Solved Examples

JEE Main/Boards

Example 1: Find the r^{th} term if the p^{th} term of an AP is q and the q^{th} term is p .

Sol: Using $T_n = a + (n - 1) d$, we can obtain the p^{th}, q^{th} and r^{th} terms.

Let the initial term and common difference of the given AP be a and d, respectively.

As given,

$q = a + (p - 1) d$ ... (i)

$p = a + (q - 1) d$ ... (ii)

Subtracting (i) by (ii), we find that

$q - p = (p - q) d$

$\therefore d = -1$

Putting $d = -1$ in (i), we get

$a = q + p - 1$

$\therefore t_r = a + (r - 1)d$

$= (q + p - 1) - r + 1 = p + q - r$

Example 2: Find out the number of terms in a given AP 20, 25, 30, 35, ... 100.

Sol: We know that $T_n = a + (n - 1) d$.

Given, $a = 20$, $d = 5$ and $T_n = 100$. Therefore, by solving the equation, we will get the number of terms.

Let the number of terms be n.

Given, $T_n = 100$, $a = 20$, $d = 5$

$T_n = a + (n - 1) d$

$\Rightarrow 100 = 20 + (n - 1) 5 \Rightarrow 80 = (n - 1) 5$

$\Rightarrow 16 = (n - 1) \Rightarrow n = 17$

Example 3: Solve the following series:

99 + 95 + 91 +87 + ... to 20 terms

Sol: Using $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we can solve the given problem.

We know that the terms of the given series are in AP. Given,

D = – 4, a = 99 and n = 20

$$\because S_n = \frac{n\left(2a + (n-1)d\right)}{2}$$

$$S_{20} = \frac{20}{2}[2 \times 99 + (20 - 1)(-4)]$$

$= 10[198 + 19 \times (-4)] = 10(198 - 76) = 1220$

Example 4: Find out the G.P. if the fifth and second terms of a G.P. are 81 and 24, respectively.

Sol: We know that in GP, the n^{th} term is given by $T_n = a.r^{n-1}$. Thus, by using this formula, we can find the GP.

Given, $T_5 = 81$ and $T_2 = 24$

$\therefore \quad 81 = ar^4$ ... (i)

and $24 = ar$ ... (ii)

Dividing (i) by (ii), we get

$$\frac{81}{24} = r^3 \Rightarrow r^3 = \frac{27}{8} \Rightarrow r^3 = \left(\frac{3}{2}\right)^3 \Rightarrow r = \frac{3}{2}$$

Substituting the value of r in (ii), we get, a = 16

Thus, the required G.P. is 16, 24, 36, 54 ,....

Example 5: If the sum of four numbers in AP is 50 and the greatest of them is four times the least, then find the numbers.

Sol: Let the four numbers in AP be a, a +d, a + 2d, a +3d with d > 0.

As given, sum of the numbers is 50.

$a + (a + d) + (a + 2d) + (a + 3d) = 50$

$\therefore 4a + 6d = 50$

$\Rightarrow 2a + 3d = 25$ ...(i) and

$a + 3d = 4a$

$\Rightarrow 3d = 3a$

$\therefore d = a$

$\therefore$ Equation (i) becomes 5a = 25

Thus, a = 5 = d

Therefore, the four number are 5, 10, 15 and 20.

Example 6: If $S_1, S_2, S_3, \ldots, S_p$ are the sum of p infinite geometric progression whose first terms are 1, 2, 3,...., p and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{p+1}$, respectively, then prove that $S_1 + S_2 + ... + S_p = \frac{p(p+3)}{2}$.

Sol: As we know $S_\infty = \frac{a}{1-r}$, therefore by using this formula we can obtain the value of $S_1, S_2, ...S_p$.

We know that $S_\infty = \frac{a}{1-r}$

$$\therefore \quad S_1 = \frac{1}{1-\frac{1}{2}} = 2;\ S_2 = \frac{2}{1-\frac{1}{3}} = 3$$

$$S_p = \frac{p}{1-\frac{1}{p+1}} = p+1$$

$$S_1 + S_2 + ... + S_p = \frac{p}{2}[2\times 2 + (p-1)1] = \frac{p}{2}[p+3]$$

Example 7: Solve the series $1 + 2\cdot2 + 3\cdot2^2 + 4\cdot2^3 + ... + 100\cdot2^{99}$.

Sol: Let $S = 1 + 2\cdot2 + 3\cdot2^2 + 4\cdot2^3 + ... + 100\cdot2^{99}$. Therefore, by multiplying 2 on both the sides and then taking the difference, we can solve the given problem.

Given,

$$S = 1 + 2\cdot2 + 3\cdot2^2 + 4\cdot2^3 + ... + 100\cdot2^{99} \quad ...\ (i)$$

Multiplying 2 on both the sides,

$$2S = 1\cdot2 + 2\cdot2^2 + 3\cdot2^3 + ... + 99\cdot2^{99} + 100\cdot2^{100} \quad ...\ (ii)$$

Subtracting (ii) from (i), we get

$$-S = 1 + 1\cdot2 + 1\cdot2^2 + 1\cdot2^3 + ... + 1\cdot2^{99} - 100\cdot2^{100}$$

$$-S = \frac{1-2^{100}}{1-2} - 100\cdot2^{100};$$

$$\Rightarrow S = 99\cdot2^{100} + 1$$

Example 8: If (5n + 4) : (9n + 6) is the ratio of the sums of the n^{th} terms of two APs, then find out the ratio of their 13^{th} terms.

Sol: Let a_1 and a_2 be the first terms of the two APs and d_1 and d_2 be their respective common difference.

Applying $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we can solve the given problem.

Given,

$$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \quad \frac{a_1 + \frac{n-1}{2}d_1}{a_2 + \frac{n-1}{2}d_2} = \frac{5n+4}{9n+6} \quad ...\ (i)$$

The ratio of the 13^{th} terms is $\frac{a_1 + 12d_1}{a_2 + 12d_2}$ [which is obtained from (i) with n = 25]

$$\therefore \quad \frac{a_1 + 12d_1}{a_2 + 12d_2} = \frac{129}{231}$$

Example 9: If the 7^{th} and 8^{th} terms of an H.P. are 8 and 7, respectively, then find its 15^{th} term.

Sol: We know that $t_n = \frac{1}{a + (n-1)d}$. Therefore, by using this formula we can solve the given problem.

Given, T_7 = 8 =and T_8 = 7

$$\therefore \frac{1}{a+6d} = 8 \Rightarrow 8a + 48d - 1 = 0 \quad ..(i)$$

$$\frac{1}{a+7d} = 7 \Rightarrow 7a + 49d - 1 = 0 \quad ..(ii)$$

By solving these two equations, we find that d = a

$$\therefore \text{From eq.(i), we get } \frac{1}{7a} = 8$$

$$\Rightarrow a = d = \frac{1}{56}$$

$$\therefore \quad T_{15} = \frac{1}{a+14d} = \frac{56}{15}$$

Example 10: Suppose x y and z are positive real numbers, which are different from 1.

If $x^{18} = y^{21} = z^{28}$, then show that 3, $3\log_y(x)$. $3\log_z(y)$ and $7\log_x(z)$ are in AP.

Sol: By applying log on $x^{18} = y^{21} = z^{28}$, we can find the values of $\log_y x$, $\log_z y$ and $\log_x z$.

Given, $x^{18} = y^{21} = z^{28}$

Taking log, we find that

18 log x = 21 log y = 28 log z

$$\log_y x = \frac{\log x}{\log y} = \frac{7}{6}$$

$$\Rightarrow 3\log_y x = \frac{7}{2} \quad ...\ (i)$$

$$\log_z y = \frac{\log y}{\log z} = \frac{4}{3}; \quad 3\log_z y = 4 \quad ...\ (ii)$$

$\log_x z = \frac{\log z}{\log x} = \frac{9}{14}$

$\Rightarrow 7\log_x z = \frac{9}{2}$ ... (ii)

The numbers 3, $\frac{7}{2}$, 4 and $\frac{9}{2}$ are in AP with common difference = $\frac{1}{2}$.

$\therefore$ 3, $\log_y x$, $3\log_z y$ and $7\log_x z$ are in AP.

Example 11: If $\sqrt[x]{a} = \sqrt[y]{b} = \sqrt[z]{c}$ and if a, b and c are positive and in GP, then prove that x, y and z are in AP.

Sol: This problem can be solved by taking log on $\sqrt[x]{a} = \sqrt[y]{b} = \sqrt[z]{c}$.

$a^{1/x} = b^{1/y} = c^{1/z}$

$\Rightarrow \frac{\log a}{x} = \frac{\log b}{y} = \frac{\log c}{z} = k$

$\Rightarrow$ log a = kx, log b = ky, log c = kz ... (i)

a, b and c are in GP

$\Rightarrow b^2 = ac$

$\therefore$ 2log b = log a + log c

$\Rightarrow$ 2ky = kx + kz by (i)

$\Rightarrow$ 2y = x + z

$\Rightarrow$ x, y and z are in AP.

Example 12: Determine the relation between x, y and z if 1, $\log_y x$, $\log_z y$, $-15\log_x z$ are in AP.

Sol: By considering the common difference as d and obtaining its value by $\log_y x = 1 + d$ and $\log_z y = 1 + 2d$, we can determine the required relation.

Suppose d be the common difference of the given AP, then

$\log_y x = 1 + d \Rightarrow x = y^{1+d}$ ... (i)

$\log_z y = 1 + 2d \Rightarrow y = z^{1+2d}$ ... (ii)

$15\log_x z = -(1 + 3d)$

$\Rightarrow z = x^{\frac{1+3d}{-15}}$ ... (iii)

Elimination y and z from equations (i), (ii) and (iii), we get

$x = x^{\frac{(1-d)(1+2d)(1+3d)}{-15}}$

$\therefore 1 = \frac{(1+d)(1+2d)(1+3d)}{-15}$

or (1 + d) (1 + 2d) (1 + 3d) + 15 = 0

or $(d + 2)(6d^2 + 5d + 8) = 0$

$\Rightarrow d = -2$

The other factors do not give any real solution.

$\therefore x = y^{-1}, y = z^{-3}, z = x^{1/3}$ or $x = y^{-1} = z^3$

Example 13: There are four numbers of which the first three are in G.P. and the last there are in AP, with a common difference of 6. If the first number and the last number are equal, then find the numbers.

Sol: Let the four numbers be a, a – 2d, a – d, a, where d = 6

a, a – 12, a – 6 are in GP.

$\Rightarrow a(a-6) = (a-12)^2$

$\Rightarrow a^2 - 6a = a^2 - 24a + 144$

$\Rightarrow 18a = 144$

$\Rightarrow a = 8$

The numbers are 8, – 4, 2 and 8.

Example 14: a, b and c are the p^{th}, q^{th} and r^{th} terms of both an AP and a GP, respectively, then prove that $a^{b-c}.\ b^{c-a}.\ c^{a-b} = 1$ (both progressions have the same first term.)

Sol: By using formula $T_n = a + (n-1)d$ and $T_n = a.r^{n-1}$, we can obtain the p^{th}, q^{th} and r^{th} terms of both an AP and a GP.

$T_p = a = a_1 + (p-1)d_1 = a_1(r_1)^{p-1}$ ... (i)

$T_q = b = a_1 + (q-1)d_1 = a_1(r_1)^{q-1}$ ... (ii)

$T_r = c = a_1 + (r-1)d_1 = a_1(r_1)^{r-1}$ ... (iii)

From (i), (ii), (iii)

$a - b = (p - q)\, d_1$

$b - c = (q - r)\, d_1$

$c - a = (r - p)\, d_1$

Therefore, $a^{b-c}.\ b^{c-a}.\ c^{a-b}$

$= (a_1 r_1^{p-1})^{b-c}\, (a_1 r_1^{q-1})^{c-a}\, (a_1 r_1^{r-1})^{a-b}$

$= a_1^{b-c+c-a+a-b}.r_1^{(p-1)(b-c)+(q-1)(c-a)+(r-1)(a-b)}$

$= a_1^0.\ r_1^{(p-1)(q-r)d_1+(q-1)(r-p)d_1+(r-1)(p-q)d_1}$

$= a_1^0.r_1^0 = 1$

JEE Advanced/Boards

Example 1: If the sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3, the first term of each being 1 and the common differences being 1, 2 and 3, respectively, then prove that $S_1 + S_3 = 2S_2$

Sol: Using $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, we can get the values of S_1, S_2 and S_3.

Given, $a = 1$, $d_1 = 1$, $d_2 = 2$, $d_3 = 3$

$S_1 = \frac{n}{2}[2a + (n-1)\,d_1]$

$= \frac{n}{2}[2 \times 1 + (n-1)\,1] \quad = \frac{n}{2}[1 + n]$

$S_2 = \frac{n}{2}[2a + (n-1)d_2] = \frac{n}{2}[2 \times 1 + (n-1)\,2] = n^2$

$S_3 = \frac{n}{2}[2a + (n-1)\,d_3] = \frac{n}{2}[2 \times 1 + (n-1)\,3]$

$= \frac{n}{2}[3n-1]$

$S_1 + S_3 = \frac{n}{2}[1 + n + 3n - 1] = 2n^2 = 2S$

Example 2: Calculate the sum to n terms of the series: 8 +88 + 888 + ...

Sol: We can solve this problem by taking 8 as common from given series and applying various operations.

Let $S_n = 8 + 88 + 888 + \ldots\ldots$ to n terms $= 8\,[1 + 11 + 111 + \ldots\ldots]$

$= \frac{8}{9}[9 + 99 + 999 + \ldots\ldots] = \frac{8}{9}[(10-1) + (100-1) + (1000-1) + \ldots\ldots$ to n terms$]$

$= \frac{8}{9}[10 + 100 + 1000 + \ldots +$ to n terms$] - \frac{8}{9}n = \frac{8}{9}\frac{(10^n-1)}{9} - \frac{8n}{9}$

$= \frac{8}{81}[10^{n+1} - 9n - 10]$

Example 3: If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in AP, where $a_i > 0$ for all I, then show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots\ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

Sol: We can write $x - y = p$ as

$\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right) = p$.

Thus, by following this method we can represent difference of $a_1, a_2, a_3, \ldots\ldots, a_n$.

$a_1, a_2, a_3, \ldots\ldots, a_n$ are in AP.

$a_2 - a_1 = a_3 - a_2 = \ldots\ldots = a_n - a_{n-1} = d$ (say)

$\Rightarrow \quad \left(\sqrt{a_2}+\sqrt{a_1}\right)\left(\sqrt{a_2}-\sqrt{a_1}\right)$

$= \left(\sqrt{a_3}+\sqrt{a_2}\right)\left(\sqrt{a_3}-\sqrt{a_2}\right)\ldots\ldots$

$= \left(\sqrt{a_n}+\sqrt{a_{n-1}}\right)\left(\sqrt{a_n}-\sqrt{a_{n-1}}\right) = d$

$\Rightarrow \frac{1}{\sqrt{a_2}+\sqrt{a_1}} = \frac{1}{d}\left(\sqrt{a_2}-\sqrt{a_1}\right), \frac{1}{\sqrt{a_3}+\sqrt{a_2}}$

$= \frac{\sqrt{a_3}-\sqrt{a_2}}{d}, \ldots\ldots$

$\frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}} = \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{d}$

$\text{LHS} = \frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots\ldots\ldots +$

$= \frac{1}{d}\left(\sqrt{a_n}-\sqrt{a_1}\right)$

$\because\ d = \frac{a_n - a_1}{n-1}$

$\Rightarrow \text{LHS} = \frac{\sqrt{a_n}-\sqrt{a_1}}{a_n - a_1}(n-1) = \frac{n-1}{\sqrt{a_n}+\sqrt{a_1}} = \text{RHS}$

Example 4: A series of natural numbers is divided into groups: (1); (2, 3, 4); (5, 6, 7, 8, 9) and so on. Prove that the sum of the numbers in the n^{th} group is $(n-1)^3 + n^3$.

Sol: In this problem, the last term of each group is the square of the corresponding number of the group. Thus, the first term of the n^{th} group is $(n-1)^2 + 1 = n^2 - 2n + 2$. Hence, by using $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, we can solve the problem.

The number of terms in the first group = 1

The number of terms in the second group = 3

The number of terms in the third group = 5

$\therefore$ The number of terms in the n^{th} group = $2n-1$

The common difference of the numbers in the n^{th} group = 1

The required sum = $\frac{2n-1}{2}$ [2(n^2 – 2n + 2) + (2n – 2) 1]

$= \frac{2n-1}{2}[2n^2 - 2n + 2] = (2n-1)[n^2 - n + 1]$

$= 2n^3 - 3n^2 + 3n - 1 = n^3 + (n-1)^3$

Example 5: Find the sum of the series

$1 + \frac{1}{5} + \frac{3}{5^2} + \frac{5}{5^3} + \ldots$ to ∞.

Sol: We can write given series as $S_\infty = 1 + S_1$, where $S_1 = \frac{1}{5} + \frac{3}{5^2} + \frac{5}{5^3} + \ldots$ to ∞ Thus, by multiplying $\frac{1}{5}$ on both the sides and subtracting, we can obtain the required sum.

Let $S_\infty = 1 + \frac{1}{5} + \frac{3}{5^2} + \frac{5}{5^3} + \ldots$ to ∞

$\therefore S_\infty = 1 + S_1$ ....(i)

Where

$\therefore \frac{1}{5}S_1 = \frac{1}{5^2} + \frac{3}{5^3} + \frac{5}{5^4} + \ldots$ to ∞ ....(ii)

Subtracting, (ii) from (i)

$\therefore \frac{4}{5}S_1 = \frac{1}{5} + 2\left(\frac{1}{5^2} + \frac{1}{5^3} + \ldots \text{to } \infty\right)$

$= \frac{1}{5} + 2\frac{\frac{1}{5^2}}{1-\frac{1}{5}} = \frac{1}{5} + \frac{2}{4}\left(\frac{1}{5}\right) = \frac{3}{10}$,

$S_1 = \frac{3}{8}$

From (i), $S_\infty = 1 + \frac{3}{8} = \frac{11}{8}$

Example 6: If $n \in N$ and $n > 1$, then prove that

(a) $n^n \geq 1.3.5 \ldots(2n-1)$ (b) $2^n \geq 1 + n\,2^{\frac{n-1}{2}}$

Sol: (a) Use the inequality A.M. ≥ GM.

(b) By solving $\frac{1+2+2^2+\ldots+2^{n-1}}{n} \geq (1.2.2^2 \ldots 2_{n-1})^{1/n}$, we can prove the given equation.

(a) $\therefore \frac{1+3+5+\ldots+(2n-1)}{n} \geq \left[1.3.5 \ldots (2n-1)\right]^{1/n}$

$\Rightarrow \frac{\frac{n}{2}\left[1+(2n-1)\right]}{n} \geq \left[1.3.5 \ldots (2n-1)\right]^{1/n}$

$\Rightarrow n = \left[1.3.5 \ldots (2n-1)\right]^{1/n}$

$\therefore n^n = 1.3.5 \ldots (2n-1)$

(b) $\frac{1+2+2^2+\ldots+2^{n-1}}{n} \geq (1.2.2^2 \ldots 2_{n-1})^{1/n}$

$\Rightarrow \frac{2^n-1}{2-1} \times \frac{1}{n} \geq \left(2^{\frac{n(n-1)}{2}}\right)^{1/n}$

$\Rightarrow \frac{2^n-1}{n} \geq 2^{n-1/2}$

$\Rightarrow 2^n \geq n\sqrt{2^{n-1}} + 1$

JEE Main/Boards

Exercise 1

Q.1 In a G.P. sum of n terms is 364. First term is 1 and common ratio is 3. Find n.

Q.2 The sum of an infinite geometric progression is 2 and the sum of the geometric progression made from the cubes of this infinite series in 24. Then find the series.

Q.3 Sum of n terms of the series,

(i) 0.7 + 0.77 + 0.777 + ...(ii) 6 + 66 + 666 + ...

Q.4 If a, b, c are in A.P., prove that

(i) b + c, c + a, a + b are also in A.P.

(ii) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in A.P.

(iii) $a^2(b+c), b^2(c+a), c^2(a+b)$ are also in A.P.

(iv) $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are also in A.P.

Q.5 The sum of three numbers in A.P. is 12 and the sum of their cubes is 288. Find the numbers.

Q.6 Find the sum of the integers between 1 and 200 which are

(i) Multiple of 3 (ii) Multiple of 7

(iii) Multiple of 3 and 7

Q.7 The sum of first n terms of two A.P.'s are in the ratio (3n – 3): (5n + 21). Find the ratio of their 24^{th} terms.

Q.8 If the p^{th} term of an A.P. is x and q^{th} term is y, show that the sum first (p + q) terms is $\frac{p+q}{2}\left\{x+y+\frac{x-y}{p-q}\right\}$

Q.9 If a, b, c are in H.P. prove that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P.

Q.10 Find the sum of n terms of the series $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+......$

Q.11 Let a, b, c, d, e be five real numbers such that a, b, c are in A.P.; b, c, d are in G.P.; c, d, e are in H.P. If a = 2 and e = 18, find all possible values of b, c and d.

Q.12 Find the sum of first n terms of the series: $1\cdot2\cdot3 + 2\cdot3\cdot4 + 3\cdot4\cdot5 + ...$

Q.13 Find the sum of first 2n terms of the series: $1^2 + 2 + 3^2 + 4 + 5^2 + 6 + ...$

Q.14 The H.M of two numbers is 4 and their A.M. (A) and G.M. (G) satisfy the relation $2A + G^2 = 27$. Find the numbers.

Q.15 Find the sum of first 10 terms of the series: $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + ...$

Q.16 Find the sum of first 20 terms of the series: $1\cdot3^2 + 2\cdot5^2 + 3\cdot7^2 + ...$

Q.17 Find three numbers a, b, c between 2 and 18 such that:

(i) Their sum is 25.

(ii) The numbers 2, a, b are consecutive terms of an A.P. and

(iii) The numbers b, c, 18 are consecutive terms of a G.P.

Q.18 If a > 0, b > 0 and c > 0, prove that:

$(a + b + c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \ge 9$

Q.19 If A_1, A_2, G_1, G_2 ; and H_1, H_2 be two A.M.'s, G.M.'s and H.M's between two numbers, then prove that:

$$\frac{G_1G_2}{H_1H_2} = \frac{A_1+A_2}{H_1+H_2}$$

Q.20 Find the coefficient of x^{99} and x^{98} in the polynomial:

(x – 1) (x – 2) (x – 3) ... (x – 100).

Q.21 The interior angles of a polygon are in A.P. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon

Q.22 A number consists of three digits in G.P.. The sum of the digits at units and hundreds place exceeds twice the digit at tens place by 1 and the sum of the digits at tens and hundreds place is two third of the sum of the digits at tens and units place. Find the number.

Q.23 25 trees are planted in a straight line at intervals of 5 meters. To water them the gardener must bring water for each tree separately from a well 10 meters from the first tree in line with the trees. How far he will have to cover in order to water all the tree beginning with the first if he starts from the well.

Q.24 Natural numbers have been grouped in the following way 1 ; (2, 3) ; (4, 5, 6); (7, 8, 9, 10) ; ...... Show that the sum of the numbers in the nth group is $\frac{n(n^2+1)}{2}$.

Q.25 In three series of GP's, the corresponding numbers in G.P. are subtracted and the difference of the numbers are also found to be in G.P. Prove that the three sequences have the same common ratio.

Q.26 If a_1, a_2, a_3 ,.... Are in A.P such that $a_i \ne 0$, show that

$$S = \frac{1}{a_1a_2}+\frac{1}{a_2a_3}+.....+\frac{1}{a_na_{n+1}} = \frac{n}{a_1a_{n+1}}$$

Also evaluate $\lim_{a\to\infty} S$.

Q.27 If 9 arithmetic means and 9 harmonic means be inserted between 2 and 3, prove that $A+\frac{6}{H}=5$, where A is any arithmetic mean and H the corresponding harmonic mean.

Q.28 If x + y + z = 1 and x, y, z are positive numbers, show that $(1-x)(1-y)(1-z) \geq 8xyz$.

Q.29 Show that any positive integral power (greater than 1) of a positive integer m, is the sum of m consecutive odd positive integers. Find the first odd integer for $m^r (r > 1)$.

Exercise 2

Single Correct Choice Type

Q.1 If a, b, c are distinct positive real in H.P., then the value of the expression, $\frac{b+a}{b-a}+\frac{b+c}{b-c}$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

Q.2 The sum of infinity of the series

$\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}+........$ is equal to

(A) 2 (B) 5/2 (C) 3 (D) None

Q.3 Along a road lies an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is

(A) 15 (B) 29 (C) 31 (D) 35

Q.4 If $S = 1^2 + 3^2 + 5^2 + ... + (99)^2$ then the value of the sum $2^2 + 4^2 + 6^2 + ... + (100)^2$ is

(A) S + 2550 (B) 2S (C) 4S (D) S +5050

Q.5 In an A.P. with first term and the common difference d (a, d ≠ 0), the ratio 'S' of the sum of the first n terms to sum of n terms succeeding them does not depend on n. Then the ratio "a/d" and the ratio 'ρ', respectively are

(A) $\frac{1}{2},\frac{1}{4}$ (B) $2,\frac{1}{3}$ (C) $\frac{1}{2},\frac{1}{3}$ (D) $\frac{1}{2},2$

Q.6 If x ∈ R, the numbers $(5^{1+x} + 5^{1-x})$, $a/2\ (25^x + 25^{-x})$ form an A.P. then 'a' must lie in the interval

(A) [1, 5] (B) [2, 5] (C) [5, 12] (D) [12, ∞]

Q.7 If the sum of the first 11 terms of an arithmetical progression equals that of the first 19 terms, then the sum of its first 30 terms, is

(A) Equal to 0 (B) Equal to – 1

(C) Equal to 1 (D) Non unique

Q.8 Let s_1, s_2, s_3 ..... and t_1, t_2, t_3 .... are two arithmetic sequence such that $s_1 = t_1 \neq 0$; $s_2 = 2t_2$ and $\sum_{i=1}^{10} s_i = \sum_{i=1}^{15} t_i$.

Then the value of $\frac{s_2 - s_1}{t_2 - t_1}$ is

(A) 8/3 (B) 3/2 (C) 19/8 (D) 2

Q.9 Let a_n, n ∈ I be the n^{th} term an A.P. with common difference 'd' and all whose terms are non-zero. If n approaches infinity, then the sum

$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+.....+\frac{1}{a_na_{n+1}}$ will approach

(A) $\frac{1}{a_1d}$ (B) $\frac{2}{a_1d}$ (C) $\frac{1}{2a_1d}$ (D) a_1d

Q.10 The sum of the first three terms of an increasing G.P. is 21 and the sum of their squares is 189. Then the sum of its first n term is

(A) $3(2^n - 1)$ (B) $12\left(1-\frac{1}{2^n}\right)$

(C) $6\left(1-\frac{1}{2^n}\right)$ (D) $6(2^n - 1)$

Q.11 The sum $\sum_{n=1}^{\infty}\left(\frac{n}{n^4+4}\right)$ is equal to

(A) 1/4 (B) 1/3 (C) 3/8 (D) 1/2

Q.12 If a ≠ 1 and $(\ln a^2) + (\ln a^2)^2 + (\ln a^2)^3 + ... = 3[\ln a + (\ln a)^2 + (\ln a)^3 + (\ln a)^4 + ...]$, then 'a' is equal to

(A) $e^{1/5}$ (B) $e^{1/2}$ (C) $3e^{1/2}$ (D) $e^{1/4}$

Previous Years' Questions

Q.1 If a, b, c d and p are distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \le 0$, then a, b, c, d ***(1987)***

(A) Are in A.P. (B) Are in G.P.

(C) Are in H.P. (D) Satisfy ab = cd

Q.2 Sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+....$ is equal to

(A) $2^n - n - 1$ (B) $1 - 2^{-n}$

(C) $n + 2^{-n} - 1$ (D) $2^n + 1$

Q.3 If x > 1, y > 1, z > 1 are in G.P. then $\frac{1}{1+\ln x},\frac{1}{1+\ln y},\frac{1}{1+\ln z}$ are in ***(1998)***

(A) AP (B) H.Π. (C) G.Π. (D) None

Q.4 If a, b, c, d are positive real number such that a + b + c + d = 2, then M = (a + b) (c +d) satisfies the relation ***(2000)***

(A) $0 < M \le 1$ (B) $1 \le M \le 2$

(C) $2 \le M \le 3$ (D) $3 \le M \le 4$

Q.5 Let the positive numbers a, b, c, d be in A.P. then abc, abd, acd, bcd are ***(2001)***

(A) not in AP/GP/HP (B) in AP

(C) in GP (D) in HP

Q.6 Suppose a, b, c are in AP and a^2, b^2, c^2 are in G.P. If a < b < c and a + b + c = $\frac{3}{2}$, then the value of a is ***(2002)***

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$

(C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

Q.7 An infinite G.P. has first term x and sum 5, then x belongs to ***(2004)***

(A) $x < -10$ (B) $-10 < x < 0$

(C) $0 < x < 10$ (D) $x > 10$

Q.8 If the sum of first n terms of an AP is cn^2, then the sum of squares of these n terms is ***(2009)***

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

Q.9 The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is ***(2008)***

(A) –4 (B) –12 (C) 12 (D) 4

Q.10 The sum to the infinity of the series is

$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+.......$ ***(2009)***

(A) 2 (B) 3 (C) 4 (D) 6

Q.11 If m is the A. M. of two distinct real numbers l and n(l, n > 1) and G1, G2 and G3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals: ***(2015)***

(A) $4\, l^2\, mm$ (B) $4\, lm^2 n$ (C) $4\, lmn^2$ (D) $4\, l^2m^2n^2$

Q.12 The sum of first 9 terms of the series is

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+.......$ ***(2015)***

(A) 71 (B) 96 (C) 142 (D) 192

Q.13 If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is: ***(2016)***

(A) $\frac{4}{3}$ (B) 1 (C) $\frac{7}{3}$ (D) $\frac{8}{5}$

Q.14 If the sum of the first ten terms of the series is $\frac{16}{5}m$ then m is equal to $\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+.......$ ***(2016)***

(A) 101 (B) 100 (C) 99 (D) 102

Q.15 Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is ***(2014)***

(A) $2-\sqrt{3}$ (B) $2+\sqrt{3}$ (C) $\sqrt{2}+\sqrt{3}$ (D) 3 + 2

Assertion Reasoning Type

Q.16 Statement-I: The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement-II: $\sum_{k=1}^{n}(k^3-(k-1)^3)=n^3$ for any natural number n. ***(2012)***

(A) Statement-I is false, statement-II is true

(B) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I

(C) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I

(D) Statement-I is true, statement-II is false

Q.17 Statement-I: The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement-II: $\sum_{k=1}^{n}(k^3-(k-1)^3)=n^3$ for any natural number n.

Q.18 If 100 times the 100th term of an AP with non zero common difference equals the 50 times its 50th term, then the 150th term of this AP is ***(2012)***

(A) –150 (B) 150 times its 50th term

(C) 150 (D) Zero

Q.19 The real number k for which the equation, 2x3 + 3x + k = 0 has two distinct real roots in [0, 1] ***(2013)***

(A) Lies between 1 and 2

(B) Lies between 2 and 3

(C) Lies between -1 and 0

(D) Does not exist.

Q.20 If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, a,b,c $\in$ R, have a common root, then a : b : c is ***(2013)***

(A) 1 : 2 : 3 (B) 3 : 2 : 1 (C) 1 : 3 : 2 (D) 3 : 1 : 2

Q.21 The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is ***(2013)***

(A) $\frac{7}{81}(179-10^{-20})$ (B) $\frac{7}{9}(99-10^{-20})$

(C) $\frac{7}{81}(179+10^{-20})$ (D) $\frac{7}{9}(99+10^{-20})$

Q.22 If x, y, z are in A.P. and tan-1x, tan-1y and tan-1z are also in A.P., then ***(2013)***

(A) x = y = z (B) 2x = 3y = 6z

(C) 6x = 3y = 2z (D) 6x = 4y = 3z

JEE Advanced/Boards

Exercise 1

Q.1 (i) The harmonic mean of two numbers is 4. The arithmetic mean A & the geometric mean G satisfy the relation $2A + G^2 = 27$. Find the two numbers.

(ii) The A.M. of two numbers exceeds their G.M. by 15 and HM by 27. Find the numbers.

Q.2 If the 10^{th} term of an H.P. is 21 and 21^{st} term of the same H.P. is 10, then find the 210^{th} term.

Q.3 If sinx, $\sin^2 2x$ and cosx.sin4x form an increasing geometric sequence, then find the numerical value of cos2x. Also find the common ratio of geometric sequence.

Q.4 If a, b, c, d, e be 5 numbers such that a, b, c are in AP; b, c, d are in G.P. & c, d, e are in H.P. then,

(i) Prove that a, c, e are in GP

(ii) Prove that $e = (2b - a)^2/a$

(iii) If a = 2 & e = 18, find all possible values of b, c, d.

Q.5 Let a_1 and a_2 be two real values of α for which the numbers $2\alpha^2$, α^4, 24 taken in that order form an arithmetic progression. If β_1 and β_2 are two real values of β for which the numbers 1, β^2, $6 - \beta^2$ taken in that order form a geometric progression, then find the value of $(\alpha_1^2+\alpha_2^2+\beta_1^2+\beta_2^2)$.

Q.6 Two distinct, real infinite geometric series each have a sum of 1 and have the same second term. The third term of one of the series is 18. If the second term of both the series can be written in the form $\frac{m-n}{p}$, where m, n and p are positive integers, and m is not divisible by the square of any prime, find the value of 100m + 10n+p.

Q.7 Let $S = \sum_{n=1}^{99} \frac{5^{100}}{(25)^n + 5^{100}}$. Find [s].

Where [y] denotes largest integer less than or equal to y.

Q.8 Given that the cubic $ax^3 - ax^2 + 9bx - b = 0$ ($a \neq 0$) has all three positive roots. Find the harmonic mean of the roots independent of a and b, hence deduce that the root are all equal. Find also the minimum value of (a + b), if a and $b \in N$.

Q.9 A computer solved several problems in succession. The time it took the computer to solve each successive problem was the same number of times smaller than the time it took to solve the preceding problem. How many problems were suggested to the computer if it spent 63.5 min to solve all the problems except for the first, 127 min to solve all the problems except for the last one, an 31.5 min to solve all the problems except for the first two?

Q.10 The sequence $a_1, a_2, a_3, \ldots\ldots a_{98}$ satisfies the relation $a_{n+1} = a_n + 1$ for 1,2,3,.... 97 and has the sum equal to 4949. Evaluate $\sum_{k=1}^{49} a_{2k}$.

Q.11 Let a and b be positive integers. The value of xyz is 55 or $\frac{343}{55}$, according as a, x, y, z, b are in arithmetic progression or harmonic progression resp.. Find the value of $(a^2 + b^2)$.

Q.12 If the roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in harmonic progression, then find c and all the roots.

Q.13 If a, b, c be in G.P. & $\log_c a$, $\log_b c$, $\log_a b$ be in AP, then find the common difference of the AP if $\log_a c = 4$.

Q.14 The first term of a geometric progression is equal to b – 2, then third term is b + 6, and the arithmetic mean of the first and third term to the second term is in the ratio 5: 3. Find the positive integral value of b.

Q.15 In a G.P. the ratio of the sum of the first eleven terms to the sum of the last eleven terms is 1/8 and the ratio of the sum of all the terms without the first nine to the sum of all the terms with out the last nine is 2. Find the number of terms in the GP.

Q.16 If sum of first n terms of an AP (having positive terms) is given by $S_n = (1 + 2T_n)(1 - T_n)$ where T_n is the n^{th} term of series then $T_2^2 = \frac{\sqrt{a} - \sqrt{b}}{4}$ (a, $b \in N$). Find the value of (a + b).

Q.17 Given a three digit number whose digits are three successive terms of a G.P. If we subtract 792 form it, we get a number written by the same digits in the reverse order. Now if we subtract four from the hundred's digit of the initial number and leave the other digits unchanged, we get a number whose digits are successive terms of an A.P. Find the number.

Q.18 For $0 < \theta < \frac{\pi}{4}$, let $S(\theta) = 1 + (1 + \sin\theta)\cos\theta + (1 + \sin\theta + \sin^2\theta)\cos^2\theta + \ldots\ldots \infty$.

Then find the value of $\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) S\left(\frac{\pi}{4}\right)$.

Q.19 If $\tan\left(\frac{\pi}{12} - x\right), \tan\frac{\pi}{12}, \tan\left(\frac{x}{12} + x\right)$ in order are three consecutive terms of a G.P., then sum of all the solutions in [0, 314] is $k\pi$. Find the value of k.

Exercise 2

Single Correct Choice Type

Q.1 The arithmetic mean of the nine numbers in the give set {9, 99,999,......999999999} is a 9 digit number N, all whose digits are distinct. The number N does not contain the digit

(A) 0 (B) 2 (C) 5 (D) 9

Q.2 $\sum_{k=1}^{360} \left(\frac{1}{k\sqrt{k+1} + (k+1\sqrt{k})} \right)$ is the ratio of two relative prime positive integers m and n. The value of (m + n) is equal to

(A) 43 (B) 41 (C) 39 (D) 37

Q.3 The sum $\sum_{k=1}^{100} \frac{k}{k^4 + k^2 + 1}$ is equal to

(A) $\frac{4950}{10101}$ (B) $\frac{5050}{10101}$

(C) $\frac{5151}{10101}$ (D) None of these

Q.4 A circle of radius r is inscribed in a square. The mid point of sides of the square have been connected by line segment and a new square resulted. The sides of these square were also connected by segments so that a new square was obtained and so on, then the radius of the circle inscribed in the nth square is

(A) $\left[2^{\frac{1-n}{2}}\right]r$ (B) $\left[2^{\frac{3-3n}{2}}\right]r$

(C) $\left[2^{\frac{n}{2}}\right]r$ (D) $\left[2^{\frac{5-3n}{2}}\right]r$

Assertion Reasoning Type

(A) Statement-I is true, statement-II is true and statment-II is correct explanation for statment-I.

(B) Statement-I is true, statement-II is true and statment-II is NOT the correct explanation for statment-I.

(C) Statement-I is true, statement -II is false.

(D) Statement-I is false, statement-II is true

Q.5 Statement-I: If 27 abc ≥ (a + b + c)^3 and 3a + 4b + 5c = 12 then $\frac{1}{a^2}+\frac{1}{b^3}+\frac{1}{c^5}=10$, where a, b, c are positive real numbers.

Statement-II: For positive real numbers A.M. ≥ G.M.

Multiple Correct Choice Type

Q.6 Let a_1, a_2, a_3 .... and b_1, b_2, b_3 .... be arithmetic progressions such that a_1 = 25, b = 75 and a_{100} + b_{100} = 100. Then,

(A) The difference between successive terms in progression 'a' is opposite of the difference in progression 'b'.

(B) a_n + b_n = 100, for any n.

(C) (a_1 + b_1), (a_2 + b_2), (a_3 + b_3),... are in AP

(D) $\sum_{r=1}^{100}(a_r+b_r)=10000$

Q.7 If sin (x – y), sin x and sin (x + y) are in H.P. then the value of sin x.sec $\frac{y}{2}$ =

(A) 2 (B) $2^{1/2}$ (C) – 2 (D) $-2^{1/2}$

Q.8 The sum of the first three terms of the G.P. in which the difference between the second and the first term is 6 and the difference between the fourth and the third term 54, is

(A) 39 (B) –10.5 (C) 27 (D) –27

Q.9 If the roots of the equation $x^3 + px^2 + qx - 1 = 0$ form an increasing GP, where p and q are real, then

(A) p + q = 0

(B) p ∈ (–3, ∞)

(C) One of the roots is unity

(D) One root is smaller than 1

Q.10 If the triplets log a, log b, log c and (log a – log 2b), (log 2b – log3c), (log 3c – loga) are in arithmetic progression then

(A) 18 $(a + b + c)^2 = 18(a^2 + b^2 + c^2)$ + ab

(B) a, b, c are in GP

(C) a, 2b, 2c are in HP

(D) a, b, c can be the lengths of the sides of a triangle.

(Assume all logarithmic terms to be defined)

Q.11 x_1, x_2 are the roots of the equation $x^2 - 3x + A = 0$; x_3, x_4 are roots of the equation $x^2 - 12x + B = 0$, such that x_1, x_2, x_3, x_4 form an increasing G.P. then

(A) A = 2 (B) B = 32

(C) $x_1 + x_3 = 5$ (D) $x_2 + x_4 = 10$

Previous Years' Question

Q.1 If the first and the (2n – 1)th term of an AP, G.P. and H.P. are equal and their nth terms are a, b, and c respectively, then ***(1988)***

(A) a = b = c (B) a ≥ b ≥ c

(C) a + c =b (D) ac – b^2 = 0

Q.2 Let S_1, S_2.... be squares such that for each n ≥ 1 the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1 sq. cm ? ***(1999)***

(A) 7 (B) 8 (C) 9 (D) 10

Q.3 Let S_k, k = 1,2,....., 100, denotes the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}|(k^2-3k+1)S_k|$ is ***(2010)***

Q.4 Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{(k-2)}$, for k = 3, 4,...., 11. If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots a_{11}}{11}$ is equal to ***(2010)***

Q.5 Le $a_1, a_2, a_3, \ldots a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is .... ***(2011)***

Paragraph 1: Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For n > 2, let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

Q.6 Which one of the following statements is correct?

(A) $G_1 > G_2 > G_3 > \ldots$

(B) $G_1 < G_2 < G_3 < \ldots$

(C) $G_1 = G_2 = G_3 = \ldots$

(D) $G_1 < G_3 < G_5 < \ldots$ and $G_2 > G_4 > G_6 > \ldots$

Q.7 Which one of the following statements is correct?

(A) $A_1 > A_2 > A_3 > \ldots$

(B) $A_1 < A_2 < A_3 < \ldots$

(C) $A_1 > A_3 > A_5 > \ldots$ and $A_2 < A_4 < A_6 < \ldots$

(D) $A_1 < A_3 < A_5 < \ldots$ and $A_2 > A_4 > A_6 > \ldots$

Q.8 Which one of the following statements is correct?

(A) $H_1 > H_2 > H_3 > \ldots$

(B) $H_1 < H_2 < H_3 < \ldots$

(C) $H_1 > H_3 > H_5 > \ldots$ and $H_2 < H_4 < H_6 < \ldots$

(D) $H_1 < H_3 < H_5 < \ldots$ and $H_2 > H_4 > H_6 > \ldots$

Paragraph 2: Let V_r denote the sum of the first 'r' terms of an arithmetic progression (A.P.), whose first term is 'r' and the common difference is (2r – 1). Let $T_r = V_{r+1} - V_{r-2}$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2, ... ***(2007)***

Q.9 The sum $V_1 + V_2 + \ldots + V_n$ is

(A) $\frac{1}{12}$ n(n + 1)(3n² – n + 1)

(B) $\frac{1}{12}$ n(n + 1)(3n² + n + 2)

(C) $\frac{1}{2}$ n(2n² – n + 1)

(D) $\frac{1}{3}$ (2n³ – 2n + 3)

Q.10 T_r is always

(A) An odd number (B) An even number

(C) A prime number (D) A composite number

Q.11 Which one of the following is a correct statement?

(A) $Q_1, Q_2, Q_3 \ldots$ are in A.P. with common difference 5.

(B) $Q_1, Q_2, Q_3 \ldots$ are in A.P. with common difference 6.

(C) $Q_1, Q_2, Q_3 \ldots$ are in A.P. with common difference 11.

(D) $Q_1 = Q_2 = Q_3 = \ldots$

Q.12 Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$ for n = 1, 2, 3, ... Then, ***(2008)***

(A) $S_n < \frac{\pi}{3\sqrt{3}}$ (B) $S_n > \frac{\pi}{3\sqrt{3}}$

(C) $T_n < \frac{\pi}{3\sqrt{3}}$ (D) $T_n > \frac{\pi}{3\sqrt{3}}$

Q.13 Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let b1 = a_1, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$. ***(2008)***

Statement-I: The numbers b_1, b_2, b_3, b_4 are neither in A.P. Nor in G.P.

Statement-II: The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I

(B) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I.

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Q.14 If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is ***(2009)***

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

Q.15 Let S_k, k = 1, 2,...., 100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right|$ is ***(2010)***

Q.16 Let a_1, a_2, a_3, ..., a11 be real numbers satisfying a1 = 15, 27 − $2a_2$ > 0 and ak = $2a_k$−1 − a_k−2 for k = 3, 4, ...,11. If $\frac{a_1^2+a_2^2+....+a_{11}^2}{11}=90$, then the value of $\frac{a_1+a_2+....+a_{11}}{11}$ is equal to ***(2010)***

Q.17 Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$ is ***(2011)***

(A) 6 (B) 7 (C) $\frac{6}{7}$ (D) ∞

Q.18 Let a_1, a_2, a_3, ... be in harmonic progression with a_1 = 5 and a_{20} = 25. The least positive integer n for which an< 0 is ***(2012)***

(A) 22 (B) 23 (C) 24 (D) 25

Q.19 Let $S_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$. Then S_n can take value(s) ***(2013)***

(A) 1056 (B) 1088 (C) 1120 (D) 1332

Q.20 Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2+a-14}{a+1}$ is ________ ***(2014)***

Q.21 Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is ***(2015)***

Q.22 Let b_i > 1 for i = 1, 2, ..., 101. Suppose loge b1 loge b_2, ..., loge b_{101} are in Arithmetic Progression (A.P.) with the common difference loge 2. Suppose $a_1, a_2, ..., a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + ...+ b_{51}$ and $s = a_1 + a_2 + ... + a_{51}$, then ***(2016)***

(A) s > t and $a_{101} > b_{101}$ (B) s > t and $a_{101} < b_{101}$

(C) s < t and $a_{101} > b_{101}$ (D) s < t and $a_{101} < b_{101}$

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.3 Q.11 Q.14
Q.17 Q.21 Q.25
Q.27

Exercise 2

Q.2 Q.4 Q.10
Q.13

Previous Years' Questions

Q.2 Q.5 Q.8

JEE Advanced/Boards

Exercise 1

Q.6 Q.9 Q.12
Q.15 Q.17

Exercise 2

Q.1 Q.4 Q.5 Q.12

Previous Years' Questions

Q.1 Q.3 Q.4
Q.6 Q.7 Q.8

Answer Key

JEE Main/Boards

Exercise 1

Q.1 n = 6

Q.2 $3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \ldots\ldots$ is the series.

Q.3 (a) $\frac{7n}{9} - \frac{7}{81}\left(1 - \left(\frac{1}{10}\right)^n\right)$; (b) $\frac{2}{27}[10^{n+1} - 9n - 10]$

Q.5 (2, 4, 6) or (6, 4, 2)

Q.6 (i) 6633 (ii) 2842 (iii) 945

Q.7 69: 128

Q.10 $\frac{35}{16} - \frac{3}{16(5^{n-2})} - \frac{(3n-2)}{4(5^{n-1})}$

Q.11 c = 6, b = 4, d = 9; b = – 2, c = – 6, d = – 18

Q.12 $\frac{1}{4}n(n+1)(n+2)(n+3)$

Q.13 $\frac{1}{3}n(4n^2 + 3n + 2)$

Q.14 6,3

Q.15 4960

Q.16 188090

Q.17 a = 5, b = 8, c = 12.

Q.20 –5050, $\frac{1}{2}[(5050)^2 - 338350]$

Q.21 9

Q.22 469

Q.23 3370 m

Q.26 $\frac{1}{a_1(a_2 - a_1)}$

Exercise 2

Single Correct Choice Type

Q.1 B	**Q.2** A	**Q.3** C	**Q.4** D	**Q.5** C	**Q.6** D
Q.7 A	**Q.8** C	**Q.9** A	**Q.10** A	**Q.11** C	**Q.12** D

Previous Years' Questions

Q.1 B	**Q.2** C	**Q.3** B	**Q.4** A	**Q.5** D	**Q.6** D
Q.7 C	**Q.8** C	**Q.9** B	**Q.10** B	**Q.11** B	**Q.12** B
Q.13 A	**Q.14** A	**Q.15** B	**Q.16** B	**Q.17** D	**Q.18** D
Q.19 A	**Q.20** C	**Q.21** A			

JEE Advanced/Boards

Exercise 1

Q.1 (i) 6, 3 ; (ii) 120, 30 **Q.2** 1 **Q.3** $\frac{\sqrt{5}-1}{2};\sqrt{2}$

Q.4 (iii) b = 4, c = 6, d = 9 or b = – 2, c = – 6, d = – 18 **Q.5** 12 **Q.6** 518 **Q.7** 49

Q.8 28 **Q.9** 8 problems, 127.5 minutes **Q.10** 2499 **Q.11** 50

Q.12 C = 9; (3, –3/2, – 3/5) **Q.13** 13/4 **Q.14** 3 **Q.15** n = 38

Q.16 6 **Q.17** 931 **Q.18** 2 **Q.19** 4950

Exercise 2

Single Correct Choice Type

Q.1 A **Q.2** D **Q.3** B **Q.4** A

Assertion Reasoning Type

Q.5 D

Multiple Correct Choice Type

Q.6 A, B, C, D **Q.7** B, C **Q.8** A, B **Q.9** A, C, D **Q.10** B, D **Q.11** A, B, C, D

Previous Years' Questions

Q.1 A, B, D	**Q.2** B, C, D	**Q.3** 4	**Q.4** 0	**Q.5** 3 or 9	**Q.6** C
Q.7 A	**Q.8** B	**Q.9** B	**Q.10** D	**Q.11** B	**Q.12** A, D
Q.13 C	**Q.14** C	**Q.15** 3	**Q.16** 0	**Q.17** B	**Q.18** D
Q.19 A, D	**Q.20** 6	**Q.21** 9	**Q.22** B		

Solutions

JEE Main/Boards

Exercise 1

Sol 1: Sum of n terms is 364

$a + ar + ar^2$ .... $ar^{n-1} = 364$

$\frac{a(r^n - 1)}{r - 1} = 364$

given r = 3, a = 1

$\Rightarrow \frac{(3^n - 1)}{(3 - 1)} = 364 \Rightarrow 3^n - 1 = 728$

$\Rightarrow 3^n = 729$

$\Rightarrow \boxed{n = 6}$

Sol 2: Sum of infinite G.P. is 2 $\Rightarrow \frac{a}{-r+1} = 2$

$\Rightarrow a = -2(r - 1)$

Series is a, ar, ar^2 .... (|r| < 1)

$\Rightarrow a^3, (ar)^3, (ar^2)^3$ ....(2)

First term of this infinite series is a^3 and ratio is r^3

Hence sum of this infinite series is $\frac{a^3}{-r^3+1}$

Given $\frac{a^3}{-r^3+1} = 24$

$\frac{8(r-1)^3}{\left(-r^3+1\right)} = 24 \Rightarrow \frac{(r-1)^2}{(r^2+r+1)} = 3$

$\Rightarrow r^2 + 1 - 2r = 3r^2 + 3r + 3$

$\Rightarrow 2r^2 + 5r + 2 = 0 \Rightarrow 2r^2 + 4r + r + 2 = 0$

$\Rightarrow (2r + 1)(r + 2) = 0$

$\Rightarrow r = -2, r = -\frac{1}{2}$

$|r| < 1 \Rightarrow r = -\frac{1}{2} \Rightarrow a = +3$

Series is $3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, ...$

Sol 3: (a) Sum upto n terms

$S_n = 0.7 + 0.77 + 0.777$ .... n terms

$= 7[0.1 + 0.11 + 0.111 + ...] = \frac{7}{9}[0.9 + 0.99 + 0.999 ...]$

$= \frac{7}{9}[1 - 0.1 + 1 - 0.01 + 1 - 0.001 ...]$

$= \frac{7}{9}[n - (0.1 + 0.01 + 0.001 ...)]$

$= \frac{7}{9}\left[n - \frac{0.1(1-(0.1)^n)}{1-0.1}\right] = \frac{7}{9}\left[n - \frac{1}{9}(1-(0.1)^n)\right]$

$= \frac{7n}{9} - \frac{7}{81}\left(1 - \left(\frac{1}{10}\right)^n\right)$

(b) $6 + 66 + 666 = 6[1 + 11 + 111 ...]$

$= \frac{6}{9}[9 + 99 + 999 + ...] = \frac{2}{3}[10 - 1 + 100 - 1 + 1000 - 1 ...]$

$= \frac{2}{3}\left[\frac{10(10^n - 1)}{10 - 1} - n\right] = \frac{2}{3} \times \frac{10}{9}(10^n - 1) - \frac{2n}{3}$

$= \frac{2}{27}\left(10^{n+1} - 10\right) - \frac{2n \times 9}{3 \times 9} = \frac{2}{27}\left[10^{n+1} - 9n - 10\right]$

Sol 4: a, b, c are in AP

(i) b + c, c + a, a + b are also in AP

a, b, c are in AP $\Rightarrow$ 2b = a + c

$\Rightarrow b - a = c - b$

$\Rightarrow \boxed{a - b = b - c}$

Difference between term of given AP = a − b, b − c which are equal by equation (i)

Hence b + c, c + a, a + b is an AP

(ii) $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are also in AP

Common difference = $\frac{(b-a)}{cab}, \frac{(c-b)}{abc}$

By equation (i) b – a = c – b ie difference between terms is same

Hence the given series is in AP

(iii) $a^2(b + c), b^2(c + a), c^2(a + b)$

Difference = $\underbrace{b^2c + b^2a - a^2b - a^2c}_{d_1}, \underbrace{c^2a + c^2b - b^2c - b^2a}_{d_2}$

$d_1 = c(b^2 - a^2) + ab(b - a) = (ca + ab + cb)(b - a)$

$= (ca + ab + bc)(c - b)$ [from eq.(i)

$d_2 = a(c^2 - b^2) + bc(c - b) = (ac + ab + bc)(c - b)$

$d_1 = d_2$

Hence given series is an AP

(iv) $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$

$\Rightarrow d_1 = \frac{1}{c}(b-a) + \frac{b}{a} - \frac{a}{b} \Rightarrow d_2 = \frac{1}{a}(c-b) + \frac{c}{b} - \frac{b}{c}$

$d_1 = \frac{b-a}{c} + \frac{(b-a)(b+a)}{ab}$

$d_2 = \frac{c-b}{a} + \frac{(c-b)(c+b)}{bc} = (b-a)\left[\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right]$

$= (b-a)\left[\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right] = (c-b)\left[\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right]$

From eq^n (i) $\Rightarrow d_1 = d_2$

Hence given series is also an AP

Sol 5: Sum of first 3 numbers in AP is 12

Let a – r, a, a + r be the first 3 numbers

$3a = 12 \Rightarrow a = 4$

$\Rightarrow (a - r)^3 + a^3 + (a + r)^3 = 288$

$\Rightarrow 2a^3 + 6ar^2 + a^3 = 288 \Rightarrow 3a^3 + 6ar^2 = 288$

$\Rightarrow 6 \times 4 \times r^2 = 288 - 3(4^3) \Rightarrow 24r^2 = 96$

$\Rightarrow r = \pm 2$

So numbers are, 4 – 2, 4, 4 + 2 = (2, 4, 6) (for r = 2)

(4 + 2, 4, 4 – 2) for (r = – 2)

$\Rightarrow$ (6, 4, 2)

Sol 6: (i) Sum of integers between 1 & 200 which are multiple of 3

3, 6, 9, ... 198 $\Rightarrow$ n = 66

Hence sum = $\frac{n}{2}[a + l]$

$= \frac{(66)}{2}[3 + 198] = 33[201] = 6633$

(ii) Multiple of 7

7, 14, 21 ... 196 $\Rightarrow$ n = 28

Now sum = $\frac{n}{2}[a+l] = \frac{28}{2}[7 + 196] = 14[203] = 2842$

(iii) Multiple of 3 and 7

21, 42, 63 ... 189 $\Rightarrow$ n = 9

Sum = $\frac{9}{2}[21 + 189] = \frac{9}{2} . 210 = 945$

Sol 7: Sum of first n terms of 2 AP's are in ratio

$= \frac{3n-3}{5n+21}$

$\Rightarrow$ Let the AP be $a_1, a_1 + d_1 \ldots$

2^{nd} AP be $a_2, a_2 + d_2 \ldots$

$$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_1]} = \frac{3n-3}{5n+21}$$

$$\Rightarrow \frac{a_1 + \frac{(n-1)}{2}d_1}{a_2 + \frac{(n-1)d_2}{2}} = \frac{3n-3}{5n+21} \quad .....(i)$$

Ratio of 24^{th} term well be $\frac{a_1 + 23d_1}{a_2 + 23d_2}$

Putting n = 47 in equation (i), we well get desired ratio

$$\frac{a_1 + 23d_1}{a_2 + 23d_2} = \frac{3(47)-3}{5(47)+21} = \frac{138}{256} = \frac{69}{128}$$

Sol 8: given

$a + (p - 1)d = x$ ...(i)

$a + (q - 1)d = y$ ...(ii)

sum of first (p + q) terms

$= \frac{p+q}{2}[2a + (p+q-1)d]$ ...(iii)

Subtracting (ii) from (i)

$(p - q)d = x - y$

$\Rightarrow d = \frac{x-y}{p-q}$ and putting this value in equation (i)

$a + \frac{(p-1)(x-y)}{p-q} = x$

$\Rightarrow a = x - \frac{(px - py - x + y)}{p-q} = \frac{-qx + py + x - y}{p-q}$

Putting values of a and d in equation (iii)

$$S_{p+q} = \frac{p+q}{2}\left[\frac{2x - 2y - 2qx + 2py + (p+q-1)(x-y)}{p-q}\right]$$

$$= \frac{p+q}{2}\left[\frac{x-y-qx+py+px-qy}{p-q}\right]$$

$$= \frac{p+q}{2}\left[(x+y)\frac{(p-q)}{(p-q)}+\frac{(x-y)}{p-q}\right] = \frac{p+q}{2}\left[x+y+\frac{x-y}{p-q}\right]$$

Sol 9: a, b, c are in HP

i.e $\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow b = \frac{2ac}{a+c}$ ...(i)

$\frac{2a_1c_1}{a_1+c_1}$ [where a_1 & c_1 are 1st & 3rd terms of given series]

$$= \frac{\frac{2ac}{(a+b)(b+c)}}{\frac{a}{b+c}+\frac{c}{a+b}} = \frac{2ac}{a^2+ab+c^2+bc} = \frac{2ac}{a^2+c^2+b(a+c)}$$

$= \frac{2ac}{a^2+c^2+(2ac)}$ (from equation (i)

$$= \frac{2ac}{(a+c)(a+c)} = \frac{b}{a+c} = b_1$$

Middle term of given series, hence $\frac{2a_1c_1}{a_1+c_1} = b_1$ ie given Series is H.P

Sol 10: $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3}$

Sum of first n terms

$S_n = \frac{1}{5^0} + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} \ldots \frac{1+(n-1)3}{5^{n-1}}$ ...(i)

$\frac{S_n}{5} = \frac{1}{5} + \frac{4}{5.5} + \frac{7}{5.5^2} + \frac{10}{5.5^3} \ldots \frac{1+(n-1)3}{5^n}$ ...(ii)

Subtracting (ii) from (i)

$$= 1 + \frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} \ldots - \left(\frac{3n-2}{5^n}\right)$$

$$= 1 - \frac{(3n-2)}{5^n} + 3\left[\frac{1}{5}+\frac{1}{5^2}\ldots\frac{1}{5^{n-2}}\right]$$

$$= 1 - \frac{(3n-2)}{5^n} + 3.\frac{1}{5}\left[\frac{\left(1-\frac{1}{5^{n-1}}\right)}{\left(1-\frac{1}{5}\right)}\right]$$

$$= 1 - \left(\frac{3n-2}{5^n}\right) + \frac{3}{5}\frac{(5^{n-1}-1)\times 5}{5^{n-1}\times 4}$$

$$\frac{4s_n}{5} = 1 - \left(\frac{3n-2}{5^n}\right) + \frac{15}{4}\frac{(5^{n-1}-1)}{5^n}$$

$$S_n = \frac{5}{4} - \frac{(3n-2)}{4.5^{n-1}} + \frac{75}{16}\left(\frac{1}{5}-\frac{1}{5^n}\right)$$

$$= \frac{5\times 4}{4\times 4} + \frac{15}{16} - \frac{3}{16\times 5^{n-2}} \quad \frac{-(3n-2)}{4\times 5^{n-1}}$$

$$= \frac{35}{16} - \frac{3}{16(5^{n-2})} \quad \frac{-(3n-2)}{4(5^{n-1})}$$

Sol 11: a, b, c are in AP

$\Rightarrow 2b = a + c$ ...(i)

b, c, d in GP

$c^2 = bd$ ...(ii)

c, d, e are in HP

$d = \frac{2ce}{c+e}$ ...(ii)

Given that a = 2, e = 18

We have (2b – 2) = c from (i) and $\frac{(2b-2)^2}{b}$ = d from (ii)

and also $\frac{(2b-2)^2}{b} = \frac{2\times(2b-2)18}{(2b-2)+18}$ from (iii)

$\Rightarrow (2b-2) = \frac{36b}{2b+16} \Rightarrow (b-1)(b+8) = 9b$

$\Rightarrow b^2 + 7b - 8 = 9b \Rightarrow b^2 - 2b - 8 = 0$

$\Rightarrow b = 4, -2$

$\Rightarrow c = 6, -6$

$\Rightarrow d = 9, -18$

b, c, d = [4, 6, 9] and [–2, –6, – 18]

Sol 12: $S_n = 1.2.3 + 2.3.4 + 3.4.5 \ldots$

$T_n = n(n+1)(n+2) = n(n^2+3n+2)$

$= n^3 + 3n^2 + 2n = n^3 + 3n^2 + 2n$

$\Sigma_n = \Sigma T_n = \Sigma n^3 + 3\Sigma n^2 + 2\Sigma n$

$$= \left[\frac{n(n+1)}{2}\right]^2 + 3\frac{(n)(n+1)(2n+1)}{6} + 2\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{3(2n+1)}{3}+2\right]$$

$= \frac{n(n+1)}{2}\left[\frac{n^2+n+4n+2+4}{2}\right]$

$= \frac{n(n+1)}{4}(n^2 + 5n + 6) = \frac{n(n+1)(n+2)(n+3)}{4}$

Sol 13: $S_n = 1^2 + 2 + 3^2 + 4 + 5^2 + 6$

(first 2n numbers)

$1^2 + 3^2 + 5^2$... n terms + 2 + 4 + 6... n terms

$= 2(1 + 2 + 3 \ldots + n) + 1^2 + 3^2 + 5^2 \ldots$

$= 2n\frac{(n+1)}{2} + 1^2 + 2^2 + 3^2 \ldots (2n-1)^2 - 2^2$

$- 4^2 - 6^2 \ldots (2n-2)^2$

$= n(n+1) + \frac{(2n-1)(2n-1+1)(4n-2+1)}{6}$

$- 2^2\{1^2 + 2^2 + \ldots (n-1)^2\}$

$= n(n+1) + \frac{(2n-1)n(4n-1)}{3}$

$- 2^2\frac{(n-1)n(2n-1)}{6}$

$= n(n+1) + \frac{(2n-1)(n)(4n-1)}{3}$

$- \frac{2^2(n-1)n(2n-1)}{6}$

$= n(n+1) + \frac{n}{3}[(2n-1)(4n-1) - 2(n-1)(2n-1)]$

$= n(n+1) + \frac{n}{3}(2n-1)[2n+1]$

$= n(n+1) + \frac{(4n^2-1)n}{3} = \frac{n}{3}[4n^2 + 3n + 2]$

Sol 14: HM of 2 numbers is 4 ie $\frac{2ab}{a+b} = 4$

$\boxed{ab = 2a + 2b}$ ...(i)

$AM = \frac{a+b}{2}$ and $G.M. = \sqrt{ab}$

We have $2A + G^2 = 27 \Rightarrow a + b + ab = 27$

$a + b + 2a + 2b = 27$ [from (i)]

$\Rightarrow a + b = 9$ and $ab = 18$

[a = 6, b = 3]; [a = 3, b = 6]

Sol 15: $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots$

$= 3^3 + 5^3 + 7^3 + \ldots + 21^3$

$- 2^3(1 + 2^3 + 3^3 \ldots 10^3) + 1 - 1$

$= 1^3 + 2^3 + 3^3 + 4^3 \ldots 21^3 - 2^4(1 + 2^3 \ldots 10^3) - 1$

$= \left[\frac{21\times(21+1)}{2}\right]^2 - 2^4\left[\frac{(10)(10+1)}{2}\right]^2 - 1$

$= (21 . 11)^2 - 16(5 . 11)^2 - 1$

$= 11^2(21 . 21 - 16 . 25) - 1$

$= 121 \times 41 - 1 = 4961 - 1 = 4960$

Sol 16: $S_n = 1.3^2 + 2.5^2 + 3.7^2 + \ldots$

$T_n = n(2n+1)^2 = 4n^3 + n + 4n^2$

$\Sigma_n = \Sigma T_n = 4\Sigma n^3 + \Sigma n + 4\Sigma n^2$

$= 4\left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)}{2} + 4\frac{n(n+1)(2n+1)}{6}$

$= n^2(n+1)^2 + \frac{n(n+1)}{2} + \frac{2}{3}n(n+1)(2n+1)$

$= n(n+1)\left[n^2 + n + \frac{1}{2} + \frac{4n}{3} + \frac{2}{3}\right]$

$= n(n+1)\left[n^2 + \frac{7n}{3} + \frac{7}{6}\right]$

$S_{20} = 20 \times 21\left[20^2 + \frac{7}{3}\times 20 + \frac{7}{6}\right]$

$= \frac{420}{6}\left[2400 + 280 + 7\right] = 70(2687) = 188090$

Sol 17: $a, b, c \in (2, 18)$

$a + b + c = 25$ ...(i)

$2a = 2 + b$ ...(ii)

$c^2 = 18b$ ...(iii)

$b = 2a - 2$

$c = 25 - a - 2a + 2 = 27 - 3a$

$\Rightarrow (27 - 3a)^2 = 18(2a - 2)$

$\Rightarrow (9 - a)^2 = 4(a - 1) \Rightarrow a^2 + 81 - 18a = 4a - 4$

$\Rightarrow a^2 - 22a + 85 = 0$

a = 17, 5

b = 32, 8

c = 24, 12

Numbers are (5, 8, 12)

Sol 18: a > 0, b > 0, c > 0

To prove $(a + b + c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$

We know that A.M. ≥ G.M. ≥ HM

Therefore, A.M. ≥ HM

For 3 numbers a, b, c

$AM = \frac{a+b+c}{3}$, $HM = \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\Rightarrow \frac{a+b+c}{3} \geq \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\Rightarrow (a + b + c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$

Sol 19: Let the 2 number be a, b

a, A_1, A_2, b are in AP, (a, G_1, G_2, b) are in GP,

(a, H_1, H_2, b) are in HP

$A_1 + A_2 = a + b$ ...(i)

$G_1 G_2 = ab$ ...(ii)

By properties of respective series

$$\frac{H_1H_2}{H_1+H_2} = \frac{ab}{a+b}$$

$$\frac{H_1H_2}{H_1+H_2} = \frac{G_1G_2}{A_1+A_2}$$

$\frac{G_1G_2}{H_1H_2} = \frac{A_1+A_2}{H_1+H_2}$ Hence proved

Sol 20: (x –1) (x – 2) (x – 3) ... (x – 100)

Coefficient of $x^{99} = \frac{-b}{a}$

We can see that a = 1

and $b = 1 + 2 + 3 \ldots 100 = \frac{100\times101}{2} = 5050$

$\therefore$ Coefficient of $x^{99} = -5050$

Coefficient of x^{98}

$$= 1\times2+1\times3+\ldots.+1\times100+2\times3+2\times4+\ldots.+99\times100$$

$$=\frac{1}{2}\left\{\left(1+2+\ldots.+100\right)^2-\left(1^2+2^2+100^2\right)\right\}$$

$$=\frac{1}{2}\left\{\left(5050\right)^2-338350\right\}$$

Sol 21: Given that,for a polygon of "n" sides, we have

$\alpha = 120^o$; d = 5

Sum of interior angle

$(n – 2)\, 180^o = a + (n – 1)d$

$$= \frac{n}{2}\left[2(120)+(n-1)5\right] = \frac{n}{2}\left[5n+235\right]$$

$5n^2 + 235n = 360n – 720$

$\Rightarrow 5n^2 – 125n + 720 = 0$

$\Rightarrow n^2 – 25n + 144 = 0 \Rightarrow n = 16, 9$

If n = 16, then interior angle will be greater than 180°.

Hence the answer is 9.

Sol 22: Let the number be a, b, c

$b^2 = ac$, $a + c = 2b + 1$, $b + a = \frac{2}{3}(b + c)$

$\Rightarrow a = 2b + 1 – c$

$\Rightarrow 3b + 1 – c = \frac{2}{3}(b+c)$

$\Rightarrow 2b + 2c = 9b + 3 – 3c$

$\Rightarrow 7b = 5c – 3$

$c = \frac{7b+3}{5} \Rightarrow a = 2b + 1 - \frac{(7b+3)}{5}$

$a = \frac{3b+2}{5} \Rightarrow b^2 = \frac{(3b+2)(7b+3)}{25}$

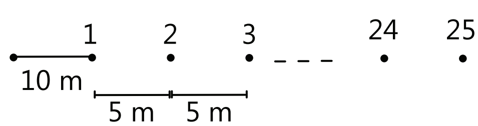
$\Rightarrow 25b^2 = 21b^2 + 23b + 6 \Rightarrow 4b^2 – 23b – 6 = 0$

$\Rightarrow 4b^2 – 24b + b – 6 = 0$

$\Rightarrow b = 6, c = 9$

$\Rightarrow a = 4$ Number is 469

Sol 23:



Sn = 10 + 10 + 15 + 15 + 20 + 20

... 85 + 85 + ... + 125 + 125 + 130

= 2[10 + 15 + 20 ... 130] – 130

= 10[2 + 3 + 4 ... 26] – 130

$= 10\left[\frac{26\times27}{2}-1\right] - 130 = 10 \times 350 - 130$

= 3500 – 130 = 3370 m

Sol 24: Number of elements in nth group = n

First number in the group will be $\frac{n(n-1)+2}{2}$

$S_n = \frac{n}{2}[n(n-1) + 2 + (n-1)1]$

$= \frac{n}{2}[n^2 - n + 2 + n - 1] = \frac{n}{2}[n^2 + 1]$

Sol 25: Let the 3 number in G.P. be a, ar, ar^2 & other 3 numbers be a_1, a_1r_1, $a_1r_1^2$

$(a_1r_1 - ar)^2 = (a_1r_1^2 - ar^2)(a_1 - a)$

$a_1^2r_1^2 + a^2r^2 - 2aa_1rr_1 = a_1^2r_1^2 - aa_1r^2 - aa_1r^2 + a^2r^2$

$2rr_1 = r^2 + r_1^2$

$\Rightarrow (r_1 - r)^2 = 0$

$\Rightarrow r = r_1$

Ratio for third G.P. $= \frac{a_1r_1 - ar}{a_1 - a} = \frac{(a_1 - a)r}{(a_1 - a)} = r$

Hence ratio of the three G.P. is same

Sol 26: $S = \frac{1}{a_1a_2} + \frac{1}{a_2a_3} \ldots \frac{1}{a_na_{n+1}}$

$= \frac{1}{a_2 - a_1}\left[\frac{a_2 - a_1}{a_1a_2} + \frac{a_3 - a_2}{a_2a_3}\ldots\right]$

As $a_2 - a_1 = a_3 - a_2 = \ldots a_{n+1} - a_n$

$= \frac{1}{a_2 - a_1}\left[\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} \ldots + \frac{1}{a_n} - \frac{1}{a_{n+1}}\right]$

$= \frac{1}{a_2 - a_1}\left[\frac{1}{a_1} - \frac{1}{a_{n+1}}\right] = \frac{a_{n+1} - a_1}{(a_2 - a_1)a_1a_{n+1}}$

$= \frac{nd}{d(a_1a_{n+1})} = \frac{n}{a_1a_{n+1}}$

$S = \frac{n}{a_1a_{n+1}} = \frac{n}{a_1(a_1 + nd)}$

$\lim_{n\to\infty} S = \lim_{n\to\infty} \frac{n}{a_1(a_1 + nd)} = \lim_{n\to\infty} \frac{1}{a_1\left(d + \frac{a_1}{n}\right)} = \frac{1}{a_1d}$

Sol 27: 2, AM_1, AM_2 ... AM_9, 3

2, HM_1, HM_2 ... HM_9, 3

Suppose we take AM_n and HM_n

$\Rightarrow 2 + 10d = 3 \Rightarrow d = \frac{1}{10}$

$AM_n = 2 + nd = 2 + \frac{n}{10}$

$\frac{1}{2}, \frac{1}{HM_1} \ldots \frac{1}{HM_9}, \frac{1}{3}$ in AP

$\Rightarrow \frac{1}{2} + 10d = \frac{1}{3} \Rightarrow d = \frac{-1}{60}$

$HM_n = \frac{60}{30 - n}$

$A + \frac{6}{H} = 2 + \frac{n}{10} + \frac{6}{60}(30 - n)$

$= 2 + \frac{n}{10} + 3 - \frac{n}{10} = 5$

Hence proved

Sol 28: $x + y + z = 1$

For x, y, z. A.M. $\geq$ HM

$\Rightarrow \frac{x + y + z}{3} \geq \frac{3}{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \geq 9$

Therefore, $xy + yz + zx - 9xyz \geq 0$

Hence proved.

Sol 29: We must prove that for some m and p;

$M^p = \frac{m}{2}[2a + (m - 1)2]$, for some odd a

$= m[a + m - 1]$

Let us prove this by induction

Taking, P = 2

$m^2 = m[a + m - 1] \Rightarrow a = 1$ is the required a.

$m^{p+1} = m^p.m$

$= m[a + m - 1]\, m = m[ma + m^2 - m]$

$= m[ma + m^2 - 2m + 1 + m - 1]$

$= m\,[ma + (m - 1)^2 + m - 1]$

$= m\,[A + m - 1]$

We must prove that A is odd.

A is odd

For even m, ma is even and $(m - 1)^2$ is odd $\Rightarrow$ A is odd
for odd m, ma is odd and $(m - 1)^2$ is even $\Rightarrow$ A is odd

$\therefore$ By induction hypothesis,

$M^p = \frac{m}{2}[2a + (m - 1)2]$, with odd a.

Hence proved

Exercise 2

Single Correct Choice Type

Sol 1: (B) $b = \frac{2ac}{a+c}$ [a, b, c are in HP] ...(i)

$= \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{\frac{2ac}{a+c}+a}{\frac{2ac}{a+c}-c} + \frac{\frac{2ac}{a+c}+c}{\frac{2ac}{a+c}-c}$

$= \frac{3ac+a^2}{ac-a^2} + \frac{3ac+c^2}{ac-c^2} = \frac{3c+a}{c-a} + \frac{(3a+c)}{a-c}$

$= \frac{3c+a}{c-a} - \frac{(3a+c)}{c-a} = \frac{2c-2a}{c-a} = 2$

Sol 2: (A) Given summation is, $\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3}$

... $T_n = \frac{2}{n(n+1)} \Rightarrow T_n = 2\left[\frac{1}{n} - \frac{1}{n+1}\right]$

$S_n = ST_n = 2\left[\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} ... \frac{1}{\infty}\right] = 2$

Sol 3: (C)

• • • • • • •

←10m→ ←middle→ n stones

n stones

$\Rightarrow 2[20 + 40 + 60 \ldots n \text{ terms}] = 4800$

$\Rightarrow 120 = 1 + 2 \ldots n$ terms

$\Rightarrow 120 = \frac{n(n+1)}{2} \Rightarrow n^2 + n = 240$

$\Rightarrow n = 15$

Total no of stone = 2n+1 = 31 [C]

Sol 4: (D) $S = 1^2 + 3^2 + 5^2 \ldots 99^2$

$S_1 = 2^2 + 4^2 + 6^2 \ldots (100)^2$

$S_1 - S = 2^2 - 1^2 + 4^2 - 3^2 + 6^2 - 5^2 \ldots$

$= 2+1+4+3+6+5+ \ldots +100+99$

$= \frac{100}{2}[1+100] = 50\,[101] = 5050$

Sol 5: (C) a, a + d, ... a + (n – 1)d, a + nd, ... a + (2n – 1) d

$S_1 = \frac{n}{2}[2a + (n-1)d]$

$S_2 = \frac{n}{2}[2a + 2nd + (n-1)d]$

$\frac{s_1}{s_2} = \frac{2a+(n-1)d}{2a+2nd+(n-1)d}$

$\Rightarrow \frac{2+(n-1)\frac{d}{a}}{2+(3n-1)\frac{d}{a}} = s$

$\Rightarrow 2 + (n-1)\frac{d}{a} = 2s + (3n-1)\frac{d}{a}s$

$\Rightarrow 2s - 2 = \frac{d}{a}[n - 3ns + s - 1]$

$(2s-2)\frac{a}{d} = n(1-3s) + s - 1$

This is independent of n ie coefficient of n will be zero

$\Rightarrow s = \frac{1}{3}; \Rightarrow \frac{a}{d} = \frac{1}{2}$

Sol 6: (D) $a = 5^{1+x} + 5^{1-x} + 25^x + 25^{-x}$

$\Rightarrow a = 5.5^x + \frac{5}{5^x} + (5^x)^2 + (5^{-x})^2$

Let $5^x = t \Rightarrow a = 5t + \frac{5}{t} + t^2 + \frac{1}{t^2}$

$\frac{5t+\frac{5}{t}}{2} \ge 5; \frac{t^2+\frac{1}{t^2}}{2} \ge 1$ hence $a \ge 10 + 2$

$\therefore \quad a \ge 12$

Sol 7: (A) $S_{11} = S_{19}$

$\frac{11}{2}[2a + 10d] = \frac{19}{2}[2a + 18d]$

$16a = -232\,d \Rightarrow \frac{a}{d} = \frac{-29}{2}$

$S_{30} = \frac{30}{2}[2a + 29d] = 30\left[a + \frac{29d}{2}\right] = 0$

Sol 8: (C) $S_2 = 2t_2$

$S_1 + d_s = 2(t_1 + d_t)$

$d_s = t_1 + 2d_t$

$\Rightarrow \frac{10}{2}[2s_1 + 9d_s] = \frac{15}{2}[2t_1 + 14d_t]$

$\Rightarrow 18d_s = 2t_1 + 42d_t \Rightarrow 18d_s = 2d_s - 4d_t + 42d_t$

$\Rightarrow 16d_s = 38d_t$

$\frac{d_s}{d_t} = \frac{19}{8}$

Sol 9: (A) The given expression is equal to

$$\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\ldots\right) = \frac{1}{a_1 d}$$

Sol 10: (A) $a + ar + ar^2 = 21$ ...(i)

$a^2 + a^2r^2 + a^2r^4 = 189$ ...(ii)

Squaring equation (i) & then dividing by (ii)

$$\Rightarrow \frac{a^2(1+r+r^2)^2}{a^2(1+r^2+r^4)} = \frac{441}{189}$$

$$\Rightarrow \frac{(1+r+r^2)(1+r+r^2)}{(1+r+r^2)(1-r+r^2)} = \frac{441}{189}$$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow r = 2, \frac{1}{2} \Rightarrow a = 3, 12$

GP is 3, 6, 12 ... $S_n = 3\frac{(2^n-1)}{2-1} = 3(2^n - 1)$ Hence, (A) is the correct choice

Sol 11: (C) $\sum_{n=1}^{\infty}\frac{n}{n^4+4} = \frac{1}{n\left(n^2+\frac{4}{n^2}+4-4\right)}$

$$= \sum\frac{1}{n\left[\left(n+\frac{2}{n}\right)^2-4\right]} = \sum\frac{1}{n\left[n+\frac{2}{n}-2\right]\left[n+\frac{2}{n}+2\right]}$$

$$= \sum\frac{n}{(n^2+2-2n)(n^2+2n+2)}$$

$$= \frac{1}{4}\sum\left[\frac{1}{n^2-2n+2}-\frac{1}{n^2+2n+2}\right]$$

$$= \frac{1}{4}\sum_{n=1}^{\infty}\left[\frac{1}{(n-1)^2+1}-\frac{1}{(n+1)^2+1}\right]$$

$$= \frac{1}{4}\left(1+\frac{1}{2}+\sum_{n=1}^{\infty}\left[\frac{1}{(n+1)^2+1}-\frac{1}{(n+1)^2+1}\right]\right) = \frac{3}{8}$$

Sol 12: (D) $\ln a^2 + (\ln a^2)^2 + (\ln a^2)^3$ .....

$= 3\{\ln a + (\ln a)^2 + (\ln a)^3 + \ldots\}$

$$\Rightarrow \frac{2\ln a}{1-2\ln a} = \frac{3\ln a}{1-\ln a}$$

$\Rightarrow 2 - 2\ln a = 3 - 6\ln a$

$\Rightarrow 1 = 4\ln a$

$\Rightarrow a = e^{1/4}$

Previous Years' Questions

Sol 1: (B) Here, $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \le 0$

$\Rightarrow (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \le 0$

$\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$

(Since, sum of squares is never less than zero)

$\Rightarrow$ Each of the squares is zero

$\therefore (ap - b)^2 = (bp - c)^2 = (cp - d)^2 = 0$

$\Rightarrow p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$\therefore$ a, b, c are in G.P.

Sol 2: (C) Sum of the n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots$ upto n terms can be written as

$\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)\ldots$ upto n terms

$$= n-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+n\text{terms}\right)$$

$$= n-\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} = n+2^{-n}-1$$

Sol 3: (B) Let the common ratio of the G.P. be r. Then,

$Y = xr$ and $z = xr^2$

$\Rightarrow \ln y = \ln x + \ln r$ and $\ln z = \ln x + 2\ln r$

Let $A = 1 + \ln x$, $D = \ln r$

Then, $\frac{1}{1+\ln x} = \frac{1}{A}$,

$$\frac{1}{1+\ln y} = \frac{1}{1+\ln x+\ln r} = \frac{1}{A+D}$$

and $\frac{1}{1+\ln z} = \frac{1}{1+\ln x+2\ln r} = \frac{1}{A+2D}$

Therefore, $\frac{1}{1+\ln x},\frac{1}{1+\ln y},\frac{1}{1+\ln z}$ are in H.P.

Sol 4: (A) Since A.M. $\ge$ G.M., then

$$\frac{(a+b)+(c+d)}{2} \ge \sqrt{(a+b)(c+d)}$$

$\Rightarrow M \le 1$

Also, $(a + b) + (c + d) > 0$ ($\because$ a, b, c, d > 0)

$\therefore 0 < M \le 1$

Sol 5: (D) Since a, b, c, d are in A.P.

$\Rightarrow \frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd}$ are in AP

$\Rightarrow \frac{1}{bcd}, \frac{1}{cda}, \frac{1}{abd}, \frac{1}{abc}$ are in A.P.

$\Rightarrow$ bcd, cda, abd, abc are in HP.

$\Rightarrow$ abc, abd, cda, bcd are in HP.

Sol 6: (D) Since a, b, c are in AP.

Leta $= A - D$, $b = A$, $c = A + D$

Given, $a + b + c = \frac{3}{2}$

$\Rightarrow (A - D) + A + (A + D) = \frac{3}{2} \Rightarrow 3A = \frac{3}{2} \Rightarrow A = \frac{1}{2}$

$\therefore$ The numbers are $\frac{1}{2} - D, \frac{1}{2}, \frac{1}{2} + D$

Also, $\left(\frac{1}{2} - D\right)^2, \frac{1}{4}, \left(\frac{1}{2} + D\right)^2$ are in GP.

$$\Rightarrow \left(\frac{1}{4}\right)^2 = \left(\frac{1}{2} - D\right)^2 \left(\frac{1}{2} + D\right)^2 \Rightarrow \frac{1}{16} = \left(\frac{1}{4} - D^2\right)^2$$

$$\Rightarrow \frac{1}{4} - D^2 = \pm\frac{1}{4} \Rightarrow D^2 = \frac{1}{2} \Rightarrow D = \pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow a = \frac{1}{2} + \frac{1}{\sqrt{2}} \text{ or } \frac{1}{2} - \frac{1}{\sqrt{2}}$$

So, out of the given values, $a = \frac{1}{2} - \frac{1}{\sqrt{2}}$ is the right choice

Sol 7: (C) We know that, the sum of infinite term of G.P. is

$$S_\infty = \begin{cases} \frac{a}{1-r}, & |r| < 1 \\ \infty, & |r| \le 1 \end{cases}$$

$\therefore S_\infty = \frac{x}{1-r} = 5$ (thus $|r| < 1$)

or $1 - r = \frac{x}{5} \Rightarrow r = \frac{5-x}{5}$ exists only when $|r| < 1$

i.e., $-1 < \frac{5-x}{5} < 1$

or $-10 < -x < 0 \Rightarrow 0 < x < 10$

Sol 8: (C) Let $S_n = cn^2$

$S_{n-1} = c(n-1)^2 = cn^2 + c - 2cn$

$\therefore T_n = 2cn - c$ ($\because T_n = S_n - S_{n-1}$)

$T_n^2 = (2cn - c)^2 = 4c^2n^2 + c^2 - 4c^2n$

$\therefore$ Sum $= ST_n^2$

$$= \frac{4c^2n(n+1)(2n+1)}{6} + nc^2 - 2c^2n(n+1)$$

$$= \frac{2c^2n(n+1)(2n+1) + 3nc^2 - 6c^2n(n+1)}{3}$$

$$= \frac{nc^2(4n^2 + 6n + 2 + 3 - 6n - 6)}{3} = \frac{nc^2(4n^2 - 1)}{3}$$

Sol 9: (B) Let a, ar, ar^2, ...

$a + ar = 12$ ... (i)

$ar^2 + ar^3 = 48$ ... (ii)

Dividing (ii) by (i), we have

$$\frac{ar^2(1+r)}{a(r+1)} = 4$$

$\Rightarrow r^2 = 4$ if $r \ne -1$

$\therefore r = -2$

Also, $a = -12$ (using (i)).

Sol 10: (B) Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots$ ... (i)

$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \ldots$ ... (ii)

Dividing (i) and (ii)

$$S\left(1 - \frac{1}{3}\right) = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots$$

$$\frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2}\left(1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\right)$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2}\left(\frac{1}{1 - \frac{1}{3}}\right) = \frac{4}{3} + \frac{4}{3^2}\frac{3}{2} = \frac{4}{3} + \frac{2}{3} = \frac{6}{2}$$

$$\Rightarrow \frac{2}{3}S = \frac{6}{3} \Rightarrow S = 3$$

Sol 11: (B) $M = \frac{\ell + n}{2}$

ℓ, G1, G2, G3, n are in G.P.

$$r = \left(\frac{n}{\ell}\right)^{\frac{1}{4}}$$

$G_1 = \ell\left(\frac{n}{\ell}\right)^{\frac{1}{4}} \quad G_2 = \ell\left(\frac{n}{\ell}\right)^{\frac{1}{2}} \quad G_3 = \ell\left(\frac{n}{\ell}\right)^{\frac{3}{4}}$

$G_1^4 + 2G_2^4 + G_3^4$

$= \ell^4 \times \frac{n}{\ell} + 2\ell^4 \times \frac{n^2}{\ell^2} + \ell^4 \times \frac{n^3}{\ell^3}$

$= \ell^3 n + 2\ell^2 n^2 + \ell n^3$

$= n\ell(\ell^2 + 2n\ell + n^2)$

$= n\ell(\ell + n)^2$

$= 4m^2 n\ell$

Sol 12: (B) $T_n = \frac{\frac{n^2(n+1)^2}{4}}{n^2}$

$T_n = \frac{1}{4}(n+1)^2$

$T_n = \frac{1}{4}[n^2 + 2n + 1]$

$S_n = \sum_{n=1}^{n} T_n$

$S_n = \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n(n+1) + n\right]$

$n = 9$

$S_9 = \frac{1}{4}\left[\frac{9 \times 10 \times 19}{6} + 9 \times 10 + 9\right] = \frac{1}{4}[285 + 90 + 9] = \frac{384}{4} = 96$

Sol 13: (A) a + d, a + 4d, a + 8d → G.P

∴ $(a + 4d)^2 = a^2 + 9ad + 8d2$

⇒ $8d^2 = ad$ ⇒ a = 8d

∴ 9d, 12d, 16d → G.P.

Common ratio $r = \frac{12}{9} = \frac{4}{3}$

Sol 14: (A)

$\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 + \left(\frac{24}{5}\right)^2 + \ldots\ldots.$

$\frac{8^2}{5^2} + \frac{12^2}{5^2} + \frac{16^2}{5^2} + \frac{20^2}{5^2} + \frac{24^2}{5^2} + \ldots\ldots$

$T_n = \frac{(4n+4)^2}{5^2}; S_n = \frac{1}{5^2}\sum_{n=1}^{10} 16(n+1)^2 = \frac{16}{25}\sum_{n=1}^{10}(n^2 + 2n + 1)$

$= \frac{16}{25}\left[\frac{10 \times 11 \times 21}{6} + \frac{2 \times 10 \times 11}{2} + 10\right] = \frac{16}{25} \times 505 = \frac{16}{5} m$

⇒ m = 101

Sol 15: (B) a, ar, ar^2 → G.P.

a, 2ar, ar^2 → A.P.

$2 \times 2ar = a + ar^2$

$4r = 1 + r^2$

⇒ $r^2 - 4r + 1 = 0$

$r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

$\boxed{r = 2 + \sqrt{3}}$

$r = 2 - \sqrt{3}$ is rejected

∴ (r > 1)

G.P. is increasing.

Sol 16: (B) Statement-I has 20 terms whose sum is 8000 and statement-II is true and supporting statement-I.

∴ k^{th} bracket is $(k-1)^2 + k(k-1) + k^2 = 3k^2 - 3k + 1$.

Sol 17: (D) $100(T_{100}) = 50(T_{50})$ ⇒ 2[a + 99d] = a + 49d ⇒ a + 149d = 0 ⇒ $T_{150} = 0$

Sol 18: (D) $f(x) = 2x^3 + 3x + k$

$f'(x) = 6x^2 + 3 > 0 \quad \forall x \in R$

⇒ f(x) is strictly increasing function

⇒ f(x) = 0 has only one real root, so two roots are not possible

Sol 19: (A) $x^2 + 2x + 3 = 0$ ... (i)

$ax^2 + bx + c = 0$ ... (ii)

Since equation (i) has imaginary roots

So equation (ii) will also have both roots same as (i). Thus

$\frac{a}{1} = \frac{b}{2} = \frac{c}{3} \Rightarrow a = \lambda, b = 2\lambda, c = 3\lambda$

Hence 1 : 2 : 3

Sol 20: (C) $\frac{7}{10} + \frac{77}{100} + \frac{777}{10^3} + \ldots\ldots. +$ up to 20 terms

$= 7\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{10^3} + \ldots\ldots. + \text{up to 20 terms}\right]$

$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\ldots. + \text{up to 20 terms}\right]$

$$= \frac{7}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+.......+\text{up to 20 terms}\right]$$

$$= \frac{7}{9}\left[20-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^{20}\right)}{1-\frac{1}{10}}\right] == \frac{7}{9}\left[20-\frac{1}{9}\left(1-\left(\frac{1}{10}\right)^{20}\right)\right]$$

$$= \frac{7}{9}\left[\frac{179}{9}+\frac{1}{9}\left(\frac{1}{10}\right)^{20}\right] = \frac{7}{81}\left[179+(10)^{-20}\right]$$

Sol 21: (A) 2y = x + z

2 tan-1 y = tan-1 x + tan-1 (z)

$$\tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\frac{x+z}{1-y^2} = \frac{x+z}{1-xz}$$

$\Rightarrow$ y2 = xz or x + z = 0 $\Rightarrow$ x = y = z

JEE Advanced/Boards

Exercise 1

Sol 1: (i) Let 2 numbers be a, b

Given H.M. = $\frac{2ab}{a+b}$ = 4 $\Rightarrow$ a + b = $\frac{ab}{2}$

We have A.M. = $\frac{a+b}{2}$ and G.M. = $\sqrt{ab}$

$(G.M.)^2$ = ab ... (i)

2A.M. = a + b ... (ii)

$2A + G^2 = 27$

a + b + ab = 27 using (i) and (ii)

$\frac{3ab}{2}$ = 27 $\Rightarrow$ ab = 18

a + b = 9

$\Rightarrow$ a, b = 3, 6

(ii) A.M. = G.M. + 15 = H.M. + 27

$\frac{a+b}{2} = \frac{2ab}{a+b} + 27 = \sqrt{ab} + 15$

HM = $\frac{G^2}{AM}$ (we know this)

$\Rightarrow (A-27)(A) = (A-15)^2$

$\Rightarrow$ 27A = 225 – 30 A

A = 75 ...(i)

G.M. = 60 ...(ii)

H.M. = 48

a + b = 150 using (i) and (ii)

ab = 3600

a = 120

b = 30

Sol 2: $H_{10} = 21$, $H_{21} = 10$

$\frac{1}{H_{10}} = \frac{1}{H_1} + 9d = \frac{1}{21}$

$\frac{1}{H_{10}} = \frac{1}{H_1} + 20d = \frac{1}{10}$

$\Rightarrow 11d = \frac{11}{210} \Rightarrow d = \frac{1}{210}$

$\frac{1}{H_1} + \frac{9}{210} = \frac{10}{210}$

$\frac{1}{H_1} = \frac{1}{210}$

$\frac{1}{H_{210}} = \frac{1}{H_1} + 209d = \frac{1}{210} + \frac{209}{210} = \frac{210}{210} = 1$

Sol 3: sin x, $\sin^2 2x$, cos x sin 4x are in GP.

$\sin^4 2x$ = sin x cos x sin 4x

$(2 \sin x \cos x)^4$= sin x cos x 2 sin 2x cos 2x

$16 \sin^4 x \cos^4 x = 4 \sin^2 x \cos^2 x \cos 2x$

$4 \sin^2 x \cos^2 x = \cos 2x$

$\text{Sin}^2 2x = \cos 2x$

$1 - \cos^2 2x = \cos 2x$

$\cos^2 2x + \cos 2x - 1 = 0$

$\cos 2x = \frac{-1\pm\sqrt{5}}{2}$

$\therefore$ cos θ can never be equal to $\frac{-1-\sqrt{5}}{2}$ i.e

$\therefore \cos 2x = \frac{-1+\sqrt{5}}{2}$

Common ratio = $\frac{\sin^2 2x}{\sin x} = \frac{4\sin^2 x \cos^2 x}{\sin x}$

$= 4\cos^2 x \sin x = 2\cos x \sin 2x$

$= 2\sqrt{\dfrac{1+\cos 2x}{2}}\sqrt{1-\cos^2 2x}$

$= \sqrt{2}\sqrt{1-\dfrac{(6-2\sqrt{5})}{4}}\sqrt{1+\left(\dfrac{\sqrt{5}-1}{2}\right)}$

$= \sqrt{2}\sqrt{\dfrac{\sqrt{5}-1}{2}} \cdot \sqrt{\dfrac{\sqrt{5}+1}{2}} = \sqrt{2} \cdot \sqrt{\dfrac{4}{4}} = \sqrt{2}$

Sol 4: a, b, c, d, e be 5 numbers

a b c in AP, b c d in GP, c d e in HP

$2b = a + c, c^2 = bd, d = \dfrac{2ce}{c+e}$, Let b be b

c be br, d be br^2

$br^2 = \dfrac{2bre}{br+e} \Rightarrow br^2 + er = 2e \Rightarrow e = \dfrac{br^2}{2-r}$

$a = 2b - br = b(2 - r)$

$ae = b^2r^2 = c^2$ hence a,c,e are in GP ...(i)

(ii) $\Rightarrow \dfrac{(2b-a)^2}{a} = \dfrac{c^2}{a} = \dfrac{b^2r^2}{b(2-r)} = \dfrac{br^2}{2-r} = e$

Hence proved.

(iii) $a = 2\ e = 18 \Rightarrow c = \pm 6$

bc, d, e = (4, 6, 9); (–2, –6, –18)

$\Rightarrow b = 4, -2 \Rightarrow d = 9, -18$

Sol 5: $2\alpha^2, \alpha^4, 24$ form A.P.

$\alpha^4 = \alpha^2 + 12$ ...(i)

$\alpha^4 - \alpha^2 = 12$

$\Rightarrow \alpha = 2, -2\ (\alpha_1, \alpha_2 = 2, -2$

$(\beta^2)^2 = 1(6 - \beta^2)$

$\beta^4 + \beta^2 = 6$

$\beta^4 + \beta^2 - 6 = 0$

$\beta^4 + 3\beta^2 - 2\beta^2 - 6 = 0$

$\beta^2 = 2$

$\beta_1, \beta_2 = \sqrt{2}, -\sqrt{2}$

$\alpha_1^2 + \alpha_2^2 + \beta_1^2 + \beta_2^2 = 4 + 4 + 2 + 2 = 12$

Sol 6: 2G.P.s

$\dfrac{a_1}{1-r_1} = 1 \Rightarrow a_1 = 1 - r_1$

$\dfrac{a_2}{1-r_2} = 1 \Rightarrow a_2 = 1 - r_2$

$a_1r_1 = a_2r_2 \Rightarrow (1 - r_1)r_1 = (1 - r_2)r_2$

$\Rightarrow r_1 - r_2 = (r_1 - r_2)(r_1 + r_2)$

$\Rightarrow r_1 + r_2 = 1$

$a_1r_1^2 = 1/8 \Rightarrow (1 - r_1)r_1^2 = 1/8$

$\Rightarrow (2r_1 - 1)(4r_1^2 - 2r_1 - 1) = 0$

If $r_1 = ½$ then $r_1 = r_2$

$\Rightarrow 4r_1^2 - 2r_1 - 1 = 0$

$\Rightarrow r_1 = \dfrac{1\pm\sqrt{5}}{4}$

If $r_1 = \dfrac{1-\sqrt{5}}{4}$ then, $r_2 > 1$.

$\Rightarrow r_1 = \dfrac{1+\sqrt{5}}{4}$

$\therefore a_1r_1 = (1 - r_1)r_1 = \left(1-\left(\dfrac{1+\sqrt{5}}{4}\right)\right)\left(\dfrac{1+\sqrt{5}}{4}\right)$

$= \left(\dfrac{3-\sqrt{5}}{4}\right)\left(\dfrac{1+\sqrt{5}}{4}\right) = \dfrac{\sqrt{5}-1}{8} = \dfrac{\sqrt{m}-n}{p}$

$\therefore 100m + 10n + p = 500 + 10 + 8 = 518$

Sol 7: $S = \displaystyle\sum_{n=1}^{99}\frac{5^{100}}{25^n + 5^{100}}$

$T_1 = \dfrac{5^{100}}{5^2 + 5^{100}}$

$T_{99} = \dfrac{5^{100}}{5^{2\times 99} + 5^{100}} = \dfrac{5^{100}}{\dfrac{5^{200}}{5^2} + 5^{100}} = \dfrac{5^2}{5^2 + 5^{100}}$

$T_1 + T_n = 1$

$S = T_1 + T_2 + \ldots + T_{99} = 1 + 1 \ldots T_{50}$

$= 49 + T_{50} = 49 + \dfrac{5^{100}}{5^{100} + 5^{100}} = 49 + {}^1/2$

$[S] = 49$

Sol 8: $ax^3 - ax^2 + 9bx - b = 0$

HM roots $= \dfrac{3}{\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}} = \dfrac{3\alpha\beta\gamma}{\alpha\beta+\beta\gamma+\gamma\alpha}$

$\alpha + \beta + \gamma = 1$

$\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{9b_1}{a_1}$

$\alpha\beta\gamma = \frac{b_1}{a_1}$

H.M .= $\frac{1}{3}$

$\frac{\alpha+\beta+\gamma}{3} \geq$ H.M.$_{(abc)}$

$\frac{\alpha+\beta+\gamma}{3} \geq \frac{1}{3}$

$\alpha+\beta+\gamma \geq 1$ ...(i)

Its given $\alpha+\beta+\gamma=1$

Equality of equation (i) holds only if $\alpha=\beta=\gamma$

i.e all the roots are $\frac{1}{3}$

$\frac{b}{a} = \alpha^3 = \left(\frac{1}{3}\right)^3$

b = 27a

b + a = 28a

$\therefore$ a is an integer, min (a + b) = 28

Sol 9: Let time taken to solve 1st problem be S time to solve second problem will be $\frac{S}{r}$

$\frac{S}{r} + \ldots + \frac{S}{r^{n-1}} = 63.5$ ...(i)

$S_n = 127 = S + \frac{s}{r} \ldots \frac{s}{r^{n-2}}$ ...(ii)

$31.5 = \frac{s}{r^2} + \ldots \frac{s}{r^{n-1}}$ ...(iii)

$\frac{S}{r} = 32$ ...(iv)

$127 + \frac{S}{r^{n-1}} = 63.5 + S$

$63.5 + \frac{S}{r^{n-1}} = S$

$\frac{S}{r} - \frac{S}{r^n} = 63.5\left(1-\frac{1}{r}\right)$

$32 - \frac{s}{r^n} = 63.5 - \frac{63.5}{r}$

$\frac{63.5}{r} \frac{S}{r^n} = 31.5$ ...(v)

$127 = \frac{S\left(1-\frac{1}{r^{n-1}}\right)}{\left(1-\frac{1}{r}\right)}$

$127 - \frac{127}{r} = S - \frac{S}{r^{n-1}}$

$127\left(1-\frac{1}{r}\right) = 32r - r\left(\frac{63.5}{r} - 31.5\right)$

from (v)

$127 - \frac{127}{r} = 63.5\,r - 63.5$

$\Rightarrow r = 2$

$\therefore$ s = 64

$\frac{s}{r^{n-1}} = \frac{1}{2}$

$2^{n-2} = 64$

$\Rightarrow n = 8$

Sol 10: $a_n + 1 = a_n + 1$ for n = 1 ... 97

$\Rightarrow a_2 = a_1 + 1$

$\Rightarrow a_3 = a_2 + 1 = a_1 + 2$

$\Rightarrow a_4 = a_1 + 3$

$a_n = a_1 + (n-1)$

$\Rightarrow a_1 + a_2 \ldots a_{98} = 4949 = \frac{98}{2}[2a_1 + 97.1]$

$101 = 2a_1 + 97 \Rightarrow a_1 = 2$

Now, we can write here Σa_{2k}

$= a_2 + a_4 + a_6 \ldots a_{98} = a_1 + 1 + a_1 + 3 \ldots a_1 + 97$

$= \frac{49}{2}[2a_1 + 2 + 48\times 2] = 49[a_1 + 49] = 49 \times 51$

= 2499

Sol 11: xyz = 55 or $\frac{343}{55}$ acc to a, x, y, z, b in AP/HP

For a, x, y, z, b in AP

x = a + d; y = a + 2d z = a + 3d

$b = a + 4d \Rightarrow d = \frac{b-a}{4}$

(a + d) (a + 2d) (a + 3d) = 55 ...(i)

For a, x, y, z, b in HP

$\frac{1}{x} = \frac{1}{a} + d_H;\ \frac{1}{b} = \frac{1}{a} + 4d_H \Rightarrow d_H = \frac{1}{4}\left[\frac{1}{b} - \frac{1}{a}\right]$

$\frac{1}{y} = \frac{1}{a} + 2d_H$

$\frac{1}{z} = \frac{1}{a} + 3d_H$

$$\frac{1}{xyz} = \left(\frac{1}{a}+d_H\right)\left(\frac{1}{a}+2d_H\right)\left(\frac{1}{a}+3d_H\right) = \frac{55}{343} \quad ...(ii)$$

Equation (i) can be written as

$$\left(a+\frac{b-a}{4}\right)\left(a+\frac{(b-a)2}{4}\right)\left(a+\frac{(b-a)3}{4}\right)=55$$

$$\frac{(3a+b)(2a+2b)(a+3b)}{64} = 55$$

Equation (ii) can be written as

$$\left(\frac{3}{4a}+\frac{1}{4b}\right)\left(\frac{1}{2a}+\frac{1}{2b}\right)\left(\frac{1}{4a}+\frac{3}{4b}\right) = \frac{55}{343}$$

$$\frac{(3b+a)(2b+2a)(b+3a)}{64a^2b^2ab} = \frac{55}{343}$$

$\Rightarrow a^3b^3 = 343$

$\Rightarrow ab = 7$

a & b are integers

i.e a = 1 , b = 7or a = 7, b = 1

i.e $a^2 + b^2 = 50$

Sol 12: $10x^3 - cx^2 - 54x - 27 = 0$

Let α, β, γ be the roots

$$\alpha + \beta + \gamma = \frac{c}{10} \quad ...(i)$$

$$a\beta + by + \gamma\alpha = -\frac{54}{10} \quad ...(ii)$$

$$ab\gamma = \frac{27}{10} \quad ...(iii)$$

α β & γ are in harmonic progression

i.e $\beta = \dfrac{2\alpha\gamma}{\alpha+\gamma}$

$$b\alpha + b\gamma = 2\alpha\gamma \quad ....(iv)$$

Putting this in equation (iii)

$\beta = -3/2$ this in equation (iv)

$$(\alpha + \gamma)\left(\frac{-3}{2}\right) = \frac{-3.6}{10}$$

$$\alpha + \gamma = \frac{12}{5}$$

$\Rightarrow \alpha = 3;\ \gamma = \dfrac{-3}{5}$

The 3 roots are 3, $\dfrac{-3}{2}, \dfrac{-3}{5}$

$$\frac{C}{10} = \alpha + \beta + y = \frac{12}{5} - \frac{3}{2} = \frac{9}{10} \Rightarrow C = 9$$

Sol 13: We have $b^2 = ac$

Also, $\log_c a$, $\log_b c$ and $\log_a b$ are in AP

We can write $\log_a b = \log_c a + (3 - 1)d$

$$\Rightarrow d = \frac{\log_a b - \log_c a}{2}$$

$$= \frac{\log_a \sqrt{ac} - \log_c a}{2}$$

$$= \frac{1 + 3\log_a c}{4}$$

Given that $\log_a c = 4$

$$\therefore d = \frac{1+3\times 4}{4} = \frac{13}{4}$$

Sol 14: $a = b - 2$

$ar^2 = b + 6$

$$\frac{a+ar^2}{2ar} = \frac{5}{3}$$

$$\frac{2b+4}{2ar} = \frac{5}{3}$$

$$\frac{3}{5}(b + 2) = ar$$

$$\frac{9}{25}(b + 2)^2 = (b + 6)(b - 2)$$

$\Rightarrow 9(b^2 + 4 + 4b) = 25(b^2 + 4b - 12)$

$16b^2 + 64b - 336 = 0$

$b^2 + 4b - 21 = 0$

$b^2 + 7b - 3b - 21 = 0$

$b = 7, 3 \Rightarrow$ +ve integral value of b is 3.

Sol 15: $\dfrac{S_{1-11}}{S_{n-10-n}} = \dfrac{1}{8}$ ... (i)

$$\frac{S_{10-n}}{S_{(n-8)-n}} = 2 \quad ... (ii)$$

$$S_{1-11} = a\frac{\left(r^{11}-1\right)}{r-1} \quad S_{10-n} = ar^9\frac{\left(r^{n-9}-1\right)}{r-1}$$

$$S_{(n-10)-n} = ar^{n-11}\frac{\left(r^{11}-1\right)}{r-1} \quad S_{(n-8)-n} = a\frac{\left(r^{n-9}-1\right)}{r-1}$$

Putting these values in equation (i) and equation (ii)

$$\frac{1}{r^{n-11}} = \frac{1}{8}$$

$r^9 = 2 \Rightarrow r = 2^{\frac{1}{9}}$

$\Rightarrow r^{n-11} = 2^3$

$\Rightarrow 2^{\frac{n-11}{9}} = 2^3$

$\Rightarrow n = 11 + 27 = 38$

Sol 16: $S_n = (1 + 2T_n)(1 - T_n)$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= [1 + 2a + (n-1) 2d] [1 - a - (n-1)d]$

$T_n = a + (n-1)d$

$S_n = 1 + T_n - 2T_n^2$

$S_1 = 1 + T_1 - 2T_1^2 = T_1$

$T_1 = \frac{1}{\sqrt{2}}$

$S_2 = 1 + T_2 - 2T_2^2 = T_1 + T_2$

$1 - \frac{1}{\sqrt{2}} = 2T_2^2$

$\left(\frac{\sqrt{2}-1}{2\sqrt{2}}\right) = T_2^2 = \frac{2-\sqrt{2}}{4} = \frac{\sqrt{4}-\sqrt{2}}{4}$

$\Rightarrow a = 4$ and $b = 2$

$a + b = 6$

Sol 17: Let number be abc $(a > b > c)$

$a = a,\ b = \frac{a}{r},\ c = \frac{a}{r^2}$

$\Rightarrow 100a + \frac{10a}{r} + \frac{a}{r^2} - 100\frac{a}{r^2} - \frac{10a - a}{r} = 792$

$\Rightarrow 99a - \frac{99a}{r^2} = 792 \Rightarrow a - \frac{a}{r^2} = 8$

$\therefore$ New number $= 100(a-4) + 10b + c$

$\Rightarrow 2b = a - 4 + c \Rightarrow \frac{2a}{r} = a - 4 + \frac{a}{r^2}$

$\Rightarrow \frac{2a}{r} = a - 4 + a - 8$

$\Rightarrow 2a - 12 = \frac{2a}{r} \Rightarrow a = \frac{a}{r} + 6 = \frac{a}{r^2} + 8$

$\Rightarrow r = \frac{a}{a-6}$

$\Rightarrow \left(\frac{a}{a-6}\right)^2 = \frac{a}{a-8}$

$\Rightarrow a(a-8) = (a-6)^2 \Rightarrow a = 9,\ r = 3$

So the number is 931

Sol 18: $S(\theta) = 1 + (1 + \sin\theta)\cos\theta$

$+ (1 + \sin\theta + \sin^2\theta)\cos^2\theta \ldots \infty$

$= 1 + \cos\theta + \cos^2\theta \ldots$

$+ \sin\theta(\cos\theta + \cos^2\theta \ldots) + \sin^2\theta$

$= \frac{1}{1-\cos\theta} + \frac{\sin\theta}{1-\cos\theta}\cos\theta + \frac{\sin^2\theta\cos^2\theta}{1-\cos\theta}$

$= \frac{1}{1-\cos\theta}[1 + \sin\theta\cos\theta + \sin^2\theta\cos^2\theta \ldots]$

$S(\theta) = \frac{1}{(1-\sin\theta\cos\theta)(1-\cos\theta)}$

$S_{\left(\frac{\pi}{4}\right)} = \frac{1}{\left(1-\frac{1}{2}\right)}\left(1-\frac{1}{\sqrt{2}}\right) = \frac{2\sqrt{2}}{\left(\sqrt{2}-1\right)}$

Sol 19: $\tan\left(\frac{\pi}{12}-x\right), \tan\frac{\pi}{12}, \tan\left(\frac{\pi}{12}+x\right)$ are in GP

$\tan^2\frac{\pi}{12} = \tan\left(\frac{\pi}{12}-x\right)\ \tan\left(\frac{\pi}{12}+x\right)$

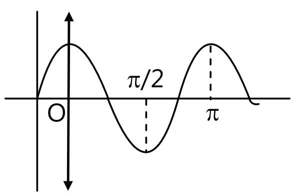
$= \frac{\sin\left(\frac{\pi}{12}+x\right)\sin\left(\frac{\pi}{12}-x\right)}{\cos\left(\frac{\pi}{12}+x\right)\cos\left(\frac{\pi}{12}-x\right)} \Rightarrow \frac{\cos 2x - \cos\frac{\pi}{6}}{\cos 2x + \cos\frac{\pi}{6}} = \tan^2\frac{\pi}{12}$

$\cos 2x = \frac{\cos\frac{\pi}{6}\tan^2\frac{\pi}{12} + \cos\frac{\pi}{6}}{1-\tan^2\frac{\pi}{12}} = \frac{\cos\frac{\pi}{6}\left[\tan^2\frac{\pi}{12}+1\right]}{1-\tan^2\frac{\pi}{12}}$

$= \cos\frac{\pi}{6}\left[\frac{\sin^2\left(\frac{\pi}{12}\right)+\cos^2\left(\frac{\pi}{12}\right)}{\cos^2\left(\frac{\pi}{12}\right)-\sin^2\left(\frac{\pi}{12}\right)}\right] = \frac{\sqrt{3}}{2}\left[\frac{1}{\cos\left(\frac{\pi}{6}\right)}\right] = 1$

$= \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = 1$

$\therefore \cos 2x = 1$

$\Rightarrow$ 

Solutions are O, π, 2π, 3π ... 99π

$\frac{99}{2}[2\pi + 98\pi]$ = 50 π . 99 = 4950 π

K = 4950

Exercise 2

Single Correct Choice Type

Sol 1: (A) A.M. = 9 + 99 ... 999999999/9

⇒ 9[1 + 11 + 111 + ... 111111111]/9

= 123456789

This does not contain 0

Sol 2: (D) Given $\sum_{k=1}^{360} \frac{1}{k\sqrt{k+1}+(k+1)\sqrt{k}} = \frac{m}{n}$

$$\sum_{k=1}^{360} = \frac{1}{\sqrt{k+1}\sqrt{k}}\left[\frac{1}{\sqrt{k}+\sqrt{k+1}}\frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k+1}-\sqrt{k}}\right]$$

$$\sum_{k=1}^{360} = \frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k+1}\sqrt{k}}$$

$$\sum_{k=1}^{360} = \sum\left[\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}\right]$$

$$\therefore \sum_{k=1}^{360} T_k = \frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \dots - \frac{1}{\sqrt{361}}$$

$= 1 - \frac{1}{19} = \frac{18}{19} = \frac{m}{n}$ (given)

⇒ m + n = 18 + 19 = 37

Sol 3: (B) $\sum_{k=1}^{100} \frac{k}{k^4+k^2+1} = \sum \frac{k}{(k^2+1)^2-k^2}$

$$= \sum \frac{k}{(k^2+1+k)(k^2+1-k)}$$

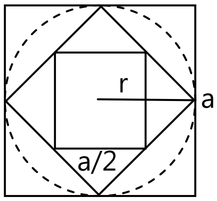
$$= \sum \frac{k}{2k}\left[\frac{1}{k^2+k+1} + \frac{1}{k^2-k+1}\right]$$

$$= \sum \frac{1}{2}\left[\frac{1}{k^2-k+1} - \frac{1}{k^2+k+1}\right]$$

$$= \sum \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{7} \dots \frac{1}{100^2+100+1}\right]$$

$$= \frac{1}{2}\left[\frac{100^2+100}{10101}\right] = \frac{5050}{10101}$$

Sol 4: (A)



Circle inscribed in 1st circle = r = a/2

In 2nd circle = $r_1 = \frac{a}{2\sqrt{2}}$

In 3rd circle = $\frac{a}{2 \times 2}$ this is G.P. with common ratio $\frac{1}{\sqrt{2}}$

$a_r = \frac{a}{2}\left(\frac{1}{\sqrt{2}}\right)^{n-1} = r\left(\frac{1}{2}\right)^{\frac{n-1}{2}} = r2^{\left(\frac{1-n}{2}\right)}$ [A]

Assertion Reasoning Type

Sol 5: (D) Statement-I: If $(a + b + c)^3 \le 27$ abc

3a + 4b + 5c = 12

Statement-II: ⇒ A.M. ≥ G.M. (True)

We Know A.M. ≥ G.M.

For three numbers a + b + c

$\Rightarrow \frac{a+b+c}{3} \ge (abc)^{1/3}$

$\Rightarrow (a + b + c)^3 \ge 27$ abc

Given $(a + b + c)^3 \le 27$ abc

⇒ a = b = c & $(a + b + c)^3 = 27$ abc

3a + 4a + 5a = 12 ⇒ a = b = c = 1

$\frac{1}{a^2} + \frac{1}{b^3} + \frac{1}{b^5} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3 \ne 10$

So statement-II is false.

Multiple Correct Choice Type

Sol 6: (A, B, C, D) $a_1 = 25$, $b_1 = 75$, $a_{100} + b_{100} = 100$

$a_1 + 99d_a + b_1 + 99d_b = 100$

$\Rightarrow d_a + d_b = 0$

$\Rightarrow d_a = -d_B$

(a) Hence a is correct

(b) $a_n + b_n = a_1 + (n-1)d_a + b_1 + (n-1)d_B$

$= a_1 + b_1 + (n-1)(d_a + d_B) = a_1 + b_1 = 100$

Hence, correct

(c) $(a_1 + b_1)(a_2 + b_2)(a_3 + b_3)$ ...

= 100, 100, 100 is in AP

(d) $\sum_{r=1}^{100}(a_r + b_r) = \sum_{r=1}^{100}(100) = 10000$

Sol 7: (B, C) sin (x – y), sin x, sin (x + y) are in HP

$\sin x = \frac{2\sin(x-y)\sin(x+y)}{\sin(x-y)+\sin(x+y)}$

$\Rightarrow 2\sin^2 x \cos y = 2\cos^2 y - 2\cos^2 x$

$\Rightarrow \sin^2 x(\cos y - 1) = \cos^2 y - 1$

$\Rightarrow \sin^2 x = 1 + \cos y = 2\cos^2 y/2$

$\Rightarrow \sin x \sec\left(\frac{y}{2}\right) = \sqrt{2}$

Sol 8: (A, B) Given series is a ar ar^2 ar^3...

Given that $ar - a = 6$ and $ar^3 - ar^2 = 54$

$\Rightarrow ar^2(r-1) = 54 \Rightarrow a(r-1) = 6$

Dividing these (2)

$r^2 = 9$

$r = \pm 3$

a = 3, for r = 3

ar = 9

$ar^2 = 27$

a = 3

sum = 39 [A]

for $r = -3$; $a = \frac{-3}{2}$

$ar = \frac{9}{2}$; $ar^2 = \frac{-27}{2}$

$\therefore$ sum $= \frac{-3}{2} + \frac{9}{2} - \frac{27}{2} = 3 - \frac{27}{2} = \frac{-21}{2}$

Sol 9: (A, C, D) $x^3 + px^2 + qx - 1$

Roots form increasing GP

Roots be $\frac{a}{r}$, a, ar

$\frac{a}{r} + a + ar = -p$

$\frac{a^2}{r} + a^2 r + a^2 = q$

$a^3 = 1 \Rightarrow a = 1$

$\left(r + 1 + \frac{1}{r}\right) = -p$ ... (i)

$\left(\frac{1}{r} + r + 1\right) = q \Rightarrow q = -p \Rightarrow p + q = 0$

$\frac{r + 1 + \frac{1}{r}}{3} \geq 1$ [AM ≥ GM]

$r + 1 + \frac{1}{r} \geq 3$

$p \in (-\infty, 3)$ [B is incorrect]

one root (a) is unity

one root is $\frac{1}{r}$ & other is r, so if 1 root is greater than 1 and other less than [ACD]

Sol 10: (B, D) log a, log b, log c, log $\frac{a}{2b}$, log $\frac{2b}{3c}$, log $\frac{3c}{a}$ are in AP

$a_1, a_2, a_3, a_4, a_5, a_6$

$2a_5 = a_4 + a_6$

$\frac{4b^2}{9c^2} = \frac{3c}{2b}$

$\frac{b}{c} = \frac{3}{2} \Rightarrow b = \frac{3c}{2}$

$a = \frac{3b}{2} = \frac{9c}{4}$

$a + b = \frac{15c}{4} > c$

$a + c = \frac{13c}{4} > \frac{3c}{2}$ (b)

$b + c = \frac{5c}{2} > \frac{9c}{4}$ (a)

Hence a, b, c can form Δ

log b – log a = log c – log b

2log b = log a + log c

$b^2 = ac$ ie a, b, c are in GP[B]

$a = a, b = ar, c = ar^2$

$18(a + b + c)^2 - 18a^2 - 18b^2 - 18c^2$

$= 18(2ab + 2bc + 2ac)$

$= 36(ab + bc + ac) > ab$ so A is incorrect

Sol 11: (A, B, C, D) $x^2 - 3x + A = 0$

$x_1 + x_2 = 3$

$x_1x_2 = A$

$x^2 - 12x + B = 0$

$x_3\, x_4 = B$

$x_3 + x_4 = 12$

$x_1 = a\,;\ x_2 = ar\,;\ x_3 = ar^2\,;\ x_4 = ar^3$

$a^2r = A$

$a(1 + r) = 3$

$a^2r^5 = B$

$ar^2(1 + r) = 12$

$r = \pm 2$

$a = 1, -3$

$A = a^2r = 1 \times 2 = 2$

$a^2r^5 = 2^5 = 32 = B$

$x_1 + x_3 = a(1 + r^2) = 5$

$x_2 + x_4 = ar\,(1 + r^2) = 2.5 = 10$

Previous Years' Questions

Sol 1: (A, B, D) Since, first and (2n – 1)the terms are equal. Let first term be x and $(2n - 1)^{th}$ term by y. whose middle term is t_n.

Thus in arithmetic progression ; $t_n = \dfrac{x+y}{2} = a$

In geometric progression : $\sqrt{xy} = b$

In harmonic progression ; $t_n = \dfrac{2xy}{x+y} = c$

$\Rightarrow b^2 = ac$ and $a \ge b \ge c$ (using A.M. ≥ G.M. ≥ HM)

Here, equality holds (ie, a = b = c) only if all terms are same.

Sol 2: (B, C, D) Let a_n denotes the length of side of the square S_n.

We are given a_n = length of diagonal of S_{n+1}.

$\Rightarrow a_n = \sqrt{2}a_{n+1} \Rightarrow a_{n+1} = \dfrac{a_n}{\sqrt{2}}$

This show that $a_1, a_2, a_3 \ldots$ Form a G.P. with common ratio $1/\sqrt{2}$

Therefore, $a_n = a_1\left(\dfrac{1}{\sqrt{2}}\right)^{n-1}$

$\Rightarrow a_n = 10\left(\dfrac{1}{\sqrt{2}}\right)^{n-1}$ $(Qa_1 = 10$ given)

$\Rightarrow a_n^2 = 100\left(\dfrac{1}{\sqrt{2}}\right)^{2(n-1)}$

$\Rightarrow \dfrac{100}{2^{n-1}} \le 1\ (\because\ a_n^2 \le 1 \text{ given})$

$\Rightarrow 100 \le 2^{n-1}$

This is possible for $n \ge 8$. So (b), (c), (d) are the answer.

Sol 3: (4) We have $S_k = \dfrac{\frac{k-1}{k!}}{1-\frac{1}{k}} = \dfrac{1}{(k-1)!}$

Now, $(k^2 - 3k + 1)\, S_k = \{(k - 2)(k - 1) - 1\} \times S_k$

$= \dfrac{1}{(k-3)!} - \dfrac{1}{(k-1)!} \Rightarrow \sum_{k=1}^{100} |(k^2 - 3k + 1)S_k|$

$= 1 + 1 + 2 - \left(\dfrac{1}{99!} + \dfrac{1}{98!}\right) = 4 - \dfrac{100^2}{100!}$

$\Rightarrow \dfrac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k + 1)S_k| = 4$

Sol 4: (0) $a_k = 2a_{k-1} - a_{k-2}$

$\Rightarrow a_1, a_2, \ldots, a_{11}$ are in AP

$\therefore \dfrac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = \dfrac{11a^2 + 35 \times 11d^2 + 10ad}{11} = 90$

$\Rightarrow 225 + 35d^2 + 150d = 90$

$\Rightarrow 35d^2 + 150d + 135 = 0$

$\Rightarrow d = -3, -\dfrac{9}{7}$

Given $a_2 < \dfrac{27}{2}$ $\quad\therefore d = -3$ and $d \ne -\dfrac{9}{7}$

$\Rightarrow \dfrac{a_1 + a_2 + \ldots + a_{11}}{11} = \dfrac{11}{2}[30 - 10 \times 3] = 0$

Sol 5: $a_1, a_2, a_3, \ldots A_{100}$ is an A.P.

$a_1 = 3,\ Sp = \sum_{i=1}^{p} a_1,\ 1 \le p \le 100$

$\dfrac{S_m}{S_n} = \dfrac{S_{5n}}{S_n} = \dfrac{\frac{5_n}{2}\left(6 + (5n - 1)d\right)}{\frac{n}{2}\left(6 - d + nd\right)}$

Sm is independent of n of 6 – d = 0 $\Rightarrow$ d = 6

$a_1 = 3$

$a_2 = 3 + 6 = 9$

$a_2 = 9$

Sol 6: (C) Let a and b are two numbers. Then,

$$A_1 = \frac{a+b}{3}; G_1 = \sqrt{ab}; H_1 = \frac{2ab}{a+b}$$

$$A_n = \frac{A_{n-1} + H_{n-1}}{2}, G_n = \sqrt{A_{n-1}H_{n-1}},$$

$$H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1} + H_{n-1}}$$

Clearly, $G_1 = G_2 = G_3 = ... = \sqrt{ab}$

Sol 7: (A) A_2 is A.M. of A_1 and H_1 and $A_1 > H_1$

$\Rightarrow A_1 > A_2 > H_1$

A_3 is A.M. of A_2 and H_2 and $A_2 > H_2$

$\Rightarrow A_2 > A_3 > A_4$

: : :

$\therefore A_1 > A_2 > A_3 >$.......

Sol 8: (B) As above $A_1 > H_2 > H_1$, $A_2 > H_3 > H_2$

$\therefore H_1 < H_2 < H_2 <$ ......

Sol 9: (B) Here, $V_r = \frac{r}{2}[2r + (r-1)(2r-1)] = \frac{1}{2}(2r^3 - r^2 + r)$

$$\therefore \Sigma V_r = \frac{1}{2}[2\Sigma r^3 - \Sigma r^2 + \Sigma r]$$

$$= \frac{1}{2}\left[2\left(\frac{n(n+1)}{2}\right)^2 - \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{12}[3n(n+1) - (2n+1) + 3]$$

$$= \frac{1}{12}n(n+1)(3n^2 + n + 2)$$

Sol 10: (D) $V_{r+1} - V_r = (r+1)^3 - r^3 - \frac{1}{2}[(r+1)^2 - r^2] + \frac{1}{2}(1)$

$= 3r^2 + 2r - 1$

$\therefore T_r = 3r^2 + 2r - 1 = (r + 1)(3r - 1)$

Which is a composite number.

Sol 11: (B) Since,

$T_r = 3r^2 + 2r - 1$

$\therefore T_{r+1} = 3(r + 1)^2 + 2(r + 1) - 1$

$\therefore Q_r = T_{r+1} - T_r = 3[2r + 1] + 2[1]$

$Q_r = 6r + 5$

Sol 12: (A, D) $S_n < \lim_{n\to\infty} S_n = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot\frac{1}{1 + k/n + (k/n)^2}$

$$= \int_0^1 \frac{dx}{1 + x + x^2} = \frac{\pi}{3\sqrt{3}}$$

Now, $T_n > \frac{\pi}{3\sqrt{3}}$ as $h\sum_{k=0}^{n-1} f(kh) > \int_0^1 f(x)dx > h\sum_{k=1}^{n} f(kh)$

Sol 13: (C) $b_1 = a_1$, $b_2 = a_1 + a_2$, $b_3 = a_1 + a_2 + a_3$, $b_4 = a_1 + a_2 + a_3 + a_4$

Hence b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P. nor in H.P.

Sol 14: (C) $t_n = c\{n^2 - (n - 1)^2\} = c(2n - 1)$

$$\Rightarrow t_n^2 = c^2(4n^2 - 4n + 1)$$

$$\Rightarrow \sum_{n=1}^{n} t_n^2 = c^2\left\{\frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n\right\}$$

$$= \frac{c^2 n}{6}\{4(n+1)(2n+1) - 12(n+1) + 6\}$$

$$= \frac{c^2 n}{3}\{4n^2 + 6n + 2 - 6n - 6 + 3\} = \frac{c^2}{3}n(4n^2 - 1)$$

Sol 15: (3) $\sum_{k=2}^{100}\left|(k^2 - 3k + 1)S_k\right|$

for k = 2 $|(k^2 - 3k + 1) S_k| = 1$

$$\sum_{k=3}^{100}\left|\frac{k-1}{(k-2)!} - \frac{k-1+1}{(k-1)!}\right|$$

$$\sum_{k=3}^{100}\frac{1}{(k-3)!} + \frac{1}{(k-2)!} - \frac{1}{(k-2)!} - \frac{1}{(k-1)!}$$

$$\sum_{k=3}^{100}\left(\frac{1}{(k-3)!} - \frac{1}{(k-1)!}\right)$$

$$S=1+\left(1-\frac{1}{2!}\right)+\left(\frac{1}{1!}-\frac{1}{3!}\right)+\left(\frac{1}{2!}-\frac{1}{4!}\right)+\left(\frac{1}{3!}-\frac{1}{5!}\right)+\left(\frac{1}{4!}-\frac{1}{6!}\right)+$$

$$....\left(\frac{1}{94!}-\frac{1}{96!}\right)+\left(\frac{1}{95!}-\frac{1}{97!}\right)+\left(\frac{1}{96!}-\frac{1}{98!}\right)+\left(\frac{1}{97!}-\frac{1}{99!}\right)$$

$$=2-\frac{1}{98!}-\frac{1}{99!}$$

$$\therefore E=\frac{100^2}{100!}+3-\frac{1}{98!}-\frac{1}{99.98!}$$

$$=\frac{100^2}{100!}+3-\frac{100}{99!}=\frac{100^2}{100.99!}+3-\frac{100}{99!}=3$$

Sol 16: (0) $a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_1, a_2, ..., a_{11}$ are in A.P.

$$\therefore \frac{a_1^2+a_2^2+....+a_{11}^2}{11}=\frac{11a^2+35\times 11d^2+10ad}{11}=90$$

$\Rightarrow 225 + 35d2 + 150d = 90$

$35d2 + 150d + 135 = 0 \Rightarrow d = -3, -9/7$

Given $a^2 < \frac{27}{2}$ $\therefore$ $d = -3$ and $d \neq -9/7 \Rightarrow$

$$\frac{a_1+a_2+....+a_{11}}{11}=\frac{11}{2}\left[30-10\times 3\right]=0$$

Sol 17: (B) $ax^2 + bx + c = 0 \Rightarrow x^2 + 6x - 7 = 0$

$\Rightarrow \alpha = 1, \beta = -7$

$$\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n=\sum_{n=0}^{\infty}\left(\frac{1}{1}-\frac{1}{7}\right)^n=7$$

Sol 18: (D) Corresponding A.P.

$\frac{1}{5}$,.........................$\frac{1}{25}$(20th term)

$$\frac{1}{25}=\frac{1}{5}+19d \qquad \Rightarrow d=\frac{1}{19}\left(\frac{-4}{25}\right)=-\frac{4}{19\times 25}$$

$$a_n<0$$

$$\frac{1}{5}-\frac{4}{19\times 25}\times(n-1)<0$$

$$\frac{19\times 5}{4}<n-1$$

$$n>24.75$$

Sol 19: (A, D) $S_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$

$$=\sum_{r=0}^{(n-1)}((4r+4)^2+(4r+3)^2-(4r+2)^2-(4r+1)^2)$$

$$=\sum_{r=0}^{(n-1)}(2(8r+6)+2(8r+4)) \;=\sum_{r=0}^{(n-1)}(32r+20)$$

$$= 16(n-1)n + 20n = 4n(4n + 1) =\begin{cases}1056 \text{ for } n=8\\ 1332 \text{ for } n=9\end{cases}$$

Sol 20: (6) $\frac{b}{a}=\frac{c}{b}=(\text{integer})$

$$b^2=ac\Rightarrow c=\frac{b^2}{a}$$

$$\frac{a+b+c}{3}=b+2$$

$$a+b+c=3b+6\Rightarrow a-2b+c=6$$

$$a-2b+\frac{b^2}{a}=6\Rightarrow 1-\frac{2b}{a}+\frac{b^2}{a^2}=\frac{6}{a}$$

$$\left(\frac{b}{a}-1\right)^2=\frac{6}{a}\Rightarrow a=6\,\text{only}$$

Sol 21: (9) Let seventh term be 'a' and common difference be 'd'

Given, $\frac{S_7}{S_{11}}=\frac{6}{11}\Rightarrow a=15d$

Hence, $130 < 15d < 140$

$\Rightarrow$ d = 9

Sol 22: (B) $\log(b_2) - \log(b_1) = \log(2)$

$\Rightarrow \frac{b_2}{b_1}=2 \Rightarrow$ b_1, b_2, ... are in G.P. with common ratio 2

$\therefore t = b_1 + 2b_1 + ... + 250\, b_1 = b_1 (251 - 1)$

$S = a_1 + a_2 + ... + a_{51}$

$$=\frac{51}{2}(a_1+a_{51}) \quad =\frac{51}{2}(b_1+b_2) \quad =\frac{51}{2}b_1(1+2^{50})$$

$$S-t=b_1\left(\frac{51}{2}+51\times 2^{49}-2^{51}+1\right)=b_1\left(\frac{53}{2}+2^{49}\times 47\right)$$

$\Rightarrow S > t$

$b_{101} = 2_{100}\, b_1$

$a_{101} = a_1 + 100\, d = 2\,(a_1 + 50d) - a_1$

$= 2a_{51} - a_1 = 2b_{51} - b_1 = (2 \times 2_{51} - 1)\, b_1 = (2_{51} - 1)\, b_1$

$\therefore b_{101} > a_{101}$