Solved Examples

JEE Main/Boards

Example 1: Find the coefficient of $\frac{1}{y^2}$ in $\left(\frac{c^3}{y^2}+y\right)^{10}$

Sol: By using formula of finding general term we can easily get coefficient of $\frac{1}{y^2}$.

In the binomial expansion, (r + 1)th term is

$$T_{r+1} = {}^{n}C_r\left(y\right)^r\left(\frac{c^3}{y^2}\right)^{n-r} \quad : n = 10$$

$$\Rightarrow T_{r+1} = {}^{10}C_r\left(y\right)^r\left(c^3\right)^{10-r}\left(\frac{1}{y^2}\right)^{10-r}$$

$$= {}^{10}C_r\, c^{30-3r}y^{3r-20} \qquad \text{...(i)}$$

$$\therefore 3r - 20 = -2; r = 6$$

$\therefore$ 7th term will contain y^{-2} and from (i) the coefficient of y^{-2} is = 210 c^{12}

Example 2: Use Binomial theorem to find the value of $(10.1)^5$.

Sol: After reducing $(10.1)^5$ into the form of $(10 + 0.1)^n$ we can use binomial expansion to get required result.

$$(10.1)^5 = (10 + 0.1)^5$$

$$= \left(10\right)^5 + {}^5C_1\left(10\right)^4\left(0.1\right) + {}^5C_2\left(10\right)^3\left(0.1\right)^2$$

$$+ {}^5C_3\left(10\right)^2\left(0.1\right)^3 + {}^5C_4 10\left(0.1\right)^4 + \left(0.1\right)^5$$

$$= \left(10\right)^5 + 5\left(10^3\right) + 10\left(10\right)^3\left(0.01\right) + 10\left(10\right)^2$$

$$\left(0.001\right) + 5\left(10\right)\left(0.0001\right) + \left(0.00001\right)$$

$$= 100000 + 5000 + 100 + 1 + 0.005 + 0.00001$$

$$= 105101.00501$$

Example 3: Find the middle term(s) in the expansion of $\left(2x^2 - \frac{1}{x}\right)^7$.

Sol: Since n = 7 is a odd number. Therefore, find the $\frac{n+1}{2}$ th and $\frac{n+3}{2}$ th term.

The total number of terms in the expansion are 8. Therefore $\frac{7+1}{2}$ th and $\frac{7+3}{2}$ th i.e. 4th and 5th terms are the two middle terms. 4th term = ${}^7C_3\left(2x^2\right)^{7-3}\left(-\frac{1}{x}\right)^3$

$$= -\frac{7!}{3!4!}16x^{8-3} = -560x^5$$

and 5th term = ${}^7C_4\left(2x^2\right)^{7-4}\left(\frac{-1}{x}\right)^4 = 280x^2$

Hence the two middle terms are $-560x^5$ and $280x^2$.

Example 4: The coefficient of (r – 1)th, rth and (r + 1)th term in the expansion of $(x + 1)^n$ are in the ratio 1:3:5. Find n and r.

Sol: In this problem, by using the formula of general term we will get the equation of given terms and by taking ratios of these terms we can get the value of n and r.

Coefficient of (r – 1)th term is ${}^nC_{r-2}$

Coefficient of rth term is ${}^nC_{r-1}$

Coefficient of (r + 1)th term is ${}^nC_r$

Coefficient are in ratio of 1 : 3 : 5

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{3} \text{ and } \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{5}$$

$$\text{or } \frac{r-1}{n-r+2} = \frac{1}{3} \text{ and } \frac{r}{n-r+1} = \frac{3}{5}$$

i.e. n – 4r + 5 = 0 and 3n – 8r + 3 = 0

Solving both we get n = 7 & r = 3

Example 5: Find the remainder when $27^{10} + 7^{51}$ is divided by 10

Sol: We can obtain the remainder by reducing $27^{10} + 7^{51}$ into the form of $10\lambda + a$, where λ is any integer and a is an integer less than 10.

We have $27^{10} = 3^{30} = 9^{15} = (10 - 1)^{15}$

$7^{51} = 7.7^{50} = 7.(49)^{25} = 7\,(50 - 1)^{25}$

$27^{10} = 10m_1$ ...(i)

$7^{51} = 7(50 - 1)^{25} = 10m_2 - 7$ ...(ii)

Adding (i) and (ii)

$27^{10} + 7^{51} = (10m_1 - 1) + (10m_2 - 7) = 10m_1 + 10m_2 - 8$

$= 10m_1 + 10m_2 - 10 + 2$

Thus, the remainder is 2 when $27^{10} + 7^{51}$ is divided by 10.

Example 6: If A be the sum of odd numbered terms and B the sum of even numbered terms in the expansion of $(x + a)^n$ prove that $A^2 - B^2 = (x^2 - a^2)^n$

Sol: Do it yourself.

$$(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a$$

$$x + {}^nC_2x^{n-2}a^2 + ... + {}^nC_na^n = A + B$$

When $A = {}^nC_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4 + ...$

$$B = {}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + {}^nC_5x^{n-5}a^5 + ...$$

$$\therefore (x-a)^n = A - B,\ A^2 - B^2 = (A-B)(A+B)$$

$$= (x-a)^n (x+a)^n = (x^2 - a^2)^n$$

Example 7: If C_r denotes the binomial coefficient ${}^nC_r$, prove that :

$$C_0^2 + C_1^2 + ... + C_n^2 = \frac{2n!}{(n!)^2}.$$

Sol: Multiply the expansion of $(x+1)^n$ and $(1+x)^n$ and compare the coefficients of x^n on both sides.

We know that $(1+x)^n = {}^nC_0 + {}^nC_1x$

$$+{}^nC_2x^2 + ... + {}^nC_{n-1}x^{n-1} + {}^nC_nx^n$$

$$(x+1)^n = {}^nC_0x^n + {}^nC_1x^{n-1}$$

$$+{}^nC_2x^{n-2} + ... + {}^nC_{n-1}x + {}^nC_n$$

Multiplying these equations side by side, we get

$$(1+x)^n (x+1)^n = \left(C_0 + C_1x + C_2x^2 + ... + C_{n-1}x^{n-1} + C_nx^n\right)$$

$$\times\left(C_0x^n + C_1x^{n-1} + C_2x^{n-2} + ... + C_{n-1}x + {}^nC_n\right)$$

Coefficient of x^n on R.H.S. is equal to

$$C_0^2 + C_1^2 + C_2^2 + ... + C_{n-1}^2 + C_n^2$$

Coefficient of x^n in L.H.S. is ${}^{2n}C_n = \frac{2n!}{n!n!}$.

This proves the required identity.

Example 8: If $\left(1 + x + x^2\right)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$ show that

(i) $a_0 + a_1 + a_2 + ... + a_{2n} = 3^n$

(ii) $a_0 - a_1 + a_2 - a_3 + ... + a_{2n} = 1$

(iii) $a_0 + a_3 + a_6 + ... = 3^{n-1}$

Sol: By putting x = 1, -1, and ω, ω^2

Respectively in the expansion of $(1 + x + x^2)^n$ we will get the result.

Given $(1 + x + x^2)^n$

$$= a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n} \qquad ....(i)$$

(i) Putting x = 1, we get

$$3^n = a_0 + a_1 + a_2 + ... + a_{2n} \quad .....(A)$$

(ii) Putting x = –1 in (i), we get

$$1 = a_0 - a_1 + a_2 - a_3 ..... + a_{2n}$$

(iii) Putting $x = \omega, \omega^2$ successively in (i), we get $0 = a_0 + a_1\omega + a_2\omega^2 + a_3$

$+a_4\omega + a_5\omega^2 + ... + a_{2n}\omega^{2n}$ .....(B) $0 = a_0 + a_1\omega^2 + a_2\omega + a_3$

$+a_4\omega^2 + a_5\omega + a_6 + ... + a_{2n}\omega^{4n}$ ...(C)

Adding (A), (B) and (C) we have

$$3^n = 3\left(a_0 + a_3 + a_6 + ...\right)$$

$$\therefore a_0 + a_3 + a_6 + ... = 3^{n-1}$$

Example 9: If $(1+x)^n = C_0 + C_1x +$

$$C_2x^2 + C_3x^3 + ... + C_nx^n$$

then prove that $C_1^2 + 2C_2^2 + 3C_3^2 + ... + nC_n^2 = \frac{(2n-1)!}{\left((n-1)!\right)^2}$

Sol: Expanding $(1+x)^n$ and $(x+1)^n$ and multiplying these two expansion and comparing the coefficient of x^{n-1} we will prove above equation.

Given $(1+x)^n = C_0 + C_1x +$

$$C_2x^2 + C_3x^3 + ... + C_nx^n$$

Differentiating both sides w. r. t. to x, we get

$$n(1+x)^{n-1} = 0 + C_1 + 2C_2x + 3C_3x^2 + ... + nC_nx^{n-1}$$

$$\Rightarrow n(1+x)^{n-1} = C_1 + 2C_2x$$

$$+3C_2x^2 + ... + nC_nx^{n-1} \qquad ....(i)$$

and $(x+1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2}$

$$+C_3x^{n-3} + C_4x^{n-4} + ... + C_n \qquad ....(ii)$$

Multiplying (i) and (ii), we get

$$n(1+x)^{2n-1} = \left(C_1 + 2C_2x + 3C_3x^2 + ... + nC_nx^{n-1}\right)$$

$$\times\left(C_0x^n + C_1x^{n-1} + C_2x^{n-2} + C_3x^{n-3} + ... + C_n\right) \qquad ....(iii)$$

Now, coefficient of x^{n-1} on R.H.S.

$= C_1^2 + 2C_2^2 + 3C_3^2 + ... + nC_n^2$ and coefficient of x^{n-1} on

L.H.S. = $n.^{2n-1}C_{n-1}$

$$= n\frac{(2n-1)!}{(n-1)!n!} = \frac{(2n-1)!}{(n-1)!(n-1)!} = \frac{(2n-1)!}{\left[\left((n-1)!\right)^2\right]}$$

But (iii) is an identity, therefore the coefficient of x^{n-1} in R.H.S. = coefficient of x^{n-1} in R.H.S.

$$\Rightarrow C_1^2 + 2C_2^2 + 3C_3^2 + ... + nC_n^2 = \frac{(2n-1)!}{\left((n-1)!\right)^2}$$

Example 10: Find the numerically greatest term in the expansion of $(3 - 5x)^{15}$ when x = 1/5.

Sol: Follow the algorithm for the greatest term.

Using standard notations w.r.t. $(x + a)^n$

$$\frac{n+1}{1+\left|\frac{x}{a}\right|} = \frac{16}{1+\left|\frac{3}{(-1)}\right|} = 4$$

T_4 and T_5 are numerically equal to each other and are greater than any other term.

Example 11: If $(1 + x + x^2) = a_0 + a_1x + a_2x^2 + a_3x^3 ... + a_{2n}x^{2n}$

Then show that

$$a_0 + a_3 + a_6 + ... = a_1 + a_4 + a_7 + ... = 3^{n-1}.$$

Sol: By Putting x = 1 ω, ω² respectively in the given equation and adding these values we can prove it.

$$3^n = a_0 + a_1 + a_2 + a_3 + a_4 + ... \qquad ...(i)$$

$$0 = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + a_4\omega^4 + ... \qquad ....(ii)$$

Because $1 + \omega + \omega^2 = 0$

$$0 = a_0 + a_1\omega^2 + a_2\omega^4 + a_3\omega^6 + a_4\omega^8 + ... \qquad ...(iii)$$

Adding these

$$3^n = 3(a_0) + a_1\left(1+\omega+\omega^2\right) + a_2$$

$$\left(1+\omega^2+\omega^4\right) + a_3\left(1+\omega^3+\omega^6\right)$$

$$+.... = 3\left(a_0 + a_3 + a_6 + ...\right)$$

$$\therefore a_0 + a_3 + a_6 + ... = 3^{n-1}$$

From (i) + (ii) × ω² (iii) × ω,+we get,

$$3^n + 0\times\omega^2 + 0\times\omega$$

$$= a_0\left(1+\omega^2+\omega\right) + a_1\left(1+\omega^3+\omega^3\right)$$

$$+a_2\left(1+\omega^4+\omega^5\right) + a_3\left(1+\omega^5+\omega^7\right)$$

$$+a_4\left(1+\omega^6+\omega^9\right) + ...$$

$$\therefore 3^n = 3\left(a_1 + a_4 + a_7 + ...\right)$$

Because coefficient of each is

$1 + \omega + \omega^2 = 0$, using $\omega^3 = 1$

$$\therefore a_1 + a_4 + a_7 + ... = 3^{n-1}$$

Again, from (i) + (ii) ω + (iii) × ω³, we get

$$= 3^n = a_0\left(1+\omega+\omega^2\right) + a_1\left(1+\omega^2+\omega^4\right)$$

$$+a_2\left(1+\omega^3+\omega^3\right) + ... = 3\left(a_2 + a_5 + a_8 + ...\right)$$

Example 12: Sum the series

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1}$$

Sol: Expanding $(1+x)^n$ integrating it from 0 to 1 or by using summation method we will get result.

$$\text{Sum} = \sum_{r=1}^{n+1}\frac{C_{r-1}}{r} = \sum_{r=1}^{n+1}\frac{1}{n+1}.^{n+1}C_r$$

$$= \frac{1}{n+1}\left(^{n+1}C_0 + ^{n+1}C_1 + ... + ^{n+1}C_{r+1} - ^{n+1}C_0\right)$$

$$= \frac{1}{n+1}\left(2^{n+1} - 1\right)$$

Alternative method

$$(1+x)^n = C_0 + C_1 + C_2x^2 + ... + C_nx^n$$

Integrating both sides w.r.t. x from 0 to 1

$$\int_0^1 (1+x)^n\,dx = \int_0^1 \left(C_0 + C_1x + ... + C_nx^n\right)dx$$

$$\frac{2^{n+1}-1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1}$$

Example 13: Find the last three digits of 27^{26}.

Sol: By reducing 27^{26} into the form $(730-1)^n$ and using simple binomial expansion we will get required digits.

We have $27^2 = 729$.

Now $27^{26} = (729)^{13} = (730-1)^{13}$

$$= {}^{13}C_0(730)^{13} - {}^{13}C_1(730)^{12} + {}^{13}C_2(730)^{11}$$

$$-.....-{}^{13}C_{10}(730)^3 - {}^{13}C_{12}(730)^2$$

$$-{}^{13}C_{12}(730)+1$$

$$=1000m+\frac{(13)(12)}{2}(14)^2-(13)(730)+1$$

Where m is a positive integer

= 1000 m + 15288 – 9490 + 1

= 1000 m + 5799

Thus, the last three digits of 17^{256} are 799.

JEE Advanced/Boards

Example 1: Find the coefficient of x^4 in the expansion of

(i) $(1 + x + x^2 + x^3)^{11}$

(ii) $(2 - x + 3x^2)^6$

Sol: By expanding given equation using expansion formula we can get the coefficient x^4 .

(i) $1 + x + x^2 + x^3 = (1 + x) + x^2(1 + x) = (1 + x)(1 + x^2)$

$$\therefore \left(1+x+x^2+x^3\right)^{11} = (1+x)^{11}\left(1+x^2\right)^{11}$$

$$=\left(1+{}^{11}C_1x+{}^{11}C_2x^2+{}^{11}C_3x^3+{}^{11}C_4x^4+.......\right)$$

$$\left(1+{}^{11}C_1x^2+{}^{11}C_2x^4+......\right)$$

To find term in x^4 from the product of two brackets on the right-hand-side, consider the following products terms as

$$1\times{}^{11}C_2x^4+{}^{11}C_2x^2\times{}^{11}C_1x^2+{}^{11}C_4x^4$$

$$=\left[{}^{11}C_2+{}^{11}C_2\times{}^{11}C_1+{}^{11}C_4\right]x^4$$

$$\left[55+605+330\right]x^4 = 990x^4$$

$\therefore$ The coefficient of x^4 is 990.

(ii) $\left(2-x+3x^2\right)^6 = \left[2-x(1-3x)\right]^6$

$$= [2^6 - {}^6C_1 \times 2^5 \times x(1-3x) + {}^6C_2 2^4$$

$$\times x^2(1-3x)^2 - {}^6C_3 2^3 \times x^3(1-3x)^3$$

$$+{}^6C_4 2^2 \times x^4(1-3x)^4 - 2\times{}^6C_5$$

$$\times x^5(1-3x)^5 + {}^6C_6 \times x^6(1-3x)^6]$$

The term in x^4 will come only from the three terms, viz.

(a) ${}^6C_2 \times 2^4 \times x^2(1-3x)^2 = 15\times16x^2\left(1-6x+9x^2\right)$

$\therefore$ The term in x^4 is (15) (16) $(9x^4)$

(b) $-{}^6C_3 2^3 \times x^3(1-3x)^3$

$$= -20\times8\times x^3\left[1-9x+27x^2-27x^3\right]$$

$\therefore$ The term in x^4 is $-20\times(-9)\times(8)x^4$

(c) ${}^6C_4 2^2 x^4(1-3x)^4 = 15\times4x^4\left(1-4\times3x+.......\right)$

$\therefore$ The term in x^4 is $15\times4\times x^4$

$\therefore$ The total term in x^4 is

$$\left[15\times16\times9+20\times8\times9+15\times4\right]\times x^4$$

$$=\left[2160+1440+60\right]x^4 = 3660x^4$$

$\therefore$ The coefficient of x^4 is 3660.

Example 2: Show that $\sum_{r=0}^{n} r(n-r)C_r^2 = n^2.{}^{2n-2}C_n$

Sol: By expanding and differentiating $(1+x)^n$ and $(x+1)^n$ and then multiplying these expansion we can prove given equations by comparing coefficient of x^{n-2} on both side.

We have

$$(1+x)^n = C_0 + C_1x + C_2x + ... + C_nx^n \qquad ....(i)$$

Differentiating both side w.r.t x, we get

$$n(1+x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + ... + {}^nC_nx^n \qquad ... (ii)$$

(i) can also the be written as

$$(1+x)^n = (x+1)^n = C_0x^n$$

$$+C_1x^{n-1} + C_2x^{n-2} + ... + C_{n-1}x + C_n \qquad ....(iii)$$

Differentiating both sides w.r.t. x, we get

$$n(1+x)^{n-1} = nC_0x^{n-1} + (n-1)$$

$$C_1x^{n-2} + (n-2)C_2x^{n-3} + ... + C_{n-1} \qquad ....(iv)$$

Multiplying (ii) and (iv), we have

$$n^2\left(1+x\right)^{n-1}\left(x+1\right)^{n-1} = n^2\left(1+x\right)^{2n-2}$$

$$= [C_1 + 2C_2 + 3C_3x^2 + ... + {}^rC_nx^{n-1}]$$

$$x\ [nC_0x^{n-1} + (n-1)C_1x^{n-2} + (n-2)$$

$$C_2x^{n-3} + ... + C_{n-2}x + C_{n-1}] \qquad ....(v)$$

The coefficient of x^{n-2} on the LHS of (v) is

$$n^2.{}^{2n-2}C_{n-2} = n^2.{}^{2n-2}C_n$$

The coefficient of x^{n-2} on the RHS of (v) is

$$1.\left(n-1\right)C_1^2 + 2.\left(n-2\right)C_2^2 + ... + \left(n-1\right).1C_{n-1}^2$$

$$= \sum_{r=0}^{n-1} r\left(n-r\right)C_r^2 = \sum_{r=0}^{n} r\left(n-r\right)C_r^2$$

Hence, $\sum_{r=0}^{n} r\left(n-r\right)C_r^2 = n^2\left({}^{2n-2}C_n\right)$

Example 3: Prove that

(i) $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$

(ii) $2.C_0 + 2^2.\frac{C_1}{2} + 2^3.\frac{C_2}{3} + ...$
$+ 2^{n+1}.\frac{C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$

(iii) $C_0 - \frac{1}{2}C_1 + \frac{1}{3}C_2 - \frac{1}{4}C_3 + ...$
$+\left(-1\right)^n \frac{C_n}{n+1} = \frac{1}{n+1}$

(iv) $\frac{C_0}{1.2} + \frac{C_1}{2.3} + \frac{C_2}{3.4} + ...$
$+ \frac{C_n}{\left(n+1\right).\left(n+2\right)} = \frac{2^{n+2}-n-3}{\left(n+1\right)\left(n+2\right)}$

(v) $C_0 + \frac{C_2}{3} + \frac{C_4}{5} + ... = \frac{2^n}{n+1}$

Sol: Expand $\left(1+x\right)^n$ and integrate it within the limit 0 to 1, 0 to 2, -1 to 0 and -1 to 1 respectively to prove these equations

$$\left(1+x\right)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + ... + C_nx^n \qquad ....(i)$$

(i) Integrating both sides of equation (i) within limits 0 to 1, we get

$$\int_0^1 (1+x)^n dx =$$

$$\int_0^1 (C_0 + C_1x + C_2x + C_2x^2 + C_3x^3 + ...C_nx^n)\ dx$$

$$\left(\frac{\left(1+x\right)^{n+1}}{n+1}\right)_0^1 = C_0x + C_1\frac{x^2}{2} +$$

$$C_2\frac{x^3}{3} + ... + C_n\frac{x^{n+1}}{n+1}\Bigg]_0^1$$

$$\frac{2^{n+1}-1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1}$$

(ii) Integrating both sides of equation (i) within limits 0 to 2.

$$\int_0^2 \left(1+x\right)^n = \int_0^2 \left(C_0 + C_1x + C_2x^2 + C_3x^3 + ...C_nx^n\right)dx$$

$$\text{or}\left(\frac{\left(1+x\right)^{n+1}}{n+1}\right)_0^2 = \left[C_0x + C_1\frac{x^2}{2} + C_2\frac{x^3}{3} + ... + C_n\frac{x^{n+1}}{n+1}\right]_0^2$$

$$\text{or}\ \frac{3^{n+1}-1}{n+1} = C_0.2 + 2^2. + \frac{C_1}{2} + 2^3.\frac{C_2}{3} + ... + 2^{n+1}.\frac{C_n}{n+1}$$

(iii) Integrating both sides of equation (i) within limits –1 to 0,

$$\int_{-1}^{0} \left(1+x\right)^n dx = \int_{-1}^{0} \left(C_0 + C_1x + C_2x^2 + C_3x^3 + ...C_nx^n\right)dx$$

$$\left(\frac{\left(1+x\right)^{n+1}}{n+1}\right)_{-1}^{0} = C_0x + C_1\frac{x^2}{2} + C_2\frac{x^3}{3} + ... + C_n\frac{x^{n+1}}{n+1}\Big|_{-1}^{0}$$

$$\frac{1}{n+1} - 0 = 0 - \left[-C_0 + \frac{C_1}{2} - \frac{C_2}{3} + ... + (-1)^{n+1}\frac{C_n}{n+1}\right]$$

$$\frac{1}{n+1} = C_0 - \frac{C_1}{2} + \frac{C_2}{3} + ... + \left(-1\right)^n\frac{C_n}{n+1}$$

(iv) General term of L.H.S $= \frac{{}^nC_k}{(k+1)(k+2)}$

$$= \frac{{}^{n+1}C_{k+1}}{(n+1)\ (k+2)} = \left[\because \frac{{}^nC_r}{n} = \frac{{}^{n-1}C_{r-1}}{r}\right] = \frac{{}^{n+2}C_{k+2}}{\left(n+1\right)\left(n+2\right)}$$

∴ The sum of terms on L.H.S.

$$= \sum_{k=0}^{n}\frac{{}^{n+2}C_{k+2}}{\left(n+1\right)\left(n+2\right)} = \frac{1}{\left(n+1\right)\left(n+2\right)}\cdot\sum_{k=0}^{n}{}^{n+2}C_{k+2}$$

$$= \frac{1}{(n+1)(n+2)}\left[2^{n+2} - {}^{n+2}C_0 - {}^{n+2}C_1\right]$$

$$= \frac{1}{(n+1)(n+2)}\left[2^{n+2} - 1 - (n+2)\right] = \frac{2^{n+2} - n - 3}{(n+1)(n+2)}$$

(v) Integrating both sides of equation (i) within limits –1 to 1, we get

$$\int_{-1}^{1} (1+x)^n \, dx =$$

$$\int_{-1}^{1} \left(C_0 + C_1x + C_2x^2 + C_3x^3 + ... + C_nx^n\right)dx$$

$$\left(\frac{(1+x)^{n+1}}{n+1}\right)_{-1}^{1} = C_0x + C_1\frac{x^2}{2} + C_2\frac{x^3}{3} + ... + C_n\frac{x^{n+1}}{n+1}\Bigg|_{-1}^{1}$$

$$\frac{2^{n+1} - 0}{n+1} = \left[C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1}\right] - \left[-C_0 + \frac{C_1}{2} - \frac{C_2}{3} + ...\right]$$

$$\frac{2^{n+1}}{n+1} = 2\left[C_0 + \frac{C_2}{3} + \frac{C_4}{5} + ...\right]$$

$$\Rightarrow \frac{2^n}{n+1} = C_0 + \frac{C_2}{3} + \frac{C_4}{5} + ...$$

Example 4: Prove, by binomial expansion, that

(i) $\displaystyle\sum_{k=1}^{n} k^2 \cdot {}^nC_k = n(n+1)2^{n-2}$

(ii) $\displaystyle\prod_{k=1}^{n} \left(C_{k-1} + C_k\right) = \frac{C_0C_1........C_{n-1}(n+1)^n}{n!}$

Sol: Expanding $(1+x)^n$ and differentiating it twice we will prove given equation (i) and by multiplying and dividing by $C_0C_1C_2.......C_{n-1}$ in L.H.S. of equation (ii) we can prove it.

(i) Now $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + ... + C_nx^n$

Differentiating twice w.r.t. x, we get

$$n(n-1)(1+x)^{n-2} = 2.C_2 + 3.2.C_3x$$
$$+4.3C_4x^2 + ... + n(n-1)C_nx^{n-2}$$

Substituting x = 1, we get

$$n(n-1)2^{n-2} = \sum_{k=1}^{n} k(k-1)(C_k)$$

$$\therefore \sum_{k=1}^{n} \left(k^2\right)\left({}^nC_k\right) = n(n-1)2^{n-2} + n.2^{n-1}$$

$$\left[\because k.{}^nC_k = n.{}^{n-1}C_{k+1}\right]$$

$$= 2^{n-2}\left[n^2 - n + 2n\right]$$

$$\therefore \sum_{k=1}^{n} k^2 \, {}^nC_k = n(n+1)2^{n-2}$$

(ii) To prove $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)......$

$$(C_{n-1} + C_n) = \frac{C_0C_1......C_{n-1}(n+1)^n}{n!}$$

Multiply and divide L.H.S. by $C_0C_1C_2.......C_{n-1}$; then,

$$\text{L.H.S.} = C_0C_1C_2.......C_{n-1}\left(1 + \frac{C_1}{C_0}\right)$$
$$\left(1 + \frac{C_2}{C_1}\right)........\left(1 + \frac{C_n}{C_{n-1}}\right)$$

On using $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$ we have,

$$\text{L.H.S.} = C_0C_1C_2.....C_{n-1}$$
$$\left(1 + \frac{C_1}{C_0}\right)\left(1 + \frac{C_2}{C_1}\right)......\left(1 + \frac{C_n}{C_{n-1}}\right)$$

$$= C_0C_1C_2...C_{n-1}(1+n)\left(\frac{1+n}{2}\right)\left(\frac{1+n}{3}\right)...\left(\frac{n+1}{n}\right)$$

$$= \frac{C_0C_1...C_{n-1}(1+n)^n}{1.2.3........n} = \frac{C_0C_1.......C_{n-1}(n+1)^n}{n!} = \text{R.H.S.}$$

Example 5: If $(1+x)^n = C_0 + C_1x + C_2x^2$
$+C_3x^3 + ... + C_nx^n$

Then find the value of $\displaystyle\sum_{0\le i}\ \sum_{<j\le n}\left(C_i + C_j\right)^2$

Sol: By using summation and coefficients properties we can prove given equations.

$$\sum_{0\le i}\ \sum_{<j\le n}\left(C_i + C_j\right)^2$$

$$= (C_0 + C_1)^2 + (C_0 + C_2)^2 + ... +$$

$$(C_0 + C_n)^2 + (C_1 + C_2)^2 + ... + ... +$$

$$(C_1 + C_n)^2 + (C_2 + C_3)^2 + ... + ... + (C_2 + C_n)^2$$

$$+ \ldots\ldots. + (C_{n-1} + C_n)^2$$

$$= n\left(C_0^2 + C_1^2 + C_2^2 + \ldots.. + C_n^2\right) + 2\sum_{0\le i}\ \sum_{<j\le n} C_i \cdot C_j$$

The square of the sum of n terms is given by

$$= \left(C_0 + C_1 + C_2 + C_3 + \ldots\ldots + C_n\right)^2$$

$$= \left(C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2\right) + 2\sum_{0\le i}\ \sum_{<j\le n} C_i \cdot C_j$$

$$\therefore 2\sum_{0\le i}\ \sum_{<j\le n} C_i \cdot C_j$$

$$= \left[\left(C_0 + C_1 + C_2 + C_3 + \ldots\ldots +\right)^2 - \left(C_0^2 + C_1^2 + \ldots.. + C_n^2\right)\right]$$

$$= \left(2^n\right)^2 - {}^{2n}C_n$$

$$\therefore \sum_{0\le i}\ \sum_{<j\le n}\left(C_i + C_j\right)^2 = \left[n.{}^{2n}C_n\right] + \left[2^{2n} - {}^{2n}C_n\right],$$

$$= (n-1)\,{}^{2n}C_n + 2^{2n}$$

Example 6: Show that

$$\frac{C_0}{1} - \frac{C_1}{4} + \frac{C_2}{7} - \frac{C_3}{10} + \ldots.. + 3\frac{(-1)^n C_n}{2n+1} = \frac{3^n n!}{1.4.7\ldots\ldots.(3n+1)}$$

Sol: By expanding $\left(1-x^3\right)^n$ using binomial expansion and integrating it within a limit 0 to 1 we will prove given equation.

$$\left(1-x^3\right)^n = C_0 - C_1x^3 + C_2x^6$$

$$-C_3x^9 + C_4x^{12} + \ldots. + (-1)^n C_n x^{3n}$$

Integrating both sides between limits 0 and 1, we get

$$\int_0^1 \left(1-x^3\right)^n dx = C_0 - \frac{C_1}{4} + \frac{C_2}{7} - \frac{C_3}{10} + \ldots.. + \frac{(-1)^n C_n}{3n+1} \qquad \ldots(i)$$

$$\text{Also } I_n = \int_0^1 \left(1-x^3\right)^n dx$$

$$= \left[x\left(1-x^3\right)^n\right]_0^1 - \int_0^1 n\left(1-x^3\right)^{n-1}; \qquad \left(-3x^2\right).x\,dx$$

$$= 3n\int_0^1 x^3\left(1-x^3\right)^{n-1} dx$$

$$= 3n\int_0^1 \left(x^3 - 1 + 1\right)\left(1-x^3\right)^{n-1} dx$$

$$= 3nI_{n-1} - 3nI_n;\ (1+3n)I_n = 3nI_{n-1} \therefore\ I_n = \frac{3n}{3n+1}I_{n-1}$$

Replacing n by 1, 2, 3, 4, ... n–1 successively in the above reduction formula, we get

$$I_n = \frac{3n}{3n+1}\frac{3(n-1)}{3n-2};\ \frac{3(n-2)}{3n-5}\ldots\frac{3}{4}I_0 \qquad \ldots(ii)$$

$$\text{But } I_0 = \int_0^1 \left(1-x^3\right)^0 dx = \int_0^1 dx = 1$$

Hence, from (ii),

$$I_n = \frac{3^n n!}{(3n+1)(3n-2)(3n-5)\ldots\ldots7.4}$$

Using (i)

$$\frac{C_0}{1} - \frac{C_1}{4} + \frac{C_2}{7} - \frac{C_3}{10} + \ldots\ldots + \frac{(-1)^n C_n}{3n+1} = \frac{3^n n!}{1.4.7\ldots..(3n+1)}$$

Example 7: Prove that

$$\frac{1}{m!}C_0 + \frac{n}{(m+1)!}C_1\frac{n(n-1)}{(m-2)!}C_2 + \ldots.. + \frac{n(n-1)\ldots\ldots2.1}{(m+n)!}C_n$$

$$= \frac{(m+n+1)(m+n+2)\ldots\ldots(m+2n)}{(m+n)!}$$

Sol: As $(1+x)^{m+n}.(1+x)^n = (1+x)^{m+2n}$ and expanding this by using expansion formula and equating the coefficient of x^n we can prove given equation.

$$\Rightarrow \left({}^{m+n}C_0 + {}^{m+n}C_1x + {}^{m+n}C_2x^2 + \ldots. + {}^{m+n}C_{m+n}x^{m+n}\right)$$

$$\times\left(C_0 + C_1x + C_2x^2 + \ldots.. + C_nx^n\right) = (1+x)^{m+2n}$$

Equating the coefficients of x^n on both sides, we find

$${}^{m+n}C_n.C_0 + {}^{m+n}C_{n-1}.C_1 + {}^{m+n}C_{n-2}.C_2$$

$$+ \ldots.. + {}^{m+n}C_0.C_n = {}^{m+2n}C_n$$

$$\Rightarrow \frac{(m+n)!}{m!n!}C_0 + \frac{(m+n)!}{(n-1)!(m+1)!}C_1$$

$$+ \frac{(m+n)!}{(n-2)!(m+2)!}C_2 + \ldots. + \frac{(m+n)!}{(m+n)!}C_n = \frac{(m+2n)!}{(m+n)!n!}$$

Dividing both sides by (m + n)!/n! we find

$$\frac{1}{m!}C_0 + \frac{n}{(m+1)!}C_1 + \frac{n(n-1)}{(m+2)!}C_2 + \ldots\ldots + \frac{n(n-1)\ldots..2.1}{(m+n)!}C_n$$

$$=\frac{(m+2n)!}{(m+n)!(m+n)!}=\frac{(m+n+1)(m+n+2).....(m+2n)}{(m+n)!}$$

Example 8: Find the sum of the following series

$$S=C_1^2+\frac{1+2}{2}C_2^2+\frac{1+2+3}{3}C_3^2+...\text{Upto n term}$$

Sol: In this problem, first obtain the rth term and then by using binomial expansion and coefficient property we can get required sum.

The rth term of the given series

$$=\frac{1+2+......+r}{r}C_r^2=\frac{r(r+1)}{2r}C_r^2=\frac{1}{2}(r+1)C_r^2$$

$$\therefore S=\frac{1}{2}(1+1)C_1^2+\frac{1}{2}(2+1)C_2^2+\frac{1}{2}$$

$$(3+1)C_2^3+....+\frac{1}{2}(n+1)C_n^2$$

We know that

$$C_0+C_1x+C_2x^2+....+C_nx^n=(1+x)^n$$

$$\Rightarrow C_0x+C_1x^2+C_2x^3+....+C_nx^{n+1}=x(1+x)^n$$

Differentiating both sides w.r.t. x we get

$$C_0+2C_1x+3C_2x^2+4C_3x^3$$

$$+....+(n+1)C_nx^n=(1+x)^n$$

$$+nx(1+x)^{n-1} \qquad ...(i)$$

Also

$$C_0+C_1\left(\frac{1}{x}\right)+C_2\left(\frac{1}{x}\right)^2+C_3\left(\frac{1}{x}\right)^3$$

$$+....+C_n\left(\frac{1}{x}\right)^n=\left(1+\frac{1}{x}\right)^n \qquad ...(ii)$$

Now, $C_0^2+2C_1^2+3C_2^2+4C_3^2+....+(n+1)C_n^2$

= Coefficient of constant term in

$$\left[C_0+2C_1x+3C_2x^2+4C_3x^3+.....+(n+1)C_nx^n\right]\times$$

$$\left[C_0+C_1\left(\frac{1}{x}\right)+C_2\left(\frac{1}{x}\right)^2+.....+C_n\left(\frac{1}{x}\right)^n\right]$$

= Coefficient of constant term in

$$\left[(1+x)^n+nx(1+x)^{n-1}\right](1+1/x)^n$$

= Coefficient of x^n in

$$\left[(1+x)^n+nx(1+x)^{n-1}\right](x+1)^n$$

= Coefficient of x^n in

$$\left[(1+x)^{2n}+nx(1+x)^{2n-1}\right]={}^{2n}C_n+n.{}^{2n-1}C_{n-1}$$

$$=\frac{(2n)!}{n!n!}\left(1+\frac{n}{2}\right)={}^{2n}C_n\left(1+\frac{n}{2}\right)$$

$$\Rightarrow 2C_1^2+3C_2^2+4C_3^2+......+(n+1)C_n^2$$

$$={}^{2n}C_n\left(1+\frac{n}{2}\right)-1 \qquad [\because C_0=1]$$

$$\Rightarrow S=\frac{1}{2}\left[{}^{2n}C_n\left(1+\frac{n}{2}\right)-1\right]$$

Example 9: If n be a positive integer, then prove that the integral part I of $(5+2\sqrt{6})^n$ is an odd integer. If f be the fractional part of $(5+2\sqrt{6})^n$ prove that I = $\frac{1}{1-f}$ – f.

Sol: By using expansion formula we can expand the given binomial and separating its integral and fractional part we can prove given equations.

Let $P=(5+2\sqrt{6})^n=I+f$

Or $\quad I+f=5^n+C_15^{n-1}(2\sqrt{6})$

$$+C_25^{n-2}(2\sqrt{6})^2+......+C_n(2\sqrt{6})^n \qquad ...(i)$$

$$0<5-2\sqrt{6}<1\Rightarrow 0<(5-2\sqrt{6})^n<1$$

Let $(5-2\sqrt{6})^n$ = f′, where 0 < f′ < 1.

$$f'=5^n-C_15^{n-1}(2\sqrt{6})$$

$$+C_25^{n-2}(2\sqrt{6})^2-C_35^{n-3}(2\sqrt{6})^3+.... \qquad ...(ii)$$

Adding (i) and (ii) $\quad I+f+f'=$

$$2\left[5^n+{}^nC_25^{n-2}(2\sqrt{6})^2+{}^nC_45^{n-1}(2\sqrt{6})^4....\right]$$

Or I + f + f′ = even integer

Now $0\le f<1$ and 0 < f′ < 1.

$$\therefore 0<f+f'<2$$

$\therefore f + f' = 1$ and $\therefore$ I is an odd integer

Now $I + f = \left(5 + 2\sqrt{6}\right)^n$,

$\left(5 - 2\sqrt{6}\right)^n = f' = 1 - f \Rightarrow \left(I + f\right)\left(1 - f\right) = 1$

$\therefore \left(I + f\right) = \dfrac{1}{1 - f} \qquad \therefore I = \dfrac{1}{1 - f} - f$

Example 10: If $\left(1 + x + x^2\right) = a_0 + a_1 x$

$+ a_2 x^2 + a_3 x^3 .... + a_{2n} x^{2n}$

Then show that

$a_0 + a_3 + a_6 + ... = a_1 + a_4 + a_7 + ... = 3^{n-1}$.

Sol: By using properties of binomial coefficients and cube root unity $1, \omega,\ \omega^2$ we can prove given problem.

The r^th term of the given series

Putting $x = 1, \omega, \omega^2$, where ω is a non real cube root of unity.

$3^n = a_0 + a_1 + a_2 + a_3 + a_4 + ...$ ...(i)

$0 = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + a_4\omega^4 + ...$ ...(ii)

Because $1 + \omega + \omega^2 = 0$

$0 = a_0 + a_1\omega^2 + a_2\omega^4 + a_3\omega^6 + a_4\omega^8 + ...$ ...(iii)

Adding these

$3^n = 3\left(a_0\right) + a_1\left(1 + \omega + \omega^2\right) + a_2$

$\left(1 + \omega^2 + \omega^4\right) + a_3\left(1 + \omega^3 + \omega^6\right)$

$+ .... = 3\left(a_0 + a_3 + a_6 + ...\right)$

$\therefore a_0 + a_3 + a_6 + ... = 3^{n-1}$

From (i) + (ii) × ω^2 + (iii) × ω,

we get, $3^n + 0 \times \omega^2 + 0 \times \omega$

$= a_0\left(1 + \omega^2 + \omega\right) + a_1\left(1 + \omega^2 + \omega^3\right)$

$+ a_2\left(1 + \omega^4 + \omega^5\right) + a_3\left(1 + \omega^5 + \omega^7\right)$

$+ a_4\left(1 + \omega^6 + \omega^9\right) + ...$

$\therefore 3^n = 3\left(a_1 + a_4 + a_7 + ...\right)$

Because coefficient of each is

$1 + \omega + \omega^2 = 0$, using $\omega^2 = 1$

$\therefore a_1 + a_4 + a_7 + ... = 3^{n-1}$

Again, from (i) + (ii) ω + (iii) × ω^3, we get

$= 3^n = a_0\left(1 + \omega + \omega^2\right) + a_1\left(1 + \omega^2 + \omega^4\right)$

$+ a_2\left(1 + \omega^3 + \omega^3\right) + ... = 3\left(a_2 + a_5 + a_8 + ...\right)$

Example 11: Find the

(i) Last digit

(ii) Last two digits and

(iii) Last three digits of 17^{256}.

Sol: By reducing 17^{256} into the form $\left(x - 1\right)^n$ and using simple binomial expansion we will get required digits.

Since

$17^{256} = \left(17^2\right)^{128} = \left(289\right)^{128} = \left(290 - 1\right)^{128}$

$\therefore 17^{256} = {}^{128}C_0\left(190\right)^{128} - {}^{128}C_1\left(290\right)^{127}$

$+ {}^{128}C_2\left(290\right)^{126} - ... - {}^{128}C_{125}\left(290\right)^3$

$+ {}^{128}C_{126}\left(290\right)^2 - {}^{128}C_{127}\left(290\right) + 1$

$[{}^{128}C_0\left(290\right)^{128} - {}^{128}C_1\left(290\right)^{127}$

$+ {}^{128}C_2\left(290\right)^{126} - ...... - {}^{128}C_{125}\left(290\right)^3]$

$+ {}^{128}C_{126}\left(290\right)^2 - {}^{128}C_{127}\left(290\right) + 1$

$= 1000m + {}^{128}C_2\left(290\right)^2 - {}^{128}C_1\left(290\right) + 1 \ \left(m \in I_+\right)$

$= 1000m + \dfrac{(128)(127)}{2}\left(290\right)^2 - 128 \times 290 + 1$

= 1000 m + (128) (127) (290) (145) – 128 × 290 + 1

= 1000 m + (128) (290) (127 × 145 –1) + 1

= 1000 m + (128) (290) (18414) + 1

= 1000 (m + 683527) + 681

Hence last three digits of 17^{256} must be 681. As result last two digits of 17^{256} or 81 and last digit of 17^{256} is 1.

Example 12: If $32^{32^{32}}$ is divided by 7, then find the remainder

Sol: Here in this problem, we can obtain required remainder by reducing $32^{32^{32}}$ into the form of $7\lambda + a$, where λ is any integer and a is an integer less than 7.

We have 32 = 2^5

$\therefore (32)^{32} = \left(2^5\right)^{32} = 2^{160}; \ (32)^{32} = (3-1)^{160}$

$= {}^{160}C_0 3^{160} - {}^{160}C_1 3^{159} + ... + {}^{160}C_{159} 3 + {}^{160}C_{160} + 1$

$= 3\left(3^{159} - {}^{160}C_1 3^{158} + ... - {}^{160}C_{159}\right) + 1$

$= 3m + 1, \quad m \in I^+$

Now, $32^{32^{32}} = 32^{3m+1} = 2^{5(3m+1)} = 2^{15m+5}$

$\therefore 32^{32^{32}} = 2^{3(5m+1)}.2^2 = 4.(8)^{5m+1}$

$= 4.(7+1)^{5m+1}$

$= 4.({}^{5m+1}C_0 (7)^{5m+1} + {}^{5m+1}C_1 (7)^{5m}$

$+ {}^{5m+1}C_2 (7)^{5m-1} + ... +$

${}^{5m+1}C_{5m} 7 + {}^{5m+1}C_{5m+1})$

$= 4[7\{{}^{5m+1}C_0 - 7^{5m} + {}^{5m+1}C_1 7^{5m-1}$

$+ {}^{5m+1}C_2 7^{5m-2} + ... + {}^{5m+1}C_{5m}\} + 1]$

$= 4\left[7n+1\right], \qquad n \in I_+ \qquad = 28\ n + 4$

This show that where $32^{32^{32}}$ is divided by 7, then remainder is 4.

JEE Main/Boards

Exercise 1

Q.1 Expand $\left(x^2 + 2a\right)^5$ by binomial theorem.

Q.2 Expand $(a+b)^6 - (a-b)^6$. Hence find the value of $\left(\sqrt{2}+1\right)^6 - \left(\sqrt{2}-1\right)^6$.

Q.3 Show that $(101)^{50} > (100)^{50} + (99)^{50}$

Q.4 If x > 1 and the third term in the expansion of $\left(\frac{1}{x} + x^{\log_{10} x}\right)^5$ is 1000, find the value of x.

Q.5 Find the sum of rational terms in the expansion of $\left(\sqrt{2} + 3^{1/5}\right)^{10}$.

Q.6 Find the middle term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^7$

Q.7 Find the middle term in the expansion of $\left(1 - 2x + x^2\right)^n$.

Q.8 Show that the greatest coefficient in the expansion of $\left(x + \frac{1}{x}\right)^{2n}$ is $\frac{1.3.5....(2n-1).2^n}{n!}$.

Q.9 Given that the 4th term in the expansion of $\left(px + \frac{1}{x}\right)^n$ is $\frac{5}{2}$, find n and p.

Q. 10 If in the expansion of $(1 + x)^m (1 - x)^n$ the coefficient of x and x^2 are 3 and –6 respectively then find m.

Q.11 If the coefficients of a^{r-1}, a^r, a^{r+1} in the binomial expansion of $(1 + a)^n$ are in A.P., prove that $n^2 - n(4r+1) + 4r^2 - 2 = 0$.

Q.12 If n be a positive integer, then prove that $6^{2n} - 35n - 1$ is divisible by 1225.

Q.13 Using binomial theorem, show that $3^{4n+1} + 16n - 3$ is divisible by 256 if n is a positive integer.

Q.14 If a_1, a_2, a_3 and a_4 be any four consecutive coefficients in the expansion of $(1 + x)^n$, prove that

$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$$

Q.15 If 3 consecutive coefficients in the expansion of $(1 + x)^n$ are in the ratio 6 : 33 : 110, find n and r.

Q.16 If a, b, c be the three consecutive coefficients in the expansion of a power of (1 + x), prove that the index of the power is $\frac{2ac + b(a+c)}{b^2 - ac}$

Q.17 Expand $\left(x-\frac{1}{y}\right)^{11}$, $y \neq 0$

Q.18 Expand $(1 - x + x^2)^4$

Q.19 Which number is larger, $(1.2)^{4000}$ or 800?

Q.20 If in the expansion of $(1 + x)^n$, the coefficients of 14th, 15th and 16th terms are in A.P., find n.

Q.21 If three consecutive coefficient in the expansion of $(1 + x)^n$ be 165, 330 and 462, find n and the position of the coefficient.

Q.22 Find the greatest term in the expansion of; $(7 - 5x)^{11}$, where $x = \frac{2}{3}$

Q.23 Find the coefficient of x^{-1} in $(1 + 3x^2 + x^4)\left(1+\frac{1}{x}\right)^8$

Q.24 Find the value of k so that the term independent of x in $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$ of 405.

Q.25 If A be the sum of odd terms and B the sum of even terms in the expansion of $(x + a)^n$, prove that

$$2\left(A^2+B^2\right)=\left(x+a\right)^{2n}+\left(x-a\right)^{2n}$$

Q.26 Find the coefficient of x^{40} in the expansion of $\left(1+2x+x^2\right)^{27}$

Q.27 Find the term independent of x in $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

Q.28 If $(1 + ax)^n = 1 + 8x + 24x^2 + ...$ Find a and n.

Exercise 2

Single Correct Choice Type

Q.1 Given that the term of the expansion $\left(x^{1/3}-x^{-1/2}\right)^{15}$ which does not contain x is 5 m where $m \in N$, then m =

(A) 1100 (B) 1010 (C) 1001 (D) None

Q.2 If the coefficients of x^7 & x^8 in the expansion of $\left[2+\frac{x}{3}\right]^n$ are equal, then the value of n is:

(A) 15 (B) 45 (C) 55 (D) 56

Q.3 The coefficient of x^{49} in the expansion of $\left(x-1\right)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2^2}\right)..\left(x-\frac{1}{2^{49}}\right)$ is equal to

(A) $-2\left(1-\frac{1}{2^{50}}\right)$ (B) + ve coefficient of x

(C) – ve coefficient of x (D) $-2\left(1-\frac{1}{2^{49}}\right)$

Q.4 The last digit of $(3^P + 2)$ is

(A) 1 (B) 2 (C) 4 (D) 5

Where $P = 3^{4n}$ and $n \in N$

Q.5 The sum of the binomial coefficient of $\left[2x+\frac{1}{x}\right]^n$ is equal to 256. The constant term in the expansion is :

(A) 1120 (B) 2110 (C) 1210 (D) None

Q.6 The coefficient of x^4 in $\left[\frac{x}{2}-\frac{3}{x^2}\right]^{10}$ is

(A) $\frac{405}{256}$ (B) $\frac{504}{259}$ (C) $\frac{450}{263}$ (D) $\frac{405}{512}$

Q.7 If $(11)^{27} + (21)^{27}$ when divided by 16 leaves the remainder

(A) 0 (B) 1 (C) 2 (D) 14

Q.8 Last three digits of the number $N = 7^{100} - 3^{100}$ are

(A) 100 (B) 300 (C) 500 (D) 000

Q.9 The last two digits of the number 3^{400} are:

(A) 81 (B) 43 (C) 29 (D) 01

Q.10 If $\left(1+x+x^2\right)^{25} = a_0 + a_1x + a_2x^2 + .. + a_{50}.x^{50}$ then $a_0 + a_2 + a_4 + ... + a_{50}$ is:

(A) Even

(B) Odd and of the form 3n

(C) Odd and of the form (3n –1)

(D) Odd and of the form (3n+1)

Q.11 The sum of the series

$(1^2+1).1!+(2^2+1).2!+(3^2+1).3!+....+(n^2+1).n!$ is

(A) (n + 1). (n + 2)! (B) n.(n + 1)!

(C) (n + 1). (n + 1)! (D) None of these

Q.12 Let P_m stand for ${}^nP_m$. Then the expression $1.P_1+2.P_2+3.P_3+.....+n.P_n=$

(A) (n + 1)! –1 (B) (n + 1)! + 1

(C) (n + 1)! (D) None of these

Q.13 The expression

$$\frac{1}{\sqrt{4x+1}}\left[\left[\frac{1+\sqrt{4x+1}}{2}\right]^7-\left[\frac{1-\sqrt{4x+1}}{2}\right]^7\right]$$

is a polynomial in x of degree

(A) 7 (B) 5 (C) 4 (D) 3

Q.14 If the second term of the expansion $\left[a^{1/13}+\frac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$ then the value of $\frac{{}^nC_3}{{}^nC_2}$ is

(A) 4 (B) 3 (C) 12 (D) 6

Q.15 If $(1+x)(1+x+x^2)$

$(1+x+x^2+x^3).....(1+x+x^2+x^3+....+x^n)$

$$=a_0+a_1x+a_2x^2+a_3x^3+....+a_mx^m$$

Then $\sum_{r=0}^{m}a_r$ has the value equal to

(A) n! (B) (n + 1)!

(C) (n –1)! (D) None of these

Q.16 In the expansion of $(1+x)^{43}$ if the coefficient of the (2r + 1)th and the (r + 2)th terms are equal, the value of r is :

(A) 12 (B) 13 (C) 14 (D) 15

Q.17 The positive value of a so that the coefficient of x^5 is equal to that of x^{15} in the expansion of $\left(x^2+\frac{a}{x^3}\right)^{10}$ is

(A) $\frac{1}{2\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) $2\sqrt{3}$

Q.18 In the expansion of $x^2+\left(\frac{9}{43}\right)^{10}$ the term which does not contain x is :

(A) ${}^{10}C_0$ (B) ${}^{10}C_7$ (C) ${}^{10}C_4$ (D) None of these

Q.19 If the 6th term in the expansion of the binomial $\left[\frac{1}{x^{8/3}}+x^2\log_{10}x\right]^8$ is 5600, then x equals to

(A) 5 (B) 8 (C) 10 (D) 100

Q.20 $(1+x)(1+x+x^2)(1+x+x^2+x^3).....$ $(1+x+x^2+....+x^{100})$ when written in the ascending power of x then the highest exponent of x is______.

(A) 4950 (B) 5050 (C) 5150 (D) None of these

Q.21 Let $(5+2\sqrt{6})^n=p+f$ where $n\in N$ and $p\in N$ and $0<f<1$ then the value of, $f^2-f+pf-p$ is

(A) A natural number (B) A negative integer

(C) A prime number (D) Are irrational number

Q.22 Number of rational terms in the expansion of $(\sqrt{2}+\sqrt[4]{3})^{100}$ is-

(A) 25 (B) 26 (C) 27 (D) 28

Q.23 The greatest value of the term independent of x in the expansion of $\left(x\sin\theta+\frac{\cos\theta}{x}\right)^{10}$ is

(A) ${}^{10}C_5$ (B) 2^5 (C) $2^5.\ {}^{10}C_5$ (D) $\frac{{}^{10}C_5}{2^5}$

Q.24 If $(1+x-3x^2)^{2145}=a_0+a_1x+a_2x^2+....$ then $a_0-a_1+a_2-a_3+....$ end with

(A) 1 (B) 3 (C) 7 (D) 9

Q.25 Coefficient of x^6 in the binomial expansion $\left(\frac{4x^2}{3}-\frac{3}{2x}\right)^9$ is

(A) 2438 (B) 2688 (C) 2868 (D) None

Q.26 The expression

$$\left[x+\left(x^3-1\right)^{1/2}\right]^5+\left[x-\left(x^3-1\right)^{1/2}\right]^5$$

is a polynomial of degree

(A) 5 (B) 6 (C) 7 (D) 8

Q.27 Given $\left(1-2x+5x^2-10x^3\right)\left(1+x\right)^n$ = 1 + a_1x + a_2x^2 + ... and that $a_1^2=2a_2$ then the value of n is

(A) 6 (B) 2 (C) 5 (D) 3

Q.28 The sum of the series

$$aC_0+\left(a+b\right)C_1+\left(a+2b\right)C_2+....+\left(a+nb\right)C_n$$

is where C_r denotes combinatorial coefficient in the expansion of $(1+x)^n$, $n\in N$

(A) $(a+2nb)2^n$
(B) $(2a+nb)2^n$
(C) $(a+nb)2^{n-1}$
(D) $(2a+nb)2^{n-1}$

Previous Years' Questions

Q.1 Given positive integers r > 1, n > 2 and the coefficient of (3r)th and (r + 2)th terms in the binomial expansion of $(1+x)^{2n}$ are equal. Then ***(1980)***

(A) n = 2r
(B) n = 2r + 1
(C) n = 3r
(D) None of these

Q.2 If C_r stands for ${}^nC_r$, then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\cdot\left[C_0^2-2C_1^2+3C_2^2-....+\left(-1\right)^n\left(n+1\right)C_n^2\right]$$

Where n is an even positive integer, is equal to ***(1986)***

(A) $\left(-1\right)^{n/2}\left(n+2\right)$
(B) $\left(-1\right)^n\left(n+1\right)$
(C) $\left(-1\right)^{n/2}\left(n+1\right)$
(D) None of these

Q.3 The expression

$$\left[x+\left(x^3-1\right)^{1/2}\right]^5+\left[x-\left(x^3-1\right)^{1/2}\right]^5$$

is a polynomial of degree ***(1992)***

(A) 5 (B) 6 (C) 7 (D) 8

Q.4 For $2\le r\le n,\ {}^nC_r+2\,{}^nC_{r-1}+{}^nC_{r-2}$
Is equal to ***(2000)***

(A) ${}^{n+1}C_{r-1}$
(B) $2\,{}^{n+1}C_{r+1}$
(C) $2\,{}^{n+2}C_r$
(D) ${}^{n+2}C_r$

Q.5 Let T_n denotes the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1}-T_n=21$, then n equals ***(2001)***

(A) 5 (B) 7 (C) 6 (D) 4

Q.6 If ${}^{n-1}C_r=\left(k^2-3\right){}^nC_{r+1}$, then k belongs to ***(2004)***

(A) $\left(-\infty,-2\right]$
(B) $\left[-2,-\sqrt{3}\right)\cup\left(\sqrt{3},2\right]$
(C) $\left[-\sqrt{3},\sqrt{3}\right]$
(D) $\left(\sqrt{3},\infty\right]$

Q.7 ${}^{30}C_0\,{}^{30}C_{10}-{}^{30}C_1\,{}^{30}C_{11}+......{}^{30}C_{20}\,{}^{30}C_{30}$ is equal to ***(2005)***

(A) ${}^{30}C_{11}$ (B) ${}^{60}C_{10}$ (C) ${}^{30}C_{10}$ (D) ${}^{65}C_{55}$

Q.8 For r = 0, 1, ..., let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10}A_r\left(B_{10}B_r-C_{10}A_r\right)$ is equal to ***(2010)***

(A) $B_{10}-C_{10}$
(B) $A_{10}\left(B_{10}^2-C_{10}A_{10}\right)$
(C) 0
(D) $C_{10}-B_{10}$

Q.9 If the coefficients of x^3 and x^4 in the expansion of $\left(1+ax+bx^2\right)\left(1-2x\right)^{18}$ in powers of x are both zero, then (a, b) is equal to: ***(2014)***

(A) $\left(16,\frac{251}{3}\right)$
(B) $\left(14,\frac{251}{3}\right)$
(C) $\left(14,\frac{272}{3}\right)$
(D) $\left(16,\frac{272}{3}\right)$

Q.10 The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is: ***(2015)***

(A) $\frac{1}{2}\left(3^{50}+1\right)$ (B) $\frac{1}{2}\left(3^{50}\right)$

(C) $\frac{1}{2}\left(3^{50}-1\right)$ (D) $\frac{1}{2}\left(2^{50}+1\right)$

Q.11 If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n, x\neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is: ***(2016)***

(A) 64 (B) 2187 (C) 243 (D) 729

JEE Advanced/Boards

Exercise 1

Q.1 Let $f(x)=1-x+x^2-x^3.....+x^{16}-x^{17}$

$=a_0+a_1(1+x)+a_2(1+x)^2+....+a_{17}(1+x)^{17}$,

Find the value of a_2.

Q.2 (a) Find the term independent of x in the expansion of

(i) $\left[\sqrt{\frac{x}{3}}+\frac{\sqrt{3}}{2x^2}\right]^{10}$ (ii) $\left[\frac{1}{2}x^{1/3}+x^{-1/5}\right]^8$

(b) Find the value of x for which the fourth term in the expansion,

$\left(5^{\frac{2}{5}\log_5\sqrt{4^x+44}}+\frac{1}{5^{\log_5\sqrt[3]{2^{x-1}+7}}}\right)^8$ is 336.

Q.3 Find the coefficients:

(i) x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$

(ii) x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$

(iii) Find the relation between a and b, so that these coefficients are equal.

Q.4 (a) If the coefficients of the rth, (r + 1)th & (r + 2)th terms in the expansion of $(1 + x)^{14}$ are in AP, find r.

(b) If the coefficients of 2nd, 3rd & 4th terms in the expansion of $(1 + x)^{2n}$ are in AP, show that $2n^2 - 9n + 7 = 0$.

Q.5 Let a and b be the coefficient of x^3 in $(1 + x + 2x^2 + 3x^3)^4$ and $(1 + x + 2x^2 + 3x^3 + 4x^4)^4$ respectively. Find the value of (a – b).

Q.6 Prove that the ratio of the coefficient of x^{10} in $(1 - x^2)^{10}$ & the term independent of x in $\left(x-\frac{2}{x}\right)^{10}$ is 1 : 32.

Q.7 Find the coefficient of

(a) $x^2y^3z^4$ in the expansion of $(ax - by + cz)^9$.

(b) $a^2b^3c^4d$ in the expansion of $(a - b - c + d)^{10}$.

Q.8 Given $S_n=1+\frac{q+1}{2}+\left(\frac{q+1}{2}\right)^2+....+\left(\frac{q+1}{2}\right)^n$,

$q\neq 1$, prove that ${}^{n+1}C_1+{}^{n+1}C_2.S_1+{}^{n+1}C_3.S_2$

$+....+{}^{n+1}C_{n+1}.S_n=2^n.S_n$.

Q.9 Find numerically the greatest term in the expansion of

(i) $(2 + 3x)^9$ when $x=\frac{3}{2}$

(ii) $(3 - 5x)^{15}$ when $x=\frac{1}{5}$

Q.10 Given that

$\left(1+x+x^2\right)^n=a_0+a_1x+a_2x^2+....+a_{2n}x^{2n}$,

Find the values of :

(i) $a_0+a_1+a_2+....+a_{2n}$;

(ii) $a_0-a_1+a_2-a_3....+a_{2n}$;

(iii) $a_0^2-a_1^2+a_2^2-a_3^2+....+a_{2n}^2$

Q.11 For which positive values of x is the fourth term in the expansion of $(5 + 3x)^{10}$ is the greatest.

Q.12 Find the index n of the binomial $\left(\frac{x}{5}+\frac{2}{5}\right)^n$ if the 9th term of the expansion has numerically the greatest coefficient $(n \in N)$.

Q.13 Find the number of divisors of the number

$$N = {}^{2000}C_1 + 2.{}^{2000}C_2 + 3.{}^{2000}C_3 + ... + 2000.{}^{2000}C_{2000}$$

Q.14 Find number of different dissimilar terms in the sum

$$(1+x)^{2012} + (1+x^2)^{2011} + (1+x^3)^{2010}$$

Q.15 Find the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$.

Q.16 Let f(n) = $\sum_{r=0}^{n}\sum_{k=r}^{n}\binom{k}{r}$. Find the total number of divisors of f (11).

Q.17 Find the sum $\sum_{j=0}^{11}\sum_{i=j}^{11}\binom{i}{j}$.

[Note : $\binom{n}{r} = {}^nC_r$]

Q.18 Let $(1+x^2)^2.(1+x)^n = \sum_{k=0}^{n+4} a_k.x^k$. If a_1, a_2 and a_3 are in AP, find n.

Q.19 Prove that $\sum_{K=0}^{n} {}^nC_k \sin kx.\cos(n-k)x = 2^{n-1}\sin nx$.

Q.20 Find the sum of the roots (real or complex) of the equation $x^{2001} + \left(\frac{1}{2}-x\right)^{2001} = 0$.

Q.21 If for $n \in N$, $\sum_{k=0}^{2n} (-1)^k \, ({}^{2n}C_k)^2 = A$, then what will be the value of $\sum_{k=0}^{2n} (-1)^k \, (k - 2n) \, ({}^{2n}C_k)^2$?

Paragraph for questions. 22 and 23

A path of length n is a sequence of points $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$ with integer coordinates such that for all i between 1 and n – 1 both inclusive, either $x_{i+1} = x_i + 1$ and $y_{i+1} = y_i$ (in which case we say the ith step is rightward) or $x_{i+1} = x_i$ and $y_{i+1} = y_i + 1$ (in which case we say that the ith step is upward).

This path is said to start at (x_1, y_1) and end at (x_n, y_n). Let P (a, b), for a and b non negative integers, denotes the number of paths that start at (0, 0) and end at (a, b)

Q.22 The value of $\sum_{i=0}^{10} P(i, 10-i)$, is

(A) 1024 (B) 512 (C) 256 (D) 128

Q.23 Number of ordered pairs (i, j) where $i \neq j$ for which P (i, 100 – i) = P(j, 100 – j), is

(A) 50 (B) 99 (C) 100 (D) 101

Q.24 If $(6\sqrt{6}+14)^{2n+1}$ = N & F be the fractional part of N, prove that NF = 20^{2n+1} $(n \in N)$.

Q.25 Let P = $(2+\sqrt{3})^5$ and f = P – [P], where [P] denotes the greatest integer function. Find the value of $\left(\frac{f^2}{1-f}\right)$.

Q.26 If C_0, C_1, C_2, ... , C_n are the combinatorial coefficients in the expansion of $(1 + x)^n$, $n \in N$ then prove the following:

(a) $C_1 + 2C_2 + 3C_3 + ... + n.C_n = n.2^{n-1}$

(b) $C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n = (n+2)2^{n-1}$

(c) $C_0 + 3C_1 + 5C_2 + ... + (2n+1)C_n = (n+1)2^n$

(d) $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)....(C_{n-1} + C_n)$

$$= \frac{C_0.C_1.C_2....C_{n-1}(n+1)^n}{n!}$$

(e) $1.C_0^2 + 3.C_1^2 + 5.C_2^2 + ... + (2n+1)C_n^2 = \frac{(n+1)(2n)!}{n!n!}$

Q.27 Let I denotes the integral part and F the proper fractional part of $(3+\sqrt{5})^n$ where $n \in N$ and if ρ denotes the rational part and σ the irrational part of the same, show that

$$\rho = \frac{1}{2}(I+1) \text{ and } \sigma = \frac{1}{2}(I+2F-1)$$

Q.28 Prove that

(a) $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + ... + \frac{n.C_n}{C_{n-1}} = \frac{n(n+1)}{2}$

(b) $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$

(c) $2.C_0 + \frac{2^2.C_1}{2} + \frac{2^3.C_2}{3} + \frac{2^4.C_3}{4}$

$+.... + \frac{2^{n+1} \cdot C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$

(d) $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - .... + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$

Q.29 Prove the following identities using the theory of permutation where C_0, C_1, C_2, ... , C_n are the combinatorial coefficients in the expansion of $(1+x)^n$, $n \in N$, the prove the following :

(a) $C_0C_1 + C_1C_2 + C_2C_3 + ... +$

$C_{n-1}C_n = \frac{2n!}{(n+1)!(n-1)!}$

(b) $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... + C_{n-r}C_n = \frac{2n!}{(n-r)!(n+r)!}$

(c) $\sum_{r=0}^{n-2} \left({}^nC_r . {}^nC_{r+2}\right) = \frac{(2n)!}{(n-2)!(n+2)!}$

(d) ${}^{100}C_{10} + 5.{}^{100}C_{11} + 10.{}^{100}C_{12} + 10.$

${}^{100}C_{13} + 5.{}^{100}C_{14} + {}^{100}C_{15} = {}^{105}C_{90}$

Q.30 If a_0, a_1, a_2,..... be the coefficients in the expansion of $(1 + x + x^2)^n$ in ascending powers of x, then prove that :

(i) $a_0a_1 - a_1a_2 + a_2a_3 - .... = 0$

(ii) $a_0a_2 - a_1a_3 + a_2a_4 - .... + a_{2n-2}a_{2n} = a_{n+1}$ or a_{n-1}

(iii) $E_1 = E_2 = E_3 = 3_{n-1}$;

Where $E_1 = a_0 + a_3 + a_6 + ...; E_2 = a_1 + a_4 + a_7$

$+..... \& E_3 = a_2 + a_5 + a_8 + ...$

Q.31 Let $\sum_{r=0}^{100}\sum_{s=0}^{100}\left(C_1^2 + C_s^2 + C_rC_s\right) = m\left({}^{2n}C_n\right) + 2^p$

Where m, n and p are even natural numbers and C_r represents the coefficient of x_r in the expansion of $(1 + x)^{100}$. Find the value of (m + n + p).

Q.32 The expressions $1 + x$, $1 + x + x^2$, $1 + x + x^2 + x^3$, ... $1 + x + x^2 + ... + x^n$ are multiplied together and the terms of the product thus obtained are arranged in increasing powers of x in the form of $a_0 + a_1x + a_2x^2 + ...$, then

(a) How many terms are there in the product.

(b) Show that the coefficients of the terms in the product, equidistant from the beginning and end are equal.

(c) Show that the sum of the odd coefficients = the sum of the even coefficients = $\frac{(n+1)!}{2}$

Q.33 Let $S_1 = \sum_{0 \le i < j \le 100}\sum C_iC_j, S_2 =$

$\sum_{0 \le j < i \le 100}\sum C_iC_j$ and $S_3 = \sum_{0 \le i = j \le 100}\sum C_iC_j$

Where C_r represents coefficient of x^r in the binomial expansion of $(1 + x)^{100}$.

If $S_1 + S_2 + S_3 = a^b$ where a, b $\in$ N then find the least value of (a + b).

Exercise 2

Single Correct Choice Type

Q.1 In the binomial $(2^{1/3} + 3^{-1/3})$, if the ratio of the seventh term from the beginning of the expansion to the seventh term from its end is 1/6, then n =

(A) 6 (B) 9 (C) 12 (D) 15

Q.2 The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is

(A) 4 (B) 15 (C) 0 (D) 18

Q.3 The value of $4\left\{{}^nC_1 + 4.{}^nC_2 + 4^2.{}^nC_3 + ... + 4^{n-1}\right\}$ is

(A) 0 (B) $5^n + 1$ (C) 5^n (D) $5^n - 1$

Q.4 If n be a positive integer such that $n \ge 3$, then the value of the sum to n terms of the series

$1.n - \frac{(n-1)}{1!}(n-1) + \frac{(n-1)(n-2)}{2!}$

$(n-2) - \frac{(n-1)(n-2)(n-3)}{3!}(n-3) + ...$ is

(A) 0
(B) 1
(C) –1
(D) None of these

Q.5 If the 6th term in the expansion of the binomial $\left[\frac{1}{x^{8/3}} + x^2 \log_{10} x\right]^8$ is 5600, then x equals to-

(A) 5
(B) 8
(C) 10
(D) 100

Q.6 Coefficient of α^t in the expansion of,

$$(\alpha+p)^{m-1} + (\alpha+p)^{m-2}(\alpha+q) + (\alpha+p)^{m-3}(\alpha+q)^2 + ... + (\alpha+q)^{m-1}$$

Where $\alpha \neq -q$ and $p \neq q$ is:

(A) $\frac{{}^mC_t\left(p^t - q^t\right)}{p-q}$
(B) $\frac{{}^mC_t\left(p^{m-t} - q^{m-t}\right)}{p-q}$
(C) $\frac{{}^mC_t\left(p^t + q^t\right)}{p-q}$
(D) $\frac{{}^mC_t\left(p^{m-t} + q^{m-t}\right)}{p-q}$

Q.7 If $\left(1+x-3x^2\right)^{2145} = a_0 + a_1x + a_2x^2 + ...$ then $a_0 - a_1 + a_2 - a_3 + ...$ end with

(A) 1
(B) 3
(C) 7
(D) 9

Q.8 Coefficient of x^6 in the binomial expansion $\left(\frac{4x^2}{3} - \frac{3}{2x}\right)^9$ is

(A) 2438
(B) 2688
(C) 2868
(D) None of these

Q.9 The term independent of 'x' in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, x > 0, is α times the corresponding binomial coefficient. Then 'α' is:

(A) 3
(B) $\frac{1}{3}$
(C) $-\frac{1}{3}$
(D) 1

Q.10 The expression

$$\left[x + \left(x^3-1\right)^{1/2}\right]^5 + \left[x - \left(x^3-1\right)^{1/2}\right]^5$$

Is a polynomial of degree

(A) 5
(B) 6
(C) 7
(D) 8

Q.11 Value of the expression $C_0^2 + C_1^2 + C_2^2 + ... + C_n^2$ is

(A) 2^{2n-1}
(B) 2n ($^{2n}C_n$)
(C) $^{2n}C_n$
(D) None of these

Q.12 The sum of the series

$aC_0 + (a+b)C_1 + (a+2b)\ C_2 + ... + (a+nb)C_n$ is

where C_r denotes combinatorial coefficient in the expansion of $(1 + x)^n$, $n \in N$

(A) $(a + 2nb)2^n$
(B) $(2a + nb)2^n$
(C) $(a + nb)2^{n-1}$
(D) $(2a + nb)2^{n-1}$

Previous Years' Questions

Q.1 Prove that $C_1^2 - 2.C_2^2 + C_3^2 - ..... - 2n.C_{2n}^2 = (-1)^n n.C_n$ ***(1979)***

Q.2 Given,

$$S_n = 1 + q + q^2 + ... + q^n$$

$$S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n, q \neq 1$$

Prove that $^{n+1}C_1 + {}^{n+1}C_2S_1 + {}^{n+1}C_3S_2 + ... + {}^{n+1}C_{n+1}S_n = 2^nS_n$ ***(1984)***

Q.3 Find the sum of the series

$$\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} \text{...upto m terms}\right]$$

(1985)

Q.4 If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r$ and $a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$ ***(1992)***

Q.5 Let n be a positive integer and

$\left(1+x+x^2\right)^n = a_0 + a_1x + \ldots . + a_{2n}x^{2n}$.

Show that $a_0^2 - a_1^2 + \ldots . + a_{2n}^2 = a_n$ ***(1994)***

Q.6 Prove that ***(2003)***

$$2^k\,{}^nC_0\,{}^nC_k - 2^{k-1}\,{}^nC_1\,{}^{n-1}C_{k-1} + 2^{k-2}\,{}^nC_2\,{}^{n-2}C_{k-2} - \ldots\ldots + (-1)^k\,{}^nC_k\,{}^{n-k}C_0 = {}^nC_k$$

Q.7 For r = 0, 1, ..., 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}, (1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10}\left(B_{10}B_r - C_{10}A_r\right)$ is equal to ***(2010)***

(A) B_{10}-C_{10}

(B) $A10\left(B_{10}^2 - C_{10}A_{10}\right)$

(C) 0

(D) C_{10}-B_{10}

Q.8 The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n =__ ***(2013)***

Q.9 Coefficient of x^{11} in the expansion of $\left(1+x^2\right)^4\left(1+x^3\right)^7\left(1+x^4\right)^{12}$ is ***(2014)***

(A) 1051 (B) 1106 (C) 1113 (D) 1120

Q.10 The coefficient of x^9 in the expansion of $(1+x)\left(1+x^2\right)\left(1+x^3\right)\ldots .\left(1+x^{100}\right)$ is ***(2015)***

Q.11 Let $z = \frac{-1+\sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s \in \{1,2,3\}$.

Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is ***(2016)***

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q. 3 Q. 16 Q. 19 Q. 23

Q. 28 Q. 32 Q. 34

Exercise 2

Q. 7 Q. 13 Q. 15 Q. 21

Q. 22 Q. 25 Q. 29

Previous Years' Questions

Q. 2 Q. 3 Q. 5 Q. 6

Q. 8

JEE Advanced/Boards

Exercise 1

Q. 14 Q. 23 Q. 26 Q. 31

Q. 34 Q. 35

Exercise 2

Q. 2 Q. 4 Q. 12

Previous Years' Questions

Q. 3 Q. 4

Answer Key

JEE Main/Boards

Exercise 1

Q.1 $x^{10}+10x^8a+40x^6a^2+80x^4a^3+80x^2a^4+32a^5$ **Q.2** $4ab\left[3a^4+10a^2b^2+3b^4\right]$, $140\sqrt{2}$

Q.4 100 **Q.5** 41 **Q.6** $280x^2$ **Q.7** $\frac{2n!}{n!n!}\left(-1\right)^n x^n$

Q.9 $n = 6, p = \frac{1}{2}$ **Q.10** $m = 12$ **Q.15** $n = 12, r = 1$ **Q.20** 34, 23

Q.21 11, T_{3+1}, T_{3+2}, T_{3+3} **Q.23** 232 **Q.24** $k = \pm 3$ **Q.26** $^{54}C_{14}$

Q.27 $\frac{7}{18}$ **Q.28** $n = 4, a = 2$

Exercise 2

Single Correct Choice Type

Q.1 C **Q.2** C **Q.3** A **Q.4** D **Q.5** A **Q.6** A

Q.7 A **Q.8** D **Q.9** D **Q.10** A **Q.11** B **Q.12** A

Q.13 D **Q.14** A **Q.15** B **Q.16** C **Q.17** A **Q.18** C

Q.19 C **Q.20** B **Q.21** B **Q.22** B **Q.23** D **Q.24** B

Q.25 B **Q.26** C **Q.27** A **Q.28** D

Previous Years' Questions

Q.1 A **Q.2** A **Q.3** C **Q.4** D **Q.5** B **Q.6** B

Q.7 C **Q.8** D **Q.9** D **Q.10** A **Q.11** D

JEE Advanced/Boards

Exercise 1

Q.1 816 **Q.2** (a) (i) $\frac{5}{12}$ (ii) $T_6 = 7$, (b) $x = 0$ or 1

Q.3 (i) $^{11}C_5\frac{a^6}{b^5}$ (ii) $^{11}C_6\frac{a^5}{b^6}$ (iii) $ab = 1$ **Q.4** (a) $r = 5$ or 9

Q.5 0 **Q.7** (a) $-1260\,a^2b^3c^4$; (b) -12600

Q.9 (i) $T_7 = \frac{7.3^{13}}{3}$ (ii) 455×3^{12} **Q.10** (i) 3^n (ii) 1, (iii) a_n

Q.11 $\frac{5}{8} < x < \frac{20}{21}$

Q.12 $n = 12$

Q.13 8016

Q.14 4023

Q.15 $\frac{17}{54}$

Q.16 24

Q.17 4095

Q.18 $n = 2$ or 3 or 4

Q.20 500

Q.22 A

Q.23 C

Q.25 722

Q.31 502

Q.32 (a) $\frac{n^2+n+2}{2}$, (b) $a_0 = a\frac{n(n+1)}{2}$, (c) $\frac{(n+1)!}{2}$

Q.33 66

Exercise 2

Single Correct Choice Type

Q.1 B

Q.2 C

Q.3 D

Q.4 A

Q.5 C

Q.6 B

Q.7 B

Q.8 B

Q.9 D

Q.10 C

Q.11 C

Q.12 D

Previous Years' Questions

Q.3 $\frac{2^{mn}-1}{2^{mn}\left(2^n-1\right)}$

Q.9 C

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $(x^2 + 2a)^5$

$= {}^5C_0\,(x^2)^5 + {}^5C_1\,(x^2)^{5-1}(2a)^1 + {}^5C_2(x^2)^{5-2}$

$(2a)^3 + {}^5C_3(x^2)^{5-3}(2a)^3 + {}^5C_4(x^2)^{5-4}$

$(2a)^4 + {}^5C_5(x^2)^{5-5}(2a)^5$

$= x^{10} + 5x^8(2a) + 10x^6(2a)^2 + 10x^4(2a)^3 + 5x^2(2a)^4 + (2a)^5$

$= x^{10} + 10x^8a + 40x^6a^2 + 80x^4a^3 + 80x^2a^4 + 32a^5$

Sol 2: $(a+b)^6 - (a-b)^6$

${}^6C_0a^6 + {}^6C_1a^5b + {}^6C_2a^4b^2 + {}^6C_3a^3b^3$

$+ {}^6C_4a^2b^4 + {}^6C_5ab^5 + {}^6C_6b^6$

$-({}^6C_0a^6 - {}^6C_1a^5b + {}^6C_2a^4b^2 - {}^6C_3a^3b^3$

$+ {}^6C_4a^2b^4 - {}^6C_5a^4b^5 + {}^6C_0b^6)$

$= 2[6a^5b + 20a^3b^3 + 6ab^5] = 4ab[3a^4 + 10a^2b^2 + 3b^4]$

For finding the value, put $a = \sqrt{2}$ $b = 1$

$\therefore \sqrt{2}\,(12 + 20 + 3)$

$\Rightarrow 140\sqrt{2}$

Sol 3: $(101)^{50} > (100)^{50} + (99)^{50}$

$(100+1)^{50} > (100)^{50} + (100-1)^{50}$

$= (100+1)^{50} - (100-1)^{50} > 100^{50}$

Both binomial will cancel every odd terms of each others rest of the even terms are.

$= 2[{}^{50}C_1(100)^{49} + {}^{50}C_3(100)^{47}] + {}^{50}C_5(100)^{45} +$

$...... + {}^{50}C_{49}100]$

$= 100(100)^{49} + 2[{}^{50}C_3(100)^{47} + ... + {}^{50}C_{49}(100)]$

$= (100)^{50} + 2[{}^{50}C_3(100)^{47} + \ldots + {}^{50}C_{47}(100)] > 100^{50}$

Which is always true

So $(101)^{50}-(99)^{50} > (100)^{50}$

$= (101)^{50} > 100^{50} - (99)^{50}$

Sol 4: $x > 1$

$\left(\frac{1}{x} + {}^{x\log_{10} x}\right)^5$ and $T4 = {}^7C_3\ (2x^2)^3 \cdot \left(\frac{1}{x}\right)^4 = 280y^2$

$T_3 = T_{2+1} = {}^5C_2\left(\frac{1}{x}\right)^{5-2}\left(x^{\log_{10} x}\right)^2 = 1000$

$= 10\left(\frac{1}{x^3}\right)\left(x^{2\log_{10} x}\right) = 1000 \Rightarrow x^{\log_{10} x^2} = 100x^3$

Assume $x = 10^y$

$\Rightarrow 10^{y\log_{10}(10^y)^2} = 100(10^y)^3 = 10^{2+3y}$

$\Rightarrow 10^{2y(\log_{10} 10^y)} = 10^{2y^2} = 10^{2+3y}$

$\Rightarrow 2y^2=2+3y \Rightarrow 2y^2-3y-2=0$

$\Rightarrow (y-2)(2y+1)=0$

$\Rightarrow y=2$ or $y = -\frac{1}{2}$

$\Rightarrow x=10^2$ or $x = 10^{-1/2} \Rightarrow x=100$ or $x = \frac{1}{\sqrt{10}}$

But $x>1$ so $x=100$

Sol 5: $(\sqrt{2} + 3^{1/5})^{10}$

For rational number

$(\sqrt{2})^y \to y = 2n, n \in N$

$(3^{1/5})^z \to z = 5n, n \in N$

Rational terms

${}^{10}C_0(\sqrt{2})^{10} + {}^{10}C_{10}(\sqrt{2})^0(3^{1/5})^{10} = 2^5+3^2=32+9=41$

Sol 6: $\left(2x^2 - \frac{1}{x}\right)^7$

Middle terms are $T_4 = T_{3+1}$ and $T_5 = T_{4+1}$

$T_{4+1} = {}^7C_4(2x^2)^{7-4}\left(-\frac{1}{x}\right)^4 = \frac{7\times6\times5}{1.2.3}(2x^2)^3\,\frac{1}{x^4}$

$= 35\times8\times\frac{x^6}{x^4} = 280x^2$

Sol 7: $(1\text{-}2x+x^2)^n = (1\text{-}2x+x^2)^n$

$=(-1+x)^{2n} = (x-1)^{2n}$

Middle term= $T_{n+1} = {}^{2n}C_n(x)^{2n-n}(-1)^n$

$= \frac{2n!}{n!n!}x^n(-1)^n$

Sol 8: $\left(x + \frac{1}{x}\right)^{2n}$

Greatest coefficient = ${}^{2n}C_n$

$= \frac{2n!}{n!(2n-n)!} = \frac{2n!}{n!n!}$

$= \frac{2n(2n-1)(2n-2)(2n-3)(2n-4).....3.2.1}{n!(n(n-1)(n-2)(n-3)..........3.2.1)}$

$= \frac{2^n[n(n-1)(n-2)(n-3)\cdots1]1.3.5.7........(2n-1)}{n!(n(n-1)......3.2.1)}$

$= \frac{2^n1.3.5.......(2n-1)}{n!}$

Sol 9: $= \left(px + \frac{1}{x}\right)^n$

Given = 4th term = $\frac{5}{2}$

$T_4 = T_{3+1} = {}^nC_3(px)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2}$

$\Rightarrow {}^nC_3p^{n-3}x^{n-3+3(-1)} = \frac{5}{2}x^0$

$\Rightarrow n = 6 \Rightarrow {}^6C_3p^{6-3}x^0 = \frac{5}{2}$

$\Rightarrow \frac{6\times4\times5}{1.2.3}\ p^3 = \frac{5}{2}$

$\Rightarrow p^3 = \frac{1}{8} = \left(\frac{1}{2}\right)^3 \Rightarrow p = \frac{1}{2}$ and $n = 6$

Sol 10: $(1+x)^m\ (1\text{-}x)^n$

$= ({}^mC_0x^0+{}^mC_1x^1+{}^mC_2x^2+\ldots\ldots. +{}^mC_mx^m)$

$({}^nC_0+{}^nC_1(-x)+{}^nC_2(-x)^2+\ldots.\ .\ {}^nC_2(-x)^n)$

terms of $x= {}^mC_0\ {}^nC_1\ (-1) + {}^nC_0\ {}^mC_1$

$= (-n)+m=m-n=3$ (given) .... (i)

terms of

$x^2= {}^mC_0\,{}^nC_2(-1)^2+ {}^mC_2\ {}^nC_0+{}^mC_1{}^nC_1(-1)$

$$= 1.\frac{n\times(n-1)}{1.2}1+\frac{m(m-1)}{1.2}1+m(-n)$$

$$= \frac{n^2-n}{2}+\frac{m^2-m}{2}-mn=-6 \qquad \text{.... (ii)}$$

In equation (i) m–n=3 ⇒ n = (m-3)

Put the values of n in eq. (ii)

$$\frac{(m-3)^2-(m-3)}{2}+\frac{m^2-m}{2}-m(m-3)=-6$$

$m^2 + 3^2 - 3(2)(m) - m + 3 + m^2 - m - 2m^2 + 6m = 12$

$2m^2 - 6m + 9 + 3 - 2m - 2m^2 + 6m = -12 12 - 2m = -12$

$\Rightarrow 2m = 24 \Rightarrow m = 12$ and $n = 9$

Sol 11: Coefficient of a^{r-1}, a^r, a^{r+1} in the binomial expansion of $(1+a)^n$ are in A. P. so

Terms of $a^{r-1} = T_r = {}^nC_{r-1}(a)^{r-1}$

Terms of $a^r = T_{r+1} = {}^nC_r\, a^r$

Terms of $a^{r+1} = T_{r+2} = {}^nC_{r+1}a^{r+1}$

Coefficients of T_r, T_{r+1}, T_{r+2} are in A. P. so

$${}^nC_{r-1}+{}^nC_{r+1}=2\,{}^nC_r$$

$$\frac{n!}{(r-1)!(n-r+1)!}+\frac{n!}{(r+1)!(n-r-1)!}=2\frac{n!}{r!(n-r)!}$$

$$\Rightarrow\frac{1}{(r-1)!(n-r+1)(n-r)(n-r-1)!}$$

$$+\frac{1}{(r+1)r(r-1)!(n-r-1)!}=\frac{2}{r(r-1)!(n-r)(n-r-1)!}$$

$$\Rightarrow\frac{1}{(n-r)(n-r+1)}+\frac{1}{r(r+1)}=\frac{2}{r(n-r)}$$

$$\Rightarrow\frac{r(r+1)+(n-r)(n-r+1)}{r(r+1)(n-r)(n-r+1)}=\frac{2}{r(n-r)}$$

$\Rightarrow r^2 + r + n^2 - nr + n - nr + r^2 - r = 2(r + 1)(n - r + 1)$

$\Rightarrow 2r^2 + n^2 - 2nr + n = 2rn - 2r^2 + 2r + 2n - 2r + 2$

$\Rightarrow n^2 + 4r^2 - 4rn - n - 2 = 0$

$\Rightarrow n^2 - n(4r + 1) + 4r^2 - 2 = 0$

Sol 12: n is a positive integer

$\Rightarrow 6^{2n}-35n-1 = (6^2)^n-35n-1=(36)^n-35n-1$

$= (35+1)^n - 35n-1$

$$= {}^nC_0 35^n+{}^nC_1 35^{n-1}+\ldots\ldots+{}^nC_{n-2}35^2$$

$$+\cancel{{}^nC_{n-1}35}+\cancel{{}^nC_n 35^0}-\cancel{35}n-\cancel{1}$$

And $1225 = 35^2$

so each term is a multiple of 35^2 and is divisible by 1225

Sol 13: 3^{4n+1} +16n–3is divisible by 256

$256 = 2^8 = 4^4$

$= 3^{4n+1}+16n-3$

$= 3.\ 3^{4n} +16n-3$

$= 3[4-1]^{4n}+16n-3$

$$= 3[\,{}^{4n}C_0 4^{4n}+{}^{4n}C_1 4^{4n-1}(-1)+\ldots..+$$

$$^{4n}C_{4n-2}(4)^{4n-4n+2}-{}^{4n}C_{4n-1}(4)^{4n-4n+1}+{}^{4n}C_{4n}]+16n-3$$

= all terms which is multiple of 4^4 is divisible by 256. So rest of the terms

$$=3[-^{4n}C_{4n-2}(4)^2-{}^{4n}C_{4n-1}(4)^1+1]+16n-3$$

$$3[\frac{4n(4n-1)}{1.2}\times4^2-4n\times4+1]+16n-3$$

$= 128n^2.\ 3-128n=128(3n^2-1)$

= 128n(3n-1) and (3n–1) is always even

so $128n(3n-1)=128\times2^{x+1}$ (assume), $x\in N$

$= 256\times2^x$, which is divisible by 256

Sol 14: a_1, a_2, a_3 and a_4 are any four consecutive coefficients in the expansion of $(1+x)^n$

$a_1= {}^nC_r$, $a_2={}^nC_{r+1}$

$a_3={}^nC_{r+2}$, $a_4={}^nC_{r+3}$

L. H. S.

$$=\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}$$

$$=\frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}+\frac{{}^nC_{r+2}}{{}^nC_{r+2}+{}^nC_{r+3}}$$

$$=\frac{{}^nC_r}{{}^{n+1}C_{r+1}}+\frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}}$$

$$=\frac{n!(r+1)!(n-r)!}{(n+1)!(r)!(n-r)!}+\frac{n!(r+3)!(n-r-2)!}{(n+1)!(r+2)!(n-r-2)!}$$

$$= \frac{(r+1)}{(n+1)}+\frac{(r+3)}{(n+1)} = \frac{2r+4}{n+1} = \frac{2(r+2)}{n+1}$$

$$= 2\frac{n!(r+2)!(n-r-1)!}{(n+1)!(r+1)!(n-r-1)!} = 2\frac{{}^{n}C_{r+1}}{{}^{n+1}C_{r+2}}$$

$$= 2\frac{{}^{n}C_{r+1}}{{}^{n}C_{r+2}+{}^{n}C_{r+1}} = \frac{2a_2}{a_2+a_3}$$

Sol 15: 3 consecutive coefficients in the expansion of $(1+x)^n$ are in the ratio 6 : 33 : 110

$= T_{r+1} : T_{r+2} : T_{r+3}$

$= {}^{n}C_{r} : {}^{n}C_{r+1} : {}^{n}C_{r+2}$

$$= \frac{n!}{r!n-r!} : \frac{n!}{r+1!n-r-1!} : \frac{n!}{r+2!n-r-2!}$$

$$= \frac{1}{(n-r)(n-r-1)} : \frac{1}{(r+1)(n-r-1)} : \frac{1}{(r+1)(r+2)}$$

= 6 : 33 : 110

$$\Rightarrow \frac{(r+1)(n-r-1)}{(n-r)(n-r-1)} = \frac{6}{33} = \frac{2}{11} \Rightarrow \frac{r+1}{n-r} = \frac{2}{11}$$

$\Rightarrow 11r+11 = 2n-2r \Rightarrow 2n–13r–11=0$ .... (i)

And $\frac{(r+1)(r+2)}{(r+1)(n-r-1)} = \frac{33}{110} = \frac{3}{10} \Rightarrow \frac{r+2}{n-r-1} = \frac{3}{10}$

$\Rightarrow 10r + 20 = 3n - 3r - 3 \Rightarrow 3n - 13r - 23 = 0$ .... (ii)

Subtracting equation (i) from (ii), we get n = 12

Putting n = 12 in equation (i)

$13r = 2n - 11 = 2(12) - 11 = 24 - 11 = 13 \Rightarrow r = 1$

So terms are $T_{r+1}, T_{r+2}, T_{r+3}$

Sol 16: a, b, c are three consecutive coefficients in the expansion of power (say n) of $(1+x)^n$

So $a = {}^{n}C_r$, $b = {}^{n}C_{r+1}$, $c = {}^{n}C_{r+2}$

$\frac{a}{b} = \frac{(r+1)}{(n-r)} \Rightarrow an-ar = br+b$

$\Rightarrow r = \frac{an-b}{a+b}$ ......(i)

$\frac{b}{c} = \frac{(r+2)}{(n-r-1)}$

$\Rightarrow bn-br-b = cr+2c$

$\Rightarrow r = \frac{bn-b-2c}{b+c}$ .... (ii)

From (i) and (ii)

$$\frac{an-b}{a+b} = \frac{bn-b-b-2c}{b+c}$$

$\Rightarrow abn - b^2 + acn - bc = abn - 2ab - 2ac + b^2n - 2b^2 - 2bc$

$\Rightarrow n(ac-b^2) = -ab-2ac-bc$

$$\Rightarrow n = \frac{ab+2ac+bc}{b^2-ac} = \frac{2ac+b(a+c)}{b^2-ac}$$

Sol 17: $\left(x-\frac{1}{y}\right)^{11}$, $y \neq 0$

$$= {}^{11}C_0x^{11} + {}^{11}C_1x^{11-1}\left(-\frac{1}{y}\right) + {}^{11}C_2x^{11-2}\left(-\frac{1}{y}\right)^2$$

$$+ {}^{11}C_3x^{11-3}\left(-\frac{1}{y}\right)^3 + \cdots + {}^{11}C_{11}\left(-\frac{1}{y}\right)^{11}$$

Sol 18: $(1–x+x^2)^4 = ((1–x)+x^2)^4$

$= {}^4C_0(1–x)^4 + {}^4C_1(1–x)^3x^2 + {}^4C_2(1–x)^2(x^2)^2$

$+ {}^4C_3(1–x)(x^2)^3 + {}^4C_4(x^2)^4$

$= (1–x)^4 + 4x^2(1–x)^3 + 6(1+x^2–2x)x^4 + 4x^6(1–x) + x^8$

$= ({}^4C_0 – {}^4C_1x + {}^4C_2x^2 – {}^4C_3x^3 + {}^4C_4x^4)$

$+ 4x^2({}^3C_0 - {}^3C_1x + {}^3C_2x^2 - {}^3C_3x^3)$

$+ (6+6x^2-12x)x^4 + 4x^6 - 4x^7 + x^8$

$= 1 - 4x + 6x^2 - 4x^3 + x^4 + 4x^2 - 12x^3$

$+ 12x^4 - 4x^5 + 6x^4 + 6x^6 - 12x^5 + 4x^6 - 4x^7 + x^8$

$= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$

Sol 19: $(1.2)^{4000} = (1+0.2)^{4000}$

$= {}^{4000}C_0(0.2)^0 + {}^{4000}C_1(0.2)^1 + {}^{4000}C_2(0.2)^2 + \ldots\ldots.$

$= 1+4000(0.2)+\ldots\ldots\ldots = 1+800+\ldots. = 801+\ldots\ldots.$

So $(1.2)^{4000}$ is greater than 800

Sol 20: For $(1+x)^n$

T_{14}, T_{15} and T_{16} are in A. P.

$\Rightarrow T_{14} + T_{16} = 2T_{15} \Rightarrow {}^{n}C_{13} + {}^{n}C_{15} = 2\,{}^{n}C_{14}$

$\Rightarrow {}^{n}C_{13} + {}^{n}C_{15} = 2\,{}^{n}C_{14}$

$$\frac{n!}{13!(n-13)!}+\frac{n!}{15!(n-15)!}=2\frac{n!}{14!(n-14)!}$$

$$\frac{1}{(n-13)(n-14)}+\frac{1}{15\times 14}=\frac{2}{14(n-14)}$$

$$\frac{15\times 14+(n-13)(n-14)}{15\times 14(n-13)(n-14)}=\frac{2}{14(n-14)}$$

$210 + n^2 + 182 - n(13 + 14) = 2 \times 15(n - 13)$

$n^2 + 392 - 27n = 30n - 390$

$n - 57n + 782 = 0$

$$n=\frac{57\pm\sqrt{57^2-4(1)(782)}}{2(1)}=\frac{57\pm\sqrt{121}}{2}=\frac{57\pm 11}{2}$$

$n = 34, 23$

Sol 21: Given

${}^nC_r=165;\ {}^nC_{r+1}=330;\ {}^nC_{r+2}=462$

$$\frac{{}^nC_r}{{}^nC_{r+1}}=\frac{165}{330}=\frac{1}{2}$$

$$\Rightarrow \frac{n!(r+1)!(n-r-1)!}{r!(n-r)!n!}=\frac{1}{2}$$

$$\Rightarrow \frac{(r+1)}{(n-r)}=\frac{1}{2}$$

$\Rightarrow 2r+2=n-r$

$\Rightarrow 3r=n-2$ .... (i)

$$\frac{{}^nC_{r+2}}{{}^nC_{r+1}}=\frac{462}{330}=\frac{7}{5}$$

$$\frac{(r+1)!n!(n-r-1)!}{n!(r+2)!(n-r-2)!}=\frac{n-r-1}{r+2}=\frac{7}{5}$$

$\Rightarrow 5n - 5r - 5 = 7r + 14$

$\Rightarrow 12r = 5n - 19$ .... (ii)

From eq (i) and (ii)

$4(n2) = 5n19 \qquad \Rightarrow n = 11$

So $3r = 11 - 2 = 9 \quad \Rightarrow r = 9/3 = 3$

Position of coefficients are $T_{3+1}, T_{3+2}, T_{3+3}$

Sol 22: $(7-5x)^{11},\ x=\frac{2}{3}$

$$\frac{n+1}{1+\left|\frac{x}{a}\right|}=\frac{11+1}{1+\left(\frac{7\times 3}{5\times 2}\right)}=\frac{12}{1+\frac{21}{10}}=\frac{12}{3.1}=3.87$$

So greatest is 4^{+n}.

$$|T_4|=|T_{3+1}|={}^{11}C_3\,(7)^{11-3}\left(5\times\frac{2}{3}\right)^3$$

$$=\frac{11\times 10\times 9}{1.2.3}\times 7^8\ \ 5^3\times\frac{2^3}{3^3}=\frac{11}{9}2^3 5^4 7^8$$

Sol 23: $(1+3x^2+x^4)\left(1+\frac{1}{x}\right)^8$

For Coefficient of x^{-1}

$$=(1)\ {}^8C_1\left(\frac{1}{x}\right)+3x^2\ {}^8C_3\frac{1}{x^3}+x^4\ {}^8C_5\frac{1}{x^5}$$

$$=\frac{8}{x}+\frac{3\times 8\times 7\times 6}{1\times 2\times 3}\times x^{-1}+\frac{8\times 7\times 6}{1.2.3}x^{-1}$$

$$=\frac{1}{x}[8+168+56]=\frac{232}{x}$$

Coefficient of $x^{-1}=232$

Sol 24: $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$

$$T_{r+1}={}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{k}{x^2}\right)^r={}^{10}C_r(x)^{\frac{10-r}{2}}k^r x^{-2r}$$

T_{r+1} is independent of x

So $\frac{10-r}{2}-2r=0 \Rightarrow 10 - 5r = 0 \Rightarrow r=\frac{10}{5}=2$

So Coefficient is $= {}^{10}C_2k^2$

$$=\frac{10\times 9}{1\times 2}\times k^2=405 \Rightarrow k^2=9 \Rightarrow k=\pm 3$$

Sol 25: $(x+a)^n$

A = Sum of odd terms

B = Sum of even terms

(ii) $2(A^2+B^2)=(x+a)^{2n}+(x-a)^{2n}$

$A = {}^nC_0x^n+{}^nC_2x^{n-2}a^2+....\ {}^nC_nx^0a^n$

$B = {}^nC_1x^{x-1}a+{}^nC_3x^{n-3}a^3+......+{}^nC_{n-1}+xa^{n-1}$

$2(A^2+B^2) = (A+B)^2 + (A-B)^2 = (x+a)^{2n}+(x-a)^{2n}$

L. H. S. = R. H. S.

Sol 26: $(1+2x+x^2)^{27} = ((1+x)^2)^{27} = (1+x)^{54}$

$T_{r+1}={}^{54}C_r x^r$

Coefficient of $x^{40} \Rightarrow r=40$

Coefficient = ${}^{54}C_{40}={}^{54}C_{54-40}= {}^{54}C_{14}$

Sol 27: $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$

$$T_{r+1}={}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$$

For independence of x

$2(9-r)-r=18-2r-r=18-3r=0$

Coefficient $r=6$

$$T_{r+1}=T_{6+1}={}^9C_6\left(\frac{3}{2}\right)^{9-6}\left(-\frac{1}{3}\right)^6$$

$$= {}^9C_3\times\frac{3^3}{2^3}\frac{1}{3^6}=\frac{9\times8\times7}{1.2.3}\times\frac{(1)}{2^3 3^3}=\frac{7\times3}{2.3^3}=\frac{7}{18}$$

Sol 28: $(1+ax)^n = 1+8x+24x^2+\ldots\ldots$

${}^nC_0 + {}^nC_1ax + {}^nC_2(ax)^2 + \ldots . = 1 + 8x + 24x^2 + \ldots\ldots$

So ${}^nC_1a = 8$ and ${}^nC_2a^2 = 24$

$$na=8 \text{ and } \frac{n(n-1)}{2}a^2=24$$

$a^2n^2 - na^2 = 48 \Rightarrow (8)^2 - 8a = 48 \Rightarrow 64 - 8a = 48$

$\Rightarrow a = 2 \Rightarrow n = 4$

Exercise 2

Single Correct Choice Type

Sol 1: (C) $(x^{1/3}-x^{-1/2})^{15}$

$T_{r+1}= {}^{15}C_r\,(x^{1/3})^{15-r}(-x^{-1/2})^r$

$$\text{Power of } x=\frac{15-r}{3}-\frac{r}{2}=0 \text{ for } x^0$$

$2(15-r) -3r=0$

$30 - 2r - 3r = 0 \Rightarrow 5r = 30 \Rightarrow r = 30/5 = 6$

Coefficient $T_{r+1}={}^{15}C_6\times1=5005$

$5m = 5005 \Rightarrow m = 1001$

Sol 2: (C) In the expansion $\left(2+\frac{x}{3}\right)^n$ the coefficients of x^7 & x^8 are equal

$${}^nC_7(2)^{n-7}\left(\frac{1}{3}\right)^7={}^nC_8(2)^{n-8}\left(\frac{1}{3}\right)^8$$

$$\frac{6}{(n-7)}=\frac{1}{8}\Rightarrow n-7=48\Rightarrow n=48+7=55$$

Sol 3: (A) $(x-1)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2^2}\right)\ldots\ldots\left(x-\frac{1}{2^{49}}\right)$

Max power of x = 50

Coefficient of x^{49}

$$=-1-\frac{1}{2}-\frac{1}{2^2}-\frac{1}{2^3}\cdots-\frac{1}{2^{49}}$$

$$=\left[\frac{1-\left(\frac{1}{2}\right)^{50}}{1-\frac{1}{2}}\right]=-2\left[1-\frac{1}{2^{50}}\right]$$

Sol 4: (D) (3^P+2)

$P=3^{4n},\ n\in N = 3^{3^{4n}}+2$

$3^0=1, 3^1=3,\ 3^2=9,\ 3^3=27, 3^4=81$

Last digit = 1,3,9,7

Last digit of 3^x repeat after every power of 4 so 3^{4n} last digit =1

$3^1=3$

$3^1+2=5$

So last digit of $3^{3^{4n}}+2$ is 5

Sol 5: (A) $\left(2x+\frac{1}{x}\right)^n$

Sum of binomial coefficient= $2^n=256$

$2^n=2^8 \Rightarrow n=8$

Constant term =

$${}^8C_4(2x)^4.\left(\frac{1}{x}\right)^4=\frac{8\times7\times6\times5}{1.2.3.4}\times2^4x^{4-4}=1120$$

Sol 6: (A) $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$

$$T_{r+1}={}^{10}C_r\left(\frac{x}{2}\right)^{10-r}\left(-\frac{3}{x^2}\right)^r$$

Power of x=10–r–2(r)=4 (given)

$\Rightarrow 10 - 3r = 4 \Rightarrow 3r = 10 - 4 = 6 \Rightarrow r = 6/3 = 2$

Coefficient $T_{r+1} = {}^{10}C_2\left(\frac{1}{2}\right)^{10-2}(-3)^2$

$$= \frac{10\times 9}{1.2}2^{-8}3^2 = \frac{5\times 9\times 9}{2^8} = \frac{405}{256}$$

Sol 7: (A) $11^{27}+21^{27}$

$= (16-5)^{27}+(16+5)^{27}$

$= 2\left[{}^{27}C_0 16^{27} + ... + {}^{27}C_{26}16\right]$

= 2. 16k = 32k

Always divisible by 16

Sol 8: (D) $N=7^{00}-3^{100}$

$N = (5+2)^{100} - (5-2)^{200}$

$N = 2\left[{}^{100}C_1 5^{99}.2 + ... + {}^{100}C_{99}5.2^{99}\right]$

$N = [{}^{100}C_1 5^{97}.100 + 10^3\ {}^{100}C_3 5^{94}$

$+ ...... + {}^{100}C_{99}10.2^{99}]$

$N = 1000.[10.5^{97} + ... + 2^{99}]$

Integer

Last three digits = 000

Sol 9: (D) $3^{400} = (3^2)^{200}$

$(9)^{200} = (10-1)^{200}$

$= {}^{200}C_0 10^{200} - {}^{200}C_1 10^{199} + ... {}^{200}C_{199}10^1 + {}^{200}C_{200}.1$

$= 10m + 1 \quad (m \in N)$

Last 2 digits are 01

Sol 10: (A) $(1+x+x^2)^{25} = a_0 + a_1x + ... + a_{50}x^{50}$

x = 1

$3^{25} = a_0 + a_1 + a_2 + ... + a_{50}$ ...(i)

x = –1

$(1 - 1 + 1)^{25} = 1$

$= a_0 - a_1 + a_2 - a_3 + ... + a^{50}$ .... (ii)

Sum of both eq^n.

$3^{25} + 1 = 2(a_0 + a_2 + a_4 + ... + a_{50})$

$a_0 + a_2 + a_4 + ... + a_{50} = \frac{1}{2}(3^{25}+1)$

$= \frac{1}{2}\left[(4-1)^{25}+1\right]$

$= \frac{1}{2}\left[\left({}^{25}C_0 4^{25} - {}^{25}C_1 4^{24} + ... + {}^{25}C_{24}4 - 1\right) + 1\right]$

= 2 m always even. (∵ divisible by 2)

Sol 11: (B) $(1^2+1)1!+(2^2+1)2!+(3^2+1)3!+......+(n^2+1)n!$

$T_n = (n^2 + 1)\ n! = n(n + 1)! - (n - 1)\ n!$

$S_n = n(n+1)!$

Sol 12: (A) $P_{m} \rightarrow {}^{n}P_{m}$

$1P_1 + 2P_2 + 3P_3 + ... + n.\ P_n$

$= 1.\ n + 2.\ n(n - 1) + 3n(n - 1)(n - 2)$

$+ 4n(n - 1)(n - 2)(n - 3) + ... + n.\ n!$

Add (+1 and –1)

$= 1 + {}^nC_1 + 2\,{}^nC_2 2! + 3\,{}^nC_3 3! + 4\,{}^nC_4 4! + ...n\,{}^nC_n n! - 1$

$= -1 + 1 + \sum_{i=0}^{n} i\,{}^nC_i(i)!$

$= -1 + 1\sum_{i=0}^{n} iP_i$

When $1 + 1P_1 + 2P_2 + 3P_3 + ... + nP_n = (n+1)!$

$= -1 + 1(n+1)! - 1$

$= (n+1)! - 1$

Sol 13: (D)

$$\frac{1}{\sqrt{4x+1}}\left[\left[\frac{1+\sqrt{4x+1}}{2}\right]^7 - \left[\frac{1-\sqrt{4x+1}}{2}\right]^7\right]$$

$= \frac{1.2}{2^7\sqrt{4x+1}}\left[{}^7C_1\sqrt{4x+1} + {}^7C_3(\sqrt{4x+1})^3\right.$

$\left. + ... + {}^7C_7(\sqrt{4x+1})^7\right]$

$= 2^{-6}\left[{}^7C_1 + {}^7C_3(4x+1) + {}^7C_5\right.$

$\left.(4x+1)^2 + {}^7C_7(4x+1)^3\right]$

$\Rightarrow$ Max. power of $x = 3$

Sol 14: (A)

$$\left(a^{1/13}+\frac{a}{\sqrt{a^{-1}}}\right)^{n}=\left(a^{1/13}+a^{1+1/2}\right)^{n}=\left(a^{1/13}+a^{3/2}\right)^{n}$$

$$T_2={}^{n}C_1(a^{1/13})^{n-1}+(a^{3/2})^{1}=14a^{5/2}$$

$$\Rightarrow n\ a^{\frac{n-1}{13}+\frac{3}{2}}=14a^{5/2}\ \Rightarrow n=14$$

$$\frac{{}^{14}C_3}{{}^{14}C_2}=\frac{14-3+1}{3}=\frac{12}{3}=4$$

Sol 15: (B) $(1+x)(1+x+x^2)(1+x+x^2+x^3)$

$\ldots.\ (1+x+\ldots.\ +x^n)$

$= a_0+a_1x+a_2x^2+\ldots\ldots+a_mx^m$

$$\sum_{r=0}^{m}ar=a_0+a_1+a_2+\ldots\ldots+a_m$$

At $x=1$

$= 2.3.4.5.6\ldots..(n+1) = (n+1)!$

Sol 16: (C) $(1+x)^{43}$

Given $T_{2r+1}=T_{r+2}$

${}^{43}C_{2r}={}^{43}C_{r+1}={}^{43}C_{43-(r+1)}$

$$\Rightarrow 2r=43-r-1=42-r\ \Rightarrow 3r=42\Rightarrow r=\frac{42}{3}=14$$

Sol 17: (A) $\left(x^2+\dfrac{a}{x^3}\right)^{10}$

Coefficient of x^5 is equal to that of x^{15}

$$T_{r+1}={}^{10}C_r(x^2)^{10-r}\left(\frac{a}{x^3}\right)^{r}$$

Power of $x=2(10-r)-3r$ $=20-5r$

$20-5r=5\Rightarrow r=3$

$20-5r=15\Rightarrow r=1$

$T_{3+1}=T_{1+1}$

${}^{10}C_3a^3={}^{10}C_1a$

$$\frac{10\times9\times8}{1.2.3}a^2=10$$

$$a^2=\frac{1}{12}\Rightarrow a=\frac{1}{\sqrt{12}}=\frac{1}{2\sqrt{3}}$$

Sol 18: (C) $\left(x^2+\dfrac{a}{x^3}\right)^{10}$

Power of x for term

$$T_{r+1}={}^{10}C_r(x^2)^{10-r}\left(\frac{a}{x^3}\right)^{r}$$

Power of $x=2(10-r)-3r=20-5r=0$

$\Rightarrow 5r=20=r=20/5=4$

$T_{4+1}={}^{10}C_4$ binomial coefficient

Sol 19: (C) $\left(\dfrac{1}{x^{8/3}}+x^2\log_{10}x\right)^{8}$

$$T_6=T_{5+1}={}^{8}C_5\left(\frac{1}{x^{8/3}}\right)^{8-5}\left(x^2\log_{10}x\right)^{5}=5600$$

$$\Rightarrow\frac{8\times7\times6}{1.2.3}\times\left(x^{-8/3}\right)^{3}x^{10}(\log_{10}x)^5=5600$$

$x^{-8+10}(\log_{10}x)^5=100\Rightarrow x^2(\log_{10}x)^5=100$

Assume $x=10^y$

So $10^{2y}(\log_{10}10^y)^5=10^2\Rightarrow 10^{2y-2}\ y^5=1$

$\Rightarrow y=1\Rightarrow x=10$

Sol 20: (B) $(1+x)(1+x+x^2)(1+x+x^2+x^3)$

$\ldots.\ (1+x+\ldots.\ x^{100})$

Highest power of x $=1+2+3+\ldots.\ +100$

$$=\frac{100(100+1)}{2}=50\times101=5050$$

Sol 21: (B) $(5+2\sqrt{6})^n=p+f$

$p=[(5+2\sqrt{6})^n]-f$

$f^2-f+pf-p=f(f-1)+p(f-1)=(f-1)(f+p)$

Assume $F=(5-2\sqrt{6})^n=\left(\dfrac{1}{5+2\sqrt{6}}\right)^{n}$

$0<f<1,\ 0<F<1$

$F+f+p=(5+2\sqrt{6})^n+(5-2\sqrt{6})^n=$ integer $=2I$

$F+f=2I-p$ = Integer

$0<F+f<2\ \Rightarrow\ F+f=1\Rightarrow F=1-f$

$(F)(f+p)=(5-2\sqrt{6})^n(5+2\sqrt{6})^n=-1$

Sol 22: (B) $(\sqrt{2}+\sqrt[4]{3})^{100} = (2^{1/2}+3^{1/4})^{100}$

L. C. M. of 2 and 4 =4

Total terms = $n+1=100+1=101$

T rational = ${}^{100}C_{4n}\left(2^{1/2}\right)^{100-4n}\left(3^{1/4}\right)^{4n}$

$\Rightarrow\ 0\le 100-4n\le 100$

$\Rightarrow\ 0\le n\le 25\ n\in N$

$n=\{0,1,2,3.....25\}$

Total number for $n=26$

Sol 23: (D) $\left(x\sin\theta+\frac{\cos\theta}{x}\right)^{10}$

$T_{r+1}={}^{10}C_r(x\sin\theta)^{10-r}\left(\frac{\cos\theta}{x}\right)^r$

Power of $x=10-r+r(-1)=10-2r=0$ (given)

$\Rightarrow$ r = 5

$T_{r+1}={}^{10}C_5(\sin\theta)^5(\cos\theta)^5={}^{10}C_5\ (\sin\theta\cos\theta)^5$

$= {}^{10}C_5\left(\frac{\sin2\theta}{2}\right)^5$. Max value when $\sin2\theta=1$

$\therefore$ Max. value = $\frac{{}^{10}C_5}{2^5}$

Sol 24: (B) $(1+x-3x^2)^{2145}=a_0+a_1x+a_2x^2+$

At x= – 1

$(1-1-3)^{2145}=-(3)^{2145}$

$=a_0-a_1+a_2-a_3+.......$

L. H. S. = 3^{2145} = 3.3^{2144} = $3[9]^{1072}$

Even power of 9 ends with 1. Hence 3^{2145} ends with 3.

Sol 25: (B) $\left(\frac{4x^2}{3}-\frac{3}{2x}\right)^9$

$T_{r+1}={}^9C_r\left(\frac{4x^2}{3}\right)^{9-r}\left(-\frac{3}{2x}\right)^r$

Power of $x=2(9-r)+(-1)r = 18-3r=6$

$\Rightarrow\ 3r=18-6=12\ \Rightarrow\ r = 4$

Coefficient ${}^9C_4\left(\frac{4}{3}\right)^{9-4}\left(\frac{-3}{2}\right)^4=\frac{9\times8\times7\times6}{1.2.3.4}\left(\frac{4}{3}\right)^5\left(\frac{3}{2}\right)^4$

$=9\times2\times7\times\frac{2^{10}\times3^4}{3^5\times2^4} = 21\times2^7 = 2688$

Sol 26: (C) $[x+(x^3-1)^{1/2}]^5+[x(x^3-1)^{1/2}]^5$

$= 2[{}^5C_0x^5+{}^5C_2x^{5-2}(x^3-1)+{}^5C_4x^{5-4}(x^3-1)^2]$

Max power of x = 7

Sol 27: (A) $(1-2x+5x^2-10x^3)(1+x)^n$

$= 1+a_1x-1a_2x^2+......$ and $a_1^2=2a_2$

Coefficient of $x=a_1={}^nC_1-2=n-2$

Co-efficient of $x^2=a_2=5+{}^nC_2-2{}^nC_1= 5+\frac{n(n-1)}{2}-2n$

$= 5+\frac{n^2-n-4n}{2}=5+\frac{n^2-5n}{2}$

$a_1^2=2a_2\ \Rightarrow\ (n-2)^2=2\left[\frac{10+n^2-5n}{2}\right]$

$\Rightarrow n^2+4-4n=10+n^2-5n \Rightarrow n=6$

Sol 28: (D) $aC_0+(a+b)C_1+(a+2b)C_2+.... +(a+nb)C_n$

$a(C_0+C_1+C_2+..... +C_n)$

$+b(C_1+2C_2+......+nC_n)$

$= a2^n+b[n2^{n-1}] = 2^{n-1}[2a+nb]$

Previous Years' Questions

Sol 1: (A) In the expansion

$(1+x)^{2n}, t_{3r} = {}^{2n}C_{3r-1}(x)^{3r-1}$

$t_{r+2} = {}^{2n}C_{r+1}(x)^{r+1}$

Since, binomial coefficient of t_{3r} and t_{r+2} are equal.

$\Rightarrow {}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$

$\Rightarrow 3r-1=r+1$ or $2n=(3r-1)+(r+1)$

$\Rightarrow 2r=2$ or $2n=4r$

$\Rightarrow r=1$ or $n=2r$

But r > 1,

$\therefore$ We take n = 2r

Sol 2: (A) We have $C_n^2 - 2C_1^2 + 3C_2^2 - 4C_3^2$

$$+.... + \left(-1\right)^n \left(n+1\right)C_n^2$$

$$= \{C_0^2 - C_1^2 + C_2^2 - C_3^2 + ... + \left(-1\right)^n C_n^2\}$$

$$-\{C_1^2 - 2C_2^2 + 3C_3^2 - .... + \left(-1\right)^n nC_n^2\}$$

$$= \left(-1\right)^{n/2} \cdot \frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!} - \left(-1\right)^{\frac{n}{2}-1} \frac{n}{2}\frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}$$

$$= \left(-1\right)^{n/2} \frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}\left(1 + \frac{n}{2}\right)$$

$$\therefore \frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}$$

$$\{C_0^2 - 2C_1^2 + 3C_2^2 - .... + \left(-1\right)^r \left(n+1\right)C_n^2\}$$

$$= \frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left(-1\right)^{n/2} \frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}\frac{\left(n+2\right)}{2} = \left(-1\right)^{n/2}\left(n+2\right)$$

Sol 3: (C) We know that $\left(a+b\right)^5 + \left(a-b\right)^5$

$$= {}^5C_0a^5 + {}^5C_1a^4b + {}^5C_2a^3b^2$$

$$+ {}^5C_3a^2b^3 + {}^5C_4ab^4 + {}^5C_5b^5 + {}^5C_0a^5 - {}^5C_1a^4b$$

$$+ {}^5C_2a^3b^2 - {}^5C_3a^2b^3 + {}^5C_4ab^4 - {}^5C_5b^5$$

$$= 2\left[a^5 + 10a^3b^2 + 5ab^4\right]$$

$$\therefore \left[x + \left(x^3 - 1\right)^{1/2}\right]^5 + \left[x - \left(x^3 - 1\right)^{1/2}\right]^5$$

$$= 2\left[x^5 + 10x^3\left(x^3 - 1\right) + 5x\left(x^3 - 1\right)^2\right]$$

Therefore, the given expression is a polynomial of degree 7.

Sol 4: (D) ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$

$$= \left({}^nC_r + {}^nC_{r-1}\right) + \left({}^nC_{r-1} + {}^nC_{r-2}\right)$$

We know that

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

$$\therefore {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r$$

Sol 5: (B) $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2}$

$$= \left[\binom{n}{r} + \binom{n}{r-1}\right] + \left[\binom{n}{r-1} + \binom{n}{r-2}\right]$$

$$= \binom{n+1}{r} + \binom{n+1}{r-1} = \binom{n+2}{r}$$

According to given condition, $T_n = {}^nC_3$

and $T_{n+1} - T_n = 21$

$$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$$

$$\Rightarrow \frac{1}{6}\left(n+1\right)\left(n\right)\left(n-1\right) - \frac{1}{6}n\left(n-1\right)\left(n-2\right) = 21$$

$$\Rightarrow \frac{n\left(n-1\right)}{6}\left[\left(n+1\right) - \left(n-2\right)\right] = 21$$

$$\Rightarrow \frac{n\left(n-1\right)}{6} = 21 \Rightarrow n\left(n-1\right) = 42$$

$$\Rightarrow n = 7$$

Sol 6: (B) Given, ${}^{n-1}C_r = \left(k^2 - 3\right){}^nC_{r+1}$

$$\Rightarrow {}^{n-1}C_r = \left(k^2 - 3\right)\frac{n}{r+1}\,{}^{n-1}C_r$$

$$\Rightarrow k^2 - 3 = \frac{r+1}{n}$$

$$(\text{Since}, n \ge r \Rightarrow \frac{r+1}{n} \le 1 \text{ and } n, r > 0)$$

$$\Rightarrow 0 < k^2 - 3 \le 1$$

$$\Rightarrow 3 < k^2 \le 4$$

$$\Rightarrow k \in \left[-2, -\sqrt{3}\right) \cup \left(\sqrt{3}, 2\right]$$

Sol 7: (C) Let $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11}$

$$+ \binom{30}{2}\binom{30}{12} - .... + \binom{30}{20}\binom{30}{30}$$

$$\therefore A = {}^{30}C_0.{}^{30}C_{10} - {}^{30}C_1.{}^{30}C_{11}$$

$$+ {}^{30}C_2.{}^{30}C_{12} - .... + {}^{30}C_{20}.{}^{30}C_{30}$$

= Coefficient of x^{20} in $(1 + x)^{30}.(1 - x)^{30}$

= Coefficient of x^{20} in $(1 - x^2)^{30}$

= Coefficient of x^{20} in $\sum_{r=0}^{30}(-1)^r\,{}^{30}C_r\left(x^2\right)^r$

∴ For coefficient of x^{20} clearly $2r = 20 \Rightarrow r = 10$

Put $(r = 10) = {}^{30}C_{10}$

Sol 8: (D) A_r = Coefficient of x^r in $(1 + x)^{10} = {}^{10}C_r$

B_r = Coefficient of x^r in $(1 + x)^{2n} = {}^{20}C_r$

C_r = Coefficient of x^r in $(1 + x)^{30} = {}^{30}C_r$

$$\therefore \sum_{r=1}^{10} A_r\left(B_{10}B_r - C_{10}A_r\right) = \sum_{r=1}^{10} A_rB_{10}B_r - \sum_{r=1}^{10} A_rC_{10}A_r$$

$$= \sum_{r=1}^{10} {}^{10}C_r\,{}^{20}C_{10}\,{}^{20}C_r - \sum_{r=1}^{10} {}^{10}C_r\,{}^{30}C_{10}\,{}^{10}C_r$$

$$= \sum_{r=1}^{10} {}^{10}C_{10-r}\cdot{}^{20}C_{10}\,{}^{20}C_r - \sum_{r=1}^{10} {}^{10}C_{10-r}\,{}^{30}C_{10}\,{}^{10}C_r$$

$$= {}^{20}C_{10}\sum_{r=1}^{10} {}^{10}C_{10-r}.{}^{20}C_r - {}^{30}C_{10}\sum_{r=1}^{10} {}^{10}C_{10-r}\,{}^{10}C_r$$

$$= {}^{20}C_{10}\left({}^{30}C_{10} - 1\right) - {}^{30}C_{10}\left({}^{20}C_{10} - 1\right)$$

$$= {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$$

Sol 9: (D) $\left(1 + ax + bx^2\right)$

$$\left[1 - {}^{18}C_1 2x + {}^{18}C_2(2x)^2 - {}^{18}C_3(2x)^3 + {}^{18}C_4(2x)^4\right]$$

Coefficient of x^3 is

$$-{}^{18}C_3\left(2^3\right) + a\left({}^{18}C_2 \times 4\right) - b\left({}^{18}C_1 \times 2\right) = 0 \qquad ...(i)$$

Coefficient of x^4 is

$${}^{18}C_4\left(2^4\right) + a\left(-{}^{18}C_3 \times 2^3\right) + {}^{18}C_2 b2^2 = 0 \qquad ....(ii)$$

or solving both these equation

a = 16 and b = 272/3.

Sol 10: (A)

$$\left(1 - 2\sqrt{x}\right)^{50} = {}^{50}C_0 - {}^{50}C_1\left(2\sqrt{x}\right)^1 + {}^{50}C_2\left(2\sqrt{x}\right)^2$$
$$-{}^{50}C_3\left(2\sqrt{x}\right)^3 + {}^{50}C_4\left(2\sqrt{x}\right)^4$$

So, sum of coefficient Integral powers of x

$$S = {}^{50}C_0 + {}^{50}C_2.2^2 + {}^{50}C_4.2^4 + ... + {}^{50}C_{50}.2^{50}$$

Now,

$$(1 + x)^{50} = 1 + {}^{50}C_1x + {}^{50}C_2x^2 + {}^{50}C_3x^3 + {}^{50}C_4x^4$$
$$+..... + {}^{50}C_{50}x^{50}$$

Put x = 2, - 2

$$3^{50} = 1 + {}^{50}C_1.2 + {}^{50}C_2.2^2 + {}^{50}C_3.2^3$$
$$+{}^{50}C_4.2^4 + ... + {}^{50}C_{50}.2^{20} \qquad ....(i)$$

$$1 = 1 - {}^{50}C_1.2 + {}^{50}C_2.2^2 - {}^{50}C_3.2^3$$
$$+{}^{50}C_4 2^4 - .... + {}^{50}C_{50}.2^{50} \qquad ....(ii)$$

(i) + (ii)

$$3^{50} + 1 = 2\left[1 + {}^{50}C_2.2^2 + {}^{50}C_4.2^4 + ... + {}^{50}C_{50}.2^{50}\right]$$

$$\therefore \frac{3^{50} + 1}{2} = 1 + {}^{50}C_2.2^2 + {}^{50}C_4.2^4 + ... + {}^{50}C_{50}.2^{50}$$

Sol 11: (D) Number of terms $= \frac{(n+1)(n+2)}{2} = 28$

$\Rightarrow n = 6$

$$\therefore a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + ... + \frac{a_{2n}}{x^{2n}} = \left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$$

Put x = 1, n = 6,

$$a_0 + a_1 + a_2 + ... + a_{2n} = 3^6 = 729$$

JEE Advanced/Boards

Exercise 1

Sol 1: $f(x)=1-x+x^2-x^3+......x^{16}-x^{17}$

$= a_0+a_1(1+x)+a_2(1+x)^2 +....... +a_{17}(1+x)^{17}$

Differentiating both sides

$-1 + 2x\ ...\ -17x^{16} = a_1 + 2a_2(1 + x) + ... + 17a_{17}(1 + x)^{16}$

Again differentiating

$2 - 6x + ... = 2a_2 + 6a_3(1 + x) + ...$

Putting x = –1

$\Rightarrow 2 + 6 + 12 + 20 + \ldots\ldots. + 17 \times 16 = 2a_2$

$2a_2 = 1.\ 2 + 2.\ 3 + 3.\ 4\ldots\ldots. + 16.\ 17$

T_n for 1. 2+2. 3+3. 4 is $T_n = n(n+1)$

$$2a_2 = \sum_{i=1}^{16} T_n = \sum_{i=1}^{16} n^2 + \sum_{i=1}^{16} n$$

$$= \frac{(2(16)+1)16(16+1)}{6} + \frac{16(16+1)}{2} = 1632$$

$\Rightarrow a_2 = 816$

Sol 2: (a) (i) $\left(\sqrt{\frac{x}{3}} + \frac{\sqrt{3}}{2x^2}\right)^{10}$

$$T_{r+1} = {}^{10}C_r \left(\frac{\sqrt{x}}{\sqrt{3}}\right)^{10-r} \left(\frac{\sqrt{3}}{2x^2}\right)^r$$

$$= {}^{10}C_r \left(\frac{1}{\sqrt{3}}\right)^{10-r} x^{\frac{10-r}{2}-2r} \left(\frac{\sqrt{3}}{2}\right)^r$$

For term independent of x-

$\frac{10-r}{2} - 2r = 0 \Rightarrow 10 - r - 4r = 0 \Rightarrow r = 2$

So $T_3 = T_{2+1} = {}^{10}C_2 \left(\frac{1}{\sqrt{3}}\right)^{10-2} \left(\frac{\sqrt{3}}{2}\right)^2$

$$= \frac{10 \times 9}{2} \times \frac{1}{3^4} \times \frac{3}{4} = \frac{5}{12}$$

(ii) $\left[\frac{1}{2}x^{1/3} + x^{-1/5}\right]^8$

$$T_{r+1} = {}^8C_r \left(\frac{1}{2}x^{1/3}\right)^{8-r} (x^{-1/5})^r$$

Power of x = $\frac{8-r}{3} - \frac{r}{5} = 0$ for independence

$\Rightarrow 5(8 - r) - 3r = 0 \Rightarrow 40 - 5r - 3r = 0 \Rightarrow r = 5$

$$T_{5+1} = T_6 = {}^8C_5 \left[\frac{1}{2}(x^{1/3})\right]^{8-5} .\ (x^{-1/5})^5 = \frac{8 \times 7 \times 6}{1.2.3} \times \left(\frac{1}{2}\right)^3 = 7$$

(b) $\left(5^{\frac{2}{5}\log_5 \sqrt{4^x+44}} + \frac{1}{5^{\log_5 \sqrt[3]{2^{x-1}+7}}}\right)^8$

$= (a_1+a_2)^8$ assume

$$T_4 = T_{3+1} = {}^8C_3 (a_1)^{8-3} (a_2)^3$$

$$a_1 = 5^{\log_5 (\sqrt{4^x+44})^{2/5}} = ((4^x + 44)^{1/2})^{2/5} = (4^x + 44)^{1/5}$$

$$a_2 = \frac{1}{5^{\log_5 (2^{x-1}+7)^{1/3}}} = \frac{1}{(2^{x-1} + 7)^{1/3}} = (2^{x-1} + 7)^{-1/3}$$

$$T_4 = {}^8C_3 (4^x + 44)^{5/5} (2^{x-1} + 7)^{-3/3}$$

$$= \frac{8 \times 7 \times 6}{1.2.3} \times (4^x + 44)(2^{x-1} + 7)^{-1} = 336$$

$$\Rightarrow \frac{4^x + 44}{2^{x-1} + 7} = \frac{336}{8x7} = 6$$

$\Rightarrow 4^x + 44 = 6 \times 2^{x-1} + 6 \times 7 = 3.2^x + 42$

$(2^x)^2 - 3(2)^x + 44 - 42 = 0$

Assume $2^x = y$

$y^2 - 3y + 2 = 0 \Rightarrow (y - 2)(y - 1) = 0$

$\Rightarrow y = 1$ or $y = 2 \Rightarrow x = 0$ or $x = 1$

Sol 3: $\left(ax^2 + \frac{1}{bx}\right)^{11}$

$$T_{r+1} = {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx}\right)^r$$

Power of $x = 2(11 - r) + r(-1) = 7$ (given)

$\Rightarrow 22 - 2r - r = 7 \Rightarrow r = 5$

Coefficient $T_{5+1} = {}^{11}C_5 (a)^{11-5} \left(\frac{1}{b}\right)^5 = {}^{11}C_5 a^6 b^{-5}$

(ii) $\left(ax - \frac{1}{bx^2}\right)^{11}$

$$T_{r+1} = {}^{11}C_r (ax)^{11-r} \left(-\frac{1}{bx^2}\right)^r$$

Power of x = $11 - r - 2r = -7$ (given)

$\Rightarrow r = 6$

Coefficient $T_{r+1} = T_{6+1} = {}^{11}C_6 a^{11-6} \left(-\frac{1}{b}\right)^6 = {}^{11}C_6 a^5 b^{-6}$

(iii) Given that both coefficient are equal

$\Rightarrow {}^{11}C_5 a^6 b^{-5} = {}^{11}C_6 a^5 b^{-6} \Rightarrow ab = 1$

Sol 4: (a) $(1+x)^{14}$

Coefficients $T_r = T_{(r-1)+1} = {}^{14}C_{r-1}$

$T_{r+1}= {}^{14}C_r;\ T_{(r+1)+1}={}^{14}C_{r+1}$

Its given that they are in A. P. so. $T_r+T_{r+2}=2T_{r+1}$

${}^{14}C_{r-1}+{}^{14}C_{r+1}=2\ {}^{14}C_r$

$$\frac{14!}{(r-1)!(14-r+1)!}+\frac{14!}{(r+1)!(14-r-1)!}=2\frac{14!}{r!(14-r)!}$$

$$\Rightarrow \frac{1}{(15-r)(14-r)}+\frac{1}{(r+1)r}=\frac{2}{r(14-r)}$$

$$\Rightarrow \frac{r(r+1)+(15-r)(14-r)}{r(r+1)(14-r)(15-r)}=\frac{2}{r(14-r)}$$

$\Rightarrow\ r^2+r+210-14r-15r+r^2=2(r+1)(15-r)$

$\Rightarrow\ 2r^2-28r+210=30r-2r^2+30-2r$

$\Rightarrow\ 4r^2-56r+180=0 \Rightarrow\ r^2-14r+45=0$

$\Rightarrow\ (r-9)(r-5)=0 \Rightarrow r=9$ or $r=5$

(b) $(1+x)^{2n}$

Coefficients $T_2=T_{1+1}={}^{2n}C_1=2n$

$$T_3=T_{2+1}={}^{2n}C_2=\frac{2n(2n-1)}{1.2}=n(2n1)$$

$$T_4={}^{2n}C_3=\frac{2n(2n-1)(2n-2)}{1.2.3}=\frac{n(2n-1)(2n-2)}{3}$$

They all are in A. P.

So, $T_2+T_4=2T_3$

$$2n+\frac{n(2n-1)(2n-2)}{3}=2n(2n-1)$$

$\Rightarrow 3+(n-1)(2n-1)=3(2n-1)=6n-3$

$\Rightarrow 3+2n^2-2n-n+1=6n-3$

$\Rightarrow 2n^2-9n+7=0$

Sol 5: a = Coefficient of x^3 in $(1+x+2x^2+3x^3)^4$

b = Coefficient of x^3 in $(1+x+2x^2+3x^3+4x^4)^4$

$4x^4$ has no effect on the coefficient of x^3.

Hence a = b

∴ a − b = 0

Sol 6: $(1-x^2)^{10}$

$T_{r+1}={}^{10}C_r(-x^2)^r$

Given that $2r=10 \Rightarrow r=5$

So coefficient is= $(1)^r\ {}^{10}C_r = {}^{10}C_5$

And in $\left(x-\frac{2}{x}\right)^{10}$

$$T_{r+1}={}^{10}C_r(x)^{10-r}\left(-\frac{2}{x}\right)^r$$

Power of x = $10-r-r=0$

$\Rightarrow\ 10-2r=0\ \Rightarrow\ r=5$

Coefficient = ${}^{10}C_5(-2)^5=-{}^{10}C_5 2^5$

Ratio of both coefficients = $\frac{{}^{10}C_5}{{}^{10}C_5 2^5}=\frac{1}{2^5}=\frac{1}{32}$

Sol 7: (a) $(ax-by+cz)^9$

General term $=\frac{9!}{r_1!r_2!r_3!}(ax)^{r_1}(-by)^{r_2}(cz)^{r_3}$

$r_1+r_2+r_3=9$

For coefficient of $x^2y^3z^4$ so $\Rightarrow r_1=2,\ r_2=3,\ r_3=4$

So Coefficient $=\frac{9!}{2!3!4!}\times a^2\cdot b^3c^4$

$= -1260\ a^2\cdot b^3\cdot c^4$

(b) $(a-b-c+d)^{10}$

General Term= $\frac{10!}{r_1!r_2!r_3!r_4!}(a)^{r_1}(-b)^{r_2}(-c)^{r_3}(d)^{r_4}$

$r_1+r_2+r_3+r_4=10$

It given that $r_1=2,\ r_2=3,\ r_3=4,\ r_4=1$

Coefficient $\frac{10!}{2!3!4!1!}(-1)^3(-1)^4$

$$=-\frac{10\times9\times8\times7\times6\times5}{2!3!}=-12600$$

Sol 8: $s_n=1+q+q^2+\ldots\ldots+q^n=$

$$S_n=1+\frac{q+1}{2}+\left(\frac{(q+1)}{2}\right)^2+\ldots+\left(\frac{q+1}{2}\right)^n,q\neq1$$

$={}^{n+1}C_1+{}^{n+1}C_2\ s_1+{}^{n+1}C_3 s_2$

$+\ldots\ldots+\ {}^{n+1}C_{n+1}s_n$

Constant term

${}^{n+1}C_1+{}^{n+1}C_2+\ldots\ldots+{}^{n+1}C_{n+1}=2^{n+1}-1$

In S_n constant term = $1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+.....\left(\frac{1}{2}\right)^n$

$$=\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}=2\left(1-\frac{1}{2}\right)^{n+1}=\frac{(2^{n+1}-1)}{2^n}$$

So $(2^{n+1}-1)=(2^n),\left(\frac{(2^{x+1}-1)}{2^n}\right)$

$$^{n+1}C_1+{}^{n+1}C_2S_1+{}^{n+1}C_3S_n=2^n\,S_n$$

We can prove this with other terms also.

Sol 9: (i) $(2+3x)^9, x=\frac{3}{2}$. Now we have

$$\frac{n+1}{1+\left|\frac{a}{x}\right|}=\frac{9+1}{1+\frac{2\times 2}{3\times 3}}=\frac{10}{1+\frac{4}{9}}$$

$$=\frac{10}{1+0.44}=\frac{10}{1.44}=6.944$$

Greatest terms is

$T_7=T_{6+1}={}^9C_6(2)^{9-6}(3x)^6=\frac{9\times 8\times 7}{1.2.3}2^3\times 3^6\left(\frac{3}{2}\right)^6$

$$=\frac{3^2.7.3^6.3^6}{1.2.3}=\frac{7.3^{13}}{2}$$

(ii) $(3–5x)^{15}$ When $x=\frac{1}{5}$

$$\frac{n+1}{1+\left|\frac{a}{x}\right|}=\frac{15+1}{1+\left|\frac{3\times 5}{5\times 1}\right|}=\frac{16}{1+3}=\frac{16}{4}=4$$

So T_4 and T_{4+1} are same greatest term. $T_4={}^{15}C_4(3)^{15-4}(-5x)^4$

$$=\frac{15\times 14\times 13\times 12}{1.2.3.4}3^{11}\left(\frac{-5}{5}\right)^4=455.\,3^{12}$$

Sol 10: (i) $(1+x+x^2)^n=a_0++a_1x+.......+a_{2n}x^{2n}$

(i) at x=1

$a_0+a_1+a_2+a_3+.......+a_{2n}=(1+1+1)^n=3^n$

(ii) at x=–1 $\Rightarrow$ $[1–1+(–1)^2]^n=1$

$\Rightarrow 1=a_0–a_1+a_2–a_3+.......+a_{2n}$

(iii) $(1+x+x^2)^n(x^2–x+1)^{2n}$

$=(a_0x^{2n}–a_1x^{2n–1}+.......)(a_0–a_1x+.....)$

Compare $x\to -x$ in

$(x^2+x+1)\to(x^2–x+1)\ a_0-a_1x+a_2x^2-a_3x^3+.....+a_{2x}\,x^{2x}$

$[(1+x+x^2)(x^2–x+1)]^n$

$=a_0^2x^{2n}-a_1^2x^{2n}-a_3\ x^{2n}+a_4\cdot x^{2n}-\ .....+a_{2n}\,x^{2n}$

$\therefore$ For x = 1

$a_0^2-a_1^2-a_3^2+....+a_{2n}^2=3^n$

Sol 11: $(5+3x)^{10}$

$T_4={}^{10}C_3(5)^{10-3}(3x)^3$

$={}^{10}C_35^73^3x^3\ \ {}^{10}C_35^73^3x^3$ is the greatest term

So, $\frac{n+1}{1+\left|\frac{a}{x}\right|}=\frac{10+1}{1+\left|\frac{5}{3x}\right|}$

For greatest term to be T_4

$$=3<\frac{10+1}{1+\frac{5}{3x}}<4$$

$$3<\frac{33x}{3x+5}<4$$

$3(3x+5)<33x<4(3x+5)$

$9x+15<33x<12x+20$

Solving each inequality separately we get

$9x+15<33x$

$\Rightarrow 24x>15$

$\Rightarrow x>\frac{15}{24}$

$\Rightarrow x>\frac{5}{8}$

Also, $12x+20>334$

$\Rightarrow x<\frac{20}{21}$

$\therefore\ \frac{5}{8}<x<\frac{20}{21}$

Sol 12: In the expansion of $\left(\frac{x}{5}+\frac{2}{5}\right)^n$, we have

$$T_9={}^nC_8\left(\frac{x}{5}\right)^{n-8}\left(\frac{2}{5}\right)^8$$

Coefficient = ${}^nC_8(5)^{8-n}2^85^{-8}$ = ${}^nC_8 5^{-n}2^8$

Which is greatest coefficient

$8 < \frac{n+1}{1+\left|\frac{x}{a}\right|} < 9$ assume x=1 for find

$8 < \frac{n+1}{1+\frac{1\times5}{5\times2}} < 9$ greatest coefficient

$8 < \frac{n+1}{1+\frac{1}{2}} < 9 = 8 < \frac{n+1}{\frac{3}{2}} < 9$

$8\times\frac{3}{2} < n+1 < 9\times\frac{3}{2}$

$12 < n+1 < \frac{27}{2}$

$17 < n < \frac{22}{2} - 1 = \frac{25}{2} = 12.5$

$11 < n<12.5$

There is only one natural no. in region i.e., 12

Sol 13: N=${}^{2000}C_1$+2. ${}^{2000}C_2$+3. ${}^{2000}C_3$+......+2000${}^{2000}C_{2000}$

N=n. 2^{n-1} here n = 2000

⇒ N= 2000 x2^{n-1}

$\Rightarrow N = 2\times(2\times5)^3\times2^{n-1} = 2^3\times5^32^n$

⇒ N=2×$(2\times5)^3$×2^{n-1}=2^3×$5^3 2^n$

N = $2^{n+3}5^3$

Number of divisors = (n + 3 + 1)(3 + 1)

= (2000 + 4)(4) = 2004 × 4 = 8016

Sol 14: $(1+x)^{2012}+(1+x^2)^{2011}+(1+x^3)^{2010}$

Number of different dissimilar terms

= 2012+2011+2010 (no. of terms which is common in $(1+x)^{2012}$ and $(1+x^2)^{2011}$

of terms which is similar $(1+x)^{2012}$ and $(1+x^3)^{2010}$)

(no. of terms which is similar in $(1+x^2)^{2011}+(1+x^3)^{2010}$)

$$= 2012+2011+2010-\left[\frac{2011}{2}\right]-\left[\frac{2012+1}{3}\right]-\left[\frac{1005}{3}\right]+1$$

Where (+1) for constant term. And [x] is a singularity function. [1. 35] = 1

= 6034 – 1005 – 671 -335 = 4023

Sol 15: $(1+x+2x^3)\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$

For independent terms

Coefficient of x^0 in $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$ = A_0

Coefficient of x^{-1} in $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$ = A_1

Coefficient of x^{-3} in $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$ = A_2

$$T_{r+1} = {}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(-\frac{1}{3x}\right)^r$$

Power of x = 2(9–r) –r = 18 – 2r – r = 18 – 3r

$x^0 \Rightarrow 18-3r=0 \Rightarrow r=\frac{18}{3}=6$

$$T_6 = T_{5+1} = A_0 = {}^9C_6\left(\frac{3}{2}\right)^{9-5}\left(-\frac{1}{3}\right)^6$$

$A_0 = \frac{7}{18}$

For x^{-1} = $18-3r=-1$

$3r = 18+1 = 19$

r=19/3 not natural no.

So $A_1 = 0$

For x^{-3}

18–3r=–3 ⇒ 3r=18+3=21 ⇒ $r = \frac{21}{3} = 7$

So $A_2 = {}^9C_7\left(\frac{3}{2}\right)^{9-7}\left(-\frac{1}{3}\right)^7 = \frac{-1}{27}$

So coefficient of x^0 in $(1+x+2x^3)\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$

$$= \frac{7}{18}+2\times\left(-\frac{1}{27}\right) = \frac{21-2(2)}{54} = \frac{17}{54}$$

Sol 16: $f(n)=\sum_{r-0}^{n}\sum_{k-r}^{n}{}^{k}C_{r}$

For $f(11)=\sum_{r-0}^{11}\sum_{k-r}^{11}{}^{k}C_{r}$

$= ({}^{0}C_{0}+{}^{1}C_{0}+{}^{2}C_{0}+.......+{}^{11}C_{0})$

$+{}^{1}C_{1}+{}^{2}C_{1}+.........+{}^{1}C_{1}$

$+{}^{2}C_{2}+{}^{3}C_{2}+........+{}^{11}C_{2}$ :::: :::: ::::

${}^{10}C_{10}+{}^{11}C_{10}$

${}^{11}C_{11}$

$= {}^{11}C_{0}+.......+{}^{11}C_{11} \ +{}^{10}C_{0}+......+{}^{10}C_{10}$ ::

$+{}^{1}C_{1} \ +{}^{0}C_{0}$

$= 2^{11}+2^{10}+2^{9}+.......+2^{1}+2^{0}$

$= \frac{2^{11+1}-1}{2-1}=4095 \ = 4095 = 5^{1}.3^{2}\,7^{1}.\,13^{1}$

No. of divisors = (1+1). (2+1). (1+1)(1+1)

= 2×3×4 = 24

Sol 17: $\sum_{j-0}^{11}\sum_{i-j}^{11}{}^{i}C_{j}$

$={}^{0}C_{0}+\left({}^{1}C_{0}+{}^{1}C_{1}\right)+\left({}^{2}C_{0}+....+{}^{2}C_{2}\right)$

$+\left({}^{3}C_{0}+....+{}^{3}C_{3}\right)+.....+\left({}^{11}C_{0}+{}^{11}C_{1}+....+{}^{11}C_{11}\right)$

$=2^{0}+2^{1}+....+2^{11}$

$=2^{12}-1$

Sol 18: $(1+x^{2}).(1+x)^{n}=\sum_{k=0}^{n+4}a_{k}.x^{k}$

a_1, a_2 and a_3 are in A.P

$(1+x^{4}+2x^{2})({}^{n}C_{0}+{}^{n}C_{1}x+{}^{n}C_{2}x^{2}+{}^{n}C_{3}x^{3}+......{}^{n}C_{n}x^{n})$

$= a_{0}+a_{1}x+a_{2}x^{2}+a_{3}x^{3}+....$

Compare terms of x^{r} in both side

$x^{1}\Rightarrow$ L. H. S. = ${}^{n}C_{1}=n$

R. H. S. = a_{1}

$\Rightarrow n=a_{1}$ .... (i)

$x^{2}\Rightarrow 2{}^{n}C_{0}+{}^{n}C_{2}=a_{2}$

$2+\frac{n(n-1)}{2}=a_{2}$ .... (ii)

$x^{3}\Rightarrow 2\,{}^{n}C_{1}+{}^{n}C_{3}=a_{3}$

$2n+\frac{n(n-1)(n-2)}{1.2.3}=a_{3}$ .... (iii)

It's given that a_1, a_2, a_3 are in A.P

$2a_2 = a_1 + a_3$

$4+n(n-1)=n+2+\frac{n(n-1)\,(n-2)}{6}$

$=\frac{6n+12n+n(n-1)\,(n-2)}{6}$

24 + 6n(n–1) = 18n + n(n–1) (n–2)

Solving this we get, n = 2 or 3 or 4

Sol 19: $\sum_{k=0}^{n}{}^{n}C_{k}$ sin kx. cos (n-k) x = 2^{n-1} sin nx

L. H. S. = $\sum_{k=0}^{n}{}^{n}C_{k}\ \sin kx\cos(n-k)x$

We know that 2 sin A cos B = sin(A+B) +sin (A–B)

∵ A + B = kx + (n – k)x = kx + nx – kx = nx

A – B = kx – (n – k)k = kx – nx + kx = 2kx – nx

So, $\sum_{k=0}^{n}\frac{1}{2}{}^{n}C_{k}[\sin nx+\sin(2kx-nx)]$

$=\sum_{k=0}^{n}\frac{1}{2}{}^{n}C_{k}\sin nx+\sum_{k=0}^{n}\frac{1}{2}{}^{n}C_{k}\sin(2kx-nx)$

$=\frac{1}{2}\sin nx\ \sum_{k=0}^{n}{}^{n}C_{k}+\frac{1}{2}$

$[{}^{n}C_{0}\sin(-nx)+......+{}^{n}C_{n}\sin(nx)]$

$=\frac{1}{2}\sin nx.2^{n}+0= (\sin nx)2^{n-1}= 2^{n-1}\sin nx$

Sol 20: $x^{2001}+\left(\frac{1}{2}-x\right)^{2001}=0$

$= x^{2001}+\left[{}^{2001}C_{0}\left(\frac{1}{2}\right)^{2001}+.....+{}^{2001}C_{1999}\right.$

$\left.\left(\frac{1}{2}\right)^{2}(-x)^{1999}+{}^{2001}C_{2000}\left(\frac{1}{2}\right)(-x)^{2000}\right]$

$= x \ +{}^{2001}C_{2001}\left(\frac{1}{2}\right)^{0}(-x)^{2001}$

$$= x^{2001} + ... + {}^{2001}C_{1999}\frac{(-x)^{1999}}{4}$$

$$+ {}^{2001}C_{2000}\frac{(-x)^{2000}}{2} - x^{2001}$$

= Now maximum pointer of x = 2000

Sum of all solution is = $\frac{\text{Coefficient of } x^{2001-1}}{\text{Coefficient of } x^{2000}}$

$$= \frac{{}^{2001}C_{1999}\times\frac{1}{4}}{{}^{2001}C_{2000}\times\frac{1}{2}} = \frac{2001\times2000\times1}{1.2\times2001\times1}\times\frac{1}{2} = 500$$

Sol 21: Let

$$S = \sum_{k=0}^{2n}(-1)^k\ (k - 2n)\ ({}^{2n}C_k)^2 \qquad ...(i)$$

$$\Rightarrow \quad S = \sum_{k=0}^{2n}(-1)^{2n-k}\ (2n - k)\ ({}^{2n}C_{2n-k})^2$$

Writing the terms in S in the reverse order, we get

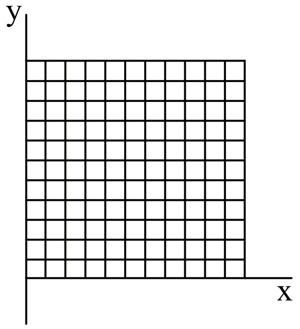
$$S = \sum_{k=0}^{2n}(-1)^k\ k\ ({}^{2n}C_k)^2 \qquad ...(ii)$$

Adding (i) and (ii) we get

$$2S = 2n\sum_{k=0}^{2n}(-1)^k\ ({}^{2n}C_k)^2 = -2nA$$

$$\Rightarrow \quad S = -nA$$

Sol 22: (A) $(\sum_{i=0}^{10}P(i,10-i)$



$P(0,10)+P(1,9)+......+P(10,0)$

$$= 1+(9+1)(1)+\frac{10\times9}{2}+....$$

$$= {}^{10}C_0 - {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + ... + {}^{10}C_{10}$$

$= 2^{10} = 1024$

Sol 23: (C) P(i,100–i) = P(j,100–j)

${}^{100}C_i = {}^{100}C_j$ and $i \ne j$

${}^{100}C_i = {}^{100}C_{100-j}$

I = 100 – j

i+j = 100

$i, j \in N, 0$

(0,100)(1,99)....... (99,1)(100,1)

Total no. of ordered pairs

(i,j) = 100

Sol 24: $(6\sqrt{6}+14)^{2n+1} = I + F$

(assume)

$$(6\sqrt{6}-14) = \frac{(6\sqrt{6}^{2})-(14)^2}{6\sqrt{6}+14}$$

$$= \frac{20}{6\sqrt{6}+14}$$

$I = [(6\sqrt{6}+14)^{2n+1}] = 0 < F < 1$

$e = (6\sqrt{6}-14)^{2n+1} = 0 < e < 1$

$I + F - e = (6\sqrt{6}+14)^{2n+1}$

$-(6\sqrt{6}-14)^{2n+1}$

$= 2({}^{n}C_1(6\sqrt{6})^{2n+1}14$

$+{}^{n}C_3(6\sqrt{6})^{2n-11-3}14^3 + ...)$

= 2K (K is const. integer)

$0 \le F - e < 1$

F – e=2K–I= Integer

$F - e = 0 = e = F = F = e = (6\sqrt{6}-14)^{2n+1}$

$$F = \frac{(20)^{2n+1}}{(6\sqrt{6}+14)^{2n+1}}$$

$(I+F)F = (6\sqrt{6}+14)^{2n+1}$

$$\frac{20^{2n+1}}{(6\sqrt{6}+14)^{2n+1}}$$

$(I+F)F = 20^{2n+1}$

Sol 25: $P=(2+\sqrt{3})^5$

$f = P - [P]$

$$2-\sqrt{3}=\frac{2^2-(\sqrt{3})^2}{2+\sqrt{3}}=\frac{1}{2+\sqrt{3}}$$

$\because 0<2-\sqrt{3}<1$

$\because 0<f<1$

$$(2+\sqrt{3})^5=\left(\frac{1}{2-\sqrt{3}}\right)^5=\frac{1}{F}$$

$$[P]+f+f=(2+\sqrt{3})^5+(2-\sqrt{3})^5$$

$$=\ 2[{}^5C_0 2^5+{}^5C_2 2^{5-2}(\sqrt{3})^2$$

$$+\ldots\ldots+{}^5C_4 2^{5-4}(\sqrt{3})^4]$$

$$2\left[2^5+\frac{5\times 4}{2}\times 2^3\times 3+5\times 2\times 3^2\right]$$

f+f = Integer

$0\le f+f<2$

$\therefore$ f+f = 1

$f=1-f=(2-\sqrt{3})^5$

$f=1-(2\sqrt{3})^5$

$$\frac{f^2}{1-f}=\frac{f^2-1^2+1}{1-f}=\frac{(f-1)^{-1}(f+1)}{(1-f)}+\frac{1}{(1-f)}$$

$$=\ -(f+1)+\frac{1}{f}$$

$f=1-f=f-2=-(f+1)$

$$=\ f-2+\frac{1}{f}$$

$$=\ (2+\sqrt{3})^5-2+(2-\sqrt{3})^5$$

$= 2[32+15\times2^4+5\times2\times3^2]-2$

$= 724 - 2 = 722$

Sol 26: $(1+x)^n={}^nC_0+C_1+$

$C_2x^2+C_3x^3+\ldots.\ +C_nx^n$ ... (i)

(a) Differentiating at both sides

$n(1+x)^{n-1}=C_1+2C_2x+\ldots.+{}^3C_3\,x^{2n}$

$x=1$

Put $n\cdot 2^{n-1}=C_1+2C_2+\ldots\ldots+nC_n$ ... (ii)

(b) Sum of eq. (i) and (ii)

$n2^{n-1}+(1+x)^n=C_1+{}^2C_2+\ldots..+{}^nC_n$

$+C_0+C_1x+C_2x^2+\ldots..+C_nx^n$

At $x=1$

$2^{n-1}(n+2)=C_0+2C_1$

$+3C_2+\ldots\ldots+(n+1)C_n$

(c) Eq. (1) + 2 × eq. (2)

At $x=1$

$2^n+n2^n=C_0+C_1(1\times2+1)$

$+C_2(2\times2+1)+\ldots\ldots.+C_n(2n+1)$

$2^{n-1}(n+2)=C_0+2C_1$

$+7C_3+\ldots\ldots..+(2n+1)C_n$

(d) $(C_0+C_1)(C_1+C_2)\ldots\ldots(C_{n-1}+C_n)=$

$$\frac{C_0C_1C_2.C_{n-1}(n+1)^{3n}}{n!}$$

Multiply and divide L. H. S. by

$C_0C_1C_2C_3\ldots\ldots.\ C_{n-1}$

$$=C_0C_1C_2\ldots\ldots..C_{n-1}\left(1+\frac{C_1}{C_0}\right)\ldots\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$$

On using $\dfrac{{}^nC_r}{{}^nC_{r-1}}=\dfrac{n-r+1}{r}$ = L. H. S.

$$C_0C_1C_2\ldots..C_{n-1}(1+n)\left(\frac{1+n}{2}\right)\left(\frac{1+n}{3}\right)\ldots.\left(\frac{1+n}{n}\right)$$

$$=\ \frac{C_0C_1C_2\ldots..C_{n-1}(n+1)^n}{n!}$$

(e) $1C_0^2+3C_1^2+5.C_2^2+\ldots..+$

$$(2n+1)C_n^2=\frac{(n+1)(2n)!}{n!n!}$$

We know that (part (C))

$C_0+3C_1+5C_2+\ldots\ldots+$

$(2n+1)C_n+(n+1)2^n$

$C_0+3C_1+5C_2x^2+\ldots\ldots.$

$C_0+{}^3C_1+{}^5C_2x^2+\ldots\ldots$

$+(2n+1)C_n x^n$

$= (n+1)(1+x)^n = (n+1)(1+x)^n$

Multiply with

$C_0x^n + C_1x^{n-1} + ... + C_n = (x+1)^n$

= and compare x^n and coefficient

$C_0 + 3C_1^2 + 5C_2^2 + ... + (2n+1)C_n^2$

= Coefficient of x^n in $(n+1)\ (1+x)^{n+n}$

$= (n+1)^{2n}C_n =$

L. H. S. = R. H. S.

Sol 27: $I = [(3\sqrt{5})^n]$

$I + F = (3+\sqrt{5})^n$

P = rational part

σ = irrational part

$$3-\sqrt{5} = \frac{9-(\sqrt{5})^2}{3+\sqrt{5}} = \frac{4}{3+\sqrt{5}}$$

$0 < 3-\sqrt{5} < 1$

$F = (3-\sqrt{5})^n$

$I + F + F = (3+\sqrt{5})^n + (3-\sqrt{5})^n$

$= 2\left({}^nC_0 3^n + {}^nC_2 3^{n-2}(\sqrt{5})^2 + ...\right)$

Rational part

0<F+F<2

F+F is 1 only integer between0and 2

$I + 1 = 2F = P = \frac{1}{2}(I+1)$

$I + F - F = (3+\sqrt{5})^n - (3-\sqrt{5})^n$

$I + F + F - F - F = 2({}^nC_1 3^{n-1}$

$(\sqrt{5})^1 + {}^nC_3 3^{n-3}(\sqrt{5})^3 + ...)$

$I + 2F - (F+F) = 2\sigma$

$I + 2F - 1 = 2\sigma$

$$\sigma = \frac{1}{2}(I + 2F - 1)$$

Sol 28:

(a) $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + ... + \frac{nC_n}{C_{n-1}} = \frac{n(n+1)}{2}$

We know that $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

$\Rightarrow \frac{r\,{}^nC_r}{{}^nC_{r-1}} = (n-r+1)$

L. H. S. $= (n-1+1) + (n-2+1)$

$+(n-3+1) + ...(n-n+1)$

$n^2 + n - (1+2+3+....n)$

$= n^2 + n - \frac{n(n+1)}{2}$

$= \frac{n^2+n}{2} = \frac{n(n+1)}{2}$

(b) $2C_0 + \frac{2^2C_1}{2} + \frac{2^3C_2}{3}$

$+ \frac{2^{n+1}C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$

(c) In equation (i) from above que.

$x = 2$

$$2C_0 + \frac{2^2C_1}{2} + ... + \frac{2^{n+1}C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$$

(d) In eq. (i) $x = -1$

$\frac{(0)^{n+1}-1}{n+1} = C_0(-1) + \frac{C_1}{2} - \frac{C_2}{3}$

$+...... + (-1)^{n+1}\frac{C_n}{n+1} = C_0 - \frac{C_1}{2} + \frac{C_2}{3} + ...$

$+(-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$

Sol 29: (a) In equation (ii) compare coefficient of x^{n-1}

${}^{2n}C_{n-1} = C_0C_1 + C_1C_2 + ... + C_{n-1}C_n$

${}^{2n}C_{n-1} = \frac{2n!}{(n-1)!(n+1)!}$

$\because$ 2n–(n–1) = n+1

L. H. S. = R. H. S.

(b) In some equ. (ii) compare coefficient of x^{n-r}

$${}^{2n}C_{n-r} = C_0C_r + ... + C_{n-r}C_n \qquad .... (ii)$$

$${}^{2n}C_{n-r} = \frac{2n!}{(n+r)!(n-r)!}$$

L. H. S. = R. H. S.

(c) $\sum_{r=0}^{n-2}({}^nC_r\ {}^nC_{r+2}) = \frac{2n!}{(n-2)!(n+2)!}$

In equ. (iii) if $r = 2$

$$= \ {}^{2n}C_{n-2} = C_0C_2 + C_1C_3 + ... + C_{n-2}C_n$$

$$= \ {}^{2n}C_{n-2} = \frac{2n!}{(n-2)!(n+2)!} \qquad ....(iii)$$

L. H. S. = R. H. S.

(d) ${}^{100}C_{10} + 5.{}^{100}C_{11} + 10.{}^{100}C_{12}$

$+10.{}^{100}C_{13} + 5.{}^{100}C_{14} + {}^{100}C_{18}$

$$= \ {}^{105}C_{90} = {}^{105}C_{105-90} = {}^{105}C_{15} = \frac{105!}{90!15!}$$

$$\text{L. H. S.} = \frac{100!}{90!10!} + \frac{5100!}{11!89!} + \frac{10\times100!}{12!88!}$$

$$+\frac{10\times1001}{13!87!} + \frac{5\times100!}{14!80!} + \frac{100!}{15!85!}$$

$$100!\left[\frac{15\times14\times13\times12\times11}{90!15!} + \right.$$

$$\frac{5\times90\times15\times14\times13\times12}{15!90} + \frac{10\times90\times89\times15\times14\times13}{15!90!}$$

$$+ \frac{10\times15\times14\times90\times89\times88}{70!15!} + \frac{5.90\times89.8887}{15!90!}$$

$$\left. + \frac{90.89.88.87.86}{90!15!}\right]$$

$$= \frac{100\times101\times102\times103\times104\times105}{90!15!} = \frac{105!}{90!15!}$$

$$= {}^{105}C_{15} = {}^{105}C_{90}$$

Sol 30: (i) $(1+x+x^2)^n = a_0+a_1x + a_2x^2+......+a_{2n}x^{2n}$

$(1+x+x^2)^n = (x^2+x+1)^n$

So $a_0 = a_{2n}$

$a_1 = a_{2n-1}$

$a_{n-1} = a_{n+1}$

So, $a_0a_1+a_2a_3+a_4a_5+.... +$

$= a_{2n}a_{2n-1} + a_{2n-2}a_{2n-3}$

$+....... + a_1a_2+a_3a_4$

$a_0a_1-a_1a_2+a_2a_3......=0$

(ii) $(1-x+x^2)^n$

$= a_0-a_1x+a_2x^2-a_3x^3 +.... .$

$(1+x+x^2)^n(1-x+x^2)^n = (a_0 - a_1x + ...)$

$(a_0x^{2n} + a_1x^{2n-1} + ...)$

$(1+x^2+x^4)^n = a_0a_2x^{2n-2} - a_1a_3x^{2n-2} + ...$

Compare x^{2n-2} coefficient

$a_{n+1} = a_{n-1} = a_0a_2 - a_1a_3 + ...$

$(\because$ in $(1+x+x^2)^n$ $x \to x^2$

So Coefficient of $x^{2(n-1)} = a_{n-1} = a_{n+1}$)

(iii) $(1+x+x^2)^n = a_0 + a_1x + ... + a_{2n}x^{2n}$

Put x = 1

$3^n = a_a + a_1 + a_2 + ... + a_{2n}$ 1

$x = \omega$

$0 = a_0 + a_1\omega + a_2\omega^2 + a_3 + ...a_{2n}\omega^{2n}$ . . 2

$x = \omega^2$

$0 = a_0 + a_1\omega^2 + a_2\omega + ... + a_{2n}\omega^{4n}$ ...3

$A + B + C =$

$3^n = 3(a_0 + a_3 + a_6 + ...)$

$$a_0 + a_3 + a_6 + ...3^{n-1} \qquad ... (i)$$

$x(1+x+x^2)^n =$ $a_0x + a_1x^2 + ... + a_{2n}x^{2n+1}$ .... A

$x = \omega =$ $0 = a_0\omega + a_1\omega^2 + a_2 + ... + a_{2n}\omega^{2n+1}$ ...B

$x = \omega^2 =$ $0 = a_0\omega^2 + a_1\omega + a_2 + ... + a_{2n}\omega^{4n+2}$ . . C

A,B,C,= $3^n = 3(a_2 + a_5 + a_8 + ...)$

$$a_2 + a_5 + a_8 = 3^{n-1} \qquad ... (ii)$$

Sum as above

$x^2(1+x+x^2)^n = a_0x^2$

$x^2(1+x+x^2)^n = a_0x^2 + a_1x^3 + ... + a_2nx^{2n+1}$

x = w =

$0 = a_0\omega^2 + a_1\omega^3 + ... + a_{2n}\omega^{2n+1}$ .... B_2

$x = \omega^2 = 0 = a_0\omega^4 + a_1 + ... + a_{2n}$ .... C_2

$A + B_2 + C_2 \ \ (a_1 + a_4 + a_7 + ...)$

$= \frac{3^n}{3} = 3^{n-1}$ ... (iii)

From (i), (ii) and (iii)

$E = E_2 = E_3 = 3^{n-1}$

Sol 31: $\sum_{r=0}^{100}\sum_{s=0}^{100} (C_r^2 + C_s^2 + C_rC_s) = m(^{2n}C_n) + 2P$

M, n and p are even natural number

$(1+x)^{100}$

C_r = coefficients of x^r in $(1+x)^{100}$

$= \sum_{r=0}^{100} [C_r^2 \times 101 + C_0^2 + C_1^2 + ... + C_{100}^2$

$+C_r(C_0 + C_1 + ... + C_{100})]$

$= 101\left(\sum C_r^2\right) + 101(^{2n}C_n) + \sum Cr(2^n)$

$= 101\ ^{2n}C_n + 101\ ^{2n}C_n + 2^n(2^n)$

$= 202\ ^{2n}C_n + 2^{100+100}$

$= 202\ ^{2n}C_n + 2^{200} = m(^{2n}C_n) + 2^P$

n=100, m=202, P=200

Hence, n+m+p = 200+100+202=502

Sol 32: $(1+x)(1+x+x^2)......(1+x+x^2+.... . +x^n)$

Max. power of x

$= 1+2+3+....... n = \frac{n(n+1)}{2}$

(a) Total terms = $\frac{1+n(n+1)}{2} = \frac{n^2+n+2}{2}$

(b) 1+x=x+1

$1+x+x^2 = x^2+x+7$, So now product is

$= (x+1)\ (x^1+x+1) = ...$

$(x^n + x^{n-1} + ... + x^2 + x + 1)$

So, $a_0 = \frac{a_{n(n+1)}}{2}$

Or if $x = \frac{1}{y}$

$(y^-)\frac{n(n+1)}{2}$

$(y+1)(y^2+y+1).... .\ (y^n+y^2+y+1)$

$= a_0y\frac{n(n+1)}{2} + ... + a\frac{n(n+1)}{2}$

$a_0 = a\frac{n(n+1)}{2}$

(c) Odd coefficient = $a_1+a_3+a_5+......$

At $x = 1$

2. 3. 4. . . .. (n+1) = (n+1)!

$= a_0a_1 + a_2 + ... + a_{\frac{n(n+1)}{2}}$

$x = -1 = a_0-a_1+a_2-a_3=0$

$= a_0 + a_2 + a_4 + ... = a_1 + a_3 + a_5 + ...$

Assume P = Q

$P + Q = 2P = 2Q = \ (n+i)! = P = Q = \frac{(n+1)!}{2}$

Sol 33: $S_1 = \sum_{0\le i}<\sum_{j\le 100} C_iC_{j.}$

$S_2 = \sum_{0\le j}<\sum_{i\le 100} C_iC_j$

$S_3 = \sum_{0\le i}=\sum_{j\le 100} C_iC_j$

$(1+x)^{100} \Rightarrow$ n = 100

$S_1 + S_2 + S_3 = a^b$

$S_1 + S_2 + S_3 =$

$\sum_{0\le i}<\sum_{j\le 100} C_iC_j + \sum_{0\le j}<\sum_{i\le 100} C_iC_j + \sum_{0\le i}\sum_{j\le 100} C_iC_j$

$S_1 = S_2 \because \sum_{0\le i}\sum_{j\le 100} C_iC_j < \sum_{0\le i}\sum_{j\le 100} C_jC$

$S_3 = S_1 + C_0C_0 + C_1C_1 + C_2^2 + ... + C_{100}^2$

$S_3 = S_1 + {}^{2n}C_n$

$S_1 + S_2 + S_3 = 2S_1 + 2^nC_n$

$$= [\,C_0(C_1 + C_2 + \ldots\ldots + C_{100}) + C_1(C_2 + \ldots\ldots\ldots + C_{100}) + \ldots\ldots\ldots\,]$$

$$= {}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + \ldots\ldots\ldots + {}^{2n}C_{2n}$$

When $C_1 + C_2 + \ldots\ldots + C_{100} = 2^n$

$$= 2^{2n} = 2^{200} = 4^{100} = 16^{50} = a^b$$

$$a + b = 16 + 50 = 66$$

Exercise 2

Sol 1: (B) Given binomial is $(2^{1/3}+3^{-1/3})^n$

$\therefore T_7 = T_{6+1} = {}^nC_6(2^{1/3})^{n-6}(3^{-1/3})^6$

$T_7{}'$ from end $= {}^nC_{n-6}(3^{-1/3})^{n-6}(2^{1/3})^6$

$$\Rightarrow \frac{T_7}{T_7{}'} = \frac{1}{6} = \frac{{}^nC_6\, 2^{n/3}2^{-2}3^{-2}}{{}^nC_6\, 3^{-n/3}3^2 2^2} = \frac{(2.3)^{n/3}}{(6)^{2+2}} = (6)^{(n/3)-4}$$

$$6^{(n/3)-4} = \frac{1}{6} \Rightarrow \frac{n}{3} - 4 = -1 \Rightarrow n=9$$

Sol 2: (C) We have $15^{23}+23^{23} = (19–4)^{23}+(19+4)^{23}$

$$= 2\left[\,{}^{23}C_0 19^{23} + {}^{23}C_2 19^{21} + \ldots + {}^{23}C_{22} 19\right]$$

= 2. 19K always divisible by 19

So the remainder is zero

Sol 3: (D) $4\{{}^nC_1 + 4\,{}^nC_2 + 4^2.\,{}^nC_3 + \ldots\ldots + 4^{n-1}\}$

$$= \{4\,{}^nC_1 + 4^2\,{}^nC_2 + 4^3\,{}^nC_3 + \ldots\ldots + 4^n\,{}^nC_n\}$$

$$= (1+x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots + x^n\,{}^nC_n$$

At $x = 4$

$$5^n = 1 + 4C_1 + 4\,{}^2C_2 + \ldots$$

So $\;{}^{4n}C_1 + 4^2\,{}^nC_2 + 4^3\,{}^nC_3 + \ldots\ldots + 4^n\,{}^nC_n = 5^n - 1$

Sol 4: (A) $n \ge 3$

$$n - \frac{(n-1)}{1!}(n-1) + \frac{(n-1)(n-2)}{2!}(n-2)$$

$$-\frac{(n-1)(2-n)(n-3)}{3!}(n-3) + \ldots\ldots$$

At $n = 3$

$$= \frac{1.3-(3-1)}{1}(3-1) + \frac{(3-1)(3-2)}{2!}(3-2) - 0$$

$$= 3 - 2\times 2 + \frac{2\times 1}{2!} = 3 + 1 - 4 = 0$$

Or

$$= n - {}^{n-1}C_1(n-1) + {}^{n-1}C_2\,(n-2) + \ldots\ldots$$

$$+3\,{}^{n-1}C_{n-3}(-1)^{n-3} + 2\,{}^{n-1}C_{n-2}(-1)^{n-2}$$

$$= n - {}^{n-1}C_0 + 2\,{}^{n-1}C_1 + 3\,{}^{n-1}C_2 + \ldots\ldots\ldots$$

$$+(-1)^{n-1}\,{}^{n-1}C_{n-1}$$

$$= n - ({}^{n-1}C_1 + {}^{n-1}C_0) + 0 = n - (n - 1 + 1) = 0$$

Sol 5: (C) t_6 in $\left[x^{-8/3} + x^2 \log_{10}^x\right]^8 = 5600$

$${}^8C_5\left(x^{-8/3}\right)^3\left(x^2 \log_{10}^x\right) = 5600$$

$$\Rightarrow x^2\left(\log_{10}^x\right)^5 = 100$$

$$\Rightarrow x = 10$$

Sol 6: (B) $(\alpha+p)^{m-1} + (\alpha+p)^{m-2}(\alpha+q)$

$$+(\alpha+p)^3(\alpha+q)^2 + \ldots\ldots + (\alpha+q)^{m-1}$$

Coefficient of ^{t}

$$= (\alpha+p)^{m-1}\left[1 + \left(\frac{\alpha+q}{\alpha+p}\right) + \ldots\ldots + \left(\frac{\alpha+q}{\alpha+p}\right)^{m-1}\right]$$

$$= (\alpha+p)^{m-1}\left[\frac{1-\left(\frac{\alpha+q}{\alpha+p}\right)^m}{1-\left(\frac{\alpha+q}{\alpha+p}\right)}\right]$$

$$= (\alpha+p)^{m-1}\frac{\left[1-\frac{(\alpha+q)}{\alpha+p}\right]}{\alpha+p-\alpha-q}(\alpha+p)$$

$$= (\alpha+p)^m\left[\frac{1-\left(\frac{\alpha+q}{\alpha+p}\right)^m}{p-q}\right] = \left[\frac{(\alpha+p)^m - (\alpha+q)^m}{p-q}\right]$$

Coefficient of $\alpha^t = \dfrac{{}^mC_t[p^{m-t} - q^{m-t}]}{p-q}$

Sol 7: (B) $\left(1 + x - 3x^2\right)^{2145} = a_0 + a_1x + a_2\,x^2 + \ldots$

Put x = - 1

$$\Rightarrow (1-1-3)^{2145} = a_0 - a_1 + a_2 - a_3 + ...$$

$$\Rightarrow a_0 - a_1 + a_2 - a_3 + ...(-3)^{2145}$$

Last digit of $(-3)^{2145}$ is 3.

Sol 8: (B) $\left(\frac{4x^2}{3} - \frac{3}{2x}\right)^9$

$$T_{r+1} = {}^9C_r\left(\frac{4x^2}{3}\right)^{9-r}\left(-\frac{3}{2x}\right)^r$$

Power of $x = 2(9-r)+(-1)r$

$\Rightarrow 18-2r-r = 18-3r = 6$ (given)

$\Rightarrow 3r = 18-6 = 12 \Rightarrow r = 12/3 = 4$

Coefficient $${}^9C_4\left(\frac{4}{3}\right)^{9-4}\left(\frac{-3}{2}\right)^4 = \frac{9\times8\times7\times6}{1.2.3.4}\left(\frac{4}{3}\right)^5\left(\frac{3}{2}\right)^4$$

$$= 9\times2\times7\times\frac{2^{10}\times3^4}{3^5\times2^4} = 21\times2^7 = 2688$$

Sol 9: (D) $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}, x > 0$

$$T_{r+1} = {}^{18}C_r(9x)^{18-r}\left(\frac{1}{\sqrt{9x}}\right)^r$$

Power of $x = 18 - r - \frac{r}{2} = 18 - \frac{3r}{2} = 0$ (given)

$$\alpha = 9^{18-r}\left(\frac{1}{\sqrt{9}}\right)^r = (9)^{18-r-\frac{r}{2}} = (9)^{18-\frac{3r}{2}} = 9^0 = 1$$

Sol 10: (C) $\left[x+\sqrt{x^3-1}\right]^5 + \left[x-\sqrt{x^3-1}\right]^5$

$$= 2\left[{}^5C_0x^5 + {}^5C_2x^3\left(x^3-1\right) + {}^5C_4x\left(x^3-1\right)^2\right]$$

$\Rightarrow$ Highest power is 7.

Sol 11: (C) We have

$$C_0^2 + C_1^2 + C_2^2 + ... + C_n^2 = \sum_{r=0}^{n}({}^nC_r)({}^nC_r)$$

$$= \sum_{r=0}^{n}({}^nC_r)({}^nC_{n-r}) \qquad [\because {}^nC_r = nC_{n-r}]$$

= Number of ways of choosing n persons out of n men and n women

= Number of ways of choosing n person out of 2n persons

= ${}^{2n}C_n$

Sol 12: (D) $aC_0 + (a+b)C_1 + ... + (a+nb)Cn$

$$= [C_0 + C_1 + ... + C_n]$$

$$+b[0\times C_0 + 1\times C_1 + 2\times C_2 + ... + nC_n]$$

$$= a2^n + bn2^{n-1}$$

$$= (2a+nb)2^{n-1}$$

Previous Years' Questions

Sol 1: We know, $(1 + x)^{2n} = C_0 + C_1x + C_2x^2 + ... + C_{2n}x^{2n}$

On differentiating both sides w.r.t. x, we get

$$2n(1+x)^{2n-1} = C_1 + 2.C_2x$$

$$+3.C_3x^2 + ... + 2nC_{2n}x^{2n-1} \qquad ... (i)$$

And

$$\left(1-\frac{1}{x}\right)^{2n} = C_0 - C_1.\frac{1}{x} + C_2.\frac{1}{x^2}$$

$$-C_3.\frac{1}{x^3} + ... + C_{2n}.\frac{1}{x^{2n}} \qquad ... (ii)$$

On multiplying Eqs. (i) and (ii), we get

$$2n(1+x)^{2n-1}\left(1-\frac{1}{x}\right)^{2n}$$

$$= \left[C_1 + 2.C_2x + 3.C_3x^2 + ... + 2n.C_{2n}x^{2n-1}\right]$$

$$\times\left[C_0 - C_1\left(\frac{1}{x}\right) + C_2\left(\frac{1}{x^2}\right) - .... + C_{2n}\left(\frac{1}{x^{2n}}\right)\right]$$

The coefficient of $\left(\frac{1}{x}\right)$ on the LHS

= Coefficient of $\frac{1}{x}$ in $2n\left(\frac{1}{x^{2n}}\right)(1+x)^{2n-1}(x-1)^{2n}$

= Coefficient of x^{2n-1} in $2n\left(1-x^2\right)^{2n-1}(1-x)$

$$= 2n\left(-1\right)^{n-1}.^{(2n-1)}C_{n-1}$$

$$= \left(-1\right)^{n}\left(2n\right)\frac{\left(2n-1\right)!}{\left(n-1\right)!n!}$$

$$= -\left(-1\right)^{n} n\cdot\frac{(2n)!}{(n!)^{2}}\cdot n \qquad \text{... (iii)}$$

$$= -(-1)^{n}\ n\cdot C_{n}$$

Again, the coefficient of $\left(\frac{1}{x}\right)$ on the RHS

$$= -\left(C_1^2 - 2.C_2^2 + 3.C_3^2 + ... + -2nC_{2n}^2\right) \qquad \text{... (iv)}$$

From Eqs. (iii) and (iv), we get

$$C_1^2 - 2.C_2^2 + 3.C_3^2 - .... - 2n.C_{2n}^2 = \left(-1\right)^{n} n.C_{n}$$

Sol 2: $^{n+1}C_1 + {}^{n+1}C_2 s_1 + {}^{n+1}C_3 s_2$

$$+.... + {}^{n+1}C_{n+1}s_n = \sum_{r=1}^{n+1} {}^{n+1}C_r s_{r-1}$$

Where $s_n = 1 + q + q^2 + ... + q^n = \frac{1-q^{n+1}}{1-q}$

$$\therefore \sum_{r=1}^{n+1} {}^{n+1}C_r\left(\frac{1-q^r}{1-q}\right)$$

$$= \frac{1}{1-q}\left(\sum_{r=1}^{n+1} {}^{n+1}C_r - \sum_{r=1}^{n+1} {}^{n+1}C_r q^r\right)$$

$$= \frac{1}{1-q}\left[(1+1)^{n+1} - (1+q)^{n+1}\right]$$

$$= \frac{1}{1-q}\left[2^{n+1} - \left(1+q\right)^{n+1}\right] \qquad \text{... (i)}$$

Also, $S_n = 1\left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$

$$= \frac{1-\left(\frac{q+1}{2}\right)^{n+1}}{1-\left(\frac{q+1}{2}\right)} = \frac{2^{n+1}-(q+1)^{n+1}}{2^n(1-q)} \qquad \text{... (ii)}$$

From eqs. (i) and (ii), we get

$$^{n+1}C_r + {}^{n+1}C_2 s_1 + {}^{n+1}C_3 s_2 + ... + {}^{n+1}C_{n+1}s_n = 2^n s_n$$

Sol 3: $\sum_{r=0}^{n}\left(-1\right)^r\ {}^nC_r$

$$\left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + ...\text{upto m terms}\right]$$

$$\sum_{r=0}^{n}\left(-1\right)^r\ {}^nC_r\left(\frac{1}{2}\right)^r +$$

$$\sum_{r=0}^{n}\left(-1\right)^r\ {}^nC_r\left(\frac{3}{4}\right)^r + \sum_{r=0}^{n}\left(-1\right)^r\ {}^nC_r\left(\frac{7}{8}\right)^r + ...$$

Upto m terms

$$\left\{\text{using}\sum_{r=0}^{n}\left(-1\right)^r\ {}^nC_r x^r = \left(1-x\right)^n\right\}$$

$$= \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + ...$$

Upto m terms

$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n + \left(\frac{1}{8}\right)^n + ...$$

Upto m terms

$$= \left(\frac{1}{2}\right)^n\left[\frac{1-\left(\frac{1}{2^n}\right)^m}{1-\frac{1}{2^n}}\right] = \frac{2^{mn}-1}{2^{mn}\left(2^n-1\right)}$$

Sol 4: Let y = (x – a)^m, where m is a positive integer, $r \le m$,

Now, $\frac{dy}{dx} = m\left(x-a\right)^{m-1}$

$$\Rightarrow \frac{d^2y}{dx^2} = m\left(m-1\right)\left(x-a\right)^{m-2}$$

$$\Rightarrow \frac{d^4y}{dx^4} = m\left(m-1\right)\left(m-2\right)\left(m-3\right)\left(x-a\right)^{m-4}$$

....

On differentiating r times, we get

$$\frac{d^ry}{dx^r} = m\left(m-1\right)....\left(m-r+1\right)\left(x-a\right)^{m-r}$$

$$= \frac{m!}{\left(m-r\right)!}\left(x-a\right)^{m-r} = r!\left({}^mC_r\right)\left(x-a\right)^{m-r}$$

And for r > m, $\frac{d^ry}{dx^r} = 0$

Now,

$$\sum_{r=0}^{2n} a_r\left(x-2\right)^r = \sum_{r=0}^{2n} b_r\left(x-3\right)^r \text{ (given)}$$

On differentiating both sides n times w.r.t. x, we get

$$\sum_{r=n}^{2n} a_r\left(n!\right)^r C_n\left(x-2\right)^{r-n}$$

$$= \sum_{r=n}^{2n} b_r\left(n!\right)^r C_n\left(x-3\right)^{r-n}$$

On putting x = 3, we get

$$\sum_{r=n}^{2n} a_r\left(n!\right)^r C_n = \left(b_n\right)n!$$

$$\Rightarrow b_r = \frac{1}{n!}\sum_{r=n}^{2n} a_r\left(n!\right)^r C_n$$

$$= {}^{2n+1}C_n$$

$$= {}^{2n+1}C_{n+1}$$

Sol 5: $\left(1+x+x^2\right)^n = a_0 + a_1x + ... + a_{2n}x^{2n}$ ... (i)

Replacing x by –1/x, we get

$$\left(1-\frac{1}{x}+\frac{1}{x^2}\right)^n$$

$$= a_0 - \frac{a_1}{x} + \frac{a_2}{x^2} - \frac{a_3}{x^3} + ... + \frac{a_{2n}}{x^{2n}} \quad \text{... (ii)}$$

Now, $a_0^2 - a_1^2 + a_2^2 - a_3^2 + ... + a_{2n}^2$ = coefficient of the term independent of x in

$$\left[a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}\right]$$

$$\times\left[a_0 - \frac{a_1}{x} + \frac{a_2}{x^2} - .... + \frac{a_{2n}}{x^{2n}}\right]$$

= Coefficient of the term independent of x in

$$\left(1+x+x^2\right)^n\left(1-\frac{1}{x}+\frac{2}{x^2}\right)^n$$

Now, RHS $= \left(1+x+x^2\right)^n\left(1-\frac{1}{x}+\frac{1}{x^2}\right)^n$

$$= \frac{\left(1+x+x^2\right)^n\left(x^2-x+1\right)^n}{x^{2n}}$$

$$= \frac{\left[\left(x^2+1\right)^2 - x^2\right]^n}{x^{2n}} = \frac{(1+2x^2+x^4-x^2)^n}{x^{2n}}$$

$$= \frac{\left(1+x^2+x^4\right)^n}{x^{2n}}$$

Thus, $a_0^2 - a_1^2 + a_2^2 - a_3^2 + ... + a_{2n}^2$

= Coefficient of the term independent of x in

$$\frac{1}{x^{2n}}\left(1+x^2+x^4\right)^n$$

= Coefficient of x^{2n} in $\left(1+x^2+x^4\right)^n$

= Coefficient of t^n in $\left(1+t+t^2\right)^n = a_n$

Sol 6: To show that

$$2^k.{}^nC_0.{}^nC_k - 2^{k-1}.{}^nC_1.{}^{n-1}C_{k-1} + 2^{k-2}.{}^nC_2.{}^{n-2}C_{k-2} - .... +$$

$$\left(-1\right)^k\,{}^nC_k\,{}^{n-k}C_0 = {}^nC_k$$

Taking LHS

$$2^k.{}^nC_0.{}^nC_k - 2^{k-1}.{}^nC_1.{}^{n-1}C_{k-1} + ... + \left(-1\right)^k.{}^nC_k = {}^nC_k$$

$$= \sum_{r=0}^{k}\left(-1\right)^r.2^{k-r}.{}^nC_r.{}^{n-r}C_{k-r}$$

$$= \sum_{r=0}^{k}\left(-1\right)^r.2^{k-r}.\frac{n!}{r!(n-r)!}.\frac{\left(n-r\right)!}{\left(k-r\right)!\left(n-k\right)!}$$

$$= \sum_{r=0}^{k}\left(-1\right)^r.2^{k-r}.\frac{n!}{\left(n-k\right)!k!}.\frac{k!}{r!\left(k-r\right)!}$$

$$= \sum_{r=0}^{k}\left(-1\right)^r.2^{k-r}.{}^nC_k.{}^kC_r$$

$$= 2^k.{}^nC_k\left\{\sum_{r=0}^{k}\left(-1\right)^r.\frac{1}{2^r}.{}^kC_r\right\}$$

$$= 2^k.{}^nC_k\left(1-\frac{1}{2}\right)^k = {}^nC_k = \text{RHS}$$

Sol 7: Let y = $\sum_{r=1}^{10} A_r(B_{10}\,Br - C_{10}A_r)$

$\sum_{r=1}^{10} A_rB_r$ = coefficient of x^{20} in $((1 + x)^{10} (x + 1)^{20}) - 1$

$= C_{20} - 1 = C_{10} - 1$ and $\sum_{r=1}^{10}\left(A_r\right)^2$ = coefficient of x^{10} in $((1 + x)^{10} (x + 1)^{10}) - 1 = B_{10} - 1$

$\Rightarrow y = B_{10}(C_{10} - 1) - C_{10}(B_{10} - 1) = C_{10} - B_{10}.$

Sol 8: Let T_{r-1}, T_r, T_{r+1} are three consecutive terms of $(1 + x)^{n+5}$

$T_{r-1} = {}^{n+5}C_{r-2}\,(x)^{r-2}$, $T_r = {}^{n+5}C_{r-1}x^{r-1}$, $T_{r+1} = {}^{n+5}C_r x^r$

Where, ${}^{n+5}C_{r-2} : {}^{n+5}C_{r-1} : {}^{n+5}C_r = 5 : 10 : 14$.

$$\text{So } \frac{{}^{n+5}C_{r-2}}{5} = \frac{{}^{n+5}C_{r-1}}{10} \Rightarrow n - 3r = -3 \qquad \text{... (i)}$$

$$\frac{{}^{n+5}C_{r-1}}{10} = \frac{{}^{n+5}C_r}{14} \Rightarrow 5n - 12r = -30 \qquad \text{... (ii)}$$

From equation (i) and (ii) n = 6

Sol 9: $2x_1 + 3x_2 + 4x_3 = 11$

Possibilities are (0, 1, 2); (1, 3, 0); (2, 1, 1); (4, 1, 0).

∴ Required coefficients

$= ({}^4C_0 \times {}^7C_1 \times {}^{12}C_2) + ({}^4C_1 \times {}^7C_3 \times {}^{12}C_0) + ({}^4C_2 \times {}^7C_1 \times {}^{12}C_1) + ({}^4C_4 \times {}^7C_1 \times 1)$

$= (1 \times 7 \times 66) + (4 \times 35 \times 1) + (6 \times 7 \times 12) + (1 \times 7)$

$= 462 + 140 + 504 + 7 = 1113.$

Sol 10: x^9 can be formed in 8 ways

ii.e. x^9, x^{1+8}, x^{2+7}, x^{3+6}, x^{4+5}, $x^{1+2} + 6$, x^{1+3+5}, x^{2+3+4} and coefficient in each case is 1.

$$\Rightarrow \text{Coefficient of } x^9 = 1 + 1 + 1 + \underbrace{\ldots\ldots\ldots\ldots}_{8 \text{ times}} + 1 = 8$$

Sol 11: $Z = \dfrac{-1 + i\sqrt{3}}{2} = \omega$

$$P = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$$P^2 = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}\begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$$= \begin{bmatrix} (-\omega)^{2r} + (\omega^{2s})^2 & \omega^{2s}(-\omega)^r + \omega^r\omega^{2s} \\ \omega^{2s}(-\omega) + \omega^r\omega^{2s} & \omega^{4s} + \omega^{2r} \end{bmatrix}$$

$$= \begin{bmatrix} \omega^{4s} + \omega^{2r} & \omega^{2s}(\omega^r + (-\omega)^r \\ \omega^{2s}(\omega^r + (-\omega)^r) & \omega^{4s} + \omega^{2r} \end{bmatrix}$$

= – I (Given)

$\omega^{4s} + \omega^{2r} = -1$ and $\omega^{2s}(\omega^r + (-\omega)^r) = 0$

$\omega^r + (-\omega)^r = 0$

r	s	r	s
1	1	1	1
2	2	3	3

Total no. pairs = 1