MasterJEE

IIT-JEE | Medical | Foundations

Time : 3 hrs. **M.M. : 360**

Answers & Solutions

for

JEE (MAIN)-2019

(Online CBT Mode)

(Physics, Chemistry and Mathematics)

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 **(four)** marks for each correct response.
4. *Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for a question in the answer sheet.*
5. There is only one correct response for each question.

PHYSICS

1. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass $m(m << M)$. When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to

(1) $\frac{g}{30}$ (2) $\frac{g}{5}$

(3) $\frac{g}{20}$ (4) $\frac{g}{10}$

Answer (2)

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}}$

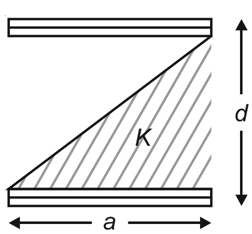
$$v' = \sqrt{\frac{m\sqrt{g^2+a^2}}{\mu}}$$

$$\Rightarrow \frac{v'}{v} = \sqrt{\frac{\sqrt{g^2+a^2}}{g}}$$

$$\Rightarrow a \simeq 1.83$$

$$\Rightarrow a = \frac{g}{5}$$

2. A parallel plate capacitor is made of two square plates of side a, separated by a distance $d(d << a)$. The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is

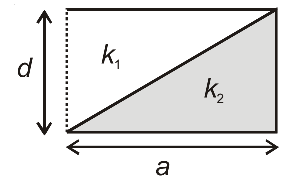


(1) $\frac{K\varepsilon_0 a^2}{d(K-1)}\ln K$ (2) $\frac{K\varepsilon_0 a^2}{d}\ln K$

(3) $\frac{K\varepsilon_0 a^2}{2d(K+1)}$ (4) $\frac{1}{2}\frac{K\varepsilon_0 a^2}{d}$

Answer (1)

Sol. $C_{eq} = \dfrac{\in_0 k_1k_2a^2 \ln\frac{k_1}{k_2}}{(k_1-k_2)d}$



$$\Rightarrow C_{eq} = \frac{\in_0 ka^2 \ln\frac{1}{k}}{d(1-k)}$$

as $k_1 = 1$, $k_2 = k$

$$\Rightarrow C_{eq} = \frac{\in_0 ka^2}{d(k-1)}\ln k$$

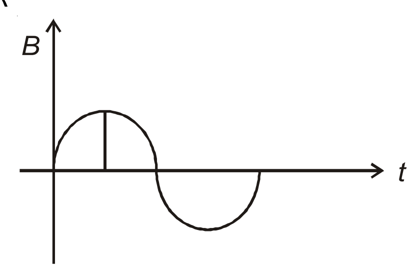
3. A conducting circular loop made of a thin wire, has area 3.5×10^{-3} m^2 and resistance 10 Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4T)\sin(50\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0$ s and $t = 10$ ms is close to

(1) 7 mC (2) 21 mC

(3) 6 mC (4) 14 mC

Answer (Bonus)

Sol. $\Delta Q = \frac{\Delta\phi}{R}$



$$B(t) = 0.4\sin 50\pi t$$

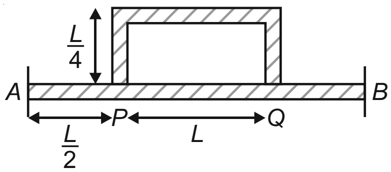
$$\Rightarrow \frac{2\pi}{T} = 50\pi$$

$$T = \frac{1000}{25}\text{ms} \Rightarrow T = 40 \text{ ms}$$

$$\Delta Q = \frac{0.4\times3.5\times10^{-3}}{10}$$

$$\Delta Q = 1.4\times10^{-4}\,\text{C}$$

4. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length $2L$. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (see figure). In steady state, temperature difference between P and Q will be close to



(1) 35°C (2) 45°C

(3) 60°C (4) 75°

Answer (2)

Sol. $$\frac{2(T_A - T_P)}{L} = \left(\frac{T_P - T_Q}{L}\right) + \frac{2}{3}\left(\frac{T_P - T_Q}{L}\right)$$

$$2(T_A - T_P) = \frac{5}{3}(T_P - T_Q) \qquad ....(1)$$

$$2(T_Q - T_B) = \frac{5}{3}(T_P - T_Q) \qquad ....(2)$$

From (1) and (2)

$$2(T \quad T) \quad 2\,T \quad T \quad \frac{10}{}\,T \quad T$$

$$\Rightarrow \quad 2 \times 120 = \frac{16}{3}(T_P - T_Q)$$

$$\Rightarrow \quad T_P - T_Q = \frac{2 \times 120 \times 3}{16} = 45°\text{C}$$

5. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is

(1) 1.1 cm away from the lens

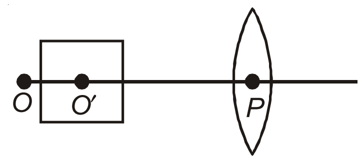
(2) 0.55 cm towards the lens

(3) 0

(4) 0.55 cm away from the lens

Answer (4)

Sol. $2f = 10\text{ cm} \Rightarrow f = 5\text{cm}$



Shift $= OO' = 1.5\left(1 - \frac{2}{3}\right)$

= 0.5 cm

$\Rightarrow O'P = 9.5\text{ cm}$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} + \frac{1}{9.5} = \frac{1}{5}$$

$\Rightarrow$ v = 10.55 cm

Shift = 0.55 cm away

6. A mixture of 2 moles of helium gas (atomic mass = 4 u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$, is close to

(1) 2.24

(2) 0.45

(3) 3.16

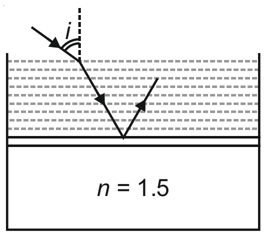
(4) 0.32

Answer (3)

Sol. $\frac{v_{rms(helium)}}{v_{rms(argon)}} = \sqrt{\frac{M_{ar}}{M_{He}}}$ at same temperature

$$\frac{v_{rms(helium)}}{v_{rms(argon)}} = \sqrt{\frac{40}{4}} = 3.16$$

7. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is



(1) $\frac{4}{3}$ (2) $\sqrt{\frac{5}{3}}$

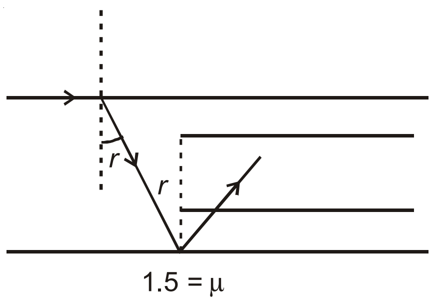
(3) $\frac{3}{\sqrt{5}}$ (4) $\frac{5}{\sqrt{3}}$

Answer (3)

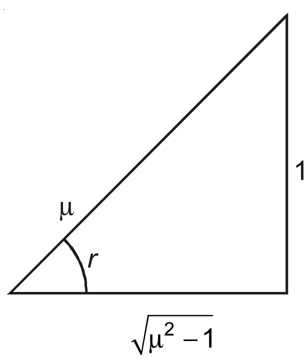
Sol. For air-medium interface

Max. angle of incidence = 90°

$$\frac{\sin 90^\circ}{\sin r} = \mu$$



$$\sin r = \frac{1}{\mu}$$



$$\tan i_B = \frac{3}{2\mu} \text{ (Brewster's law)}$$

$$\Rightarrow \frac{1}{\sqrt{\mu^2 - 1}} < \frac{3}{2\mu}$$

$$\Rightarrow \mu \geq \frac{3}{\sqrt{5}}$$

$$\Rightarrow \mu_{min} = \frac{3}{\sqrt{5}}$$

8. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is

(1) 0.5%

(2) 2.0%

(3) 2.5%

(4) 1.0%

Answer (4)

Sol. Al = Constant

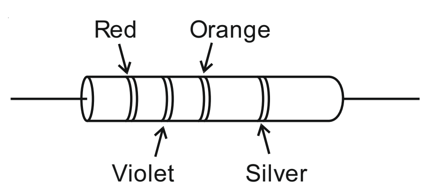
$$R = \rho\frac{l}{A}$$

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} + \frac{\Delta A}{A}$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{2\Delta l}{l} = 2 \times 0.5\%$$

$$\Rightarrow \frac{\Delta R}{R}\% = 1\%$$

9. A resistance is shown in the figure. Its value and tolerance are given respectively by



(1) 27 kΩ, 20% (2) 270 Ω, 5%

(3) 27 kΩ, 10% (4) 270 Ω, 10%

Answer (3)

Sol. R V O S

↓ ↓ ↓ ↓

2 7 3 ±10%

$\Rightarrow R = 27 \times 10^3 \pm 10\%$

10. Surface of certain metal is first illuminated with light of wavelength λ_1 = 350 nm and then, by light of wavelength λ_2 = 540 nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to

(Energy of photon = $\frac{1240}{\lambda(\text{in nm})}$eV)

(1) 1.8 (2) 5.6

(3) 2.5 (4) 1.4

Answer (1)

Sol. $\frac{1}{2}m(2v)^2 = \frac{hc}{350} - \phi_0$

and, $\frac{1}{2}mv^2 = \frac{hc}{540} - \phi_0$

$\Rightarrow \quad 4\left(\frac{hc}{540}\right) - \left(\frac{hc}{350}\right) = 3\phi_0$

$\Rightarrow$ 9.12 – 3.54 = $3\phi_0$

$\Rightarrow \quad \phi_0 \simeq 1.8\,\text{eV}$

11. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an *n*-type semiconductor, the density of electrons is 10^{19} m^{-3} and their mobility is 1.6 m^2/(V.s) then the resistivity of the semiconductor (since it is an *n*-type semiconductor contribution of holes is ignored) is close

(1) 2 Ωm (2) 0.2 Ωm

(3) 0.4 Ωm (4) 4 Ωm

Answer (3)

Sol. $I = n.e\,A\,\mu\,E$

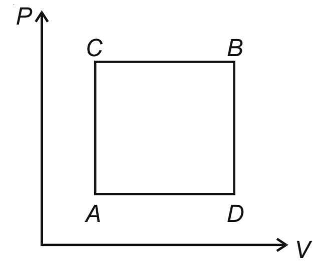
$I = \frac{n.eA\mu V}{L}$

$\frac{V}{I} = \frac{L}{n.eA\mu} = \rho\frac{L}{A}$

$\Rightarrow \quad \rho = \frac{1}{ne\mu} = \frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 1.6} = \frac{1}{2.56}$

$\Rightarrow$ ρ = 0.4 Ωm

12. A gas can be taken from *A* and *B* via two different processes *ACB* and *ADB*.



When path *ACB* is used 60 J of heat flows into the system and 30 J of work is done by the system. If path *ADB* is used work done by the system is 10 J. The heat Flow into the system in path *ADB* is

(1) 100 J

(2) 80 J

(3) 20 J

(4) 40 J

Answer (4)

Sol. $Q_{ACB} = W_{ACB} + U_{ACB}$

$\Rightarrow$ 60 = 30 + U_{ACB}

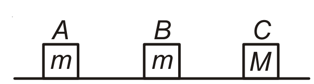
$\Rightarrow$ 30J = $U_{ACB} = U_{ADB}$

And , $Q_{ADB} = W_{ADB} + U_{ADB}$

= (10 + 30) J

= 40 J

13. Three blocks *A*, *B* and *C* are lying on a smooth horizontal surface, as shown in the figure. *A* and *B* have equal masses, *m* while *C* has mass *M*. Block *A* is given an initial speed ν towards *B* due to which it collides with *B* perfectly inelastically. The combined mass collides with *C*, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of *M*/*m*?



(1) 3 (2) 4

(3) 2 (4) 5

Answer (2)

Sol. $\Delta E_1 = \frac{1}{2}\mu v_{rel}^2$

$= \frac{1}{2}\frac{m}{2}v^2 = \frac{1}{4}mv^2$

Velocity after collision = $\frac{v}{2}$

$\Delta E_2 = \frac{1}{2}\left(\frac{2m \cdot M}{2m+M}\right)\left(\frac{v}{2}\right)^2$

$\Delta E_1 + \Delta E_2 = \frac{5}{6}\left(\frac{1}{2}mv^2\right)$

$\Rightarrow \quad \frac{1}{2}\frac{m}{2}v^2 + \left(\frac{1}{2}\right)\frac{m}{2}v^2 \cdot \frac{M}{2m+M} = \frac{5}{6}\left(\frac{1}{2}mv^2\right)$

$\Rightarrow \quad 1 + \frac{M}{2m+M} = \frac{10}{6} = \frac{5}{3}$

$\Rightarrow \quad \frac{M}{m} = 4$

14. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\,j$ V/m. The corresponding magnetic field $\vec{B}$, at that point will be

(1) $18.9 \times 10^{-8}\,\hat{k}$ T

(2) $2.1 \times 10^{-8}\,\hat{k}$ T

(3) $6.3 \times 10^{-8}\,\hat{k}$ T

(4) $18.9 \times 10^{8}\,\hat{k}$ T

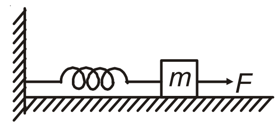
Answer (2)

Sol. $\vec{B} = \left|\frac{\vec{E}}{c}\right|\hat{k}$

$= \frac{6.3}{3 \times 10^8}\hat{k}$

$= 2.1 \times 10^{-8}\,\hat{k}$

15. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is



(1) $\frac{F}{\pi\sqrt{mk}}$ (2) $\frac{\pi F}{\sqrt{mk}}$

(3) $\frac{2F}{\sqrt{mk}}$ (4) $\frac{F}{\sqrt{mk}}$

Answer (4)

Sol. $T = 2\pi\sqrt{\frac{m}{k}}$

$\omega = \sqrt{\frac{k}{m}}$

$V_{max} = A.\omega$

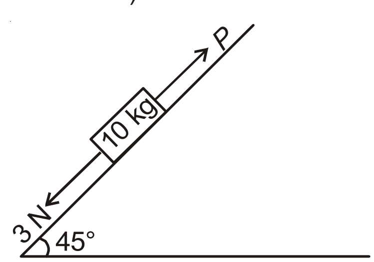
$\because\ F = kA,\ A = \frac{F}{k}$

$V_{max} = \frac{F}{k} \cdot \sqrt{\frac{k}{m}}$

$\Rightarrow\ V_{max} = \frac{F}{\sqrt{km}}$

16. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward?

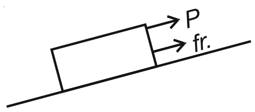
(take g = 10 ms^{-2})



(1) 25 N (2) 32 N

(3) 18 N (4) 23 N

Answer (2)

Sol. 

Friction force should be acting upward along the plane. So for state of impending motion.

$$3+10\times10\frac{1}{\sqrt{2}}=P+10\times10\frac{1}{\sqrt{2}}\times\frac{6}{10}$$

$\Rightarrow$ 73.71 – 42.42 = P

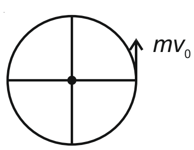
$\Rightarrow$ P = 31.28 $\approx$ 32 N

17. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is

(1) $\frac{L}{m}$ (2) $\frac{4L}{m}$

(3) $\frac{L}{2m}$ (4) $\frac{2L}{m}$

Answer (3)

Sol. 

$\therefore$ mv_0R = L

$\therefore$ $v_0 = \frac{L}{mR}$

$\therefore$ $T = \frac{2\pi R}{v_0}$

Area velocity = $\frac{\pi R^2}{T}$

$$\Rightarrow \frac{dA}{dt}=\frac{\pi R^2 v_0}{2\pi R}=\frac{Rv_0}{2}=\frac{L}{2m}$$

18. A particle is moving with a velocity $\vec{v}=K(y\hat{i}+x\hat{j})$, where K is a constant. The general equation for its path is

(1) $y^2 = x$ + constant (2) $y = x^2$ + constant

(3) $y^2 = x^2$ + constant (4) xy = constant

Answer (3)

Sol. $\frac{dx}{dt}=ky$

$\frac{dy}{dt}=kx$

$\Rightarrow \frac{dy}{dx}=\frac{x}{y}$

$\therefore$ $ydy = xdx \Rightarrow ydy - xdx = 0$

$\Rightarrow$ $y^2 - x^2$ = constant

Or, $y^2 = x^2$ + constant

19. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is

(1) 520 A/m (2) 2600 A/m

(3) 1200 A/m (4) 285 A/m

Answer (2)

Sol. 

Field inside solenoid

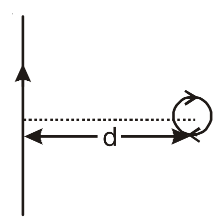
$B = \mu_0 ni$ Here $n = \frac{100\times10}{2} = 500$ m

$\therefore$ $B = \mu_0 \times 500i$

So, corresponding H = 500 × i = 500 × 5.2 A

= 2600 A/m

20. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d (d>>a). If the loop applies a force F on the wire then :



(1) $F = 0$ (2) $F \propto \left(\frac{a}{d}\right)^2$

(3) $F \propto \left(\frac{a}{d}\right)$ (4) $F \propto \left(\frac{a^2}{d^3}\right)$

Answer (2)

Sol. For shifting of loop along x-direction $PE(x) = -\vec{\mu}\cdot\vec{B}$

$$\therefore\ PE(x) = -\pi a^2 i\,\frac{\mu_0 I_0}{2\pi x}$$

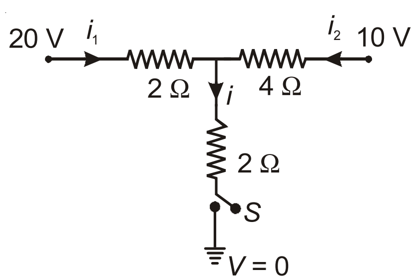
$$\therefore\ U(x) = -\frac{\mu_0 i I_0 a^2}{2x}$$ (PE decreases as it comes closer to wire)

So, attractive force $F(x) = \frac{-dU}{dx} = \frac{\mu_0 i I_0 a^2}{2}\left(\frac{-1}{x^2}\right)$

$$\therefore\ F(x) = \frac{\mu_0 i I_0 a^2}{2d^2} \text{ (Attractive)}$$

$$\therefore\ F \propto \frac{a^2}{d^2}$$

21. When the switch S, in the circuit shown, is closed, then the value of current i will be



(1) 2 A

(2) 5 A

(3) 4 A

(4) 3 A

Answer (2)

Sol. $20 - 2i_1 - 2(i_1 + i_2) = 0 \Rightarrow 20 - 2i_1 = 10 - 4i_2$

$10 - 4i_1 - 2(i_1 + i_2) = 0 \Rightarrow \frac{10 - 2i_1}{4} = i_2$

$\therefore\ 20 = 2i_1 + 2i_1 + 2i_2 \Rightarrow 20 = 4i_1 + 5 - i_1$

$\Rightarrow\ 3i_1 = 15\ \therefore\ i_1 = 5$ and $i_2 = 0$

$i_1 + i_2 = 5\ A$

22. A rod, length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by $\Delta T k$. Young's modulus, Y, for this metal is

(1) $\frac{F}{A\alpha(\Delta T - 273)}$

(2) $\frac{F}{A\alpha\Delta T}$

(3) $\frac{2F}{A\alpha\Delta T}$

(4) $\frac{F}{2A\alpha\Delta T}$

Answer (2)

Sol. $\Delta L_{\text{Thermal}} = L_0\,\alpha\,\Delta T$ (+ve)

$\Delta L_{\text{Mechanical}} = \frac{FL_0}{AY}$ (–ve)

$\Delta L_{\text{eff}} = 0\ \therefore\ L_0\,\alpha\,\Delta T = \frac{FL_0}{AY}$

$\Rightarrow\ Y = \frac{F}{A\alpha\Delta T} = \frac{F}{A\alpha\Delta T}$

23. Three charges $+Q$, q, $+Q$ are placed respectively, at distance, 0, $d/2$ and d from the origin, on the x-axis. If the net force experienced by $+Q$, placed at $x = 0$, is zero then value of q is

(1) $\frac{+Q}{2}$

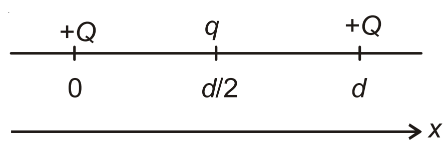
(2) $\frac{-Q}{2}$

(3) $\frac{-Q}{4}$

(4) $\frac{+Q}{4}$

Answer (3)

Sol.



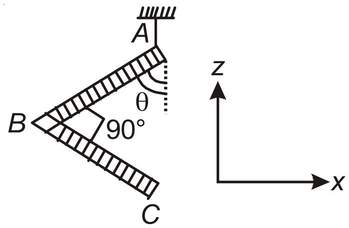
$$F_Q = \frac{-Q^2}{4\pi\epsilon_0 d^2} + \frac{Qq\times 4}{4\pi\epsilon_0 d^2} = 0$$

$\Rightarrow$ q must be (–ve)

$$\Rightarrow \frac{Qq.4}{4\pi\epsilon_0 d^2} = \frac{Q^2}{4\pi\epsilon_0 d^2}$$

$$\Rightarrow\ q = \frac{Q}{4}\ (-\text{ve})$$

24. An L- shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If $AB = BC$, and the angle made by AB with downward vertical is θ, then

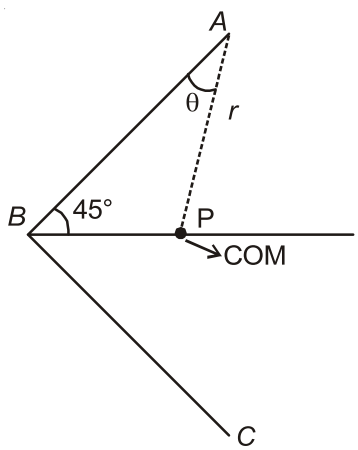


(1) $\tan\theta = \frac{1}{2}$ (2) $\tan\theta = \frac{2}{\sqrt{3}}$

(3) $\tan\theta = \frac{1}{3}$ (4) $\tan\theta = \frac{1}{2\sqrt{3}}$

Answer (3)

Sol. Position of COM from 'B' is $\frac{L}{2}\frac{1}{\sqrt{2}} = \frac{L}{2\sqrt{2}}$



Now, $r^2 = L^2 + \frac{L^2}{8} - 2.L.\frac{L}{2\sqrt{2}}\frac{1}{\sqrt{2}}$

$\Rightarrow r^2 = \frac{9L^2}{8} - \frac{L^2}{2} = \frac{5L^2}{8}$

$\therefore r = \sqrt{\frac{5}{8}}\cdot L$

$\therefore r\cos\theta = L - \frac{L}{4} \Rightarrow r\cos\theta = \frac{3L}{4}$

$\therefore \cos\theta = \frac{3L\sqrt{8}}{4\sqrt{5}\times L} = \frac{3\times 2\sqrt{2}}{4\sqrt{5}} = \frac{3}{\sqrt{10}}$

$\therefore \tan\theta = \frac{1}{3}$

25. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If the electron density in copper is $9 \times 10^{28}/\text{m}^3$ the value of v in mm/s is close to (Take charge of electron to be $= 1.6 \times 10^{-19}$ C)

(1) 0.02 (2) 0.2

(3) 3 (4) 2

Answer (1)

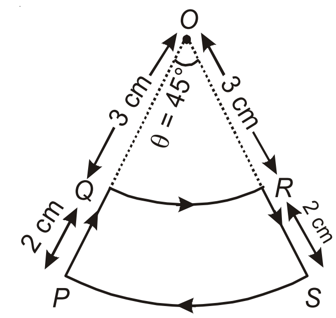
Sol. $\vec{J} = n\, e\, v_d$

$\therefore \frac{1.5}{5\times10^{-6}} = 9\times10^{28}\times1.6\times10^{-19}\times v$

$\Rightarrow v = \frac{1.5}{(5\times9\times1.6\times10^{3})} = 2.08\times10^{-5}\text{m/s}$

$\Rightarrow v = 0.02$ mm/s

26. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to



$i = 10$ A

(1) 1.5×10^{-7} T (2) 1.0×10^{-5} T

(3) 1.5×10^{-5} T (4) 1.0×10^{-7}

Sol. $B = \frac{\mu_0 i}{2R}\left(\frac{\theta}{360^\circ}\right)$

So $B_{\text{eff}} = B_{\odot\odot} - B_{\otimes\otimes} = \frac{\mu_0 i}{2\times3}\frac{45}{360} - \frac{\mu_0 i}{2\times5}\times\frac{45}{360}$

$\Rightarrow B_{\text{eff}} = \frac{\mu_0 i\times45}{2\times360}\left[\frac{1}{3}-\frac{1}{5}\right] = \frac{\mu_0 i\times45}{2\times360}\times\frac{2}{15}$

$B = \frac{\mu_0 i}{120} = 1.047\times10^{-7} \approx 1.0\times10^{-7}$ Tesla

27. A sample of radioactive material A, that has an activity of 10 mCi(1 Ci = 3.7×10^{10} decays/s), has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B would then be respectively:

(1) 10 day and 40 days (2) 20 day and 5 days

(3) 20 day and 10 days (4) 5 day and 10 days

Answer (2)

Sol. 10 mCi = $\lambda_A N_A(t)$ ...(1)

20 mCi = $\lambda_B N_B(t)$ ...(2)

As $N_A(t) = 2N_B(t)$

$$\therefore \quad \frac{1}{2} = \frac{\lambda_A . N_A(t)}{\lambda_B . N_B(t)} \Rightarrow \frac{1}{2} = \frac{\lambda_A}{\lambda_B} \cdot 2$$

$$\Rightarrow \lambda_B = 4\lambda_A \qquad \Rightarrow t_{1/2}(B) = \frac{t_{1/2}(A)}{4}$$

$$t_{\frac{1}{2}}(A) = 4t_{\frac{1}{2}}(B)$$

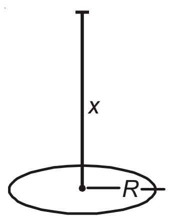
28. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is :

(1) $\frac{R}{\sqrt{2}}$ (2) $\frac{R}{\sqrt{5}}$

(3) R (4) $R\sqrt{2}$

Answer (1)

Sol. $E(x) = \dfrac{Q.x}{4\pi \in_0 \left(R^2 + x^2\right)^{\frac{3}{2}}}$



$\because \dfrac{dE}{dx} = 0$ for maximum

$$\Rightarrow \frac{Q}{4\pi \in_0}\left[\frac{(R^2+x^2)^{\frac{3}{2}} - x \cdot \frac{3}{2}\, R^2 + x^2 \, ^{\frac{1}{2}} .2x}{(R^2+x^2)^3}\right] = 0$$

$$\Rightarrow \frac{(R^2+x^2)^{\frac{1}{2}} Q}{4\pi \in_0 .(R^2+x^2)^3}\left(x^2 + R^2 - 3x^2\right) = 0$$

$$\Rightarrow x = \frac{R}{\sqrt{2}} \qquad \Rightarrow h = \frac{R}{\sqrt{2}}$$

29. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio:

(1) 25 : 9 (2) 4 : 1

(3) 16 : 9 (4) 5 : 3

Answer (1)

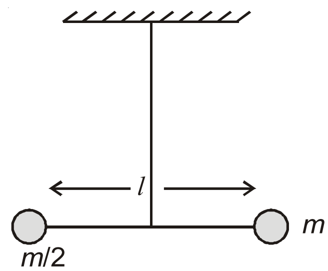
Sol. $\dfrac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = 16$

$$\Rightarrow \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = 4$$

$$\Rightarrow \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{4+1}{4-1} = \frac{5}{3}$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{25}{9}$$

30. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be:

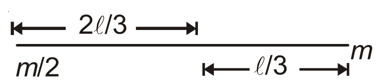


(1) $\frac{k\theta_0^2}{2l}$ (2) $\frac{k\theta_0^2}{l}$

(3) $\frac{2k\theta_0^2}{l}$ (4) $\frac{3k\theta_0^2}{l}$

Answer (2)

Sol. $I = \dfrac{m\ell^2}{9} + \dfrac{m}{2} \cdot \dfrac{4\ell^2}{9}$



$$\Rightarrow I = \frac{m\ell^2}{9}(1+2) = \frac{m\ell^2}{3}$$

$$\therefore \quad \text{PE at } (\theta_0) = \frac{1}{2}k\theta_0^2$$

$$\text{Now, } \frac{1}{2}k\theta_0^2 = \frac{1}{2}\frac{m\ell^2}{3} \cdot \omega^2$$

$$\Rightarrow \frac{3k\theta_0^2}{m\ell^2} = \omega^2$$

$$T = m\omega^2 \frac{\ell}{3} = \frac{m \cdot 3k\theta_0^2}{m\ell^2} \cdot \frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

❑ ❑ ❑

CHEMISTRY

1. Two complexes $[Cr(H_2O)_6]Cl_3$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding them is

   (1) Δ_0 values of (A) and (B) are calculated from the energies of violet and yellow light, respectively

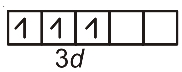
   (2) Both are paramagnetic with three unpaired electrons

   (3) Δ_0 value for (A) is less than that of (B)

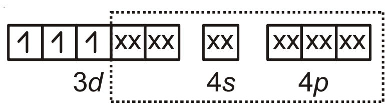
   (4) Both absorb energies corresponding to their complementary colors

Answer (1)

Sol. Cr^{3+} : $3d^3$

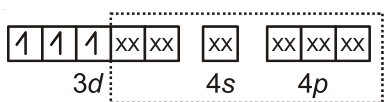


$\underset{(A)}{[Cr(H_2O)_6]^{3+}}$:



d^2sp^3 hybridisation

$\underset{(B)}{[Cr(NH_3)_6]^{3+}}$:



d^2sp^3 hybridisation

Both (A) and (B) are paramagnetic with 3 unpaired electrons each. The splitting energy (Δ_0) values of (A) and (B) are calculated from the wavelengths of light absorbed and not from the wavelengths of light emitted. H_2O is a weak field ligand causing lesser splitting than NH_3 which is relatively stronger field ligand.

2. In general, the properties that decrease and increase down a group in the periodic table, respectively, are

   (1) Electronegativity and electron gain enthalpy

   (2) Atomic radius and electronegativity

   (3) Electron gain enthalpy and electronegativity

   (4) Electronegativity and atomic radius

Answer (4)

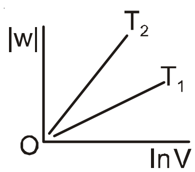
Sol. Down the group

Electronegativity decrease as size increases

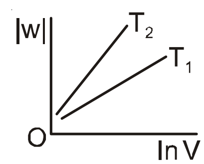
$$EN \propto \frac{1}{size}$$

3. Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 ($T_1 < T_2$). The correct graphical depiction of the dependence of work done (w) on the final volume (V) is

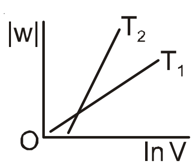
(1)



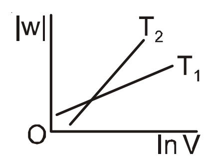
(2)



(3)



(4)



Answer (2)

Sol. Work done in an isothermal reversible expansion is given by

$$W = -2.303\ nRT\log\frac{V_2}{V_1}$$

$$|W| = nRT\ln\frac{V_2}{V_1}$$

$|W| \propto T$ for the same values of V_1 and V_2

$|W|$ = +ve if V_2 = 1 L and V_1 is fraction of a litre

4. The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexes is

   (1) 5.92 (2) 6.93

   (3) 4.90 (4) 3.87

Answer (1)

Sol. The transition metal atom/ion in a complex may have unpaired electrons ranging from zero to 5. So, maximum number of unpaired electrons that may be present in a complex is 5. Magnetic moment is given as

$\mu = \sqrt{n(n+2)}$ BM [no. of unpaired electrons = n)

Maximum value of magnetic moment

$= \sqrt{5(5+2)} = \sqrt{35} = 5.92\,\text{BM}$

5. The correct match between Item-I and Item-II is

	Item-I (drug)		**Item-II (test)**
A.	Chloroxylenol	P.	Carbylamine test
B.	Norethindrone	Q.	Sodium hydrogen Carbonate test
C.	Sulphapyridine	R.	Ferric chloride test
D.	Penicillin	S.	Bayer's test

(1) $A \rightarrow Q$, $B \rightarrow P$, $C \rightarrow S$, $D \rightarrow R$

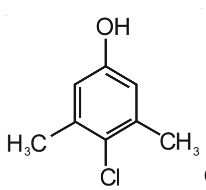
(2) $A \rightarrow R$, $B \rightarrow S$, $C \rightarrow P$, $D \rightarrow Q$

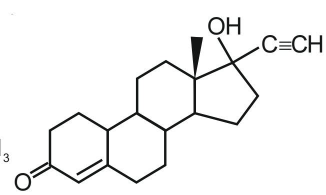
(3) $A \rightarrow Q$, $B \rightarrow S$, $C \rightarrow P$, $D \rightarrow R$

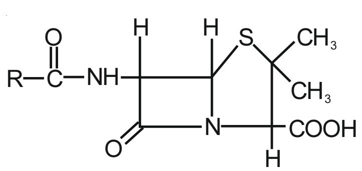
(4) $A \rightarrow R$, $B \rightarrow P$, $C \rightarrow S$, $D \rightarrow Q$

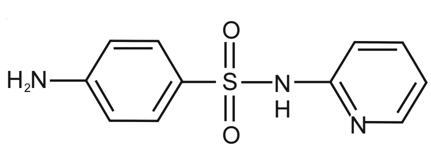
Answer (2)

Sol.
- Chloroxylenol is dettol contain phenolic group so give $FeCl_3$ test
- Norethindrone has double bond so will give Baeyer's reagent test
- Sulphapyridine has – NH_2 group it give carbyl amine test
- Penicillin has – COOH group so will respond to $NaHCO_3$ test

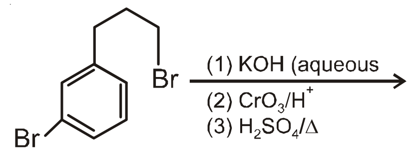

Chloroxylenol

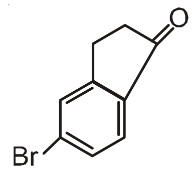

Norethindrone

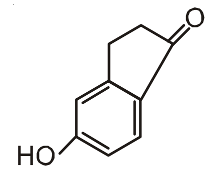

Pencillin

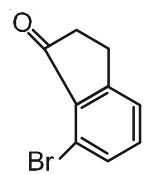

Sulphapyridine

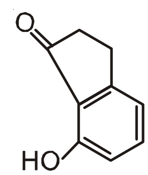
6. The major product of the following reaction is



(1) 

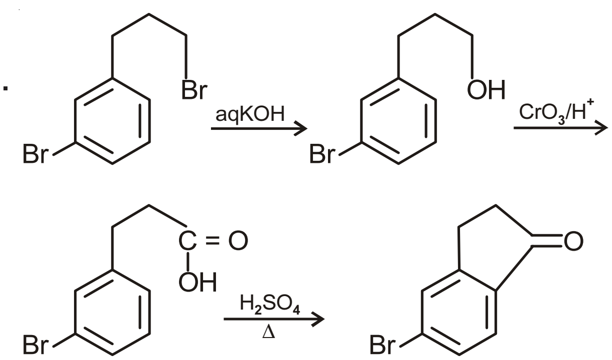
(2) 

(3) 

(4) 

Answer (1)

Sol.



7. The ore that contains both iron and copper is

(1) Copper pyrites (2) Dolomite

(3) Malachite (4) Azurite

Answer (1)

Sol.

Copper pyrites	$CuFeS_2$
Dolomite	$MgCO_3 \cdot CaCO_3$
Malachite	$CuCO_3 \cdot Cu(OH)_2$
Azurite	$2CuCO_3 \cdot Cu(OH)_2$

Copper pyrites contains both copper and iron

8. Correct statements among a to d regarding silicones are

(a) They are polymers with hydrophobic character

(b) They are biocompatible

(c) In general, they have high thermal stability and low dielectric strength

(d) Usually, they are resistant to oxidation and used as greases

(1) (a), (b) and (d) only (2) (a), (b), (c) and (d)

(3) (a), (b) and (c) only (4) (a) and (b) only

Answer (1)

Sol. Silicones are polymer with Si–O–Si linkages and are strongly hydrophobic. They are highly thermally stable with high dielectric strength. Now a days silicone greases are commony used.

9. Which amongst the following is the strongest acid?

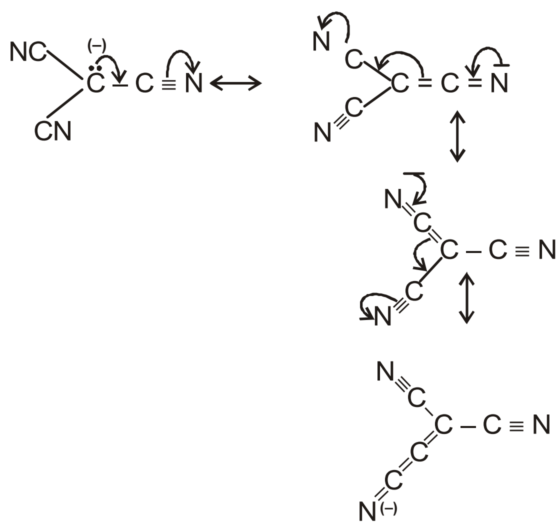
(1) $CHBr_3$ (2) $CH(CN)_3$

(3) CHI_3 (4) $CHCl_3$

Answer (2)

Sol. Of the given compounds $CH(CN)_3$ is strongest acid because its conjugate base is stabilised by resonance

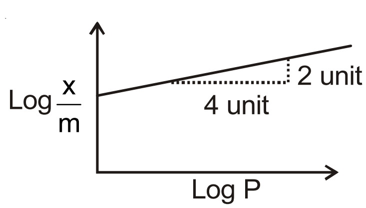
$$CH(CN)_3 \rightleftharpoons H^+ + \bar{C}(CN)_3$$



$CHBr_3$ and CHI_3 are less stable as their conjugate bases are stabilised by inductive effect of halogens. Conjugate base of $CHCl_3$ involves back bonding between 2*p* and 3*p* orbitals.

10. Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure p.

$\frac{x}{m}$ is proportional to



(1) p^2 (2) p

(3) $p^{1/4}$ (4) $p^{1/2}$

Answer (4)

Sol. In Freundlich adsorption of a gas on the surface of solid, the extent of adsorption(x/m) is related to pressure of gas (P) as

$$\frac{x}{m} = K(P)^{1/n}$$

Or $\log\frac{x}{m} = \log K + \frac{1}{n}\log P$

The slope of plot of $\log\frac{x}{m}$ versus log $P = \frac{2}{4} = \frac{1}{2}$

$$\therefore \frac{x}{m} \propto (P)^{1/2}$$

11. The increasing order of pKa of the following amino acids in aqueous solution is

Gly, Asp, Lys, Arg

(1) Gly < Asp < Arg < Lys

(2) Arg < Lys < Gly < Asp

(3) Asp < Gly < Arg < Lys

(4) Asp < Gly < Lys < Arg

Answer (2)

Sol. Structures of the given α-amino acids are

$H_3\overset{+}{N}-CH_2-COO^-$
Glycine (Gly)

$HOOC-CH_2-CH(\overset{+}{N}H_3)-COO^-$
Aspartic acid (Asp)

$H_2N-(CH_2)_4-CH(\overset{+}{N}H_3)-COO^-$
Lysine (Lys)

$H_2N-C(=NH)-NH-(CH_2)_3-CH(\overset{+}{N}H_3)-COO^-$
Arginine (Arg)

Aspartic acid is acidic, glycine is neutral and lysine & arginine are basic α-amino acids with arginine being more basic due to stronger basic functional group. Their pK_a value is directly proportional to basic strength, i.e., Arg > Lys > Gly > Asp.

12. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^- ?

(1) Li_2^+ is unstable and Li_2^- is stable

(2) Li_2^+ is stable and Li_2^- is unstable

(3) Both are stable

(4) Both are unstable

Answer (3)

Sol. Electronic configurations of Li_2^+ and Li_2^- are

$Li_2^+ : \sigma1s^2\ \sigma^*1s^2\ \sigma2s^1$

$Li_2^- : \sigma1s^2\ \sigma^*1s^2\ \sigma2s^2\ \sigma^*2s^1$

Bond order of $Li_2^+ = \frac{1}{2}(3-2) = \frac{1}{2}$

Bond order of $Li_2^- = \frac{1}{2}(4-3) = \frac{1}{2}$

Since both Li_2^+ and Li_2^- have +ve bond order, both are stable (reference : NCERT)

13. The correct decreasing order for acid strength is

(1) $FCH_2COOH > NCCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$

(2) $CNCH_2COOH > O_2NCH_2COOH > FCH_2COOH > ClCH_2COOH$

(3) $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

(4) $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$

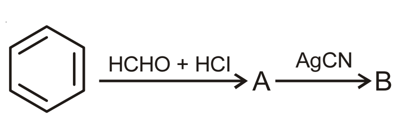
Answer (3)

Sol. The acidic strength of the given compounds is decided on the basis of (–I) effect of the substituents of carboxylic acids. Higher the (–I) effect of substituent, higher will be the acidic strength. The decreasing order of (–I) effect of the given substituents is $NO_2 > CN > F > Cl$.

Therefore, correct decreasing order of acidic strength

$O_2NCH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

14. The compounds A and B in the following reaction are, respectively



(1) A = Benzyl alcohol, B = Benzyl isocyanide

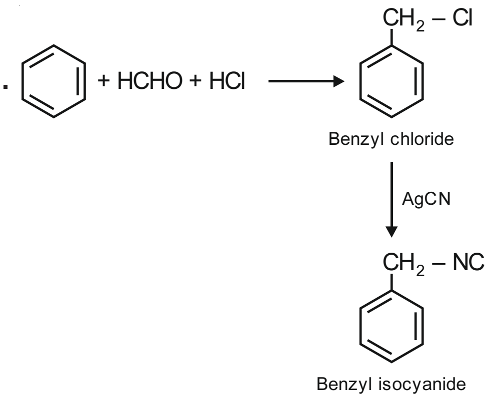
(2) A = Benzyl chloride, B = Benzyl cyanide

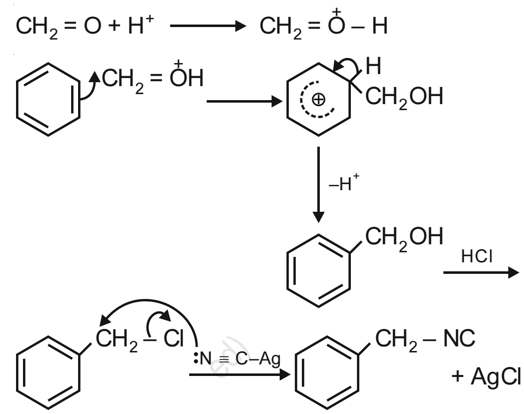
(3) A = Benzyl chloride, B = Benzyl isocyanide

(4) A = Benzyl alcohol, B = Benzyl cyanide

Answer (3)

Sol.





15. The one that is extensively used as a piezoelectric material is

(1) Tridymite

(2) Mica

(3) Quartz

(4) Amorphous silica

Answer (3)

Sol. Quartz exhibits piezoelectricity. It is fact based.

16. The isotopes of hydrogen are

(1) Tritium and protium only

(2) Deuterium and tritium only

(3) Protium and deuterium only

(4) Protium, deuterium and tritium

Answer (4)

Sol. Hydrogen has three isotopes :

Protium $_1H^1$

Deuterium $_1H^2$

Tritium $_1H^3$

Their natural abundance is in order H > D > T.

17. The following results were obtained during kinetic studies of the reaction ; 2 A + B → Products

Experiment	[A] (in mol L^{-1})	[B] (in mol L^{-1})	Initial Rate of reaction (in mol L^{-1} min^{-1})
I	0.10	0.20	6.93×10^{-3}
II	0.10	0.25	6.93×10^{-3}
III	0.20	0.30	1.386×10^{-2}

The time (in minutes) required to consume half of A is

(1) 100 (2) 1

(3) 5 (4) 10

Answer (3)

Sol. From experiment I and II, it is observed that order of reaction w.r.t. 3 is zero.

From experiment II and III,

$$\frac{1.386\times10^{-2}}{6.93\times10^{-3}} = \left(\frac{0.2}{0.1}\right)^{\alpha}$$

∴ $\alpha = 1$

Rate = $K[A]^1$

$6.93 \times 10^{-3} = K(0.1)$

$K = 6.93 \times 10^{-2}$

For 2A + B → products

$$2\ Kt = \ln\frac{[A]_0}{[A]}$$

$$t_{1/2} = \frac{0.693}{2K}$$

$$= \frac{0.693}{6.93\times10^{-2}\times2}$$

$= 5$

18. The alkaline earth metal nitrate that does not crystallise with water molecules, is

(1) $Ba(NO_3)_2$ (2) $Ca(NO_3)_2$

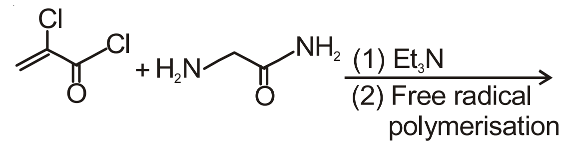
(3) $Mg(NO_3)_2$ (4) $Sr(NO_3)_2$

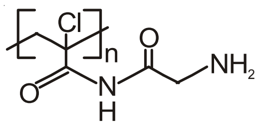
Answer (1)

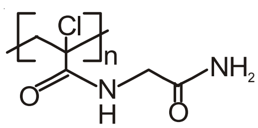
Sol. Down the group as the charge density decreases so chances of formation of hydrate decreases.

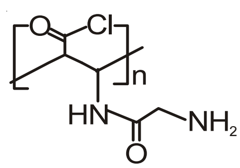
So, $Ba(NO_3)_2$ does not crystallise with water molecules.

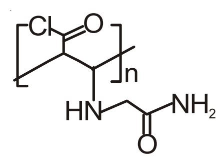
19. Major product of the following reaction is



(1) 

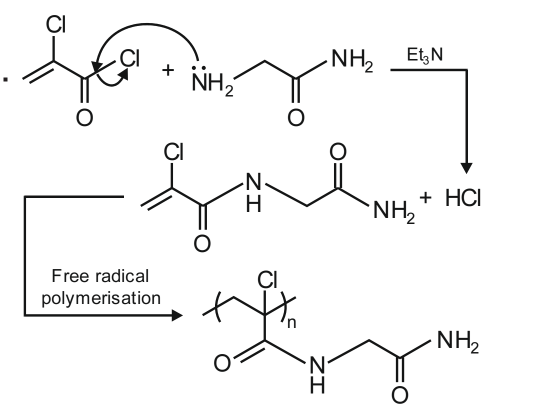
(2) 

(3) 

(4) 

Answer (2)

Sol.



20. Which one of the following statements regarding Henry's law is not correct?

(1) Different gases have different K_H (Henry's law constant) values at the same temperature

(2) The value of K_H increases with increase of temperature and K_H is function of the nature of the gas

(3) The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution

(4) Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids.

Answer (4)

Sol. Solubility decreases with the increase in value of K_H.

21. The major product of following reaction is

$$R-C\equiv N\xrightarrow[(2)\ H_2O]{(1)\ AlH(i\text{-}Bu)_2}?$$

(1) RCH_2NH_2 (2) RCHO

(3) $RCONH_2$ (4) RCOOH

Answer (2)

Sol. (2) $R-C\equiv N\xrightarrow[(2)H_2O]{(1)AlH(i-Bu)_2}R-CHO$

$AlH(i-Bu)_2$ is DIBALH which reduces nitrites to aldehydes.

22. A water sample has ppm level concentration of the following metals: Fe = 0.2 ; Mn = 5.0 ; Cu = 3.0 ; Zn = 5.0. The metal that makes the water sample unsuitable for drinking is:

(1) Cu (2) Mn

(3) Zn (4) Fe

Answer (2)

Sol. Prescribed level of Mn is 0.5 ppm. So water sample containing Mn = 5ppm is water unsuitable for drinking.

23. 20 ml of 0.1 M H_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is : [pK_b of NH_4OH = 4.7]

(1) 9.0 (2) 5.2

(3) 5.0 (4) 9.4

Answer (1)

Sol.

$$\underset{\substack{20\ ml\\0.1\ M}}{H_2SO_4} + \underset{\substack{30\ ml\\0.2\ M}}{2NH_4OH}$$

$$\downarrow$$

$$\underset{\substack{20\ ml\\0.1\ M}}{(NH_4)_2SO_4} + \underset{\substack{10\ ml\\0.2\ M}}{NH_4OH}$$ (remain unreacted)

$$pOH = pK_b + \log\frac{[Salt]}{[Base]}$$

$$= 4.7 + \log\left(\frac{2\times 20\times 0.1}{10\times 0.2}\right)$$

= 4.7 + log2

= 5

pH = 14 – pOH = 9

24. For emission line of atomic hydrogen from n_i = 8 to n_f = n, the plot of wave number ($\bar{\nu}$) against $\left(\frac{1}{n^2}\right)$ will be (The Rydberg constant, R_H is in wave number unit)

(1) Linear with slope R_H

(2) Linear with intercept $-R_H$

(3) Non-linear

(4) Linear with slope $-R_H$

Answer (1)

Sol. $$\bar{\nu} = R_H\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)Z^2\ \ (Z=1)$$

$$\bar{\nu} = R_H\left(\frac{1}{n^2}-\frac{1}{8^2}\right)$$

$$\bar{\nu} = \frac{R_H}{n^2}-\frac{R_H}{64}$$

y = mx + c

$x = \frac{1}{n^2}$, m = R_H (slope)

25. A solution of sodium sulfate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in mol kg^{-1} is:

(1) 16 (2) 4

(3) 8 (4) 12

Answer (2)

Sol. 92g of Na^+ = $\frac{92}{23} = 4$ moles

$$\text{Molality} = \frac{\text{number of moles}}{\text{mass of solvent (in kg)}}$$

$$= \frac{4}{1} = 4\ \text{mol kg}^{-1}$$

26. 0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a container of volume 10 m^3 at 1000 K. Given R is the gas constant in JK^{-1} mol^{-1}, x is

(1) $\frac{2R}{4-R}$ (2) $\frac{2R}{4+R}$

(3) $\frac{4-R}{2R}$ (4) $\frac{4+R}{2R}$

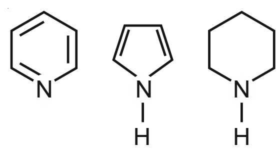
Answer (3)

Sol. PV = nRT (ideal gas equation)

$200 \times 10 \times 1000 = (0.5 + x) \times R \times 1000 \times 1000$

$x = \dfrac{4-R}{2R}$

27. Arrange the following amines in the decreasing order of basicity

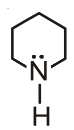


I II III

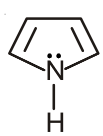
(1) III > II > I (2) I > III > II

(3) III > I > II (4) I > II > III

Answer (3)

Sol.  most basic

 lone pair is not involved in resonance but N atom is sp^2 hybridised.

 lone pair of nitrogen is involved in aromaticity.

28. The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is (Molar mass of $PbSO_4$ = 303 g mol^{-1})

(1) 7.6 (2) 15.2

(3) 11.4 (4) 22.8

Answer (1)

Sol. $PbSO_4 \longrightarrow Pb^{+4} + 2e^-$

$w = Z \times I \times t$

$= \dfrac{303}{2 \times F} \times 0.05F$

$= 7.575$

$\approx 7.6g$

29. Aluminium is usually found in +3 oxidation state. In contrast, thallium exists in +1 and +3 oxidation states. This is due to

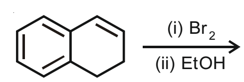
(1) Lattice effect (2) Lanthanoid contraction

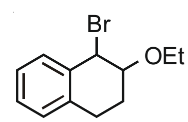
(3) Diagonal relationship (4) Inert pair effect

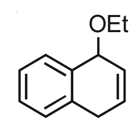
Answer (4)

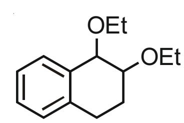
Sol. +1 is more stable form of Thallium due to inert pair effect. For Tl +1 > +3 oxidation state.

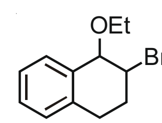
30. The major product of the following reaction is



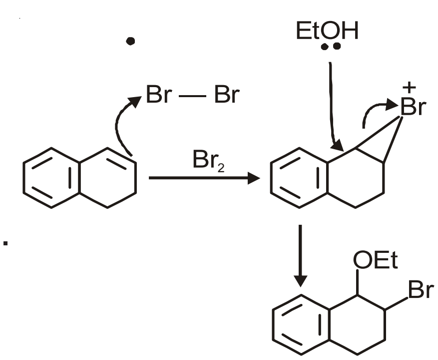
(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. 

❑ ❑ ❑

MATEMATICS

1. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true?

   (1) The lines are all parallel

   (2) The lines are not concurrent

   (3) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

   (4) Each line passes through the origin

Answer (3)

Sol. $px + qy + r = 0$

$\Rightarrow 4px + 4qy + 4r = 0$

$\Rightarrow 4px - 3p + 4qy - 2q + 3p + 2q + 4r = 0$

$\Rightarrow 4px - 3p + 4qy - 2q = 0$

$\Rightarrow p(4x - 3) + q(4y - 2) = 0$

i.e. $(4x - 3) + \lambda(4y - 2) = 0 \quad \left(\text{Where } \lambda = \frac{q}{p}\right)$

$\therefore$ Set of lines are passing through $x = \frac{3}{4}, y = \frac{1}{2}$

2. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is

   (1) 18

   (2) 20

   (3) 22

   (4) 16

Answer (2)

Sol. Given, Variance = $\sigma^2 = \frac{\sum x_i^2}{N} - (\bar{x})^2$

$\Rightarrow 18 = \frac{\sum x_i^2}{5} - (150)^2$

$\Rightarrow \sum x_i^2 = 112590$

$V_{\text{New}} = \frac{112590 + (156)^2}{6} - \left(\frac{750 + 156}{6}\right)^2$

$= 22821 - 22801$

$= 20$

3. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to

   (1) 8 (2) 4

   (3) 6 (4) 14

Answer (1)

Sol. $2^{403} = 8\left(2^4\right)^{100} = 8(16)^{100}$

$= 8(1 + 15)^{100}$

$= 8 + 15\lambda$

When divided by 15, remainder is 8.

Hence fractional part is $\frac{8}{15}$

$\therefore$ Value of K is 8

4. Three circles of radii a, b, c $(a < b < c)$ touch each other externally. If they have x-axis as a common tangent, then

   (1) a, b, c are in A.P.

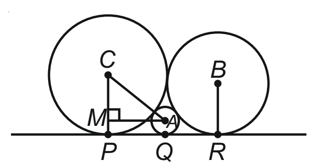
   (2) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

   (3) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

   (4) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

Answer (2)

Sol.



$AM^2 = AC^2 - MC^2$

$= (a + c)^2 - (a - c)^2 = 4ac$

$\because AM = PQ$

$\Rightarrow PQ = 2\sqrt{ac}$

Similarly, $QR = 2\sqrt{ba}$ and $PR = 2\sqrt{bc}$

$\Rightarrow PR = PQ + QR$

$\Rightarrow 2\sqrt{bc} = 2\sqrt{ac} + 2\sqrt{ba}$

$\Rightarrow \frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

5. Let $A = \left\{ \theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary} \right\}$.

Then the sum of the elements in A is

(1) $\frac{5\pi}{6}$ (2) π

(3) $\frac{3\pi}{4}$ (4) $\frac{2\pi}{3}$

Answer (4)

Sol. Let $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$

As z is purely imaginery, $z + \bar{z} = 0$

$$\frac{3+2i\sin\theta}{1-2i\sin\theta} + \frac{3-2i\sin\theta}{1+2i\sin\theta} = 0$$

$$\Rightarrow \frac{(3+2i\sin\theta)(1+2i\sin\theta)+(3-2i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2\theta} = 0$$

$$\Rightarrow \sin^2\theta = \frac{3}{4}$$

$$\Rightarrow \cos^2\theta = \frac{1}{4}$$

$$\theta = n\pi \pm \frac{\pi}{3}$$

$\theta = \frac{2\pi}{3}$ is the only possible value in $\left(\frac{-\pi}{2}, \pi\right)$

6. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to

(1) –512 (2) 512

(3) 256 (4) –256

Answer (4)

Sol. $x = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

Let $\alpha = -1 + i$, $\beta = -1 - i$

$\alpha^{15} + \beta^{15} = (-1 + i)^{15} + (-1 - i)^{15}$

$$= \left(\sqrt{2}e^{i\frac{3\pi}{4}}\right)^{15} + \left(\sqrt{2}e^{-i\frac{3\pi}{4}}\right)^{15}$$

$$= \left(\sqrt{2}\right)^{15}\left[e^{\frac{i45\pi}{4}} + e^{\frac{-i45\pi}{4}}\right]$$

$$= \left(\sqrt{2}\right)^{15}\left[e^{\frac{i5\pi}{4}} + e^{\frac{-i5\pi}{4}}\right]$$

$$= \left(\sqrt{2}\right)^{15} . 2\cos\frac{5\pi}{4} = \frac{-2}{\sqrt{2}}\left(\sqrt{2}\right)^{15}$$

$$= -2\left(\sqrt{2}\right)^{14} = -256$$

7. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is

(1) 200 (2) 350

(3) 500 (4) 300

Answer (4)

Sol. Firstly select 2 girls by $^5C_2$ ways.

3 boys can be selected in 3 ways.

(i) Selection of A and selection of any 2 other boys (except B) in $^5C_2$ ways

(ii) Selection of B and selection of any 2 two other boys (except A) in $^5C_2$ ways

(iii) Selection of 3 boys (except A and B) in $^{15}C_3$ ways

$\Rightarrow$ Number of ways = $^5C_2\,(^5C_2 + {}^5C_2 + {}^5C_3)$

$= 300$

8. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is

(1) $\frac{32}{3}$

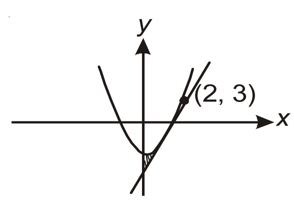
(2) $\frac{8}{3}$

(3) $\frac{56}{3}$

(4) $\frac{14}{3}$

Answer (2)

Sol.



Tangent at $(2, 3): \frac{y+3}{2} = 2x - 1$

$\Rightarrow y + 3 = 4x - 2 \Rightarrow 4x - y - 5 = 0$

$$\text{Area} = \int_0^2 \left[\left(x^2-1\right)-\left(4x-5\right)\right]dx$$

$$= \int_0^2 \left(x^2 - 4x + 4\right)dx$$

$$= \left[\frac{x^3}{3} - 2x^2 + 4x\right]_0^2 = \frac{8}{3} - 8 + 8 = \frac{8}{3}$$

9. For $x \in R - \{0,1\}$, let $f_1(x) = \frac{1}{x}, f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then $J(x)$ is equal to

(1) $f_1(x)$ (2) $\frac{1}{x} f_3(x)$

(3) $f_2(x)$ (4) $f_3(x)$

Answer (4)

Sol. $(f_2 o\ J o\ f_1)(x) = f_3(x) = \frac{1}{1-x}$

$$\Rightarrow (f_2 o J) f_1(x) = \frac{1}{1-x}$$

$$\Rightarrow (f_2 o J)\left(\frac{1}{x}\right) = \frac{1}{1-\frac{1}{\frac{1}{x}}} = \frac{\frac{1}{x}}{\frac{1}{x}-1}$$

$$\Rightarrow (f_2 o J)(x) = \frac{x}{x-1}$$

$$\Rightarrow f_2(J(x)) = \frac{x}{x-1}$$

$$\Rightarrow 1 - J(x) = \frac{x}{x-1} = 1 + \frac{1}{x-1} = 1 - \frac{1}{1-x}$$

$$\therefore J(x) = \frac{1}{1-x} = f_3(x)$$

10. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval

(1) (2, 3]

(2) (3/2, 2]

(3) (1, 3/2]

(4) (3, ∞)

Answer (4)

Sol. $a^2 = \cos^2\theta$, $b^2 = \sin^2\theta$

$e > 2 \Rightarrow 1 + b^2/a^2 > 4 \Rightarrow 1 + \tan^2\theta > 4$

$\Rightarrow \sec^2\theta > 4 = \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Latus rectum,

$$LR = \frac{2b^2}{a} = \frac{2\sin^2\theta}{\cos\theta} = 2(\sec\theta - \cos\theta)$$

$$\frac{d(LR)}{d\theta} = 2(\sec\theta\tan\theta + \sin\theta) > 0\ \forall\theta\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

$$\therefore \text{ min (LR) } = 2\left(\sec\frac{\pi}{3} - \cos\frac{\pi}{3}\right) = 2\left(2 - \frac{1}{2}\right) = 3$$

max (LR) tends to infinity as $\theta \to \frac{\pi}{2}$

11. The value of $\int_0^{\pi} |\cos x|^3\, dx$ is :

(1) 0 (2) $\frac{2}{3}$

(3) $-\frac{4}{3}$ (4) $\frac{4}{3}$

Answer (4)

Sol. $I = \int_0^{\pi} |\cos x|^3\, dx$

$$= 2\int_0^{\pi/2} \cos^3 x\, dx$$

$$= \frac{2}{4}\int_0^{\pi/2} (3\cos x + \cos 3x)\, dx$$

$$= \frac{1}{2}\left(3\sin x + \frac{\sin 3x}{3}\right)_0^{\pi/2}$$

$$= \frac{1}{2}\left(3 - \frac{1}{3}\right)$$

$$= \frac{4}{3}$$

12. Let $a_1, a_2 \ldots a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to

(1) 47 (2) 57

(3) 52 (4) 42

Answer (3)

Sol. $S = \frac{30}{2}[2a_1 + 29d]$

$T = \frac{15}{2}[2a_1 + 14(2d)]$

According to Question $S - 2T = 75$

$\Rightarrow$ $30\,a_1 + 29.15d \quad -30a_1 - 30.14d = 75$

$\Rightarrow$ $d = 5$

Also, $a_5 = 27 \Rightarrow a_1 + 4d = 27 \Rightarrow a_1 = 7,\ d = 5$

So $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

13. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\,\vec{c} = 4$, then $|\vec{c}|^2$ is equal to:

(1) $\frac{17}{2}$ (2) $\frac{19}{2}$

(3) 9 (4) 8

Answer (2)

Sol. $|\vec{a} \times \vec{c}|^2 = |\vec{a}|^2 |\vec{c}|^2 - (\vec{a} \cdot \vec{c})^2$

$\Rightarrow |-\vec{b}|^2 = 2|\vec{c}|^2 - 16$

$\Rightarrow 3 = 2|\vec{c}|^2 - 16$

$\Rightarrow |\vec{c}|^2 = \frac{19}{2}$

14. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals:

(1) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(2) $13 - 4\cos^2\theta + 6\cos^4\theta$

(3) $13 - 4\cos^6\theta$

(4) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

Answer (3)

Sol. $3(1 - 2\sin\theta\cos\theta)^2 + 6(1 + 2\sin\theta\cos\theta) + 4\sin^6\theta$

$= 3(1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta) + 6 + 12\sin\theta\cos\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta\cos^2\theta + 4\sin^6\theta$

$= 9 + 12\cos^2\theta(1 - \cos^2\theta) + 4(1 - \cos^2\theta)^3$

$= 9 + 12\cos^2\theta - 12\cos^4\theta + 4(1 - \cos^6\theta - 3\cos^2\theta + 3\cos^4\theta)$

$= 9 + 4 - 4\cos^6\theta$

$= 13 - 4\cos^6\theta$

15. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X = 1) + P(X = 2)$ equals:

(1) $\frac{24}{169}$ (2) $\frac{25}{169}$

(3) $\frac{49}{169}$ (4) $\frac{52}{169}$

Answer (2)

Sol. X = number of aces drawn

$\therefore\ P(X = 1) + P(X = 2)$

$= \left\{\frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52}\right\} + \left\{\frac{4}{52} \times \frac{4}{52}\right\}$

$= \frac{24}{169} + \frac{1}{169}$

$= \frac{25}{169}$

16. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

(1) (4, –4)

(2) $(5, 2\sqrt{6})$

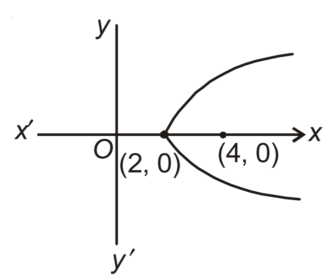
(3) $(6, 4\sqrt{2})$

(4) (8, 6)

Answer (4)

Sol. Vertex and focus of given parabola is (2, 0) and (4, 0) respectively

Equation of parabola is



$(y - 0)^2 = 4 \times 2(x - 2)$

$y^2 = 8x - 16$

Clearly (8, 6) does not lies on given parabola.

17. The system of linear equations

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$

(1) has infinitely many solutions for $a = 4$

(2) is inconsistent when $|a| = \sqrt{3}$

(3) has a unique solution for $|a| = \sqrt{3}$

(4) is inconsistent when $a = 4$

Answer (2)

Sol. The equations are

$x + y + z = 2$ ...(1)

$2x + 3y + 2z = 5$ ...(2)

$2x + 3y + (a^2 - 1)z = a + 1$ ...(3)

$$\text{Here, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & a^2-3 \end{vmatrix}$$

$= a^2 - 3$

$a^2 - 3 = 0 \quad \Rightarrow \quad |a| = \sqrt{3}$

If $a^2 = 3$, then plane represented by (2) and (3) are parallel.

$\therefore$ Given system of equation is inconsistent.

18. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point:

(1) (3, 2, 1) (2) (3, 3, –1)

(3) (–3, 0, –1) (4) (–3, 1, 1)

Answer (1)

Sol. Equation of plane through intersection of planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$(2x + 3y - z + 4) + \lambda(x + y + z - 1) = 0$

$(2 + \lambda)x + (3 + \lambda)y + (-1 + \lambda)z + (4 - \lambda) = 0$ ...(1)

$\because$ This plane is parallel to y-axis.

$\Rightarrow 0 \times (2 + \lambda) + 1 \times (3 + \lambda) + 0 \times (-1 + \lambda) = 0$

$\Rightarrow \lambda = -3$

$\therefore$ Equation of required plane

$-x - 4z + 7 = 0$

$x + 4z - 7 = 0$

$\therefore$ (3, 2, 1) lies on the plane.

19. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is:

(1) $\sqrt{3}y = x + 3$ (2) $2\sqrt{3}y = 12x + 1$

(3) $\sqrt{3}y = 3x + 1$ (4) $2\sqrt{3}y = -x - 12$

Answer (1)

Sol. A tangent to parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$ ...(1)

This line is also the tangent to circle

$x^2 + y^2 - 6x = 0$

$\therefore$ Centre of circle = (3, 0)

radius of circle = 3

$$\therefore \frac{\left|3m + \frac{1}{m}\right|}{\sqrt{1+m^2}} = 3$$

$$\therefore m = \pm\frac{1}{\sqrt{3}}$$

Equation of common tangents are: $y = \pm\frac{1}{\sqrt{3}}x \pm \sqrt{3}$

$\therefore \sqrt{3}y = x + 3$ is one of the common tangent

20. If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to

(1) $\frac{13}{16}$ (2) $\frac{7}{64}$

(3) $\frac{1}{4}$ (4) $\frac{49}{16}$

Answer (4)

Sol. $\because \quad x\frac{dy}{dx} + 2y = x^2$

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

$$\text{I.F.} = e^{\int\frac{2}{x}dx} = e^{2\ln x} = x^2.$$

Solution of differential equation is:

$$y \cdot x^2 = \int x \cdot x^2 dx$$

$$y \cdot x^2 = \frac{x^4}{4} + c$$

$\because \quad y(1)=1$

$\therefore \quad c=\frac{3}{4}$

$y \cdot x^2=\frac{x^2}{4}+\frac{3}{4}$

$\therefore \quad y=\frac{x^2}{4}+\frac{3}{4x^2}$

$\therefore \quad y\left(\frac{1}{2}\right)=\frac{1}{16}+3=\frac{49}{16}$

21. $\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

(1) Exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

(2) Does not exist

(3) Exists and equals $\frac{1}{4\sqrt{2}}$

(4) Exists and equals $\frac{1}{2\sqrt{2}}$

Answer (3)

Sol. $\ell = \lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

$$= \lim_{y\to 0}\frac{\left(\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}\right)\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0}\frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0}\frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \lim_{y\to 0}\frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \frac{1}{2\sqrt{2}\times 2}=\frac{1}{4\sqrt{2}}$$

22. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to :

(1) $\frac{8}{15}$ (2) $\frac{7}{17}$

(3) $\frac{8}{17}$ (4) $\frac{4}{9}$

Answer (1)

Sol. $y = 10 - x^2$ ...(1)

$y = 2 + x^2$ ...(2)

For intersection point of (1) and (2)

(1) + (2)

$2y = 12 \Rightarrow y = 6$

from (1) $x = \pm 2$

differentiate with respect to x equation (1)

$$\frac{dy}{dx}=-2x\Rightarrow\left(\frac{dy}{dx}\right)_{(2,6)}=-4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)}=4$$

differentiate with respect to x equation (2)

$$\frac{dy}{dx}=2x\Rightarrow\left(\frac{dy}{dx}\right)_{(2,6)}=4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)}=-4$$

$$\Rightarrow \tan\theta = \left(\frac{-4-4}{1+(-4)\times 4}\right)=\frac{-8}{15}\Rightarrow|\tan\theta|=\frac{8}{15}$$

23. Let $f : R \to R$ be a function defined as

$$f(x)=\begin{cases} 5, & \text{if } x\le 1 \\ a+bx, & \text{if } 1<x<3 \\ b+5x, & \text{if } 3\le x<5 \\ 30, & \text{if } x\ge 5 \end{cases}$$

Then, f is:

(1) Continuous if $a = -5$ and $b = 10$

(2) Continuous if $a = 5$ and $b = 5$

(3) Continuous if $a = 0$ and $b = 5$

(4) Not continuous for any values of a and b

Answer (4)

Sol. If $f(x)$ is continuous at $x = 1$, then

$f(1^-) = f(1) = f(1^+)$

$\Rightarrow 5 = a + b$ ...(1)

If $f(x)$ is continuous at $x = 3$, then

$f(3^-) = f(3) = f(3^+)$

$\Rightarrow a + 3b = b + 15$ ...(2)

If $f(x)$ is continuous at $x = 5$, then

$f(5^-) = f(5) = f(5^+)$

$\Rightarrow$ $b + 25 = 30$ ...(3)

From (3) $b = 5 \Rightarrow$ from (1), $a = 0$

but $a = 0$, $b = 5$ do not satisfy equation (2)

$\Rightarrow f(x)$ is not continuous for any values of a and b

24. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix

A^{-50} when $\theta = \frac{\pi}{12}$, is equal to:

(1) $\begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Answer (3)

Sol. $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $|A| = 1$

$$\text{adj}(A) = \begin{bmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{bmatrix}^T$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = B$$

$$B^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$B^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

$$\Rightarrow A^{-50} = B^{50} = \begin{bmatrix} \cos(50\theta) & \sin(50\theta) \\ -\sin(50\theta) & \cos(50\theta) \end{bmatrix}$$

$$(A^{-50})_{\theta = \frac{\pi}{12}} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

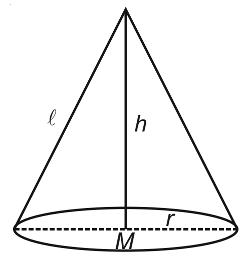
25. The maximum volume (in cu. m) of the right circular cone having slant height 3 m is :

(1) $\frac{4}{3}\pi$ (2) $2\sqrt{3}\,\pi$

(3) $3\sqrt{3}\,\pi$ (4) $6\,\pi$

Answer (2)

Sol.



$h^2 + r^2 = \ell^2 = 9$ ...(1)

$V = \frac{1}{3}\pi r^2 h$ ...(2)

From (1) and (2),

$$V = \frac{1}{3}\pi\,(9 - h^2)h$$

$$V = \frac{1}{3}\pi\,(9h - h^3)$$

For maxima/minima,

$$\frac{dV}{dh} = 0 \Rightarrow \frac{1}{3}\pi(9 - 3h^2) = 0$$

$$\Rightarrow h = \pm\sqrt{3} \Rightarrow h = \sqrt{3} \quad (\because\ h > 0)$$

$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(-6h)$$

$$\left(\frac{d^2V}{dh^2}\right)_{\text{at } h = \sqrt{3}} < 0$$

$\Rightarrow$ at $h = \sqrt{3}$, volume is maximum

$$\Rightarrow V_{\text{max.}} = \frac{1}{3}\pi(9 - 3)\sqrt{3} = 2\sqrt{3}\pi$$

26. If the Boolean expression

$(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to

$p \wedge q$, where $\oplus$, $\odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is

(1) $(\vee, \wedge)$ (2) $(\vee, \vee)$

(3) $(\wedge, \wedge)$ (4) $(\wedge, \vee)$

Answer (4)

Sol. Check by options

(1) $(p \vee q) \wedge (\sim p \wedge q) = (\sim p \wedge q)$

(2) $(p \vee q) \wedge (\sim p \vee q) = q$

(3) $(p \wedge q) \wedge (\sim p \wedge q) = F$

(4) $(p \wedge q) \wedge (\sim p \vee q) = p \wedge q$

27. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$, then x is equal to :

(1) $\frac{\sqrt{146}}{12}$

(2) $\frac{\sqrt{145}}{12}$

(3) $\frac{\sqrt{145}}{10}$

(4) $\frac{\sqrt{145}}{11}$

Answer (2)

Sol. $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} \qquad \left(x > \frac{3}{4}\right)$

$\Rightarrow \cos^{-1}\left(\frac{2}{3x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{3}{4x}\right)$

$\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\left(\frac{2}{3x}\right) = \sin^{-1}\left(\frac{3}{4x}\right)$

$\Rightarrow \cos^{-1}\left(\frac{2}{3x}\right) = \cos^{-1}\left(\frac{\sqrt{16x^2-9}}{4x}\right)$

$\because \sin^{-1}\left(\frac{3}{4x}\right) = \cos^{-1}\left(\frac{\sqrt{16x^2-9}}{4x}\right)$

$\Rightarrow \frac{2}{3x} = \frac{\sqrt{16x^2-9}}{4x}$

$\Rightarrow x^2 = \frac{64+81}{9\times16}$

$\boxed{x = \frac{\sqrt{145}}{12}} \qquad \left(\because x > \frac{3}{4}\right)$

28. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}}\,dx$ is equal to :

(where c is a constant of integration)

(1) $\frac{1}{2}\log_e\left|\sec(x^2-1)\right| + c$

(2) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$

(3) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$

(4) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right| + c$

Answer (2 and 3 both are correct)

Sol. $I = \int x\sqrt{\frac{2\sin(x^2-1) - 2\sin(x^2-1)\cos(x^2-1)}{2\sin(x^2-1) + 2\sin(x^2-1)\cos(x^2-1)}}\,dx$

$I = \int x\sqrt{\frac{1-\cos(x^2-1)}{1+\cos(x^2-1)}}\,dx$

$I = \int x\left|\tan\left(\frac{x^2-1}{2}\right)\right|dx$, Now let $\frac{x^2-1}{2} = t$

$\frac{2x}{2}dx = dt$

$I = \int|\tan(t)|dt$

$I = \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c = \frac{1}{2}\ln\left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$

29. If a, b and c be three distinct real numbers in G.P. and $a + b + c = xb$, then x cannot be:

(1) 2 (2) –3

(3) –2 (4) 4

Answer (1)

Sol. $\because$ a, b, c, are in G.P.

$\boxed{b^2 = ac}$

Now $a + b + c = xb$

$\Rightarrow a + c = (x-1)b$ Now square it

$\Rightarrow a^2 + c^2 + 2ac = (x-1)^2b^2$

$\Rightarrow a^2 + c^2 = (x-1)^2 ac - 2ac$

$\Rightarrow a^2 + c^2 = ac[(x-1)^2 - 2]$

$a^2 + c^2 = ac[x^2 - 2x - 1]$

$\because$ $a^2 + c^2$ are positive

and $b^2 = ac$ which is also positive

so $x^2 - 2x - 1$ would be positive

but for $x = 2$, $x^2 - 2x - 1$ is negative

so x cannot take 2.

30. The equation of the line passing through (–4, 3, 1), parallel to the plane x + 2y – z – 5 = 0 and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is :

(1) $\frac{x-4}{2} = \frac{y+3}{2} = \frac{z+1}{4}$

(2) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

(3) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(4) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

Answer (3)

Sol. Let any point on the intersecting line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1} = \lambda$$

$\Rightarrow (-3\lambda - 1,\ 2\lambda + 3,\ -\lambda + 2)$

which also lie on a line which passes through (–4, 3, 1)

So D.R. of line

$= \langle -3\lambda - 1 + 4, 2\lambda + 3 - 3, -\lambda + 2 - 1 \rangle$

$= \langle -3\lambda + 3, 2\lambda, -\lambda + 1 \rangle$

and this line is parallel to the plane

$x + 2y - z - 5 = 0$

so perpendicular vector to the line is $\hat{i} + 2\hat{j} - \hat{k}$

Now $(-3\lambda + 3)(1) + (2\lambda)(2) + (-\lambda + 1)(-1) = 0$

$\boxed{\lambda = -1}$

Now D.R. of line = $\langle 3, -1, 1 \rangle$

Now equation of line is

$$\frac{(x+4)}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$