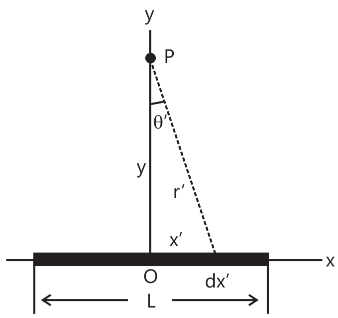
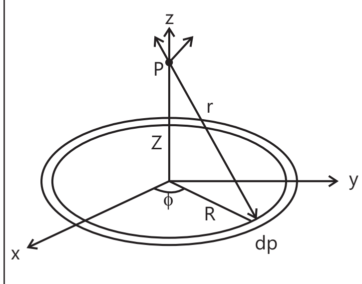
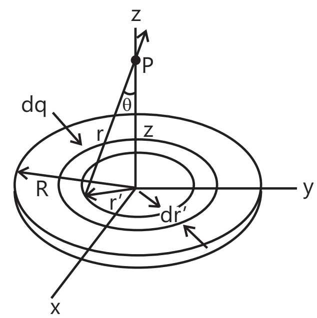
PROBLEM-SOLVING TACTICS

Below we illustrate how the above methodologies can be employed to compute the electric potential for a line of charge, a ring of charge and a uniformly charged disk.

	Charged Rod	**Charged Ring**	**Charged disk**
Figure	 **Figure 19.49**	 **Figure 19.50**	 **Figure 19.51**
(2) Express dq in terms of charge density	$dq = \lambda dx'$	$dq = \lambda dl$	$dq = \sigma dA$
(3) Substitute dq into expression for dV	$dV = k_e \frac{\lambda dx'}{r}$	$dV = k_e \frac{\lambda dl}{r}$	$dV = k_e \frac{\sigma dA}{r}$
(4) Rewrite r and the differential element in terms of the appropriate coordinates	dx' $r = \sqrt{x'^2 + y^2}$	$dl = R d\phi$ $r = \sqrt{R^2 + z^2}$	$dA = 2\pi r' dr'$ $r = \sqrt{r'^2 + z^2}$
(5) Rewrite dV	$dV = k_e \frac{\lambda dx'}{(x'^2 + y^2)^{1/2}}$	$dV = k_e \frac{\lambda R d\phi'}{(R^2 + z^2)^{1/2}}$	$dV = k_e \frac{2\pi\sigma r' dr'}{(r'^2 + z^2)^{1/2}}$
(6) Integrate to get V	$V = \frac{\lambda}{4\pi\varepsilon_0} \int_{-\ell/2}^{\ell/2} \frac{dx^1}{\sqrt{x'^2 + y^2}}$ $= \frac{\lambda}{4\pi\varepsilon_0} \ln \left[\frac{(\ell/2) + \sqrt{(\ell/2)^2 + y^2}}{-(\ell/2) + \sqrt{(\ell/2)^2 + y^2}} \right]$	$V = k_e \frac{R\lambda}{(R^2 + z^2)^{1/2}} \oint d\phi'$ $= k_e \frac{(2\pi R\lambda)}{\sqrt{R^2 + z^2}}$ $= k_e \frac{Q}{\sqrt{R^2 + z^2}}$	$V = k_e 2\pi\sigma \int_0^R \frac{r' dr'}{(r'^2 + z^2)^{1/2}}$ $= 2k_e \pi\sigma \left(\sqrt{z^2 + R^2} - \lvert z \rvert \right)$ $= \frac{2k_e Q}{R^2} \left(\sqrt{z^2 + R^2} - \lvert z \rvert \right)$
Derive E from V	$E_y = -\frac{\partial V}{\partial y}$ $= \frac{\lambda}{2\pi\varepsilon_0 y} \frac{\ell/2}{\sqrt{(\ell/2)^2 + y^2}}$	$E_z = -\frac{\partial V}{\partial z} = \frac{k_e Q_Z}{\left(R^2 + z^2\right)^{3/2}}$	$E_z = -\frac{\partial V}{\partial z}$ $= \frac{2k_e Q}{R^2} \left(\frac{z}{\lvert z \rvert} - \frac{z}{\sqrt{z^2 + R^2}} \right)$
Point- charge limit for E	$E_y \approx \frac{k_e Q}{y^2} \quad y >> \ell$	$E_z \approx \frac{k_e Q}{z^2} \; z >> R$	$E_z \approx \frac{k_e Q}{z^2} \; z >> R$

For any given combination, one may proceed as follows:

Step 1: Identify the two points between which the equivalent capacitance is to be calculated. Call any one of them as P and the other as N.

Step 2: Connect (mentally) a battery between P and N with the positive terminal connected to P and the negative terminal to N. Send a charge +Q from the positive terminal of the battery.

Step 3: Write the charges appearing on each of the plates of the capacitors. The charge conservation principle may be used. The facing surfaces of a capacitor will always have equal and opposite charges. Assume variables Q_1, Q_2 ..., etc., for charges wherever needed.

Step 4: Take the potential of the negative terminal N to be zero and that of the positive terminal P to be V. Write the potential of each of the plates. If necessary, assume variables V_1, V_2....

Step 5: Write the capacitor equation Q = CV for each capacitor. Eliminate Q_1, Q_2 ...and V_1, V_2...., etc., to obtain the equivalent capacitance C=Q/V.

FORMULAE SHEET

S. No	FORMULA
1.	$q = CV$
2.	$\varepsilon_0 \oint \vec{E}.d\vec{A} = q.$
3.	$V_1 - V_2 = \int_i^f \vec{E}.d\vec{S}.$
4.	$V = \int_-^+ Eds = E\int_0^d ds = Ed$
5.	$q = \varepsilon_0 EA.$
6.	$C = \frac{\varepsilon_0 A}{d}$ (parallel-plate capacitor)
7.	$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$ (cylindrical capacitor)
8.	$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$ (spherical capacitor)
9.	$C = 4\pi\varepsilon_0 R$ (isolated sphere)
10.	$C_{eq} = \sum_{j=1}^{n} C_J$ (n capacitors in parallel)
11.	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_J}$ (n capacitors in series)

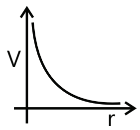
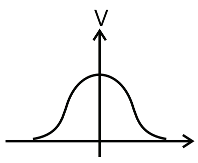
S. No	FORMULA
12.	$U = \frac{1}{2}CV^2$ (potential energy)
13.	$U = \frac{q^2}{2C}$ (potential energy)
14.	$u = \frac{1}{2}\varepsilon_0 E^2$ (energy density)
15.	$E = \frac{q}{4\pi K\varepsilon_0 r^2}$
16.	$\varepsilon_0 \oint K\vec{E}.d\vec{A} = q$ (Gauss' law with dielectric).
17.	Force on a Dielectric Slab inside a Capacitor $F = \frac{\varepsilon_0 bV^2(K-1)}{2d}$

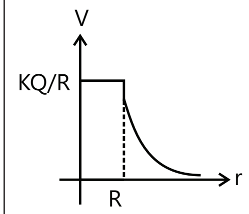
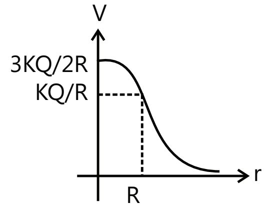
Electric Potential Formulae

S. No	Term	Description
1	Electric Potential energy	ΔU = -W Where ΔU = Change in potential energy and W= Work done by the electric lines of force. For a system of two particles $U(r) = q_1q_2 / 4\pi\varepsilon r$ Where r is the separation between the charges. We assume U to be zero at infinity. Similarly for a system of n charges U = Sum of potential energy of all the distinct pairs in the system For example for three charges $U = (1/4\pi\varepsilon)(q_1q_2/r_{12} + q_2q_3/r_{23} + q_1q_3/r_{13})$
2	Electric PE of a charge	= qV where V is the potential.
3	Electric Potential	Like Electric field intensity is used to define the electric field; we can also use Electric Potential to define the field. Potential at any point P is equal to the work done per unit test charge by the external agent in moving the test charge from the reference point (without Change in KE) $V_p = W_{ext} / q$. So for a point charge V0 = $Q / 4\pi\varepsilon r$ Where r is the distance of the point from charge.

S. No	Term	Description
4.	Some points about Electric potential	1. It is scalar quantity 2. Potential at a point due to system of charges will be obtained by the summation of potential of each charge at that point $V = V_1 + V_2 + V_3 + V_4$ 3. Electric forces are conservative forcew so work done by the electric force between two points is independent of the path taken 4. $V_2 - V_1 = -\int E.dr$
		5. In Cartesian coordinates system $dV = -E.dr;\ dV = -(E_x\,dx + E_y\,dy + E_z\,dz)$ So $E_x = \partial V / \partial x$, $E_y = \partial V / \partial y$ and $E_z = \partial V / \partial z$, Also $E = \left[\left(\partial V / \partial x\right)i + \left(\partial V / \partial y\right)i + \left(\partial V / \partial z\right)k\right]$ 6. Surface where electric potential is same everywhere is called equipotential surface. Electric field components parallel to equipotential surface are always zero.
5	Electric dipole	A combination of two charges +q and –q separated by a distance d has a dipole moment p = qd, where d is the vector joining negative to positive charge.
6	Electric potential due to dipole	$V = (1/4\pi\varepsilon) \times (p\cos\theta / r^2)$ Where r is the distance from the center and θ is angle made by the line from the axis of dipole.
7	Electric field due to dipole	$E_\theta = (1/4\pi\varepsilon)(p\sin\theta / r^3);\ E_r = (1/4\pi\varepsilon) \times (2p\cos\theta / r^3)$ Total $E = \sqrt{E_\theta^{\,2} + E_r^{\,2}} = (p/4\pi\varepsilon r^3)(\sqrt{(3\cos^2\theta + 1})$ Torque on dipole = p × E Potential Energy U = –p.E
8	Few more points	1. $\int E.dl$ over closed path is zero 2. Electric potential in the spherical charge conductor is $Q / 4\pi\varepsilon R$ where R is the radius of the shell and the potential is same everywhere in the conductor. 3. Conductor surface is a equipotential surface

Electric potential due to various charge distributions

Name/Type	Formula	Note	Graph
Point Charge	$\frac{Kq}{r}$	• q is source charge • r is the distance of the point from the point charge.	
Ring (uniform/non uniform charge distribution)	At centre, $\frac{KQ}{R}$ At axis, $\frac{KQ}{\sqrt{R^2 + x^2}}$	• Q is source charge • x is distance of the point from centre.	

Name/Type	Formula	Note	Graph
Uniformly charged hollow conducting/ non - conducting sphere or solid conducting sphere	For $r \geq R, V = \frac{KQ}{R}$ For $r \leq R, V = \frac{KQ}{R}$	• R is radius of sphere • r is distance of the point from centre of the sphere • Q is total charge ($(= \sigma 4\pi R^2)$)	
Uniformly charged solid non - conducting sphere (insulating material)	For $r \geq R$, $V = \frac{KQ}{r}$ For $r \leq R$, $V = \frac{KQ(3R^2 - r^2)}{2R^3}$ $= \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$	• R is radius of sphere • r is distance of point from centre of the sphere. • Q is total charge $(= \rho \frac{4}{3}\pi R^3)$ • $V_{centre} = \frac{3}{2} V_{surface}$ • Inside sphere potential varies parabolically. • Outside potential varies hyperbolically.	
Line charge	Not defined	• Absolute potential is not defined • Potential difference between two points is given by formula $V_B - V_A = -2K\lambda \ln(r_B / r_A)$, where lambda is the charge per unit length	
Infinite nonconducting thin sheet	Not defined	• Absolute potential Is not defined • Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{2\varepsilon_0}(r_B - r_A)$, where sigma is the charge density	
Infinite charged conducting thin sheet	Not defined	• Absolute potential is not defined • Potential difference between two point is given by formula $V_B - V_A = -\frac{\sigma}{\varepsilon_0}(r_B - r_A)$, where sigma is the charge density	

Electric dipole moment: $\vec{p} = qd\hat{z}$, where two charges of charge $\pm q$ are placed along the z axis at $z = \pm\frac{d}{2}$

Electric dipole field: Along the z axis (z>>d): $\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{p}{|z|^3}\hat{z}$, in the +z direction.

Along the x axis(x>>d): $\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{p}{|x|^3}\hat{x}$ in the +x direction.

Along the x axis(y>>d): $\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{p}{|y|^3}\hat{y}$ in the +y direction.