MasterJEE

IIT-JEE | Medical | Foundations

Time : 3 hrs. M.M. : 360

Answers & Solutions

for

JEE (MAIN)-2019

(Online CBT Mode)

(Physics, Chemistry and Mathematics)

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 **(four)** marks for each correct response.
4. *Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for a question in the answer sheet.*
5. There is only one correct response for each question.

PHYSICS

1. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot?

   (1) 2750 kHz

   (2) 2900 kHz

   (3) 2000 kHz

   (4) 2250 kHz

Answer (3)

Sol. f_c = 2500 kHz, f_m = 250 kHz

$f_c + f_c$ = 2750 kHz

$f_c - f_m$ = 2250 kHz

For accepted frequency, two bandwidths do not overlap

$f_2 = f_c + 2f_m$ or $f_c - 2\ f_m$ = 3000 kHz or 2000 kHz

2. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is:

   (1) 1.3×10^{-5} N

   (2) 1.8×10^{-5} N

   (3) 6.5×10^{-5} N

   (4) 3.6×10^{-5} N

Answer (3)

Sol. $\dfrac{B_V}{B_H} = \tan 45^\circ$

$$B_V = B_H = 18 \times 10^{-6}\,\text{T}$$

$$\tau_{BV} = \left[18 \times 10^{-6} \times 1.8 \times \frac{0.12}{2}\right] \times 2$$

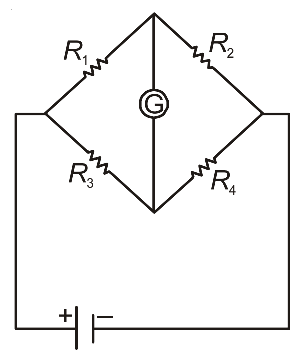
$$T_f = F_V \times \frac{0.12}{2}$$

$$F_v = 18 \times 1.8 \times 10^{-6} \times 2$$

$$\simeq 6.5 \times 10^{-5}\ \text{N}$$

3. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80 Ω and 40 Ω respectively.

   Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be:



   (1) Brown, Blue, Black

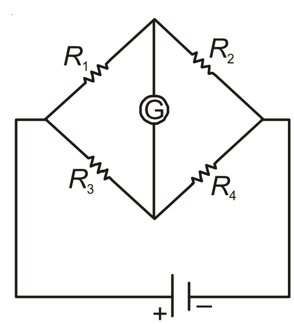
   (2) Red, Green, Brown

   (3) Grey, Black, Brown

   (4) Brown, Blue, Brown

Answer (4)

Sol. $R_1(O, R, B) = 320\ \Omega$



$R_2 = 80\ \Omega$

$R_4 = 40\ \Omega$

Clearly, $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4} \Rightarrow R_3 = \dfrac{320 \times 40}{80}$

$R_3 = 160\ \Omega \quad \therefore \quad 16 \times 10^1$

$\Rightarrow$ Colour for R_3 $\Rightarrow$ Brown, Blue, Brown

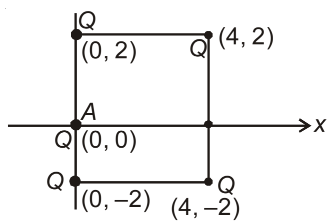
4. Four equal point charges Q each are placed in the *xy* plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth charge *Q* at the origin of the coordinate system will be:

(1) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (2) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$

(3) $\frac{Q^2}{4\pi\varepsilon_0}$ (4) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

Answer (4)

Sol. $V_A = \frac{kQ}{2}+\frac{kQ}{2}+\frac{kQ}{2^2+4^2}+\frac{kQ}{\sqrt{2^2+4^2}}$



$$V_A = \frac{kQ}{2}+\frac{kQ}{2}+\frac{kq}{\sqrt{2^2+4^2}}+\frac{kQ}{\sqrt{2^2+4^2}}$$

$$V_A = kQ+\frac{2kQ}{2\sqrt{5}}$$

$$PE = kQ^2\left(1+\frac{1}{\sqrt{5}}\right)=\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$$

5. The electric field of a plane polarized electromagnetic wave in free space at time *t* = 0 is given by an expression

$$\vec{E}\,(x,y)=10\hat{j}\cos[(6x+8z)]$$

The magnetic field $\vec{B}$ (*x, z, t*) is given by : (*c* is the velocity of light)

(1) $\frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\cos[(6x+8z-10ct)]$

(2) $\frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\cos[(6x+8z+10ct)]$

(3) $\frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\cos[(6x-8z+10ct)]$

(4) $\frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\cos[(6x+8z-10ct)]$

Answer (4)

Sol. $\vec{E}.\vec{B}=0$

$\Rightarrow$ $\vec{B}$ is in *xz* plane

$\vec{E}\times\vec{B}$ is parallel to $\left(6\hat{i}+8\hat{k}\right)$

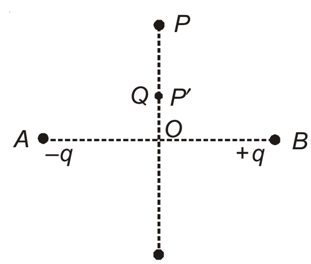
let $\vec{B}=\left(x\hat{i}+z\hat{k}\right)$

then, $\hat{j}\times\left(x\hat{i}+z\hat{k}\right)=6\hat{i}+8\hat{k}$

$\Rightarrow z = 6$ and $x = -8$

$\therefore$ $\vec{B}=\frac{1}{C}\left(6\hat{k}-8\hat{i}\right)$ cos (6*x* + 8*z* – 10*ct*)

6. Charges –*q* and +*q* located at A and B, respectively, constitute an electric dipole. Distance AB = 2*a*, *O* is the mid point of the dipole and *OP* is perpendicular to *AB*. A charge *Q* is placed at *P* where *OP* = *y* and *y* >> 2*a*. The charge *Q* experiences an electrostatic force F. If *Q* is now moved along the equatorial line to *P′* such that $OP'=\left(\frac{y}{3}\right)$, the force on *Q* will be close to : $\left(\frac{y}{3}>>2a\right)$



(1) $\frac{F}{3}$ (2) 9*F*

(3) 27*F* (4) 3*F*

Answer (3)

Sol. $F=\frac{Kp}{y^3}Q$ for electric dipole

and, $F'=\frac{Kp}{(y/3)^3}Q=\frac{27\,KpQ}{y^3}$

$\therefore$ $F'=27\,F$

7. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to: (density fo water = 10^3 kg/m^3)

(1) 2.50 rad s^{-1} (2) 3.75 rad s^{-1}

(3) 5.00 rad s^{-1} (4) 1.25 rad s^{-1}

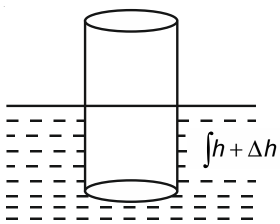
Answer (BONUS)

Sol. $mg = B_o = \rho Ah.g$

$ma = B - mg$

$-ma = \rho A(h + \Delta h)g - \rho Ahg$

$-ma = \rho Ag\Delta h$



$$a = \frac{-\rho Ag\Delta h}{m}$$

$$\omega = \sqrt{\frac{10^3 \times A \times g}{310 \times 10^{-3}}}$$

$$= \sqrt{\frac{10^3 \times \pi \times 6.25 \times 10^{-4} \times 10}{310 \times 10^{-3} \times 100}}$$

$$= \sqrt{62.5} \cong 8 \text{ rod/s}$$

8. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to :

(Gas constant R = 8.31 J/mol K)

(1) 291 J

(2) 581 J

(3) 146 J

(4) 73 J

Answer (1)

Sol. $\Delta W = P\Delta V$

$= nR\Delta T$

$= \frac{1}{2} \times 8.31\ (70)$

$\simeq 291$ J

9. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then

(1) $T_h = 0.5T_c$ (2) $T_h = T_c$

(3) $T_h = 2T_c$ (4) $T_h = 1.5T_c$

Answer (2)

Sol. $T_h = 2\pi\sqrt{\frac{I_h}{M_h \cdot B}}$

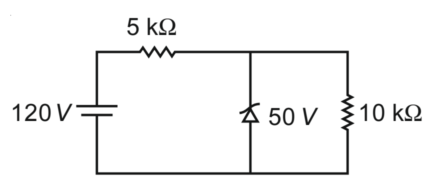
and $T_c = 2\pi\sqrt{\frac{I_C}{M_C B}}$

$\therefore \frac{T_h}{T_C} = \sqrt{\frac{I_h M_C}{I_C M_h}}$ and $M_h = 2M_C$

$\Rightarrow \frac{T_h}{T_C} = \sqrt{\frac{MR^2 \times M_C}{\frac{MR^2}{2} \times 2M_C}} = 1$

$\Rightarrow T_h = T_C$

10. For the circuit shown below, the current through the Zener diode is:



(1) Zero

(2) 9 mA

(3) 14 mA

(4) 5 mA

Answer (2)

Sol. $I_b = \frac{120 - 50}{5 \times 10^3} = \frac{70}{5} \times 10^{-3}$

$\Rightarrow I_b = 14$ mA

and $I_L = \frac{50}{10 \times 10^3} = 5$ mA

$\therefore I_Z = 14 - 5 = 9$ mA

11. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C. (Specific heat of brass is 394 J $kg^{-1}K^{-1}$)

(1) 916 J $kg^{-1}K^{-1}$ (2) 1232 J $kg^{-1}K^{-1}$

(3) 654 J $kg^{-1}K^{-1}$ (4) 458 J $kg^{-1}K^{-1}$

Answer (1)

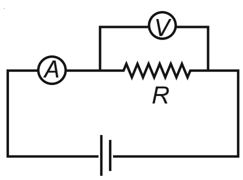
Sol. $192 \times s(100 - 21.5) = 128 \times 0.394 \times (21.5 - 8.4)$

$+ 240 \times 4.18 \times (21.5 - 8.4)$

$\Rightarrow s = \dfrac{660.65 + 13142}{15072}$

$\Rightarrow$ s ≃ 916 J $kg^{-1}K^{-1}$

12. The actual value of resistance R, shown in the figure is 30 Ω. This is measured in an experiment as shown using the standard formula $R = \dfrac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is:



(1) 570 Ω

(2) 600 Ω

(3) 350 Ω

(4) 35 Ω

Answer (1)

Sol. $R' = \dfrac{R \cdot R_V}{R + R_V}$, $R' = \dfrac{95}{100} \times 30 = 28.5$

$\Rightarrow 28.5 = \dfrac{30R_V}{30 + R_V}$

$\Rightarrow 855 + 28.5R_V = 30R_V$

$\Rightarrow R_V = \dfrac{855}{1.5}$

$\Rightarrow R_V = 570\ \Omega$

13. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1 s, the change in the energy of the inductance is:

(1) 437.5 J (2) 740 J

(3) 637.5 J (4) 540 J

Answer (1)

Sol. $\Delta E = \dfrac{1}{2}LI_f^2 - \dfrac{1}{2}LI_{in}^2$

and, $L\dfrac{di}{dt} = 25$

$\therefore L \times \dfrac{25 - 10}{1} = 25$

$\Rightarrow L = \dfrac{25}{15} = \dfrac{5}{3}$ H

$\therefore \Delta E = \dfrac{1}{2} \times \dfrac{5}{3}(625 - 100)$

$\Rightarrow \Delta E = 437.5$ J

14. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement?

(1) 15 J

(2) 9 J

(3) 12 J

(4) 10 J

Answer (1)

Sol. $\Delta W = KE_f - KE_i = \left(3\hat{i} - 12\hat{j}\right) \cdot 4\hat{i}$

$\Rightarrow KE_f = 12 + KE_{in} = 12 + 3 = 15$ J

15. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is :

(1) 560 pJ

(2) 692 pJ

(3) 508 pJ

(4) 600 pJ

Answer (3)

Sol. $U_{in} = \frac{1}{2}\times 12\times 10^{-12}\times 100$

$\Rightarrow$ U_{in} = 600 pJ

and, Q = 120 pC

$$U_{final} = \frac{1}{2}\times\frac{120\times 120\times 10^{-24}\times 2}{12\times 10^{-12}\times 13}$$

$\Rightarrow$ U_{final} = 92.3 pJ

$U_{initial} - U_{final} = \Delta W \simeq 508$ pJ

16. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A}+\vec{B})$ is 'n' times the magnitude of $(\vec{A}-\vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is :

(1) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$ (2) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

(3) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$ (4) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

Answer (2)

Sol. Clearly, $|\vec{A}+\vec{B}|^2 = A^2 + B^2 + 2AB\cos\theta$

and $|\vec{A}-\vec{B}|^2 = A^2 + B^2 - 2AB\cos\theta$

As, $|\vec{A}+\vec{B}|^2 = n^2|\vec{A}-\vec{B}|^2$

$\therefore$ $A^2 + B^2 + 2AB\cos\theta = n^2(A^2 + B^2 - 2AB\cos\theta)$

$\therefore$ $A^2 + B^2 + 2AB\cos\theta$

$= n^2A^2 + n^2B^2 - 2n^2AB\cos\theta$

$\therefore$ $\cos\theta \times 2AB(1 + n^2) = A^2(n^2 - 1) + B^2(n^2 - 1)$

$\Rightarrow$ $\cos\theta = \frac{(n^2-1)(A^2+B^2)}{2AB(1+n^2)}$

$\Rightarrow$ $\cos\theta = \frac{n^2-1}{n^2+1}$

$\Rightarrow$ $\theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

17. Consider the nuclear fission

$Ne^{20} \rightarrow 2He^4 + C^{12}$

Given that the binding energy/nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement:

(1) Energy of 12.4 MeV will be supplied

(2) 8.3 MeV energy will be released

(3) Energy of 3.6 MeV will be released

(4) Energy of 11.9 MeV has to be supplied

Answer (Bonus)

Sol. Amount of energy released

$\Delta E_{(out)}$ = 2 × (Binding energy of He^4) + (Binding energy of C^{12}) – Binding energy of Ne^{20}

$\therefore$ $\Delta E_{(out)}$ = (2 × 4 × 7.07)

+ (12 × 7.86) – (20 × 8.03) MeV

= –9.72

So, 9.72 MeV of energy will be supplied.

18. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be : (Assume that the highest frequency a person can hear is 20,000 Hz)

(1) 7 (2) 4

(3) 6 (2) 5

Answer (3)

Sol. $f_o = \frac{v}{4L}$

Also, $f_n = (2n + 1) f_o$

So $(2n + 1)$ would take value of $\frac{20,000}{1500} = 13.33$

$\Rightarrow$ 1, 3, 5, 7, 9, 11, 13

So overtones are 3, 5, 7, 9, 11, 13

Total 6 No. of overtones would be heard

19. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is

(1) 11×10^{-5} W (2) 11×10^{5} W

(3) 11×10^{-3} W (4) 11×10^{-4} W

Answer (1)

Sol. I^2R = 4.4 watt

$\Rightarrow$ $R = \frac{4.4}{4\times 10^{-6}}\,\Omega$

$P_{(dissipated)}$ for V = 11 volt

$$P = \frac{V^2}{R} = \frac{11\times 11\times 4\times 10^{-6}}{4.4}$$

$$P = \frac{11\times 11\times 4\times 10^{-6}\times 10}{44}$$

$= 1.1 \times 10^{-4}$ watt

$= 11 \times 10^{-5}$ watt

20. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be : [1 $eV = 1.6 \times 10^{-19}$ J]

(1) 10^{14} and 10 eV
(2) 10^{11} and 5 eV
(3) 10^{10} and 5 eV
(4) 10^{12} and 5 eV

Answer (2)

Sol. $\left(\frac{Nhc}{\lambda}\right)\frac{1}{\Delta t.\Delta A} = I$

$\Rightarrow \left(\frac{N}{\Delta t}\right) = \frac{16\times10^{-3}\times1\times10^{-4}\times\lambda}{hc}$ (per sec)

Given $\frac{hc}{\lambda} = 10$ eV

So total incident photons per second

$\Rightarrow \frac{N}{\Delta t} = \frac{16\times10^{-7}}{10\text{ eV}} = 9.98 \times 10^{11}$

No. of emitted electrons per sec $= \frac{1}{10}\frac{N}{\Delta t}$

$= 9.98 \times 10^{10}$

$\approx 10^{11}$

Maximum kinetic energy = 10 eV – 5 eV = 5 eV

21. Two kg of a monoatomic gas is at a pressure of 4×10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion ?

(1) 10^3 J
(2) 10^5 J
(3) 10^4 J
(4) 10^6 J

Answer (3)

Sol. $V = \left(\frac{m}{\rho}\right) = \frac{2}{8} = \frac{1}{4}\text{ m}^3$

$PV = nRT \Rightarrow 4 \times 10^4 \times \frac{1}{4} = nRT$

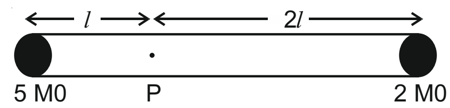
$\Rightarrow nRT = 10^4$

$\therefore$ Internal Energy $= \frac{3}{2}nRT = \frac{3}{2} \times 10^4$

$= 1.5 \times 10^4$ J

$\Rightarrow$ Order $= 10^4$ J

22. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be :



(1) $\frac{g}{2l}$
(2) $\frac{g}{3l}$
(3) $\frac{g}{13l}$
(4) $\frac{7g}{3l}$

Answer (3)

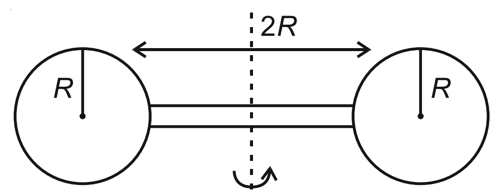
Sol. $\tau = I_p\alpha$

$5M_0gl - 4M_0gl = [2M_0(2l)^2 + 5M_0l^2]\,\alpha$

$\Rightarrow M_0gl = 13M_0l^2\alpha$

$\Rightarrow \alpha = \frac{g}{13l}$

23. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is :



(1) $\frac{152}{15}MR^2$
(2) $\frac{17}{15}MR^2$
(3) $\frac{209}{15}MR^2$
(4) $\frac{137}{15}MR^2$

Answer (4)

Sol. $I = I_{spheres} + I_{rod}$

$I_{rod} = \frac{M\times4R^2}{12} = \frac{MR^2}{3}$

$I_{spheres} = 2 \times \left[\frac{2}{5}MR^2 + 4MR^2\right]$

$= \frac{44}{5}MR^2$

$I = \left[\frac{44}{5}+\frac{1}{3}\right]MR^2$

$= \frac{137}{15}MR^2$

24. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is

(1) $\frac{4\pi}{3}$ (2) $\frac{3}{8}\pi$

(3) $\frac{8\pi}{3}$ (4) $\frac{7}{3}\pi$

Answer (3)

Sol. $a = -\omega^2 x = -\omega^2 \times 4$

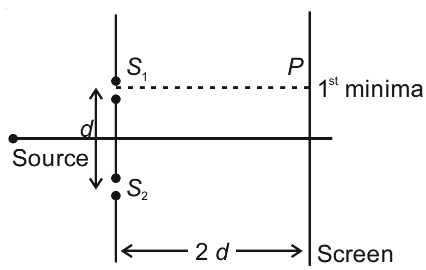
$V = \omega\sqrt{A^2 - x^2} = \omega \times \sqrt{5^2 - 4^2}$

$\because \omega^2 \times 4 = \omega \times 3$

$\Rightarrow \omega = \frac{3}{4}$

$\Rightarrow T = \frac{2\pi}{3/4} = \frac{8\pi}{3}$

25. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms wavelength λ such that the first minima occurs directly in front of the slit (S_1)?



(1) $\frac{\lambda}{2(\sqrt{5}-2)}$ (2) $\frac{\lambda}{2(5-\sqrt{2})}$

(3) $\frac{\lambda}{(5-\sqrt{2})}$ (4) $\frac{\lambda}{(\sqrt{5}-2)}$

Answer (1)

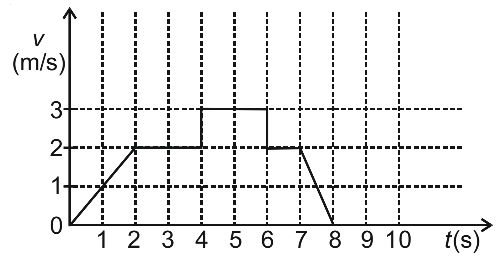
Sol. For 1st minima

$\sqrt{d^2 + (2d)^2} - 2d = \frac{\lambda}{2}$

$\Rightarrow \sqrt{5}\,d - 2d = \frac{\lambda}{2}$

$\Rightarrow d = \frac{\lambda}{2(\sqrt{5}-2)}$

26. A particle starts from the origin at time $t = 0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle time $t = 5$ s?



(1) 9 m (2) 6 m

(3) 10 m (4) 3 m

Answer (1)

Sol. Displacement = Area under v - t graph

Area = $\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1$

= 9 m

27. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is:

(Take Gravitational constant

$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2})

(1) 2.8×10^5 m/s (2) 1.4×10^5 m/s

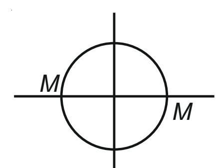
(3) 2.4×10^4 m/s (4) 3.8×10^4 m/s

Answer (1)

Sol. $2R = d = 2 \times 10^{11}$ m

$\Rightarrow R = 10^{11}$ m

$\therefore V(\text{at centre}) = -\frac{GM}{R} \times 2$



$\therefore$ Let v_0 be the minimum speed of meteorite

then $\frac{1}{2}mv_0^2 - \frac{2GMm}{R} = 0$

$v_0 = \sqrt{\frac{4GM}{R}} = \sqrt{\frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}}$

$\Rightarrow v_0 = 2.83 \times 10^5$ m/s $\approx 2.8 \times 10^5$ m/s

28. Two forces P and Q, of magnitude $2F$ and $3F$, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is:

(1) 30° (2) 90°

(3) 60° (4) 120°

Answer (4)

Sol. Let θ be the angle between them

$$|\vec{P}+\vec{Q}|^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$|\vec{P}+\vec{R}|^2 = 4F^2 + 9F^2 + 12F^2\cos\theta = F_R^2 \quad ...(i)$$

Also, $|\vec{P}+2\vec{Q}|^2 = 4F^2 + 36F^2 + 24F^2\cos\theta = 4F_R^2$ ...(ii)

Then (i) and (ii)

$\therefore\ F^2 + 9F^2 + 6F^2\cos\theta = 4F^2 + 9F^2 + 12F^2\cos\theta$

$\therefore\ -3F^2 = 6F^2\cos\theta \qquad \Rightarrow \cos\theta = \left(-\frac{1}{2}\right)$

$\Rightarrow\ \theta = 120°$

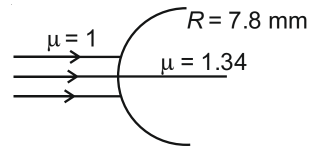
29. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34, Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

(1) 4.0 cm (2) 1 cm

(3) 3.1 cm (4) 2 cm

Answer (3)

Sol. $\frac{1.34}{v} - \frac{1}{u} = \frac{1.34-1}{(7.8\text{ mm})}$



As $u = -\infty$ $\therefore$ $v = f$

$\therefore\ \frac{1.34}{f} = \frac{0.34}{7.8\text{ mm}} \qquad \Rightarrow f = \left(\frac{1.34\times7.8}{0.34}\right)\text{mm}$

$\Rightarrow\ f = 3.07 \approx 3.1$ cm

30. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures?

(1) 4264 ± 81 cm^3 (2) 4300 ± 80 cm^3

(3) 4260 ± 80 cm^3 (4) 4264.4 ± 81.0 cm^3

Answer (3)

Sol. $V = \frac{\pi D^2 h}{4} = \frac{\pi}{4}\times 34.2\times(12.6)^2 = 4264.39\text{ cm}^3$

$\therefore\ \frac{\Delta V}{V} = \frac{2\Delta D}{D} + \frac{\Delta h}{h} = \frac{2\times0.1}{12.6} + \frac{0.1}{34.2}$

$\Rightarrow\ \Delta V = 80.157$

Reducing the answers to proper significant digit then we should write it as

$V = 4260 \pm 80\text{ cm}^3$

❑ ❑ ❑

CHEMISTRY

1. The process with negative entropy change is :
   (1) Sublimation of dry ice
   (2) Dissociation of $CaSO_4(s)$ to $CaO(s)$ and $SO_3(g)$
   (3) Synthesis of ammonia from N_2 and H_2
   (4) Dissolution of iodine in water

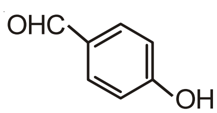
Answer (3)

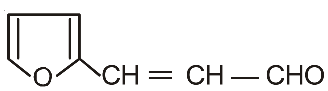
Sol. $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

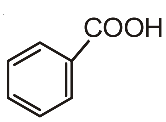
$\because$ Number of gaseous moles are decreasing

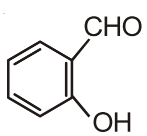
$\therefore$ Change in entropy will be negative.

2. An aromatic compound 'A' having molecular formula $C_7H_6O_2$ on treating with aqueous ammonia and heating forms compound 'B'. The compound 'B' on reaction with molecular bromine and potassium hydroxide provides compound 'C' having molecular formula C_6H_7N. The structure 'A' is :

(1) 

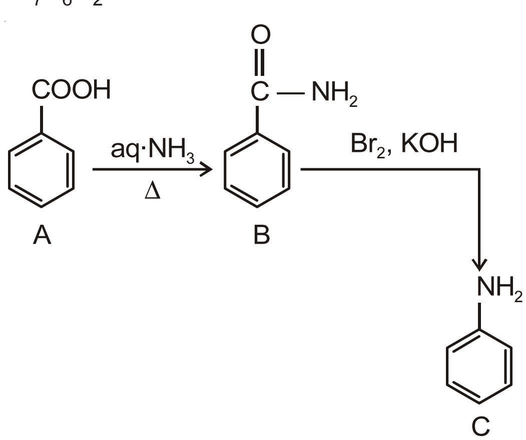
(2) 

(3) 

(4) 

Answer (3)

Sol. $C_7H_6O_2$ – A



$\therefore$ A is Benzoic Acid

3. The ground state energy of hydrogen atom is –13.6 eV. The energy of second excited state of He^+ ion in eV is :

(1) –27.2	(2) –6.04
(3) –54.4	(4) –3.4

Answer (2)

Sol. Energy in n^{th} state as per Bohr's model

$$= -13.6 \times \frac{Z^2}{n^2}\text{ eV}$$

$\therefore$ 2^{nd} excited state

$\Rightarrow$ $n = 3$

$$\therefore \quad E_{3,He^+} = -13.6 \times \frac{2^2}{3^2}\text{ eV}$$

$$= -6.04\text{ eV}$$

4. In the cell

$Pt(s)|H_2(g, 1bar)|HCl(aq)|AgCl(s)|Ag(s)|Pt(s)$ the cell potential is 0.92 V when a 10^{-6} molal HCl solution is used. The standard electrode potential of $(AgCl/Ag, Cl^-)$ electrode is:

$$\left\{\text{Given, } \frac{2.303RT}{F} = 0.06\text{ V at 298 K}\right\}$$

(1) 0.20 V	(2) 0.40 V
(3) 0.76 V	(4) 0.94 V

Answer (1)

Sol. Cell : $Pt(s)|H_2(g, 1bar)|HCl(aq)|AgCl(s)|Ag(s)|Pt(s)$

$E_{cell} = 0.92$ V

$$= E^{\circ}_{H_2(g)|H^+(aq)} + E^{\circ}_{AgCl(s)|Ag(s),Cl^-} - \frac{0.06}{n}\log Q$$

Cell reaction : $\frac{1}{2}H_2(g) \longrightarrow H^+(aq) + e^-$

$AgCl(s) + e^- \longrightarrow Ag(s) + Cl^-$

Net cell reaction:

$$\frac{1}{2}H_2(g) + AgCl(s) \longrightarrow Ag(s) + H^+_{(aq)} + Cl^-(aq)$$

$$\therefore \quad Q = \frac{[H^+][Cl^-]}{(P_{H_2})^{\frac{1}{2}}}$$

10^{-6} molal HCl solution is taken

Assuming molality is same as molarity

$$Q = \frac{10^{-6} \times 10^{-6}}{1} = 10^{-12}$$

$$\therefore \quad 0.92 = E^{\circ}_{AgCl(s)|Ag(s),Cl^-} - \frac{0.06}{1} \log 10^{-12}$$

$$\therefore \quad E^{\circ}_{AgCl(s)|Ag(s),Cl^-} = 0.92 + [0.06 \times (-12)]$$

$$= 0.92 - 0.72$$

$$= 0.20 \text{ V}$$

5. The correct match between item 'I' and item 'II' is :

Item 'I' (compound)	**Item 'II' (reagent)**
(A) Lysine	(P) 1-naphthol
(B) Furfural	(Q) ninhydrin
(C) Benzyl alcohol	(R) $KMnO_4$
(D) Styrene	(S) Ceric ammonium nitrate

(1) (A) → (R); (B) → (P); (C) → (Q); (D) → (S)

(2) (A) → (Q); (B) → (P); (C) → (S); (D) → (R)

(3) (A) → (Q); (B) → (R); (C) → (S); (D) → (P)

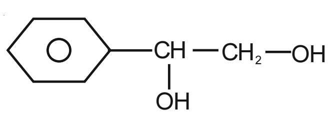
(4) (A) → (Q); (B) → (P); (C) → (R); (D) → (S)

Answer (2)

Sol. – Lysine (amino - acid) reacts with ninhydrin to give a coloured product (blue purple)

– In furfural test (to distinguish between glucose and fructose) dilute sugar solution is added to 1-naphthol (in alcohol) and conc. HCl.

– Benzyl alcohol is oxidised to aldedydes using ceric ammonium nitrate

Styrene is converted to 

using $KMnO_4$

6. Among the following reactions of hydrogen with halogens, the one that requires a catalyst is :

(1) $H_2 + Cl_2 \rightarrow 2HCl$ (2) $H_2 + I_2 \rightarrow 2HI$

(3) $H_2 + Br_2 \rightarrow 2HBr$ (4) $H_2 + F_2 \rightarrow 2HF$

Answer (2)

Sol. Reaction of I_2 with H_2 requires catalyst.

While all other halogens reacts with H_2 without catalyst.

7. The electrolytes usually used in the electroplating of gold and silver, respectively, are :

(1) $[Au(CN)_2]^-$ and $[AgCl_2]^-$

(2) $[Au(NH_3)_2]^+$ and $[Ag(CN)_2]^-$

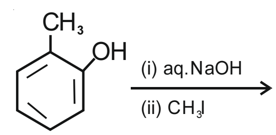
(3) $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$

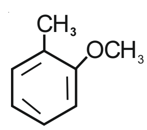
(4) $[Au(OH)_4]^-$ and $[Ag(OH)_2]^-$

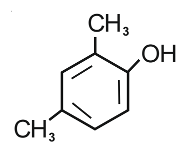
Answer (3)

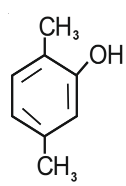
Sol. $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$ are used in the electroplating of Au and Ag respectively.

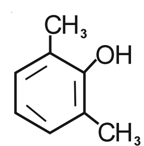
8. The major product of the following reaction is :



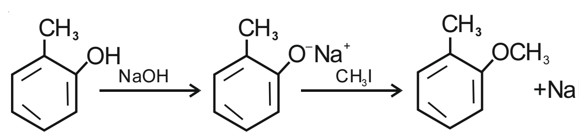
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

9. A reaction of cobalt (III) chloride and ethylenediamine in a 1 : 2 mole ratio generates two isomeric products A (violet coloured) and B (green coloured). A can show optical activity, but, B is optically inactive. What type of isomers does A and B represent?

(1) Ionisation isomers (2) Coordination isomers

(3) Geometrical isomers (4) Linkage isomers

Answer (3)

Sol. $[CoCl_6]^{3-} + 2(en) \longrightarrow [CoCl_2(en)_2]^+$

(1 : 2 mole ratio) (cis-trans isomer)

A - optically active (cis-isomer)

B - optically inactive (trans isomer)

10. Sodium metal on dissolution in liquid ammonia gives a deep blue solution due to the formation of

(1) Ammoniated electrons

(2) Sodium-ammonia complex

(3) Sodium ion-ammonia complex

(4) Sodamide

Answer (1)

Sol. Sodium metal on dissolution in liquid ammonia gives a deep blue solution due to ammoniated electrons.

11. In the reaction of oxalate with permanganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is

(1) 1

(2) 10

(3) 2

(4) 5

Answer (1)

Sol. $\overset{+3}{C_2}O_4^{2-} \longrightarrow 2\overset{+4}{C}O_2 \quad +2e^-$

O.N. of one C-atom ↑ by 1

∴ No. of electrons involved in producing one mole of CO_2 is 1.

12. 5.1 g NH_4SH is introduced in 3.0 L evacuated flask at 327ºC. 30% of the solid NH_4SH decomposed to NH_3 and H_2S as gases. The K_p of the reaction at 327ºC is (R = 0.082 L atm $mol^{-1}K^{-1}$, Molar mass of S = 32 g mol^{-1}, molar mass of N = 14 g mol^{-1})

(1) 4.9×10^{-3} atm^2 (2) 0.242 atm^2

(3) 1×10^{-4} atm^2 (4) 0.242×10^{-4} atm^2

Answer (2)

Sol. $NH_4SH \longrightarrow NH_3(g) + H_2S(g)$

Initial moles $\frac{5.1}{51} = 0.1$ mol

Moles at equilibrium $\quad NH_4SH \rightarrow NH_3(g) + H_2S(g)$

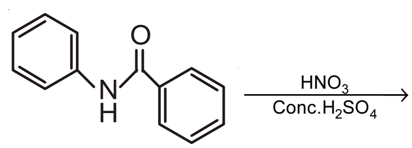
$0.1(1-0.3) \quad 0.1 \times 0.3 \quad 0.1 \times 0.3$

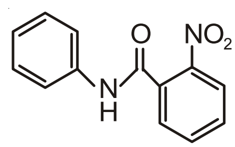
$\therefore \quad K_C = [NH_3][H_2S] = \left(\frac{0.03}{3}\right)^2 = 10^{-4}$

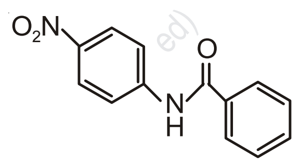
$K_P = K_C.(RT)^{\Delta n_g} = 10^{-4} \times (0.082 \times 600)^2$

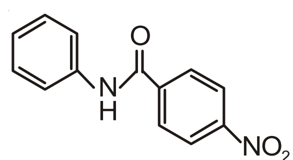
$= 0.242\ atm^2$

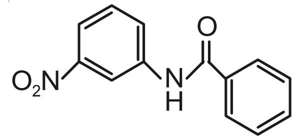
13. What will be the major product in the following mononitration reaction?



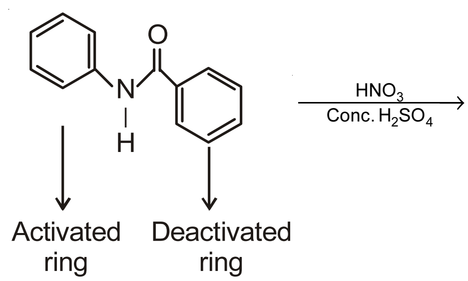
(1) 

(2) 

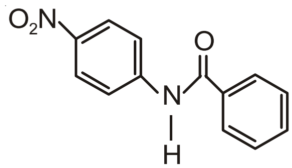
(3) 

(4) 

Answer (2)

Sol. 

∴ Major product will be formed as per activating group.

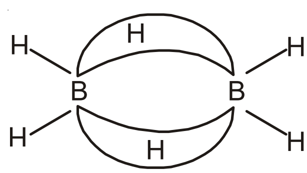
∴ Product formed is 

14. The number of 2-centre-2-electron and 3-centre-2-electron bonds in B_2H_6 respectively are:

(1) 4 and 2 (2) 2 and 2

(3) 2 and 4 (4) 2 and 1

Answer (1)

Sol. 

No. of 2-C-2-e^- bond = 4

No. of 3-C-2-e^- bond = 2

15. The 71st electron of an element X with an atomic number of 71 enters into the orbital:

(1) 5 d (2) 6 p

(3) 4 f (4) 6 s

Answer (1)

Sol. Atomic number = 71

Electronic configuration:

$[Xe]6s^2\,4f^{14}\,5d^1 \leftarrow$ last electron

∴ Orbital occupied by last e^- is 5d

16. Which of the following tests cannot be used for identifying amino acids?

(1) Barfoed test (2) Biuret test

(3) Xanthoproteic test (4) Ninhydrin test

Answer (1)

Sol. Barfoed test is used for carbohydrate to cheak reducing nature of sugar.

17. The amount of sugar ($C_{12}H_{22}O_{11}$) required to prepare 2L of its 0.1 M aqueous solution is:

(1) 136.8 g (2) 17.1 g

(3) 34.2 g (4) 68.4 g

Answer (4)

Sol. Molarity = $\dfrac{\text{Mole of sugar}}{\text{Volume of solution (in L)}}$

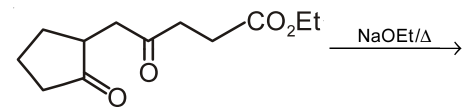
$\Rightarrow 0.1 = \dfrac{\text{Mole of sugar}}{2\,L}$

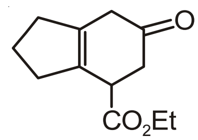
Mole of sugar = 0.2 mol

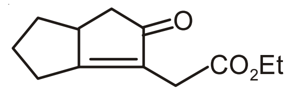
Mass of sugar = Mole × Molar mass of sugar

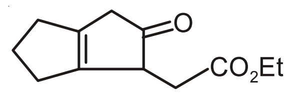
= 0.2 × 342 = 68.4 g

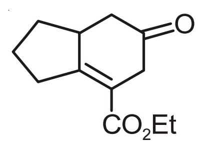
18. The major product obtained in the following reaction is:



(1) 

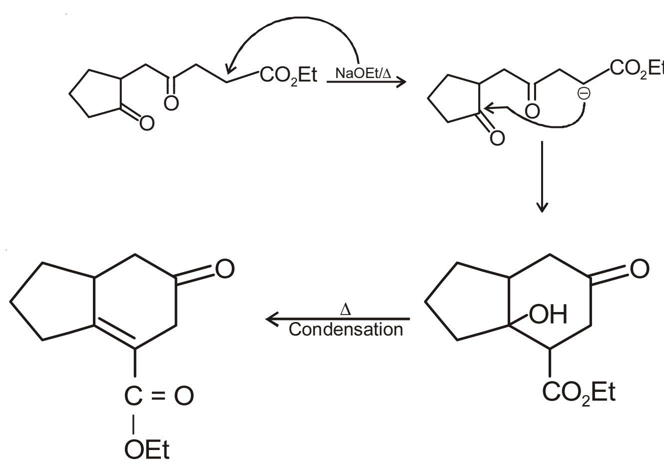
(2) 

(3) 

(4) 

Answer (4)

Sol.



19. The difference in the number of unpaired electrons of a metal ion in its high spin and low-spin octahedral complexes is two. The metal ion is:

(1) Ni^{2+} (2) Mn^{2+}

(3) Fe^{2+} (4) Co^{2+}

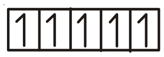
Answer (1)

Sol. Ni^{2+} = |Ar| $4s^0\ 3d^8$

3 d^8 4 s 4 p

No of unpaired electron remain two with S.F.L or W.F.L

Mn^{2+} = |Ar| $4s^\circ\ 3d^5$

 → 5 unpaired electron with W.F.L

Fe^{2+} = |Ar| $4s^\circ\ 3d^6$

 → 4 unpaired electron with W.F.L

Co^{2+} = |Ar| $4s^\circ\ 3d^7$

 → 3 unpaired electron with W.F.L

20. Haemoglobin and gold sol are examples of :

(1) negatively charged sols

(2) positively charged sols

(3) positively and negatively charged

(4) negatively and positively charged sols, respectively

Answer (3)

Sol. Haemoglobin → Positive sol

Metal → Negative sol

21. For an elementary chemical reaction, $A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2A$, the expression for $\frac{d[A]}{dt}$ is :

(1) $k_1[A_2] + k_{-1}[A]^2$ (2) $2k_1[A_2] - 2k_{-1}[A]^2$

(3) $2k_1[A_2] - k_{-1}[A]^2$ (4) $k_1[A_2] - k_{-1}[A]^2$

Answer (2)

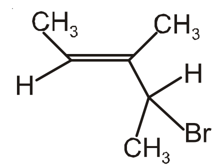
Sol. $A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2A$

$$-\frac{1}{2}\frac{d[A]}{dt} = k_{-1}[A]^2 - k_1[A_2]$$

$$\frac{d[A]}{dt} = -2k_{-1}[A]^2 + 2k_1[A_2]$$

$$\Rightarrow \frac{d[A]}{dt} = 2k_1[A_2] - 2k_{-1}[A]^2$$

22. What is the IUPAC name of the following compound?



(1) 3-Bromo-1, 2-dimethylbut-1-ene

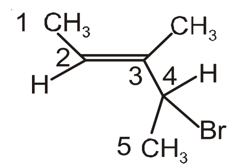
(2) 4-Bromo-3-methylpent-2-ene

(3) 3-Bromo-3-methyl-1, 2- dimethylprop-1-ene

(4) 2-Bromo-3-methylpent-3-ene

Answer (2)

Sol.



4-Bromo-3-methylpent-2-ene

23. An ideal gas undergoes isothermal compression from 5 m^3 to 1 m^3 against a constant external pressure of 4 Nm^{-2}. Heat released in this process is used to increase the temperature of 1 mole of Al. If molar heat capacity of Al is 24 J $mol^{-1}K^{-1}$, the temperature of Al increases by:

(1) $\frac{3}{2}$ K (2) 1 K

(3) 2 K (4) $\frac{2}{3}$ K

Answer (4)

Sol. $w = -P_{ext}(V_f - V_i) = -4\ Nm^{-2}\ (1 - 5)m^3$

$= 16\ Nm \Rightarrow 16\ J$

For isothermal process

$\Delta U = q + w \Rightarrow q = -w = -16\ J$

($\because$ $\Delta U = 0$ for isothermal process)

From calorimetry

Heat given = $nc\Delta T$

$$16 = \frac{1 \times 24\ J \times \Delta T}{mol\ K}$$

$$\Delta T = \frac{2}{3} K$$

24. The pair that contains two P-H bond in each of the oxoacids is :

(1) $H_4P_2O_5$ and $H_4P_2O_6$

(2) $H_4P_2O_5$ and H_3PO_3

(3) H_3PO_2 and $H_4P_2O_5$

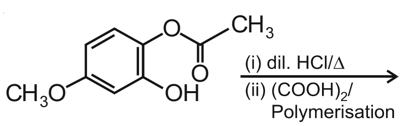
(4) H_3PO_3 and H_3PO_2

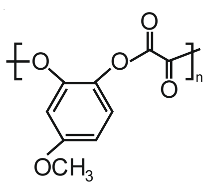
Answer (3)

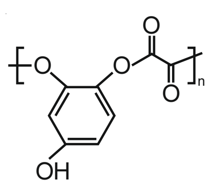
Sol.

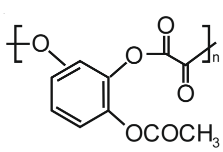
Acid	No of P-H bond
$H_4P_2O_5$	2
$H_4P_2O_6$	0
H_3PO_3	1
H_3PO_2	2

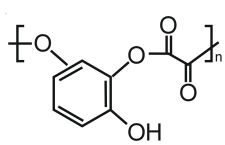
25. The major product of the following reaction



(1) 

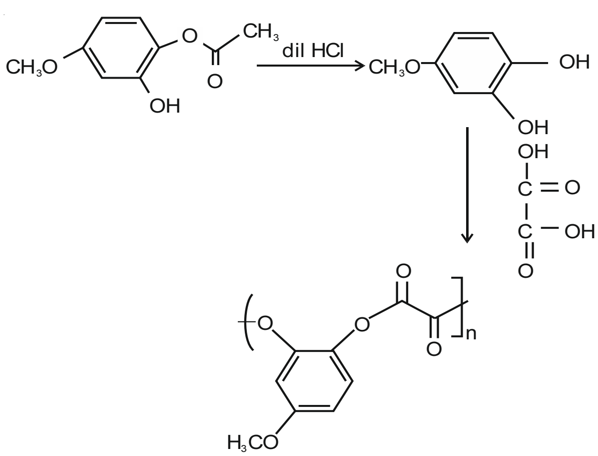
(2) 

(3) 

(4) 

Answer (1)

Sol.



26. Elevation in the boiling point for 1 molal solution of glucose is 2 K. The depression in the freezing point for 2 molal solution of glucose in the same solvent is 2 K. The relation between K_b and K_f is :

(1) $K_b = 0.5\ K_f$ (2) $K_b = 2\ K_f$

(3) $K_b = 1.5\ K_f$ (4) $K_b = K_f$

Answer (2)

Sol. $\Delta T_b = k_b m \Rightarrow k_b(1) = 2 \Rightarrow k_b = 2\ km^{-1}$

$\Delta T_f = k_f m \Rightarrow k_f(2) = 2 \Rightarrow k_f = 1\ km^{-1}$

$k_f = 0.5\ k_b \Rightarrow k_b = 2\ k_f$

27. A compound of formula A_2B_3 has the hcp lattice. Which atom forms the hcp lattice and what fraction of tetrahedral voids is occupied by the other atoms:

(1) hcp lattice - B, $\frac{1}{3}$ Tetrahedral voids - A

(2) hcp lattice - A, $\frac{2}{3}$ Tetrahedral voids - B

(3) hcp lattice - B, $\frac{2}{3}$ Tetrahedral voids - A

(4) hcp lattice - A, $\frac{1}{3}$ Tetrahedral voids - B

Answer (1)

Sol. A_2B_3 can be written as $\Rightarrow A_4B_6$

H.C.P has Six atom so 'B' form

H.C.P lattice and A is present in void.

Total tetrahedral void = 12

Fraction of tetrahedral void occupied by

A = 4/12 = 1/3

28. The reaction that is NOT involved in the ozone layer depletion mechanism in the stratosphere is :

(1) $HOCl(g) \xrightarrow{h\nu} \dot{O}H(g) + \dot{C}l(g)$

(2) $CH_4 + 2O_3 \longrightarrow 3CH_2 = O + 2H_2O$

(3) $Cl\dot{O}(g) + O(g) \longrightarrow \dot{C}l(g) + O_2(g)$

(4) $CF_2Cl_2(g) \xrightarrow{uv} \dot{C}l(g) + \dot{C}F_2Cl(g)$

Answer (2)

Sol. $CH_4 + 2O_3 \rightarrow 3CH_2 = O + 3H_2O$ not involved in the ozone layer.

29. Which is the most suitable reagent for the following transformation?

$$CH_3-CH=CH-CH_2-\underset{|}{\overset{OH}{C}}H-CH_3 \longrightarrow CH_3-CH=CH-CH_2CO_2H$$

(1) $I_2/NaOH$ (2) Alkaline $KMnO_4$

(3) Tollen's reagent (4) CrO_2Cl_2/CS_2

Answer (1)

Sol.

$$CH_3-CH=CH-CH_2-\underset{\underset{OH}{|}}{CH}-CH_3$$

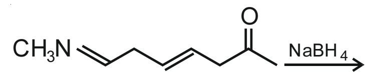
$$\downarrow I_2/NaOH$$

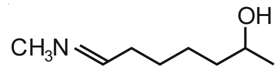
$$CH_3-CH=CH-CH_2-\underset{\underset{O}{||}}{C}-CH_3$$

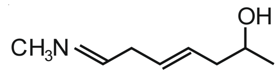
$$\downarrow I_2/NaOH$$

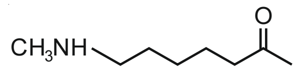
$$CH_3-CH=CH-CH_2-\underset{\underset{O}{||}}{C}-ONa + CHI_3$$

30. The major porduct of the following reaction is :



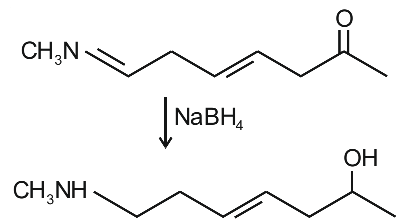
(1) 

(2) 

(3) 

Answer (4)

Sol.



❑ ❑ ❑

MATHEMATICS

1. Let $\vec{\alpha} = (\lambda - 2)\,\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\,\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is

(1) – 3 (2) – 4

(3) 3 (4) 4

Answer (2)

Sol. $\vec{\alpha}$ and $\vec{\beta}$ are collinear

$\Rightarrow \vec{\alpha} = t\vec{\beta}$

$(\lambda - 2)\,\vec{a} + \vec{b} = t\left((4\lambda - 2)\,\vec{a} + 3\vec{b}\right)$

$(\lambda - 2 - t(4\lambda - 2))\vec{a} + \vec{b}(1 - 3t) = \vec{0}$

$\vec{a}$ and $\vec{b}$ are non-collinear

$\Rightarrow \lambda - 2 - t(4\lambda - 2) = 0,\ 1 - 3t = 0$

$\Rightarrow t = \frac{1}{3}$ and $\lambda - 2 - \frac{1}{3}(4\lambda - 2) = 0$

$3\lambda - 6 - 4\lambda + 2 = 0$

$\boxed{\lambda = -4}$

Option (2) is correct

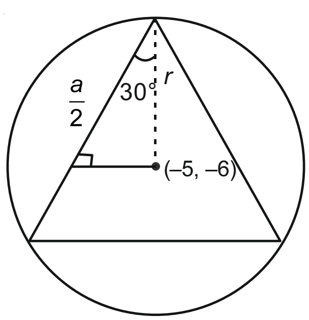
2. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units

(1) 13 (2) 25

(3) – 25 (4) 20

Answer (2)

Sol.



Let side of equilateral Δ is a

$\Rightarrow \cos 30° = \frac{a}{2r}$

$a = \sqrt{3}r$

$\Delta = 27\sqrt{3} = \frac{\sqrt{3}}{4} \cdot a^2 = \frac{\sqrt{3}}{4} \times 3r^2$

$r^2 = 36$

$\Rightarrow r = 6$

$(-5)^2 + (-6)^2 - c = 36$

$c = 25$

Option (2) is correct.

3. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$

has a non-trivial solution, is:

(1) Four (2) One

(3) Three (4) Two

Answer (4)

Sol. For non-trivial solution

$\Delta = 0$

$$\begin{vmatrix} \text{Sin}\,3\theta & \cos 2\theta & 2 \\ 1 & 3 & 7 \\ -1 & 4 & 7 \end{vmatrix} = 0$$

$\sin 3\theta(21 - 28) - \cos 2\theta(7 + 7) + 2\,(4 + 3) = 0$

$\sin 3\theta + 2\cos 2\theta - 2 = 0$

$3\sin\theta - 4\sin^3\theta + 2 - 4\sin^2\theta - 2 = 0$

$4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$

$\sin\theta\,(4\sin^2\theta + 4\sin\theta - 3) = 0$

$\sin\theta\,(4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3) = 0$

$\sin\theta\,[2\sin\theta\,(2\sin\theta - 1) + 3\,(2\sin\theta - 1)] = 0$

$\sin\theta = 0,\ \sin\theta = \frac{1}{2}$ $\left(\because\ \sin\theta \neq -\frac{3}{2}\right)$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Option (4) is correct.

4. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is

(1) 3 (2) 4

(3) $\sqrt{5}$ (4) $2\sqrt{2}$

Answer (2)

Sol. Coefficient of x^2 in $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

= co-efficient of x° in $\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

General term in $\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10} = {}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r$

for constant term

$$\frac{10-r}{2}-2r=0$$

$\Rightarrow$ $r = 2$

$\Rightarrow$ Co-efficient of x^2 in expression

$= {}^{10}C_2\lambda^2 = 720$

$\Rightarrow$ $\lambda^2 = \frac{720}{5\times 9} = 16$

$\lambda = 4$

Option (2) is correct.

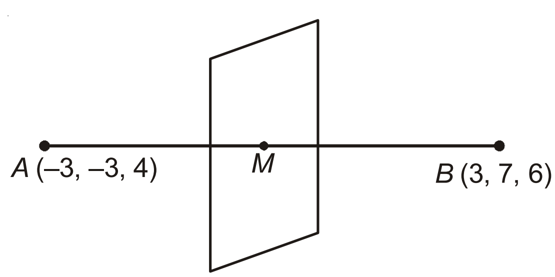
5. The plane which bisects the line segment joining the points (–3, –3, 4) and (3, 7, 6) at right angles, passes through which one of the following points?

(1) (–2, 3, 5) (2) (4, 1, –2)

(3) (2, 1, 3) (4) (4, –1, 7)

Answer (2)

Sol.



M (0, 2, 5)

D.R's of normal to the plane is $\vec{n} = 6\hat{i}+10\hat{j}+2\hat{k}$

$\Rightarrow$ equation of the plane is

$(x – 0)6 + (y–2)10 + (z – 5)2 = 0$

$3x + 5y – 10 + z – 5 = 0$

$3x + 5y + z = 15$ ...(i)

plane (i) passes through (4, 1, –2)

option (2) is correct.

6. Let $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z, then

(1) $I(z) = 0$ (2) $R(z) > 0$ and $I(z) > 0$

(3) $R(z) < 0$ and $I(z) > 0$ (4) $R(z) = – 3$

Answer (1)

Sol. $z = (e^{i\frac{\pi}{6}})^5 + (e^{-i\frac{\pi}{6}})^5 = 2\cos\frac{\pi}{6} = \sqrt{3}$

$\Rightarrow$ $I(z) = 0,\ \text{Re}(z) = \sqrt{3}$

$\Rightarrow$ Option (1) is correct

7. If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is

(1) 4 (2) 5

(3) 3 (4) 6

Answer (2)

Sol. Let the number of independent shots required to hit the target at least once is n

$\Rightarrow$ $1-\left(\frac{2}{3}\right)^n > \frac{5}{6}$

$\left(\frac{2}{3}\right)^n < \frac{1}{6}$

Least value of n is 5

Option (2) is correct.

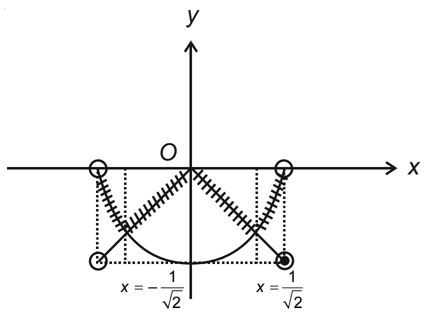
8. Let $f:(-1, 1)\to R$ be a function defined by $f(x)=\max\{-|x|,-\sqrt{1-x^2}\}$. If K be the set of all points at which f is not differentiable, then K has exactly

(1) Three elements (2) Two elements

(3) One element (4) Five elements

Answer (1)

Sol. $f(x)=\max\{-|x|,-\sqrt{1-x^2}\}$



$f(x)$ is not differentiable at $x\in\left\{-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right\}$

$\Rightarrow K=\left\{-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right\}$

9. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is

(1) $\frac{19}{21}$ (2) $\frac{23}{22}$

(3) $\frac{22}{23}$ (4) $\frac{21}{19}$

Answer (4)

Sol. $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$

$$=\cot\left(\sum_{n=1}^{19}\cot^{-1}(1+n(n+1))\right)$$

$$=\cot\left(\sum_{n=1}^{19}\tan^{-1}\left(\frac{(n+1)-n}{1+(n+1)n}\right)\right)$$

$$=\cot\left(\sum_{n=1}^{19}(\tan^{-1}(n+1)-\tan^{-1}n)\right)$$

$$=\cot(\tan^{-1}20-\tan^{-1}1)$$

$$=\cot\left(\tan^{-1}\left(\frac{20-1}{1+20\times1}\right)\right)$$

$$=\cot\left(\tan^{-1}\left(\frac{19}{21}\right)\right)=\cot\cot^{-1}\left(\frac{21}{19}\right)=\frac{21}{19}$$

10. Consider the following three statements:

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true?

(1) $(\sim P)\wedge(\sim Q\wedge R)$

(2) $(\sim P)\vee(Q\wedge R)$

(3) $P\vee(\sim Q\wedge R)$

(4) $(P\wedge Q)\vee(\sim R)$

Answer (3)

Sol. P is True

Q is False

R is True

(1) $(\sim P)\wedge(\sim Q\wedge R)\equiv F\wedge(T\wedge T)\equiv F\wedge T=F$

(2) $(\sim P)\vee(Q\wedge R)\equiv F\vee(F\wedge T)\equiv F\vee F=F$

(3) $P\vee(\sim Q\wedge R)\equiv T\vee(T\wedge T)\equiv T\vee T=T$

(4) $(P\wedge Q)\vee(\sim R)\equiv(T\wedge F)\vee(F)\equiv F\vee F=F$

11. The curve amongst the family of curves represented by the differential equation, $(x^2-y^2)dx+2xydy=0$ which passes through (1, 1) is

(1) A hyperbola with transverse axis along the x-axis.

(2) A circle with centre on the y-axis.

(3) An ellipse with major axis along the y-axis.

(4) A circle with centre on the x-axis.

Answer (4)

Sol. $(x^2-y^2)dx+2xy\,dy=0$

$y^2dx-2xydy=x^2dx$

$2xydy-y^2dx=-x^2dx$

$$\frac{xd(y^2)-y^2d(x)}{x^2}=-dx$$

$$d\left(\frac{y^2}{x}\right)=-dx$$

$$\int d\left(\frac{y^2}{x}\right) = -\int dx$$

$\frac{y^2}{x} = -x + C$ passes through (1, 1) $\Rightarrow C = 2$

$y^2 = -x^2 + 2x$

$\Rightarrow$ $y^2 = -(x-1)^2 + 1$

$(x-1)^2 + y^2 = 1$, circle with centre (1, 0)

centre lies on x-axis

12. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is

(1) $3\sqrt{2}$ (2) $6\sqrt{3}$

(3) $2\sqrt{11}$ (4) $8\sqrt{2}$

Answer (2)

Sol. For intersection point $A(x_1, y_1)$ and $B(x_2, y_2)$

$$x - \sqrt{2}\frac{x^2}{4} + 4\sqrt{2} = 0$$

$$\sqrt{2}x^2 - 4x - 16\sqrt{2} = 0$$

$$x_1 + x_2 = 2\sqrt{2}, x_1x_2 = -16, (x_1 - x_2)^2 = 8 + 64 = 72$$

$$\because \quad x_1 - \sqrt{2}y_1 + 4\sqrt{2} = 0$$

$$x_2 - \sqrt{2}y_2 + 4\sqrt{2} = 0$$

$$(x_2 - x_1) = \sqrt{2}(y_2 - y_1) \Rightarrow (x_2 - x_1)^2 = 2(y_2 - y_1)^2$$

$$\Rightarrow \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + \frac{(x_2 - x_1)^2}{2}}$$

$$= |x_2 - x_1| \cdot \frac{\sqrt{3}}{\sqrt{2}} = 6\sqrt{2} \times \frac{\sqrt{3}}{\sqrt{2}} = 6\sqrt{3}$$

13. If $\int x^5 e^{-4x^3} dx = \frac{1}{48}e^{-4x^3} f(x) + C$, where C is a constant of integration, then $f(x)$ is equal to

(1) $4x^3 + 1$ (2) $-4x^3 - 1$

(3) $-2x^3 + 1$ (4) $-2x^3 - 1$

Answer (2)

Sol. $I = \int x^5 e^{-4x^3} dx$

Put $-4x^3 = t$

$\Rightarrow$ $-12x^2\, dx = dt$

$\Rightarrow$ $x^2 dx = -\frac{dt}{12}$

$\Rightarrow$ $I = \int \frac{1}{48} te^t\, dt = \frac{1}{48}[t\, e^t - e^t] + C$

$I = \frac{1}{48}e^{-4x^3}(-4x^3 - 1) + C$

$\Rightarrow$ $f(x) = -4x^3 - 1$

14. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$?

(1) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

(2) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

(3) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

(4) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

Answer (1)

Sol. Any point on the line $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ is $P(2\lambda + 4, 2\lambda + 5, \lambda + 3)$ lies on the plane $x + y + z = 2$

$\Rightarrow$ $2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$

$\Rightarrow$ $5\lambda = -10 \Rightarrow \lambda = -2$

$\Rightarrow$ Point of intersection is (0, 1, 1)

which lies on the line $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

15. Let $S=\left\{(x,y)\in R^2:\frac{y^2}{1+r}-\frac{x^2}{1-r}=1\right\}$, where $r\neq\pm1$. Then S represents

(1) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when $r > 1$.

(2) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$.

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when $0 < r < 1$.

(4) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

Answer (2)

Sol. When $r > 1$

$$\frac{x^2}{r-1}+\frac{y^2}{r+1}=1 \quad \text{(Ellipse)}$$

$$(r-1)=(r+1)(1-e^2) \Rightarrow 1-e^2=\frac{(r-1)}{(r+1)}$$

$$\Rightarrow e^2=1-\frac{(r-1)}{(r+1)}=\frac{2}{(r+1)} \Rightarrow e=\sqrt{\frac{2}{(r+1)}}$$

when $0 < r < 1$

$$\frac{x^2}{1-r}-\frac{y^2}{1+r}=-1 \quad \text{(Hyperbola)}$$

$$(1-r)=(1+r)(e^2-1) \Rightarrow e^2=1+\frac{(r-1)}{(r+1)}=\frac{2r}{(r+1)}$$

$$\Rightarrow e=\sqrt{\frac{2r}{r+1}}$$

Option (2) is correct.

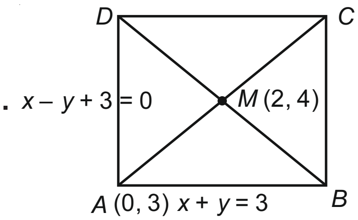
16. Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at (2, 4) then one of its vertex is :

(1) (2, 1) (2) (3, 5)

(3) (2, 6) (4) (3, 6)

Answer (4)

Sol.



$x - y + 3 = 0$ and $x + y = 3$ are perpendicular lines intersection point of $x - y + 3 = 0$ and $x + y = 3$ is $A(0, 3)$.

$\Rightarrow$ M is mid-point of AC $\Rightarrow C(4, 5)$

Let $D(x_1, x_1 + 3)$ and $B(x_2, 3 - x_2)$

M is mid-point of DB

$\Rightarrow x_1 + x_2 = 4$, $x_1 + 3 + 3 - x_2 = 8$

$\Rightarrow x_1 = 3, x_2 = 1$

Option (4) is correct.

17. Let $a_1, a_2, a_3 \ldots, a_{10}$ in G.P with $a_i > 0$ for $i = 1, 2, \ldots, 10$ and S be the set of pairs (r, k), $r, k\in N$ (the set of natural numbers for which

$$\begin{vmatrix}\log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k\\ \log_e a_4^r a^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k\\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k\end{vmatrix}=0$$

(1) 2 (2) 10

(3) 4 (4) Infinitely many

Answer (4)

Sol. Let common ratio of G.P. is R

$\Rightarrow a_2 = a_1R$, $a_3 = a_1R^2, \ldots a_{10} = a_1R^9$

$C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$$\Delta=\begin{vmatrix}\ln\left(\frac{a_1^r a_2^k}{a_2^r a_3^k}\right) & \ln\left(\frac{a_2^r a_3^k}{a_3^r a_4^k}\right) & \ln a_3^r a_4^k\\ \ln\left(\frac{a_4^r a_5^k}{a_5^r a_6^k}\right) & \ln\left(\frac{a_5^r a_6^k}{a_6^r a_7^k}\right) & \ln a_6^r a_7^k\\ \ln\frac{a_7^r a_8^k}{a_8^r a_9^k} & \ln\left(\frac{a_8^r a_9^k}{a_9^r a_{10}^k}\right) & \ln a_9^r a_{10}^k\end{vmatrix}$$

$$\Delta=\begin{vmatrix}\ln\frac{1}{R^{r+k}} & \ln\frac{1}{R^{r+k}} & \ln a_3^r a_4^k\\ \ln\frac{1}{R^{r+k}} & \ln\frac{1}{R^{r+k}} & \ln a_6^r a_7^k\\ \ln\frac{1}{R^{r+k}} & \ln\frac{1}{R^{r+k}} & \ln a_9^r a_{10}^k\end{vmatrix}=0\ \forall\ r,k\in N$$

$\Rightarrow$ No. of elements in S is infinitely many

$\Rightarrow$ Option (4) is correct.

18. With the usual notation, in ΔABC, if $\angle A$ + $\angle B$ = 120°, $a=\sqrt{3}+1$ and $b=\sqrt{3}-1$, then the ratio $\angle A$: $\angle B$, is :

(1) 7 : 1 (2) 3 : 1

(3) 9 : 7 (4) 5 : 3

Answer (1)

Sol. $\because$ $A + B = 120°$

$\Rightarrow C = 180° - 120° = 60°$

$$\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$=\frac{2}{2\sqrt{3}}(\cot 30°)=\frac{1}{\sqrt{3}}\times\sqrt{3}=1$$

$\Rightarrow \frac{A-B}{2}=\frac{\pi}{2}$

$\Rightarrow A - B = 90°$

$\Rightarrow A = 105°$, $B = 15°$

$\Rightarrow \angle A : \angle B : : 7 : 1$

Option (1) is correct.

19. Let $A=\begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$ where $b > 0$. Then

(1) $-\sqrt{3}$ (2) $\sqrt{3}$

(3) $2\sqrt{3}$ (4) $-2\sqrt{3}$

Answer (3)

Sol. $|A|=\begin{vmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{vmatrix}$

$=2(2b^2+2-b^2)-b(2b-b)+1(b^2-b^2-1)$

$= 2b^2 + 4 - b^2 - 1 = b^2 + 3$

$\frac{|A|}{b}=b+\frac{3}{b}$

$\because \frac{b+\frac{3}{b}}{2}\geq\left(b\cdot\frac{3}{b}\right)^{\frac{1}{2}}$

$\frac{|A|}{b}\geq 2\sqrt{3}$

Minimum value of $\frac{|A|}{b}$ is $2\sqrt{3}$.

Option (3) is correct.

20. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{dx}{[x]+[\sin x]+4}$, where $[t]$ denotes the greatest integer less than or equal to t, is :

(1) $\frac{1}{12}(7\pi-5)$ (2) $\frac{3}{10}(4\pi-3)$

(3) $\frac{3}{20}(4\pi-3)$ (4) $\frac{1}{12}(7\pi+5)$

Answer (3)

Sol. $I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{dx}{[x]+[\sin x]+4}$

$$=\int_{\frac{-\pi}{2}}^{-1}\frac{dx}{-2-1+4}+\int_{-1}^{0}\frac{dx}{-1-1+4}+\int_{0}^{1}\frac{dx}{0+0+4}+\int_{1}^{\frac{\pi}{2}}\frac{dx}{1+0+4}$$

$$=\left(-1+\frac{\pi}{2}\right)+\frac{1}{2}(0+1)+\frac{1}{4}(1-0)+\frac{1}{5}\left(\frac{\pi}{2}-1\right)$$

$$=\frac{3\pi}{5}-\frac{9}{20}=\frac{3}{20}(4\pi-3)$$

Option (3) is correct.

21. Let f be a differentiable function such that $f'(x)=7-\frac{3}{4}\frac{f(x)}{x}$, $(x > 0)$ and $f(1) \neq 4$. Then $\lim_{x\to 0^+} xf\left(\frac{1}{x}\right)$:

(1) Exist and equals 4. (2) Does not exist.

(3) Exists and equals $\frac{4}{7}$. (4) Exists and equals 0.

Answer (1)

Sol. Let $y = f(x)$

$$\frac{dy}{dx}+\left(\frac{3}{4x}\right)y=7$$

I.F $= e^{\int\frac{3}{4x}dx}=e^{\frac{3}{4}\ln x}=x^{\left(\frac{3}{4}\right)}$

Solution of differential equation

$$y\cdot x^{\frac{3}{4}}=\int 7\cdot x^{\frac{3}{4}}dx+C$$

$$y \cdot x^{\frac{3}{4}} = 7 \cdot \frac{x^{\frac{7}{4}}}{\left(\frac{7}{4}\right)} + C = 4x^{\frac{7}{4}} + C$$

$$f(x) = 4x + Cx^{-\frac{3}{4}}$$

$$\Rightarrow \quad f\left(\frac{1}{x}\right) = \frac{4}{x} + Cx^{\frac{3}{4}}$$

$$\Rightarrow \quad \lim_{x \to 0^+} x \cdot f\left(\frac{1}{x}\right) = \lim_{x \to 0^+} \left(4 + Cx^{\frac{7}{4}}\right) = 4$$

Option (1) is correct.

22. If $\sum_{r=0}^{25} \left\{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \right\} = K\left({}^{50}C_{25}\right)$, then K is equal to :

(1) $2^{25} - 1$ (2) $(25)^2$

(3) 2^{25} (4) 2^{24}

Answer (3)

Sol. $\sum_{r=0}^{25} \left({}^{50}C_r \cdot {}^{50-r}C_{25-r}\right) = \sum_{r=0}^{25} \left(\frac{\lfloor 50}{\lfloor 50-r \, \lfloor r} \; \frac{\lfloor 50-r}{\lfloor 25 \, \lfloor 25-r} \right)$

$$= \sum_{r=0}^{25} \left(\frac{\lfloor 50}{\lfloor 25} \times \frac{1}{\lfloor 25} \times \left(\frac{\lfloor 25}{\lfloor 25-r \, \lfloor r} \right) \right)$$

$$= {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = {}^{50}C_{25}(2^{25})$$

$\Rightarrow$ $K = 2^{25}$

$\Rightarrow$ Option (3) is correct.

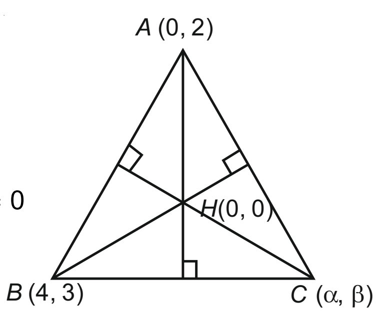
23. Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant?

(1) Fourth (2) Third

(3) First (4) Second

Answer (4)

Sol. $m_{BC} \times m_{AH} = -1$



$$\Rightarrow \quad m_{BC} = -\frac{1}{m_{AH}}$$

$$\Rightarrow \quad m_{BC} = \frac{\beta - 3}{\alpha - 4} = 0$$

$\Rightarrow$ $\beta = 3$

$m_{AB} \times m_{CH} = -1$

$$\Rightarrow \quad \frac{1}{4} \times \frac{\beta}{\alpha} = -1$$

$\Rightarrow$ $\beta = -4\alpha$

$$\Rightarrow \quad \alpha = -\frac{3}{4}$$

Vertex C Is $\left(\frac{-3}{4}, 3\right)$

$\Rightarrow$ Vertex C lies in second quadrant.

$\Rightarrow$ Option (4) is correct.

24. Let N be the set of natural numbers and two functions f and g be defined as

$f, g : N \to N$ such that

$$f(n) = \begin{cases} \frac{n+1}{2} \text{ if } n \text{ is odd} \\ \frac{n}{2} \text{ if } n \text{ is even} \end{cases}$$

and $g(n) = n - (-1)^n$. Then fog is :

(1) One-one but not onto.

(2) Onto but not one-one.

(3) Neither one-one nor onto.

(4) Both one-one and onto.

Answer (2)

Sol. $$f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2}, \text{ if } n \text{ is even} \end{cases}$$

$$g(n) = \begin{cases} 2, & n = 1 \\ 1, & n = 2 \\ 4, & n = 3 \\ 3, & n = 4 \\ 6, & n = 5 \\ 5, & n = 6 \end{cases} \Rightarrow f(g(n)) = \begin{cases} \frac{g(n)+1}{2}, \text{ if } g(n) \text{ is odd} \\ \frac{g(n)}{2}, \text{ if } g(n) \text{ is even} \end{cases}$$

$$f(g(n)) = \begin{cases} 1, & n = 1 \\ 1, & n = 2 \\ 2, & n = 3 \\ 2, & n = 4 \\ 3, & n = 5 \\ 3, & n = 6 \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \end{cases} \Rightarrow fog \text{ is onto but not one - one}$$

$\Rightarrow$ Option (2) is correct.

25. The tangent to the curve $y = xe^{x^2}$ passing through the point (l, e) also passes through the point :

(1) $(2, 3e)$ (2) $\left(\frac{4}{3}, 2e\right)$

(3) $(3, 6e)$ (4) $\left(\frac{5}{3}, 2e\right)$

Answer (2)

Sol. $y = xe^{x^2} \Rightarrow \frac{dy}{dx} = e^{x^2}.1 + x.e^{x^2}.2x$

(1,e) lies on the curve $y = xe^{x^2}$

$\Rightarrow$ equation of tangent at $(1, e)$ is

$$y - e = \left(e^{x^2}(1+2x^2)\right)_{x=1}(x-1)$$

$$y - e = 3e\,(x-1)$$

$3ex - y = 2e$ passes through the point $\left(\frac{4}{3}, 2e\right)$.

$\Rightarrow$ Option (2) is correct.

26. A helicopter is flying along the curve given by $y - x^{3/2} = 7$, $(x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is :

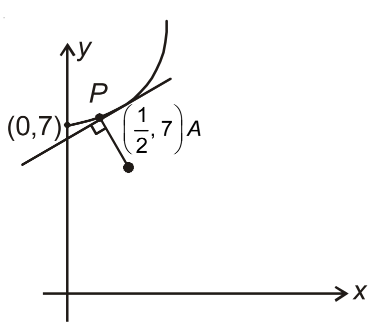
(1) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (2) $\frac{\sqrt{5}}{6}$

(3) $\frac{1}{2}$ (4) $\frac{1}{3}\sqrt{\frac{7}{3}}$

Answer (1)

Sol. $f(x) = y = x^{3/2} + 7$ $\frac{dy}{dx} \Rightarrow \frac{3}{2}\sqrt{x} > 0$

$\Rightarrow$ $f(x)$ is increasing function $\forall$ $x > 0$



Let $P\left(x_1, x_1^{3/2} + 7\right)$

$\Rightarrow$ $m_{AP} \cdot m_{\text{at } p} = -1$

$$\Rightarrow \left(\frac{x_1^{3/2}}{x_1 - \frac{1}{2}}\right) \times \frac{3}{2} x_1^{\frac{1}{2}} = -1$$

$$\Rightarrow -\frac{2}{3} = \frac{x_1^2}{x_1 - \frac{1}{2}}$$

$$\Rightarrow -3x_1^2 = 2x_1 - 1 \Rightarrow 3x_1^2 + 2x_1 - 1 = 0$$

$$\Rightarrow 3x_1^2 + 3x_1 - x_1 - 1 = 0$$

$$\Rightarrow 3x_1(x_1+1) - 1(x_1+1) = 0$$

$$\Rightarrow x_1 = \frac{1}{3} \quad (\because x_1 > 0) \Rightarrow P\left(\frac{1}{3}, 7 + \frac{1}{3\sqrt{3}}\right)$$

$$\Rightarrow \text{AP} = \sqrt{\frac{1}{27} + \frac{1}{36}} = \frac{1}{6}\sqrt{\frac{7}{3}}$$

$\Rightarrow$ Option (1) is correct.

27. If $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$, then $f'(½)$ is :

(1) $\frac{6}{25}$ (2) $\frac{24}{25}$

(3) $\frac{4}{5}$ (4) $\frac{18}{25}$

Answer (2)

Sol. $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$

$\Rightarrow$ $f(x) = 2x - x^2 f(x)$

$$\Rightarrow f(x) = \frac{2x}{1+x^2}$$

$$\Rightarrow f'(x) = \frac{2(1-x^2)}{(1+x^2)^2}$$

$$f'(1/2) = \frac{2\left(1 - \frac{1}{4}\right)}{\left(1 + \frac{1}{4}\right)^2} = \frac{3}{2} \times \frac{16}{25} = \frac{24}{25}$$

28. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations x_1, x_2, ..., x_5 and –50 is equal to :

(1) 586.5 (2) 582.5

(3) 509.5 (4) 507.5

Answer (4)

Sol. $\sum_{i=1}^{5} x_i = 10 \times 5 = 50 \Rightarrow \sum_{i=1}^{6} x_i = 50 - 50 = 0$

$$\frac{\sum_{i=1}^{5} x_i^2}{5} - (10)^2 = 3^2 = 9 \Rightarrow \sum_{i=1}^{5} x_i^2 = 545$$

$$\Rightarrow \sum_{i=1}^{6} x_i^2 = 545 + (-50)^2 = 3045$$

$$\text{Variance} = \frac{\sum_{i=1}^{6} x_i^2}{6} - \left(\frac{\sum_{i=1}^{6} x_i}{6}\right)^2$$

$$= \frac{3045}{6} - 0 = 507.5$$

29. The value of

$\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}.\ldots.\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$ is:

(1) $\frac{1}{512}$ (2) $\frac{1}{256}$

(3) $\frac{1}{2}$ (4) $\frac{1}{1024}$

Answer (1)

Sol. $E = \cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}.....\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$

$$= \frac{1}{2}\left(\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}....\cos\frac{\pi}{2^9}\sin\frac{\pi}{2^9}\right)$$

$$= \frac{1}{2^8}\left(\cos\frac{\pi}{2^2}.\sin\frac{\pi}{2^2}\right) = \frac{1}{2^9}\sin\frac{\pi}{2}$$

$$= \frac{1}{512}$$

30. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is:

(1) 2

(2) 1

(3) $\frac{15}{8}$

(4) $\frac{4}{9}$

Answer (1)

Sol. Sum of roots = $\alpha + \beta = \lambda - 3$

Product of roots = $\alpha\beta = 2 - \lambda$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (\lambda - 3)^2 - 2(2 - \lambda)$

$= \lambda^2 - 4\lambda + 5$

$= (\lambda - 2)^2 + 1$

$\lambda = 2$ for least $(\alpha^2 + \beta^2)$.

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