MasterJEE

IIT-JEE | Medical | Foundations

Time : 3 hrs.

Answers & Solutions

M.M. : 360

for

JEE (MAIN)-2019

(Online CBT Mode)

(Physics, Chemistry and Mathematics)

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 **(four)** marks for each correct response.
4. *Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for a question in the answer sheet.*
5. There is only one correct response for each question.

PHYSICS

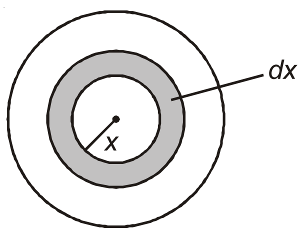
1. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is

(1) $\frac{\mu FR}{2}$ (2) $\frac{\mu FR}{3}$

(3) $\frac{\mu FR}{6}$ (4) $\frac{2}{3}\mu FR$

Answer (4)

Sol. $P = \frac{F}{\pi R^2}$

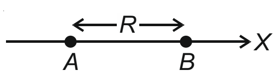


$$\tau = \frac{\mu F}{\pi R^2}\int_0^R 2\pi x^2 dx$$

$$= \frac{\mu f \times 2\pi R^3}{3\pi R^2}$$

$$= \frac{2}{3}\mu FR$$

2. Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4qa\hat{i}$ and $\vec{d}_B = -2qa\hat{i}$ are placed on the x-axis with a separation R, as shown in the figure



The distance from A at which both of them produce the same potential is

(1) $\frac{\sqrt{2}R}{\sqrt{2}-1}$ (2) $\frac{\sqrt{2}R}{\sqrt{2}+1}$

(3) $\frac{R}{\sqrt{2}-1}$ (4) $\frac{R}{\sqrt{2}+1}$

Answer (2)

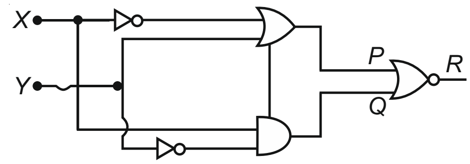
Sol. $V \propto \frac{1}{d^2}$

$$\frac{4qa}{x^2} = \frac{2qa}{(R-x)^2}$$

$$(R-x) = \frac{x}{\sqrt{2}}$$

$$\frac{\sqrt{2}R}{\left(\sqrt{2}+1\right)} = x$$

3. To get output '1' at R, for the given logic gate circuit the input values must be



(1) X = 1, Y = 1 (2) X = 0, Y = 0

(3) X = 1, Y = 0 (4) X = 0, Y = 1

Answer (3)

Sol. $A = \left[\overline{(\overline{x}+y)+\overline{x}\overline{y}}\right]$

$$= \overline{\overline{x}+y+\overline{x}+y}$$

$$A = \overline{(\overline{x}+y)}$$

Output is 1 when $x = 1$, $y = 0$

4. A solid metal cube of edge length 2 cm is moving in a positive y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is

(1) 12 mV (2) 2 mV

(3) 6 mV (4) 1 mV

Answer (1)

Sol. $E = vB$

$= 0.6$ V/m

$V = Ed$

$= 0.6 \times 2 \times 10^{-2}$

$= 12$ mV

5. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as

(1) $2\mu_1 - \mu_2 = 1$

(2) $3\mu_2 - 2\mu_1 = 1$

(3) $2\mu_2 - \mu_1 = 1$

(4) $\mu_1 + \mu_2 = 3$

Answer (1)

Sol. $\frac{1}{f_1} = (\mu_1 - 1)\frac{1}{R}$

$\frac{1}{f_2} = (\mu_2 - 1)\frac{1}{R}$

$2(\mu_1 - 1)\frac{1}{R} = (\mu_2 - 1)\frac{1}{R}$

$2\mu_1 - \mu_2 = 1$

6. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At t = 0 it was 1600 counts per second and t = 8 seconds it was 100 counts per second. The count rate observed, as counts per second, at t = 6 seconds is close to

(1) 400 (2) 360

(3) 150 (4) 200

Answer (4)

Sol. $(2)^N = \left(\frac{1600}{100}\right)$

$N = 4$

$4t = 8$

$t = 2$ s

Count Rate $= \frac{1600}{(2)^3} = 200$

7. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is

(1) 5.1 cm (2) 1.7 cm

(3) 2.9 cm (4) 4 cm

Answer (1)

Sol. $-\frac{dh}{dt} = \sqrt{2gh}$

$a\sqrt{2gh} = Q$

$10^{-4}\sqrt{2gh} = 10^{-4}$

$h = \frac{1}{2g}$

$h = 5.1\,\text{cm}$

8. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode? (Given : radius of earth = 6.4×10^6 m)

(1) 65 km (2) 80 km

(3) 40 km (4) 48 km

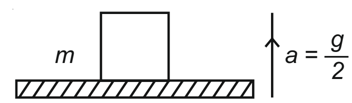
Answer (1)

Sol. $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$= 42.33 + 22.6$

$d \approx 65$ km

9. A block of mass m is kept on a platform which starts from rest with constant acceleration $\frac{g}{2}$ upward, as shown in fig. Work done by normal reaction on block in time t is



(1) $\frac{3mg^2t^2}{8}$ (2) $-\frac{mg^2t^2}{8}$

(3) 0 (4) $\frac{mg^2t^2}{8}$

Answer (1)

Sol. $N - mg = \frac{mg}{2}$

$N = \frac{3mg}{2}$

$S = \frac{1}{2}\left(\frac{g}{2}\right)t^2$

$w = \frac{3mg}{2} \times \frac{g}{4}t^2$

$w = \frac{3mg^2}{8}t^2$

10. A magnet of total magnetic moment $10^{-2}\hat{i}$ A-m^2 is placed in a time varying magnetic field, $B\hat{i}(\cos\omega t)$ where B = 1 Tesla and ω = 0.125 rad/s. The work done for reversing the direction of the magnetic moment at t = 1 second, is

(1) 0.028 J (2) 0.007 J

(3) 0.014 J (4) 0.01 J

Answer (BONUS)

Sol. $v_i = -MB\cos\left(\frac{1}{8}\right)$

$v_f = MB\cos\left(\frac{1}{8}\right)$

$w = 2MB\cos\left(\frac{1}{8}\right)$

$= 0.0198$ J

None of the option given is correct.

11. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio $\frac{f_1}{f_2}$ is

(1) $\frac{21}{20}$

(2) $\frac{20}{19}$

(3) $\frac{18}{17}$

(4) $\frac{19}{18}$

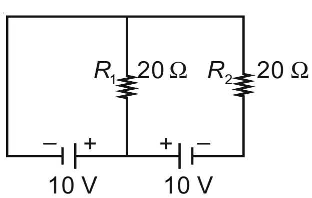
Answer (4)

Sol. $f_1 = f_0\left(\frac{340}{340-34}\right)$

$f_2 = f_0\left(\frac{340}{340-17}\right)$

$\frac{f_1}{f_2} = \frac{340-17}{340-34} = \frac{19}{18}$

12. In the given circuit the cells have zero internal resistance. The currents (in amperes) passing through resistance R_1 and R_2 respectively, are



(1) 1, 2

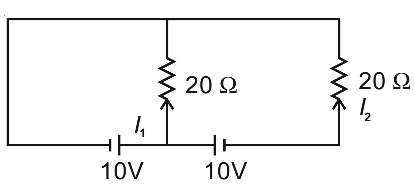
(2) 0, 1

(3) 0.5, 0

(4) 2, 2

Answer (3)

Sol.



$10 - I_1 \times 20 = 0$

$I_1 = 0.5$ A

$I_2 \times 20 = 0$

$I_2 = 0$

13. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are

(1) 380 nm, 500 nm (2) 625 nm, 500 nm

(3) 380 nm, 525 nm (4) 400 nm, 500 nm

Answer (2)

Sol. $d\theta = m_1 \lambda_1$

$d\theta = m_2 \lambda_2$

$d\theta = \frac{0.1\times10^{-3}}{40}\times10^9$ nm

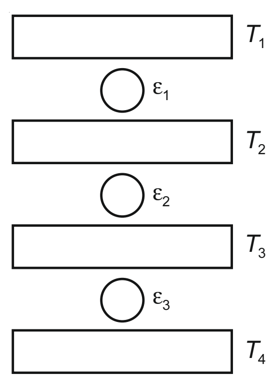
$d\theta = \frac{1}{40}\times10^5 = 25\times10^2$ nm

$d\theta$ is LCM of λ_1 and λ_2

λ_1 = 625 nm and λ_2 = 500 nm

[By observation]

14. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if



(1) $T_2 = (T_1T_4^2)^{1/3}$; $T_3 = (T_1^2T_4)^{1/3}$

(2) $T_2 = (T_1^3T_4)^{1/4}$; $T_3 = (T_1T_4^3)^{1/4}$

(3) $T_2 = (T_1T_4)^{1/2}$; $T_3 = (T_1^2T_4)^{1/3}$

(4) $T_2 = (T_1^2T_4)^{1/3}$; $T_3 = (T_1T_4^2)^{1/3}$

Answer (4)

Sol. $\eta = 1-\frac{T_2}{T_1} = 1-\frac{T_3}{T_2} = 1-\frac{T_4}{T_3}$

$T_2 = \sqrt{T_1T_3}$

$T_3 = \sqrt{T_2T_4}$

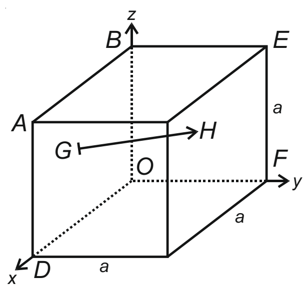
$T_3^2 = T_1^{1/2}T_3^{1/2}T_4$

$T_3^{3/2} = T_1^{1/2}T_4$

$T_3 = \left[T_1T_4^2\right]^{1/3}$

Similarly $T_2 = \left(T_1^2T_4\right)^{1/3}$

15. In the cube of side 'a' shown in the figure, the vector from the central point of the face *ABOD* to the central point of the face *BEFO* will be



(1) $\frac{1}{2}a(\hat{j}-\hat{i})$ (2) $\frac{1}{2}a(\hat{i}-\hat{k})$

(3) $\frac{1}{2}a(\hat{j}-\hat{k})$ (4) $\frac{1}{2}a(\hat{k}-\hat{i})$

Answer (1)

Sol. P.V. of G = $\frac{a}{2}(\hat{i}+\hat{k})$

P.V. of H = $\frac{a}{2}(\hat{j}+\hat{k})$

$\vec{G}H = \frac{a}{2}(\hat{j}-\hat{i})$

16. A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is

(1) 20 mA (2) 0.4 mA

(3) 100 mA (4) 63 mA

Answer (1)

Sol. *G B R Br*

Resistance = $50 \times 10^2 \pm 1\%$

$\rho = I^2R$

$2 = 5000 \times I^2$

$\frac{4}{10000} = I^2$

$I = \frac{2}{100}$

I = 20 mA

17. If the magnetic field of a plane electromagnetic wave is given by (The speed of light = 3×10^8 m/s) $B = 100\times10^{-6}\sin\left[2\pi\times2\times10^{15}\left(t-\frac{x}{c}\right)\right]$, then the maximum electric field associated with it is

(1) 6×10^4 N/C (2) 3×10^4 N/C

(3) 4.5×10^4 N/C (4) 4×10^4 N/C

Answer (2)

Sol. $E_m = CB_m = 3 \times 10^8 \times 100 \times 10^{-6} = 3\times10^4$ N/C

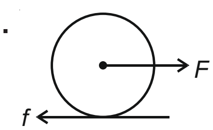
18. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is

(1) $\frac{F}{2mR}$ (2) $\frac{2F}{3mR}$

(3) $\frac{F}{3mR}$ (4) $\frac{3F}{2mR}$

Answer (2)

Sol.



$F - f = Ma$

$fR = \frac{MR^2}{2}\cdot\frac{a}{R}$

$\Rightarrow f = \frac{Ma}{2}$

$\Rightarrow F = \frac{3Ma}{2}$

$\Rightarrow a = \frac{2F}{3M}$

$\alpha = \frac{2F}{3MR}$

19. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is

(1) $2mv^2$ (2) mv^2

(3) $\frac{1}{2}mv^2$ (4) $\frac{3}{2}mv^2$

Answer (2)

Sol. $U = -2\times\frac{1}{2}mv^2$

In order to escape $U + K = 0$

$\Rightarrow K = mv^2$

20. A charge Q is distributed over three concentric spherical shells of radii a, b, c ($a < b < c$) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where $r < a$, would be

(1) $\dfrac{Q(a+b+c)}{4\pi\epsilon_0 (a^2+b^2+c^2)}$ (2) $\dfrac{Q}{4\pi\epsilon_0 (a+b+c)}$

(3) $\dfrac{Q(a^2+b^2+c^2)}{4\pi\epsilon_0 (a^3+b^3+c^3)}$ (4) $\dfrac{Q}{12\pi\epsilon_0}\dfrac{ab+bc+ca}{abc}$

Answer (1)

Sol. $Q = \sigma 4\pi[a^2+b^2+c^2]$

$$V = \frac{1}{4\pi\varepsilon_0}\sigma\left[\frac{4\pi a^2}{a}+\frac{4\pi b^2}{b}+\frac{4\pi c^2}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q(a+b+c)}{(a^2+b^2+c^2)}$$

21. An insulating, thin rod of length l has a linear charge density $\rho(x) = \rho_0\dfrac{x}{l}$ on it. The rod is rotated about an axis passing through the origin ($x = 0$) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is

(1) $\pi n\rho l^3$ (2) $n\rho l^3$

(3) $\dfrac{\pi}{4}n\rho l^3$ (4) $\dfrac{\pi}{3}n\rho l^3$

Answer (3)

Sol. $dm = \rho dx n \pi x^2$

$$\int dm = \int_0^{\ell}\frac{\rho_0 n\pi x^3 dx}{\ell}$$

$$m = \frac{\pi\rho_0 n}{\ell}\cdot\frac{\ell^4}{4} = \frac{\pi\rho_0 n\ell^3}{4}$$

22. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to

(1) 100 keV

(2) 1 keV

(3) 500 keV

(4) 25 keV

Answer (4)

Sol. Here $\lambda \simeq 7.5 \times 10^{-12}$ m

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

$$\Rightarrow K = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.6\times10^{-34})^2 ev}{2\times9.1\times10^{-31}\times(7.5\times10^{-12})^2\times1.6\times10^{-19}}$$

$\simeq$ 25 keV

23. A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 $WK^{-1}m^{-1}$, the energy flux through it in the steady state is

(1) 200 Wm^{-2} (2) 65 Wm^{-2}

(3) 120 Wm^{-2} (4) 90 Wm^{-2}

Answer (4)

Sol. $\dfrac{Q}{A} = \dfrac{K[T_1 - T_2]}{L} = \dfrac{0.1\times900}{1} = 90\text{ w/m}^2$

24. The density of a material is SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is

(1) 640 (2) 410

(3) 40 (4) 16

Answer (3)

Sol. $\rho = \dfrac{128\text{kg}}{\text{m}^3} = \dfrac{128}{\left(\dfrac{100}{25}\right)^3}\dfrac{1000}{50} = \dfrac{128}{4^3}\times20 = 40$

25. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is

(1) 1 : 4

(2) 1 : 8

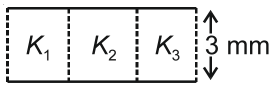
(3) 1 : 2

(4) 1 : 16

Answer (4)

Sol. $\dfrac{A_1}{A_2} = \dfrac{\pi R_{1,\max}^2}{\pi R_{2,\max}^2} = \left(\dfrac{u_1^2}{u_2^2}\right)^2 = \left(\dfrac{1}{4}\right)^2 = \dfrac{1}{16}$

26. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants K_1 = 10, K_2 = 12 and K_3 = 14. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be



(1) 36 (2) 14

(3) 12 (4) 4

Answer (3)

Sol. $C = C_1 + C_2 + C_3$

$$\frac{\varepsilon_0 K A}{d} = \frac{\varepsilon_0 K_1 A}{3d} + \frac{\varepsilon_0 K_2 A}{3d} + \frac{\varepsilon_0 K_3 A}{3d}$$

$$\Rightarrow K = \frac{K_1 + K_2 + K_3}{3} = \frac{10+12+14}{3} = 12$$

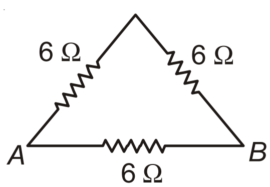
27. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is

(1) 4 Ω (2) 12 Ω

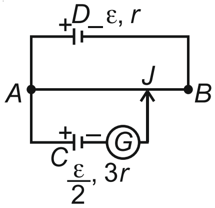
(3) 8 Ω (4) 2 Ω

Answer (1)

Sol. $R_{AB} = \frac{12\times 6}{12+6} = 4\ \Omega$



28. A potentiometer wire *AB* having length *L* and resistance 12*r* is joined to a cell *D* of emf ε and internal resistance *r*. A cell *C* having emf $\frac{\varepsilon}{2}$ and internal resistance 3*r* is connected. The length *AJ* at which the galvanometer as shown in fig. shows no deflection is



(1) $\frac{11}{12}L$ (2) $\frac{11}{24}L$

(3) $\frac{5}{12}L$ (4) $\frac{13}{24}L$

Answer (4)

Sol. $\phi = \frac{\varepsilon}{13r}\frac{12r}{L} = \frac{12\varepsilon}{13L}$

$$\frac{\varepsilon}{2} = \phi L' = \frac{12}{13L}\varepsilon \cdot L'$$

$$\Rightarrow L' = \frac{13L}{24}$$

29. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to

(1) 33.3 cm (2) 10.0 cm

(3) 16.6 cm (4) 20.0 cm

Answer (4)

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{8\times 1}{5\times 10^{-3}}}$ = 40 m/s

$$\lambda = \frac{v}{f} = \frac{40}{100} = 0.4\text{ m}$$

Separation between successive nodes = $\frac{\lambda}{2}$

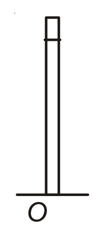
= 0.2 m ≃ 20 cm

30. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upwards, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is (g = 10 ms^{-2})

(1) 30 m (2) 40 m

(3) 20 m (4) 10 m

Answer (2)



Sol. $100 - \frac{1}{2}10.t^2 = 100t - 5t^2$

$\Rightarrow t = 1$ s

Conservation of momentum

$\Rightarrow 90 \times 0{\cdot}02 - 10 \times 0{\cdot}03 = v \times 0{\cdot}05$

$$\Rightarrow v = \frac{1.8 - 0.3}{0\cdot 05} = \frac{1.5}{0\cdot 05} = \frac{150}{5} = 30\text{ m/s}$$

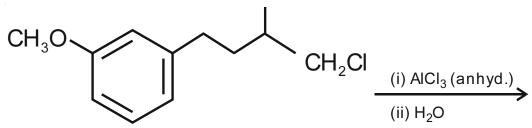
$$s_2 = \frac{30^2}{2\times 10} = \frac{30\times 30}{2\times 10} = 45\text{ m}$$

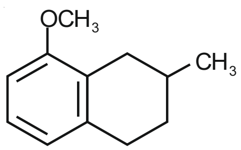
⇒ Maximum height above the building

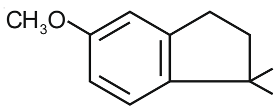
= 45 – 5 = 40 m

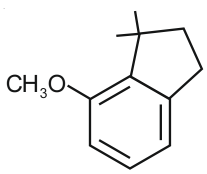
CHEMISTRY

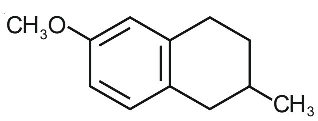
1. The major product of the following reaction is



(1) 

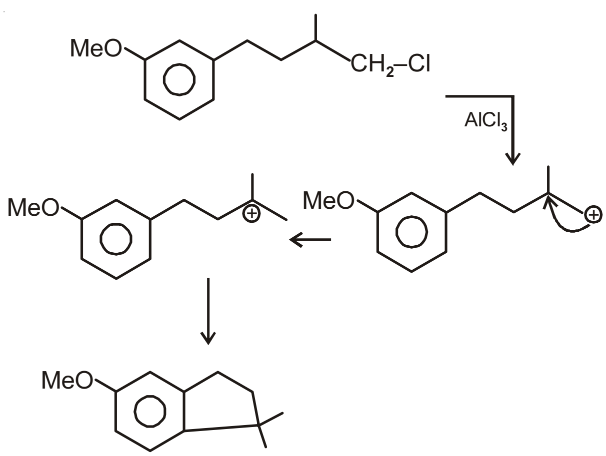
(2) 

(3) 

(4) 

Answer (2)

Sol.



2. The chemical nature of hydrogen peroxide is
   (1) Oxidising and reducing agent in both acidic and basic medium
   (2) Oxidising and reducing agent in acidic medium, but not in basic medium
   (3) Reducing agent in basic medium, but not in acidic medium
   (4) Oxidising agent in acidic medium, but not in basic medium

Answer (1)

Sol. H_2O_2 can act as both oxidising as well as reducing agent in both acidic as well as basic medium.

3. Consider the following reduction processes:

   $Zn^{2+} + 2e^- \rightarrow Zn(s)$; $E^o = -0.76$ V

   $Ca^{2+} + 2e^- \rightarrow Ca(s)$; $E^o = -2.87$ V

   $Mg^{2+} + 2e^- \rightarrow Mg(s)$; $E^o = -2.36$ V

   $Ni^{2+} + 2e^- \rightarrow Ni(s)$; $E^o = -0.25$ V

   The reducing power of the metals increases in the order :

   (1) Ca < Mg < Zn < Ni (2) Ni < Zn < Mg < Ca

   (3) Ca < Zn < Mg < Ni (4) Zn < Mg < Ni < Ca

Answer (2)

Sol. As $E^\circ_{M/M^{2+}}$ increases, reducing power increases.

$E^\circ_{Zn/Zn^{2+}}$ = 0.76 V

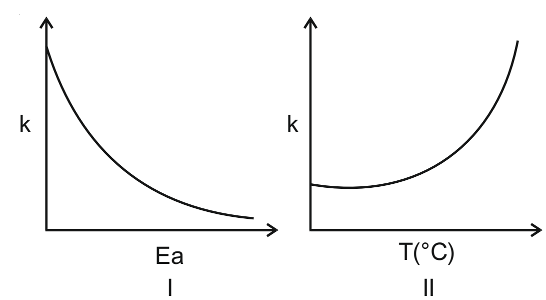
$E^\circ_{Ca/Ca^{2+}}$ = 2.87 V

$E^\circ_{Mg/Mg^{2+}}$ = 2.36 V

$E^\circ_{Ni/Ni^{2+}}$ = 0.25 V

Ca > Mg > Zn > Ni

4. Consider the given plots for a reaction obeying Arrhenius equation (0°C < T < 300°C) : (k and E_a are rate constant and activation energy, respectively)



Choose the correct option:

(1) I is wrong but II is right

(2) Both I and II are correct

(3) Both I and II are wrong

(4) I is right but II is wrong

Answer (2)

Sol. $K = Ae^{-Ea/RT}$

So, as Ea increases, K decreases.

and as T increases, K increases.

5. The electronegativity of aluminium is similar to

(1) Beryllium (2) Carbon

(3) Lithium (4) Boron

Answer (1)

Sol. Be and Al show diagonal relationship.

6. The values of $\frac{K_p}{K_c}$ for the following reactions at 300 K are, respectively (At 300 K, RT = 24.62 dm^3 atm mol^{-1})

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

(1) 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2}, 1.65×10^{-3} dm^{-6} atm^{-2} mol^2

(2) 1, 24.62 dm^3 atm mol^{-1}, 1.65×10^{-3} dm^{-6} atm^{-2} mol^2

(3) 1, 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2}

(4) 1, 4.1×10^{-2} dm^{-3} atm^{-1} mol, 606 dm^6 atm^2 mol^{-2}

Answer (2)

Sol. $\frac{K_P}{K_C} = (RT)^{\Delta n_g}$

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g), \Delta n_g = 0$

$\frac{K_P}{K_C} = 1$

$N_2O_4(g) \rightleftharpoons 2NO_2(g), \Delta n_g = 1$

$\frac{K_P}{K_C} = 24.62$

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g), \Delta n_g = -2$

$\frac{K_P}{K_C} = \frac{1}{(24.62)^2} = 1.65 \times 10^{-3}$

7. Liquids A and B form an ideal solution in the entire composition range. At 350 K, the vapor pressures of pure A and pure B are 7×10^3 Pa and 12×10^3 Pa, respectively. The composition of the vapor in equilibrium with a solution containing 40 mole percent of A at this temperature is

(1) $x_A = 0.76; x_B = 0.24$

(2) $x_A = 0.37; x_B = 0.63$

(3) $x_A = 0.28; x_B = 0.72$

(4) $x_A = 0.4; x_B = 0.6$

Answer (3)

Sol. $P_A^\circ = 7 \times 10^3$

$P_B^\circ = 12 \times 10^3$

$X_A = 0.4$

$X_B = 0.6$

$\therefore$ $P = (7 \times 0.4 + 12 \times 0.6) \times 10^3 = 10^4$

$P_A = 2.8 \times 10^3, \; P_B = 7.2 \times 10^3$

$\therefore$ $Y_A = 0.28, \; Y_B = 0.72$

8. If dichloromethane (DCM) and water (H_2O) are used for differential extraction, which one of the following statements is correct?

(1) DCM and H_2O will make turbid/colloidal mixture

(2) DCM and H_2O will be miscible clearly

(3) DCM and H_2O would stay as upper and lower layer respectively in the separating funnel (S.F.)

(4) DCM and H_2O would stay as lower and upper layer respectively in the S.F.

Answer (4)

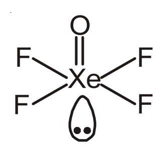
Sol. Density of dichloromethane is greater than water. So, DCM would be the lower layer and water will form the upper layer in the separating funnel.

9. The type of hybridisation and number of lone pair(s) of electrons of Xe in $XeOF_4$, respectively, are

(1) sp^3d and 2 (2) sp^3d^2 and 2

(3) sp^3d^2 and 1 (4) sp^3d and 1

Answer (3)

Sol. 

, sp^3d^2, Lone pair = 1

10. Wilkinson catalyst is (Et = C_2H_5)

(1) $[(Ph_3P)_3IrCl]$ (2) $[(Ph_3P)_3RhCl]$

(3) $[(Et_3P)_3IrCl]$ (4) $[(Et_3P)_3RhCl]$

Answer (2)

Sol. Wilkinson's catalyst is $[Rh(PPh_3)_3]Cl$

11. The metal used for making X-ray tube window is

(1) Ca

(2) Na

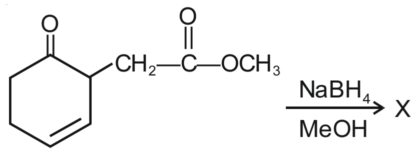
(3) Mg

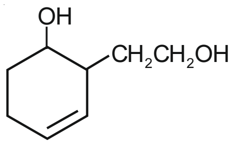
(4) Be

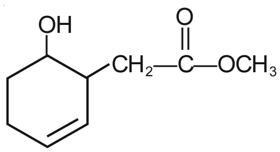
Answer (4)

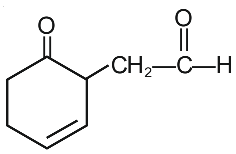
Sol. Be is used in making X-ray tube windows.

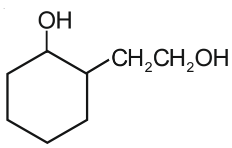
12. The major product 'X' formed in the following reaction is



(1) 

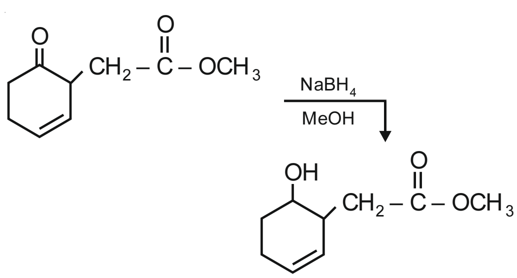
(2) 

(3) 

(4) 

Answer (2)

Sol. $NaBH_4$ selectively reduces the ketone, it does not affect alkene and ester.



13. Hall-Heroult's process is given by

(1) $ZnO + C \xrightarrow{\text{Coke, 1673K}} Zn + CO$

(2) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

(3) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

(4) $Cu^{2+}(aq) + H_2(g) \rightarrow Cu(s) + 2H^+(aq)$

Answer (3)

Sol. Hall Heroult's process is used in extraction of Al from Alumina.

Cathode : $Al^{3+} + 3e^- \longrightarrow Al$

Anode : $C + 2O^{2-} \longrightarrow CO_2 + 4e$

14. The total number of isotopes of hydrogen and number of radioactive isotopes among them, respectively, are

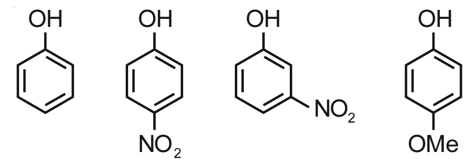
(1) 2 and 1 (2) 3 and 2

(3) 2 and 0 (4) 3 and 1

Answer (4)

Sol. There are three isotopes of H out of which only tritium is radioactive.

15. The increasing order of the pK_a values of the following compounds is



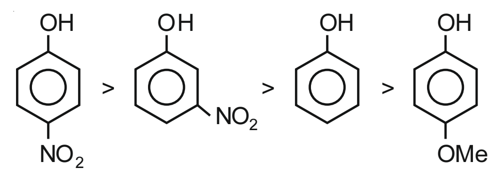
A B C D

(1) D < A < C < B (2) B < C < D < A

(3) B < C < A < D (4) C < B < A < D

Answer (3)

Sol. Acidic strength



(B) (C) (A) (D)

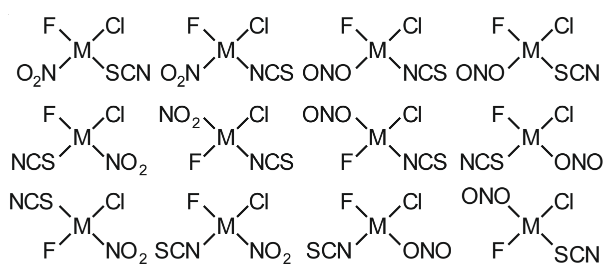
$\therefore$ pK_a : B < C < A < D

16. The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is

(1) 8 (2) 12

(3) 4 (4) 16

Answer (2)

Sol. 

$\therefore$ Answer is 12.

17. Which of the following is not an example of heterogeneous catalytic reaction?

(1) Combustion of coal

(2) Ostwald's process

(3) Hydrogenation of vegetable oils

(4) Haber's process

Answer (1)

Sol. $C + O_2 \longrightarrow CO_2$ No catalyst

Ostwald process :

$$4NH_3 + 5O_2 \xrightarrow{Pt/Rh} 4NO + 6H_2O$$

Haber's process

$$N_2 + 3H_2 \xrightarrow{Fe} 2NH_3$$

18. A process had $\Delta H = 200$ $Jmol^{-1}$ and $\Delta S = 40$ JK^{-1} mol^{-1}. Out of the values given below, choose the minimum temperature above which the process will be spontaneous.

(1) 4 K (2) 12 K

(3) 5 K (4) 20 K

Answer (3)

Sol. $\Delta H = 200$ J mol^{-1}

$\Delta S = 40$ JK^{-1} mol^{-1}

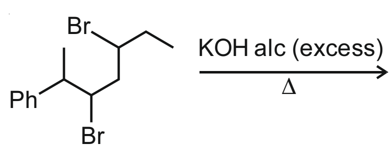
For spontaneous reaction,

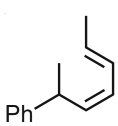
$$T \geq \frac{\Delta H}{\Delta S}$$

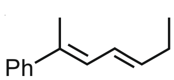
$$T \geq \frac{200}{40} \geq 5\,K$$

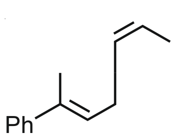
So, minimum temperature is 5 K

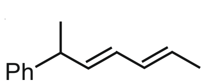
19. The major product of the following reaction is



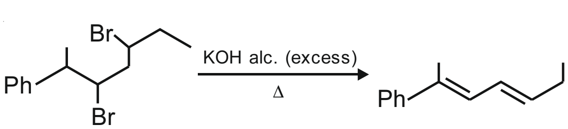
(1) 

(2) 

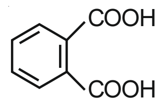
(3) 

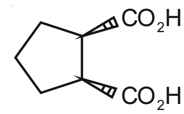
(4) 

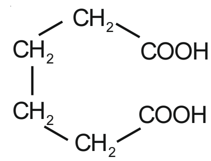
Answer (2)

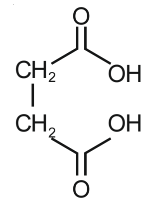
Sol. 

20. Which dicarboxylic acid in presence of a dehydrating agent is least reactive to give an anhydride?

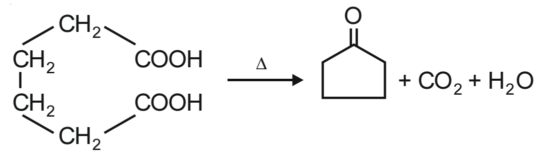
(1) 

(2) 

(3) 

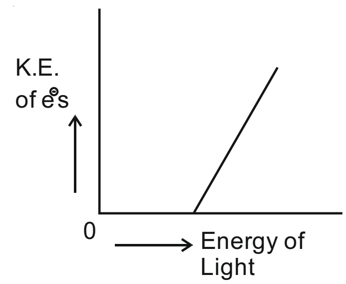
(4) 

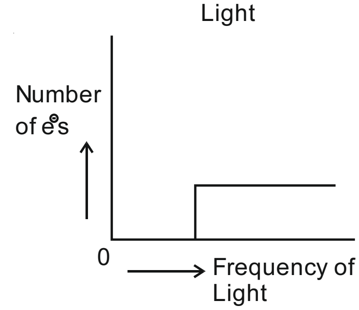
Answer (3)

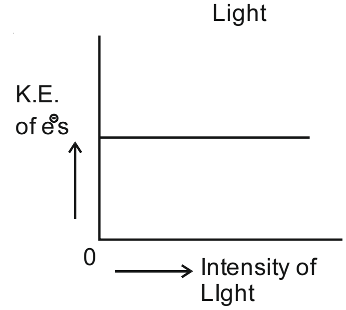
Sol. 

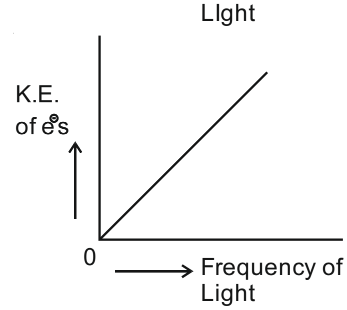
This compound does not form anhydride.

21. Which of the graphs shown below does not represent the relationship between incident light and the electron ejected from metal surface?

(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. $h\nu_{incident} = h\nu_{th} + KE$

KE is independent of intensity and number of photoelectrons does not depend on frequency of light.

$\Rightarrow KE = h\nu_{incident} - h\nu_{th}$

22. Which hydrogen in compound (E) is easily replaceable during bromination reaction in presence of light?

$$\underset{\delta}{CH_3} - \underset{\gamma}{CH_2} - \underset{\beta}{CH} = \underset{\alpha}{CH}$$

(E)

(1) γ-hydrogen (2) α-hydrogen

(3) δ-hydrogen (4) β-hydrogen

Answer (1)

Sol. $CH_3 - CH_2 - CH = CH_2 + Br_2 \xrightarrow{h\nu}$

$$CH_3 - \underset{\substack{| \\ Br}}{CH} - CH = CH_2$$

23. Water filled in two glasses A and B have BOD values of 10 and 20, respectively. The correct statement regarding them, is

(1) Both A and B are suitable for drinking

(2) A is suitable for drinking, whereas B is not

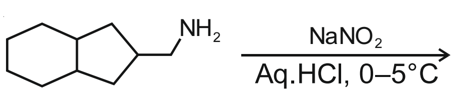
(3) B is more polluted than A

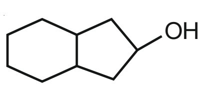
(4) A is more polluted than B

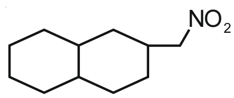
Answer (3)

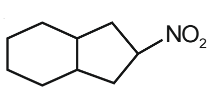
Sol. As B.O.D. increase, level of pollution in water increases.

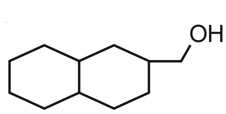
24. The major product formed in the reaction given below will be



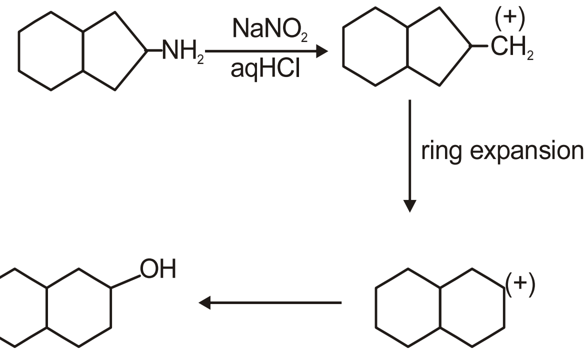
(1) 

(2) 

(3) 

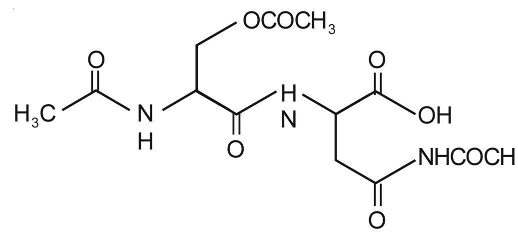
(4) 

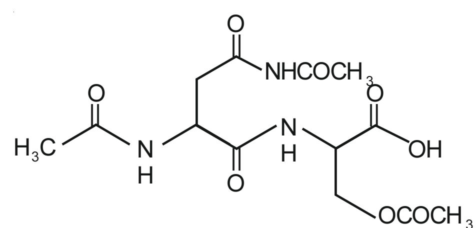
Answer (No option is correct)

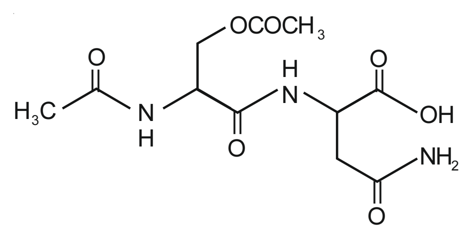
Sol. 

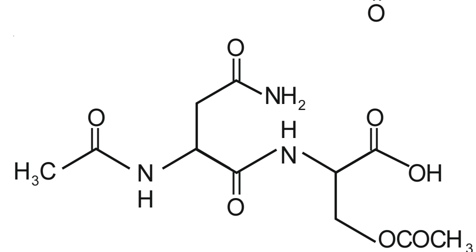
25. The correct structure of product 'P' in the following reaction is

$$Asn - Ser + \underset{(excess)}{(CH_3CO)_2O} \xrightarrow{NEt_3} P$$

(1) 

(2) 

(3) 

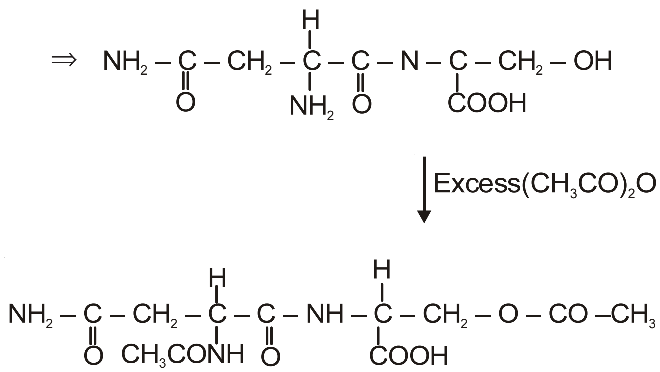
(4) 

Answer (4)

Sol. Asn $\Rightarrow NH_2 - \underset{\substack{\| \\ O}}{C} - CH_2 - \overset{\substack{H \\ |}}{\underset{\substack{| \\ NH_2}}{C}} - COOH$

Ser $\Rightarrow HO - CH_2 - \overset{\substack{H \\ |}}{\underset{\substack{| \\ NH_2}}{C}} - COOH$

Asn – Ser



26. The effect of lanthanoid contraction in the lanthanoid series of elements by and large means

(1) Increase in atomic radii and decrease in ionic radii

(2) Decrease in atomic radii and increase in ionic radii

(3) Decrease in both atomic and ionic radii

(4) Increase in both atomic and ionic radii

Answer (3)

Sol. Due to lanthanoid contraction, size of atom as well as ion of lanthanoid decrease

27. A mixture of 100 m mol of $Ca(OH)_2$ and 2 g of sodium sulphate was dissolved in water and the volume was made up to 100 mL. The mass of calcium sulphate formed and the concentration of OH^- in resulting solution, respectively, are (Molar mass of $Ca(OH)_2$, Na_2SO_4 and $CaSO_4$ are 74, 143 and 136 g mol^{-1}, respectively; K_{sp} of $Ca(OH)_2$ is 5.5×10^{-6})

(1) 1.9 g, 0.14 mol L^{-1} (2) 13.6 g, 0.28 mol L^{-1}

(3) 13.6 g, 0.14 mol L^{-1} (4) 1.9 g, 0.28 mol L^{-1}

Answer (4)

Sol. $Na_2SO_4 + Ca(OH)_2 \rightarrow CaSO_4 + 2NaOH$

mmol of Na_2SO_4 = $\frac{2\times1000}{143} \simeq 13.98$ m Mol

mmol of $CaSO_4$ formed = 13.98 m Mol

Mass of $CaSO_4$ formed = $13.98 \times 10^{-3} \times 136 = 1.90$ g

mmol of NaOH = 28 mmol

$$Ca(OH)_2 \rightleftharpoons \underset{S}{Ca^{2+}} + \underset{2S+0.28}{2OH^-}$$

Value of 'S' will be negligible so

$$[OH^-] = \frac{0.028}{0.1} = 0.28\,mol\,L^{-1}$$

28. Two pi and half sigma bonds are present in

(1) O_2^+ (2) O_2

(3) N_2^+ (4) N_2

Answer (3)

Sol. $N_2^+ = \sigma_{1s}^2\,\sigma^*{}_{1s}^2\,\sigma_{2s}^2\sigma^*{}_{2s}^2\,\pi_{2p_y}^2 = \pi_{2p_x}^2\sigma_{2p_z}^1$

B.O. 2.5 = 2π bond + 0.5 σbond

29. Which primitive unit cell has unequal edge lengths ($a \neq b \neq c$) and all axial angles different from 90°?

(1) Hexagonal (2) Monoclinic

(3) Triclinic (4) Tetragonal

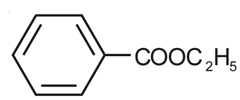
Answer (3)

Sol. For triclinic Crystal

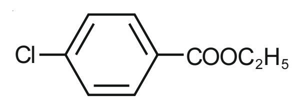
$a \neq b \neq c$ Axial distance

$\alpha \neq \beta \neq \gamma \neq 90$ Axial angle

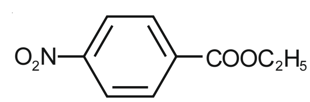
30. The decreasing order of ease of alkaline hydrolysis for the following esters is



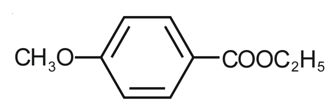
I



II



III



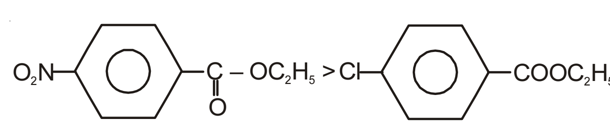
IV

(1) III > II > IV > I (2) IV > II > III > I

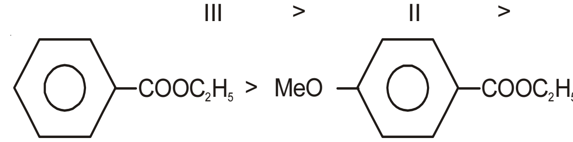
(3) II > III > I > IV (4) III > II > I > IV

Answer (4)

Sol. Rate of reaction α positive charge on carbonyl carbon so E.W.G. increase rate while E.D.G. decrease the rate.



III > II >



I > IV

1. Let $d \in R$, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix},$$

$\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is

(1) –5 (2) $2(\sqrt{2}+1)$

(3) –7 (4) $2(\sqrt{2}+2)$

Answer (1)

Sol.

$$\det(A) = \begin{vmatrix} -2 & 4+d & \sin\theta-2 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & -\sin\theta+2+2d \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 4+d & \sin\theta-2 \\ 1 & \sin\theta+2 & d \\ 1 & 0 & 0 \end{vmatrix}$$

$= d(4 + d) – (\sin^2\theta – 4)$

$\Rightarrow$ det $(A) = d^2 + 4d + 4 – \sin^2\theta = (d + 2)^2 – \sin^2\theta$

min det (A) is attained when $\sin^2\theta = 1$

$\therefore$ $(d + 2)^2 –1 = 8 \Rightarrow (d + 2)^2 = 9 \Rightarrow d + 2 = \pm 3$

$\Rightarrow$ d = –5 or 1

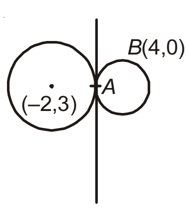
2. If a circle C passing through the point (4, 0) touches the circle $x^2 + y^2 + 4x – 6y = 12$ externally at the point (1, –1), then the radius of C is

(1) 5 (2) $2\sqrt{5}$

(3) $\sqrt{57}$ (4) 4

Answer (1)

Sol.



Let A = (1, –1) & B = (4, 0)

Equation of tangent at A to the given circle :

$3x – 4y – 7 = 0$ ....(1)

The required circle is tangent to (1) at (1, –1).

$\therefore$ $(x – 1)^2 + (y + 1)^2 + \lambda (3x – 4y – 7) = 0$ ....(2)

(2) Passes through B(4, 0)

$\Rightarrow$ $3^2 + 1^2 + \lambda(12 – 7) = 0 \Rightarrow 5\lambda + 10 = 0 \Rightarrow \lambda = –2$

(2) Becomes $x^2 + y^2 – 8x + 10y + 16 = 0$

radius = $\sqrt{(-4)^2+(5)^2-16} = 5$

3. If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals

(1) 400 (2) 100

(3) 200 (4) 50

Answer (2)

Sol.

$$\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}} = \frac{{}^{20}C_{i-1}}{{}^{20}C_i} = \frac{20!}{(i-1)!(21-i)!}\times\frac{i!(21-i)!}{21!}$$

$$\therefore \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3 = \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{(1)}{(21)^3}\sum_{i=1}^{20} i^3$$

$$= \frac{1}{(21)^3}\times\left(\frac{20\times 21}{2}\right)^2 = \frac{100}{21}$$

$\therefore K = 100$

4. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is

(1) 1465 (2) 1356

(3) 1365 (4) 1256

Answer (2)

Sol.

S = 16 + 23 + 30+......+93 = 654

S' = 12 + 19 + 26+.....+96 = 702

Required Sum = 654 + 702

= 1356

5. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$ and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals

(1) $\frac{1}{3}+e^6$ (2) $\frac{1}{3}+e^3$

(3) $\frac{1}{3}$ (4) $-\frac{4}{3}$

Answer (1)

Sol.

$$\frac{dy}{dx}=\sec^2 x(1-3y)$$

$$\Rightarrow \int\frac{dy}{(1-3y)}=\int\sec^2 x\,dx$$

$$\Rightarrow -\frac{1}{3}\ln|1-3y|=\tan x+C$$

$$y\left(\frac{\pi}{4}\right)=\frac{4}{3}$$

$$\Rightarrow -\frac{1}{3}\ln|1-4|=\tan\frac{\pi}{4}+C$$

$$\Rightarrow -\frac{1}{3}\ln 3=C+1 \Rightarrow C=-1-\frac{1}{3}\ln 3$$

$$x=\frac{\pi}{4}$$

$$\Rightarrow -\frac{1}{3}\ln|1-3y|=\tan\left(-\frac{\pi}{4}\right)+C=-1+C$$

$$=-1-1-\frac{1}{3}\ln 3$$

$\Rightarrow$ ln |1 – 3y| = 6 + ln3

$$\Rightarrow \ln\left|\frac{1}{3}-y\right|=6 \Rightarrow \left|\frac{1}{3}-y\right|=e^6 \Rightarrow y=\frac{1}{3}\pm e^6$$

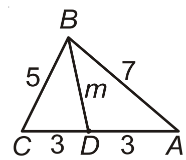
6. Consider a triangular plot *ABC* with sides *AB* = 7 m, *BC* = 5 m and *CA* = 6 m. A vertical lamp-post at the mid point *D* of *AC* subtends an angle 30° at *B*. The height (in m) of the lamp-post is

(1) $2\sqrt{21}$ (2) $7\sqrt{3}$

(3) $\frac{2}{3}\sqrt{21}$ (4) $\frac{3}{2}\sqrt{21}$

Answer (3)

Sol.



By Appolonius Theorem,

$$2\left(BD^2+\left(\frac{AC}{2}\right)^2\right)=BC^2+AB^2$$

$$\Rightarrow 2\left(m^2+3^2\right)=25+9 \Rightarrow m=2\sqrt{7}$$

$$\tan 30^\circ=\frac{\text{height of lamppost}}{\text{BD}}$$

$$\Rightarrow \text{Height}=2\sqrt{7}\times\frac{1}{\sqrt{3}}=\frac{2\sqrt{21}}{3}$$

7. Let $I=\int_a^b(x^4-2x^2)dx$. If *I* is minimum then the ordered pair (*a*, *b*) is

(1) $(-\sqrt{2},0)$ (2) $(0,\sqrt{2})$

(3) $(\sqrt{2},-\sqrt{2})$ (4) $(-\sqrt{2},\sqrt{2})$

Answer (4)

Sol.

$$I=\int_a^b\left(x^4-2x^2\right)dx$$

$$\Rightarrow \frac{dI}{dx}=x^4-2x^2=0 \quad \Rightarrow x=0,\pm\sqrt{2}$$

Also, $I=\left[\frac{x^5}{5}-\frac{2x^3}{3}\right]_a^b=x^3\left(\frac{x^2}{5}-\frac{2}{3}\right)$

|*I*| is maximum when $b=-\sqrt{2}$ and $a=\sqrt{2}$

$\therefore$ *I* is minimum when (*a*, *b*) $=\left(-\sqrt{2},\sqrt{2}\right)$

8. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1, 2, 3, ..., 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is

(1) $\frac{13}{36}$ (2) $\frac{15}{72}$

(3) $\frac{19}{36}$ (4) $\frac{19}{72}$

Answer (4)

Sol.

P (7 or 8 is the sum of two dice) = $\frac{11}{36}$

P (7 or 8 is the number of card) = $\frac{2}{9}$

Required probability = $\frac{1}{2}\times\frac{11}{36}+\frac{1}{2}\times\frac{2}{9}$

$= \frac{1}{2}\left(\frac{11+8}{36}\right)=\frac{19}{72}$

9. Consider the quadratic equation $(c-5)x^2-2cx+(c-4)=0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is

(1) 11 (2) 18

(3) 12 (4) 10

Answer (1)

Sol.

$f(0).f(3) > 0$ and $f(0).f(2) < 0$

$\Rightarrow$ $(c-4)(4c-49) > 0$ and $(c-4)(c-24) < 0$

$\Rightarrow$ $c \in (-\infty, 4) \cup \left(\frac{49}{4}, \infty\right)$ and $c \in (4, 24)$

$\Rightarrow$ $c \in \left(\frac{49}{4}, 24\right)$

$\therefore$ $S = \{13, 14, ..., 23\}$

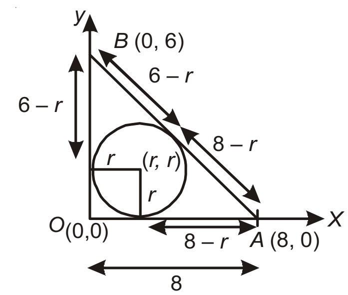
10. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin is

(1) (4, 3) (2) (3, 4)

(3) (4, 4) (4) (2, 2)

Answer (4)

Sol.



$8 - r + 6 - r = 10$

$\Rightarrow$ $2r = 4$

$\Rightarrow$ $r = 4$

$\therefore$ Incentre = (2, 2)

11. If the third term in the binomial expansion of $(1+x^{\log_2 x})^5$ equals 2560, then a possible value of x is

(1) $2\sqrt{2}$ (2) $\frac{1}{4}$

(3) $4\sqrt{2}$ (4) $\frac{1}{8}$

Answer (2)

Sol: Third term of $(1+x^{\log_2 x})^5 = 5_{C_2}(x^{\log_2 x})^2$

given, $5_{C_2}(x^{\log_2 x})^2 = 2560$

$\Rightarrow$ $(x^{\log_2 x})^2 = 256 = (\pm 16)^2$

$\Rightarrow$ $x^{\log_2 x} = 16$ or $x^{\log_2 x} = -16$ (rejected)

$\Rightarrow$ $x^{\log_2 x} = 16 \Rightarrow \log_2 x \log_2 x = \log_2 16 = 4$

$\Rightarrow$ $\log_2 x = \pm 2 \Rightarrow x = 2^2$ or 2^{-2}

$\Rightarrow$ $x = 4$ or $\frac{1}{4}$

12. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then

(1) $Im(z) = 0$ (2) $|z| = \sqrt{\frac{5}{2}}$

(3) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$ (4) $Re(z) = 0$

Answer (No option Matches)

Sol: Let $z_1 = r_1 e^{i\theta}$ and $z_2 = r_2 e^{i\phi}$

$$3|z_1| = 4|z_2| \Rightarrow 3r_1 = 4r_2$$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = \frac{3}{2}\frac{r_1}{r_2} e^{i(\theta-\phi)} + \frac{2}{3}\frac{r_2}{r_1} r^{i(\phi-\theta)}$$

$$= \frac{3}{2}\times\frac{4}{3}(\cos(\theta-\phi) + i\sin(\theta-\phi)) +$$

$$\frac{2}{3}\times\frac{3}{4}[\cos(\theta-\phi) + i\sin(\phi-\theta)]$$

$$= \left(2+\frac{1}{2}\right)\cos(\theta-\phi) + \left(2-\frac{1}{2}\right)\sin(\theta-\phi)$$

$$\therefore\ |z| = \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = \frac{\sqrt{34}}{2}$$

13. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is

(1) 4 : 9 (2) 6 : 7

(3) 10 : 3 (4) 5 : 8

Answer (1)

Sol: $x_1 + x_2 + x_3 + x_5 = 25$

$x_1 + x_2 + x_3 = 1 + 3 + 8 = 12$

$\Rightarrow x_4 + x_5 = 25 - 12 = 13$ ...(1)

$$\frac{\sum_{i=1}^{5} x_i^2}{5} - (5)^2 = 9.2 \Rightarrow \sum_{i=1}^{5} x_i^2 = 5(25 + 9.2)$$

$$= 125 + 46 = 171$$

$$\Rightarrow (1)^2 + (3)^2 + (8)^2 + x_4^2 + x_5^2 = 171$$

$\Rightarrow x_4^2 + x_5^2 = 97$ ...(2)

$\therefore\ 2_4 x_5 = 13^2 - 97 = 72 \Rightarrow x_4 x_5 = 36$ ...(3)

(1) and (3) $\Rightarrow x_4 : x_5 = \frac{4}{9}$ or $\frac{9}{4}$

14. Let $f(x) = \begin{cases} \max\{|x|, x^2\}, & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$

Let S be the set of points in the interval (–4, 4) at which f is not differentiable. Then S

(1) Equals {–2, –1, 0, 1, 2}

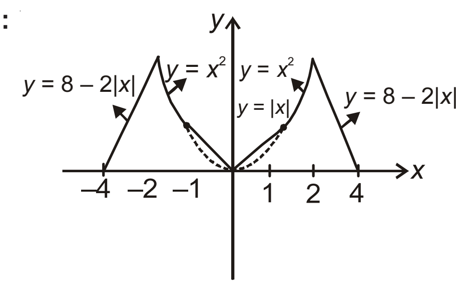
(2) Equals {–2, 2}

(3) Is an empty set

(4) Equals {–2, –1, 1, 2}

Answer (1)

Sol:



Clearly, S = {–2, –1, 0, 1, 2}

15. Consider the statement : "$P(n)$: $n^2 - n + 41$" is prime." Then which one of the following is true?

(1) $P(5)$ is false but $P(3)$ is true

(2) $P(3)$ is false but $P(5)$ is true

(3) Both $P(3)$ and $P(5)$ are false

(4) Both $P(3)$ and $P(5)$ are true

Answer (4)

Sol: $P(n) = n^2 - n + 41$

$\Rightarrow P(3) = 9 - 3 + 41 = 47$

$P(5) = 5^2 - 5 + 41 = 61$

Both 47 and 61 are prime

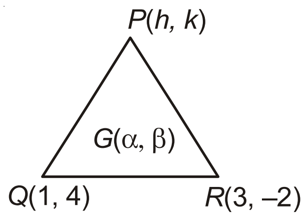
16. A point P moves on the line $2x - 3y + 4 = 0$. If $Q(1, 4)$ and $R(3, -2)$ are fixed points, then the locus of the centroid of ΔPQR is a line

(1) Parallel to y-axis (2) With slope $\frac{3}{2}$

(3) With slope $\frac{2}{3}$ (4) Parallel to x-axis

Answer (3)

Sol:



Let centroid G be (α, β)

we have $3\alpha = 1 + 3 + h \Rightarrow h = 3\alpha - 4$

$3\beta = 4 - 2 + k \Rightarrow k = 3\beta - 2$

but $P(h, k)$ lies on $2x - 3y + 4 = 0$

$\Rightarrow 2(3\alpha - 4) - 3(3\beta - 2) + 4 = 0$

$\Rightarrow 6\alpha - 9\beta - 8 + 6 + 4 = 0$

$\Rightarrow 6\alpha - 9\beta + 2 = 0$

Locus: $6x - 9y + 2 = 0$

Slope $= \frac{6}{9} = \frac{2}{3}$

17. If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a, b, c)?

(1) $\left(\frac{1}{2}, 2, 0\right)$ (2) $\left(\frac{1}{2}, 2, 3\right)$

(3) (1, 1, 0) (4) (1, 1, 3)

Answer (4)

Sol: Note: x-axis is a common normal. Hence all the options are correct for $m = 0$.

Normal to $y^2 = 8ax$ is

$y = mx - 4am - 2am^3$ ...(1)

Normal to $y^2 = 4b(x - c)$ with slope m is

$y = m(x - c) - 2bm - bm^3$ ...(2)

(1) and (2) are identical

$\Rightarrow 4am + 2am^3 = cm + 2bm + bm^3$

$\Rightarrow 4a + 2am^2 = c + 2b + bm^2$ or $m = 0$

$\Rightarrow 4a - c - 2b = (b - 2a)m^2$

or (X-axis is common normal always)

$$\Rightarrow m^2 = \frac{4a - 2b - c}{b - 2a} = -2 - \frac{c}{b - 2a}$$

$$m^2 \geq 0 \Rightarrow -2 - \frac{c}{b - 2a} \geq 0 \Rightarrow \frac{c}{b - 2a} + 2 \leq 0$$

only (1, 1, 3) satisfies $\frac{c}{b - 2a} + 2 \leq 0$

18. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is

(1) $\frac{5\pi}{4}$ (2) $\frac{\pi}{2}$

(3) $\frac{3\pi}{8}$ (4) π

Answer (2)

Sol: $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$$

$$\Rightarrow \cos^2 2\theta\left(1 - \cos^2 2\theta\right) = \frac{1}{4} \quad ...(1)$$

But $$(\cos^2 2\theta)(1 - \cos^2 2\theta) \leq \left(\frac{\cos^2 2\theta + (1 - \cos^2 2\theta)}{2}\right)^2$$

$$= \frac{1}{4} \quad ...(2)$$

Equation (1) and (2)

$\Rightarrow \cos^2 2\theta = 1 - \cos^2 2\theta$

$\Rightarrow \cos^2 2\theta = \frac{1}{2}$

$\Rightarrow \cos 2\theta = \pm \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8}$

$$\text{Sum} = \frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}$$

19. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x - y = 2$ is

(1) $x - y + 7 = 0$ (2) $x - y + 1 = 0$

(3) $x - y - 3 = 0$ (4) $x - y + 9 = 0$

Answer (2)

Sol: Slope, $m = 1$

$$\frac{x^2}{5} - \frac{y^2}{4} = 1$$

$a^2 = 5, \quad b^2 = 4$

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$$= x \pm \sqrt{5-4}$$

$\Rightarrow y = x \pm 1$

20. Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$, $x \in R$. Then $f(2)$ equals

(1) 8 (2) –4

(3) –2 (4) 30

Answer (3)

Sol: Let $f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b \quad \Rightarrow f'(1) = 3 + 2a + b$

$f''(x) = 6x + 2a \quad \Rightarrow f''(2) = 12 + 2a$

$f'''(x) = 6 \quad \Rightarrow f'''(3) = 6$

$\because f(x) = x^3 + f'(1)x^2 + f''(2)\,x + f'''(3)$

$\Rightarrow f'(1) = a \Rightarrow 3 + 2a + b = a \Rightarrow a + b = -3 \quad \text{...(1)}$

$\Rightarrow f''(2) = b \Rightarrow 12 + 2a = b \Rightarrow 2a - b = -12 \quad \text{...(2)}$

From (1) and (2)

$3a = -15 \Rightarrow a = -5 \Rightarrow b = 2$

$\Rightarrow f(x) = x^3 - 5x^2 + 2x + 6$

$\Rightarrow f(2) = 8 - 20 + 4 + 6 = -2$

21. If the system of equations

$x + y + z = 5$

$x + 2y + 3z = 9$

$x + 3y + \alpha z = \beta$

has infinitely many solutions, then $\beta - \alpha$ equals

(1) 18 (2) 21

(3) 8 (4) 5

Answer (3)

Sol.

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = 2\alpha - 9 - \alpha + 3 + 1 = \alpha - 5$$

$$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & \alpha \end{vmatrix} = 5(2\alpha - 9) - 1(9\alpha - 3\beta) + (27 - 2\beta)$$

$$= 10\alpha - 45 - 9\alpha + 3\beta + 27 - 2\beta$$

$$= \alpha + \beta - 18$$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 9 & 3 \\ 1 & \beta & \alpha \end{vmatrix} = 9\alpha - 3\beta - 5(\alpha - 3) + 1(\beta - 9)$$

$$= 9\alpha - 3\beta - 5\alpha + 15 + \beta - 9$$

$$= 4\alpha - 2\beta + 6$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \beta \end{vmatrix} = 2\beta - 27 - \beta + 9 + 5 = \beta - 13$$

$\Rightarrow$ for infinitely many solutions

$\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 0$

$\Rightarrow \alpha = 5, \beta = 13 \Rightarrow \beta - \alpha = 8$

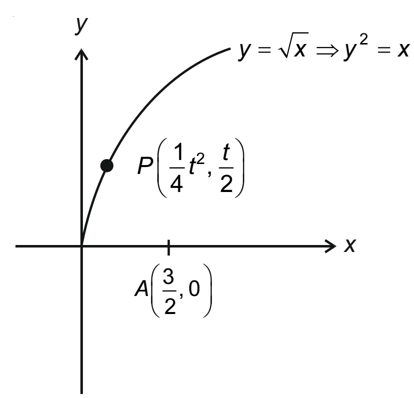
22. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}, (x > 0)$, is

(1) $\frac{3}{2}$ (2) $\frac{5}{4}$

(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{\sqrt{5}}{2}$

Answer (4)

Sol.



Normal at P to $y^2 = x$ is

$$y = -tx + \frac{1}{2}t + \frac{1}{4}t^3 \quad \text{...(1)}$$

for minimum AP, (1) passes through $A\left(\frac{3}{2}, 0\right)$

$$\Rightarrow \frac{-3}{2}t+\frac{t}{2}+\frac{t^3}{4}=0 \Rightarrow -4t+t^3=0$$

$$\Rightarrow t(t^2-4)=0 \Rightarrow t=-2,0,2 \quad \because t\ge 0 \Rightarrow t=0,2$$

if $t = 0$, $P(0, 0) \Rightarrow AP = \frac{3}{2}$

if $t = 2$, $P(1, 1) \Rightarrow AP = \frac{\sqrt{5}}{2}$

$\Rightarrow$ Shortest distance $\left(\frac{3}{2},0\right)$ and $y=\sqrt{x}$ is $\frac{\sqrt{5}}{2}$

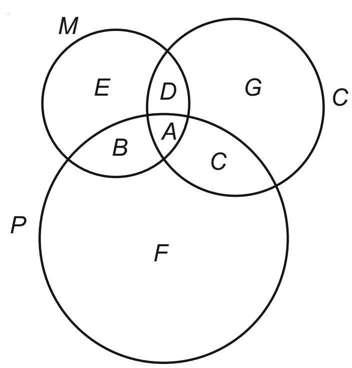
23. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :

(1) 1 (2) 38

(3) 102 (4) 42

Answer (2)

Sol.



$A = \{30, 60, 90, 120\} \Rightarrow n(A) = 4$

$B = \{6n: n \in N, 1 \le n \le 23\} - A \Rightarrow n(B) = 19$

$C = \{15n: n \in N, 1 \le n \le 9\} - A \Rightarrow n(C) = 5$

$D = \{10n: n \in N, 1 \le n \le 14\} - A \Rightarrow n(D) = 10$

$n(E) = 70 - n(A) - n(B) - n(D) = 70 - 33 = 37$

$n(F) = 46 - n(A) - n(B) - n(C) = 46 - 28 = 18$

$n(G) = 28 - n(A) - n(C) - n(D) = 28 - 19 = 9$

$\Rightarrow$ Number of required students

$= 140 - (4 + 19 + 5 + 10 + 37 + 18 + 9)$

$= 140 - 102 = 38$

24. Let $n \ge 2$ be a nutural number and $0 < \theta < \pi/2$.

Then $\int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$ is equal to

(where C is a constant of integration)

(1) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

(2) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(3) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(4) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

Answer (3)

Sol.

$$I = \int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$$

Let $\sin\theta = t$

$\cos\theta d\theta = dt$

$$I = \int \frac{(t^n - t)^{\frac{1}{n}}}{t^{n+1}}dt$$

$$= \int \frac{\left(1-\frac{1}{t^{n-1}}\right)^{\frac{1}{n}}}{t^n}dt = \int t^{-n}(1-t^{1-n})^{\frac{1}{n}}dt$$

Let $1 - t^{1-n} = u$

$-(1 - n)t^{-n}dt = du$

$$t^{-n}dt = \frac{du}{n-1}$$

$$I = \int u^{\frac{1}{n}}\cdot\frac{du}{n-1} = \frac{1}{n-1}\cdot\frac{u^{\frac{1}{n}+1}}{\frac{1}{n}+1}$$

$$= \frac{n}{n^2-1}u^{\frac{n+1}{n}}+C$$

$$= \frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$$

25. The plane passing through the point (4, –1, 2) and parallel to the lines $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ also passes through the point :

(1) (–1, –1, –1) (2) (–1, –1, 1)

(3) (1, 1, 1) (4) (1, 1, –1)

Answer (3)

Sol. Equation of required plane is

$$\begin{vmatrix} x-4 & y+1 & z-2 \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}=0$$

$$(x-4)(-3-4)-(y+1)(9-2)+(z-2)(6+1)=0$$

$$-7(x-4)-7(y+1)+7(z-2)=0$$

$$x-4+y+1-z+2=0$$

$$x+y-z-1=0$$

∴ Point (1, 1, 1) lies on the plane

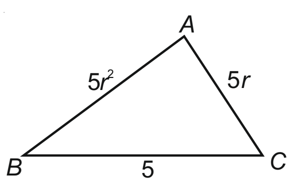
26. If 5, 5r, 5r^2 are the lengths of the sides of a triangle, then r **cannot** be equal to :

(1) $\frac{3}{4}$ (2) $\frac{7}{4}$

(3) $\frac{3}{4}$ (4) $\frac{5}{4}$

Answer (2)

Sol.



For ΔABC is possible if

$5 + 5r > 5r^2$

$1 + r > r^2$

$r^2 - r - 1 < 0$

$$\left(r-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\left(r-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)<0$$

$$\therefore\ r\in\left(\frac{-\sqrt{5}+1}{2},\frac{\sqrt{5}+1}{2}\right)$$

$$\therefore\ r\neq\frac{7}{4}$$

27. For each $t\in R$, let $[t]$ be the greatest integer less than or equal to t. Then,

$$\lim_{x\to1+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{x}{2}[1-x]\right)}{|1-x|[1-x]}$$

(1) Equals 0 (2) Equals 1

(3) Equals –1 (4) Does not exist

Answer (1)

Sol.

$$\lim_{x\to1^+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

$$=\lim_{h\to0}\frac{(1-|1+h|+\sin|1-1-h|\sin\frac{\pi}{2}[1-1-h]}{|1-1-h|[1-1-h]}$$

$$=\lim_{h\to0}\frac{(-h+\sin h)\sin\left(-\frac{\pi}{2}\right)}{h(-1)}$$

= 0

28. Let $\vec{a}=2\hat{i}+\lambda_1\hat{j}+3\hat{k},\ \vec{b}=4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\vec{c}=3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b}=2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is :

(1) (1, 3, 1) (2) (1, 5, 1)

(3) $\left(\frac{1}{2},4,-2\right)$ (4) $\left(-\frac{1}{2},4,0\right)$

Answer (4)

Sol. $\because\ \bar{b}=2\bar{a}$

$\therefore\ 4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}=4\hat{i}+2\lambda_1\hat{j}+6\hat{k}$

$\therefore\ 3-\lambda_2=2\lambda_1$ ...(1)

$\because\ \bar{a}$ is perpendicular to $\bar{c}$

$\therefore\ 6+6\lambda_1+3(\lambda_3-1)=0$

$2+2\lambda_1+\lambda_3-1=0$

$2\lambda_1+\lambda_3+1=0$

$\therefore\ \lambda_3=-2\lambda_1-1$ ...(2)

from eq (1) and (2) one of possible value of

$\lambda_1=-\frac{1}{2},\lambda_2=4$ and $\lambda_3=0$

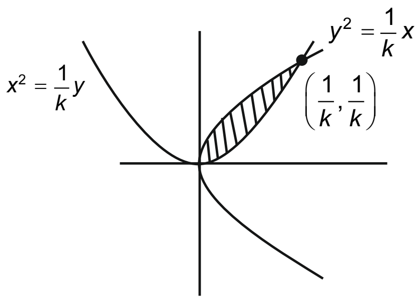
29. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k > 0)$, is 1 square unit. Then k is :

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$

(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{2}{\sqrt{3}}$

Answer (2)

Sol.



Area of shaded region = 1.

$$\therefore \int_0^{\frac{1}{k}} \left(\frac{\sqrt{x}}{\sqrt{k}} - kx^2\right) dx = 1$$

$$\left(\frac{1}{\sqrt{k}} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^{\frac{1}{k}} - \left(k \cdot \frac{x^3}{3}\right)_0^{\frac{1}{k}} = 1$$

$$\frac{2}{3\sqrt{k}} \cdot \frac{1}{k^{\frac{3}{2}}} - \frac{k}{3k^3} = 1$$

$$\frac{2}{3k^2} - \frac{1}{3k^2} = 1$$

$3k^2 = 1$

$k = \pm\frac{1}{\sqrt{3}}$ but $k > 0$

$\therefore k = \frac{1}{\sqrt{3}}$

30. Let A be a point on the line $\vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$ and $B(3, 2, 6)$ be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane $x - 4y + 3z = 1$ is :

(1) $\frac{1}{4}$

(2) $\frac{1}{2}$

(3) $\frac{1}{8}$

(4) $-\frac{1}{4}$

Answer (1)

Sol. $\because$ A be a point on given line.

$\therefore$ Position vector of

$$A = \overline{OA} = \vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$$

position vector of $B = \overline{OB} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore \overline{AB} = \overline{OB} - \overline{OA}$

$= (3\mu+2)\hat{i} + (3-\mu)\hat{j} + (4-5\mu)\hat{k}$

equation of plane is: $x - 4y + 3z = 1$

$\because \overline{AB}$ is parallel to plane.

$\therefore 1(3\mu+2) - 4(3-\mu) + 3(4-5\mu) = 0$

$3\mu + 2 - 12 + 4\mu + 12 - 15\mu = 0$

$2 - 8\mu = 0$

$\mu = \frac{1}{4}$

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