$$= \begin{vmatrix} n! & \sin\left(x+\frac{n\pi}{2}\right) & \cos\left(x+\frac{n\pi}{2}\right) \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}; \qquad \left(\frac{d^n}{dx^n}\{f(x)\}\right)_{x=0} = \begin{vmatrix} n! & \sin\frac{n\pi}{2} & \cos+\frac{n\pi}{2} \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix} = 0$$

# PROBLEM-SOLVING TACTICS

Let $\Delta(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$, then $\Delta'(x) = \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ and in general

$\Delta^n(x) = \begin{vmatrix} f_1^n(x) & f_2^n(x) & f_3^n(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ where n is any positive integer and $f^n(x)$ denotes the $n^{th}$ derivative of f(x).

Let $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ l & m & n \end{vmatrix}$, where a, b, c, $l$, m and n are constants.

$$\Rightarrow \int_a^b \Delta(x)\,dx = \begin{vmatrix} \int_a^b f(x)\,dx & \int_a^b g(x)\,dx & \int_a^b h(x)\,dx \\ a & b & c \\ l & m & n \end{vmatrix}$$

If the elements of more than one column or rows are functions of x then the integration can be done only after evaluation/expansion of the determinant.

# FORMULAE SHEET

**(a)** Determinant of order 3 × 3 = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ b_3 & b_3 \end{vmatrix}$

**(b)** In the determinant D = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, minor of $a_{12}$ is denoted as $M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ and so on.

**(c)** Cofactor of an element $a_{ij} = C_{ij} = (-1)^{i+j} M_{ij}$

**(d)** Properties of determinants:

**(i) Reflection property:** $\left|A_{i\times j}\right| = \left|A_{j\times i}\right|$

**(ii) All-zero property:** If all the elements of a row (or column) are zero, then the determinant is zero.

**(iii) Proportionality (Repetition) Property:** If all the elements of a row (or column) are proportional (identical) to the elements of some other row (or column), then the determinant is zero.

**(iv) Switching Property:** The interchange of any two rows (or columns) of the determinant changes its sign.

**(v) Scalar Multiple Property:** If all the elements of a row (or column) of a determinant are multiplied by a non-zero constant, then the determinant gets multiplied by the same constant.

**(vi)** Sum Property: $\begin{vmatrix} a_1+b_1 & c_1 & d_1 \\ a_2+b_2 & c_2 & d_2 \\ a_3+b_3 & c_3 & d_3 \end{vmatrix} = \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} + \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}$

**(vii)** Property of Invariance: $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+\alpha b_1+\beta c_1 & b_1 & c_1 \\ a_2+\alpha b_2+\beta c_2 & b_2 & c_2 \\ a_3+\alpha b_3+\beta c_3 & b_3 & c_3 \end{vmatrix}$

That is, a determinant remains unaltered under an operation of the form $C_i \to C_i + \alpha C_j + \beta C_k$, where $j, k \neq i$, or an operation of the form $R_i \to R_i + \alpha R_j + \beta R_k$, where $j, k \neq i$.

**(viii)** Triangle Property: $\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3$

**(e)** Cramer's rule : if $a_1x+b_1y+c_1z=d_1$, $a_2x+b_2y+c_2z=d_2$ and $a_3x+b_3y+c_3z=d_3$ then $x=\frac{\Delta_1}{\Delta}, y=\frac{\Delta_2}{\Delta}, z=\frac{\Delta_3}{\Delta}$ where

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\ \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},\ \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \text{ and } \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}.$$

And if $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ then $x=\frac{\Delta_1}{\Delta}\ y=\frac{\Delta_2}{\Delta}$.

Where $\Delta_1 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}$ and $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

**(f)** **(i)** lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ are concurrent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$

**(ii)** $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$

**(iii)** area of a triangle whose vertices are $(x_r, y_r)$; r = 1, 2, 3 is : $D = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

**(iv)** Equation of a straight line passing through $(x_1, y_1)$ & $(x_2, y_2)$ is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0$