) $\square\square\square\square$ Number of integer greater than 6000 may be 4 digit or 5 digit

C-1 when number is of 4 digit

C-2 when number is of 5 digit = 5! = 120

total = 120 + 72 = 192 digit

{6, 7, 8}

$\square\square\square\square$

$3\ \ 4\ \ 3\ \ 2 = 72$

Sol 33: (A) There seems to be ambiguity in the question. It should be mentained that boxes are different and one particular box has 3 balls:

Then,

Number of ways $= \frac{^{12}C_3\times 2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

Sol 34: (D) $\frac{x_1+x_2........x_{16}}{16} = 16$

If $x_1 = 16$

$$\frac{x_1+x_2........x_{10}-16+3+4+5}{18}$$

$$= \frac{16\times 10-16+12}{18} = \frac{240+12}{18} = \frac{252}{29} = 14$$

Sol 35: (A) Standard deviation of numbers 2, 3, a and 11 is 3.5

$$\therefore \quad (3.5)^2 = \frac{\Sigma x_1^2}{4} - (\bar{x})^2$$

$$\Rightarrow \quad (3.5)^2 = \frac{4+9+a^2+121}{4} - \left(\frac{2+3+a+11}{4}\right)^2$$

On solving, we get $3a^2 - 32a + 84 = 0$

Sol 36: (C) E_1 : {(4, 1), ...(4, 6)} 6 cases

E_2 : {(1, 2),........ (6, 2)} 6 cases

E_3 : 18 cases (sum of both are odd)}

$$P(E_1) = \frac{6}{36} = \frac{1}{6} = P(E_2)$$

$$P(E_3) = \frac{18}{36} = \frac{1}{2}$$

$$P(E_1 \cap E_2) = \frac{1}{36}$$

$$P(E_2 \cap E_3) = \frac{1}{12}$$

$$P(E_3 \cap E_1) = \frac{1}{12}$$

$$P(E_1 \cap E_2 \cap E_3) = 0$$

$\therefore \quad E_1, E_2, E_3$ are not independent

Sol 37: (C) SMALL

$$A\_\_\_\_\# \frac{4!}{2!} = 12$$

$$L\_\_\_\_\# 4! = 24$$

$$M\_\_\_\_\# \frac{4!}{2!} = 12$$

$$SA\_\_\_\_\# \frac{3!}{2!} = 3$$

$$SL\_\_\_\# 3! = 6$$

$\underline{S}\ \underline{M}\ \underline{A}\ \underline{L}\ L$ #1

58th position

JEE Advanced/Boards

Exercise 1

Sol 1:

I	II
5 science	3 science
3 engg.	5 engg.

E = engg. subject select

if, 3 or 5 came after thrown a dice then, select a subject from I otherwise from II

$$\Rightarrow P(E) = \left(\frac{2}{6}\right) \times \frac{3}{8} + \left(\frac{4}{6}\right) \cdot \frac{5}{8} = \frac{1}{8} + \frac{5}{12} = \frac{3+10}{24} = \frac{13}{24}$$

Sol 2: two dice throw possibility 10

$\Rightarrow$ (4, 5) (1, 5) (2, 5) (3, 5) (6, 5)......

= At least one should be greater than 4

$$\Rightarrow P = \frac{6\times1+1\times6-1}{36} + \frac{6\times1+1\times6-1}{36} - \frac{2}{36}$$

$$P = \frac{20}{36} = \frac{5}{9}$$

Sol 3: Odds for A, B, C, D

$\Rightarrow$ 1: 3, 1: 4, 1: 5, 1: 6

$$P(A) = \frac{1}{1+3} = \frac{1}{4},\ P(B) = \frac{1}{5},$$

$$P(C) = \frac{1}{6},\ P(D) = \frac{1}{7}$$

P(one of them will wins)

= P(A) + P(B) + P(C) + P(D)

$$= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} = \frac{319}{420}$$

Sol 4: Assume number of roses = x

P(sweety win when she start first) = P(S)

$$= \frac{x}{(60+x)} + \frac{60^2 x}{(60+x)^3} + \ldots$$

$$= \frac{x}{60+x}\left[1 + \left(\frac{60}{60+x}\right)^2 + \ldots\right]$$

$$= \frac{x}{60+x}\left[\frac{1}{1-\left(\frac{60}{60+x}\right)^2}\right] = \frac{60+x}{720+x}$$

So P(sweety wins) = P(w) = $1 - \frac{60+x}{120+x}$

Its given that(s) = 3P (w)

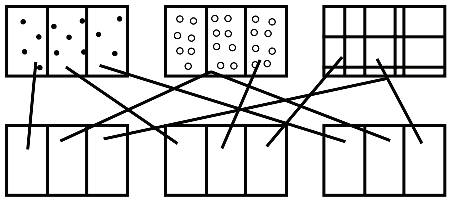
$$\frac{60+x}{120+x} = \frac{3.60}{120+x}$$

x = 180 – 160 = 120

Sol 5: Total roll = 9(3, 3, 3)

$$\frac{m}{n} = \frac{{}^3C_1{}^3C_1{}^3C_1}{{}^9C_3} \cdot \frac{{}^2C_1{}^2C_1{}^2C_1}{{}^6C_3} \cdot \frac{{}^1C_1{}^1C_1{}^1C_1}{{}^3C_3} = \frac{9}{70} = \frac{m}{n}$$

m + n = 9 + 70 = 79



Sol 6: Probability to head shown = P′

P′(A wins, when first A start)

$= P' + (1 - P')\,P'^2 + (1 - P')\,P(1 - P')\,P'^2 + \ldots$

$= P[1 + (1 - P')P'((1 - P')P')2 + ((1 - P')P')^3 + \ldots]$

$= P'\left[\frac{1}{1 - P'(1 - P')}\right] = \frac{P'}{1 - P' + P^2}$

$P(\text{B win}) = 1 - \frac{P}{1 - P' + P'^2}$

$= \frac{1 - P' + P'^2 - P'}{1 - P' + P'^2} = \frac{1 - 2P' + P'^2}{1 - P' + P'^2}$

For first to both

P(A) = P(B)

$\Rightarrow \frac{1 - 2P' + P^2}{1 - P' + P'^2} = \frac{P'}{1 - P' + P'^2}$

$\Rightarrow P'^2 - 3P' + 1 = 0$

$\Rightarrow P' + P = 1 \Rightarrow P'\,(1 - P)$

$\Rightarrow (1 - P)^2 - 3\,(1 - P) + 1 = 0$

$\Rightarrow P^2 - 2P + 1 - 3 + 3P + 1 = 0$

$\Rightarrow P^2 + P - 1 = 0$

$P = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{\sqrt{5} - 1}{2} = P$

$\because 0 < P < 1$

Sol 7: a, b, c, d → integer $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

| D | = ad – bc

if (D) is even than

ad and bc are even → a or d is even and b or c is even

ad and bc are odd → a, d are odd and b, c are odd

$P(|D| \text{ is even}) = (P)^4 + (1 - P^2)\,(1 - P^2) = \frac{1}{2}$

$\Rightarrow P^4 + P^4 - 2P^2_{+1} = \frac{1}{2}$

$4P^4 + 4P^2 + 1 = 0$

$(2P^2 - 1)^2 = 0$

$\Rightarrow 2P^2 - 1 = \Rightarrow P^2 = 1/2$ or

$P = \frac{1}{\sqrt{2}} \because 0 \le P \le 1$

Sol 8: P = P(out cons of the 5th throw was already thrown)

P = 1 – P (out com of 5th throw was first line throw)

$P = 1 - 6\left[\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right]$

$P = 1 - \frac{625}{1296} = \frac{671}{1296} = \frac{a}{b}$

a + b = 671 + 1296 = 1967

Sol 9: x = number of bomb hitting on target

n = 4

P = 0. 4(probability of hitting the target for and bomb)

$x \ge 2$ for destroy bridge

$P(x \ge 2) = P(x = 2) + P(x = 3) + P(x = 4)$

$= {}^4C_2\,(0.4)^2\,(0.6)^2 + {}^4C_3(0.4)^3\,(0.6)^1 + {}^4C_4(0.4)^4$

$= \frac{4 \times 3}{2} \times (0.24)^2 + 4\,(0.064)\,(0.6) + (0.0256) = \frac{328}{625}$

Sol 10: assume P = Ist event's probability

q= IInd event's probability

$P = q^2$

Odds against the first $= \frac{1 - P}{P}$

odds against the first $= \frac{1 - q}{q}$

$\frac{1 - P}{q} = \left(\frac{1 - q}{q}\right)^3 = \frac{1 - q^2}{q^2}$

$\Rightarrow \frac{(1 - q)(1 + q)}{q^2} = \frac{(1 - q)(1 - q)^2}{q^3}$

$q + q^2 = 1 + q^2 - 2q$

$3q = 1 \Rightarrow q = 1/3$

So $P = (q)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$; $(P, q) \rightarrow \left(\frac{1}{9}, \frac{1}{3}\right)$

Sol 11: Total tubes = 5(3G, 2D)

defective = 2(D)

(i) Test stopped on the 2^{nd} test

$$P = \frac{{}^2C_2}{{}^5C_2} = \frac{1\times 2}{5\times 4} = \frac{1}{10}$$

(ii) Test stopped on 3^{rd} test

(DGD) + (GDD) + (GGG)

$$\Rightarrow \frac{{}^2C_1 \times {}^3C_1 \times 1}{{}^5C_3} + \frac{{}^3C_1 \times 2\times 3}{5\times 4\times 3} + \frac{3\times 2\times 1}{5\times 4\times 3}$$

$$= \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$$

(iii) $P\left(\dfrac{\text{first tube is non} - \text{defective}}{\text{test stopped on } 3^{rd}\text{test}}\right) = \dfrac{\frac{1}{10}+\frac{1}{10}}{\frac{3}{10}} = \dfrac{2}{3}$

Sol 12: Hitting the plane at first = 0. 4

Hitting the plane at 2^{nd} = 0. 3

Hitting the plane at 3^{rd} = 0. 2

Hitting the plane at 4^{rd} = 0. 1

P = P(gun hits the plane)

= 1 – (1 – 0. 4) (1 – 0. 3) (1 – 0. 2) (1 – 0. 1)

= 0. 6976

Sol 13: Total articles = 10

Defective (D) = 4

Non-defective (R) = 6

Number of chosen articles = 6

$\Rightarrow {}^{10}C_6$, x = number of defective articles

P (batch will be rejected)

$$= 1 - \frac{P(x=0)+P(x=1)+P(x=2)}{1}$$

$$= 1 - \left(\frac{{}^6C_0}{{}^{10}C_6} + \frac{{}^6C_5\,{}^4C_1}{{}^{10}C_6} + \frac{{}^6C_4\,{}^4C_2}{{}^{10}C_6}\right)$$

$$= \left[\frac{1+6\times 4+15\times 6}{\frac{10\times 9\times 8\times 7}{1\cdot 2\cdot 3\cdot 4}}\right]$$

$$= 1 - \frac{115}{210} = \frac{210-115}{210} = \frac{95}{210} = \frac{19}{42}$$

Sol 14: Total side = 6

B $\Rightarrow$ one red , 2 blue, 3 green

P(second blue result occurs on or before the tenth)

= 1 – P(second blue result occurs after 10^{th})

$$= 1 - \left(\frac{4}{6}\right)^{10} - \left(\frac{4}{6}\right)^{9}\left(\frac{2}{6}\right){}^{10}C_1 = \frac{6^{10} - 4^{10} - 4^{10}\times 5}{6^{10}}$$

$$= \frac{6^{10} - 6.4^{10}}{6^{10}} = \frac{6^9 - 4^{10}}{6^9} = \frac{3^9 - 2^{11}}{3^9}$$

$$= \frac{3^P - 2^q}{3^r}\text{, P = r, q = 11, r = 9}$$

$P^2 + q^2 + r^2 = 9^2 + 9^2 + 11^2 = 283$

Sol 15: Probability of good book (G) = $\frac{1}{2}$

So P(bod book) (B) = $\frac{1}{2}$

$P\left(\frac{P}{G}\right)$ = 2/3 and $P\left(\frac{P}{B}\right) = \frac{1}{4}$

n = 2

P(at lead one book published)

= 1 – P(no-book published)

= 1 – P(GP′ BP′) – P(GP′ GP′) – P(B′P BP′)

$$= 1 - 2\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4} - \frac{1}{2}\times\frac{1}{2}\times\left(\frac{1}{3}\right)^2 - \frac{1}{2}\times\frac{1}{2}\left(\frac{3}{4}\right)^2$$

$$= 1 - \frac{169}{576} = \frac{407}{576}$$

Sol 16: P(A) = $\frac{1}{3}$

$P\left(\frac{B}{A_2}\right) = 1, P\left(\frac{B}{A_1}\right) = \frac{1}{2}, P\left(\frac{B}{A_0}\right) = 0$

Assume x = number of trial when A occur

n = 4

P(B) = 1. P(x ≥ 2) + 0: P(x = 0) + 1/2 P(x = 1)

$$= P(x = 2) + P(x = 3) + P(x = 4) + 0 + \frac{P(x=1)}{2}$$

$$= {}^4C_2 \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^2 + {}^4C_3\left(\frac{1}{3}\right)^3\frac{2}{3}$$

$$+ {}^4C_4\left(\frac{1}{3}\right)^4 + {}^4C_1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 \times \frac{1}{2}$$

$$= \frac{1}{81}\left[6\times 4+4\times 2+1+4\times\frac{4}{2}\right] = \frac{49}{81} = \frac{m}{n}$$

$\Rightarrow$ m + n = 49 + 81 = 130

Sol 17: faces $\to$ 1, 2, 3, 4

When two dice thrown together

(i) Exactly 6 on each of successive throws

$$6\begin{cases}\to (4,2)\dfrac{1}{4}\times\dfrac{2}{4}=\dfrac{2}{10}\\ \to (2,4)\dfrac{2}{4}\times\dfrac{1}{4}=\dfrac{2}{16}\\ \to (3,3)\dfrac{1}{4}\times\dfrac{1}{4}=\dfrac{1}{16}\end{cases}$$

$\Rightarrow$ For 3 throws

$$\Rightarrow \left(\frac{2}{16}+\frac{2}{16}+\frac{1}{16}\right)^3 \Rightarrow \left(\frac{5}{16}\right)^3 = \frac{125}{16^3}$$

(ii) More than 4 on at least are of the three throws.

$\Rightarrow$ 1 – P(less than 4 or 4 on all thrown)

$\Rightarrow$ Less than 4 or 4 $\to$ (2, 2)

$$\Rightarrow \frac{2}{4}\times\frac{2}{4}=\frac{1}{4}\Rightarrow 1-\left(\frac{1}{4}\right)^3 = \frac{64-1}{64}=\frac{63}{64}$$

Sol 18: Total red card out of 52 = 26 (R)

Total green = 4 (Q)

q card are red and green both (RQ)

Two cards drawn

$\Rightarrow$ P(one is red & one is green)

$\Rightarrow$ [26 × 2 + 25 × 1 + 25 × 1 – (1) (1)]

$$\Rightarrow \frac{101}{1326}$$

Sol 19: Total coin = 4

Discard those which turn up tails

P = (at least 3 coins discard after 2nd flip)

= P(3) + P(4) = 1 – P(0) – P(1) – P(1)

= 1 – [P(HHHH, HHHH)] – P[(HHHT, HHH) + (HHHH, HHHT)]

– P [(HHHH, HHTT) + (HHTT, HH) + (HHHT, HHT)]

$$= 1-\left(\frac{1}{2}\right)^8 - \frac{1}{2^8}\left[{}^4C_1\times 2\times 1+{}^4C_1\right]$$

$$-\frac{1}{2^8}\left(\frac{4!}{2!2!}+\frac{2^2.4!}{2!2!}+4\times 2\times 3\right)$$

$$= 1-\frac{1}{28}[1+12+30+24] = \frac{256-67}{256} = \frac{189}{256}$$

Sol 20: Total passengers before stop = n

P(get down) = P

P(boarding the bus at next stop) = 1 – P_0

P(n passenger are in bus after stop)

= $(1 - P)\,{}^nP_0 + {}^nC_1\,P(-P)^{n-1}(1 - P_0)$

= $(1 - P)^{n-1}\,[P_0(1 - P) + nP(1 - P_0)]$

Sol 21: Total balls = 2n (n while, n black)

n person each draw 2 balls

(i) P(each of n person drawn both balls)

$$= \frac{2^n(n\times n)(n-1)^2(n-2)^2}{2n!} - 3^2.2^2$$

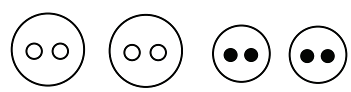
$$= \frac{n|n|2^n}{2n} = \frac{2^n}{{}^{2n}C_n} = \frac{8}{35}$$

$$n = 4 \to \frac{2^4}{{}^8C_4} = \frac{16\times 4\times 3\times 2}{1\times 7\times 1\times 5} = \frac{8}{35}$$

So n = 4

(ii) n = 4

Each of 4 draw the balls of same colour



$$P = \frac{{}^4C_1\times{}^3C_1\,{}^2C_1\,{}^1C_1\times 4!}{8! \qquad\qquad 2!2!}\times{}^4C_2.2^2$$

$$P = \frac{4!\;4\times 3\times 2\times 1}{4\times 8\times 7\times 6\times 5\times 4}\;4\times 6 = \frac{3}{35}$$

(iii) n = 7 there is 7 white balls and 7 black ball there is no way to each of the person draw balls of same colour.

$\because$ Number of same color ball is 7 (odd)

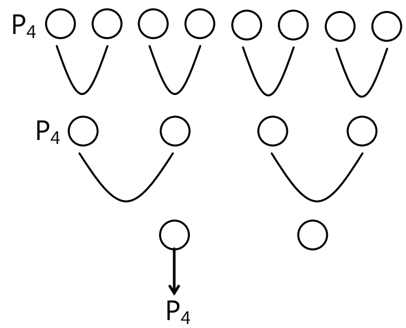
7 = 2 + 2 + 2 + 1 – ? (1)$\Rightarrow$not possible

P = 0

Sol 22: Between P_i and P_j

If i < j $\to$ P_i win

P(player P_4, reaches the Final)



to get in final for P_4 exactly 3 match will be there and P_4 all win.

So other 3 are P_5, P_6, P_7 or P_8

$$P = \frac{{}^4C_1\,{}^3C_1\,{}^2C_1.4!\,2^3}{8!}$$

$$P = \frac{4\times3\times2\times4!\,2^3}{8\times7\times6\times5\;4!} = \frac{4}{35}$$

Sol 23: $P(A) = 0.4 \rightarrow P(\bar{A}) = 1 - 0.4 = 0.6$

$P(B) = 0.8 \Rightarrow P(\bar{B}) = 1 - 0.8 = 0 - 2$

$$P(\bar{A}/\bar{B}) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = 0.1$$

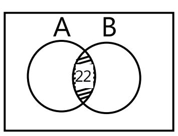
(i) $P(\bar{A} \cup B) = ?$, $P(\bar{A} \cap \bar{B}) = P(\bar{B})\ 10.1$

$= 0.2 \times 0.1 = 0.02$

$1 - P(\bar{A} \cap \bar{B}) = P(A \cup B) = 1 - 0.02 = 0.98$

$P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B)$

$P(\bar{A} \cap B) = P(\text{only } B)$



$= P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

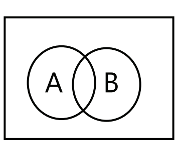
$= 0.4 + 0.8 - 0.98 = 0.22$

$P(\bar{A} \cap B) = 0.8 - 0.22 = 0.58$

$P(\bar{A} \cup B) = 0.6 + 0.8 - (0.58)$

$= 1.4 - 0.58 = 0.82$

(ii) $P[(\bar{A} \cap B) \cup (A \cap \bar{B})]$



$P(\bar{A} \cap B) = 0.58$

$P(A \cap \bar{B}) = P(\text{only } A)$

$= P(A) - P(A \cap B) = 0.4 - 0.22 = 0.18$

$P[(\bar{A} \cap B) \cap (A \cap \bar{B})] = 0$ from dia.

So $P[(\bar{A} \cap B) \cup (A \cap \bar{B})]$

$= P(\bar{A} \cap B) + P(A \cap \bar{B}) = 0.18 + 0.58 = 0.76$

Sol 24: (i) Number of ways $= {}^8C_3$

choose 3 digits from set B.

Total ways $= {}^8C_3 \times {}^9C_3$

$$P(\text{A and B have same 3-digit number}) = \frac{{}^8C_3}{{}^8C_3 \times {}^9C_3} = \frac{1}{84}$$

(ii) Case-I: Mr A's number contains 9

$$P_1 = \frac{{}^8C_2 \times {}^8C_3}{{}^9C_3 \times {}^8C_3} = \frac{{}^8C_2}{{}^9C_3}$$

Case-II: Mr A's number do not contain 9

$$P_2 = \frac{(1 - P(\text{A \& B have same number}))}{2}$$

$\times$ P(Mr A's number don't contains 9)

$$= \left(1 - \frac{1}{{}^8C_3}\right) \times \frac{1}{2} \times \frac{{}^8C_3}{{}^9C_3}$$

P(Mr A's number > Mr. B's number)

$= P_1 + P_2$

$$= \frac{{}^8C_2}{{}^9C_3} + \frac{1/2({}^8C_3 - 1)}{{}^9C_3} = \frac{111}{168} = \frac{37}{56}$$

Sol 25: One pair is selected

$$P(\text{one is mole and one female}) = \frac{10}{19}$$

assume total student is = 2m

= n boy + n girl

$$P = \frac{10}{19} = \frac{{}^nC_1.{}^nC_1}{{}^{2n}C_2} = \frac{10}{19}$$

$\Rightarrow 19\,n = 10(2n - 1) = 20\,n - 10$

$\Rightarrow 20n - 19\,n = 10$

$\Rightarrow n = 10$

Total student is $2n = 20$

Exercise 2

Single Correct Choice Type

Sol 1: (B) P(A) = P(person A lives above 35 years)

So, $P(A') = \frac{9}{9+7} = \frac{9}{16}$

And $P(B') = \frac{3}{3+2} = \frac{3}{5}$

So, P(at least one lives)

= 1 – P(no lives) = 1 – P(A') . P(B')

$= 1 - \frac{9}{16}\cdot\frac{3}{5} = \frac{80-27}{80} = \frac{53}{80}$

Sol 2: (D) Σ P = 1for a close event

$0 \le P \le 1$

$P_1 + P_2 + P_3 + P_4 = 0.2 + 0.3 + 0.4 + 0.1 = 1$

$0 \le P_1, P_2, P_3, P_4 \le 1$

Sol 3: (A) P = Probability in a group of 4 person all are born on different days of the week.

$P = \frac{7\times6\times5\times4}{7\times7\times7\times73} = \frac{120}{343}$

$\frac{1}{3} < P < \frac{1}{2}$

Sol 4: (D) x is a integer

P(x^4 ends in the digits 6)

$1^4 = 1, 2^4 = 16, 3^4 = 81, 4^4 = 256,$

$5^4 = 625, 6^4 = ...6, 7^4 = ...1$

$8^4 = ...6, 9^4 = ...1, 0^4 = 0$

There are 4 out of 6 digits which has 6 in the last for 4th power

So, $P = \frac{4}{10} = 40\%$

Sol 5: (B) 2 match each with two other teams.

x = 0, 1, 2(Points)

For one match

P(x = 0) = 0. 45

P(x = 1) = 0. 05

P(x = 2) = 0. 50

For all matches

max point = 2 × 4 = 8

$P(x \ge 7) = P(x = 7) + P(x = 8)$

$= {}^4C_1\,P(2, 2, 2, 1) + P(2, 2, 2, 2)$

$= 4 \times (0.5)^3\,(0.05) + (0.5)^4$

$= (0.5)^3\,[0.2 + 0.5] = (0.5)^3\,0.7 = 0.0875$

Sol 6: (B) Total key = n

There is only one key to open door

P(last key is the right key)

$= \frac{n-1}{n}\cdot\frac{(n-1)}{(n-1)}......\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{n}$

Sol 7: (D) A and B are independent

$P(A) = P;\ P(B) = P \Rightarrow P(A \cap B) = P(A)\,.\,P(B) = P^2$

$\therefore P\left(\frac{A}{A \cup B}\right) = \frac{A \cap (A \cup B)}{(A \cup B)} = \frac{P(A \cap A \cup (A \cap B)}{P(A \cup B)}$

$= \frac{P(A \cup (A \cap B)}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P + P - P^2 = 2P - P^2$

$\frac{P(A)}{P(A \cup B)} = \frac{P}{P(2-P)} = \frac{1}{2-P}$

Sol 8: (B) x = number of getting tails

n = bp = q = $\frac{1}{2}$

More tail than heads so x > 3 $\Rightarrow$ P(x > 3)

P(x = 4) + 0 (x = 5) + P(x = 6)

$\Rightarrow {}^6C_4\left(\frac{1}{2}\right)^6 + {}^6C_5\left(\frac{1}{2}\right)^6 + {}^6C_6$

$= \frac{1}{2^6}\left[\frac{6\times5}{1.2} + 6 + 1\right] = \frac{1}{2^6}[15+7] = \frac{22}{2^6} = \frac{22}{64}$

Sol 9: (D) Success = head = H

Success = one rupee win = 0

Lose = Tail $\rightarrow$ lose one rupee = T

P (he loses)

= P(T) + P(HT) + P(HHT) + P(HHHT) + P(HHHHT) + P(HHHHHT) + P(HHHHHH)

$= \frac{1}{2} + \frac{1}{2^6}$ (1 + 1 + 1 + 1 + 1 + 1)= $\frac{16+6}{32} = \frac{22}{32}$

Sol 10: (C) Let {1, 2, 3,......,50}

x^x should be perfect square

So x $\rightarrow$ seven

Or x $\rightarrow$ perfect

x = all even $\rightarrow$ 1, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2

$8^2 > 50$

So $x \neq 8^2$

Total number of x = 25 + 7 – (3)

because 2^2, 4^2, 6^2 all are even and perfect square both.

$$P = \frac{25+7-3}{50} = \frac{29}{50}$$

Sol 11: (D) P(A) = P(a solve problem)

P(A) = P

P(B) = 1/2

P(they will make same mistakes)= $\frac{1}{100}$

$$P\left(\frac{\text{their ans. is correct}}{\text{they get same ans.}}\right) = \frac{300}{301}$$

$\Rightarrow$ P(they get same ans.)

$$= P \times \frac{1}{2} + \frac{1}{100} = \frac{1}{100} + \frac{P}{2}$$

P(their ans. is correct) = $P \times \frac{1}{2} = \frac{P}{2}$

$$\Rightarrow \frac{300}{301} = \frac{P/2}{\frac{1}{100}+\frac{P}{2}} = \frac{100 \times P}{2+100P}$$

$\Rightarrow$ 3(2 + 100P) = 301P

$\Rightarrow$ 6 + 300P = 301P

$\Rightarrow$ P = 6, P $\neq$ 6 $\because$ = P < 1

Sol 12: (B) Two dice are thrown until a 6 appears

P(For the I[st] time, 46 appear in second throw)

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{11}{36} = \frac{275}{1296}$$

Therefore there is 1 case to show at least one 6

(6, 1) (1, 6) (2, 6) (6, 2) (3, 6) (6, 3) (4, 6) (6, 4) (5, 6) (6, 5) (6, 6)

Sol 13: (B)

A	B
3 white, 2 red	2 white, 4 red
$\downarrow$ 2 balls	$\downarrow$ 2 balls

$$\text{P(all balls are white)} = \frac{{}^3C_2}{{}^5C_2} \cdot \frac{{}^2C_2}{{}^6C_2} = \frac{3 \times 1}{\frac{5 \times 4}{2} \cdot \frac{6 \times 5}{2}}$$

$$= \frac{1}{50} = 2\%$$

Sol 14: (B) $P(\bar{A}) = 0.7$

$\Rightarrow P(A) = 1 - P(\bar{A}) = 1 - 0.7 = 0.3$

$P(\bar{B}) = a$

$\Rightarrow$ P(B) = 1 – a

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) \cdot P(B)$

$\because$ A and B are independent

$\Rightarrow$ 0. 8 = 0. 3 + 1 – a + (0. 3) (1 – a) (–1)

$\Rightarrow$ a (1 – 0. 3) = 1 + 0. 3 – 0. 3 – 0. 8 = 0. 2

$$\Rightarrow a = \frac{0.2}{0.7} = \frac{2}{7}$$

Sol 15: (D) P(son will post the letter) = $\frac{1}{2}$ = $P(5_p)$

P(letter react its destination) = $\frac{5}{6}$ = P(D)

P(R) = probability of letter was received

$$P\left(\frac{S_P^1}{R'}\right) = \frac{P(S_P^1 \cap R')}{P(R')}$$

$$P(R') = P(S_p \cap D') + P(S_P^1) = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{6} = \frac{6+1}{12} = \frac{7}{12}$$

$$P\left(\frac{S_P^1}{R'}\right) = \frac{1/2}{7/12} = \frac{6}{7}$$

Sol 16: (D) Set = {1, 2, 3...,20}

$x_1 x_2 \in$ set

$x_1 x_2$ is even

when $x_1 + x_2$ = odd

possibility for this condition = (one odd, one even)

(not of possibility /or this condition)

$= {}^{20}C_1 \cdot {}^{10}C_1 = 200$

Total possibility = 20 × 20 = 400

$$P = \frac{200}{400} = \frac{1}{2}$$

1, 2, 4, 5, 7, 8, 10, 11, 12, 14, 15, 16, 20, 21, 24, 25

3, 6, 7, 13, 17, 18, 19, 22, 23.

Previous Years' Questions

Sol 1: Since, the drawn balls are in the sequence black, black, white, white, white, red, red and red.

Let the corresponding probabilities be

$P_1, P_2, \ldots\ldots\ldots . P_9$

Then, $P_1 = \frac{2}{9}$; $P_2 = \frac{1}{8}$, $P_3 = \frac{4}{7}$;

$P_4 = \frac{3}{6}$, $P_5 = \frac{2}{5}$

$P_6 = \frac{1}{4}$, $P_7 = \frac{3}{3}$, $P_8 = \frac{2}{2}$, $P_6 = 1$

$\therefore$ Required probabilities

$P_1.P_2.P_3\ldots\ldots P_9$

$$= \left(\frac{2}{9}\right)\left(\frac{1}{8}\right)\left(\frac{4}{7}\right)\left(\frac{3}{6}\right)\left(\frac{2}{5}\right)\left(\frac{1}{4}\right)\left(\frac{3}{3}\right)\left(\frac{2}{2}\right)(1) = \frac{1}{1260}$$

Sol 2: Given, $P(A).P(B) = \frac{1}{6}$, $P(\bar{A}).P(\bar{B}) = \frac{1}{3}$

$\therefore [1 - P(A)]\ [1 - P(B) = \frac{1}{3}$

Let P(A) = x and P(B) = y

$\Rightarrow (1 - x)\ (1 - y) = \frac{1}{3}$ and $xy = \frac{1}{6}$

$\Rightarrow 1 - x - y + xy = \frac{1}{3}$ and $xy = \frac{1}{6}$

$\Rightarrow x + y = \frac{5}{6}$ and $xy = \frac{1}{6}$

$$\Rightarrow x\left(\frac{5}{6} - x\right) = \frac{1}{6}$$

$\Rightarrow 6x^2 - 5x + 1 = 0 \Rightarrow (3x - 1)\ (2x - 1) = 0$

$\Rightarrow x = \frac{1}{3}$ and $\frac{1}{2}$

$\therefore P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$

Sol 3: The total number of ways to answer the question

$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1 = 15$

P(getting marks) = P(correct answer in I chance) + P(correct answer in II change) + P(Correct answer in III chance)

$$= \frac{1}{15} + \left(\frac{14}{15}.\frac{1}{14}\right) + \left(\frac{14}{15}.\frac{13}{14}.\frac{1}{13}\right) = \frac{3}{15} = \frac{1}{5}$$

Sol 4: Let Q = 1 – P = probability of getting the tail. We have α = probability of A getting the head on tossing firstly

$= P\ (H_1$ or $T_1T_2T_3H_4$ or $T_1T_2T_3T_4T_5T_6H_7$ or .....

$= P(H)P(T) + P(H)P(T)^3 + P(H)P(T)^6$ .....

$$= \frac{P(H)}{1 - P(T)^3} = \frac{P}{1 - Q^3}$$

Also β = probability of B getting the heat on tossing secondly

$= P(T_1H_2$ or $T_1T_2T_3T_4H_5$ or $T_1T_2T_3T_4T_5T_6T_7H_8$ or ....)

$= P(H)\ P(T) + P(H)\ P(T)^4 + P(H)\ P(T)^7 + \ldots$

$= P(T)\ [P(H) + P(H)\ P(T)^3 + P(H)P(T)^6 + \ldots]$

$$= Q\alpha = (1 - P)\alpha = \frac{P(1-P)}{1 - Q^3}$$

Again, we have $\alpha + \beta + \gamma = 1$

$$\Rightarrow \gamma = 1 - (\alpha + \beta) = 1 - \frac{P + P(1-P)}{1 - Q^3} = 1 - \frac{P + P(1-P)}{1 - (1-P)^3}$$

$$\gamma = \frac{1 - (1-P)^3 - 2P + P^2}{1 - (1-P)^3} = \frac{P - P^2 + P^3}{1 - (1-P)^3}$$

$$\text{Also, } \alpha = \frac{P}{1 - (1-P)^3},\ \beta = \frac{P(1-P)}{1 - (1-P)^3}$$

Sol 5: The total no. of outcomes = 6^n

We can choose three numbers out of 6 in ${}^6C_3$ ways. By using three numbers out of 6 we can get 3^n sequences of length n. But these sequences of length n which use exactly two umbers and exactly one number.

The number of n – sequences which use exactly two numbers

$= {}^3C_2\left[2^n - 1^n - 1^n\right] = 3\left(2^n - 2\right)$ and the number of n sequence which are exactly one number.

$$= \left({}^3C_1\right)\left(1^n\right) = 3$$

Thus, the number of sequences, which use exactly three numbers

$$= {}^6C_3\left[3^n - 3\left(2^n - 2\right) - 3\right] = {}^6C_3\left[3^n - 3\left(2^n\right) + 3\right]$$

$\therefore$ Probability of the required event.

$$= {}^6C_3\left[3^n - 3\left(2^n\right) + 3\right] / 6^n$$

Sol 6: Let A_1 be the event exactly 4 white balls have been drawn. A_2. Be the event exactly 5 white balls have been drawn. A_3 be the event exactly 6 white balls have been drawn. B be the event exactly 1 white ball is drawn from two draws.

Then,

$$P(B) = P\left(\frac{B}{A_1}\right)P(A_1) + P\left(\frac{B}{A_2}\right)P(A_2) + P\left(\frac{B}{A_3}\right)P(A_3)$$

But $P\left(\frac{B}{A_3}\right) = 0$ ($\because$ there are only 6 white balls in the bag)

$$\therefore P(B) = P\left(\frac{B}{A_1}\right)P(A_1) + P\left(\frac{B}{A_2}\right)P(A_2)$$

$$= \frac{{}^{12}C_2 . {}^{6}C_4}{{}^{18}C_6} . \frac{{}^{10}C_1 . {}^{2}C_1}{{}^{10}C_2} + \frac{{}^{12}C_1 . {}^{6}C_5}{{}^{18}C_6} . \frac{{}^{11}C_1 . {}^{1}C_1}{{}^{12}C_2}$$

Sol 7: As, the statement shows problem is to be related to Baye's law.

Law C, S, B, T be the events when when person is going by car, scooter, bus or train respectively.

$\therefore P(C) = \frac{1}{7}$, $P(S) = \frac{3}{7}$, $P(B) = \frac{2}{7}$, $P(T) = \frac{1}{7}$

Again, L be the event of the person reaching office late.

$\therefore \bar{L}$ be the event of the person reaching office in time.

Then, $P\left(\frac{\bar{L}}{C}\right) = \frac{7}{9}$, $P\left(\frac{\bar{L}}{S}\right) = \frac{8}{9}$, $P\left(\frac{\bar{L}}{B}\right) = \frac{5}{9}$

And $P\left(\frac{\bar{L}}{T}\right) = \frac{8}{9}$

$$\therefore P\left(\frac{C}{\bar{L}}\right) = \frac{P\left(\frac{\bar{L}}{C}\right).P(C)}{P\left(\frac{\bar{L}}{C}\right).P(C) + P\left(\frac{\bar{L}}{S}\right).P(S) + P\left(\frac{\bar{L}}{B}\right).P(B) + P\left(\frac{\bar{L}}{T}\right).P(T)}$$

$$= \frac{\frac{7}{9}\times\frac{1}{7}}{\frac{7}{9}\times\frac{1}{7} + \frac{8}{9}\times\frac{3}{7} + \frac{5}{9}\times\frac{2}{7} + \frac{8}{9}\times\frac{1}{7}} = \frac{1}{7}$$

Sol 8 : (B) Here, $P(u_i) = K_i$, $\Sigma P(u_i) = 1$

$$\Rightarrow k = \frac{2}{n(n+1)}$$

$$\therefore \lim_{n\to\infty} P(W) = \lim_{n\to\infty} \sum_{i=t}^{n} \frac{2i^2}{n(n+1)^2}$$

$$= \lim_{n\to\infty} \frac{2n(n+1)(2n+1)}{6n(n+1)^2} = \frac{2}{3}$$

Sol 9: (A) $P\left(\frac{u_n}{W}\right) = \frac{\frac{n}{n+1}}{\frac{\Sigma i}{n+1}} = \frac{2}{n+1}$

Sol 10: (B, C, D) Since, E and F are independent events. Therefore $P(E \cap F) = P(E).P(F) \neq 0$, so E and F are not mutually exclusive events.

Now, $P(E \cap \bar{F}) = P(E) - P(E \cap F)$

$= P(E) - P(E).P(F) = P(E)[1 - P(F)] = P(E).P(\bar{F})$

and $P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$

$= 1 - [1 - P(\bar{E}).P(\bar{F})]$

($\because$ E and F are independent)

$= P(\bar{E}).P(\bar{F})$

So, E and $\bar{F}$ as well and $\bar{E}$ and $\bar{F}$ are independent events.

Now,

$$P(E/F) + P(\bar{E}/F) = \frac{P(E \cap F) + P(\bar{E} \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

Sol 11: (A, D) Both E and F happen $\Rightarrow P(E \cap F) = \frac{1}{12}$ and neither E nor F happens

$$\Rightarrow P(\bar{E} \cap \bar{F}) = \frac{1}{2}$$

But for independent events, we have

$$P(E \cap F) = P(E)\,P(F) = \frac{1}{12} \quad \text{..... (i)}$$

and $P(\bar{E} \cap \bar{F}) = P(\bar{E})P(\bar{F})$

$= \{1 - P(E)\}\ \{(1 - P(F)\}$

$= 1 - P(E) - P(F) + P(E)P(F)$

$$\Rightarrow \frac{1}{2} = 1 - \{P(E) + P(F)\} + \frac{1}{12}$$

$$P(E) + P(F) = 1 - \frac{1}{2} + \frac{1}{12} = \frac{7}{12} \quad \text{..... (ii)}$$

On solving Equation (i) and (ii), we get

either $P(E) = \frac{1}{3}$ and $P(F) = \frac{1}{4}$ or $P(E) = \frac{1}{4}$ and $P(F) = \frac{1}{3}$

Sol 12: (A, D) $P(E/F) + P(\bar{E}/F) = \frac{P(E\cap F)}{P(F)} + \frac{P(\bar{E}\cap F)}{P(F)}$

$= \frac{P(E\cap F)+P(\bar{E}\cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$

(b) $P(E/F) + P(E/\bar{F})$

$= \frac{P(E\cap F)}{P(F)} + \frac{P(E\cap \bar{F})}{P(F)}$

$= \frac{P(E\cap F)}{P(F)} + \frac{P(E\cap \bar{F})}{1 - P(F)} \neq 1$

(c) $P(\bar{E}/F) + P(E/\bar{F}) = \frac{P(\bar{E}\cup F)}{P(F)} + \frac{P(E\cap \bar{F})}{P(\bar{F})}$

$= \frac{P(\bar{E}\cap F)}{P(F)} + \frac{P(E\cap \bar{F})}{1-P(F)} \neq 1$

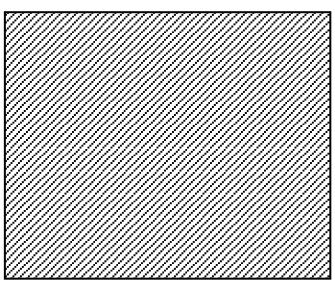
(d) $P(E/\bar{F}) + P(\bar{E}/\bar{F}) = \frac{P(E\cap \bar{F})}{P(\bar{F})} + \frac{P(\bar{E}\cap \bar{F})}{P(\bar{F})}$

$= \frac{P(E\cap \bar{F})+P(\bar{E}\cap \bar{F})}{P(\bar{F})} = \frac{P(\bar{F})}{P(\bar{F})} = 1$

Sol 13: (A, D) $P(E \cup F) - P(E \cap F) = \frac{11}{25}$ ..... (i)

(i.e. only E or only F) Neither of them occurs $= \frac{2}{25}$

$\Rightarrow P(\bar{E}\cap \bar{F}) = \frac{2}{25}$ ..... (ii)



From Eq. (i) $P(E) + P(F) - 2P(E \cap F) = \frac{11}{25}$ ..... (iii)

From eq. (ii), $(1 - P(E))(1 - P(F)) = \frac{2}{25}$

$\Rightarrow 1 - P(E) - P(F) + P(E).P(F) = \frac{2}{25}$ ..... (iv)

From Eq. (iii) and (iv), we get

$P(E) + P(F) = \frac{7}{5}$ and $P(E).P(F) = \frac{12}{25}$

$\therefore P(E).\left\{\frac{7}{5} - P(E)\right\} = \frac{12}{25}$

$\Rightarrow (P(E))^2 - \frac{7}{5}P(E) + \frac{12}{25} = 0$

$\Rightarrow \left(P(E) - \frac{3}{5}\right)\left(P(E) - \frac{4}{5}\right) = 0$

$\therefore P(E) = \frac{3}{5}$ or $\frac{4}{5} \Rightarrow P(F) = \frac{4}{5}$ or $\frac{3}{5}$

Sol 14: (C) Let E = event when each American man is seated adjacent to his wife

A = event when Indian man is seated adjacent to his wife

Now $n(A\cap E) = (4!)\times(2!)^5$

Even when each American man is seated adjacent to his wife

Again $n(E) = (5!)\times(2!)^4$

$\Rightarrow \quad P\left(\frac{A}{E}\right) = \frac{n(A\cap E)}{n(E)} = \frac{(4!)\times(2!)^5}{(5!)\times(2!)^4} = \frac{2}{5}$

Sol 15 : (D) Statement-I:

If $P(H_i \cap E) = 0$ for some I, then

$P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_i}\right) = 0$

If $P(H_i \cap E) \neq 0$ for $\forall\ i = 1, 2.....n$ then

$P\left(\frac{H_i}{E}\right) = \frac{P(H_i\cap E)}{P(H_i)} \times \frac{P(H_i)}{P(E)}$

$= \frac{P\left(\frac{E}{H_i}\right)\times P(H_i)}{P(E)} > P\left(\frac{E}{H_i}\right)\cdot P(H_i)$ [as 0 < P (E) < 1]

Hence, Statement-I may not always be true.

Statement-II: Clearly $H_1 \cup H_2 ..... \cup H_n = S$ (sample space)

$\Rightarrow \quad P(H_1) + P(H_2) + ... + P(H_n) = 1$

Sol 16 : (C) COCHIN

The second place can be filled in $^4C_1$ ways and the remaining four alphabets can be arranged in 4! Ways in four different places. The next 97th word will be COCHIN

Hence, there are 96 word before COCHIN.

Sol 17: (C) $P\left(\frac{E^c\cap F^c}{G}\right) = \frac{P(E^c\cap F^c\cap G)}{P(G)}$

$$= \frac{P(G) - P(E \cap G) - P(G \cap F)}{P(G)}$$

$$= \frac{P(G)(1 - P(E) - P(F))}{P(G)} \quad [\because P(G) \neq 0]$$

$$= 1 - P(E) - P(F)$$

$$= P(E^c) - P(F)$$

Sol 18: (D) $P(A \cap B) = \frac{4}{10} \times \frac{p}{10} = \frac{2p/5}{10}$

$\Rightarrow \quad \frac{2p}{5}$ is an integer

$\Rightarrow \quad = 5$ or 10

Sol 19: $A \to p$; $B \to s$; $C \to q$; $D \to q$

(A) ENDEA, N, O, E, L are five different letter, then permutation = 5!

(B) If E is in the first and last position then $\frac{(9-2)!}{2!} = 7 \times 3 \times 5! = 2! \times 5!$

(C) For first four letters $= \frac{4!}{2!}$

For last five letters = 5!/3!

Hence $\frac{4!}{2!} \times \frac{5!}{3!} = 2 \times 5!$

(D) For A, E and O 5!/3! And for others 4!/2!

Hence $\frac{5!}{3!} \times \frac{4!}{2!} = 2 \times 5!$

Sol 20 : (C) Coefficient of x^{10} in $(x + x^2 + x^3)^7$

Coefficient of x^3 in $(1 + x + x^2)^7$

Coefficient of x^3 in $(1 - x^3)^7 (1 - x)^{-7}$

$$= {}^{7+3-1}C_3 - 7$$

$$= {}^{9}C_3 - 7$$

$$= \frac{9 \times 8 \times 7}{6} - 7 = 77$$

Sol 21 : (A) $P(X = 3) = \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\frac{1}{6} = \frac{25}{216}$

Sol 22: (B) $\frac{25}{216}$

Required probability $= 1 - \frac{11}{36} = \frac{25}{36}$

Sol 23 : (D) For $X \geq 6$, the probability is

$$\frac{5^5}{6^6} + \frac{5^6}{6^7} + \ldots.\infty = \frac{5^5}{6^6}\left(\frac{1}{1 - 5/6}\right) = \left(\frac{5}{6}\right)^5$$

For $X \geq 3$

$$\frac{5^3}{6^4} + \frac{5^4}{6^5} + \frac{5^5}{6^6} + \ldots..\infty = \left(\frac{5}{6}\right)^3$$

Hence the conditional probability $\dfrac{\left(\frac{5}{6}\right)^6}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$.

Sol 24: (C) Event G = original signal is green

E_1 = A receives the signal correct

E_2 = B receives the signal correct

E = signal received by B is green

P(signal received by B is green)

$$= P(GE_1E_2) + P(GE_1E_2) + P(G\overline{E}_1E_2) + P(\overline{G}E_1\overline{E}_2) + P(\overline{G}\overline{E}_1E_2)$$

$$P(E) = \frac{46}{5 \times 16}$$

$$P(G/E) = \frac{40/5 \times 16}{46/5 \times 16} = \frac{20}{23}$$

Sol 25: (B) H $\to$ ball from U_1 to U_2

T $\to$ 2 ball from U_1 to U_2

E : 1 ball drawn from U_2

$$\text{P/W from } U_2 = \frac{1}{2} \times \left(\frac{3}{5} \times 1\right) + \frac{1}{2} \times \left(\frac{2}{5} \times \frac{1}{2}\right) + \frac{1}{2}$$

$$\times \left(\frac{{}^3C_2}{{}^5C_2} \times 1\right) + \frac{1}{2} \times \left(\frac{{}^2C_2}{{}^5C_2} \times \frac{1}{3}\right) + \frac{1}{2}$$

$$\times \left(\frac{{}^3C_1 \cdot {}^2C_1}{{}^5C_2} \times \frac{2}{3}\right) = \frac{23}{30}$$

Sol 26: (D) $P\left(\frac{H}{W}\right) = \frac{P(W/H) \times P(H)}{P(W/T) \cdot P(T) + (W/H) \cdot P(H)}$

$$= \frac{\frac{1}{2}\left(\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{2}\right)}{23/30} = \frac{12}{23}$$

Sol 27: (B) Number of ways

$= 3^5 - {}^3C_1 \cdot 2^5 + {}^3C_2 1^5$

$= 243 - 96 + 3 = 150$

Sol 28: (B, D) $P(X_1) = \frac{1}{2}, P(X_2) = \frac{1}{4}, P(X_3) = \frac{1}{4}$

$P(X) = P(X_1 \cap X_2 \cap X_3^C) + P(X_1 \cap X_2^C \cap X_3)$

$+ P(X_1^C \cap X_2 \cap X_3) + P(X_1 \cap X_2 X_3) = \frac{1}{4}$

(A) $P(X_1^C / X) = \frac{P(X \cap X_1^C)}{P(X)} = \frac{\frac{1}{32}}{\frac{1}{4}} = \frac{1}{8}$

(B) P [exactly two engines of the ship are

functioning | X] $= \frac{\frac{7}{32}}{\frac{1}{4}} = \frac{7}{8}$

(C) $P\left(\frac{X}{X_2}\right) = \frac{\frac{5}{32}}{\frac{1}{4}} = \frac{5}{8}$

(D) $P\left(\frac{X}{X_1}\right) = \frac{\frac{7}{32}}{\frac{1}{2}} = \frac{7}{16}$

Sol 29: (A) Favourable : D_4 shows a number and only 1 of D_1 D_2 D_3 shows same number

Or only 2 of D_1 D_2 D_3 shows same number

Or all 3 of D_1 D_2 D_3 shows same number

Required probability

$= \frac{{}^6C_1({}^3C_1 \times 5 \times 5 + {}^3C_2 \times 5 + {}^3C_3)}{216 \times 6}$

$= \frac{6 \times (75 + 15 + 1)}{216 \times 6} = \frac{6 \times 91}{216 \times 6} = \frac{91}{216}$

Sol 30: (A, B) $P(X/Y) = \frac{1}{2}$

$\frac{P(X \cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{1}{3}$

$P(Y/X) = \frac{1}{3}$

$\frac{P(X \cap Y)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{2}{3}$ (A is correct)

$P(X \cap Y) = P(X) \cdot P(X) \Rightarrow X$ and Y are independent (B is correct)

$P(X^c \cap Y) = P(Y) - P(X \cap Y)$

$= \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$ (D is not correct)

Sol 31: (A) P (at least one of them solves correctly) = 1 – P (none of them solves correctly)

$= 1 - \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}\right) = \frac{235}{256}$

Sol 32: (6) Let P (E_1) = x, P (E_2) = y and P (E_3) = z

Then (1 – x) (1 - y) (1 - z) = p

$(1-x)y(1-z) = \alpha$

$(1-x)y(1-z) = \beta$

$(1-x)(1-y)(1-z) = \gamma$

so $\frac{1-x}{x} = \frac{p}{\alpha}$ $\quad x = \frac{\alpha}{\alpha + p}$

Similarly $z = \frac{\gamma}{\gamma + p}$

So, $\frac{P(E_1)}{P(E_3)} = \frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}} = \frac{\frac{\gamma+p}{\gamma}}{\frac{\alpha+p}{\alpha}} = \frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}}$

Also given $\frac{\alpha\beta}{\alpha - 2\beta} = p = \frac{2\beta\gamma}{\beta - 3\gamma} \Rightarrow \beta = \frac{5\alpha\gamma}{\alpha + 4\gamma}$

Substituting back $\left(\alpha - 2\left(\frac{5\alpha\gamma}{\alpha + 4\gamma}\right)\right)p = \frac{\alpha \cdot 5\alpha\gamma}{\alpha + 4\gamma}$

$\Rightarrow \quad \alpha p - 6p\gamma = 5\alpha\gamma$

$\Rightarrow \quad \left(\frac{p}{\gamma} + 1\right) = 6\left(\frac{p}{\alpha} + 1\right) \Rightarrow \frac{\frac{p}{\gamma} + 1}{\frac{p}{\alpha} + 1} = 6$

Sol 33: (5)

Clearly, $1 + 2 + 3 + ... + n - 2 \le 1224 \le 3 + 4 + ...n$

$\Rightarrow \quad \frac{(n-2)(n-1)}{2} \le 1224 \le \frac{(n-2)}{2}(3+n)$

$\Rightarrow \quad n^2 - 3n - 2446 \le 0$ and $n^2 + n - 2454 \ge 0$

$\Rightarrow \ 49 < n < 51 \Rightarrow n = 50$

$\therefore \ \frac{n(n+1)}{2} - (2k+1) = 1224 \Rightarrow k = 25 \Rightarrow k - 20 = 5$

Sol 34: (A) P (required) = P (all are white) + P (all are red) + P (all are black)

$= \frac{1}{6} \times \frac{2}{9} + \frac{3}{12} + \frac{3}{6} \times \frac{3}{9} \times \frac{4}{12} + \frac{2}{6} \times \frac{4}{9} \times \frac{5}{12}$

$= \frac{6}{648} + \frac{36}{648} + \frac{40}{648} = \frac{82}{648}$

Sol 35: (D) Let A : one ball is white and other is red

E_1 : both balls are from box B_1

E_2 : both balls are from box B_2

E_3 : both balls are from box B_3

Here, P (required) $= P\left(\frac{E_2}{A}\right)$

$$= \frac{P\left(\frac{A}{E_2}\right) \cdot P(E_2)}{P\left(\frac{A}{E_1}\right) \cdot P(E_1) + P\left(\frac{A}{E_2}\right) \cdot P(E_2) + P\left(\frac{A}{E_3}\right) \cdot P(E_3)}$$

$$= \frac{\frac{{}^2C_1 \times {}^3C_1}{{}^9C_2} \times \frac{1}{3}}{\frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} \times \frac{1}{3} + \frac{{}^2C_1 \times {}^3C_1}{{}^9C_2} \times \frac{1}{3} + \frac{{}^3C_1 \times {}^4C_1}{{}^{12}C_2} \times \frac{1}{3}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{5} + \frac{1}{6} + \frac{2}{11}} = \frac{55}{181}$$

Sol 36: (A) Either a girl will start the sequence or will be at second position and will not acquire the last position as well.

Required probability $= \frac{({}^3C_1 + {}^3C_1)}{{}^3C_2} = \frac{1}{2}$

Sol 37: (C) Number of required ways

$= 5! - \{4 \cdot 4! - {}^4C_2 \cdot 3! + {}^4C_3 \cdot 2! - 1\} = 53$

Sol 38: (B) Case-I : One odd, 2 even

Total number of ways $= 2 \times 2 \times 3 + 1 \times 3 \times 3 + 1 \times 2 \times 4 = 29$

Case-II: All 3 odd

Number of ways $= 2 \times 3 \times 4 = 24$

Favourable ways = 53

Required probability $= \frac{53}{3 \times 5 \times 7} = \frac{53}{105}$

Sol 39: (C) Here $2x_2 = x_1 + x_3$

$\Rightarrow \ x_1 + x_3 = \text{even}$

Hence number of favorable ways

$= {}^2C_1 \cdot {}^4C_2 + {}^1C_1 \cdot {}^3C_1 = 11$

Sol 40: (8) Let coin was tossed 'n' times

Probability of getting atleast two heads $= 1 - \left[\frac{1}{2^n} + \frac{n}{2^n}\right]$

$\Rightarrow \ 1 - \left[\frac{n+1}{2^n}\right] \ge 0.96$

$\Rightarrow \ \frac{2^n}{n+1} \ge 25 \quad \Rightarrow \quad n \ge 8$

Sol 41: (5) n = 6! . 5! (5 girls together arranged along with 5 boys)

$m = {}^5C_4 \cdot (7! - 2.6!) \cdot 4!$

(4 out of 5 girls together arranged with others – number of cases all 5 girls are together)

$\frac{m}{n} = \frac{5 \cdot 5 \cdot 6! \cdot 4!}{6! \cdot 5!} = 5$

Sol 42: (A, B) P (Red ball) $= P(I) \cdot P(R \mid I) + P(II) \cdot P(R \mid II)$

$P(II \mid R) = \frac{1}{3} = \frac{P(II) \cdot P(R \mid II)}{P(I) \cdot P(R \mid I) + P(II) \cdot P(R \mid II)}$

$$\frac{1}{3} = \frac{\frac{n_3}{n_3 + n_4}}{\frac{n_1}{n_1 + n_2} + \frac{n_3}{n_3 + n_4}}$$

Of the given options, A and B satisfy above condition

Sol 43: (C, D) P (Red after Transfer) = P(Red Transfer) . P(Red Transfer in II Case) + P (Black Transfer) . P(Red Transfer in II Case)

$$P(R) = \frac{n_1}{n_1 + n_2} \frac{(n_1 - 1)}{(n_1 + n_2 - 1)} + \frac{n_2}{n_1 + n_2} \cdot \frac{n_1}{n_1 + n_2 - 1} = \frac{1}{3}$$

Of the given options, option C and D satisfy above condition.

Sol 44: (C) $P(T_1)=\frac{1}{5}$, $P(T_2)=\frac{4}{5}$, $P(D)=\frac{7}{100}$

$P\left(\frac{D}{T_1}\right)=10.\,P\left(\frac{D}{T_2}\right)$. Let $P\left(\frac{D}{T_2}\right)=x$

Now, $P(T_1)\times P\left(\frac{D}{T_1}\right)+P(T_2)\cdot P\left(\frac{D}{T_2}\right)=\frac{7}{100}$

$=\frac{1}{5}\times 10x+\frac{4}{5}\times x=\frac{7}{100}\Rightarrow x=\frac{1}{40}$

$\therefore \quad P\left(\frac{T_2}{D}\right)=\frac{\frac{4}{5}\times\frac{39}{40}}{\frac{93}{100}}=\frac{78}{93}$

Sol 45: (A) $={}^6C_3\times{}^4C_1\times 4+{}^6C_4\times=380$

Sol 46: (B) $P(X>Y)=\left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{1}{2}\right)=\frac{5}{12}$

Sol 47: (C) $P(X=Y)=\left(\frac{1}{2}\times\frac{1}{3}\times 2\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)=\frac{13}{36}$