17. WAVE OPTICS

1. INTRODUCTION

There have always been controversies over the nature of light. Some theories believe light to be a wave whereas some believe it to be a particle. Newton, the greatest among the great, believed that light is a collection of particles. He believed that these particles travel from a source of light in straight lines when it is not under the influence of external forces. This was one of the strongest evidence of the particle nature of light.

A Dutch physicist named Huygens (1629 – 1695), suggested that light may have a wave nature. The apparent rectilinear propagation of light explained by Newton may be just due to the fact that the wavelength of light may be much smaller than the dimensions of these obstacles. This proposal remained a dump for almost a century. Newton's theory was then challenged by the Young's double slit experiment in 1801. A series of experiments on diffraction of light conducted by French scientist Fresnel were some of the activities that put an end to the particle nature of light and established the wave nature of light.

The twist came around when the wave nature of light failed to explain the photoelectric effect in which light again behaved as particles. This again brought up the question whether light had a wave or a particle nature and an acceptance was eventually reached that light is of dual nature – particle and wave. In this material, we will focus on the study of the wave nature of light.

Key point – Light waves need no material medium to travel. They can propagate in vacuum.

1.1 Nature of Light Waves

Light waves are transverse, i.e. disturbance of the medium is perpendicular to the direction of propagation of the wave. Hence they can be polarized. If a plane light wave is travelling in the x-direction, the electric field may be along the y or z direction or any other direction in y-z plane. The equation of such a monochromatic light wave can be written as $E = E_0 \sin\omega\left(t - x/v\right)$

The speed of light is generally denoted by c. When light travels in a transparent material, the speed is decreased by a factor μ, which is called as the refractive index of the material.

$$\mu = \frac{\text{speed of light in vaccum}}{\text{speed of light in the material}}$$

The frequency of visible light varies from about 3800×10^{11} Hz to about 7800×10^{11} Hz.

Colour	Wave-length
Red	620-780 nm
Orange	590-620 nm
Yellow	570-590nm

Colour	Wave-length
Green	500-570 nm
Blue	450-500 nm
Violet	380-450 nm

Light of a single wavelength is called monochromatic light.

Illustration 1: The refractive index of glass is 1.5. Find the speed of light in glass. **(JEE MAIN)**

Sol: $\mu = \dfrac{\text{speed of light in vaccum}}{\text{speed of light in the material}}$

Thus, speed of light in glass $= \dfrac{\text{speed of light in vaccum}}{\mu} = \dfrac{3.0 \times 10^{8}\,\text{ms}^{-1}}{1.5} = 2.0 \times 10^{8}\,\text{ms}^{-1}$

2. HUYGENS'S WAVE THEORY

It has following two basic postulates:

(a) Consider all the points on a primary wave-front to be the sources of light, which emit disturbance known as secondary disturbance.

(b) Tangent envelop to all secondary wavelets gives the position of the new wave-front.

Huygens' Principle may be stated in its most general form as follows:

Various points of an arbitrary surface, when reached by a wave front, become secondary sources of light emitting secondary wavelets. The disturbance beyond the surface result from the superposition of these secondary wavelets.

Huygens' Construction

Huygens, the Dutch physicist and astronomer of the seventeenth century, gave a beautiful geometrical description of wave propagation. We can guess that, he must have seen water waves many times in the canals of his native place Holland. A stick placed in water, oscillated up and down, becomes a source of waves. Since the surface of water is two dimensional, the resulting wave fronts would be circles instead of spheres. At each point on such a circle, the water level moves up and down. Huygens' idea is that we can think of every such oscillating point on a wave front as a new source of waves. According to Huygens' principle, what we observe is the result of adding up the waves from all these different sources. These are called secondary waves or wavelets. Huygens' Principle is illustrated in (Figure) as the simple case of a plane wave.

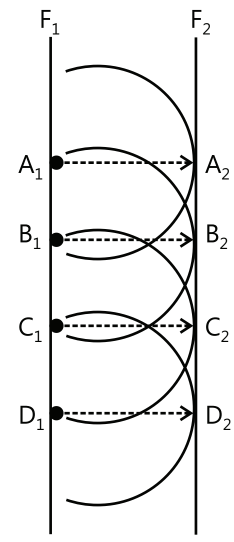


Figure 17.1

(a) At time t=0, we have a wave front F_1, F_1 separates those parts of the medium that are undisturbed from those where the wave has already reached.

(b) Each point on F_1 acts like a new source and sends out a spherical wave. After a time 't', each of these will have radius vt. These spheres are the secondary wavelets.

(c) After a time t, the disturbance would now have reached all points within the region covered by all these secondary waves. The boundary of this region is the new wavefront F_2. Notice that F_2 is a surface tangent to all the spheres. It is called the forward envelop of these secondary wavelets.

(d) The secondary wavelets from the point A_1 on F_1 touches F_2 at A_2. According to Huygens, $A_1 A_2$ is a ray. It is perpendicular to the wavefronts F_1 and F_2 and has length vt. This implies that rays are perpendicular to wavefronts. Further, the time taken for light to travel between two wavefronts is the same along any ray. In our examples, the speed 'v' of the wave has been taken to be the same at all points in the medium. In this case, we can say that the distance between two wavefronts is the same measured along any ray.

(e) This geometrical construction can be repeated starting with F_2 to get the next wavefront F_3 a time t later, and so on. This is known as Huygens' construction.

Huygens' construction can be understood physically for waves in a material medium, like the surface of water. Each oscillating particle can set its neighbors into oscillation, and therefore acts as a secondary source. But what if there is no medium, such as for light travelling in vacuum? The mathematical theory, which cannot be given here, shows that the same geometrical construction work in this case as well.

3. INTERFERENCE

When two waves of the same frequency move along the same direction in a medium, they superimpose and give rise to a phenomena called interference. Points of constructive interference have maximum intensity while points of destructive interference have minimum intensity.

3.1 Coherent and Incoherent Sources

Two light sources of light waves are coherent if the initial phase difference between the waves emitted by the sources remains constant with time. If it changes randomly with time, the sources are said to be incoherent. Two waves produce an interference pattern only if they originate from coherent sources.

3.2 Intensity and Superposition of Waves

If two waves $y_1 = A_1 \sin(\omega t)$ & $y_2 = A_2 \sin(\omega t + \theta)$ are superimposed, resultant wave is given by $y = R\sin(\omega t + \theta)$

Where $R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$ or $I = I_1 + I_2 + 2\left(\sqrt{I_1}\sqrt{I_2}\right)\cos\phi \left(\because I \propto A^2\right)$ and $\tan\theta = \dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi}$

For maxima, $\cos\phi = 1, \phi = 2n\pi \quad n = 0,1,2$

For minima, $\cos\phi = -1, \phi = (2n-1)\pi \quad n = 1,2,...$

If $I_1 = I_2 = I_0$; $I_1 = 4I_0\cos^2\dfrac{\phi}{2}$; $\quad \dfrac{I_{max}}{I_{min}} = \dfrac{(A_1+A_2)^2}{(A_1-A_2)^2} = \dfrac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}$

Note: Consider two coherent sources S_1 and S_2. Suppose two waves emanating from these two sources superimpose at point P. The phase difference between them at P is ϕ (which is constant). If the amplitude due to two individual sources at P is A_1 and A_2, then resultant amplitude at P will be, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

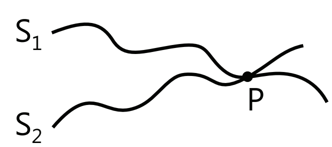


Figure 17.2

Similarly the resultant intensity at P is given by, $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$. Here, I_1 and I_2 are the intensities due to independent sources. If the sources are incoherent then resultant intensity at P is given by, $I = I_1 + I_2$

Illustration 2: Light from two sources, each of same frequency and in same direction, but with intensity in the ration 4:1 interfere. Find ratio of maximum to minimum intensity. **(JEE MAIN)**

Sol: Interference, amplitudes added and subtracted not the intensity, $A = A_1 + A_2$.

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 \left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right)^2 = \left(\frac{2+1}{2-1}\right)^2 = 9:1$$

3.3 Conditions for Interference

(a) Sources should be coherent i.e. the phase difference between them should be constant. For this, frequency of sources should be the same.

(b) The amplitudes of both the waves should be nearly equal so as to obtain bright and dark fingers of maximum contrast.

(c) The two sources should be very close to each other.

(d) The two sources of slits should be very narrow otherwise a broad source will be equivalent to a number of narrow sources emitting their own overlapping wavelets.

If the two sources are obtained from a single parent source by splitting the light into two narrow sources, they form coherent sources which produce sustained interference pattern due to a constant phase difference between the waves.

Illustration 3: In a Young's experiment, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9:1, find out: **(JEE MAIN)**

(i) The ratio of intensities

(ii) Amplitude of two interfering waves.

Sol: In Interference, $A^2 \propto I$, $A = A_1 \pm A_2$.

$$\frac{I_{max}}{I_{min}} = \frac{9}{1}; \qquad \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \frac{9}{1}; \qquad \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = \frac{3}{1}; \qquad \sqrt{I_1}+\sqrt{I_2} = 3\sqrt{I_1}-3\sqrt{I_2}$$

$$-2\sqrt{I_1} = -4\sqrt{I_2}; \quad \sqrt{\frac{I_1}{I_2}} = \frac{2}{1}; \qquad \frac{I_1}{I_2} = \frac{4}{1}; \left(I \propto a^2\right), \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}; \qquad \frac{a_1}{a_2} = \sqrt{\frac{4}{1}} = \frac{2}{1}$$

Illustration 4: Two coherent monochromatic light beams of intensity I & 4I are superimposed. What is the max & min possible intensities in the resulting wave? **(JEE MAIN)**

Sol: In Interference, $A^2 \propto I$, $A = A_1 \pm A_2$.

$$I_{max} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2 = \left(\sqrt{I}+\sqrt{4I}\right)^2 = \left(3\sqrt{I_2}\right)^2 = 9I$$

$$I_{min} = \left(\sqrt{I_1}-\sqrt{I_2}\right)^2 = \left(\sqrt{I}-\sqrt{4I}\right)^2 = I$$

3.4 Young's Double Slit Experiment

The experiment consists of a parallel beam of monochromatic light from slit S which is incident on two narrow pinhole or slits S_1 and S_2 separated by a small distance d. The wavelets emitted from these sources superimpose at the screen placed in front of these slits to produce an alternate dark and bright fringe pattern at points on the screen depending upon whether these waves reach with a phase difference $\phi = (2n-1)\pi$ producing destructive interference or $\phi = 2n\pi$ producing constructive interference respectively. If the screen is placed at a perpendicular distance D from the middle point of the slits, the point O on the screen lies at the right bisector of $S_1 S_2$ and is equidistant from S_1 and S_2. The intensity at O is maximum. Consider a point P located at a distance x_n from O on the screen as shown in the figure. The path difference of waves reaching at point P from S_2 and S_1 is given by Path difference $= S_2P - S_1P = \frac{x_n d}{D}$

$S_1P = XP$

$S_2P = S_2X + XP$

$\Rightarrow S_2P - S_1P = S_2X = d\sin\theta$

$\Rightarrow$ path difference $= d\sin\theta$

As θ is small $\sin\theta \approx \tan\theta = \frac{x_n}{D} \{\because D >> d\}$

$\therefore S_2P - S_1P = \frac{x_n d}{D}$

The point P will be bright or of a maximum intensity when the path difference is an integral multiple of wavelength λ or $\phi = 2n\pi = n\lambda$; $\therefore$ $S_2P - S_1P = \frac{x_n d}{D} = n\lambda$

The bright fringes are thus observed at distance

$x_1 = \frac{\lambda D}{d}, x_2 = \frac{2\lambda D}{d}, x_3 = \frac{3\lambda D}{d}, ..., x_n = \frac{n\lambda D}{d}$

The distance between consecutive bright fringes,

$\beta = \frac{n\lambda D}{d} - (n-1)\frac{\lambda D}{d} = \frac{\lambda D}{d}$

The point P will be dark or of minimum intensity when the path difference is an odd multiple of half wavelength or $\phi - (2n-1)\pi$;

$\therefore \quad S_2P - S_1P = \frac{x_n d}{D} = n\lambda$

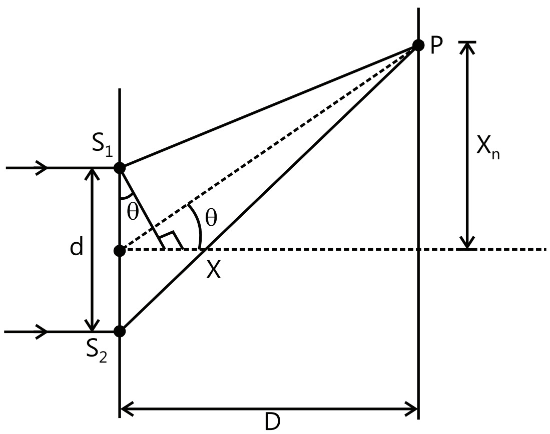


Figure 17.3

The bright fringes are thus observed at distances

$x_1 = \frac{\lambda D}{d}, x_2 = \frac{2\lambda D}{d}, x_3 = \frac{3\lambda D}{d}, ..., x_n = \frac{n\lambda D}{d}$

The distance between consecutive bright fringes, $\beta = \frac{n\lambda D}{d} - (n-1)\frac{\lambda D}{d} = \frac{\lambda D}{d}$

The point P will be dark or of minimum intensity when the path difference is an odd multiple of half wavelength or $\phi = (2n-1)\pi$; $\therefore$ $S_2P - S_1P = \frac{x_n d}{D} = (2n-1)\frac{\lambda}{2}$

Where n is an integer $\therefore x_n = (2n-1)\frac{\lambda D}{2d}$.

The dark fringes will be observed at distance $x_1' = \frac{\lambda D}{2d}, x_2' = \frac{3\lambda D}{2d}, x_3' = \frac{5\lambda D}{2d}, ..., x_n' = \frac{(2n-1)\lambda D}{2d}$

Fringe width: The spacing between any two consecutive bright or two dark fringes is equal and is called the fringe width.

The distance between two consecutive dark fringes $=\beta=\frac{(2n-1)\lambda D}{2d}-\frac{(2n-3)\lambda D}{2d}=\frac{\lambda D}{d}$

$\therefore$Fringe width $=\beta=\frac{\lambda D}{d}$

If a thin transparent plate of thickness t and refractive index μ is introduced in the path S_1P of one of the interfering waves, the entire fringes pattern is shifted through a constant distance. The path S_1P in air is increased to an air path equal to $S_1P+(\mu-1)t$

$\therefore$ The path difference $\delta=S_2P-\left[S_1P+(\mu-1)t\right]=S_2P-S_1P-(\mu-1)t=\frac{x_n d}{D}-(\mu-1)t$

For nth maxima at a distance x'_n $=\frac{x_n.d}{D}-(\mu-1)t=n\lambda;\quad x_n=\frac{D}{d}\left[n\lambda+(\mu-1)t\right]$

Thus, when a thin transparent plate of thickness t and refractive index μ is introduced in one of the paths of the waves, the path difference changes by $\frac{xd}{D}$.

$\therefore\frac{xd}{D}=(\mu-1)t;\ \ x=\frac{D}{d}(\mu-1)t$

$\therefore$The central maxima shifts by a distance equal to $\frac{D(\mu-1)t}{d}$.

MASTERJEE CONCEPTS

In Young's double slit experiment, it is important to note that energy is just redistributed over the surface of screen. It is still conserved! More energy is taken by points near bright fringes whereas dark fringes have almost no energy.

B Rajiv Reddy (JEE 2012, AIR 11)

Illustration 5: S_1 and S_2 are two coherent sources of frequently 'f' each. $(\theta_1=\theta_2=0°)$ $V_{sound}=330m/s$. Find f

(i) So that there is constructive interference at 'P'

(ii) So that there is destructive interference at 'P' **(JEE MAIN)**

Sol: Path difference for constructive and destructive interference must be λ and $\frac{\lambda}{2}$ respectively.

For constructive interference,

$K\Delta x=2n\pi$; $\frac{2\pi}{\lambda}\times 2=2n\pi$; $\lambda=\frac{2}{n}$; $V=\lambda f\Rightarrow V=\frac{2}{n}f$; $f=\frac{330}{2}\times n$ For destructive interference,

$K\Delta x=(2n+1)\pi$; $\frac{2\pi}{\lambda}.2=(2n+1)\pi$; $\frac{1}{\lambda}=\frac{(2n+1)}{4}$; $f=\frac{V}{\lambda}=\frac{330\times(2n+1)}{4}$

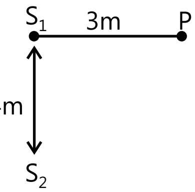


Figure 17.4

Illustration 6: In a Young's double slit experiment, the separation between the slits is 0.10 mm, the wavelength of light used is 600 nm and the interference pattern is observed on a screen 1.0 m away. Find the separation between the successive bright fringes. **(JEE MAIN)**

Sol: Path difference needs to be λ.

The separation between the successive bright fringes is, $\omega = \frac{D\lambda}{d} = \frac{1.0\text{m} \times 600 \times 10^{-9}\text{m}}{0.10 \times 10^{-3}\text{m}} = 6.0 \times 10^{-3}\text{m} = 6.0\text{mm}$

Assumptions: 2. Since d<<D, we can assume that intensities at P due to independent sources S_1 and S_2 are almost equal. or $I_1 = I_2 = I_0$ (say)

Illustration 7: When a plastic thin film of refractive index 1.45 is placed in the path of one of the interfering waves, then the central fringe is displaced through width of five fringes. Find the thickness of the film, if the wavelength of light is 5890 A. **(JEE MAIN)**

Sol: Path difference due to introducing of thin film.

$$X_0 = \frac{\beta}{\lambda}(\mu - 1)t \Rightarrow 5p = \frac{\beta(0.45)t}{5890 \times 10^{-10}} \qquad \therefore \qquad t = \frac{5 \times 589010^{-10}}{0.45} = 6.544 \times 10^{-4}\text{cm}$$

Illustration 8: Laser light of wavelength 630 nm, incident on a pair of slits produces an interference pattern in which bright fringes are separated by 8.1 mm. A second light produces an interference pattern in which the bright fringes are separated by 7.2 mm. Find the wavelength of the second light. **(JEE ADVANCED)**

Sol: The separation between the successive bright fringes, $\beta \propto \lambda$.

$$\lambda_1 = 630\text{nm} = 630 \times 10^{-9}\text{m}; \qquad \beta_1 = 8.1\text{mm} = 8.1 \times 10^{-3}\text{m}; \qquad \beta_2 = 7.2\text{mm} = 7.2 \times 10^{-3}\text{m}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{\beta_2}{\beta_1}; \qquad \lambda_2 = \frac{\beta_2}{\beta_1}\lambda_1; \qquad \lambda_2 = \frac{7.2 \times 10^{-3}}{8.1 \times 10^{-3}} \times 630 \times 10^{-9} = \frac{8}{9} \times 630 \times 10^{-9} = 560 \times 10^{-9} = 560\text{nm}$$

Illustration 9: In a Young's double slit experiment, the two slits are illuminated by light of wavelength 5890 Angstrom and the distance between the fringes obtained on the screen is $0.2°$. If the whole apparatus is immersed in water, the angular fringe width will be: (the refractive index of water is $4/3$). **(JEE ADVANCED)**

Sol: Angular fringe width, $\omega \propto \lambda$.

$$\omega_a = \lambda/d; \qquad \therefore \qquad \omega_a \propto \lambda \Rightarrow \frac{(\omega_a)_{water}}{\omega_a} = \frac{\lambda_{water}}{\lambda} \qquad \Rightarrow \frac{(\omega_a)_{water}}{\omega_a} = \frac{\lambda}{\mu_{water}\lambda} \qquad \Rightarrow (\omega_a)_{water} = 0.15°$$

Illustration 10: In a YDSE, D=1m, d=1mm and $\lambda = 1/2\text{mm}$ **(JEE ADVANCED)**

(i) Find the distance between the first and the central maxima of the screen.

(ii) Find the no. of maxima and minima obtained on the screen.

Sol: Here, $\sin\theta << 1$, not applicable. Hence $\Delta P = d\tan\theta = \frac{dy}{D}$ is used.

(i) D>>d Hence $\Delta P = d\sin\theta$; $\frac{d}{\lambda} = 2$,

Clearly, $n << \frac{d}{\lambda} = 2$ is not possible for any value of n.

Hence $\Delta p = \frac{dy}{D}$ cannot be used.

For 1st maxima, $\Delta p = d\sin\theta = \lambda \Rightarrow \sin\theta = \frac{\lambda}{d} = \frac{1}{2}; \Rightarrow \theta = 30°$

Hence, $y = D\tan\theta = \frac{1}{\sqrt{3}}$ meter

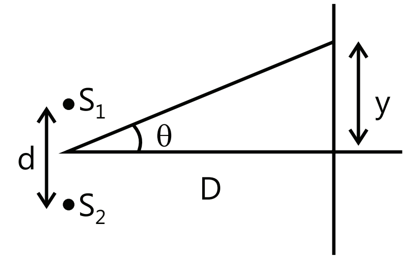


Figure 17.5

(ii) Maximum path difference

$\Delta P_{max} = d = 1mm$

$\Rightarrow$ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$

And highest order minima $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total no. of maxima $= 2n_{max} + 1^* = 5$ *(central maxima)

Total no. of minima= $= 2n_{min} = 4$

Illustration 11: Monochromatic light of wavelength 5000 A° is used in a Y.D.S.E., with slit-width, d=1mm, distance between screen and slits, D is 1m. If the intensities at the two slits are, $I_1 = 4I_0, I_2 = I_0,$ find **(JEE ADVANCED)**

(i) Fringe width β

(ii) Distance of 5th minima from the central maxima on the screen.

(iii) Intensity at $y = \frac{1}{3}$mm

(iv) Distance of the 1000th maxima from the central maxima on the screen.

(v) Distance of the 5000th maxima from the central maxima on the screen.

Sol: Refer to the formulas:-

(i) $\beta = \frac{\lambda D}{d} = \frac{5000 \times 10^{-10} \times 1}{1 \times 10^{-3}} = 0.5mm$

(ii) $y = (2n-1)\frac{\lambda D}{2d}, n = 5 \Rightarrow y = 2.25mm$

(iii) At $y = \frac{1}{3}mm, y << D$ Hence $\Delta p = \frac{d.y}{D}$

$\Delta\phi = \frac{2\pi}{\lambda}\Delta p = 2\pi\frac{dy}{\lambda D} = \frac{4\pi}{3}$

Now resultant intensity

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi;\ 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi = 5I_0 + 4I_0\cos\frac{4\pi}{3} = 3I_0$

(iv) $\frac{d}{\lambda} = \frac{10^{-3}}{0.5 \times 10^{-6}} = 2000$

n=1000 is not <<2000

Hence now Δp =d sin θ must be used

Hence, d sin $\theta = n\lambda = 1000\lambda$ $\Rightarrow \sin\theta = 1000\frac{\lambda}{d} = \frac{1}{2}$ $\Rightarrow \theta = 30°$

Y=D tan $\theta = \frac{1}{\sqrt{3}}$ meter

(v) Highest order maxima $n_{max} = \left[\frac{d}{\lambda}\right] = 2000$

Hence, n=5000 is not possible

Intensity variation on screen: If I_0 is the intensity of light beam coming from each slit, the resultant intensity at a point where they have a phase difference of ϕ is

$$I = 4I_0 \cos^2 \frac{\phi}{2}, \text{where} \phi = \frac{2\pi(d\sin\theta)}{\lambda}$$

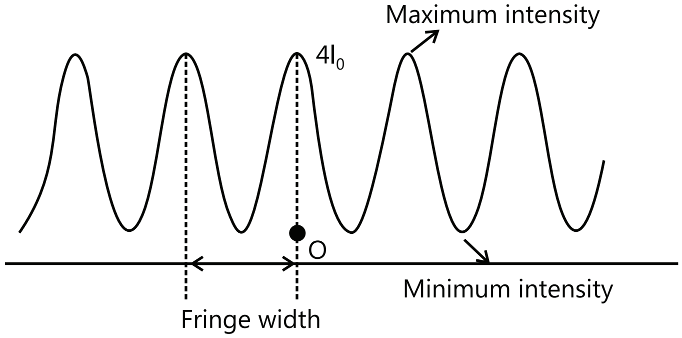


Figure 17.6

For interference in reflected rays

When $\mu_2 > \mu_1, \mu_3;$ condition for

(i) Maxima: $2t\mu_2 \cos r = \left(n - \frac{1}{2}\right)\lambda,\ n = 1, 2, ...$

(ii) Minima: $2t\mu_2 \cos r = n\lambda,\ n = 1, 2, ...$

When μ_2 is in between μ_1 & μ_2, condition for

(i) Maxima: $2t\mu_2 \cos r = n\lambda,\ n = 1, 2, ...$

(ii) Minima: $2t\mu_2 \cos r = \left(n - \frac{1}{2}\right)\lambda,\ n = 1, 2, ...$

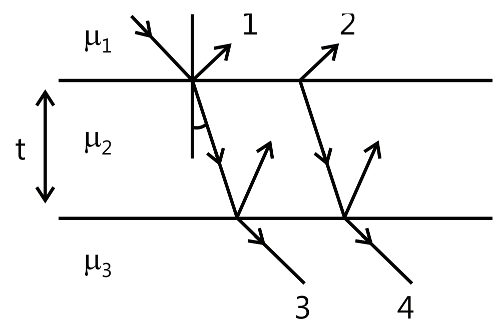


Figure 17.7

For interference in transmitted rays, 3 and 4 conditions are interchanged for maxima and minima in both cases of μ_2.

Illustration 12: When a plastic thin film of refractive index 1.45 is placed in the path of one of the interfering waves then the central fringe is displaced through width of five fringes. Find the thickness of the film, if the wavelength of light is 5890 Å. **(JEE MAIN)**

Sol: Path difference due to introducing of thin film.

$$\therefore X_0 = \frac{\beta}{\lambda}(\mu - 1)t \Rightarrow 5p = \frac{\beta(0.45)t}{5890 \times 10^{-10}}; \quad \therefore \quad t = \frac{5 \times 5890/10^{-10}}{0.45} = 6.544 \times 10^{-4}\,\text{cm}$$

Useful tips: If two slits have unequal sizes (they correspond to intensity). The intensity of the resultant is

$$I = \left(\sqrt{I_1}\right)^2 + \left(\sqrt{I_2}\right)^2 + \left(2\sqrt{I_1 I_2}\right)\cos\phi$$

$$I = I_1 + I_2 + \left(2\sqrt{I_1 I_2}\right)\cos\phi = k\left(S_1 + S_2 + 2\sqrt{S_1 S_2}\cos\phi\right)$$

Where S_1 & S_2 is the size of slits

Coherence length, $I_{coh} = \frac{\lambda^2}{\Delta\lambda}$; Coherence radius $\rho_{coh} = \frac{\lambda^2}{\phi} \quad \beta = \frac{\phi}{2}$

3.5 Optical Path

Actual distance travelled by light in a medium is called geometrical path (Δx). Consider a light wave given by the equation $E = E_0 \sin(\omega t - kx + \phi)$

If the light travels by Δx, its phase changes by $k\Delta x = \frac{\omega}{v}\Delta x$, where ω, the frequency of light does not depend on the medium, but v, the speed of light depends on the medium as $v = \frac{c}{\mu}$.

Consequently, change in phase $\Delta\phi = k\,\Delta x = \frac{w}{c}(\mu\Delta x)$

It is clear that a wave travelling a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum i.e. a path length of Δx in medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.

The quantity $\mu\Delta x$ is called the optical path length of light, Δx_{opt}. And in terms of optical path length, phase difference would be given by. $\Delta\phi = \frac{\omega}{c}\Delta x_{opt} = \frac{2\pi}{\lambda_0}\Delta x_{opt}$

where λ_0 =wavelength of light in vacuum.

However in terms of the geometrical path length Δx,

$$\Delta\phi = \frac{\omega}{c}(\mu\Delta x) = \frac{2\pi}{\lambda}\Delta x$$

Where λ = wavelength of light in the medium $\left(\lambda = \frac{\lambda_0}{\mu}\right)$.

MASTERJEE CONCEPTS

Optical path must always be linked to phase of wave, so that it's more convincing and useful. Only learning manually will make it confusing and annoying.

If a material of thickness t interrupts the path of light and distance measured from position of an end of material, then the phase of wave which is found at a distance x(<t) through the material will be same to phase at distance (mu), if there was no material.

Fringe width (w) is the distance between two successive maximas or minimas. It is given by, $w = \frac{\lambda D}{d}$; or $w \propto \lambda$ .Two conclusions can be drawn from this relation:-

(i) If a YDSE apparatus is immersed in a liquid of refractive index μ, then wavelength of light and hence, fringe width decreases μ times.

(ii) If white light is used in place of a monochromatic light then coloured fringes are obtained on the screen with red fringes of larger size than that of violet, because $\lambda_{red} > \lambda_{violet}$.

Vaibhav Gupta (JEE 2009, AIR 54)

Illustration 13: The wavelength of light coming from a sodium source is 589 nm. What will be its wavelength in water? Refractive index of water =1.33. **(JEE MAIN)**

Sol: The wavelength in water is $\lambda = \lambda_0/\mu$, where λ_0 is the wavelength in vacuum and μ is the refractive index of water. Thu $\lambda = \frac{589}{1.33} = 443\text{nm}$

3.6 Interference from Thin Films

Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble. The various colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film. Consider a film of uniform thickness t and index of refraction μ as shown in the figure. Let us assume that the rays travelling in air are nearly normal to the two surfaces of the film. To determine whether the reflected rays interfere constructively or destructively, we first note the following facts.

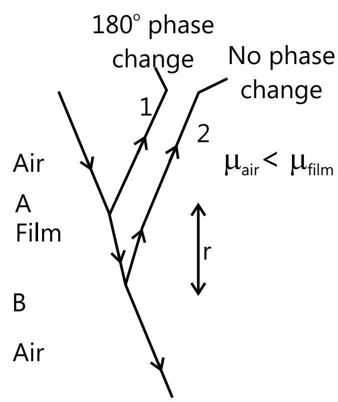


Figure 17.8

(i) The wavelength of light in a medium whose refractive index is μ is, $\lambda_\mu = \frac{\lambda}{\mu}$ Where λ is the wavelength of light in vacuum (on air)

(ii) If a wave is reflected from a denser medium, it undergoes a phase change of $180°$. Let us apply these rules to the film shown in figure. The path difference between the two rays. 1 and 2 is 2t while the phase difference between them is $180°$. Hence, condition of constructive interference will be, $2t = (2n-1)\frac{\lambda\mu}{2}$ or, $2\mu t = \left(n - \frac{1}{2}\right)\lambda$ as $\lambda_\mu = \frac{\lambda}{\mu}$

Similarly, condition of destructive interference will be $2\mu t = n\lambda$; n=0, 1, 2, ...

Illustration 14: Find the minimum thickness of a film which will strongly reflect the light of wavelength 589 nm. The refractive index of the material of the film is 1.25. **(JEE MAIN)**

Sol: Path difference due to introducing of thin film.

For strong reflection, the least optical path difference introduced by the film should be $\lambda/2$. The optical path difference between the waves reflected from the two surface of the film is $2\mu d$.

Thus, for strong reflection, $2\mu d = \lambda/2$ or, $d = \frac{\lambda}{4\mu} = \frac{589nm}{4 \times 1.25} = 118nm.$

3.7 YDSE with Glass Slab

Path difference produced by a slab

Consider two light rays 1 and 2 moving in air parallel to each other. If a slab of refractive index μ and thickness t is inserted between the path of one of the rays then a path difference $\Delta x = (\mu - 1)t$ is produced among them. This can be shown as under,

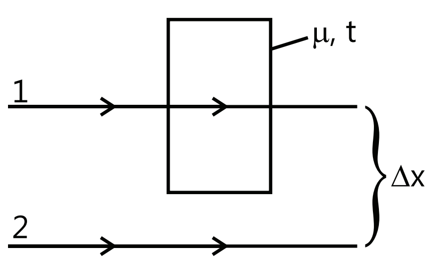


Figure 17.9

Speed of light in air = c

Speed of light in medium $= \frac{c}{\mu}$ Time taken by ray 1 to cross the slab, $t_1 = \frac{t}{c/\mu} = \frac{\mu t}{c}$ and time taken by ray 2 cross the same thickness t in air will be, $t_2 = \frac{t}{c}$ as $t_1 > t_2$

Difference in time $\Delta t = t_1 - t_2 = (\mu - 1)\frac{t}{c}$

Optical path length: Now we can show that a thickness t in a medium of refractive index μ is equivalent to a length μt in vacuum (or air). This is called optical path length. Thus,

Optical path length= μt

Shifting of fringes: Suppose a glass slab of thickness t and refractive index μ is inserted onto the path of the ray emanating from source S_1. Then, the whole fringe pattern shifts upwards by a distance $\frac{(\mu-1)tD}{d}$. This can be shown as under Geometric path difference between S_2P and S_1P is, $\Delta x_1 = S_2P - S_1P = \frac{yd}{D}$ Path difference produced by the glass slab $\Delta x_2 = (\mu-1)t$

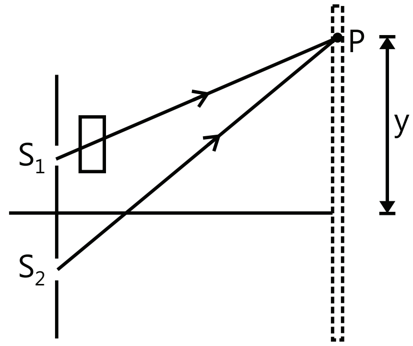


Figure 17.10

Note: Due to the glass slab, path of ray 1 gets increased by Δx_2. Therefore, net path difference between the two rays is,

$$\Delta x = \Delta x_1 - \Delta x_2 \text{ or, } \quad \Delta x = \frac{yd}{D} - (\mu-1)t$$

For n^{th} maxima on upper side, or, $\frac{yd}{D} - (\mu-1)t = n\lambda$;

$$\therefore \qquad y = \frac{n\lambda D}{d} + \frac{(\mu-1)tD}{d}$$

Earlier, it was $\frac{n\lambda D}{d}$; Shift $= \frac{(\mu-1)tD}{d}$

Following three points are important with regard to Eq. above

(a) Shift is independent of n, (the order of the fringe), i.e.,

Shift of zero order maximum = shift of 7^{th} order maximum

Shift of 5^{th} order maximum = shift of 9^{th} order minimum and so on.

(b) Shift in independent of λ, i.e., if white light is used then,

Shift of red colour fringes = shift of violet colour fringes.

(c) Number of fringes shifted = $\frac{\text{shift}}{\text{fringes width}} = \frac{(\mu-1)tD/d}{\lambda D/d} = \frac{(\mu-1)t}{\lambda}$

These numbers are inversely proportional to λ. This is because the shift is the same for all colours but the fringe width of the colour having smaller value of λ is small, so more number of fringes of this colour will shift.

Illustration 15: In a YDSE with d=1mm and D=1m, slabs of $(t = 1\mu m, \mu = 3)$ and $(t = 0.5\,\mu m, \mu = 2)$ are introduced in front of the upper slit and the lower slit respectively. Find the shift in the fringes pattern. **(JEE MAIN)**

Sol: Path difference due to introducing of thin film.

Optical path for light coming from upper slit S_1 is $S_1P + 1\mu m(2-1) = S_2P + 0.5\mu m$

Similarly optical path for light coming from S_2 is $S_2P + 0.5\mu m(2-1) = S_2P + 0.5\mu m$

Path difference $\Delta p = (S_2P + 0.5\mu m) - (S_1P + 2\mu m) = (S_2P - S_1P) - 1.5\mu m = \frac{yd}{D} - 1.5\mu m$

For central bright $\Delta p = 0 \Rightarrow y = \frac{1.5\mu m}{1mm} \times 1m = 1.5mm.$

The whole pattern is shifted by 1.5mm upwards.

Illustration 16: Bichromatic light is used in YDSE having wavelength λ_1 = 400 mm and λ_2 = 700 mm. Find minimum order of λ_1 which overlaps with λ_2. **(JEE ADVANCED)**

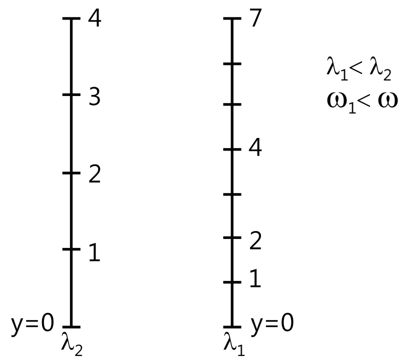


Figure 17.11

Sol: Fringe width depends on wavelength.

Let n_1 bright find of λ_1 overlaps with λ_2. Then, $\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$ Or, $\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{700}{400} = \frac{7}{4}$. The ratio $\frac{n_1}{n_2} = \frac{7}{4}$ implies that 7th bright fringes of λ_1 will overlap with 4th bright fringes of λ_2. Similarly 14th of λ_1 will overlap with 8th of λ_2 and so on.

So the minimum order of λ_1 which overlaps with λ_2 is 7.

Illustration 17: In YDSE, find the thickness of a glass slab $(\mu = 1.5)$ which should be placed before the upper slit S_1 so that the central maximum now lies at a point where 5th bright fringe was lying earlier (before inserting the slab). Wavelength of light used is 5000 Å. **(JEE MAIN)**

Sol: Path difference due to introducing of thin film.

According to the question, Shift = 5 (fringe width)

$$\therefore \frac{(\mu-1)tD}{d} = \frac{5\lambda D}{d} \therefore y = \frac{5\lambda}{\mu-1} = \frac{25000}{1.5-1} = 50{,}000 \text{ Å}$$

Tip: If the light reaching P is direct (not reflected) from two sources then P will be a bright fringe if the path difference $= n\lambda$

If the light reaching P after reflection forms a bright fringe (at P) then path difference $= (2n+1)\frac{\lambda}{2}$ $\rightarrow$ because the reflection causes an additional path difference of $\frac{\lambda}{2}$ (or phase difference = π rad.) If the interference occurs due to reflected light, central fringe (or ring in Newton's rings) will be dark. If the interference occurs due to transmitted light, central fringe (or ring in Newton's rings) will be bright.

3.8 YDSE with Oblique Incidence

In YDSE, ray is incident on the slit at an inclination of θ_0 to the axis of symmetry of the experimental set-up for points above the central point on the screen, (say for P_1)

$$\Delta p = d\sin\theta_0 + (S_2P_1 - S_1P_1)$$

$$\Rightarrow \Delta p = d\sin\theta_0 + d\sin\theta_1 \quad (\text{if } d << D)$$

and for points below O on the screen, (say for P_2)

$$\Delta p = \left|(d\sin\theta_0 + S_2P_2) - S_1P_2\right| = \left|d\sin\theta_0 - (S_1P_2 - S_2P_2)\right|$$

$$\Rightarrow \Delta p = \left|d\sin\theta_0 - d\sin\theta_2\right| \quad (\text{if } d << D)$$

We obtain central maxima at a point where, $\Delta p = 0$.

$(d\sin\theta_0 - d\sin\theta_2) = 0$ or, $\theta_2 = \theta_0$.

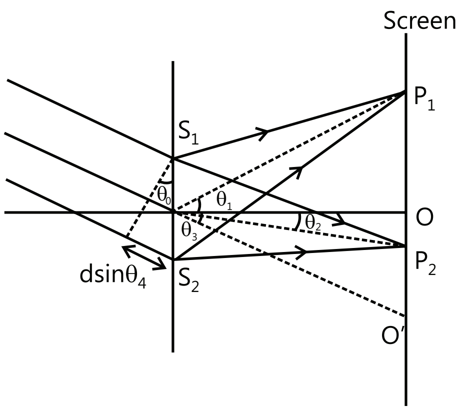


Figure 17.12

This corresponds to the point O' in the diagram. Hence, we finally have the path difference as

$$\Delta p = \begin{cases} d\left(\sin\theta_0 + \sin\theta\right) & - \quad \text{for points above O} \\ d\left(\sin\theta_0 - \sin\theta\right) & - \quad \text{for points between O \& O'} \\ d\left(\sin\theta - \sin\theta_o\right) & - \quad \text{for points below O'} \end{cases}$$

Illustration 18: In a YDSE with D=1m, d=1mm, light of wavelength 500 nm is incident at an angle of $0.57°$ w.r.t. the axis of symmetry of the experimental set up. If the centre of symmetry of screen is O as shown, find:

(i) The position of central maxima.

(ii) Intensity at point O in terms of intensity of central maxima I_0.

(iii) Number of maxima lying between O and the central maxima. **(JEE MAIN)**

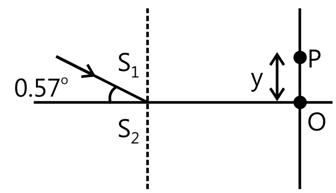


Figure 17.13

Sol: Path difference at central maxima =0.

(i) $\theta = \theta 0 = 0.57° \Rightarrow D\tan\theta = -D\theta = -1 \text{ meter} \times \left(\frac{0.57}{57}\text{rad}\right) \Rightarrow y = -1\text{cm}$

(ii) For point 0, $\theta = 0$

Hence, $\Delta p = d\sin\theta_0 \simeq d\theta_0 = 1\text{mm} \times \left(10^{-2}\text{rad}\right) = 10{,}000\text{nm} = 20 \times \left(500\text{nm}\right)$

Hence point O correspond to 20th maxima $\Rightarrow$ intensity at O=I_0

(iii) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima.

4. DIFFRACTION

The phenomenon of bending of light around the corners of an obstacle or an aperture into the region of the geometrical shadow of the obstacle is called diffraction of light. The diffraction of light is more pronounced when the dimension of the obstacle/aperture is comparable to the wavelength of the wave.

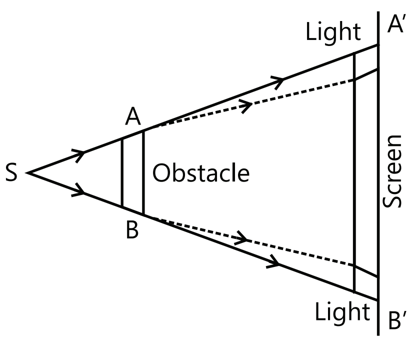


Figure 17.14

4.1 Diffraction of Light Due to Single Slit

Diverging light from monochromatic source S is made parallel after refraction through convex lens L_1. The refracted light from L_1 is propagated in the form of plane wave front WW'. The plane wave front WW' is incident on the slit AB of width 'd'. According to Huygens' Principle, each point of slit AB acts as a source of secondary disturbance of wavelets.

Path difference: To find the path difference between the secondary wavelets originating from corresponding points A and B of the plane wave front, draw AN perpendicular on BB'. The path difference between these wavelets originating from A and B is BN.

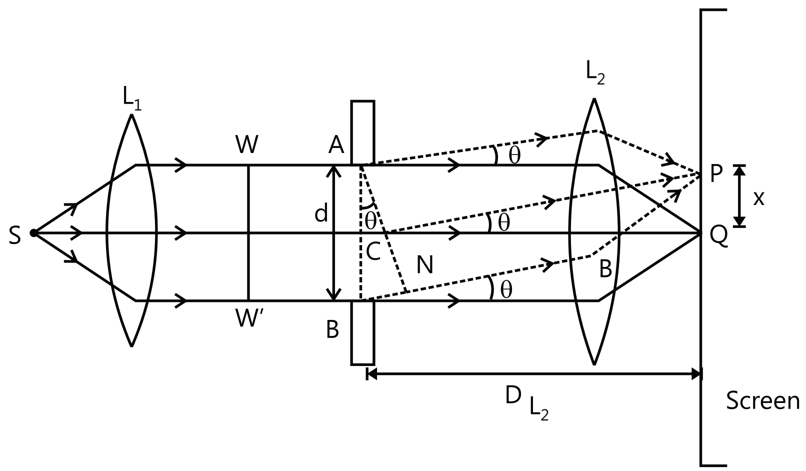


Figure 17.15

From $\Delta BAN, \frac{BN}{AB} = \sin\theta$ Or, $BN = AB\sin\theta$ $\therefore$Path difference, BN=d $\sin\theta \simeq d\theta$ $(\because \theta \text{ is small})$

(a) **For Minima:** If the path difference is equal to one wavelength i.e., BN= $d\sin\theta = \lambda$, position P will be of minimum intensity. Hence, for first minima, $d\sin\theta_1 = \lambda$

Or $\sin\theta_1 = \lambda/d$ ...(i)

Or $\theta_1 = \lambda/d$ $(\because \theta_1 \text{ is very small})$ ...(ii)

Similarly, if BN= 2λ,

Thus, for second minima, $d\sin\theta_2 = 2\lambda$ Or $\sin\theta_2 = 2\lambda/d$; $\because \sin\theta_2 = \theta_2$ Or $\theta_2 = 2\lambda/d$

In general, for minima, $d\sin\theta_m = m\lambda$ Or $\sin\theta_m = m\lambda/d$

Since θ_m is small, so sin $\theta_m = \theta_m$ $\therefore$ $\theta_m = \frac{m\lambda}{d}$ (here θ we use is in radians)

Where, θ_m is the angle giving direction of the m^{th} order minima and m=1,2,3,.... Is an integer.

Note: Condition for minima $\rightarrow$ Path difference between two waves should be $m\lambda$, where m is an integer.

(b) **For secondary maxima:** If path difference, BN= d sin θ is an odd multiple of $\lambda/2$,

i.e. $d\sin\theta_m = \left(\frac{2m+1}{2}\right)\lambda$ or $\sin\theta_m = \left(\frac{2m+1}{2}\right)\frac{\lambda}{d}$

Since θ_m is small, so $\sin\theta_m = \theta_m$; $\therefore$ $\theta_m = \left(\frac{2m+1}{2}\right)\frac{\lambda}{d}$ ...(iv)

M=1 ,2, 3, ... Is an integer

Let f be the focal length of lens L_2 and the distance of first minima on either side of the central maxima be x. Then, $\tan\theta = \frac{x}{f}$

Since the lens L_2 is very close to the slit, so f=D

$\therefore \tan\theta = \frac{x}{d}$ Since θ is very small, so $\tan\theta \approx \sin\theta$ $\therefore \sin\theta = \frac{x}{d}$ ...(i)

Also, for first minima, $d\sin\theta = \lambda$ or $\sin\theta = \frac{y}{d}$ ...(ii)

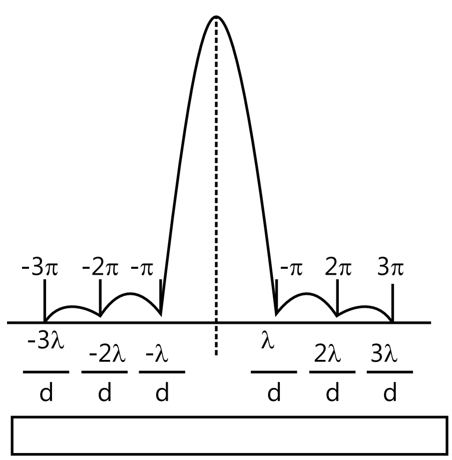


Figure 17.16

From eqns. (i) and (ii), we have $\frac{x}{D}=\frac{\lambda}{d}$ or $x=\frac{\lambda D}{d}$ ...(iii)

This is the distance of first minima on either side from the centre of the central maximum. Width of central maximum is given by: $\therefore 2x=\frac{2\lambda D}{d}$

Diffraction pattern due to a single slit consists of a central maximum flanked by alternate minima and secondary maxima is shown in figure.

Note: That $\sin\theta=0$ corresponds to central maxima while $\sin\theta=\pi$, corresponds to first minima.

Diffraction grating: It consists of a large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the direction of principal maxima is given by $d\sin\theta=n\lambda$

Here d is the distance between two consecutive slits and is called the grating element.

N=1, 2, 3,.... is the order of principal maximas.

Resolving power of the diffraction grating: The diffraction grating is most useful for measuring wavelengths accurately. Like the prism, the diffraction grating can be used to disperse a spectrum into its wavelength components. Of the two devices, the grating is the more precise if one wants to distinguish two closely spaced wavelengths. For two nearly equal wavelengths λ_1 and λ_2, between which a diffraction grating can just barely distinguish, the resolving power R of the grating is defined as

$R=\frac{\lambda}{\lambda_2-\lambda_1}=\frac{\lambda}{\Delta\lambda}$ where, $\lambda=(\lambda_1+\lambda_2)/2$ and $\Delta\lambda=\lambda_2-\lambda_1$.

Illustration 19: A parallel beam of monochromatic light of wavelength 450 nm passes through a long slit of width 0.2 mm. Find the angular divergence in which most of the light is diffracted. **(JEE MAIN)**

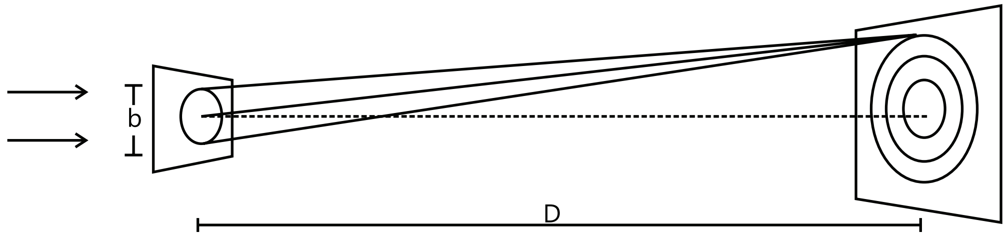
Sol: Most of the light is diffracted between the two first order minima. These minima occur at angles given by $b\sin\theta=\pm\lambda$

Or, $\sin\theta=\pm\lambda/b=\pm\frac{450\times10^{-9}m}{0.2\times10^{-3}m}=\pm2.25\times10^{-3}$ Or, $\theta=\pm2.25\times10^{-3}rad$

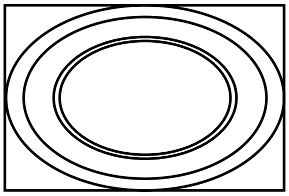
The angular divergence $=4.5\times10^{-3}rad$.

4.2 Diffraction by a Circular Aperture

Mathematical analysis shows that the first dark ring is formed by the light diffracted from the hole at an angle θ with the axis, where $\sin\theta=1.22\frac{\lambda}{b}$ Here, λ is the wavelength of the light used and b is the diameter of the hole. If the screen is at a distance D(D>>b) from the hole, the radius of the first dark ring is $R=1.22\frac{\lambda D}{b}$



(a)



(b)

Figure 17.17

If the light transmitted by the hole is converged by a converging lens at the screen placed at the focal plane of this lens, the radius of the first dark ring is $R = 1.22\frac{\lambda f}{b}$

As most of the light coming from the hole is concentrated within the first dark ring, this radius is also called the radius of the diffraction disc.

Illustration 20: A beam of light of wavelength 590nm is focused by a converging lens of diameter 10.0 cm at a distance of 20 cm from it. Find the diameter of the disc image formed. **(JEE MAIN)**

Sol: The angular radius of the central bright disc in a diffraction pattern from circular aperture is given by $\sin\theta = \frac{1.22\lambda}{b} = \frac{1.22\times 590\times 10^{-9}\,m}{10.0\times 10^{-2}\,m} = 0.7\times 10^{-5}\,rad.$

The radius of the bright disc is $0.7\times 10^{-5}\times 20cm = 1.4\times 10^{-4}\,cm$

The diameter of the disc image $= 2.8\times 10^{-4}\,cm$

4.3 Diffraction of X-Rays by Crystals

The arrangement of atoms in a crystal of NaCl is shown in the above figure. Each unit cell is a cube of length of edge a. If an incident x-ray beam makes an angle θ with one of the planes, the beam can be reflected from both the planes. However, the beam reflected from the lower plane travels farther than the beam reflected from the upper plane.

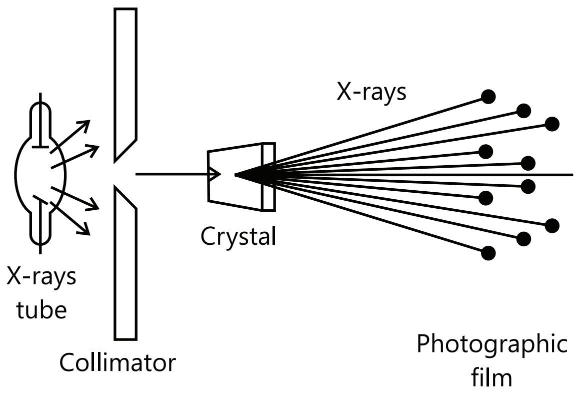


Figure 17.18

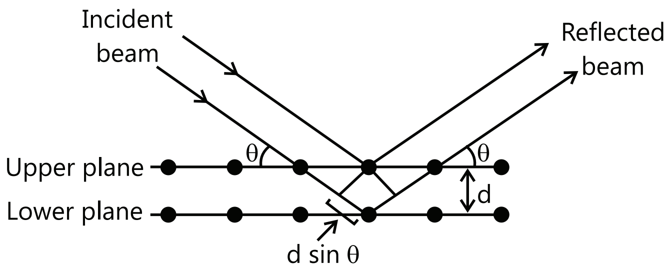


Figure 17.19

The effective path difference is $2d\ \sin\theta$. The two beams reinforce each other (constructive interference) when this path difference is equal to some integer multiple of λ. The same is true for reflection from the entire family of parallel planes. Hence, the condition for constructive interference (maxima in the reflected beam) is $2d\sin\theta = m\lambda$ where, m=1, 2, 3,...is an integer.

This condition is known as Bragg's Law, after W.L. Bragg (1890-1971), who first derived the relationship. If the wavelength and diffraction angle are measured, the above equation can be used to calculate the spacing between atomic planes.

Note: Each fringe in Young's Double Split Experiment has equal intensity while in diffraction, the intensity falls as the fringe order increases.

Important Points:

(a) Types of diffraction: The diffraction phenomenon is divided into two types viz. Fresnel diffraction and Fraunhofer diffraction. In the first type, either the source or the screen or both are at a finite distance from the diffracting device (obstacle of aperture). In the second type, both the source and screen are effectively at an infinite distance from the diffracting device. Fraunhofer diffraction is a particular limiting case of Fresnel diffraction.

(b) Difference between interference and diffraction: Both interference and diffraction are the results of superposition of waves, so they are often present simultaneously, as in Young's double slit experiment. However interference is the result of superposition of waves from two different wave fronts while diffraction results due to superposition of wavelets from different points of the same wave front.

MASTERJEE CONCEPTS

X-ray diffraction is used in crystals to find inter-atomic distance owing to the fact that wavelengths of x-rays are of order of inter-atomic distance, the required condition for diffraction.

Nivvedan (JEE 2009, AIR 113)

5. RESOLVING POWER OF OPTICAL INSTRUMENTS

When the two images cannot be distinguished, they are said to be un-resolved. If the images are well distinguished. They are said to be well resolved. On the other hand, if the images are just distinguished, they are said to be just resolved.

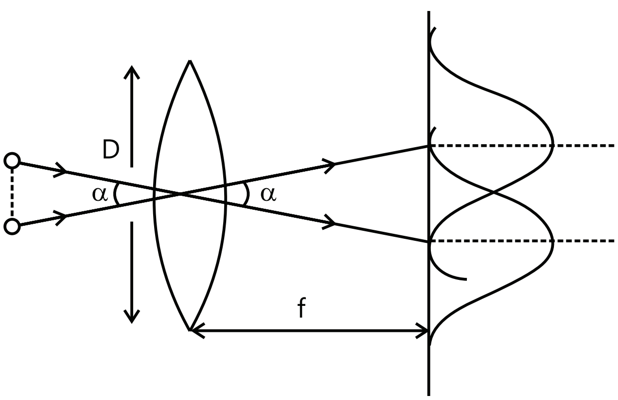


Figure 17.20

Rayleigh Criterion: According to Rayleigh, two objects or points are just resolved if the position of the central maximum of the image of one object coincides with the first minimum of the image of the other object as shown in figure (a).

5.1 Limit of Resolution

The minimum distance of separation between two points so that they can be seen as separate (or just resolved) by the optical instrument is known as its limit of resolution. Diffraction of light limits the ability of optical instruments.

MASTERJEE CONCEPTS

Resolving Power: The ability of an optical instrument to form distinctly separate images of the two closely placed points or objects is called its resolving power. Resolving power is also defined as reciprocal of the limit of resolution,

$$\text{R.P.} = \frac{1}{\text{Limit of resolution}}$$

Smaller the limit of resolution of an optical instrument, larger is its resolving power and vice-versa.

Resolving Power of Eyes: Since eye lens is a converging lens, the limit of resolution of the human eye is that of the objective lens of a telescope i.e. limit of resolution of the eye, $\alpha = \frac{1.22\gamma}{D}$

Where, D= diameter of the pupil of the eye.

$$\text{Resolving power of the eye} = \frac{1}{\alpha} = \frac{D}{1.22\lambda}$$

Resolving power of an astronomical telescope: Resolving power of a telescope,

$$\text{R.P.} = \frac{1}{\text{Limit of resolution}} \text{ or R.P.} = \frac{D}{1.22\lambda}$$

Chinmay S Purandare (JEE 2012, AIR 698)

Illustration 21: A star is seen through a telescope having objective lens diameter as 203.2cm. If the wavelength of light coming from a star is 6600 Å, find (i) the limit of resolution of the telescope and (ii) the resolving power of the telescope. **(JEE MAIN)**

Sol: Resolving power of the eye$=\frac{1}{\alpha} = \frac{D}{1.22\lambda}$ Here, D=203.2cm and $\lambda = 6600\,\text{Å} = 6600\times10^{-8}\,\text{cm} = 6.6\times10^{-5}\,\text{cm}$

(i) Limit of resolution of telescope, $\alpha = \frac{1.22\lambda}{D} = \frac{1.22\times6.6\times10^{-5}}{203.2} = 3.96\times10^{-7}\,\text{rad}$

(ii) Resolving power of telescope, $\frac{1}{\alpha} = \frac{D}{1.22\lambda} = \frac{1}{3.96\times10^{-7}} = 2.53\times10^{6}$

Resolving power of a microscope: Resolving power of a microscope$=\frac{1}{d_{min}}$ i.e.

$$\text{R.P.}_{\text{microscope}} = \frac{2n\sin\beta}{1.22\lambda}$$

6. SCATTERING OF LIGHT

Scattering of light is a phenomenon in which a part of a parallel beam of light appears in directions other than the incident radiation when passed through a gas.

Process: Absorption of light by gas molecules followed by its re-radiation in different directions.

The strength of scattering depends on the following:

(a) Loss of energy in the light beam as it passes through the gas

(b) Wavelength of light

(c) Size of the particles that cause scattering

Key point: If these particles are smaller than the wavelength, the scattering is proportional to $1/\lambda^4$. This is known as Rayleigh's law of scattering. Thus, red is scattered the least and violet is scattered the most. This is why, red signals are used to indicate dangers. Such a signal transmit to large distance without an appreciable loss due to scattering.

Practical example of scattering: The blue color of the sky is caused by the scattering of sunlight by the molecules in the atmosphere. This scattering, called Rayleigh scattering, is more effective at short wavelengths (the blue end of the visible spectrum). Therefore, the light scattered down to the earth at a large angle with respect to the direction of the sun's light, is predominantly in the blue end of the spectrum.

7. POLARIZATION OF LIGHT

The process of splitting of light into two directions is known as polarization.

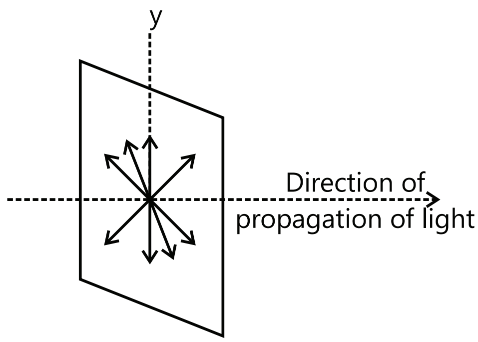
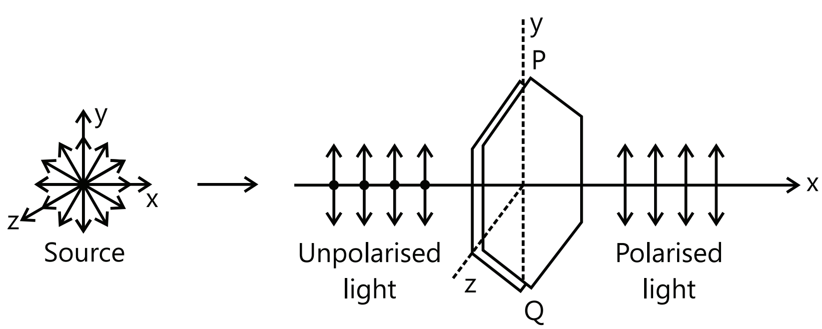


Figure 17.21

Phenomenon of polarization: The phenomenon of restricting the vibrations of a light vector of the electric field vector in a particular direction in a plane perpendicular to the direction of propagation of light is called polarization of light. Tourmaline crystal is used to polarize the light and hence it is called polarizer.



Pictorial representation of polarised light shown figure (A) and (B)

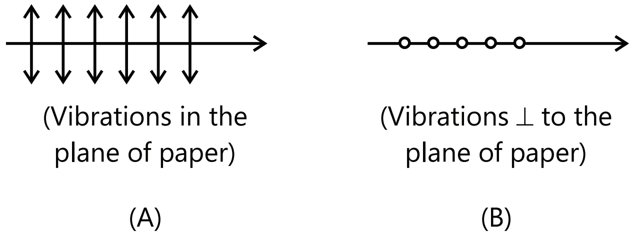


Figure 17.22

7.1 Unpolarized Light

(a) An ordinary beam of light consists of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation of electric vector E. However, because all directions are equally probable, the resulting electromagnetic wave is a superposition of waves produced by individual atomic sources. This wave is called as an unpolarized light wave.

(b) All the vibrations of an unpolarized light at a given instant can be resolved into two mutually perpendicular directions and hence an unpolarized light is equivalent to superposition of two mutually perpendicular identical plane polarized lights.

7.2 Plane Polarized Light

(a) If somehow we confine the vibrations of electric field vector in one direction perpendicular to the direction of wave motion of propagation of wave, the light is said to be plane polarized and the plane containing the direction of vibration and wave motion is called the plane of polarization.

(b) If an unpolarised light is converted into plane polarized light, its intensity reduces to half.

(c) Polarization is a proof of the wave nature of light.

Partially polarized light: If in case of unpolarised light, electric field vector in some plane is either more or less than in its perpendicular plane, the light is said to be partially polarized.

7.3 Polaroids

A polaroid is a device used to produce plane polarized light. The direction perpendicular to the direction of the alignment of the molecules of the Polaroid is known as pass-axis or the polarizing direction of the Polaroid.

Note: A Polaroid used to examine the polarized light is known as analyzer.

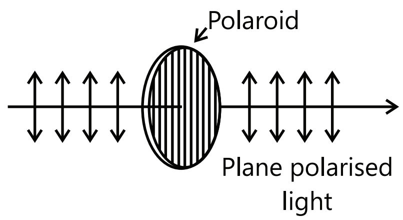


Figure 17.23

7.4 Malus' Law

This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser. Resolve E into two components: We know, intensity $\propto(\text{Amplitude})^2$

$\therefore$ Intensity of the transmitted light through the analyser is given by $I\alpha(E\cos\theta)^2$ or $I = kE^2\cos^2\theta$;

But $kE^2 = I_0$ (intensity of the incident polarised light)

$I = I_0\cos^2\theta$ Or $I \propto \cos^2\theta$ which is Malus Law.

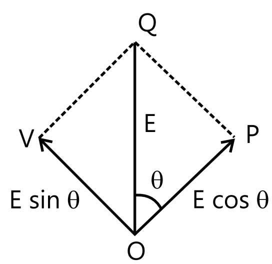


Figure 17.24

MASTERJEE CONCEPTS

Plane polarized light is used for chemical purposes in measuring optical rotations of various chemical compounds. It can also be used for stating the difference in enantiomeric compounds.

Nitin Chandrol (JEE 2012, AIR 134)

7.5 BREWSTER'S LAW

According to this law, the refractive index of the refractive medium (n) is numerically equal to the tangent of the angle of polarization $\left(I_B\right)$. i.e. $n = \tan I_B$

Illustration 22: What is the polarizing angle of a medium of refractive index 1.732? **(JEE MAIN)**

Sol: As per Brewster's law, $n = \tan i_B$ or $i_B = \tan^{-1} n = \tan^{-1} 1.732 = 60°$

PROBLEM-SOLVING TACTICS

1. Most of the questions in JEE are related to Young's Double slit experiment with minor variations. For any such problem, drawing a rough figure and writing down the given parameters is a good idea before solving the question.

2. Wave optics has a lot of derivations. It is advisable to remember the end results for faster problem solving.

3. Use the concept of optical path carefully and check for phase relations.

4. Only direct formulae related questions are asked from sections of diffraction, polarizations and scattering so these formulae must be learnt.

FORMULAE SHEET

S. No.	Term	Description
1	Wave front	It is the locus of points in the medium which at any instant are vibrating in the same phase.
2	Huygens' Principle	1 Each point on the given primary wave front acts as a source of secondary wavelets spreading out disturbance in all direction. 2 The tangential plane to these secondary wavelets constitutes the new wave front.
3	Interference	It is the phenomenon of non-uniform distribution of energy in the medium due to superposition of two light waves.
4	Condition of maximum intensity	$\phi = 2n\pi$ or $x = n\lambda$, where n=0,1,2,3,....
6	Condition of minimum intensity	$\phi = \left(2n+1\right)\pi$ or $x = \left(2n+1\right)y/2$ where n=0,1,2,3...
7	Ratio of maximum and minimum intensity	$\frac{I_{max}}{I_{min}} = \frac{\left(a_1 + a_2\right)^2}{\left(a_1 - a_2\right)^2}$
8	Distance of nth bright fringe from centre of the screen	$y_n = \frac{nD\lambda}{d}$, where d is the separation distance between two coherent sources of light, D is the distance between screen and slit, λ is the wavelength used.

S. No.	Term	Description
9	Angular position of nth bright fringe	$\theta_n = \frac{y_n}{D} = \frac{n\lambda}{d}$
10	Distance of nth dark fringes from centre of the screen	$y'_n = \frac{(2n+1)D\lambda}{2d}$
11	Angular position of nth dark fringe	$\theta'_n = \frac{y'_n}{D} = \frac{(2n+1)\lambda}{2d}$
12	Fringe width	$\beta = \frac{D\lambda}{d}$

Diffraction and polarization of light

S. No.	Term	Description
1	Diffraction	It is the phenomenon of bending of light waves round the sharp corners and spreading into the regions of the geometrical shadow of the object.
2	Single slit diffraction	Condition for dark fringes is $\sin\theta = \frac{n\lambda}{a}$ where $n = \pm1, \pm2, \pm3, \pm4.....$, a is the width of the slit and θ is the angle of diffraction. Condition for bright fringes is $\sin\theta = \frac{(2n+1)\lambda}{2a}$
3	Width of central maximum is	$\theta_0 = \frac{2\lambda D}{a}$, where D is the distance between the slit and the screen.
4	Diffraction grating	The arrangement of large number of narrow rectangular slits of equal width placed side by side parallel to each other. The condition for maxima in the interference pattern at the angle θ is $d\sin\theta = n\lambda$ where n=0, 1, 2, 3, 4.....
6	Resolving power of the grating	For nearly two equal wavelengths λ_1 and λ_2 between which a diffraction grating can just barely distinguish, resolving power is $R = \frac{\lambda}{\lambda_1 - \lambda_2} = \frac{\lambda}{\Delta\lambda}$ where $\lambda = (\lambda_1 + \lambda_2)/2$
7	Diffraction of X-Rays by crystals	The condition for constructive interference is $2d\sin\theta = n\lambda$
8	Polarisation	It is the phenomenon due to which vibrations of light are restricted in a particular plane.
9	Brewster's law	$\mu = \tan p$ where μ is refractive index of medium and p is the angle of polarisation.
10	Law of Malus	$I = I_0 \cos^2\theta$ where I is the intensity of emergent light from analyser, I_0 is the intensity of incident plane polarised light and θ is the angle between planes of transmission of the analyser and the polarizer.