Sequences and Series

PROBLEM-SOLVING TACTICS

(a) When looking for a pattern in a sequence or series, writing out several terms will help you see the pattern, do not simplify directly. If you do this way, it is often easier to spot the pattern (if you leave terms as products, sums, etc.).

(b) If each term of an AP is multiplied by (or divided by a non-zero) fixed constant C, the resulting sequence is also an AP, with a common difference C times $\left(\text{or } \frac{1}{c}\text{times}\right)$ the previous.

(c) **Tips for AP problems**

(i) When the number of terms are three, then we take the terms as a – d, a, a + d;

Five terms as a - 2d, a - d, a, a + d, a + 2d

Here, we take the middle term as 'a' and common difference as 'd'.

(ii) When the number of terms is even, then we take:

Four terms as a – 3d, a – d, a + d, a + 3d;

Six terms as a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d

Here, we take 'a – d' and 'a + d' as the middle terms and common difference as '2d'.

(iii) If the number of terms in an AP is even, then take the number of terms as 2n and if odd then take it as (2n + 1).

(d) Tips for G.P. problems

(i) When the number of terms is odd, then we take three terms as a/r, a, ar; five terms as $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$. Here, we take the middle term as 'a' and common ratio as 'r.'

(ii) When the number of terms is even, then we take four terms as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$; six terms as $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$. Here, we take $\frac{'a'}{r}$ and 'ar' as the middle terms and common ratio as 'r^2.'

(e) Tips for H.P. problems

For three terms, we take as $\frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}$

For four terms, we take as $\frac{1}{a-3d}, \frac{1}{a-d}, \frac{1}{a+d}, \frac{1}{a+3d}$

For five terms, we take as $\frac{1}{a-2d}, \frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}$

FORMULAE SHEET

Arithmetic Progression: Here, a, d, A and S_n represent the first term, common difference, A.M. and sum of the numbers, respectively, and T_n stands for the n^{th} term.

1.	$T_n = a + (n-1)d$	4.	$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$
2.	$T_n = \frac{T_{n-1}+T_{n+1}}{2}$	5.	$A = \frac{(a_1 + a_2 + \ldots + a_n)}{n}$
3.	$S_n = \frac{n}{2}(a+T_n)$	6.	Insertion of n arithmetic means between a and b is $A_n = a + \frac{n(b-a)}{n+1}$

Geometric Progression: Here, a, r, S_n and G represent the first term, common ratio, sum of the terms and G.M., respectively, and T_n stands for the n^{th} term.

1.	$T_n = a.r^{n-1}$	4.	$S_n = \frac{a(r^n-1)}{r-1}$
2.	$T_n = \sqrt{T_{n-1}\cdot T_{n+1}}$	5.	$S_\infty = \frac{a}{1-r}$ (for $-1 < r < 1$)
3.	$S_n = \frac{T_{n+1}-a}{r-1}$	6.	Insertion of n geometric means between a and b is $G_1 = ar$, $G_2 = ar^2$ ....$G_n = ar^n$ or $G_n = b/r$, where $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

Arithmetic Geometric Progression: Here, a = the first term of AP, b = the first term of GP, d = common difference and r = common ratio of GP.

1.	$S_n = ab + (a + d)br + (a + 2d)br^2 + (a + 3d)br^3 + ...$
2.	$S_n = \frac{ab}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]br^n}{1-r}$
3.	$S_\infty = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2}$ (for $-1 < r < 1$)

Harmonic Progression

1. $a_n = \frac{1}{a+(n-1)d}$, where $a = \frac{1}{a_1}$ and $d = \frac{1}{a_2} - \frac{1}{a_1}$

2. $\frac{1}{H} = \frac{1}{n}\left(\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}\right)$

3. Insertion of n harmonic means between a and b

$$\frac{1}{H_1} = \frac{1}{a} + \frac{a-b}{(n+1)ab}$$

$$\frac{1}{H_2} = \frac{1}{a} + \frac{2(a-b)}{(n+1)ab} \text{ and so on } \Rightarrow \left[\frac{1}{H_n} = \frac{1}{a} + \frac{n(a-b)}{(n+1)ab}\right]$$

1.	The sum of n natural numbers	$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$
2.	The sum of n odd natural numbers	$\sum_{r=1}^{n} (2r-1) = n^2$
3.	The sum of n even natural numbers	$\sum_{r=1}^{n} 2r = n(n+1)$
4.	The sum of squares of n natural numbers	$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$
5.	The sum of cubes of n natural numbers	$\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$