(g) If $\Delta(x) = \begin{vmatrix} f_1(x) & g_1(x) \\ f_2(x) & g_2(x) \end{vmatrix}$ then $\Delta'(x) = \begin{vmatrix} f_1'(x) & g_1'(x) \\ f_2(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1(x) \\ f_2'(x) & g_2'(x) \end{vmatrix}$ or $\begin{vmatrix} f_1'(x) & g_1(x) \\ f_2'(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(h) If $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ \alpha & \beta & \gamma \end{vmatrix}$ then $\int \Delta(x)dx = \begin{vmatrix} \int f(x)dx & \int g(x)dx & \int h(x)dx \\ a & b & c \\ \alpha & \beta & \gamma \end{vmatrix}$

Solved Examples

JEE Main/Boards

Example 1: Prove that

$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$. Use p + q + r = 0.

Sol: By using the expansion formula of determinants we can prove this.

L.H.S. $= \begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} =$

$pa\begin{vmatrix} ra & pb \\ pc & qa \end{vmatrix} - qb\begin{vmatrix} qc & pb \\ rb & qa \end{vmatrix} + rc\begin{vmatrix} qc & ra \\ rb & pc \end{vmatrix}$

$= pa(a^2qr - p^2bc) - qb(q^2ac - prb^2) + rc(pqc^2 - r^2ab)$

$= a^3pqr - p^3abc - q^3abc + b^3pqr - r^3abc$

$= pqr(a^3 + b^3 + c^3) - abc(p^3 + q^3 + r^3)$

$\because$ $p + q + r = 0$ ... (given)

$(p + q + r)^3 = 0$

$\Rightarrow p^3 + q^3 + r^3 - pqr = 0 \Rightarrow p^3 + q^3 + r^3 = 3pqr$

$\Rightarrow$ L.H.S. $= pqr(a^3 + b^3 + c^3) - abc(3pqr)$

$\Rightarrow$ L.H.S. $= pqr(a^3 + b^3 + c^3 - 3abc)$ .... (i)

R.H.S. $= pqr\left[a\begin{vmatrix} a & b \\ c & a \end{vmatrix} - b\begin{vmatrix} c & b \\ b & a \end{vmatrix} + c\begin{vmatrix} c & a \\ b & c \end{vmatrix} \right]$

$= pqr[a(a^2 - bc) - b(ca - b^2) + c(c^2 - ab)]$

$= pqr[a^3 - abc - abc + b^3 + c^3 - abc]$

$\Rightarrow$ R.H.S. $= pqr(a^3 + b^3 + c^3 - 3abc)$ .... (ii)

From eq. (i) and (ii), we get

$\therefore$ L.H.S. = R.H.S.

Example 2: Prove that the determinant

$\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

Sol: Simply by expanding the given determinant we can prove it.

We have, $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$= x\begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin\theta\begin{vmatrix} -\sin\theta & 1 \\ \cos\theta & x \end{vmatrix} + \cos\theta\begin{vmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{vmatrix}$

$x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

$= -x^3 - x + x(\sin^2\theta + \cos^2\theta) = -x^3 - x + x$

Thus, the determinant is independent of θ.

Example 3: Solve the equation $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$, $a \neq 0$.

Sol: We can expand the above determinant by applying the invariance and scalar multiple properties, and hence we can easily solve this problem.

We have, $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$

Operation: $C_1 \rightarrow C_1 + C_2 + C_3$

$\begin{vmatrix} 3x+a & x & x \\ 3x+a & x+a & x \\ 3x+a & x & x+a \end{vmatrix} = 0 \Rightarrow (3x+a)\begin{vmatrix} 1 & x & x \\ 1 & x+a & x \\ 1 & x & x+a \end{vmatrix} = 0$

Operating $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$

We get $(3x+a)\begin{vmatrix} 1 & x & x \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} = 0$

$\Rightarrow (3x+a)\begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} = 0 \qquad \Rightarrow a^2(3x+a) = 0$

$\Rightarrow 3x + a = 0$, $[\because a \neq 0] \Rightarrow x = -\dfrac{a}{3}$

Hence Proved.

Example 4: Solve, using Cramer's rule $3x - 2y + 4z = 5$; $x + y + 3z = 2$; $-x + 2y - z = 1$

Sol: By defining D, D_1, D_2, D_3 and by using Cramer's Rule we will get required result.

$D = \begin{vmatrix} 3 & -2 & 4 \\ 1 & 1 & 3 \\ -1 & 2 & -1 \end{vmatrix} = -5$

$D_1 = \begin{vmatrix} 5 & -2 & 4 \\ 2 & 1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = -33$, $D_2 = \begin{vmatrix} 3 & 5 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & -1 \end{vmatrix} = -13$

$D_3 = \begin{vmatrix} 3 & -2 & 5 \\ 1 & 1 & 2 \\ -1 & 2 & 1 \end{vmatrix} = 12$

By Cramer's Rule, $x = \dfrac{D_1}{D} = \dfrac{-33}{-5} = \dfrac{33}{5}$,

$y = \dfrac{D_2}{D} = \dfrac{-13}{-5} = \dfrac{13}{5}$; $z = \dfrac{D_3}{D} = \dfrac{12}{-5} = \dfrac{-12}{5}$

Example 5: Solve the following system of equations by Cramer's Rule

$2x - y + 3z = 9$; $x + y + z = 6$; $x - y + z = 2$

Sol: By defining $\Delta, \Delta_x, \Delta_y, \Delta_z$ and by using Cramer's Rule we will get the required result.

Here, $\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

$= 2(1 + 1) + 1(1 - 1) + 3(-1 - 1) = -2$,

$\Delta_x = \begin{vmatrix} 9 & -1 & 3 \\ 6 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 9(1 + 1) + 1(6 - 2) + 3(-6 - 2) = -2$

$\Delta_y = \begin{vmatrix} 2 & 9 & 3 \\ 1 & 6 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 2(6 - 2) - 9(1 - 1) + 3(2 - 6) = -4$

$\Delta_z = \begin{vmatrix} 2 & -1 & 9 \\ 1 & 1 & 6 \\ 1 & -1 & 2 \end{vmatrix} = 2(2 + 6) + 1(2 - 6) + 9(-1 - 1) = -6$

By Cramer's Rule

$x = \dfrac{\Delta_x}{\Delta} = 1$, $y = \dfrac{\Delta_y}{\Delta} = 2$, $z = \dfrac{\Delta_z}{\Delta} = 3$

Example 6: Show that

$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

Sol: By using invariance and scalar multiple property we can expand given determinant and can prove it.

$\Delta = \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$

$[C_1 \rightarrow C_1 + C_2 + C_3]$

$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$

$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & b+c+a & 0 \\ 0 & a & c+a+b \end{vmatrix}$

$[$ by $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1]$

$= 2(a+b+c)[1\{(b+c+a)^2 - 0\}]$

$= 2(a+b+c)(a+b+c)^2 = 2(a+b+c)^3$

Example 7: Using determinants, show that the points $(11, 7)$, $(5, 5)$ and $(-1, 3)$ are collinear.

Sol: If these points are collinear then the area of a triangle made by joining these points will be zero.

The area of the triangle formed by the given points

$$= \frac{1}{2} \begin{vmatrix} 11 & 7 & 1 \\ 5 & 5 & 1 \\ -1 & 3 & 1 \end{vmatrix}$$

Operate: $R_1 \to R_1 - R_2$; $R_2 \to R_2 - R_3$

$$= \frac{1}{2} \begin{vmatrix} 6 & 2 & 0 \\ 6 & 2 & 0 \\ -1 & 3 & 1 \end{vmatrix} = \frac{1}{2} \cdot 0 = 0$$

($\because$ R_1 and R_2 are identical)

Hence, the given points are collinear.

Example 8: If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$.

Sol: By using the multiplication property of matrices we can solve given problem.

$A^2 + B^2 = AA + BB$

= A(BA) + B(AB) [Given AB = B and BA = A]

= (AB)A + (BA)B

[Matrix multiplication is associatively]=BA + AB

[Given AB = B and BA = A]   =A+B

[Given AB = B and BA = A]

Example 9: Find the value of $\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$

Sol: By applying the invariance property we can find the value of the given determinant.

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 9 & 16 & 25 & 36 \\ 16 & 25 & 36 & 49 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 9 & 16 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 11 \\ 7 & 9 & 11 & 13 \end{vmatrix}$$

[Applying $R_4 \to R_4 - R_3, R_3 \to R_3 - R_2, R_2 \to R_2 - R_1$]

$$= \begin{vmatrix} 1 & 4 & 9 & 16 \\ 3 & 5 & 7 & 9 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{vmatrix}$$

[Applying $R_4 \to R_4 - R_3$, $R_3 \to R_3 - R_2$]

=0

JEE Advanced/Boards

Example 1: Without expanding, evaluate the determinant

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$$

Sol: By using the formula sin(A+ B) = sin A cos B + cos A sin B and invariance property of determinants we can expand the given determinant.

Let $\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$

$\Rightarrow \Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha\cos\delta+\cos\alpha\sin\delta \\ \sin\beta & \cos\beta & \sin\beta\cos\delta+\cos\beta\sin\delta \\ \sin\gamma & \cos\gamma & \sin\gamma\cos\delta+\cos\gamma\sin\delta \end{vmatrix}$

$\Rightarrow \quad \Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & 0 \\ \sin\beta & \cos\beta & 0 \\ \sin\gamma & \cos\gamma & 0 \end{vmatrix}$

[Applying $C_3 \to C_3 - \cos\delta . C_1 - \sin\delta . C_2$]

$\Rightarrow \quad \Delta = 0 \qquad$ [$\because$ C_3 consists of all zeroes]

Example 2: By using properties of determinants prove that

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

Here in this problem by using invariance and scalar multiple properties we will expand the given determinant and we will prove it.

Sol: L.H.S. = $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix}$

[Applying $C_1 \to C_1 + C_2 + C_3$]

$= (1+x+x^2) \begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$

$= (1+x+x^2) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x & x-x^2 \\ 0 & x^2-x & 1-x^2 \end{vmatrix}$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$= (1 + x + x^2)(1)\{(1-x)(1-x^2) - (x^2 - x)(x - x^2)\}$

$= (1 + x + x^2)(1-x)^2\{1 + x + x^2\}$

$= \{(1-x)(1+x+x^2)\}^2 = (1-x^3)^2$ = R.H.S.

Example 3: Show that $x = -(a+b+c)$ is one root of

the equation: $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$ and solve the

equation completely.

Sol: We can expand given determinant using the invariance and scalar multiple properties and by solving we will find out required result.

By $C_1 \to C_1 + C_2 + C_3$, we get

$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+c & a \\ x+a+b+c & a & x+b \end{vmatrix} = 0$

$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+c & a \\ 1 & a & x+b \end{vmatrix} = 0$

$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x-b+c & a-c \\ 0 & a-b & x+b-c \end{vmatrix} = 0$

$R_2 \to R_2 - R_1;\ R_3 \to R_3 - R_1$

On expanding by first column, we get

$(x+a+b+c)[(x-b+c)(x+b-c) - (a-b)(a-c)] = 0$

$\Rightarrow (x+a+b+c)[x^2 - (b-c)^2 - (a^2 - ac - ab + bc)] = 0$

$\Rightarrow (x+a+b+c)(x^2 - b^2 - c^2 + 2bc - a^2 + ac + ab - bc) = 0$

$\Rightarrow (x+a+b+c)(x^2 - a^2 - b^2 - c^2 + ab + bc + ca) = 0$

Either $x + a + b + c = 0 \Rightarrow x = -(a + b + c)$

or $x^2 - a^2 - b^2 - c^2 + ab + bc + ca = 0$

$\Rightarrow x = \pm\sqrt{a^2 + b^2 + c^2 - ab - bc - ca}$

Example 4: If the area of a triangle is 35 sq. units with vertices (2, –6), (5, 4) and (k, 4), then find k.

Sol: As we know that the area of the triangle =

$\frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$ where (a, b) (c, d) (e, f) are the vertices of

triangle. Therefore by substituting the value of vertices we will get required result.

Let the vertices of triangle be A(2, –6), B(5, 4) and C(k, 4).

Since the area of the triangle ABC is 35 sq. units, we

have, $\frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \pm 35 \Rightarrow \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 3 & 10 & 0 \\ k-2 & 10 & 0 \end{vmatrix} = \pm 35$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$\Rightarrow \frac{1}{2}\begin{vmatrix} 3 & 10 \\ k-2 & 10 \end{vmatrix} = \pm 35$ [Expanding along C_3]

$\Rightarrow \frac{1}{2}\{30 - 10(k-2)\} = \pm 35$

$\Rightarrow 30 - 10k + 20 = \pm 70 \Rightarrow 10k = 50 \mp 70$

$\Rightarrow k = +12$ or $k = -2$

Example 5: Solve the following system of equations by using determinants: $x + y + z = 1$,

$ax + by + cz = k$; $a^2x + b^2y + c^2z = k^2$

Sol: Here in this problem first define D, D_1, D_2 and D_3. then by using Cramer's rule we can solve it.

We have, $D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$

[Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$]

$= (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$

$= (b-a)(c-a).1.\begin{vmatrix} 1 & 1 \\ b+a & c+a \end{vmatrix}$

[Expanding along R_1]

$= (b-a)(c-a)(c+a-b-a)$

$= (b-a)(c-a)(a-b)$ ... (i)

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ k & b & c \\ k^2 & b^2 & c^2 \end{vmatrix} = (b-c)(c-k)(k-b)$$

[Replacing a by k in (i)]

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ a & k & c \\ a^2 & k^2 & c^2 \end{vmatrix} = (k-c)(c-a)(a-k)$$

[Replacing b by k in (i)]

and $D_3 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & k \\ a^2 & b^2 & k^2 \end{vmatrix} = (a-b)(b-k)(k-a)$

[Replacing c by k in (i)]

$$\therefore \quad x = \frac{D_1}{D} = \frac{(b-c)(c-k)(k-b)}{(b-c)(c-a)(a-b)} = \frac{(c-k)(k-b)}{(c-a)(a-b)},$$

$$y = \frac{D_2}{D} = \frac{(k-c)(c-a)(a-k)}{(b-c)(c-a)(a-b)} = \frac{(k-c)(a-k)}{(b-c)(a-b)}$$

$$z = \frac{D_3}{D} = \frac{(a-b)(b-k)(k-a)}{(b-c)(c-a)(a-b)} = \frac{(k-a)(b-k)}{(c-a)(b-c)}$$

Example 6: Show that

$$\begin{vmatrix} 1+a_1+b_1 & a_1+b_2 & a_1+b_3 \\ a_2+b_1 & 1+a_2+b_2 & a_2+b_3 \\ a_3+b_1 & a_3+b_2 & 1+a_3+b_3 \end{vmatrix}$$

$$= 1 + \sum_{i=1}^{3}(a_i+b_i) + \sum_{1\le i<j\le 3}(a_i-a_j)(b_j-b_i)$$

Sol: By putting $\alpha = a_1 - a_2$, $\beta = a_2 - a_3$, then $\alpha + \beta = a_1 - a_3$ $u = b_1 - b_2$, $v = b_2 - b_3$, then $u + v = b_1 - b_3$. Using the invariance property expand the given determinant, and then comparing it to the R.H.S. of the given problem we can prove it.

Let Now R.H.S.

$$= 1 + \sum_{i=1}^{3}(a_i+b_i) + \sum_{1\le i<j\le 3}(a_i-a_j)(b_j-b_i)$$

$$= 1 + (a_1+b_1+a_2+b_2+a_3+b_3) + (a_1-a_2)(b_2-b_1)$$

$$+ (a_2-a_3)(b_3+b_2) + (a_1-a_3)(b_3-b_1)$$

$$= 1 + (a_1+b_1+a_2+b_2+a_3+b_3) - \alpha u - \beta v - (\alpha+\beta)(u+v)$$

$$= 1 + (a_1+b_1+a_2+b_2+a_3+b_3) - 2\alpha u - 2\beta v - \beta u - \alpha v \quad \dots (i)$$

Now L.H.S. $= \begin{vmatrix} 1+\alpha & \alpha-1 & \alpha \\ \beta & 1+\beta & \beta-1 \\ a_3+b_1 & a_3+b_2 & 1+a_3+b_3 \end{vmatrix}$

$[R_1 \to R_1 - R_2, R_2 \to R_2 - R_3]$

$$= \begin{vmatrix} 2 & -1 & \alpha \\ -1 & 2 & \beta-1 \\ u & v-1 & 1+a_3+b_3 \end{vmatrix}$$

$[C_1 \to C_1 - C_2, C_2 \to C_2 - C_3]$

$$= \begin{vmatrix} 0 & -1 & \alpha \\ 3 & 2 & \beta-1 \\ u+2v-2 & v-1 & 1+a_3+b_3 \end{vmatrix} [C_1 \to C_1 + 2C_2]$$

$$= [3(1+a_3+b_3) - (u+2v-2)(\beta-1)]$$

$$+ a_2[3(v-1) - 2(u+2v-2)]$$

$$= 3 + 3(a_3+b_3) - u\beta - 2v\beta + 2\beta + u + 2v$$

$$-2 + \alpha(-v+1-2u)$$

$$= 1 + 3(a_3+b_3) + 2\beta + u + 2v + \alpha - u\beta - 2v\beta - \alpha v - 2u\alpha$$

$$= 1 + 3(a_3+b_3) + 2(a_2-a_3) + b_1 - b_2 + 2(\beta_2-b_3)$$

$$+ a - a - 2\alpha u - 2\beta v - u\beta - v\alpha$$

$$= 1 + (a_1+b_1+a_2+b_2+a_3+b_3) - 2\alpha u - 2\beta v - u\beta - v\alpha$$

$$= \text{RHS} \qquad\qquad \text{[From (i)]}$$

Example 7: Find values of c for which the equations $2x+3y=3$; $(c+2)x+(c+4)y = c+6$

$(c+2)^2 x + (c+4)^2 y = (c+6)^2$ are consistent and hence solve the equation.

Sol: Here in this problem first define given equations as Δ and solve it as $\Delta = 0$ by using the invariance method. The equation will be consistent, if

$$\begin{vmatrix} 2 & 3 & 3 \\ c+2 & c+4 & c+6 \\ (c+2)^2 & (c+4)^2 & (c+6)^2 \end{vmatrix} = 0$$

Applying $C_3 \to C_3 - C_2$, we get

$$\begin{vmatrix} 2 & 3 & 0 \\ c+2 & c+4 & 2 \\ (c+2)^2 & (c+4)^2 & 4(c+5) \end{vmatrix} = 0$$

Solving, we get $c^2 + 10c = 0$

or $\quad c = 0, -10$ $\qquad\qquad\qquad$ ... (i)

If c = 0, the system of equations becomes

$$\begin{cases} 2x+3y=3 \\ 2x+4y=6 \end{cases} \Rightarrow x = -3, y = 3 \qquad \dots (ii)$$

If c = –10, then system of equations becomes

$$\begin{cases} 2x+3y=3 \\ -8x-6y=-4 \end{cases} \Rightarrow x=-\frac{1}{2},\ y=\frac{4}{3} \qquad \dots \text{(iii)}$$

16x + 9y = 4

Hence the solutions are given by (ii) and (iii).

Example 8: If (a_r, b_r), r = 1, 2, 3 be the vertices of a triangle, prove that

$$\Delta = \begin{vmatrix} a_2-a_3 & b_2-b_3 & a_1(a_2-a_3)+b_1(b_2-b_3) \\ a_3-a_1 & b_3-b_1 & a_2(a_3-a_1)+b_2(b_3-b_1) \\ a_1-a_2 & b_1-b_2 & a_3(a_1-a_2)+b_3(b_1-b_2) \end{vmatrix} = 0 \ \cdots \text{(i)}$$

and hence show that the altitudes of a triangle are concurrent.

Sol: Using the invariance method we can expand the given determinant and using the equations of altitude we can prove it

$$\Delta = \begin{vmatrix} a_2-a_3 & b_2-b_3 & a_1(a_2-a_3)+b_1(b_2-b_3) \\ a_3-a_1 & b_3-b_1 & a_2(a_3-a_1)+b_2(b_3-b_1) \\ a_1-a_2 & b_1-b_2 & a_3(a_1-a_2)+b_3(b_1-b_2) \end{vmatrix} = 0$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ a_3-a_1 & b_3-b_1 & a_2(a_3-a_1)+b_2(b_3-b_1) \\ a_1-a_2 & b_1-b_2 & a_3(a_1-a_2)+b_3(b_1-b_2) \end{vmatrix}$$

A(a₁, b₁), F, E, B(a₂, b₂), D, C(a₃, b₃)
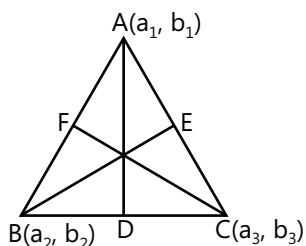
∴ Equation of altitude AD is:

$$y-b_1 = -\frac{a_2-a_3}{b_2-b_3}(x-a_1)$$

or $x(a_2-a_3)+y(b_2-b_3)=a_1(a_2-a_3)+b_1(b_2-b_3)$ ... (ii)

Similarly equation of altitudes BE and CF are

$x(a_3-a_1)+y(b_3-b_1)=a_2(a_3-a_1)+b_2(b_3-b_1)$ .... (iii)

$x(a_1-a_2)+y(b_1-b_2)=a_3(a_1-a_2)+b_3(b_1-b_2)$ .... (iv)

Altitudes (ii), (iii), (iv) are concurrent, since the determinant given by L.H.S. of (i) is zero.

Example 9: Let λ and α be real. Find the set of all values of λ and α for which the system of linear equations $\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$ $-x + (\sin\alpha)y - (\cos\alpha)z = 0$ has a non-trivial solution. If $\lambda = 1$, find all values of α.

Sol: Here in this problem first define the given equations as Δ and as we know that for non-trivial solution $\Delta = 0$.

For non-trivial solution, condition is $\Delta = 0$.

$$\Delta = \begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

or $\lambda[-\cos^2\alpha - \sin^2\alpha] - \sin\alpha[-\cos\alpha + \sin\alpha]$

$+\cos\alpha[\sin\alpha + \cos\alpha] = 0$

or $\lambda = \sin 2\alpha + \cos 2\alpha$ ∴ $\alpha \in R; |\lambda| \le \sqrt{2}$

If $\lambda = 1$, then $1 = \sin 2\alpha + \cos 2\alpha$

$$\cos\left(2\alpha - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$$

$$\Rightarrow 2\alpha - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4} : n \in I \Rightarrow \alpha = n\pi \pm \frac{\pi}{8} + \frac{\pi}{8} : n \in I$$

Example 10: For a fixed positive integer n, if

$$\Delta = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2!) & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$$

then show that $\left[\dfrac{\Delta}{(n!)^3} - 4\right]$ is divisible by n.

Sol: By using the scalar multiple property of determinants we can take $(n!)^3, (n+1)$ and (n + 2) common and using the invariance property we can solve the given problem.

$$\Delta = (n!)^3 \begin{vmatrix} 1 & n+1 & (n+2)(n+1) \\ n+1 & (n+2)(n+1) & (n+3)(n+2)(n+1) \\ (n+2)(n+1) & (n+3)(n+2)(n+1) & (n+4)(n+3)(n+2)(n+1) \end{vmatrix}$$

Taking (n+1) and (n + 2) (n + 1) common from C_2 and C_3 respectively, we get

$$\Delta = (n!)^3(n+2)(n+1)$$

$$\begin{vmatrix} 1 & 1 & 1 \\ n+1 & n+2 & n+3 \\ (n+2)(n+1) & (n+3)(n+2) & (n+4)(n+3) \end{vmatrix}$$

[Apply $C_3 \to C_3 - C_1$ and $C_2 \to C_2 - C_1$ then

$\Delta = (n!)^3 (n+1)^2 (n+2)$

$\begin{vmatrix} 1 & 0 & 0 \\ n+1 & 1 & 2 \\ (n+2)(n+1) & 2(n+2) & 4n+10 \end{vmatrix}$

$= (n!)^3 (n+1)^2 (n+2)[4n+10 - 4(n+2)]$

$= (n!)^3 (n+1)^2 (n+2) \cdot 2$

$\Delta = (n!)^3 (n^2 + 2n + 1)(2n+4) = (n!)^3 (2n^3 + 8n^2 + 10n + 4)$

$\therefore \left[\dfrac{\Delta}{(n!)^3} - 4 \right] = 2n^3 + 8n^2 + 10n$,

$2n(n^2 + 4n + 5)$, which is divisible by n.

JEE Main/Boards

Exercise 1

Q.1 Find x, if $\begin{vmatrix} -1 & 2 \\ 4 & 8 \end{vmatrix} = \begin{vmatrix} 2 & x \\ x & -4 \end{vmatrix}$.

Q.2 It matrix A = $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, find [A].

Q.3 Given $\begin{vmatrix} 4 & -1 & 0 \\ 2 & 1 & 4 \\ 1 & 0 & 3 \end{vmatrix}$, find (i) M_{23} (ii) C_{32}.

Q.4 Area of a triangle with vertices (k, 0), (1, 1) and (0, 3) is 5 sq. units. Find the value(s) of k.

Q.5 Find the area of a triangle, whose vertices are (0, 3) (−1, 4), (2, 6).

Q.6 Given determinant $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$.

Find the value of $a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{33}$.

Q.7 Find the value of p, such that the matrix $\begin{bmatrix} -1 & 24 \\ 4 & p \end{bmatrix}$ is singular.

Q.8 Given I_2. Find $|I_2|$. Also find $|3I_2|$.

Q.9 Find the value of x, such that the points (0,2), (1, x), (3, 1) are collinear.

Q.10 For two given square matrices A and B of the same order, such that |A| = 20 and |B| = −20, find |AB|.

Q.11 Find the adjoint of matrix A = $\begin{bmatrix} 3 & 1 \\ -5 & 4 \end{bmatrix}$.

Q.12 Find the inverse of matrix $\begin{bmatrix} 1 & 3 \\ -6 & -18 \end{bmatrix}$, if possible.

Q.13 Without expanding, find the value of $\begin{vmatrix} 3 & 1 & 8 \\ -4 & 2 & 16 \\ -5 & 3 & 24 \end{vmatrix}$.

Q.14 If a = $\begin{pmatrix} x & 0 & 1 \\ 2 & -1 & 4 \\ 1 & 2 & 0 \end{pmatrix}$ is a singular matrix, find x.

Q.15 Find the area of the triangle whose vertices are (3, 1) (4, 3) and (−5, 4).

Q.16 Find the value of x, if area of triangle is 35 square cms with vertices (x, 4), (2, −6) and (5, 4).

Q.17 Show that the following determinant vanishes:

$\begin{vmatrix} 5 & 15 & -25 \\ 7 & 21 & 30 \\ 8 & 24 & 42 \end{vmatrix}$.

Q.18 Using properties of determinants, prove that :

$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$.

Q.19 If points (2, 0), (0, 5) and (x, y) are collinear, then show that $\dfrac{x}{2} + \dfrac{y}{5} = 1$.

Q.20 If for matrix A, |A| = 3 find |5A|, where matrix A is of order 2 × 2.

Q.21 Given $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, such that $|A| = -10$. Find $a_{11}C_{11} + a_{12}C_{12}$.

Q.22 Without expanding prove that, the value of determinant

$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is zero.

Q.23 A is a non-singular matrix of order 3 and $|A| = -4$. Find $|\text{adj}.A|$.

Q.24 Is it possible to find the inverse of a matrix

$\begin{bmatrix} 2 & 1 & 5 \\ -1 & 0 & 3 \end{bmatrix}$? Given reasons.

Q.25 Given a square matrix A of order 3 × 3, such that $|A| = 12$, find the value of $|A.\text{adj. }A|$.

Q.26 Compute A^{-1} for the matrix $\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$ and show that $A^{-1} = \dfrac{1}{19}A$.

Q.27 Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

Verify that (i) $(\text{adj } A)^{-1}$ (ii) $(A^{-1})^{-1} = A$.

Q.28 Using matrix method, examine the system of equations: 2x + 5y = 7, 6x + 15y = 13 for consistency.

Q.29 Find the inverse of matrix $A = \begin{bmatrix} a & b \\ c & \dfrac{1+bc}{a} \end{bmatrix}$ and show that $aA^{-1} = (a^2 + bc + 1)I - aA$.

Q.30 If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$,

show that $A'A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

Q.31 If $A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$, prove that $A^{-1} = A^2 - 6A + 11\,I$.

Q.32 If $A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 1 \\ 2 & -3 & -1 \end{bmatrix}$,

verify that $(AB)^{-1} = B^{-1}A^{-1}$.

(33-38) Using properties of determinant, prove that

Q.33 $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a + b + c)(ab + bc + ca - a^2 - b^2 - c^2)$

$= 3abc - a^3 - b^3 - c^3$.

Q.34 $\begin{vmatrix} y+z & x & y \\ z+x & y & x \\ x+y & z & z \end{vmatrix} = (x + y + z)(x - z)^2$.

Q.35 $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$.

Q.36 $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ac \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$.

Q.37 $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$.

Q.38 $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2 - ac)(ax^2+2bxy+cy^2)$

Q.39 Write the minors and cofactors of the elements of second row of the following determinant:

$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$.

Q.40 Find the quadratic function defined by the equation $f(x) = ax^2 + bx + c$, if f(0) = 6(2) = 11 and f(−3) = 6, using determinants.

Q.41 Examine whether the system of equations: 2x − y = 5, 4x − 2y = 10 is consistent or inconsistent.

Q.42 Verify, whether the system of equations: 3x − y − 2z = 2, 2y − z = −1, 3x − 5y = 3 is consistent or inconsistent.

Without expanding the determinants, show that

Q.43 $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$.

Q.44 $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$.

Q.45 $\begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix} = 0$.

Q.46 Solve for x, $\begin{vmatrix} x^2 & 0 & 3 \\ x & 1 & -4 \\ 1 & 2 & 0 \end{vmatrix} = 11$.

Q.47 If $\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$, verify that $A^2 - 4A - I = O = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

and $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ and hence find A^{-1}.

Q.48 Evaluate : $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix}$.

Q.49 Show that $\{-(a + b + c)\}$ is root of the following equation:

$\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$.

Q.50 Using properties of determinants, prove

that : $\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} = 16(3x+4)$.

Q.51 Using properties of determinants, prove that:

$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$.

Q.52 Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$.

Hence find A^{-1}.

Q.53 Find matrix A if, $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & -1 \end{bmatrix}$.

Q.54 Given $A \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Compute $(AB)^{-1}$.

Q.55 For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that

$A^3 - 6A^2 + 9A - 4I = O$, hence find A^{-1}.

Exercise 2

Single Correct Choice Type

Q.1 If a, b, c are all different from zero and

$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$, then the value of $a^{-1} + b^{-1} + c^{-1}$

is:

(A) abc   (B) $a^{-1} b^{-1} c^{-1}$

(C) $-a - b - c$ (D) -1

Q.2 If a, b, c are all different and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then

(A) abc (ab + bc + ca) = a + b + c

(B) (a + b + c) (ab + bc + ca) = abc

(C) abc (a + b + c) = ab + bc + ca

(D) None of these

Q.3 If $(\sin^{-1} x + \sin^{-1} w)(\sin^{-1} y + \sin^{-1} z) = p^2$, then

$\begin{vmatrix} x^{N_1} & y^{N_2} \\ z^{N_3} & w^{N_4} \end{vmatrix}$; $(N_1, N_2, N_3, N_4 \in N)$

(A) Has a maximum value 2.

(B) Has a minimum value 2.

(C) In independent of N_1, N_2, N_3, N_4

(D) None of these

Q.4 If $(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \ldots + a_{2n} x^{2n}$ then

$\begin{vmatrix} a_{n-3} & a_{n-1} & a_{n+1} \\ a_{n-6} & a_{n-3} & a_{n+3} \\ a_{n-14} & a_{n-7} & a_{n+7} \end{vmatrix}$ is

(A) 1   (B) 2   (C) 0   (D) –1

Q.5 The absolute value of the determinant

$\begin{vmatrix} -1 & 2 & 1 \\ 3 + 2\sqrt{2} & 2 + 2\sqrt{2} & 1 \\ 3 - 2\sqrt{2} & 2 - 2\sqrt{2} & 1 \end{vmatrix}$ is

(A) $16\sqrt{2}$ (B) $8\sqrt{2}$ (C) 8   (D) None of these

Q.6 $D_1 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$, $D_2 = \begin{vmatrix} bc - a^2 & ac - b^2 & ab - c^2 \\ ac - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ac - b^2 \end{vmatrix}$,

$D_3 = \begin{vmatrix} a^2 + b^2 + c^2 & ab + bc + ca & ab + bc + ca \\ ab + bc + ca & a^2 + b^2 + c^2 & ab + bc + ca \\ ab + bc + ca & ab + bc + ca & a^2 + b^2 + c^2 \end{vmatrix}$ then

(A) $D_1 \leq 0$, if a+b+c>0   (B) $D_2^2 = D_3$

(C) $D_1^2 = D_2 = D_3$ (D) $D_2 \neq D_3 = D_1^2$

Q.7 $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$, where a, b, c are distinct

positive reals, then abc is always less than

(A) $\dfrac{1}{243}$ (B) $\dfrac{1}{729}$ (C) $\dfrac{1}{27}$ (D) $\dfrac{1}{81}$

Q.8 The value of 'a' for which the system of equations,

$(a + 1)^3 x + (a + 2)^3 y = (a + 3)^3$,

$\begin{vmatrix} \log_x xyz & \log_x y & \log_x z \\ \log_y xyz & 1 & \log_y z \\ \log_z xyz & \log_z y & 1 \end{vmatrix}$ and x + y = 1 are consistent is

(A) –2   (B) 1   (C) 0   (D) None

Q.9 The following system of equations $3x - 7y + 5z = 3$; $3x + y + 5z = 7$ and $2x + 3y + 5z = 5$ are

(A) Consistent with trivial solution

(B) Consistent with unique non-trivial solution

(C) Consistent with infinite solution

(D) Inconsistent with no solution

Q.10 The system of equations $(\sin\theta)x + 2z = 0$, $(\cos\theta)x + (\sin\theta)y = 0$, $(\cos\theta)y + 2z = a$ has

(A) Non unique solution

(B) A unique solution which is a function of a and θ

(C) A unique solution which is independent of a and θ

(D) A unique solution which is independent of θ only

Q.11 The equation

$\begin{vmatrix} (1 + x)^2 & (1 - x)^2 & -(2 + x^2) \\ 2x + 1 & 3x & 1 - 5x \\ x + 1 & 2x & 2 - 3x \end{vmatrix} +$

$\begin{vmatrix} (1 + x)^2 & 2x + 1 & x + 1 \\ (1 - x)^3 & 3x & 2x \\ 1 - 2x & 3x - 2 & 2x - 3 \end{vmatrix} = 0$

(A) Has no real solution

(B) Has 4 real solutions

(C) Has two real and two non-real solutions

(D) Has infinite number of solutions, real or non-real

Q.12 The system of equation :

$2x\cos^2\theta + y\sin 2\theta - 2\sin\theta = 0$;

$x\sin 2\theta + 2y\sin^2\theta = -2\cos\theta$;

$x\sin\theta - y\cos\theta = 0$, for all values of θ, can

(A) Have a unique nontrivial solution

(B) Not have a solution

(C) Have infinite solutions

(D) Have a trivial solution

Q.13 If x, y, z are not all simultaneously equal to zero, satisfying the system of equations $(\sin 3\theta)x - y + z = 0$; $(\cos 2\theta)x + 4y + 3z = 0$; $2x + 7y + 7z = 0$, then the number of principal values of θ is

(A) 2   (B) 4   (C) 5   (D) 6

Q.14 For a non-zero, real a, b and c

$$\begin{vmatrix} \dfrac{a^2+b^2}{c} & c & c \\ a & \dfrac{b^2+c^2}{a} & a \\ b & b & \dfrac{c^2+a^2}{b} \end{vmatrix} = \alpha\,abc$$

then the value of α is

(A) –4   (B) 0   (C) 2   (D) 4

Q.15 Number of value of 'a' for which the system of equations, $a^2x+(2-a)y = 4+a^2$; $ax+(2a-1)y = a^5-2$ possess no solution is

(A) 0   (B) 1   (C) 2   (D) Infinite

Previous Years' Questions

Q.1 The determinant

$$\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0 \qquad\qquad \textbf{(1997)}$$

(A) x, y, z are in AP   (B) x, y, z are in GP

(C) x, y, z are in HP   (D) xy, yz, zx are in AP

Q.2 If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$,

then f(100) is equal to   **(1999)**

(A) 0   (B) 1   (C) 100   (D) –100

Q.3 If the system of equations x – ky – z = 0, kx – y – z = 0, x + y – z = 0 has a non-zero solution, then possible values of k are   **(2000)**

(A) –1, 2   (B) 1, 2   (C) 0, 1   (D) –1, 1

Q.4 The number of distinct real roots of   **(2001)**

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0 \text{ in the interval } -\dfrac{\pi}{4} \le x \le \dfrac{\pi}{4} \text{ is}$$

(A) 0   (B)2   (C) 1   (D) 3

Q.5 If the system of equations x + ay = 0, az + y = 0 and ax + z = 0 has infinite solutions, then the value of a is   **(2002)**

(A) –1   (B) 1

(C) 0   (D) No real values

Q.6 The number of values of k for which the system of equations (k + 1)x + 8y = 4k, kx+ (k + 3)y = 3k – 1 has infinite solutions.

Assertion Reasoning Type

(A) Both assertion and reason are true and reason is the correct explanation of Assertion.

(B) Both assertion and reason are true and reason is not the correct explanation of assertion.

(C) Assertion is true but reason is false

(D) Assertion is false but reason is true.

Q.7 Consider the system of equations x – 2y + 3z = –1, x – 3y + 4z = 1 and –x + y – 2z = k

Statement-I: The system of equation has no solution for $k \ne 3$. and

Statement-II: The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \ne 0$, for $k \ne 0$.   **(2008)**

Q.8 Given, x = cy + bz, y = az + cx, z = bx + ay, where x, y, z are not all zero, prove that $a^2+b^2+c^2+2ab = 1$ **(1978)**

Q.9 If α be a repeated root of a quadratic equation f(x) = 0 and A(x), B(x) and C(x) be polynomials of degree 3, 4 and 5 respectively, then show that

$$\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$ is divisible by f(x), where prime

denotes the derivatives.   **(1984)**

Q.10 If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a, b, c are real positive number, abc = 1 and $A^TA = I$, then find the value of $a^3+b^3+c^3$.   **(2003)**

Q.11 The number of values of k, for which the system of equations $(k+1)x+8y = 4k$ $kx+(k+3) = 3k-1$ Has no solution, is: **(2013)**

(A) Infinite   (B) 1   (C) 2   (D) 3

Q.12 If $\alpha,\beta \ne 0$, and $f(n) = \alpha^n+\beta^n$

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+g(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+g(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)(\alpha-\beta)^2,$$

then K is equal to: *(2014)*

(A) $\alpha\beta$ (B) $\dfrac{\alpha}{\beta}$ (C) 1   (D) -1

Q.13 The set of all values of λ for which the system of linear equations:

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

Has a non – trivial solution. *(2015)*

(A) Is an empty set

(B) Is a singleton

(C) Contains two elements

(D) Contains more than two elements

Q.14 The system of linear equations *(2016)*

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for:

(A) Infinitely many values of λ

(B) Exactly one value of λ

(C) Exactly two values of λ

(D) Exactly three values of λ

JEE Advanced/Boards

Exercise 1

Q.1 Solve the following using Cramer's rule and state whether consistent or not.

(a) $x + 2y + z = 1$

     $3x + y + z = 6$

     $x + 2y = 0$

(b) $x + y + z - 6 = 0$

     $2x + y - z - 1 = 0$

     $x + y - 2z + 3 = 0$

(c) $7x - 7y + 5z = 3$

     $3x + y + 5z = 7$

     $2x + 3y + 3z = 5$

Q.2 For what value of K do the following system of equations possess a non-trivial (i.e. not all zero) solution over the set of rational Q? x + Ky + 3z = 0, 3x + Ky – 2z = 0, 2x + 3y – 4z = 0. For that value of K, find all the solutions of the system.

Q.3 The system of equations $\alpha x + y + z = \alpha - 1$ $x + \alpha y + z = \alpha - 1$; $x + y + \alpha z = \alpha - 1$ has no solution. Find α.

Q.4 If the equations a(y + z) = x, b(z + x) = y, c(x + y) = z have non-trivial solutions, then find the value of

$\dfrac{1}{1+a} + \dfrac{1}{1+b} + \dfrac{1}{1+c}$.

Q.5 Given x = cy + bz; y = az + cx; z = bx + ay where x, y, z are not all zero, prove that $a^2 + b^2 + c^2 + 2abc = 1$.

Q.6 Given $a = \dfrac{x}{y-z}$; $b = \dfrac{y}{z-x}$; $c = \dfrac{z}{x-y}$ where x, y, z are not all zero, prove that 1 + ab + bc + ca = 0.

Q.7 If $\sin q \neq \cos q$ and x, y, z satisfy the equations $x\cos p - y\sin p + z = \cos q + 1$

$x\sin p + y\cos p + z = 1 - \sin q$

$x\cos(p + q) - y\sin(p + q) + z = 2$

Then find the value of $x^2 + y^2 + z^2$.

Q.8 Investigate for what values of λ, μ the simultaneous equations x + y + z = 6; x+ 2y + 3z = 10 and $x + 2y + \lambda z = \mu$ have;

(a) A unique solution

(b) An infinite number of solutions

(c) No solution

Q.9 For what values of p, the equations: x + y + z = 1; x + 2y + 4z = p and x + 4y + 10z = p² have a solution? Solve them completely in each case.

Q.10 Solve the equations : Kx + 2y – 2z = 1; 4x + 2Ky – z = 2; 6x + 6y + Kz = 3 considering specially the case

when K = 2.

Q.11 (a) Let a, b, c, d are distinct numbers to be chosen from the set {1, 2, 3, 4, 5}. If the least possible positive solution for x to the system of equation $\left.\begin{array}{l}ax+by=1\\cx+dy=2\end{array}\right\}$ can be expressed in the form $\dfrac{p}{q}$ where p and q are relatively prime, then find the value of (p + q).

(b) Find the sum of all positive integral values of a for which every solution to the system of equations x + ay = 3 and ax + 4y = 6 satisfy the inequalities x > 1, y > 0.

Q.12 If the following system of equations (a − t)x + by + cz = 0, bx + (c − t)y + az = 0 and cx + ay + (b − t)z = 0 has non-trivial solutions for different values of t, then show that we can express product of these values of t in the form of determinant.

Q.13 Show that the system of equations 3x − y + 4z = 3, x + 2y − 3z = −2 and 6x + 5y + λz = −3 has atleast one solution for any real number λ. Find the set of solutions of $\lambda = -5$.

Q.14 Solve the system of equations:
$$\left.\begin{array}{l}z+ay+a^2x+a^3=0\\z+by+b^2x+b^3=0\\z+cy+c^2x+c^3=0\end{array}\right]$$

Q.15 (a) Consider the system of equations
$$\alpha x-y+z=\alpha$$
$$x-\alpha y+z=1$$
$$x-y+\alpha z=1$$

If L, M and N denotes the number of integral values of α in interval [−10, 10] for which the system of the equations has unique solution, no solution and infinite solutions respectively, then find the value of (L − M + N).

(b) If the system of equations is
$$2x+3y-z=0$$
$$3x+2y+kz=0$$
$$4x+y+z=0$$

have a set of non-zero integral solutions then, find the smallest positive value of z.

(c) Given a, b $\in$ {0, 1, 2, 3, 4,......, 9, 10}.

Consider the system of equations

$$x+y+z=4$$
$$2x+y+3z=6$$
$$x+2y+az=b$$

Let L: denotes number of ordered pairs (a, b) so that the system of equations has unique solution,

M: denotes number of ordered pairs (a, b) so that the system of equations has no solution and

N: denotes number of ordered pairs (a, b) so that the system of equations has infinite solutions. Find (L + M − N).

Q.16 (a) Prove that the value of the determinant
$$\begin{vmatrix} -7 & 5+3i & \frac{2}{3}-4i \\ 5-3i & 8 & 4+5i \\ \frac{2}{3}+4i & 4-5i & 9 \end{vmatrix} \text{ is real.}$$

(b) On which one of the parameter out of a, p, d or x value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend.

(c) If $\begin{vmatrix} x^3+1 & x^2 & x \\ y^3+1 & y^2 & y \\ z^3+1 & z^2 & z \end{vmatrix}=0$ and x, y, z are all different then, prove that xyz = −1.

Q.17 Prove that (a) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}=(a-1)^3$

(b) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}=[(x-y)(y-z)(z-x)(x+y+z)]$

Q.18 (a) Let $f(x)=\begin{vmatrix} x & 1 & \frac{-3}{2} \\ 2 & 2 & 1 \\ \frac{1}{x-1} & 0 & \frac{1}{2} \end{vmatrix}$ Find the minimum value of f(x) (given x > 1).

(b) If $a^2+b^2+c^2+ab+bc+ca\le 0$ $\forall$ a, b, c $\in$R, then find the value of the determinant

$$\begin{vmatrix} (a+b+2)^2 & a^2+b^2 & 1 \\ 1 & (b+c+2)^2 & b^2+c^2 \\ c^2+a^2 & 1 & (c+a+2)^2 \end{vmatrix}$$

Q.19 If $D = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ and

$D' = \begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix}$ then prove that $D' = 2D$.

Q.20 Prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Q.21 Let

$$f(x) = \begin{vmatrix} \sin x & \sin(x+h) & \sin(x+2h) \\ \sin(x+2h) & \sin x & \sin(x+h) \\ \sin(x+h) & \sin(x+2h) & \sin x \end{vmatrix}.$$

If $\underset{h \to 0}{\text{Lim}} \dfrac{f(x)}{h^2}$ has the value equation to $k(\sin 3x + \sin^3 x)$

find $k \in N$.

Q.22 Prove that

$$\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1 \\ (\gamma+\alpha+\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix}$$

$$= -64(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)(\gamma-\delta)$$

Q.23 If a, b and c are the roots of the cubic $x^3 - 3x^2 + 2 = 0$ then find the value of the determinant.

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

Q.24 Solve for x

(a) $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$

(b) $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

Q.25 If a + b + c = 0, solve for x :

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

Q.26 Let a, b, c are the solutions of the cubic $x^3 - 5x^2 + 3x - 1 = 0$, then find the value of the

determinant $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$

Q.27 Show that, $\begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix}$ is divisible by λ^2

and find the other factor.

Q.28 Prove that

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

Q.29 In a $\triangle ABC$, determine condition under which

$$\begin{vmatrix} \cos\dfrac{A}{2} & \cot\dfrac{B}{2} & \cot\dfrac{C}{2} \\ \tan\dfrac{B}{2}+\tan\dfrac{C}{2} & \tan\dfrac{C}{2}+\tan\dfrac{A}{2} & \tan\dfrac{A}{2}+\tan\dfrac{B}{2} \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Exercise 2

Single Correct Choice Type

Q.1 Let m be a positive integer &

$$D_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix} \quad (0 \le t \le m),$$

then the value of $\sum\limits_{r=0}^{m} D_r$ is given by

(A) 0   (B) m^2-1 (C) 2^m (D) $2^m \sin^2(2^m)$

Q.2 If α, β and γ are real numbers, then

$$D = \begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha - \beta) & \cos(\beta - \gamma) & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & 1 & 1 \end{vmatrix} =$$

(A) –1   (B) $\cos\alpha\cos\beta\cos\gamma$

(C) $\cos\alpha + \cos\beta + \cos\gamma$ (D) Zero

Q.3 If a, b and c are non-zero real numbers, then

$$D = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$$

(A) abc   (B) $a^2b^2c^2$

(C) bc + ca + ab   (D) Zero

Q.4 If $f'(x) = \begin{vmatrix} mx & mx-p & mx+p \\ n & n+p & n-p \\ mx+2n & mx+2n+p & mx+2n-p \end{vmatrix}$,

then y=f(x) represents

(A) A straight line parallel to x-axis

(B) A straight line parallel to y-axis

(C) Parabola

(D) A straight line with negative slope

Q.5 If $D(x) = \begin{vmatrix} x-1 & (x-1)^2 & x^3 \\ x-1 & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+1)^3 \end{vmatrix}$,

then the coefficient of x in D(x) is

(A) 5   (B) –2   (C) 6   (D) 0

Q.6 The number of integral solutions of $|D|=8$, where

$$D = \begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} \text{ is}$$

(A) 3   (B) 8   (C) 16   (D) 24

Q.7 Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$,

then the maximum value of f(x) is equal to

(A) 2   (B) 4   (C) 6   (D) 8

Q.8 If $px^4 + qx^3 + rx^2 + sx + t$

$$\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & 2-x & x-3 \\ x-3 & x+4 & 3x \end{vmatrix} \text{ then t is equal to}$$

(A) 33   (B) 0   (C) 21   (D) None

Q.9 If $D = \begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$, then D is equal to

(A) $1+a^2+b^2+c^2$ (B) $a^2+b^2+c^2$

(C) $(a+b+c)^2$ (D) None of these

Q.10 If $\alpha + \beta + \gamma = \pi$, then the value of

$$\begin{vmatrix} \sin(\alpha+\beta+\gamma) & \sin\beta & \cos\gamma \\ -\sin\beta & 0 & \tan\alpha \\ \cos(\alpha+\beta) & -\tan\alpha & 0 \end{vmatrix} \text{ is}$$

(A) 0   (B) 1

(C) 2   (D) $2.\sin\beta.\cos\gamma.\tan\alpha$

Q.11 If the entries of 3 × 3 determinant are zero or one then the value of the determinant

(A) Cannot be 3   (B) Cannot be 2

(C) Can be –2   (D) Is essentially zero

Q.12 In a third order determinant, each element of the first column consists of sum of two terms, each element of the second column consists of sum of three terms and each element of the third column consists of sum of four terms. Then it can be decomposed into n determinants, where n has the value

(A) 1   (B) 9   (C) 16   (D) 24

Q.13 If the system of equations x + 2y + 3z = 4, x + py + 2z =3, μx + 4 y + z = 3 has an infinite number of solutions, then

(A) p = 2, $\mu = 3$ (B) p = 2, $\mu = 4$

(C) 3p = 2μ (D) None of these

Q.14 Number of triplets of a, b and c for which the system of equations, ax − by = 2a − b and (c + 1)x + cy = 10 − a + 3b has infinitely many solutions and x = 1, y = 3 is one of the solutions, is

(A) Exactly one   (B) Exactly two

(C) Exactly three   (D) Infinitely many

Q.15 If the system of equations ax + y + z = 0, x + by + z = 0 & x + y + cz = 0 (a , b, c ≠ 1) has a non-trival solution , then the value of $\dfrac{1}{1-a}+\dfrac{1}{1-b}+\dfrac{1}{1-c}$ is

(A) −1   (B) 0   (C) 1   (D) None of these

Q.16 The determinant

$$\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix} \text{ is}$$

(A) 0

(B) Independent of θ

(C) Independent of ϕ

(D) Independent of θ and ϕ both

Q.17 The values θ, λ for which the following equations $x\sin\theta - y\cos\theta + (\lambda+1)z = 0$; $x\cos\theta + y\sin\theta - \lambda z = 0$; $\lambda x + (\lambda+1)y + z\cos\theta = 0$ are consistent with infinite solution, are

(A) $\theta = n\pi, \lambda \in, \lambda \in R - \{0\}$

(B) $\theta = 2n\pi, \lambda$ is any rational number

(C) $\theta = (2n+1)\pi, \lambda \in R^+, n \in I$

(D) $\theta = (2n+1)\dfrac{\pi}{2}, \lambda \in R, n \in I$

Q.18 If the system of equations, $a^2x - ay = 1-a$ and $bx + (3-2b)y = 3+a$ possess a unique solution x = 1, y = 1 then

(A) a = 1; b = −1   (B) a = −1, b = 1

(C) a = 0, b = 0   (D) None of these

Q.19 Let $D = \begin{vmatrix} ^{n+2}C_n & ^{n+3}C_{n+1} & ^{n+4}C_{n+2} \\ ^{n+3}C_{n+1} & ^{n+4}C_{n+2} & ^{n+5}C_{n+3} \\ ^{n+4}C_{n+2} & ^{n+5}C_{n+3} & ^{n+6}C_{n+6} \end{vmatrix}$ and $n \in N$

then the value of D is equal to

(A) −1   (B) 0   (C) 1

(D) (n + 2) (n + 3) (n + 4) (n + 5) (n + 6)

Q.20 The set of equations $\lambda x - y + (\cos\theta)z = 0$; $3x + y + 2z = 0$; $(\cos\theta)x + y + 2z = 0$, $0 \le \theta < 2\pi$, has nontrivial solution(s)

(A) For no value of λ and θ

(B) For all values of λ and θ

(C) For all values of λ and only two values of θ

(D) For only one value of λ and all values of θ

Multiple Correct Choice Type

Q.21 The determinant

$$\begin{vmatrix} \cos(x-y) & \cos(y-z) & \cos(z-x) \\ \cos(x+y) & \cos(y+z) & \cos(z+x) \\ \sin(x+y) & \sin(y+z) & \sin(z+x) \end{vmatrix} =$$

(A) $2\sin(x-y)\sin(y-z)\sin(z-x)$

(B) $-2\sin(x-y)\sin(y-z)\sin(z-x)$

(C) $2\cos(x-y)\cos(y-z)\cos(z-x)$

(D) $-2\cos(x-y)\cos(y-z)\cos(z-x)$

Q.22 The value of θ lying between $-\dfrac{\pi}{4}$ and $\dfrac{\pi}{2}$ and $-0 \le A \le \dfrac{\pi}{2}$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1+\cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} \text{ are}$$

(A) $A = \dfrac{\pi}{4}, \theta = -\dfrac{\pi}{8}$ (B) $A = \dfrac{3\pi}{8} = \theta$

(C) $A = \dfrac{\pi}{5} 9 \theta = -\dfrac{\pi}{8}$ (D) $A = \dfrac{\pi}{6} = \theta = \dfrac{3\pi}{8}$

Q.23 If $\begin{vmatrix} 1 & a & a^2 \\ 1 & x & x^2 \\ b^2 & ab & a^2 \end{vmatrix} = 0$

(A) x = a   (B) x = b   (C) $x = \dfrac{1}{a}$ (D) $x = \dfrac{a}{b}$

Q.24 The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$

is equal to zero, if

(A) a, b, c are in AP

(B) a, b, c are in GP

(C) α is a root of the equation $ax^2 + bx + c = 0$

(D) $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$

Q.25 The set of equations $x - y + 3z = 2$, $2x - y + z = 4$, $x - 2y + \alpha z = 3$ has

(A) Unique solution only for $\alpha = 0$

(B) Unique solution for $\alpha \neq 8$

(C) Infinite number of solution of $\alpha = 8$

(D) No solution for $\alpha = 8$

Q.26 Which of the following determinant(s) vanish(es)?

(A) $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$

(B) $\begin{vmatrix} 1 & ab & \frac{1}{a}+\frac{1}{b} \\ 1 & bc & \frac{1}{b}+\frac{1}{c} \\ 1 & ca & \frac{1}{c}+\frac{1}{a} \end{vmatrix}$

(C) $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$

(D) $\begin{vmatrix} \log_x xyz & \log_x y & \log_x z \\ \log_y xyz & 1 & \log_y z \\ \log_z xyz & \log_z y & 1 \end{vmatrix}$

Q.27 If the system of equation $a^2x - by = a^2 - b$ and $bx - b^2y = 2 + 4b$ possess an infinite number of solutions then the possible values of 'a' and 'b' are

(A) a = 1, b = –1   (B) a = 1, b = –2

(C) a = –1, b = –1   (D) a = –1, b = –2

Q.28 If p, q, r, s are in A.P, and

$$f(x) = \begin{vmatrix} p+\sin x & q+\sin x & p-r+\sin x \\ q+\sin x & r+\sin x & -1+\sin x \\ r+\sin x & s+\sin x & s-q+\sin x \end{vmatrix}$$ such that

$\int_0^2 f(x)dx = -4$, then the common difference of the A.P. can be

(A) –1   (B) ½   (C) 1   (D) None of these

Q.29 If the system of equations $x + y - 3 = 0$, $(1 + K)x + (2 + K)y - 8 = 0$ and $x - (1 + K)y + (2 + K)$ are consistent then the value of K is

(A) 1   (B) 3/5   (C) –5/3   (D) 2

Q.30 If $D = \begin{vmatrix} \dfrac{1}{z} & \dfrac{1}{z} & -\dfrac{(x+y)}{z^2} \\ -\dfrac{(y+z)}{x^2} & \dfrac{1}{x} & \dfrac{1}{x} \\ -\dfrac{y(y+z)}{x^2z} & -\dfrac{x+2y+z}{xz} & -\dfrac{y(x+y)}{xz^2} \end{vmatrix}$ then

(A) D is independent of x   (B) D is independent of y

(C) D is independent of z   (D) D is dependent of x, y, z

Previous Years' Questions

Q.1 The parameter, on which the value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$$ does not depend

upon, is   *(1997)*

(A) a   (B) p   (C) d   (D) x

Q.2 Let λ and α be real. Find the set of all values of λ for which the system of linear equations $\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$ and

$-x + (\sin\alpha)y - (\cos\alpha)z = 0$ has a non-trivial solution.

For $\lambda = 1$, find all value of α.   *(1993)*

Q.3 Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that the equation

$$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0$$

represents a straight line.

Q.4 For what value of k does the following system of equations possess a non-trivial solution over the set of rationals $x + y - 2z = 0$; $2x - 3y + z = 0$ and $x - 5y + 4z = k$. Find all the solution.   *(1979)*

Q.5 For what value of m does the system of equations $3x + my = m$ and $2x - 5y = 20$ has a solution satisfying the conditions $x > 0$, $y > 0$.   *(1979)*

Q.6 Prove that for all values of θ

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \dfrac{2\pi}{3}\right) & \cos\left(\theta + \dfrac{2\pi}{3}\right) & \sin\left(2\theta + \dfrac{4\pi}{3}\right) \\ \sin\left(\theta - \dfrac{2\pi}{3}\right) & \cos\left(\theta - \dfrac{2\pi}{3}\right) & \sin\left(2\theta - \dfrac{4\pi}{3}\right) \end{vmatrix} = 0$$ *(2000)*

Q.7 The total number of ways in which 5 balls of different colours can be distributed among 5 persons so that each person gets at least one ball is **(2012)**

(A) 75   (B) 150   (C) 120   (D) 243

Q.8 Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha\,? \qquad \textbf{(2015)}$$

(A) −4   (B) 9   (C) −9   (D) 4

Q.9 The total number of distinct $x \in R$ for

which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1=27x^3 \end{vmatrix} = 10$ is **(2016)**

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.15	Q.29	Q.38
Q.48	Q.52	Q.54
Q.55		

Exercise 2

Q.1	Q.6	Q.13

Previous Years' Questions

Q.2	Q.6	Q.7
Q.10		

JEE Advanced/Boards

Exercise 1

Q.1	Q.3	Q.8
Q.11	Q.15	Q.23

Exercise 2

Q.1	Q.11	Q.14
Q.20	Q.25	Q.30

Previous Years' Questions

Q.2	Q.5	Q.7
Q.10		

Answer Key

JEE Main/Boards

Exercise 1

Single Correct Choice Type

Q.1 $x = \pm 2\sqrt{2}$　　　**Q.2** 1　　　**Q.3** (i) 1 (ii) −16　　**Q.4** $-\dfrac{7}{2}, \dfrac{13}{2}$　　**Q.5** $\dfrac{5}{2}$ sq. units

Q.6 0　　　**Q.7** −96　　　**Q.8** 1; 9　　　**Q.9** $\dfrac{5}{3}$　　　**Q.10** −400

Q.11 $\begin{bmatrix} 4 & -1 \\ 5 & 3 \end{bmatrix}$　　**Q.12** Not possible　　**Q.13** 0　　　**Q.14** $x = \dfrac{5}{8}$　　**Q.15** $\dfrac{19}{2}$ sq. units

Q.16 -2　　　**Q.20** 75　　　**Q.21** −10　　　**Q.23** 16　　　**Q.24** No

Q.25 1728　　　**Q.26** $A^{-1} = \dfrac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$　　　**Q.28** Inconsistent

Q.29 $\begin{bmatrix} \dfrac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$　　　**Q.39** $M_{21}=39$, $M_{22}=3$, $M_{23}=-11$, $C_{21}=-39$, $C_{22}=3$, $C_{23}=11$

Q.40 $f(x) = \dfrac{1}{2}x^2 + \dfrac{3}{2}x + 6$　　　**Q.41** Consistent　　　**Q.42** Inconsistent

Q.46 $x = -\dfrac{7}{4}$ or 1　　　**Q.47** $A^{-1} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$　　　**Q.48** 0

Q.52 $A^{-1} = \dfrac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}$　　**Q.53** $\dfrac{1}{4}\begin{bmatrix} 49 & -18 \\ -23 & 10 \end{bmatrix}$　　**Q.54** $\dfrac{-1}{11}\begin{bmatrix} 3 & -19 & 12 \\ 4 & -18 & 5 \\ 4 & -29 & 27 \end{bmatrix}$

Q. 55 $\dfrac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Exercise 2

Single Correct Choice Type

Q.1 D　　　**Q.2** A　　　**Q.3** A　　　**Q.4** C　　　**Q.5** A　　　**Q.6** C

Q.7 C　　　**Q.8** A　　　**Q.9** B　　　**Q.10** B　　　**Q.11** D　　　**Q.12** B

Q.13 C　　　**Q.14** D　　　**Q.15** C

Previous Years' Questions

Q.1 B	**Q.2** A	**Q.3** D	**Q.4** C	**Q.5** A	**Q.6** 1
Q.7 A	**Q.10** 4	**Q.11** B	**Q.12** C	**Q.13** C	**Q.14** D

JEE Advanced/Boards

Exercise 1

Q.1 (a) x = 1, y = 2, z = 3; consistent   (b) x = 2, y = −1, z = 1; consistent   (c) Inconsistent

Q.2 $K = \dfrac{33}{2}, x : y : z = -\dfrac{15}{2} : 1 : -3$ **Q.3** −2   **Q.4** 2   **Q.7** 2

Q.8 (a) $\lambda \neq 3$ (b) $\lambda = 3$, $\mu = 10$ (c) $\lambda = 3, \mu \neq 10$

Q.9 x = 1 + 2K, y = −3K, z = K, when p = 1; x = 2K, y = 1 − 3K, z = K when p = 2; where $K \in R$

Q.10 If $K \neq 2$, $\dfrac{x}{2(K+6)} = \dfrac{y}{2K+3} = \dfrac{z}{6(K-2)} = \dfrac{1}{2(K^2 + 2K + 15)}$, If K = 2, then $x = \lambda$, $y = \dfrac{1 - 2\lambda}{2}$ and z = 0 where $\lambda \in R$

Q.11 (a) 19   (b) 4   **Q.12** $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Q.13 If $\lambda \neq -5$, then $x = \dfrac{4}{7}$; $y = -\dfrac{9}{7}$ and z = 0; If $\lambda = 5$ then $x = \dfrac{4 - 5K}{7}$; $y = \dfrac{13K - 9}{7}$ and z = K where $K \in R$.

Q.14 x = −(a + b + c), y = ab + bc + ca, z = − abc

Q.15 (a) 21	(b) 5	(c) 119	**Q.16** (b) p	**Q.18** (a) 4 (b) 65
Q.21 3	**Q.23** −108		**Q.24** (a) x = −1 or x = −2; (b) x = 4	

Q.25 X = 0 or $x \pm \sqrt{\dfrac{3}{2}(a^2 + b^2 + c^2)}$ **Q.26** 80

Q.27 $\lambda^2(a^2 + b^2 + c^2 + \lambda)$ **Q.29** Triangle ABC is isosceles.

Exercise 2

Single Correct Choice Type

Q.1 A	**Q.2** D	**Q.3** A	**Q.4** A	**Q.5** A	**Q.6** D
Q.7 C	**Q.8** C	**Q.9** A	**Q.10** A	**Q.11** C	**Q.12** D
Q.13 D	**Q.14** B	**Q.15** C	**Q.16** B	**Q.17** D	**Q.18** A
Q.19 A	**Q.20** A				

Multiple Correct Choice Type

Q.21 A, D   **Q.22** A, B, C, D   **Q.23** A, D   **Q.24** B, D   **Q.25** B, D   **Q.26** A, B, C, D

Q.27 A, B, C, D   **Q.28** A, C   **Q.29** A, C   **Q.30** A, B, C

Previous Years' Questions

Q.1 B   **Q.2** Zero   **Q.4** k = 0, the given system has infinitely many solutions

Q.5 $m < -\dfrac{15}{2}$ or m > 30   **Q.7** B   **Q.8 B** C   **Q.9** 2

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $\begin{vmatrix} -1 & 2 \\ 4 & 8 \end{vmatrix} = \begin{vmatrix} 2 & x \\ x & -4 \end{vmatrix}$

$-8 - 8 = 2(-4) - x^2 = -8 - x^2$

$\Rightarrow x = \pm\sqrt{8} = \pm 2\sqrt{2}$

Sol 2: $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $|A| = 1[1] - 2(0) = 1$

Sol 3: $\begin{vmatrix} 4 & -1 & 0 \\ 2 & 1 & 4 \\ 1 & 0 & 3 \end{vmatrix}$,

(i) $M_{23} = \begin{vmatrix} 4 & -1 \\ 1 & 0 \end{vmatrix} = 0 - (-1) = 1$

$C_{23} = (-1)^{2+3} = -1$

(ii) $C_{32} = (-1)^{2+3} = \begin{vmatrix} 4 & 0 \\ 2 & 4 \end{vmatrix} = -16$

Sol 4: Area of triangle, [(k, 0), (1, 1), (0, 3)] = 5 unit²

$\Rightarrow \dfrac{1}{2}\begin{vmatrix} 1 & k & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 3 \end{vmatrix} = \dfrac{1}{2}|1[3-0] + k(1-3)| = 5$

$\Rightarrow |-2k + 3| = 10$

$\Rightarrow -2k + 3 = 10$ or $2k - 3 = 10$

$\Rightarrow k = -\dfrac{7}{2}$ or $k = \dfrac{13}{2}$

Sol 5: Vertices of triangle (0, 3) (−1, 4) (2, 6)

Area = $\dfrac{1}{2}\begin{vmatrix} 1 & 0 & 3 \\ 1 & -1 & 4 \\ 1 & 2 & 6 \end{vmatrix} = \dfrac{1}{2}\left[\left[-1(6) - 8\right] + 3[2+1]\right]$

$= \dfrac{1}{2}|-14+9| = \dfrac{1}{2}|5| = \dfrac{5}{2}$ Sq. Unit

Sol 6: D $\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23}$

$= a_{11}\begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} + a_{12}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}(-1)^{2+3}$

$= a_{11}[a_{13}a_{32} - a_{33}a_{12}] + a_{12}[a_{11}a_{33} - a_{31}a_{13}] - a_{13}[a_{11}a_{32} - a_{31}a_{12}]$

$= 0$

It can be directly said as it is a property

Sol 7: $\begin{vmatrix} -1 & 24 \\ 4 & P \end{vmatrix} = A$ (assume)

$|A| = \begin{vmatrix} -1 & 24 \\ 4 & P \end{vmatrix} = -P - 4(24) = -(96 + P) = 0$

$\Rightarrow P = -96$

Sol 8: $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $|I_2| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$3I_2 = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$,

$|3I_2| = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 9$

Sol 9: $(0, 2)$, $(1, x)$ and $(3, 1)$ points are collinear

So $\begin{vmatrix} 1 & 0 & 2 \\ 1 & 1 & x \\ 1 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow [1 - 3x] + 2[3 - 1] = 0 \Rightarrow 1 - 3x + 4 = 0$

$\Rightarrow 3x = 5 \Rightarrow x = \dfrac{5}{3}$

Sol 10: $|A| = 20$, $|B| = -20$

$|AB| = |A| \, |B| = 20(-20) = -400$

Sol 11: $A = \begin{bmatrix} 3 & 1 \\ -5 & 4 \end{bmatrix}$

$C_{11} = 4$, $C_{12} = 5$, $C_{21} = -1$, $C_{22} = 3$

$adjA = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T = \begin{bmatrix} 4 & -1 \\ 5 & 3 \end{bmatrix}$

Sol 12: $A = \begin{bmatrix} 1 & 3 \\ -6 & -18 \end{bmatrix}$

$|A| = -18 - [3][-6] = -18 + 18 = 0$

So, A^{-1} does not exist

Sol 13: $\begin{vmatrix} 3 & 1 & 8 \\ -4 & 2 & 16 \\ -5 & 3 & 24 \end{vmatrix} = (8)\begin{vmatrix} 3 & 1 & 1 \\ -4 & 2 & 2 \\ -5 & 3 & 3 \end{vmatrix}$

Two columns are same, so determinant is 0

Sol 14: $a = \begin{pmatrix} x & 0 & 1 \\ 2 & -1 & 4 \\ 1 & 2 & 0 \end{pmatrix}$ is singular

So $|a| = 0$

$\Rightarrow x[-8] + 1\{2(2) - (-1)(1)\}] = -8x + 4 + 1 = 0$

$\Rightarrow 8x = 5 \rightarrow x = \dfrac{5}{8}$

Sol 15: Vertices $\rightarrow (3, 1)$ $(4, 3)$ and $(-5, 4)$

Area $= \dfrac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 1 & 4 & 3 \\ 1 & -5 & 4 \end{vmatrix}$

$= \dfrac{1}{2}[16 + 15 + 3[3 - 4] + 1[-5 - 4]]$

$= \dfrac{1}{2}[31 - 3 - 9] = \dfrac{19}{2}$ sq. unit

Sol 16: Vertices (x, y) $(2, -6)$, $(5, 4)$

Area $= \dfrac{1}{2}\begin{vmatrix} 1 & x & 4 \\ 1 & 2 & -6 \\ 1 & 5 & 4 \end{vmatrix} = \dfrac{1}{2}[8 + 30 + x[-6 - 4] + 4[5 - 2]]$

$38 - 10x + 12 = 70 \Rightarrow 10x = 50 - 70 = -20$

$\Rightarrow x = \dfrac{-20}{10} = -2$

Sol 17: $\begin{vmatrix} 5 & 15 & -25 \\ 7 & 21 & 30 \\ 8 & 24 & 42 \end{vmatrix} = (3)\begin{vmatrix} 5 & 5 & -25 \\ 7 & 7 & 30 \\ 8 & 8 & 42 \end{vmatrix}$

Two column are same so Determinants is 0

Sol18: $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ $C_2 \rightarrow C_2 + C_3$

$\Rightarrow \begin{vmatrix} 1 & a+b+c & b+c \\ 1 & a+b+c & c+a \\ 1 & a+b+c & a+b \end{vmatrix}$

$= (a+b+c)\begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$ $C_1 \rightarrow C_2 - C_1$

$= (a+b+c)\begin{vmatrix} 0 & 1 & b+c \\ 0 & 1 & a+c \\ 0 & 1 & a+b \end{vmatrix} = 0$

Sol 19: $(2, 0)$, $(0, 5)$ and (x, y) are collinear

$\Rightarrow \begin{vmatrix} 1 & 2 & 0 \\ 1 & 0 & 5 \\ 1 & x & y \end{vmatrix} = 0$

1[−5x] + 2[5 − y] = 0

−5x + 10 −2y = 0

$5x + 2y = 10 \rightarrow \dfrac{x}{2} + y - 5 = 1$

Sol 20: |A| = 3, A's order → 2 × 2

$|5A| = (5)^2 |A| = 25 \times 3 = 75$

Sol 21: $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, |A| = −10

$|A| = a_{11}C_{11} + a_{12}C_{12}$ (along first row) = |A| = −10

Sol 22: $\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix} \xrightarrow{R_2-R_1,\ R_3-R_1} \begin{vmatrix} 1 & a & b+c \\ 0 & b-a & a-b \\ 0 & c-a & a-c \end{vmatrix}$

$= (a-b)(a-c)\begin{vmatrix} 1 & a & b+c \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{vmatrix}$

Since the columns are linearly dependent, hence the value of determinant is zero.

Sol 23: |A| = −4

Order of A = 3

$|adjA| = |A|^{3-1} = (-4)^2 = 16$

Sol 24: $\begin{bmatrix} 2 & 1 & 5 \\ -1 & 0 & 3 \end{bmatrix}$

It is not a square matrix, so inverse not exist

Sol 25: |A| = 12, A's order 3 × 3

$|A.\ adjA| = |A|\ |A|^{3-1} = |12|^3 = 1728$

Sol 26: $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, $A^{-1} = \dfrac{1}{19}A$

$C_{11} = -2,\ C_{12} = -5,\ C_{21} = -3,\ C_{22} = 2$

$A^{-1} = \dfrac{adjA}{|A|} = \dfrac{1}{(-4-15)}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \dfrac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \dfrac{1}{19}A$

Sol 27: $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

$C_{11} = 15 - 1 = 14$

$C_{12} = 1 + 10 = 11$

$C_{13} = -2 - 3 = -5,$

$C_{21} = 1 + 10 = 11$

$C_{22} = 5 - 1 = 4$

$C_{23} = -2 - 1 = -3$

$C_{31} = -2 - 3 = -5,$

$C_{32} = -2 - 1 = -3$

$C_{33} = 3 - 4 = -1$

$|A| = 1[14] - 2[11] + 1[-5]$

$= 14 - 5 - 22 = -13$

$AdjA = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix},$

$A^{-1} = \dfrac{1}{|A|}adjA$

$A^{-1} = \dfrac{1}{-13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix},$

$|A^{-1}| = \dfrac{1}{|A|} = \dfrac{1}{-13}$

For adjA,

$C_{11} = \begin{vmatrix} 4 & -3 \\ -3 & -1 \end{vmatrix} = -4 - 9 = -13,$

$C_{12} = 15 + 11 = 26$

$C_{13} = -33 + 20 = -13,$

$C_{21} = 15 + 11 = 26,$

$C_{22} = -14 - 25 = -39$

$C_{21} = C_{12},\ C_{31} = C_{13},$

$C_{23} = -55 + 42 = -13,$

$C_{38} = 56 - 121 = -65$

So $|A^{-1}|^{-1} = \dfrac{1}{|A^{-1}|}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$

$= \dfrac{1}{-13}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = A$

Sol 28: $2x + 5y = 7$, $6x + 15y = 13$

$$D = \begin{vmatrix} 2 & 5 \\ 6 & 15 \end{vmatrix} = 30 - 30 = 0$$

$D = 0$. So system is inconsistent

Sol 29: $A = \begin{vmatrix} a & b \\ c & (1+bc)/a \end{vmatrix}$

$$|A| = a\left(\frac{1+bc}{a}\right) - bc = 1 + bc - bc = 1$$

$$\text{adj} A = \begin{bmatrix} (1+bc)/a & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj} A}{|A|} = \begin{bmatrix} (1+bc)/a & -b \\ -c & a \end{bmatrix}$$

$(a^2 + bc + 1)I - aA$

$$= \begin{bmatrix} a^2+bc+1 & 0 \\ 0 & a^2+bc+1 \end{bmatrix} - a\begin{bmatrix} a & b \\ c & (1+bc)/a \end{bmatrix}$$

$$= \begin{bmatrix} a^2+bc+1-a^2 & -ab \\ -ac & a^2+bc+1-(1+bc) \end{bmatrix}$$

$$= a\begin{bmatrix} (1+bc)/a & -b \\ -c & a \end{bmatrix} = aA^{-1}$$

R.H.L. = L.H.S.

Sol 30: $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$,

$$A^1 = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$|A| = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$\text{adj} A = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj} A}{|A|} = \cos^2 x \begin{bmatrix} 1 & -\dfrac{\sin x}{\cos x} \\ \dfrac{\sin x}{\cos x} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x & -\sin x \cos x \\ \sin x \cos x & \cos^2 x \end{bmatrix}$$

$$A^1 A^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}\begin{bmatrix} \cos^2 x & -\sin x \cos x \\ \sin x \cos x & \cos^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x - \sin^2 x & -\sin x \cos x - \sin x \cos x \\ \sin x \cos x + \sin x \cos x & -\sin^2 x + \cos^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix} = \text{R.H.S.}$$

Sol 31: $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$,

Assume $A - xI = 0$ ..... (i)

$|A_1 - xI| = 0$

So $\begin{bmatrix} 2-x & 0 & -1 \\ 5 & 1-x & 0 \\ 0 & 1 & 3-x \end{bmatrix} = 0$

$\Rightarrow (2-x)[x^2 + 3 - 4x] - 1[5] = 0$

$\Rightarrow -x^3 - 3x + 4x^2 + 2x^2 + 6 - 8x = 5$

$\Rightarrow x^3 - 6x^2 + 11x = 1$

$\Rightarrow x^2 - 6x + 11 = \dfrac{1}{x} = x^{-1}$

$(A - xI) = 0$

$\Rightarrow A^2 - 6A + 11I = A^{-1}$

Sol 32: $A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 1 \\ 2 & -3 & -1 \end{bmatrix}$

$$AB = \begin{bmatrix} 2+2+2 & 6+2-3 & 4+2-1 \\ -2+1+4 & -6+1-6 & -4+1-2 \\ 1-2+4 & 3-2-6 & 2-2-2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 5 & 5 \\ 3 & -11 & -5 \\ 3 & -5 & -2 \end{bmatrix}$$

For A

$C_{11} = 2 + 4 = 6$,

$C_{12} = 2 + 4 = 6$,

$C_{13} = 4 - 1 = 3$,

$C_{21} = -2 - 4 = -6$,

$C_{22} = 4 - 1 = 3$,

$C_{23} = 2 + 4 = 6$,

$C_{31} = 4 - 1 = 3,$

$C_{32} = -2 - 4 = -6,$

$C_{33} = 2 + 4 = 6$

$|A| = 2(6) + 2(6) + 1(3) = 27$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{27} \begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$$

For B

$C_{11} = -1 + 3 = 2,$

$C_{12} = 2 + 1 = 3,$

$C_{13} = -3 - 2 = -5,$

$C_{21} = -6 + 3 = -3,$

$C_{22} = -1 - 4 = -5,$

$C_{23} = -3 - 6 = -9,$

$C_{31} = 3 - 2 = 1,$

$C_{32} = 2 - 1 = 1,$

$C_{33} = 1 - 3 = -2$

$|B| = 1[2] + 3[3] + 2[-5] = 11 - 10 = 1$

$$B^{-1} = \frac{adjB}{|B|} = \begin{bmatrix} 2 & -3 & 1 \\ 3 & -5 & 1 \\ -5 & -9 & -2 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{9} \begin{bmatrix} 4-6+1 & -4-3+2 & 2+6+2 \\ 6-10+11 & -6-5+2 & 3+10+2 \\ -10-18-2 & 16-3-4 & -5+18-4 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} -1 & -5 & 10 \\ -3 & -9 & 15 \\ -30 & 3 & 9 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 5 & 5 \\ 3 & -11 & -5 \\ 3 & -5 & -2 \end{bmatrix},$$

$|AB| = 6(-3) + 5(-9) + 5(18) = -18 - 45 + 90 = +27$

$C_{11} = 22 - 25 = -3$

$C_{12} = -9$

$C_{13} = -15 + 33 = 18$

$C_{21} = -25 + 10 = -15$

$C_{22} = -12 - 15 = -27$

$C_{23} = 30 + 15 = 45$

$C_{31} = -25 + 55 = 30$

$C_{32} = -30 + 15 = 15,$

$C_{33} = -66 - 15 = -81$

$$\text{So } (AB)^{-1} = \begin{bmatrix} -3 & -15 & 30 \\ -9 & -27 & 15 \\ 18 & 45 & -81 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} -1 & -5 & 10 \\ -3 & -9 & 15 \\ -30 & 3 & 9 \end{bmatrix} = B^{-1}A^{-1}$$

Sol 33: $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= (a+b+c)(ab + bc + ca - a^2 - b^2 - c^2)$

$= 3abc - a^3 - b^3 - c^3$

$C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$

$$(a+b+c) \begin{vmatrix} 0 & b-c & c-b \\ 0 & c-a & a-b \\ 1 & a & b \end{vmatrix}$$

$= (a+b+c)[(b-a)(a-b) - (c-a)(c-b)]$

$= (a+b+c)(ab + bc + ca - a^2 - b^2 - c^2)$

$= a^2b - a^2b + \ldots\ldots + 3abc - a^3 - b^3 - c^3$

$= 3abc - a^3 - b^3 - c^3$

Sol 34: $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x + y + z)(x - z)^2$

$R \rightarrow R_1 + R_2 + R_3$

$$\begin{vmatrix} 2(x+y+z) & x+y+z & x+y+z \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= (x+y+z) \begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$C_1 \to C_1 - C_2 - C_3$

$$= (x+y+z) \begin{vmatrix} 2-1-1 & 1 & 1 \\ z+x-z-x & z & x \\ x+y-y-z & y & z \end{vmatrix}$$

$$= (x+y+z) \begin{vmatrix} 0 & 1 & 1 \\ 0 & z & x \\ x-z & y & z \end{vmatrix}$$

$= (x + y + z)(x - z)(x - z)$

Sol 35: $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab + b + ca)^3$

$C_1 \to C_1 + C_3,\ C_2 \to C_2 + C_3$

$$\begin{vmatrix} c^2 & (b+c)^2 & c^2+bc \\ (a+c)^2 & c^2 & c^2+ac \\ a^2 & b^2 & -ab \end{vmatrix}$$

$= c^2[-abc^2 - b^2(c^2 + ac)] +$

$(b+c)^2\ [a^2(c^2+ac) + ab(a+c)^2]$

$(c^2 + b)\ [b^2(a + c)^2 - c^2a^2]$

$= [-abc^4 - c^4b^2 - c^3b^2a + (b+c)^2 (a^2c^2 + a^3c + a^3b + abc^2 + 2a^2bc] + (c^2 + bc)[b^2a^2 + b^2c^2 + 2acb^2 - c^2a^2]$

This on simplification comes out to be equal to

$(ab + bc + ca)^3$

Sol 36: $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)\,(b-c)\,(c-a)$

$R_1 \to R_1 - R_3,\ R_2 \to R_2 - R_3$

$$= \begin{vmatrix} (b-c)^2-(a+b)^2 & a^2-c^2 & bc-ab \\ (c+a)^2-(a+b)^2 & b^2-c^2 & ac-ab \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$= \begin{vmatrix} (c-a)[c+a+2b] & (c-a)(-a-c) & b(c-a) \\ (b-c)[-c-b-2a] & (b-c)(b+c) & -a(b+c) \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$= (c-a)\,(b-c) \begin{vmatrix} a+2b+c & -a-c & b \\ -(2a+b+c) & b+c & -a \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$R_1 \to R_1 + R_2$

$= (c-a)(b-c)$

$$\begin{vmatrix} a+2b+(-2a-b+c) & -a-c+b+c & b-a \\ -(2a+b+c) & b+c & -a \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$= (c-a)(b-c) \begin{vmatrix} b-a & b-a & b-a \\ -2a-b-c & b+c & -a \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$\begin{vmatrix} 0 & 1 & 1 \\ 0 & b+c & -a \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$$

$C_1 \to C_1 + C_2 - 2C_3$

$= (c-a)(b-c)(b-c)$

$$\begin{vmatrix} z-z & 1 & 1 \\ -2a-b-c+b+c+2a & b+c & -a \\ a^2+b^2+2ab+c^2-2ab & c^2 & ab \end{vmatrix}$$

$= -(a-b)(b-c)(c-a)$

$$\begin{vmatrix} z-z & 1 & 1 \\ -2a-b-c+b+c+2a & b+c & -a \\ a^2+b^2+2ab+c^2-2ab & c^2 & ab \end{vmatrix}$$

$= -(a-b)(b-c)\,(c-a)\,(a^2+b^2+c^2)(-a-b-c)$

$= (a-b)(b-c)(c-a)(a^2+b^2+c^2)(a+b+c)$

Sol 37: $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix}$

$= (a\ b + c)\ (a^2 + b^2 + c^2)$

$$= \begin{vmatrix} a & b-c & c+b \\ a & b & c-a \\ a & a+b & c \end{vmatrix} + \begin{vmatrix} 0 & b-c & c+b \\ c & b & c-a \\ -b & a+b & c \end{vmatrix}$$

$$= a\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & a+b & c \end{vmatrix} + \begin{vmatrix} 0 & 1 & c+b \\ c & 1 & c-a \\ -b & b & c \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & -c & c+b \\ c & 0 & c-a \\ -b & a & c \end{vmatrix}$$

$$= a \begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & a+b & c \end{vmatrix} + b \begin{vmatrix} 0 & 1 & c+b \\ c & 1 & c-a \\ -b & 1 & c \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{vmatrix} + \begin{vmatrix} 0 & -c & c \\ c & 0 & c \\ -b & a & c \end{vmatrix}$$

Using, $C_2 \to C_2 - C_1$ and $C_3 \to C_3\text{-}C_1$

in (i), (ii), (iii)

$$a \begin{vmatrix} 1 & b-c & c+b \\ 0 & c & -a-b \\ 0 & a+c & -b \end{vmatrix} + \begin{vmatrix} 0 & 1 & c+b \\ c & 0 & -a-b \\ -b & 0 & -b \end{vmatrix}$$

$$+ c \begin{vmatrix} 0 & -c & 1 \\ c & c & 0 \\ -b & a+c & 0 \end{vmatrix}$$

$= a^2[a + b + c] + b^2[a + b + c] + c^2[a + b + c]$

$= (a^2 + b^2 + c^2)(a + b + c)$

Sol 38: $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$

$R_3 \to R_3 - xR_1 - yR_2$

$$0 = \begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ 0 & 0 & (ax^2+byx+byx+cy^2) \end{vmatrix}$$

$0 = -(ax^2 + 2bxy + cy^2)(ac - b^2)$

$0 = (b^2 - ac)(ax^2 + 2bxy + cy^2)$

$$a \begin{vmatrix} 1 & b-c & c+b \\ 0 & c & -a-b \\ 0 & a+c & -b \end{vmatrix} + b \begin{vmatrix} 0 & 1 & c+b \\ c & 0 & -a-b \\ -b & 0 & -b \end{vmatrix}$$

$$+ c \begin{vmatrix} 0 & -c & 1 \\ c & c & 0 \\ -b & a+c & 0 \end{vmatrix}$$

Sol 39: $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$

$\Rightarrow M_{21} = \begin{vmatrix} 2 & 3 \\ -7 & 9 \end{vmatrix} = 18 + 21 = 39$

$M_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 9 \end{vmatrix} = 9 - 6 = 3$

$M_{23} = \begin{vmatrix} 1 & 2 \\ 2 & -7 \end{vmatrix} = -7 - 4 = -11$

$C_{21} = -39$, $C_{22} = 3$, $C_{23} = 11$

Sol 40: $f(x) = ax^2 + bx + c$, $f(0) = 6$

$f(2) = 11$, $f(-3) = 6$

$$\begin{bmatrix} 0^2 & 0 & 1 \\ 4 & 2 & 1 \\ 9 & -3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 6 \end{bmatrix}$$

$D = -12 - 18 = -30$

$$D_a = \begin{bmatrix} 6 & 0 & 1 \\ 11 & 2 & 1 \\ 6 & -3 & 1 \end{bmatrix}$$

$= 6[2+3] + 1[-33 - 12] = 30 - 33 - 12 = -15$

$a = \dfrac{D_a}{D} = \dfrac{-15}{-30} = \dfrac{1}{2}$

$$D_b = \begin{vmatrix} 0 & 6 & 1 \\ 4 & 11 & 1 \\ 9 & 6 & 1 \end{vmatrix}$$

$= 6[9 - 4] + 1[24 - 99] = 30 + 24 - 99 = -45$

$b = \dfrac{-45}{-30} = \dfrac{3}{2}$

$$D_c = \begin{vmatrix} 0 & 0 & 6 \\ 4 & 2 & 17 \\ 9 & -3 & 6 \end{vmatrix} = 6[-12 - 18] = -30 \times 6$$

$C = \dfrac{D_C}{D} = \dfrac{-30 \times 6}{-30} = 6$

Equation $\to ax^2 + bx + c = \dfrac{x^2}{2} + \dfrac{3}{2}x + 6$

Sol 41: $2x - y = 5$

$4x - 2y = 10$

$D = \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} = 0$

$D_x = \begin{vmatrix} 5 & -1 \\ 10 & -2 \end{vmatrix} = 0$, $D_y = \begin{vmatrix} 2 & 5 \\ 4 & 10 \end{vmatrix} = 0$

So system has infinite solution (consistent).

Sol 42: $3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

$$D = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

$= 3[-5] -1[-3] -2[-6] = -15 + 3 + 12 = 0$

$$D_x = \begin{vmatrix} 2 & -1 & -2 \\ -1 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

$= 3[1 + 4] - 5[2 + 2] = 15 - 20 = -5 \neq 0$

So system is inconsistent.

Sol 43: $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

L.H.S. $= \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \dfrac{1}{abc} \begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$

$= \dfrac{abc}{abc} \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \quad C_2 \leftrightarrow C_3$

$= - \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} \quad C_1 \leftrightarrow C_2$

$= (-1)^2 \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = $ R.H.S.

Sol 44: $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

L.H.S. $= \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} \times \dfrac{abc}{abc}$

$= \dfrac{1}{abc} \begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix} = \dfrac{abc}{abc} \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = $ R.H.S.

$C_2 \leftrightarrow C_3$ and then $C_1 \leftrightarrow C_2$

Sol 45: $\begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix}$

$= -(p-q) \begin{vmatrix} q-p & q-r \\ r-p & 0 \end{vmatrix} + (p-r) \begin{vmatrix} q-p & 0 \\ r-p & r-q \end{vmatrix}$

$= +(p-q)(q-r)(r-p) - (p-q)(q-r)(r-p) = 0$

Sol 46: $\begin{vmatrix} x^2 & 0 & 3 \\ x & 1 & -4 \\ 1 & 2 & 0 \end{vmatrix} = 11$

$\Rightarrow x^2[8] + 3[2x - 1] = 11$

$\Rightarrow 4x^2 + 3x - 7 = 0$

$\Rightarrow (x - 1)(4x + 7) = 0$

$\Rightarrow (x - 1)(4x + 7) = 0$

$\therefore x = 1$ or $-\dfrac{7}{4}$

Sol 47: $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$, $A^2 - 4A - I = 0$

Assume $A - xI = 0 \rightarrow$

$\begin{bmatrix} 3-x & 2 \\ 2 & 1-x \end{bmatrix} = 0$

$(3 - x)(1 - x) - 4 = 0$

$3 + x^2 - x - 3x - 4 = 0$

$\Rightarrow x^2 - 4x - 1 = 0$

$A - xI = 0$

$\Rightarrow A^2 - 4A - I = 0$ Hence proved.

$\Rightarrow A^{-1}[A^2 - 4A - I] = 0$

$A - 4I - A^{-1} = 0$

$\Rightarrow A - 4I = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3-4 & 2 \\ 2 & 1-4 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$

Sol 48: $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ac \\ 1/c & c^2 & ab \end{vmatrix} = \dfrac{1}{(abc)} \begin{vmatrix} a/a & a^3 & abc \\ b/b & b^3 & abc \\ c/c & c^3 & abc \end{vmatrix}$

$= \dfrac{abc}{(abc)} \begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix} = 0$

Sol 49: $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$

Have to show that $x = -(a + b + c)$

$R_1 \rightarrow R_1 + R_2 + R_3$

$\begin{vmatrix} x+a+b+c & x+a+b+c & x+a+b+c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$

$(x+a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$

$x + a + b + c = 0 \Rightarrow x = -(a + b + c)$

Sol 50 : $\begin{vmatrix} x+4 & x & 2 \\ 2 & x+4 & x \\ x & x & x+4 \end{vmatrix}$

$C_1 \rightarrow C_1 - C_3, C_2 \rightarrow C_2 - C_3$

$\begin{vmatrix} 4 & 0 & x \\ 0 & 4 & x \\ -4 & -4 & x+4 \end{vmatrix}$

$= (4x + 16 + 4x) + 16x = 48x + 64$

Sol 51: $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

$C_1 \rightarrow C_1 + C_3$

$= \begin{vmatrix} a+b+c & a-b & a \\ a+b+c & b-c & b \\ a+b+c & c-a & c \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & a-b & a \\ 1 & b-c & b \\ 1 & c-a & c \end{vmatrix}$

$R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$

$= (a + b + c) \begin{vmatrix} 0 & a+c-2b & a-b \\ 0 & b-2c+a & b-c \\ 1 & c-a & c \end{vmatrix}$

$= (a + b + c) [(b - c)(a + c - 2b) - (a - b)(b + a - 2c)]$

$= a^2b + b^2a - b^2a - a^2b + \ldots \ldots + 3abc - a^3 - b^3 - c^3$

$= 3abc - a^3 - b^3 - c^3$

Sol 52: $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$

Assume $|A - xI| = 0$

$\Rightarrow \left| \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} - x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$

$\Rightarrow \left| \begin{bmatrix} 2-x & -3 \\ 3 & 4-x \end{bmatrix} \right| = |0| = 0$

$\Rightarrow (2 - x)(4 - x) + 9 = 0$

$\Rightarrow 8 + x^2 - 4x - 2x + 9 = 0$

$\Rightarrow x^2 - 6x + 17 = 0$ and $|A - xI| = 0$

So, A satisfied this equation

$\Rightarrow A^2 - 6A + 17 I = 0$

$A^{-1}[A^2 - 6A + 17I] = 0$

$\Rightarrow A - 6I + 17A^{-1} = 0$

$-17A^{-1} = (A - 6I)$

$A^{-1} = \dfrac{-1}{17} (A - 6I) = \dfrac{1}{17} \left[\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} - 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right]$

$A^{-1} = -\dfrac{1}{17} \begin{bmatrix} 2-6 & -3 \\ 3 & 4-6 \end{bmatrix} = -\dfrac{1}{17} \begin{bmatrix} -4 & -3 \\ 3 & -2 \end{bmatrix}$

$A^{-1} = \dfrac{1}{17} \begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}$

Sol 53: $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & -1 \end{bmatrix}$

Assume BAC = D

$|B| = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} = 6 - 4 = 2$

$Adj \ B = \begin{vmatrix} 3 & -4 \\ -1 & 2 \end{vmatrix}$

$B^{-1} = \dfrac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

$B^{-1}BAC = B^{-1}D$

$AC = B^{-1}D = \dfrac{1}{2}\begin{vmatrix} 3 & -4 \\ -1 & 2 \end{vmatrix}\begin{vmatrix} 1 & 6 \\ 3 & -1 \end{vmatrix}$

$AC = \dfrac{1}{2}\begin{bmatrix} 3-12 & 18+4 \\ -1+6 & -6-2 \end{bmatrix} = \dfrac{1}{2}\begin{bmatrix} -9 & 22 \\ 5 & -8 \end{bmatrix}$

$|C| = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = -2$

$adjC = \begin{bmatrix} 3 & -2 \\ -1 & 0 \end{bmatrix}, C^{-1} = \dfrac{1}{2}\begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}$

$ACC^{-1} = B^{-1}DC^{-1}$

$A = \dfrac{1}{2}\times\dfrac{1}{2}\begin{bmatrix} -9 & 22 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}$

$A = \dfrac{1}{4}\begin{bmatrix} 27+22 & -18 \\ -15-8 & 10 \end{bmatrix} = +\dfrac{1}{4}\begin{bmatrix} 49 & -18 \\ -23 & 10 \end{bmatrix}$

Sol 54: $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1}$

$C_{11} = 3 - 6 = -3,$

$C_{12} = 3 - 2 = 1,$

$C_{13} = 4 - 3 = 1,$

$C_{21} = 8,$

$C_{22} = 5 - 4 = 1,$

$C_{23} = -10,$

$C_{31} = -12,$

$C_{32} = 8 - 15 = -7,$

$C_{33} = 15$

$|A| = 5(-3) + 4(1) = -15 + 4 = -11$

$A^{-1} = \dfrac{1}{|A|}adjA = \dfrac{-1}{11}\begin{bmatrix} -3 & 8 & -12 \\ 1 & 1 & -7 \\ 1 & -10 & 15 \end{bmatrix}$

$B^{-1}A^{-1} = -\dfrac{1}{11}\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} -3 & 8 & 12 \\ 1 & 1 & -7 \\ 1 & -10 & 15 \end{bmatrix}$

$\dfrac{-1}{11}\begin{bmatrix} -3+3+3 & 8+3-30 & -12-21+45 \\ -3+4+3 & 8+4-30 & -12-28+45 \\ -3+3+4 & 8+3-40 & -12-21+60 \end{bmatrix} = \dfrac{-1}{11}\begin{bmatrix} 3 & -19 & 12 \\ 4 & -18 & 5 \\ 4 & -29 & 27 \end{bmatrix}$

Sol 55: $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix},$

Assume $(A - XI) = 0$

$\Rightarrow \begin{bmatrix} 2-x & -1 & 1 \\ -1 & 2-x & -1 \\ 1 & -1 & 2-x \end{bmatrix} = 0$

$(2 - x)[(2 - x)^2 - 1[-1 + 2 - x] + 1[1 - 2 + x] = 0$

$\Rightarrow (2 - x)[4 + x^2 - 4x - 1] - 1 + x - 1 + x = 0$

$\Rightarrow 6 - x^3 + 2x^2 - 8x - 3x + 4x^2 - 2 + 2x = 0$

$\Rightarrow -x^3 + 6x^2 - 9x + 4 = 0$

$\Rightarrow x^3 - 6x^2 + 9x - 4 = 0$

$|A - xI| = 0$, so this equation satisfied A

$\Rightarrow A^3 - 6A^2 + 9A - 4I = 0 \Rightarrow A^{-1}[A^3 - 6A^2 + 9A - 4I] = A^{-1}0 = 0$

$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0$

$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & +1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix},$

$4A^{-1} = A^2 - 6A + 9I$

$4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$4A^{-1} = \begin{bmatrix} 6-12+9 & -5+6 & 5-6 \\ -5+6 & 6-12+9 & -5+6 \\ 5-6 & -5+6 & 6-12+9 \end{bmatrix}$

$A^{-1} = \dfrac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Exercise 2

Single Correct Choice Type

Sol 1: (D) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$

$abc \begin{vmatrix} \dfrac{1}{a}+1 & \dfrac{1}{a} & \dfrac{1}{a} \\ \dfrac{1}{b} & 1+\dfrac{1}{b} & \dfrac{1}{b} \\ \dfrac{1}{c} & \dfrac{1}{c} & \dfrac{1}{c}+1 \end{vmatrix} = 0$

$R_1 \to R_1 + R_2 + R_3$

$abc \begin{vmatrix} 1+\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c} & 1+\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c} & 1+\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c} \\ \dfrac{1}{b} & 1+\dfrac{1}{b} & \dfrac{1}{b} \\ \dfrac{1}{c} & \dfrac{1}{c} & 1+\dfrac{1}{c} \end{vmatrix} = 0$

$\left(1+\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ \dfrac{1}{b} & 1+\dfrac{1}{b} & \dfrac{1}{b} \\ \dfrac{1}{c} & \dfrac{1}{c} & 1+\dfrac{1}{c} \end{vmatrix} = 0$

$C_2 \to c_2 - c_1, \ C_3 \to c_3 - c_1$

$\left(1+\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right) \begin{vmatrix} 1 & 0 & 0 \\ \dfrac{1}{b} & 1 & 0 \\ \dfrac{1}{c} & 0 & 1 \end{vmatrix} = 0$

$\left(1+\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right) = 0 \Rightarrow a^{-1} + b^{-1} + c^{-1} = -1$

Sol 2: (A) $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a & a^3 & a^4 \\ b & b^3 & b^4 \\ c & c^3 & c^4 \end{vmatrix} + \begin{vmatrix} a & a^3 & -1 \\ b & b^3 & -1 \\ c & c^3 & -1 \end{vmatrix} = 0$

$\Rightarrow abc \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} a & a^3 & 1 \\ b & b^3 & 1 \\ c & c^3 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_3, \ R_2 \to R_2 - R_3$

$\Rightarrow abc \begin{vmatrix} 0 & a^2-c^2 & a^3-c^3 \\ 0 & b^2-c^2 & b^3-c^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} a-c & a^3-c^3 & 0 \\ b-c & b^3-c^3 & 0 \\ c & c^3 & 1 \end{vmatrix}$

$\Rightarrow abc \ [(a^2-c^2)(b^3-c^3) - (b^2-c^2)(a^3-c^3)$

$= [(a-c)(b^3-c^3) - (b-c)(a^3-c^3)]$

$\Rightarrow abc \ (a-c)(b-c)[(a+c)(b^2+c^2+bc)$

$\quad -(b+c)(a^2+c^2+ac)$

$= (a-c)(b-c) \ [b^2+c^2+bc-(a^2+c^2+ac)]$

$abc \ [ab^2+ac^2+abc+cb^2+c^3+bc^2$

$\quad -ba^2-bc^2-abc-ca^2-c^3-ac^2]$

$= b^2+c^2+bc-a^2-c^2-ac$

$= (b-a)(b+a+c)$

$\Rightarrow abc \ (b-a)[ab+c(b+a)]$

$= (b-a)(a+b+c)$

$\Rightarrow abc \ [ab+bc+ca] = [a+b+c]$

Sol 3: (A) $(\sin^{-1}x + \sin^{-1}w)(\sin^{-1}y + \sin^{-1}z) = \pi^2$

$D \to \begin{vmatrix} x^{N_1} & y^{N_2} \\ z^{N_3} & w^{N_4} \end{vmatrix}$

$-1 \le (x, y, w, z) \le 1$

$x^{N_1}w^{N_4} - z^{N_3}y^{N_4}$

If $x = y = 7 = w = -1$

$\begin{vmatrix} x^{N_1} & y^{N_2} \\ z^{N_3} & w^{N_4} \end{vmatrix} \to (-1)^{N_2+N_4} - (-1)^{N_2+N_3}$

For max value

$N_1 + N_4 = 2n. \ N_2 + N_3 = 2m + 1$

$\Rightarrow n, m \in N$

Value $(-1)2n - (-1)^{2n} - (-1)^{2m+1}$

$\Rightarrow 1 - (-1) = 2$

Min value $\to -1 - 1 = -2$

Dependent of N_1, N_2, N_3, N_4

Sol 4: (C) $(1+ x + x^2)^n = a_0 + a_1x + a_2x^2 + ...+... a_{2n} x^{2n}$

$$\begin{vmatrix} a_{n-3} & a_{n-1} & a_{n+1} \\ a_{n-6} & a_{n-3} & a_{n+3} \\ a_{n-14} & a_{n-7} & a_{n+7} \end{vmatrix}$$

$(1 + x + x^2)^n = (x^2 + x + 1)^n$

$a_{n-1} = a_{n+1}$

$a_0 = a_n$

$a_{n-r} = a_{n+r} \, O \le r \le n$

So determinate $\rightarrow \begin{vmatrix} a_{n-3} & a_{n-1} & a_{n-1} \\ a_{n-6} & a_{n-3} & a_{n-3} \\ a_{n-14} & a_{n-7} & a_{n-7} \end{vmatrix}$

$C_2 \rightarrow C_2 - C_3$

$$\begin{vmatrix} a_{n-3} & 0 & a_{n-1} \\ a_{n-6} & 0 & a_{n-3} \\ a_{n-14} & 0 & a_{n-7} \end{vmatrix} = 0$$

Sol 5: (A) $\begin{vmatrix} -1 & 2 & 1 \\ 3+2\sqrt{2} & 2+2\sqrt{2} & 1 \\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1 \end{vmatrix}$ $C_1 \rightarrow C_1 - C_2 - C_3$

$(-1)^3 \begin{vmatrix} -1-2-1 & 2 & 1 \\ 3+2\sqrt{2}-2-2\sqrt{2}-1 & 2+2\sqrt{2} & 1 \\ 3-2\sqrt{2}-2+2\sqrt{2}-1 & 2-2\sqrt{2} & 1 \end{vmatrix}$

$= - \begin{vmatrix} -4 & 2 & 1 \\ 0 & 2+2\sqrt{2} & 1 \\ 0 & 2-2\sqrt{2} & 1 \end{vmatrix} = + 4 [2 + 2\sqrt{2} -(2 - 2\sqrt{2})]$

$= + 4 [4\sqrt{2}] = 16\sqrt{2}$

Sol 6: (C) $D_1 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$,

$D_2 = \begin{vmatrix} bc-a^2 & ac-b^2 & ab-c^2 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix}$

$D_1^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \begin{vmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{vmatrix}$

$= D_3$ (given)

in $D_3 \rightarrow C_1 \rightarrow C_1 - C_3, C_2 \rightarrow C_2 - C_3$

and assume $T = a^2+b^2 + c^2 - ab - bc - ca$

$$\begin{vmatrix} T & 0 & ab+bc+ca \\ +0 & T & ab+bc+ca \\ -T & -T & a^2+b^2+c^2 \end{vmatrix}$$

$= T^2 \begin{vmatrix} 1 & 0 & ab+bc+ca \\ 0 & 1 & ab+bc+ca \\ -1 & -1 & a^2+b^2+c^2 \end{vmatrix}$

$T^2[a^2+b^2+c^2+ab+bc+ca+ab+bc+ca(1)]$

$= T^2[a^2 + b^2 + c^2 + 2(ab + bc + ca)]$

$D_2 = \begin{vmatrix} bc-a^2 & ac-b^2 & ab-c^2 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix}$

$= C_1 \rightarrow C_1 + C_2 + C_3$

$D_2 = \begin{vmatrix} -T & ac-b^2 & ab-c^2 \\ -T & ab-c^2 & bc-a^2 \\ -T & bc-a^2 & ac-b^2 \end{vmatrix}$

$D_2 = T \begin{vmatrix} -1 & ac-b^2 & ab-c^2 \\ -1 & ab-c^2 & bc-a^2 \\ -1 & bc-a^2 & ac-b^2 \end{vmatrix}$

$R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$

$D_2 = T \begin{vmatrix} 0 & ac-b^2+a^2-bc & ab-c^2+b^2-ac \\ 0 & ab-c^2+a^2-bc & bc-a^2+b^2-ac \\ -1 & bc-a^2 & ac-b^2 \end{vmatrix}$

$D_2 = - T [(ac - b^2 + a^2 - bc) (bc - a^2 + b^2$

$- ac) - (ab - c^2 + b^2 - ac) (ab - c^2 + a^2 - bc)]$

$D_2 = (- T) (- T) [T + 3 (ab + bc + ca)]$

$D_2 = T^2[a^2 + b^2 + c^2 + 2(ab + bc + ca)]$

$\therefore D_1^2 = D_2 = D_3$

Sol 7: (C) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

L.H. S. $= (a - c) (b - c) (b - a)$

in L.H.S. $C_1 \rightarrow C_1 - C_3, C_2 \rightarrow C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ a-c & b-c & c \\ a^3-c^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$= (a-c)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+c^2+ac & b^2+c^2+bc & c^3 \end{vmatrix}$$

$\Rightarrow (a-c)(b-c)[b^2+c^2+bc-a^2-c^2-ac]$

$\Rightarrow (a-c)(b-c)(b-a)(b+a+c)$

$\rightarrow a+b+c = 1$

$= (a-c)(b-c)(b-a)$

$abc = a+b+c$

A.M. $\geq$ G.M

$\dfrac{a+b+c}{3} \geq (abc)^{1/3}$; $\qquad \dfrac{1}{3} \geq (abc)^{1/3}$

$\dfrac{1}{27} \geq abc \rightarrow abc$ is always less than 1/27

Sol 8: (A) $(a+1)^3x + (a+2)^3y = (a+3)^3$

$(4+1)x + (a+2)y = (a+3)$

$x + y = 1$

Here for two variable thus equation

So $D = D_x = D_{y=0}$ for consistent

$D = \begin{vmatrix} (a+1)^3 & (a+2)^3 \\ (a+1) & (a+2) \end{vmatrix}$

$= (a+1)(a+2)\begin{vmatrix} (a+1)^2 & (a+2)^2 \\ 1 & 1 \end{vmatrix}$

$= (a+1)(a+2)[a^2+1+2a-a^2-4-4a] = (a+1)(a+2)(-2a-3)$ ... (i)

$D_x = \begin{vmatrix} (a+3)^3 & (a+2)^2 \\ (a+3) & (a+2) \end{vmatrix}$

$= (a+2)(a+3)\begin{vmatrix} (a+3)^2 & (a+2)^2 \\ 1 & 1 \end{vmatrix}$

$= (a+2)(a+3)(a^2+9+6a-a^2-4-4a)$

$= (a+2)(a+3)(5+2a)$ ... (ii)

$D_y = \begin{vmatrix} (a+1)^3 & (a+2)^3 \\ (a+1) & (a+3) \end{vmatrix}$

$= (a+1)(a+3)\begin{vmatrix} (a+1)^2 & (a+3)^2 \\ 1 & 1 \end{vmatrix}$

$= (a+1)(a+3)[a^2+1+2a-a^2-9-6a]$

$= (a+1)(a+3)(-4a-8)$

$= -4(a+2)(a+1)(a+3)$

$D = D_x = D_y = 0$

$\Rightarrow a = -2$ (common solution in all)

Sol 9: (B) $3x - 7y + 5z = 3$, $3x + y + 5z = 7$

$2x + 3y + 5z = 5$

$D = \begin{vmatrix} 3 & -7 & 5 \\ 3 & 1 & 5 \\ 2 & 3 & 5 \end{vmatrix}$

$= 3[5-15] - 7[10-15] + 5[9-2]$

$= -30 + 35 + 35 = 40 \neq 0$

So system is consistent with unique non trivial solution.

Sol 10: (B) $(\sin\theta)x + 27 = 0$

$(\cos\theta)x + \sin\theta\, y = 0$

$(\cos\theta)y + 2z = 0$

$D = \begin{vmatrix} \sin\theta & 0 & 2 \\ \cos\theta & \sin\theta & 0 \\ 0 & \cos\theta & 2 \end{vmatrix}$

$D = \sin\theta(\sin\theta.2) + 2(\cos^2\theta)$

$= 2(\sin^2\theta + \cos^2\theta) = 2$ Constant

$C = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}, D_x = \begin{vmatrix} 0 & 0 & 2 \\ 0 & \sin\theta & 0 \\ a & \cos\theta & 2 \end{vmatrix}$

So system has a unique solution which is a function of a and θ

Sol 11: (D) $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix}$

$+ \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$

In 2nd determinate $R_3 \rightarrow + R_3 + R_1$

$\begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ +(2+x^2) & -(1-5x) & 3x-2 \end{vmatrix} = -|B|$ $\because 1-2x+(1+x)^2$

$= 2 + x^2 + 2x - 2x = 2 + x^2$

$R_3 \rightarrow -R_3$

$$\Rightarrow |A| - \begin{vmatrix} (1+x)^2 & (2x+1) & x+1 \\ (1-x)^2 & 3x & 2x \\ -(2+x^2) & 1-5x & 2-3x \end{vmatrix} = |B|$$

Now all rows of A is equal to columns of B

$\Rightarrow |B| = -|A|$

$|A| - |A| = 0$ (always)

For every valued of x

$|A| + |B|$ is zero

Therefore infinite solutions

Sol 12: (B) $2x \cos^2 \theta + y \sin 2\theta - 2\sin \theta = 0$ ... (i)

$x \sin 2\theta + 2y \sin^2 \theta = -2 \cos \theta$ ...(ii)

$x \sin \theta - y \cos \theta = 0$ ...(iii)

for (i) & (ii)

$$D = \begin{vmatrix} 2\cos^2 \theta & \sin 2\theta \\ \sin 2\theta & 2\sin^2 \theta \end{vmatrix}$$

$= 4 \sin^2\theta \cos^2\theta - 4 \sin^2 \theta \cos^2 \theta = 0$

$= (\sin 2\theta = 2 \sin \theta \cos \theta)$

$$D_x = \begin{vmatrix} 2\sin\theta & \sin 2\theta \\ -2\cos\theta & 2\sin^2 \theta \end{vmatrix}$$

$= 4 \sin^3\theta + 4 \sin \theta \cos^2 \theta$

$= 4 \sin \theta (\sin^2 \theta + \cos^2 \theta) = 4 \sin x$

for consistent $D_x = 0 \rightarrow 4 \sin \theta = 0$

$\theta \in n\pi, n \in I$

$$D_y = \begin{vmatrix} 2\sin^2 \theta & 2\sin\theta \\ \sin 2\theta & -\sin\theta \end{vmatrix} = 4 \cos^3\theta - 4 \sin^2\theta \cos \theta$$

$= -4 \cos \theta [\sin^2\theta + \cos^2 \theta) = -4 \cos \theta$

$D_y = 0$ $\theta = (2n + 1)\pi/2$, $n \in I$

Sin θ and cos θ both are not zero for same θ, so for every value of θ system has not a solution

Sol 13: (C) $(\sin 3\theta)x - y + z = 0$

$(\cos 2\theta)x + 4y + 3z = 0$

$2x + 7y + 7z = 0$

$(\because c = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$

So $D_x = D_y = D_z = 0$

$$D = \begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix}$$

x, y, z are not all simultaneously equal to zero so for solution (not-trivial), D = 0

$\sin 3\theta [28 - 21] - 1[6 - 7\cos 2\theta]$

$+ 1 [7 \cos 2\theta - 8] = 0$

$\Rightarrow 7 \sin 3\theta - 6 + 7 \cos^2\theta + 7 \cos 2\theta - 8 = 0$

$\Rightarrow 7 \sin 3\theta + 14 \cos 2\theta = 14$

$\Rightarrow \sin 3\theta + 2 \cos 2\theta = 2$

$\Rightarrow \sin 3\theta + 2 \cos 2\theta = 2$

$\Rightarrow 3 \sin \theta - 4 \sin^3\theta + 2[1 - 2 \sin^2\theta] = 2$

$\Rightarrow 3 \sin \theta - 4 \sin^3\theta + 2 - 4 \sin^2\theta = 2$

assume sin θ = x

$\Rightarrow 4x^3 + 4x^2 - 3x = 0$

$\Rightarrow x[4x^2 + 4x - 3] = 0$

$\Rightarrow x[4x^2 + 6x - 2x - 3] = 0$

x = 0 or 2x (2x + 3) - 1 (2x + 3) = 0

(2x + 3) (2x - 1) = 0 $\Rightarrow$ x = 1/2 or -3/2

x = 0, ½, -3/2 but $-1 \leq \sin \theta \leq 1$

sin $\theta \neq -3/2$

sin $\theta \in \{0, ½\}$, x = 0, $\pi/6$, $5\pi/6$, π, 12π

between [0, 2π]

No. of principle value = 5

Sol 14: (D) $\begin{vmatrix} \dfrac{a^2+b^2}{c} & c & c \\ a & \dfrac{b^2+c^2}{a} & a \\ b & b & \dfrac{a^2+c^2}{b} \end{vmatrix} = \alpha\, abc$

$abc \begin{vmatrix} \dfrac{a^2+b^2}{c^2} & 1 & 1 \\ 1 & \dfrac{b^2+c^2}{a^2} & 1 \\ 1 & 1 & \dfrac{a^2+c^2}{b^2} \end{vmatrix}$

$\Rightarrow abc \left[\left(\dfrac{a^2+b^2}{c^2}\right)\left(\dfrac{(b^2+c^2)(a^2+c^2)}{a^2b^2} - 1\right) \right]$

$$-\left[\frac{a^2+c^2}{b^2}-1\right]+1\left[1-\left(\frac{b^2+c^2}{a^2}\right)\right]\Bigg]$$

$$\Rightarrow abc\left[\frac{(a^2b^2+a^2c^2+b^4+b^2c^2)(a^2+c^2)-b^4a^2-a^4b^2}{a^2b^2c^2}\right.$$

$$\left.-\left(\frac{a^2+c^2-b^2}{b^2}\right)+\frac{a^2-b^2-c^2}{a^2}\right]$$

$$\Rightarrow \frac{abc}{a^2b^2c^2}\left[\begin{array}{l}a^4b^2+a^4c^2+a^2b^4+a^2b^2c^2+a^2b^2c^2+a^2c^4\\+b^4c^2+b^2c^4-a^2b^2c^2-a^4c^2-a^2c^4+a^2c^2b^2\\+b^2a^2c^2-b^4c^2-b^2c^4-a^4b^2-a^2b^4\end{array}\right]$$

$$=\frac{1}{abc}\left[4a^2b^2c^2+a^4b^2+a^2b^4\right]=4abc=2abc$$

$$\Rightarrow \alpha = 4$$

Sol 15: (C) $a^2x + (2-a)y = 4 + a^2$

$ax + (2a-1)y = a^5 - 2$

$D = \begin{vmatrix} a^2 & 2-a \\ a & 2a-1 \end{vmatrix} = a^2(2a-1) + (a-2)a$

$= 2a^3 - a^2 + a^2 - 2a$

For $D = 0 = 2a(a^2-1) \to +1, -1, 0$

$D_x = \begin{vmatrix} 4+a^2 & 2-a \\ a^5-2 & 2a-1 \end{vmatrix}$

$= (4+a^2)(2a-1) + (a-2)(a^5-2)$

$= 8a - 4 + 2a^3 - a^2 + a^6 - 2a^5 - 2a + 4$

at $a = 0$ $D_x = 0$

So $D_y = \begin{vmatrix} a^2 & 4+a^2 \\ a & a^3-2 \end{vmatrix} = \begin{vmatrix} 0 & 4+0 \\ 0 & 0-2 \end{vmatrix}$

So at $a = 0$, system has infinite solution

At $a = -1, +1$, $D = 0$, and $D_x, D_y \neq 0$

$\Rightarrow$ No solution, no. of values = 2

Previous Years Questions

Sol 1: (B) Given $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$

Applying $C_1 \to C_1 - (pC_2 + C_3)$

$\Rightarrow \begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -(xp^2+yp+yp+z) & xp+y & yp+z \end{vmatrix} = 0$

$\Rightarrow -(xp^2+2yp+z)(xz-y^2) = 0$

$\therefore$ Either $xp^2 + 2yp + z = 0$ or $y^2 = xz$

$\Rightarrow$ x, y, z are in GP.

Sol 2: (A) Given

$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$

Applying $C_3 \to C_3 - (C_1 + C_2)$

$= \begin{vmatrix} 1 & x & 0 \\ 2x & x(x-1) & 0 \\ 3x(x-1) & x(x-1)(x-2) & 0 \end{vmatrix} = 0$

$\therefore f(x) = 0 \qquad \Rightarrow f(100) = 0$

Sol 3: (D) Since, the given system has non-zero solution.

$\therefore \begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$

Applying $C_1 \to C_1 - C_2$, $C_2 \to C_2 + C_3$

$\Rightarrow \begin{vmatrix} 1+k & -k-1 & -1 \\ 1+k & -2 & -1 \\ 0 & 0 & -1 \end{vmatrix} = 0$

$\Rightarrow 2(k+1) - (k+1)^2 = 0$

$\Rightarrow (k+1)(2-k-1) = 0$

$\Rightarrow k = \pm 1$

Note: There is a golden rule in determinant that n one's $\Rightarrow$ (n – 1) zero's or n(constant) $\Rightarrow$ (n – 1) zero's for all constant should be in a single row or a single column.

Sol 4: (C) Given $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix} \sin x + 2\cos x & \cos x & \cos x \\ \sin x + 2\cos x & \sin x & \cos x \\ \sin x + 2\cos x & \cos x & \sin x \end{vmatrix}$

$$= (2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

Applying $R_2 \to C_2 - R_1, R_3 \to C_3 - R_1$

$$\Rightarrow (2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

$\Rightarrow (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

$\Rightarrow 2\cos x + \sin x = 0$ or $\sin x - \cos x = 0$

$\Rightarrow 2\cos x = -\sin x$ or $\sin x = \cos x$

$\Rightarrow \cot x = -\dfrac{1}{2}$ gives no solution in $-\dfrac{\pi}{4} \le x \le \dfrac{\pi}{4}$ and sin $x = \cos x \Rightarrow \tan x = 1$

$\Rightarrow x = \dfrac{\pi}{4}$

Sol 5: (A) Given equations

$x + ay = 0$, $az + y = 0$ and $ax + z = 0$

has infinite solutions.

$$\therefore \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1 + a^3 = 0 \text{ or } a = -1$$

Sol 6: (1) For infinitely many solution, we must have

$\dfrac{k+1}{k} = \dfrac{8}{k+3} = \dfrac{4k}{3k-1} \Rightarrow k = 1$

Sol 7: (A) The given system of equation can be expressed as

$$\begin{bmatrix} 1 & -2 & 3 \\ 1 & -3 & 4 \\ -1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ k \end{bmatrix}$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 + R_1$

$$\Rightarrow \sim \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ k-1 \end{bmatrix}$$

$$\Rightarrow \sim \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ k-3 \end{bmatrix}$$

$\Rightarrow R_3 \to R_3 - R_2$

When $k \ne 3$, the given system of equation has no solution.

$\Rightarrow$ Statement I is true. Clearly, Statement II is also true as it is rearrangement of rows and columns of

$$\begin{bmatrix} 1 & -2 & 3 \\ 1 & -3 & 4 \\ -1 & 1 & -2 \end{bmatrix}$$

Sol 8: Given systems of equation can be rewritten as

$-x + cy + bz = 0$

$cx - y + az = 0$ and $bx + ay - z = 0$

Above system of equations are homogeneous equation. Since, x, y and z are not all zero, so it has non-trivial solution.

Therefore, the coefficient of determinant must be zero

$$\therefore \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$\Rightarrow -1(1 - a^2) - c(-c - ab) + b(ca + b) = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc - 1 = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

Sol 9: Since α is repeated root of $f(x) = 0$.

$\therefore f(x) = a(x - \alpha)^2$, $a \in \text{constant} (\ne 0)$

$$\text{Let } \phi(x) = \begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

[To show $\phi(x)$ is divisible by $(x - \alpha)^2$, it is sufficient to show that $\phi(\alpha)$ and $\phi'(\alpha) = 0$].

$$\therefore \phi(\alpha) = \begin{vmatrix} A(\alpha) & B(\alpha) & C(\alpha) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix} = 0$$

[$\because R_1$ and R_2 are identical]

$$\text{Again, } \phi'(x) = \begin{vmatrix} A'(x) & B'(x) & C'(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

$$\phi'(\alpha) = \begin{vmatrix} A'(\alpha) & B'(\alpha) & C'(\alpha) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix} = 0$$

[$\because R_1$ and R_3 are identical]

Thus, α is repeated root of $\phi(x) = 0$

Hence, $\phi(x)$ is divisible by $f(x)$.

Sol 10: (4) Given $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, $abc = 1$

and $A^T A = 1$ ... (i)

Now, $A^T A = 1$

$$\Rightarrow \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow a^2+b^2+c^2 = 1$ and $ab + bc + ca = 0$ ... (ii)

We know, $a^3 + b^3 + c^3 - 3abc$

$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow a^3+b^3+c^3 = (a+b+c)(1-0)+3$

[from equation (i) and (ii)]

$\therefore a^3+b^3+c^3 = (a+b+c)+3$ ... (iii)

Now, $(a+b+c)^2 = a^2+b^2+c^2 + 2(ab+bc+ca) = 1$ ... (iv)

$\therefore$ From equation (iii), $a^3+b^3+c^3 = 1+3$

$\Rightarrow a^3+b^3+c^3 = 4$

Sol 11: (B) $\Delta = \begin{vmatrix} k+1 & 8 \\ k & k+3 \end{vmatrix} = k^2+4k+3-8k$

$= k^2 - 4k + 3$

$= (k-3)(k-1)$

$\Delta_1 = \begin{vmatrix} 4k & 8 \\ 3k-1 & k+3 \end{vmatrix} = 4k^2 + 12k - 24k + 8$

$= 4k^2 - 12k + 8 = 4(k^2-3k+2) = 4(k-2)(k-1)$

$\Delta_2 = \begin{vmatrix} k+1 & 4k \\ k & 3k-1 \end{vmatrix} = 3k^2 + 2k - 1 - 4k^2$

$= -k^2 + 2k - 1 = -(k-1)^2$

As given no solution $\Rightarrow \Delta_1$ & $\Delta_2 \neq 0$

$\Delta = 0$

$\Rightarrow k = 3$

Sol 12: (C) $\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \beta \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$

$= (1-\alpha)^2 (\alpha-\beta)^2 (\beta-1)^2$

$K = 1$

Sol 13: (C) $(2-\lambda)x_1 - 2x_1 + x_3 = 0$

$2x_1 - (3+\lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

Non-trivial solution

$\Delta = 0$

$\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$

$(1-\lambda)\{3\lambda + \lambda^2 - 4\} + 2.\{-2\lambda+2\} + (4-3-\lambda) = 0$

$\Rightarrow (6\lambda + 2\lambda^2 - 8 - 3\lambda^2 - \lambda^3 + 4\lambda) - 4\lambda + 4 + 1 - \lambda = 0$

$\Rightarrow -\lambda^2 - \lambda^2 - 5\lambda + 3 = 0$

$x^3 - \lambda^2 + 2\lambda^2 - 2\lambda - 3\lambda + 3 = 0$

$\Rightarrow \lambda^2(\lambda^2 - 1) + 2\lambda(\lambda-1) - 3(\lambda-1) = 0$

$\Rightarrow (\lambda-1)(\lambda^2 + 2\lambda - 3) = 0$

$\Rightarrow (\lambda-1)(\lambda+3)(\lambda-1) = 0$

$\lambda = 1, 1, -3$

Sol 14: (D) $x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

For non-trivial solution $\Rightarrow \Delta = 0$

$\Rightarrow \begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda + 1 - \lambda\{-\lambda^2 + 1\} - (\lambda+1) = 0$

$\Rightarrow \lambda(\lambda^2 - 1) = 0$

$\lambda = 0, \pm 1$

JEE Advanced/Boards

Exercise 1

Sol 1: (a) $x + y + z = 6$

$2x + y - z = 1$

$x + y - 2z = -3$

$$D = \begin{vmatrix} 1 & 1 & +1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{vmatrix} \quad C_1 \to C_1 - C_2$$

$$\Rightarrow \begin{vmatrix} 0 & 1 & +1 \\ 1 & 1 & -1 \\ 0 & 1 & -2 \end{vmatrix} = 1[+2 + 1] = 3$$

$$D_x = \begin{vmatrix} 6 & 1 & +1 \\ 1 & 1 & -1 \\ -3 & 1 & -2 \end{vmatrix} \quad C_3 \to C_3 + C_2$$

$$\Rightarrow \begin{vmatrix} 6 & 1 & 2 \\ 1 & 1 & 0 \\ -3 & 1 & -1 \end{vmatrix} = 1[6 - 1] + 2(1 + 3) = -5 + 8 = 3$$

$$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 1 & -1 \\ 1 & -3 & -2 \end{vmatrix}$$

$= 1[-2 - 3] + 6[-1 + 4] + 1[-6 - 1] = -5 + 18 - 7 = 6$

$$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & 1 \\ 1 & 1 & -3 \end{vmatrix}$$

$= 1[-3 - 1] + 1[1 + 6] + 6[2 - 1] = -4 + 7 + 6 = 9$

$x = \dfrac{Dx}{D} = \dfrac{3}{3} = 1$, $y = \dfrac{Dy}{D} = \dfrac{6}{3} = 2$, $z = \dfrac{Dz}{D} = \dfrac{9}{3} = 3$

Here, it is consistent

(b) $x + 2y + z = 1$

$3x + y + z = 6$

$x + 2y = 0$

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 1[6 - 1] + 1[0] = 5$$

$$D_x = \begin{vmatrix} 1 & 2 & 1 \\ 6 & 1 & 1 \\ 0 & 2 & 0 \end{vmatrix} = 2[6 - 1] = 10,$$

$y = \dfrac{D_z}{D} = \dfrac{10}{5} = 2$

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 6 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1[1 - 6] = -5,$$

$y = \dfrac{D_y}{D} = \dfrac{-5}{5} = -1$

$$D_z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 6 \\ 1 & 2 & 0 \end{vmatrix} = 1[6 - 1] + 6[0] = 5m,$$

$Z = \dfrac{D_z}{D} = \dfrac{5}{5} = 1$

(c) $7x - 7y + 5z = 3$

$3x + y + 5z = 7$

$2x + 3y + 5z = 5$

$$D = \begin{vmatrix} 7 & -7 & 5 \\ 3 & 1 & 5 \\ 2 & 3 & 5 \end{vmatrix} \quad R_1 \to R_1 - R_3; R_2 \to R_2 - R_3$$

$$\begin{vmatrix} 5 & -10 & 0 \\ 1 & -2 & 0 \\ 2 & 3 & 5 \end{vmatrix} = 5[-10 + 10] = 0$$

$$D_x = \begin{vmatrix} 3 & -7 & 5 \\ 7 & 1 & 5 \\ 5 & 3 & 5 \end{vmatrix} \quad R_1 \to R_1 - R_3; R_2 \to R_2 - R_3$$

$$\begin{vmatrix} -2 & -10 & 0 \\ 2 & -2 & 0 \\ 5 & 3 & 5 \end{vmatrix} = 5[4 + 20] = 120 \neq 0$$

$D = 0$ but $Dx \neq 0$, so, system is inconsistent

Sol 2: $x + ky + 3z = 0$ ... (i)

$3x + ky - 2z = 0$ ... (ii)

$2x + 3y - 4z = 0$ ... (iii)

Equation has non-trivial solution.

So, $D = D_x = D_y = D_z = 0$

$$D = \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix}$$

$= 1[-4k + 6] + k[-4 + 12] + 3[9 - 2k]$

$= -4k + 6 + 8k + 27 - 6k = 33 - 2k = 0$

$K = \dfrac{33}{2}$, assuming $x = t$

From equation (ii) – (i)

$2x - 5z = 0$

$z = \dfrac{2x}{5} = \dfrac{2t}{5}$, $(x = t)$

In (iii)

$\Rightarrow 2t + 3y - 4z = 0 \rightarrow 3z = 4z - 2t$

$\Rightarrow 3y = 4\left(\dfrac{2t}{5}\right) - 2t = \dfrac{8t - 10t}{5} = \dfrac{-2t}{5}$

$\Rightarrow y = \dfrac{-2t}{15}$

$(x, y, z) \Rightarrow \left(t, \dfrac{-2t}{15}, \dfrac{2t}{5}\right)$ $t \in R$

Sol 3: $\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

$D = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}$

$D = \alpha[\alpha^2 - 1] + 1[1 - \alpha] + 1[1 - \alpha]$

$= \alpha^3 - \alpha + 2 - 2\alpha = \alpha^3 - 3\alpha + 2$

$\alpha^3 - 3\alpha + 2$,

At $\alpha = 1 \Rightarrow 1 - 3 + 2 = 0$

So $(\alpha - 1)$ is a factor of $\alpha^3 - 3\alpha + 2$

Now, $\alpha^3 - 3\alpha + 2$ can be witten as

$\Rightarrow \alpha^3 - \alpha^2 + \alpha^2 - \alpha - 2\alpha + 2$

$\Rightarrow \alpha^2(\alpha - 1) + \alpha(\alpha - 1) - 2(\alpha - 1)$

$\Rightarrow (\alpha - 1)(\alpha^2 + \alpha - 2)$

$D = (\alpha - 1)(\alpha^2 + \alpha - 2)$

$D = (\alpha - 1)(\alpha^2 + 2\alpha - \alpha - 2)(\alpha - 1)$

$D = (\alpha - 1)[\alpha(\alpha + 2) - 1(\alpha + 2)]$

$D = (\alpha - 1)(\alpha + 2)(\alpha - 1)$

For $D = 0$, $\alpha = 1$ or -2

For $\alpha = 1$,

$D_x = \begin{vmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$, so consistent

So on D_y and $D_t = 0$ $\quad\quad \therefore \alpha \neq 1 \Rightarrow \alpha = -2$

Sol 4: $a(y + z) = x \rightarrow x - ay - az = 0$

$b(z + x) = y \rightarrow bx - y + bz = 0$

$c(x + y) = z \rightarrow cx + cy - z = 0$

$c = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, so $D_x = D_y = D_z = 0$

So for non-trivial solution, $D = 0$

$D = \begin{vmatrix} 1 & -a & -a \\ b & -1 & b \\ c & c & -1 \end{vmatrix}$ $C_1 \rightarrow C_1 - C_3; C_2 \rightarrow C_2 - C_3$

$D = \begin{vmatrix} 1+a & 0 & -a \\ 0 & -(1+b) & b \\ 1+c & 1+c & -1 \end{vmatrix}$

$\rightarrow \left(\dfrac{1}{1+a} + \dfrac{1}{1+b} + \dfrac{1}{1+c}\right) = 0$

or from equation

$a = \dfrac{x}{y+z}$, $b = \dfrac{y}{x+z}$, $c = \dfrac{z}{x+y}$

$1 + a = \dfrac{x+y+z}{y+z}$; $1 + b = \dfrac{x+y+z}{x+z}$;

$1 + c = \dfrac{x+y+z}{x+y}$

$\dfrac{1}{1+a} + \dfrac{1}{a+b} + \dfrac{1}{1+c} = \dfrac{x+y+y+z+z+x}{x+y+z}$

$= \dfrac{2(x+y+z)}{(x+y+z)} = 2$

Sol 5: $x = cy + bz \rightarrow x - cy - bz = 0$

$y = az + cx \rightarrow cx - y + az = 0$

$z = bx + ay \rightarrow bx + ay - z = 0$

$c = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow$

$D_x = D_y = D_z = 0$,

But system has solution. So $D = 0$

$D = \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 1[1-a^2] + c[-c - ab] - b[ac + b] = 0$

$1 - a^2 - c^2 - abc - abc - b^2 = 0$

$a^2 + b^2 + c^2 + 2ab = 1$

Sol 6: $a = \dfrac{x}{y - z} \rightarrow x - ay + az = 0$

$b = \dfrac{y}{z - x} \rightarrow bx + y - bz = 0$

$c = \dfrac{z}{x - y} \rightarrow cx - cy - z = 0$

$c = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, so $D_x = D_y = D_z = 0$,

For solution $\rightarrow D = 0$

$D = \begin{vmatrix} 1 & -a & a \\ b & 1 & -b \\ c & -c & -1 \end{vmatrix}$

$= 1[-1 - bc] - a[-bc + b] + a[-bc-c]$

$= -1 - bc + abc - ab - abc - ac$

$= -1(ab + bc + ca + 1) = 0$

$= ab + bc + ca + 1 = 0$

Sol 7: $\sin q \neq \cos q$

$X\cos p - y\sin p + z = \cos q + 1$ ... (i)

$x\sin p + y\cos p + z = 1 - \sin q$ ... (ii)

$x\cos(p + q) - y\sin(p + q) + z = 2$ ... (iii)

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\sin(p + q) = \sin p \cos q + \cos p \sin q$

equation (i)^2 + equation (ii)^2

$\Rightarrow x^2(\sin^2 p + \cos^2 p) + y^2(\cos^2 p + \sin^2 p)$

$-2xy \cos p \sin p + 2x \cos p z - 2yz \sin p + 2xy \sin p \cos p$

$+ 2xz \sin p + 2z^2 + 2yz \cos p$

$= z + 1 - 2\sin q + 2\cos q$

$\Rightarrow x^2 + y^2 + z^2 + 2xy \cos p - 2yz \sin p$

$+ 2xz \sin p + z^2 + 2yz \cos p$

$= 2 + 1 - 2\sin q + 2\cos q$

From equation (iii) and (i)

$= x^2 + y^2 + z^2 + 2z (1 + \cos q - z) + 2q(1 - \sin q - z) z^2$

$= 3 - 2(\sin q - \cos q)$

$= x^2 + y^2 + z^2 + 2z(2 + \cos q - \sin q - 2z)$

$= 3 + 2(\cos q - \sin q)$

For equation (iii)

$\Rightarrow 2z(2 + \cos q - \sin q - 2z) = 1 + 2\,|\cos q - \sin q|$

$\therefore x^2 + y^2 + z^2 = 2$

Sol 8: $x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

(a) A unique solution, $D \neq 0$

$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix}$

$= 1[2\lambda - 6] + 1[-\lambda + 3] + 0$

$= 2\lambda - 6 + 3 - \lambda = \lambda - 3 \neq 0$

$\lambda \neq 3$

(b) Infinite solution

So $D = 0$, $Dx = Dy = Dz = 0$

$D = 0 \rightarrow \lambda = 3$

$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 3 \end{vmatrix}$

$\Rightarrow 6[0] + 1[3\mu - 30] + [20 - 2\mu]$

$\Rightarrow (\mu - 10)5 = 0$

$\mu = 10$

(c) No solution $\rightarrow D = 0$, $D_x \neq 0$

$\lambda = 3$, $\mu \neq 10$

Sol 9: $x + y + z = 1$

$x + 2y + 4z = p$

$x + 4y + 10z = p^2$

$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix}$

$D = 1[20 - 16] + 1[4 - 10] + 1[4 - 2] = 4 - 6 + 2 = 0$

So for solution, $D_x = D_y = D_z = 0$

$D_x = \begin{vmatrix} 1 & 1 & 1 \\ p & 2 & 4 \\ p^2 & 4 & 10 \end{vmatrix}$

$= 1[20 - 16] + 1[4p^2 - 10p] + 1[4p - 2p^2] = 0$

$= 4 + 4p^2 - 10p + 4p - 2p^2 = 0$

$2p^2 - 6p + 4 = 0$

$p^2 - 3p + 2 = 0$

$p^2 - 2p - p + 2 = 0$

$(p - 2)(p - 1) = 0 \Rightarrow p = 1$ or 2

For p = 1

$\Rightarrow x + y + z = 1$ ... (i)

$x + 2y + 4z = 1$ ... (ii)

$x + 4y + 10z = 1$ ... (iii)

Assume that x = k

Equation (ii) – ii(i)

$-x + 2z = -1$

$\Rightarrow 2z = x - 1 \Rightarrow z = \dfrac{k-1}{2}$

So $y = 1 - z - x = 1 - k - \dfrac{(k-1)}{2}$

$y = \dfrac{2 - 2k - k + 1}{2} = \dfrac{3 - 3k}{2}$

$(x,y,z) = \left(k, \dfrac{3-3k}{2}, \dfrac{k-1}{2}\right)$

At p = 2

$x + y + z = 1$ ... (1)

$x + 2y + 4z = 2$ ... (2)

$x + 4y + 10z = 4$ ... (3)

Assume x = k

Equation (2) – 2(1)

$-x + 2z = 0$

$\Rightarrow x = 2z = k \Rightarrow z = \dfrac{k}{2}$

$y = 1 - x - z = 1 - k - \dfrac{k}{2}$

$= 1 - \dfrac{3k}{2} = \dfrac{2 - 3k}{2}$

Sol 10: $Kx + 2y - 2z = 1$

$4x + 2Ky - z = 2$

$6x + 6y + Kz = 3$

$D = \begin{bmatrix} K & 2 & -2 \\ 4 & 2K & -1 \\ 6 & 6 & K \end{bmatrix}$ at K = 2 (given)

$= \begin{vmatrix} 2 & 2 & -2 \\ 4 & 4 & -1 \\ 6 & 6 & 2 \end{vmatrix} = 0$

$\Rightarrow 2x + 2y - 2z = 1$ ... (i)

$4x + 4y - z = 2$ ... (ii)

$6x + 6y + 2z = 3$ ... (iii)

Assume x = l

Equation (iii), (ii) – (iii).(ii)

$7z = 0 \rightarrow z = 0$

$2y = 1 + 2z - 2x = 1 - 2\lambda$

$(x, y, z) = (\lambda, 1-2\lambda, \lambda)$

If $K \neq 2$

$D = \begin{bmatrix} K & 2 & -2 \\ 4 & 2K & -1 \\ 6 & 6 & K \end{bmatrix}$

$= K[2K^2 + 6] + 2[-6 - 4K] - 2[24 - 12K]$

$= 2K^3 + 6K - 12 - 8K - 48 + 24K$

$= 2K^3 + 22K - 60 = 2(K^3 + 11K - 30)$

At K = 2

$\Rightarrow 2(8 + 11(2) - 30) = 0$

So (K – 2) is a factor

$\dfrac{k^3 + 11k - 30}{k - 2} = K^2 + 2K + 15$

$D = 2(K - 2)(K^2 + 2K + 15)$

$D_x = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 2K & -1 \\ 3 & 6 & K \end{bmatrix}$

$= 2K^2 + 6 + 2[-3 - 2K] - 2[12 - 6K]$

$= 2K^2 + 6 - 6 - 4K - 24 + 12K$

$= 2K^2 + 8K - 24 = 2[K^2 + 4K - 12]$

$= 2[K^2 + 6K - 2K - 12] = 2[K(K+6) - 2(K + 6)]$

$= 2(K - 2)(K + 6)$

Similarly, $D_y = (K - 2)(2K + 3)$ and $D_z = 6(K - 2)^2$

if $K \neq 2$,

$\dfrac{x}{2(K+6)} = \dfrac{y}{2K+3} = \dfrac{z}{6(K-2)} = \dfrac{1}{2(K^2+2K+15)}$

Sol 11: (a) a, b, c, d are distinct no.

a, b, c, d $\in$ {1, 2, 3, 4, 5}

$ax + by = 1$

$cx + dy = 2$

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \qquad \text{... (i)}$$

$$D_x = \begin{vmatrix} 1 & b \\ 2 & d \end{vmatrix} = d - 2b,$$

$$x = \frac{D_x}{D} = \frac{d - 2b}{ad - bc}$$

for least possible +ve value of x

d − 2b = 1 (least natural number)

(d, b) → (3, 1) or (5, 2)

ad − bc should be maximum for least x

(a, b) → (3, 1) (ad − bc) → (3a − c)

$a, c \in \{7, 4, 5\}$

Max. → 3(5) − 2 = 15 − 2

$$x = \frac{1}{13}$$

If a, b → (5, 2),

ad − bc → 5a − 2c,

$a, c \in \{1, 3, 4\}$

Max. 5a − 2c → 5(4) − 2(1) = 18

$$\rightarrow x = \frac{1}{18} = \frac{p}{q} \text{ (min.)}$$

p + q = 1 + 18 = 19

(b) x + ay = 3 and ax + 4y = 6 → x > 1, y > 0

$$D = \begin{vmatrix} 1 & a \\ a & 4 \end{vmatrix} = 4 - a^2,$$

$$D_x = \begin{vmatrix} 3 & a \\ 6 & 4 \end{vmatrix} = 12 - 6a$$

$$D_y = \begin{vmatrix} 1 & 3 \\ a & 6 \end{vmatrix} = 6 - 3a,$$

$$x > 0, \quad \frac{D_x}{D} > 1 \rightarrow \frac{6(2-a)}{(2-a)(2+a)} > 1$$

$$\frac{6}{(2-a)} > 1,$$

2 + a < 6 → a = 1, 3

$$y = \frac{D_y}{D} = \frac{3(2-a)}{6(2-a)} = \frac{3}{6} = \frac{1}{2} + ve$$

So a is 1 and 3

1 + 3 = 4

Sol 12: (a − t)x + by + cz = 0

bx + (c − t)y + az = 0

cx + ay + (b − t)z = 0

Has non-trivial solution,

So D = 0

$$D = \begin{vmatrix} a-t & b & c \\ b & c-t & a \\ c & a & b-t \end{vmatrix} = 0$$

Assume $D = a_0 t^3 + b_0 t^2 + c_0 t + d_0 = 0$

So $t_1 t_2 t_3 = \dfrac{-d_0}{a_0}$

At t = 0, $D = d_0$

$$\text{So } d_0 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

And a_0 is coefficient of t^3 = (−1)(−1)(−1) = −1

$$t_1 t_2 t_3 = \frac{-d_0}{-1} = d_0 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Sol 13: 3x − y + 4z = 3

X + 2y − 3z = −2

6x + 5y + λz = −3,

$$D = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{vmatrix}$$

⇒ 3(2λ + 15) −1[−18 − λ) + 4[5 − 12]

⇒ 6λ + 45 + 18 + λ − 28 = 7λ + 35 = 7(λ + 5)

D = 7(λ + 5)

$$D_x = \begin{vmatrix} 3 & -1 & 4 \\ -2 & 2 & -3 \\ -3 & 5 & \lambda \end{vmatrix}$$

= 3[2λ + 15] + 1[−2λ − 9] + 4[−10 + 6]

= 6λ + 45 − 2λ − 9 − 16

= 4λ + 20 = 4(λ + 5);

$$x = \frac{D_x}{D} = \frac{4(\lambda + 5)}{7(\lambda + 5)} = \frac{4}{7}$$

$$D_y = \begin{vmatrix} 3 & 3 & 4 \\ 1 & -2 & -3 \\ 6 & -3 & \lambda \end{vmatrix}$$

$= 3[-2\lambda - 9] + 3[-18 - \lambda] + 4[-3 + 12]$

$= -6\lambda - 27 - 54 - 3\lambda + 36$

$= -9\lambda - 45 = -(\lambda + 5)$

$\Rightarrow y = \dfrac{D_y}{D} = \dfrac{-9(\lambda + 5)}{7(\lambda + 5)} = \dfrac{-9}{7}$

$D_z = \begin{vmatrix} 3 & -1 & 3 \\ 1 & 2 & -2 \\ 6 & 5 & -3 \end{vmatrix}$

$= 3[-6 + 10] + 1[-3 + 12] + 3[5 - 12]$

$= 12 + 9 - 21 = 0,$

$z = \dfrac{D_z}{D} = 0$

So x, y, z is not dependent on λ

(if $\lambda \neq -5$)

At $\lambda = -5$

$3x - y + 4z = 3$ ... (i)

$x + 2y - 3z = -2$ ... (ii)

$6x + 5y - 5z = -3$ ... (iii)

Assume z = k, (iii) − (ii)(i)

$7y - 13z = -9$

$\Rightarrow y = \dfrac{13k - 9}{7}$

So, $x = 3z - 2y - z = 3k - \dfrac{2}{7}(13k - 9) - z = \dfrac{4 - 5k}{7}$

$(x, y, z) \left(\dfrac{4 - 5k}{7}, \dfrac{13k - 9}{7}, k \right)$

Sol 14: $z + ay + a^2x + a^3 = 0$

$z + by + b^2x + b^3 = 0$

$z + cy + c^2x + c^3 = 0$

Now, $c = \begin{bmatrix} a^3 \\ b^3 \\ c^3 \end{bmatrix}$

$D = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$

$x = \dfrac{D_x}{D}, y = \dfrac{D_y}{D}, z = \dfrac{D_z}{D}$

$D_x = \begin{vmatrix} 1 & a & -a^3 \\ 1 & b & -b^3 \\ 1 & c & -c^3 \end{vmatrix}$

$= -(a + b + c)(a - b)(b - c)(c - a);$

$\therefore x = -(a + b + c)$

$D_y = \begin{vmatrix} 1 & -a^3 & a^2 \\ 1 & -b^3 & b^2 \\ 1 & -c^3 & c^2 \end{vmatrix}$

$= (ab + bc + ca)(a - b)(b - c)(c - a)$

$\therefore y = [ab + bc + ca]$

$D_z = \begin{vmatrix} -a^3 & a & a^2 \\ -b^3 & b & b^2 \\ -c^3 & c & c^2 \end{vmatrix}$

$= -abc(a - b)(b - c)(c - a)$

$\therefore z = -abc$

Sol 15: (a) $\alpha x - y + z = \alpha$

$x - \alpha y + z = 1$

$x - y + \alpha z = 1$

$D = \alpha[-\alpha^2 + 1] - 1[1 - \alpha] + [-1 + \alpha]$

$= -\alpha^3 + \alpha - 2 + 2a$

$= (-\alpha^3 + 3\alpha - 2) = -(\alpha^3 - 3\alpha + 2)$

At $\alpha = 1$

$D = -(1 - 3 + 2) = 0$

So $(\alpha - 1)$ is a factor

$\dfrac{\alpha^3 - 3\alpha + 2}{\alpha - 1} = \alpha^2 + \alpha - 2$

So $D = -(\alpha - 1)(\alpha^2 + \alpha - 2)$

$= -(\alpha - 1)(\alpha^2 + 2\alpha - \alpha - 2)$

$= -(\alpha - 1)[\alpha(\alpha + 2) - 1(\alpha + 2)]$

$= -(\alpha - 1)(\alpha - 1)(\alpha + 2)$

$\alpha \in [-10, 10]$

So, α has an integral value

$D_x = \begin{vmatrix} \alpha & -1 & 1 \\ 1 & -\alpha & 1 \\ 1 & -1 & \alpha \end{vmatrix}$

So x = 1,

$D_x = -(\alpha - 1)^2(\alpha + 2)$

$D_y = \begin{vmatrix} \alpha & \alpha & 1 \\ 1 & 1 & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0,$

$D_z = \begin{vmatrix} \alpha & -1 & \alpha \\ 1 & -\alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$

(a) Unique solution,

So $D \neq 0 \to \alpha \neq 1, -2$

Number of values for α in

$[-10, 10] = 21 - 2 = 19 = L$

(b) Number solution is not possible for every value of α, system has atleast one solution. So M = 0

(c) Infinite solution $\to D = 0$

$\alpha = 1, -2 \to N = 2$

$L - M + N = 19 + 2 = 21$

(b) $2x + 3y - z = 0$

$3x + 2y + kz = 0$

$4x + y + z = 0$

Has non-trivial solution

So, $D = 0 \Rightarrow \begin{vmatrix} 2 & 3 & -1 \\ 3 & 2 & k \\ 4 & 1 & 1 \end{vmatrix}$

$= 2[2 - k] + 3[4k - 3] - 1[3 - 8] = 0$

$4 - 2k + 12k - 9 + 5 = 10k = 0$

$\Rightarrow k = 0$

$\Rightarrow 2x + 3y - z = 0$ ... (i)

$3x + 7y = 0$ ... (ii)

$4x + y + z = 0$ ... (iii)

(iii) − (ii) (i) →

$-5y + 32 = 0 \to 3z = 5y$

$3x = -2y \to x = \dfrac{-2y}{3}$,

$z = \dfrac{5}{3}y$, $y = y$

x, y, z are integer, so at for x and z to be integer x = n

$= -\dfrac{2}{3}y$

$\to y = \dfrac{3n}{-2}$ (also an integer)

So at n = −2, −7y = 3, z = 5 (minimum +ve value)

(c) a, b ∈ {0, 1, 2, ..., 10}

$x + y + z = 4$

$2x + y + 3z = 6$

$x + 2y + az = b$

$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & a \end{vmatrix}$

$= 1(a - 6) + 1(3 - 2a) + 1(4 - 1)$

$= a - 6 + 3 - 2a + 3 = -9$

$D_x = \begin{vmatrix} 4 & 1 & 1 \\ 6 & 1 & 3 \\ b & 2 & a \end{vmatrix}$

$= 4(a-6) + 1(3b-6a) + 1(12-b)$

$= 4a - 24 + 3b - 6a + 12 - b$

$= -2a + 2b - 12$

$D_y = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 6 & 3 \\ 1 & b & a \end{vmatrix}$

$= 6a - 3b + 4[3 - 2a] + b - 6$

$= 6a - 3b + 12 - 8a + b - 6$

$= -2a - 2b + 6$

$D_z = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 1 & 6 \\ 1 & 2 & b \end{vmatrix}$

$= b - 12 + 6 - 2b + 4[4 - 1]$

$= b - 12 + 6 - 2b + 12$

$= -b + 6$

(i) Unique solution so $D \neq 0$

$\to a \neq 0$

∴ a ∈ {1, 2, ..., 10},

b ∈ {0, 1, ..., 10}

L = 10 × 11 = 110

(ii) Number solution D = 0, a = 0

$D_x \neq 0 \to 2b \neq 12 \to b \neq 6$,

and $D_y \neq 0 \to b \neq 3$

M = 1(11 − 2) = 9

(iii) Infinite solution D=0 → a=0,

$D_x = D_y = D_z = 0$

But D_x and D_z can't be zero at same times, so no possible common solution N = 0

L + M − N = 110 + 9 − 0 = 119

Sol 16: $\begin{vmatrix} -7 & 5+3i & \frac{2}{3}-4i \\ 5-3i & 8 & 4+5i \\ \frac{2}{3}+4i & 4-5i & 9 \end{vmatrix}$

(a) Assume $z_1 = 5 + 3i$, $z_2 = \frac{2}{3} + 4i$

$z_3 = 4 + 5i$

$(z^3)^2 = 4^2 + 5^2 = 41$

$\Rightarrow \begin{vmatrix} -7 & z_1 & \overline{z_2} \\ \overline{z_1} & 8 & z_3 \\ z_2 & \overline{z_3} & 9 \end{vmatrix}$

$= -7[72 - z_3\overline{z_3}] + z_1[z_2z_3 - 9\overline{z_2}] + \overline{z_2}[\overline{z_1}\,\overline{z_3} - 8z_2]$

$= -7[72 - 41] + (5 + 3i)$

$\left[\left(\frac{2}{3} + 4i\right)(4 + 5i) - 9\left(\frac{2}{3} - 4i\right) \right]$

$+ \left(\frac{2}{3} - 4i\right)\left[(5 - 3i)(4 - 5i) - 8\left(\frac{2}{3} + 4i\right) \right]$

$= -7(31) + (5 + 3i)$

$\left[\frac{8}{3} + 16i + \frac{10}{3}i - 20 - 6 + 36i \right]$

$+ \left(\frac{2}{3} - 4i\right)\left[20 - 15 - 12i - 25i - \frac{16}{3} - 32i \right]$

$= -217 + (5 + 3i)\left[\frac{-70}{3} + \frac{160i}{3} \right] + \left(\frac{2}{3} - 4i\right)\left[-\frac{1}{3} - 69i \right]$

Coefficient of i

$= -70 + \frac{800}{3} - 46 + \frac{4}{3} = -106 + \frac{804}{3}$

(b) $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$

$= 1[\cos px \sin (p+d)x - \cos(p+d)x \sin px]$

$+ a[\cos(p+d)x \sin (p - d)x - ws(p - d)x \sin (p + d)x]$

$+ a^2[\cos (p -d)x \sin px - \cos px \sin (p -d)x]$

$= [\sin x (p+d - p) + a [\sin x (p - d - p - d)]$

$+ a^2[\sin x (p - p + d)]$

$= \sin x\, d + a \sin (- 2d) + a^2 \sin dx$

It dose not depend upon p

(c) $\begin{vmatrix} x^3+1 & x^2 & x \\ y^3+1 & y^2 & y \\ z^3+1 & z^2 & z \end{vmatrix} = \begin{vmatrix} x^3 & x^2 & x \\ y^3 & y^2 & y \\ z^3 & z^2 & z \end{vmatrix} + \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix}$

$= xyz \begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} + \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix}$

$= (xyz + 1)(x - y)(y - z)(z - x) = 0$

(given) x, y, z are all different

So $(xyz + 1) = 0 \Rightarrow xyz = -1$

Sol 17: (a) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

$R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$\begin{vmatrix} a^2+2a-3 & 2a+1 & 0 \\ 2a-2 & a-2 & 0 \\ 3 & 3 & 1 \end{vmatrix}$

$= (a - 1)(a^2 + 2a - 3) - 4(a - 1)^2$

$= (a - 1)[(a^2 + 3a - a - 3) - 4(a - 1)]$

$(a - 1)[(a - 1)(a + 3) - 4(a - 1)]$

$= (a - 1)^2[a + 3 - 4] = (a - 1)^3$

(b) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^2 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ x-2 & y-2 & 2 \\ x^3-z^3 & y^3-z^3 & z^3 \end{vmatrix}$

$= (x - z)(y - z) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 2 \\ x^2+y^2+xz & y^2+z^2+yz & z^3 \end{vmatrix}$

$\therefore a^2 - b^3 = (a - b)(a^2 + b^2 + ab)$

$= (x - z)(y - z)(y^2 + z^2 + yz - x^2 - z^2 - xz)$

$= (x - z)(y - z)[z(y - x) + (y^2 - x^2)$

$= (x - y)(y - z)(z - x)(x + y + z)$

Sol 18: (a) $f(x) = \begin{vmatrix} x & 1 & -3/2 \\ 2 & 2 & 1 \\ \dfrac{1}{x-1} & 0 & 1/2 \end{vmatrix}$ $\quad x > 1$

$f(x) = x[1 - 0] + 1\left[\dfrac{1}{x-1} - 1\right] + \dfrac{3}{2}\left[\dfrac{2}{x-1}\right]$

$= x + \dfrac{1}{x-1} - 1 + \dfrac{3}{x-1}$

$= (x-1) + \dfrac{3}{x-1} = \dfrac{(x-1)^2 + 3}{x-1}$

$= \dfrac{x^2 + 1 + 3 - 2x}{x-1} = \dfrac{x^2 - 2x + 4}{x-1}$

$F'(x) = 1 - \dfrac{3}{(x-1)^2} \Rightarrow 01 = \dfrac{3}{(x-1)^2}$

$\Rightarrow (x-1)^2 = 3 \Rightarrow x = 1 \pm \sqrt{3}$

But x . so x = $1 \pm \sqrt{3}$

$f''(x) = \dfrac{-6}{(x-z)^3}$, at x = 1 + $\sqrt{3}$

$f''(x) = \dfrac{-6}{3\sqrt{3}} > 0$ so minima

$f(1 + \sqrt{3}) = \sqrt{3} + \dfrac{3}{\sqrt{3}} = 2\sqrt{3}$

But if x is integer for min. volue of f(x)

$\Rightarrow x = [1 + \sqrt{3}] = 2$

$F(x) = f(2) = 1 + \dfrac{3}{1} = 4$

(b) $a^2 + b^2 + c^2 + ab + bc + ca \le O \ \forall \ a,b,c \in R$

$\begin{vmatrix} (a+b+2)^2 & a^2+b^2 & 1 \\ 1 & (b+c+2)^2 & b^2+c^2 \\ c^2+b^2 & 1 & (c+a+2)^2 \end{vmatrix}$

$(a + b)^2 + (b + 1)^2 + (c + a)^2 \ge 0$

(always & square is +ve)

$= 2(a^2 + b^2 + c^2 + bc + ca + ab)$

Its given that $a^2+b^2+c^2 + bc + ca + ab \le 0$

So $0 \le a^2 + b^2 + c^2 + ab + ca + ab \le 0$

$\Rightarrow (a + b)^2 + (b + 1)^2 + (c + a)^2 = 0$

$\Rightarrow a = b = c = 0$

$\begin{vmatrix} 2^2 & 0 & 1 \\ 1 & 2^2 & 0 \\ 0 & 1 & 2^2 \end{vmatrix} = 4[16] + [1] = 65$

Sol 19: $D = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, $D' = \begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix}$

$D' = \begin{vmatrix} b & c+a & a+b \\ a & b+c & c+a \\ c & a+b & b+c \end{vmatrix} + \begin{vmatrix} c & c+a & a+b \\ b & b+c & c+a \\ a & a+b & b+c \end{vmatrix}$

$C_2 \to C_2 + C_1 - C_3, C_2 \to C_2 - C_1,$

$C_3 \to C_3 - C_1 C_3 \to C_3 - C_2$

$D' = \begin{vmatrix} b & c & a \\ a & b & c \\ c & a & b \end{vmatrix} + \begin{vmatrix} c & a & b \\ b & c & a \\ a & b & c \end{vmatrix}$

After swapping rows according to D

$D' = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} + \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = 2D$

Sol 20: $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

$C_1 \to C_1 - BC_3, C_2 \to C_1 + AC_3$

$\begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1-a^2+b^2 & 2a \\ b+b+a^2b+b^3 & -a+a^3-ab^2 & 1-a^2-b^2 \end{vmatrix}$

$R_3 \to R_3 + aR_2 - bR_1$

$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ 0 & 0 & 1+a^2+b^2 \end{vmatrix}$

$= (1 + a^2 + b^2)^3$

Sol 21: $f(x) = \begin{vmatrix} \sin x & \sin(x+h) & \text{sain}(x+2h) \\ \sin(x+2h) & \sin x & \sin(x+h) \\ \sin(x+h) & \sin(x+2h) & \sin x \end{vmatrix}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin (x + nh) = \sin x \cos (nh) + \sin nh \cos x$

$\Rightarrow \lim_{h \to 0} \sin (x + nh) = (\sin x)\, 1 + (nh) \cos x$

$\Rightarrow f(x) \begin{vmatrix} \sin x & \sin x + n\cos x & \sin x + 2h\cos x \\ \sin x + 2h\cos x & \sin x & \sin x + h\cos x \\ \sin x + h\cos x & \sin x + 2h\cos x & \sin x \end{vmatrix}$

$C_1 \to C_1 \to C_3,\ C_2 \to C_2 - C_3$

$f(x) = \begin{vmatrix} -2h\cos x & -h\cos_2 x & \sin x + 2h\cos x \\ h\cos x & -h\cos x & \sin x + h\cos x \\ \cos x & 2h\cos x & \sin x \end{vmatrix}$

$f(x) = h^2 \begin{vmatrix} -2\cos x & -\cos x & \sin x + 2h\cos x \\ \cos x & -\cos x & \sin x + h\cos x \\ \cos x & 2\cos x & \sin x \end{vmatrix}$

$\lim_{h \to 0} \dfrac{f(x)}{h^2} = \begin{vmatrix} -2\cos x & -\cos x & \sin x \\ \cos x & -\cos x & \sin x \\ \cos x & 2\cos x & \sin x \end{vmatrix}$

$R_1 \to R_1,\ R_3,\ R_2 \to R_2 - R_3$

$= \begin{vmatrix} -3\cos x & -3\cos x & 0 \\ 0 & -3\cos x & 0 \\ \cos x & 2\cos x & \sin x \end{vmatrix}$

$= \sin x\,(9\cos^2 x) = \sin x\,(9 - 9\sin^2 x)$

$= 9\sin x - 9\sin^3 x = 3\,(3\sin x - 3\sin^3 x)$

$= 3\,[\sin 3x + \sin^3 x] = k(\sin 3x + \sin^3 x)$

$\Rightarrow K = 3$

Sol 22: $\begin{vmatrix} (\beta + \gamma - \alpha - \delta)^4 & (\beta + \gamma - \alpha - \delta)^2 & 1 \\ (\gamma + \alpha - \beta - \delta)^4 & (\gamma + \alpha - \beta - \delta)^2 & 1 \\ (\alpha + \beta - \gamma - \delta)^4 & (\alpha + \beta - \gamma - \delta)^2 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_3,\ R_2 \to R_2 - R_3$

$\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 - (\alpha+\beta-\gamma-\delta)^4 & (\beta+\gamma-\alpha-\delta)^4 - (\alpha+\beta-\gamma-\delta)^2 & 0 \\ (\gamma+\alpha-\beta-\delta)^4 - (\alpha+\beta-\gamma-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 - (\alpha+\beta-\gamma-\delta)^2 & 0 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix}$

$= ((\beta + \gamma - \alpha - \delta)^2 - (\alpha + \beta - \gamma - \delta)^2$

$- ((\gamma + \alpha - \beta - \delta)^2 - (\alpha + \beta - \gamma - \delta)^2)$

$\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^2 + (\alpha^2+\beta-\gamma-\delta)^2 & 1 & 0 \\ (\gamma+\alpha-\beta-\delta)^2 + (\alpha+\beta-\gamma-\delta)^2 & 1 & 0 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix}$

$= ((\beta - \gamma - \delta - \delta)^2 - (\alpha + \beta - \gamma - \delta)^2)$

$((\gamma + \alpha - \beta - \delta)^2 - (\alpha + \beta - \gamma - \delta)^2)$

$[(\beta + \gamma - \alpha - \delta)^2 + (\delta^2 + \beta^2 - \gamma - \delta)^2$

$(\gamma + \alpha - \beta - \delta)^2 - (\alpha + \beta - \rho - \delta)^2]$

$= -2\,(\alpha - \beta)2\,(\alpha - \gamma)2\,(\alpha - \gamma)2\,(\beta - \gamma)2$

$(\beta - \delta)2\,(\gamma - \delta)\,(-1)^6$

$= -64(\alpha - \beta)(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \gamma)(\gamma - \delta)$

Sol 23: $x^3 - 3x^2 + 2 = 0$

At $x = 1 \Rightarrow 1 - 3 + 2 = 0$.

So $(x - 1)$ is a factor of $x^3 - 3x^2 + 2 = 0$

$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$

$\Rightarrow x^3 - 3x^2 + 2 = (x - 1)\,(x^2 - 2x - 2)$

$a = 1$, and $bc = -2$, $b + c = 2$

$\Rightarrow bc = 1 \pm \sqrt{3},\ c^2b^2 = 4 \pm 2\sqrt{3}$

$= \begin{vmatrix} 2^2 & 1 & 1 \\ 4+2\sqrt{3} & (2-\sqrt{3})^2 & 4+2\sqrt{3} \\ 4-2\sqrt{3} & 4-2\sqrt{3} & (2+\sqrt{3})^2 \end{vmatrix}$

$= \begin{vmatrix} 4 & 1 & 1 \\ 4+2\sqrt{3} & 7-4\sqrt{3} & 4+2\sqrt{3} \\ 4-2\sqrt{3} & 4-2\sqrt{3} & 7+4\sqrt{3} \end{vmatrix}$

$= 4[49 - 48] - [16 - 12] - 1\,[16 + 12) + 28$

$+ 30\sqrt{3} + 24] + [16 - 12 - (28 + 24 - \sqrt{3}\,(30) = -108$

Sol 24: (a) $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$

$= \dfrac{1}{2} \begin{vmatrix} x+2 & 4x+6 & 3x+4 \\ 2x+3 & 6x+8 & 4x+5 \\ 3x+5 & 10x+16 & 10x+17 \end{vmatrix}$

$C_2 \to C_2 - C_1 - C_3$

$= \dfrac{1}{2} \begin{vmatrix} x+2 & 0 & 3x+4 \\ 2x+3 & 0 & 4x+5 \\ 3x+5 & -3x-6 & 10x+17 \end{vmatrix}$

$\Rightarrow \dfrac{[3x+6]}{2}\,[(3x + 4)\,(2x + 3) - (x + 2)\,(4x + 5)] = 0$

$\Rightarrow (3x + 6)[6x^2 + 17x + 12 - 4x^2 - 13x - 10]=0$

$\Rightarrow (3x + 6) [2x^2 + 4x + 2] = 0$

$\Rightarrow (x + 2) (x^2 + 2x + 1) = 0$

$\Rightarrow (x + 2) (x + 1)^2 = 0$

$\Rightarrow x = -2, -1$

(b) $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

$R_1 \to R_2 - R_3,\ R_2 \to R_2 - R_3$

$\Rightarrow \begin{vmatrix} 6 & 24 & 60 \\ 4 & 18 & 48 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 1 & 4 & 10 \\ 2 & 9 & 24 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

$= 9(3x - 64) - 24(2x - 27) + 4 [24 (x - 8)$

$- 2 (3x - 64)] + 10 [2(2x - 27) - 9(x - 8)]$

$= (6 - 48 + 96 - 24 + 40 - 90) (x - 4) = 0$

$\Rightarrow x = 4$

Sol 25: $a + b + c = 0$

$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

$C_1 \to C_1 + C_2 + C_3$

$\begin{vmatrix} a+b+c-x & c & b \\ a+b+c-x & b-x & a \\ a+b+c-x & a & c-x \end{vmatrix}$

$= (a + b + c - x) \begin{vmatrix} 1 & c & b \\ 1 & b.x & a \\ 1 & a & c-x \end{vmatrix} = 0$

$a + b + c = 0\ (a + b + c - x) = -x = 0$

$\begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix}$

$= (b - x) (c - x) - a^2 + c (a - c + x) + b (c - b + x] = 0$

$bc - x (b + 1) + x^2 - a^2 + a (- c^2 + cx + ba - b^2 + bx = 0$

$x^2 + x(b + c - b - c) = a^2 + b^2 + c^2 - (ab + bc + ca)$

$x^2 = a^2 + b^2 + c^2 - (ab + bc + ca)$

$\therefore a + b + c = 0 \Rightarrow (a + b + c)^2 = 0$
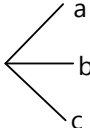
$a^2 + b^2 + c^2 + 2 (ab + bc + ca)$

$\Rightarrow ab + bc + ca = \dfrac{-(a^2 + b^2 + c^2)}{2}$

$x^2 = a^2 + b^2 + c^2 + \dfrac{a^2 + b^2 + c^2}{2} = \dfrac{3}{2} (a^2 + b^2 + c^2)$

$x = \pm \sqrt{\dfrac{3}{2}(a^2 + b^2 + c^2)}$

$x = 0\ \begin{vmatrix} a & c & b \\ c & b & a \\ b & c & a \end{vmatrix} = a^3 + b^3 + c^3 - 3abc = 0$

$\therefore a + b + c = 0$

Sol 26: $x^3 - 5x^2 + 3x - 1 = 0$ ⟨ a, b, c

$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}\ C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$

$= 5 \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix} = 5 \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & a+c & a+b \end{vmatrix}$

$= 5[(b - c) (a + b) + (a - c) (a + c) + 2 (bc - ab - bc + c^2)]$

$= 5[ab - ac + b^2 - bc + a^2 - c^2 - 2ab + 2c^2]$

$= 5[a^2 + b^2 + c^2 - (ab + bc + ca)]$

$a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$

$= 25 - 2 (3) = 19$

$= 5[19 - 3] = 5.\ 16 = 80$

Sol 27: $\begin{vmatrix} a^2 + \lambda & ab & ac \\ ab & b^2 + \lambda & bc \\ ac & b^2 & c^2 + \lambda \end{vmatrix}$

$= \dfrac{1}{abc} \begin{vmatrix} (a^2 + \lambda) & ab^2 & ac^2 \\ a^2b & b(b^2 + \lambda) & bc^2 \\ a^2c & b^2c & c(c^2 + \lambda) \end{vmatrix}$

$$= \frac{abc}{abc} \begin{vmatrix} a^2 + \lambda & b^2 & c^2 \\ a^2 & b^2 + \lambda & c^2 \\ a^2 & b^2 & c^2 + \lambda \end{vmatrix}$$

$R_1 \to R_1 - R_3,\ R_2 \to R_2 - R_3$

$$= \begin{vmatrix} \lambda & 0 & -\lambda \\ 0 & \lambda & -\lambda \\ a^2 & b^2 & c^2 + \lambda \end{vmatrix} = \lambda^2 \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ a^2 & b^2 & (c^2 + \lambda) \end{vmatrix}$$

$= \lambda^2 (c^2 + \lambda + b^2 - 1[-a^2])$

$= \lambda^2 (a^2 + b^2 + c^2 + \lambda)$

Sol 28: $= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

$C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$

$$4 \begin{vmatrix} a^2 & b^2 - a^2 & c^2 - a^2 \\ a & b - a & c - a \\ 1 & 1 & 1 \end{vmatrix}$$

$= 4\left[\left(b^2 - a^2\right)(c - a) - \left(c^2 - a^2\right)(b - a) \right]$

$= -4\ (c - a)(b - a)(b - c)$

$= 4(c - b)(b - c)(c - a)$

Sol 29:

$$\begin{vmatrix} \cot\dfrac{A}{2} & \cot\dfrac{B}{2} & \cot\dfrac{C}{2} \\ \tan\dfrac{B}{2} + \tan\dfrac{C}{2} & \tan\dfrac{C}{2} + \tan\dfrac{A}{2} & \tan\dfrac{A}{2} + \tan\dfrac{B}{2} \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_3,\ C_2 \to C_2 - C_3$

$$\begin{vmatrix} \cot\dfrac{A}{2} - \cot\dfrac{C}{2} & \cot\dfrac{B}{2} - \cot\dfrac{C}{2} & \cot\dfrac{C}{2} \\ \tan\dfrac{C}{2} - \tan\dfrac{A}{2} & \tan\dfrac{C}{2} - \tan\dfrac{B}{2} & \tan\dfrac{A}{2} + \tan\dfrac{B}{2} \\ 0 & 0 & 1 \end{vmatrix}$$

$= \left(\tan\dfrac{C}{2} - \tan\dfrac{B}{2} \right)\left(\cot\dfrac{A}{2} - \cot\dfrac{C}{2} \right)$

$- \left(\cot\dfrac{B}{2} - \cot\dfrac{C}{2} \right)\left(\tan\dfrac{C}{2} - \tan\dfrac{A}{2} \right)$

$= \tan\dfrac{C}{2}\cot\dfrac{A}{2} - 1 - \tan\dfrac{B}{2}\cot\dfrac{A}{2} + \tan\dfrac{B}{2}\cot\dfrac{C}{2}$

$- \cot\dfrac{B}{2}\tan\dfrac{C}{2} + \tan\dfrac{A}{2}\cot\dfrac{B}{2} + 1 - \tan\dfrac{A}{2}\cot\dfrac{C}{2}$

We know that $\tan\dfrac{A}{2} = \dfrac{1}{\cot\dfrac{A}{2}}$

$= \dfrac{\tan\dfrac{C}{2}}{\tan\dfrac{A}{2}} - \dfrac{\tan\dfrac{B}{2}}{\tan\dfrac{A}{2}} - \dfrac{\tan\dfrac{C}{2}}{\tan\dfrac{B}{2}} + \dfrac{\tan\dfrac{A}{2}}{\tan\dfrac{B}{2}}$

$- \dfrac{\tan\dfrac{A}{2}}{\tan\dfrac{C}{2}} + 1 - 1 + \dfrac{\tan\dfrac{B}{2}}{\tan\dfrac{C}{2}} = 0$

$\Rightarrow \dfrac{1}{\tan\dfrac{A}{2}}\left[\tan\dfrac{C}{2} - \tan\dfrac{B}{2} \right] + \dfrac{1}{\tan\dfrac{B}{2}}\left[\tan\dfrac{A}{2} - \tan\dfrac{C}{2} \right]$

$+ \dfrac{1}{\tan\dfrac{C}{2}}\left[\tan\dfrac{B}{2} - \tan\dfrac{A}{2} \right] = 0$

It can only happen when two angles are equal.

$\Rightarrow \triangle ABC$ is isosceles

Exercise 2

Single Correct Choice Type

Sol 1: (A) $D_r = \begin{vmatrix} 2r - 1 & {}^mC_r & 1 \\ m^2 - 1 & 2^m & 1 + m \\ \sin^2(m^2) & \sin^2 m & \sin^2(m+1) \end{vmatrix}$

$\displaystyle\sum_{r=0}^{m} D_r =$

$$\begin{vmatrix} \displaystyle\sum_{r=0}^{m}(2r-1) & \displaystyle\sum_{r=0}^{m}{}^mC_r & m+1 \\ (m+1)(m^2-1) & (m+1)2^m & (m+1)^2 \\ (m+1)\sin^2(m^2) & (m+1)\sin^2 m & (m+1)\sin^2(m+1) \end{vmatrix}$$

$$= \begin{vmatrix} (m+1)(m-1) & 2^m & (m+1) \\ (m+1)(m^2-1) & (m+1)2^m & (m+1)^2 \\ (m+1)\sin^2 m^2 & (m+1)\sin^2 m & (m+1)\sin^2(m+1) \end{vmatrix}$$

Common (m + 1) from C_1, C_3 and R_2

$$= (m+1)^3 \begin{vmatrix} m-1 & 2^m & 1 \\ m-1 & 2^m & 1 \\ \sin^2 m^2 & (m+1)\sin^2 m & \sin^2(m+1) \end{vmatrix} = 0$$

Sol 2: (D) $D = \begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix}$

$D = 1 - \cos(\beta-\gamma)\cos(\gamma-\beta) + \cos(\beta-\alpha)$

$[\cos(\gamma-\beta)\cos(\alpha-\gamma) - \cos(\alpha-\beta)]$

$+ \cos(\gamma-\alpha)[\cos(\alpha-\beta)\cos(\beta-\gamma)$

$- \cos(\alpha-\gamma)]$

$D = 1 - \cos^2(\beta-\gamma) + \cos(\beta-\alpha)\cos(\gamma-\beta)$

$\cos(\alpha-\gamma) - \cos^2(\beta-\alpha) + \cos(\gamma-\alpha)$

$\cos(\alpha-\beta)\cos(\beta-\gamma) - \cos^2(\gamma-\alpha)$

$D = 1 + 2\,\dfrac{\cos(\beta-\gamma)}{2}[\cos(\gamma-\beta) + \cos$

$(\gamma-\beta-2\alpha)] - \cos^2(\gamma-\alpha) - \cos^2(\beta-\alpha) - \cos^2(\beta-\gamma)$

$D = 1 + \cos^2(\beta-\gamma) + \left[\dfrac{\cos 2(\beta-\alpha) + \cos 2(\gamma-\alpha)}{2}\right]$

$- \cos^2(\beta-\gamma) - \cos^2(\gamma-\alpha) - \cos^2(\beta-\alpha)$

$= 1 + \dfrac{1}{2}\Big(2\cos^2(\beta-\alpha) - 1 + 2\cos^2(\gamma-\alpha) - 1\Big)$

$- \cos^2(\beta-\alpha) - \cos^2(\gamma-\alpha)$

$= 1 - \left(\dfrac{2}{2}\right) = \cos^2(\beta-\alpha) + \cos^2(\gamma-\alpha)$

$- \cos^2(\beta-\alpha) - \cos^2(\gamma-\alpha) = 1 - 1 = 0$

Sol 3: (A) $D = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$

$= \dfrac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & a(b+c) \\ bc^2a^2 & abc & b(a+c) \\ ca^2b^2 & abc & c(a+b) \end{vmatrix}$

$= \dfrac{abc.abc}{abc}\begin{vmatrix} bc & 1 & a(b+c) \\ ac & 1 & b(a+c) \\ ab & 1 & c(a+b) \end{vmatrix}$

$C_3 \rightarrow C_3 + C_1$

$= abc\begin{vmatrix} bc & 1 & ab+bc+ca \\ ac & 1 & ab+bc+ca \\ ab & 1 & ab+bc+ca \end{vmatrix}$

$= (abc)(ab+bc+ca)\begin{vmatrix} bc & 1 & 1 \\ ac & 1 & 1 \\ ab & 1 & 1 \end{vmatrix} = 0$

Sol 4: (A)

$f'(x) = \begin{vmatrix} mx & mx-p & mx+p \\ n & n+p & n-p \\ mx+2n & mx+2n+p & mx+2n-p \end{vmatrix}$

$C_2 \rightarrow C_2 + C_3$

$f'(x) = \begin{vmatrix} mx & 2mx & mx+p \\ n & 2n & n-p \\ mx+2n & 2(mx+2n) & mx+2n-p \end{vmatrix}$

$C_2 \rightarrow C_2 - 2C_1$

$f'(x) = \begin{vmatrix} mx & 0 & mx+p \\ n & 0 & n-p \\ mx+2n & 0 & mx+2n-p \end{vmatrix} = 0$

$y = f(x)$

$y' = 0$

$y = K$

It is a straight line parallel to x–axis.

Sol 5: (A) $D(x) = \begin{vmatrix} x-1 & (x-1)^2 & x^3 \\ x-1 & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+1)^3 \end{vmatrix}$

Assume $D(x)\ a_0 + a_1x + \ldots$

$D'(x) = a_1 + 2a_2x$

At $x = 0$ $D'(0) = a_1$

$D'(x) = \begin{vmatrix} +1 & (x-1)^2 & x_3 \\ +1 & x^2 & (x+1)^3 \\ 1 & (x+1)^2 & (x+1)^3 \end{vmatrix} + \begin{vmatrix} x-1 & 2(x-1) & x^3 \\ x-1 & 2x & (x+1)^3 \\ x & 2(x+1) & (x+1)^3 \end{vmatrix}$

$+ \begin{vmatrix} x-1 & (x-1)^2 & 3x^2 \\ x-1 & x^2 & 3(x+1)^2 \\ x & (x+1)^2 & 3(x+1)^2 \end{vmatrix}$ at $x = 0$

$$D'(0) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} -1 & -2 & 0 \\ -1 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} + \begin{vmatrix} -1 & 1 & 0 \\ -1 & 0 & 3 \\ 0 & 1 & 3 \end{vmatrix}$$

$= 1[-1] + 1\,[1 - 1] - 1[-2] - 2\,(1) - 1\,(-3) + 1[3]$

$= -1 + 0 + 2 - 2 + 3 + 3 = -1 + 6 = 5$

Sol 6: (D) $D = \begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$, $|D| = 8$

$R_1 \to R_1 - R_2 - R_3$

$R_2 \to R_2 - R_3$

$$D = \begin{vmatrix} 0 & -2x & -2x \\ z-y & z & -y \\ y & x & x+y \end{vmatrix}$$

$$D = (-2x) \begin{vmatrix} 0 & 1 & 1 \\ z-y & z & -y \\ y & x & x+y \end{vmatrix}$$

$C_2 \to c_2 - c_3$

$$D = (-2x) \begin{vmatrix} 0 & 0 & 1 \\ z-y & z+y & -y \\ y & -y & x+y \end{vmatrix} = (-2x)\,[-y(z-y) - y\,(z+y)]$$

$= -2x\,[-yz + y^2 - y^2 - yz] = \}4xyz| = 8$ given $|xyz| = 2$

For $\to 2 \to (2, 1, 1)\ (-2, 1, -1)\ (2, -1, -1)$

$\Rightarrow \dfrac{3!}{2!} + 3! + \dfrac{3!}{2!} = 12$

For $\to -2 \to (2, 1, -1),\ (-2, -1, -1),\ (-2, 1, 1) = 12$

$3 + 6 + 3 = 12$

Total solution $= 12 + 12 = 24$

Sol 7: (C) $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$

$R_1 \to R_1 - R_3,\ R_2 \to R_2 - R_3$

$$f(x) = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

$= 1 + 4\sin 2x + \cos^2 x - 1(-\sin^2 x)$

$= 1 + (\sin^2 x + \cos^2 x) + 4\sin 2x = 2 + 4\sin 2x$

For max value

Sin $2x = 1$

$\Rightarrow 2 + 4\sin 2x = 2 + 4 = 6$

Sol 8: (C) $\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & 2-x & x-3 \\ x-3 & x+4 & 3x \end{vmatrix} = px^4 + qx^3 + rx^2 + 5x + t$

At $x = 0$

$\begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix} = t$

$t = 1\,[-9] + 3\,[4 + 6] = 30 - 9 = 21$

Sol 9: (A) $D = \begin{vmatrix} a^2+1 & ab & ac \\ ba & 1+b^2 & bc \\ ca & cb & c^2+1 \end{vmatrix}$

$= \dfrac{1}{abc} \begin{vmatrix} a^3+a & a^2b & a^2c \\ b^2a & b+b^3 & b^2c \\ c^2a & c^2b & c+c^3 \end{vmatrix}$

$R_1 \to aR_1,\ R_2 \to bR_2,\ R_3 \to cR_3$

$$D = \dfrac{abc}{abc} \begin{vmatrix} 1+a^2 & a^2 & a^2 \\ b^2 & 1+b^2 & b^2 \\ c^2 & c^2 & 1+c^2 \end{vmatrix}$$

$C_1 \to C_1 - C_3,\ C_2 \to C_2 - C_3$

$\begin{vmatrix} 1 & 0 & a^2 \\ 0 & 1 & b^2 \\ -1 & -1 & 1+c^2 \end{vmatrix} = 1[1 + c^2 + b^2] + a^2\,[+1]$

$= 1 + a^2 + b^2 + c^2$

Sol 10: (A) $\alpha + \beta + \gamma = \pi$

$\begin{vmatrix} \sin(\alpha+\beta+\gamma) & \sin\beta & \cos\gamma \\ \sin\beta & 0 & \tan\alpha \\ \cos(\alpha+\beta) & \tan\alpha & 0 \end{vmatrix}$

$\sin \pi = 0$

$\alpha + \beta = \pi - \gamma,\ \cos(\pi - \gamma) = -\cos\gamma$

$$\begin{vmatrix} 0 & \sin\beta & \cos\gamma \\ -\sin\beta & 0 & \tan\alpha \\ -\cos\alpha & -\tan\alpha & 0 \end{vmatrix}$$

$$= \sin\beta\left[-\left(\frac{\sin\alpha}{\cos\alpha}\right)\cos\gamma\right] + \cos\gamma\left[\sin\beta\left(\frac{\sin\alpha}{\cos\alpha}\right)\right]$$

$$= \frac{\sin\beta\sin\alpha\cos\gamma}{\cos\gamma}$$

Sol 11: (C) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, $a_{ij} \in \{0, 1\}$

$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} \Rightarrow 1[-1] + 1[-1] = -2$

$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -1[-1] + 1[1] = 2$

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$, $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$= 1[-1] - 1[-1] + 1[-1]$

Cannot be $\rightarrow 3$

D = $a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{21} a_{32} a_{13} - a_{31} a_{22} a_{13} - a_{32} a_{23} a_{11} - a_{21} a_{12} a_{33} < 3$

As, for it to be 3, atleast one terms must be 0 but there sum would not be 3

Sol 12: (D) Order 3 × 3

First column consists of sum of 2 terms

2nd "3"

3rd "4"

Total no. of determinats = 2.3.4 = 24

Sol 13: (D) x + 2y + 3z = 4

x + py + 2z = 3

μx + 4y + z = 3

$D = \begin{vmatrix} 1 & 2 & 3 \\ 1 & p & 2 \\ \mu & 4 & 1 \end{vmatrix}$, $C = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$

D = p – 8 + 2 [2μ 4 – 1] + 3 [4 – pμ)

D = p – 8 + 4μ – 2 + 12 – 3pμ = p + 4μ – 3pμ + 2

For infinite solution D= 0, $D_x = D_y = D_z = 0$

P + 4μ – 3pμ + 2 = 0

$D_x = \begin{vmatrix} 4 & 3 & 3 \\ 3 & p & 2 \\ 3 & 4 & 1 \end{vmatrix}$ = 4[p–8] + 2[6–3] + 3 [12 – 3p]

= 4p – 32 + 6 + 36 – 9p = 0

$\rightarrow$ p = 10/5 = 2

$D_y = \begin{vmatrix} 1 & 4 & 3 \\ 1 & 3 & 2 \\ \mu & 3 & 1 \end{vmatrix}$ = 1[3–6] + 4[2μ–1] +3[3–3μ]=0

– 3 + 8μ – 4 + 9 – 9μ = 0

2 = μ

For equation (i) p + 4(2) – 3p (2) + 2 = 0

P + 8 – 6p + 2 = 0

$\rightarrow$ p = 10/5 = 2

$D_z = \begin{vmatrix} 1 & 2 & 4 \\ 1 & p & 3 \\ \mu & 4 & 3 \end{vmatrix}$

= 3p – 12 + 2(3μ – 3) + 4 (4 – pμ)

= 3p – 12 + 6μ – 6 + 16 – 4pμ

= 3p + 6μ – 4pμ – 2

At p = 2, μ = 2

$\Rightarrow$ 3(2) + 6(2) – 4(2) (2) – 2

$\Rightarrow$ 6 + 12 – 16 – 2 = 0

At p = 2, μ = 2, system has infinite solutions.

Sol 14: (B) ax – by = 2a – b

(c + 1)x + cy = 10 – a + 3b

For infinitely many solution

$D = \begin{vmatrix} a & -b \\ c+1 & c \end{vmatrix}$ = 0 ac + b (c + 1) = 0

ac + bc + b = 0 ...(i)

$D_{x=0} \begin{vmatrix} 2a-b & -b \\ 10-a+3b & c \end{vmatrix}$

= c(2a – b) + b (10 – a + 3b) = 0

2ac – bc + 10b – ba + 3b² = 0 ... (ii)

$D_{y=0} = \begin{vmatrix} a & 2a-b \\ c+1 & 10-a+3b \end{vmatrix}$

$\Rightarrow$ a(10 – a + 3b) + (2a – b) (–1 – c) = 0

$\Rightarrow$ 10a – a² + 3ba – 2a – 2ac + b + bc = 0

At x = 1, y = 3

a – 3b = 2a – b

o = a + 2b $\Rightarrow$ a = – 2b

c + 1 + 3c = 10 – a + 3b(∵ – a = 2b)

4c = 9 + 2b + 3b = 9 + 5b

4c = 9 + 5b

In equation (i) ac + bc + b = 0

$(-2b)\dfrac{(9+5b)}{4} + b\dfrac{(9+5b)}{4} + b = 0$

– 18b – 10b² + 9b + 5b² + 4b = 0

– 5b² – 5b = 0

b² + b = 0

b = – 1 or o

a = 2 or 0

c = 1 or 9/4

(a, b, c) → exactly $\Rightarrow$ (–1, 2, 1) or (0, 0, 9/4)

Sol 15: (C) ax + y + z = 0

x + by + z = 0

x + y + cz = 0 a, b, c ≠ 1

$D = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}, C = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$

So, $D_x = D_y = D_z = 0$

But system has nontrival solution

So, D = 0 and a, b, c ≠ 1

$D = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$

$C_1 \to C_1 - C_3, C_2 \to C_2 - C_3$

$D = \begin{vmatrix} a-1 & 0 & 1 \\ 0 & b-1 & 1 \\ 1-c & 1-c & c \end{vmatrix}$

$= (1-a)(1-b)(1-c)\begin{vmatrix} -1 & 0 & \dfrac{1}{1-a} \\ 0 & -1 & \dfrac{1}{1-b} \\ 1 & 1 & \dfrac{c}{1-c} \end{vmatrix} = 0$

And a, b, c ≠ 1 , So

$\begin{vmatrix} -1 & 0 & \dfrac{1}{1-a} \\ 0 & -1 & \dfrac{1}{1-b} \\ 1 & 1 & \dfrac{c}{1-c} \end{vmatrix} = 0$

$\Rightarrow -1\left[\dfrac{-c}{1-c} - \dfrac{1}{1-b}\right] + \dfrac{1}{1-a}[1] = 0$

$\Rightarrow \dfrac{1}{1-a} + \dfrac{c+1-1}{1-c} + \dfrac{1}{1-b} = 0$

$\Rightarrow \dfrac{1}{1-a} + \dfrac{1}{1-b} + \dfrac{1}{1-c} - \dfrac{(1-c)}{(1-c)} = 0$

$\Rightarrow \dfrac{1}{1-a} + \dfrac{1}{1-b} + \dfrac{1}{1-c} = 1$

Sol 16: (B) $\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos^2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$

$\Rightarrow$ cos (θ + φ) [cos θ cos φ – sin θ sin φ] +

sin (θ + φ) [sin θ – cos φ] + sin φ cos θ]

+ cos2φ (sin²θ + cos²θ)

$\Rightarrow$ cos² (θ + θ) + sin² (θ + φ) + cos 2φ

= 1 + cos 2φ

So determinant is only dependent of φ

Sol 17: (D) x sin θ – y cos θ + (λ + 1)z = 0

x cos θ + y sin θ – λ . z = 0

λ x + (λ + 1)y + z cos θ = 0

$D = \begin{vmatrix} \sin\theta & -\cos\theta & \lambda+1 \\ \cos\theta & \sin\theta & -\lambda \\ \lambda & \lambda+1 & \cos\theta \end{vmatrix}$

D = sin θ [sin θ cos θ + λ² + λ]

+ cos (cos²θ + λ²] + (λ + 1) (λ cos θ + cos θ – λ sin θ)

D = (sin²θ + cos²θ) cos θ + sin θ (λ² + λ

– λ² – λ) + cosθ (λ² + λ² + λ + λ + 1)

D = cos θ (2λ² + 2λ + 2) = cos θ [λ² + 1 + (λ + 1)²]

So for D = 0   (∵ System has infinite solution)

cos θ = 0, θ ∈ (2n + 1) π/2, λ ∈ R, n ∈ I

Sol 18: (A) $a^2x - ay = 1 - a$

$bx + (3 - 2b)y = 3 + a$ $C = \begin{bmatrix} 1 - a \\ 3 + a \end{bmatrix}$

Unique solution $x = 1, y = 1$

$D = \begin{vmatrix} a^2 & -a \\ b & 3 - 2b \end{vmatrix}$, at $(x, y) \Rightarrow (1, 1)$

$a^2 - a = 1 - a$

$a^2 = 1 \Rightarrow a = \pm 1$

And $b + 3 - 2b = 3 + a$

$3 - b = 3 + a$

$a = -b$

So $(a, b) \Rightarrow (1, -1)$ or $(-1, 1)$

At $(-1, 1) \Rightarrow x + y = 1 - (-1) = 2$

$x + y = 2$

And $x + (3 - 2)y = 3 - 1 = 2$

$x + y = 2$

Both equations are same so, $D = 0$ at $(-1, 1)$

So it is not unique solution

$(a, b) \neq (-1, 1) \because (a, b) = (1, -1)$

Sol 19: (A) $D = \begin{vmatrix} {}^{n+2}C_n & {}^{n+3}C_{n+1} & {}^{n+4}C_{n+2} \\ {}^{n+3}C_{n+1} & {}^{n+4}C_{n+2} & {}^{n+5}C_{n+3} \\ {}^{(n+4)}C_{n+2} & {}^{n+5}C_{n+3} & {}^{n+6}C_{n+6} \end{vmatrix}$

$D = \begin{vmatrix} \dfrac{(1+n)(n+2)}{2} & \dfrac{(n+3)(n+2)}{2} & \dfrac{(n+4)(n+3)}{2} \\ \dfrac{(n+3)(n+2)}{2} & \dfrac{(n+4)(n+3)}{2} & \dfrac{(n+5)(n+4)}{3} \\ \dfrac{(n+4)(n+3)}{2} & \dfrac{(n+5)(n+4)}{2} & \dfrac{(n+6)(n+5)}{2} \end{vmatrix}$

At $n = 1$

$D = \begin{vmatrix} 3 & 6 & 10 \\ 6 & 10 & 15 \\ 10 & 15 & 21 \end{vmatrix}$

$= 3[210 - 225) + 6(150 - 126) + 10(90 - 100)$

$= -45 + 144 - 100 = -1$

There is only one option (A)

Which satisfied the ans.

Using, $C_1 \to C_1 - C_2$, $C_3 \to C_3 - C_2$,

$R_3 \to R_3 - R_2$, we get

$\begin{vmatrix} -(n+2) & \dfrac{(n+3)(n+2)}{2} & n+3 \\ -1 & n+3 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -1$

Sol 20: (A) $\lambda x - y + \cos\theta\, z = 0$

$3x + y + 2z = 0$

$\cos x + y + 2z = 0$

$0 \le \theta \le 2\pi$

$C = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

So $D_x = D_y = D_z = 0$

$D = \begin{vmatrix} \lambda & -1 & \cos\theta \\ 3 & 1 & 2 \\ \cos\theta & 1 & 2 \end{vmatrix}$

For non-trivial solution

$D = 0 \therefore D_x = D_y = D_z = 0$

$\lambda[2-2]+1[6 - 2\cos\theta]+\cos\theta[3 - \cos\theta] = 0$

$\Rightarrow 6 - 2\cos\theta + 3\cos\theta - \cos^2\theta = 0$

$\Rightarrow \cos^2\theta - \cos^2\theta - 6 = 0$

$\Rightarrow \cos^2\theta - 3\cos\theta + 2\cos\theta - 6 = 0$

$\Rightarrow \cos\theta(\cos\theta - 3) + 2(\cos\theta - 3) = 0$

$\Rightarrow (\cos\theta - 3)(\cos\theta + 2) = 0$

$\Rightarrow \cos\theta = 3$ or $\cos\theta = -2$

But $-1 \le \cos\theta \le 1$

So $\cos\theta \neq 3, -2$

There is no solution for non-trivial solution

Multiple Correct Choice Type

Sol 21: (A, D) $\begin{vmatrix} \cos(x-y) & \cos(y-z) & \cos(z-x) \\ \cos(x+y) & \cos(y+z) & \cos(z+x) \\ \sin(x+y) & \sin(y+z) & \sin(z+x) \end{vmatrix}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\sin(A + B) = \cos B \sin A + \sin B \cos A$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$= \cos(x - y)[\cos(y + z)\sin(z + x) - \cos(z + x)$

$\sin(y + z)] + \cos(y - z)[\cos(z + x)\sin(x + y) - \sin(x + z)$

cos (x + y)] + cos (z − x) [cos (x + y) sin (y + z) − cos (y + z) cos (x + y)]

= cos (x − y) [sin (z + x − y − z)]+ cos (y − z) [sin (x + y − z − x)]+ cos (z − x) [sin (y + z − x − y)]

= cos (x − y) sin (x − y) + cos (y − z) sin (y − z) + cos (z − x) sin (z − x)

$= \dfrac{1}{2}$ [sin 2 (x −y) + sin (2(y − z) + sin 2(z − x)]

$= \dfrac{1}{2}$ [2 cos (x + z − zy) sin (x + z) − 2 sin (x − z) cos (x − z)]

$= \dfrac{1}{2}$ sin (x − z) [cos (x + z − 2y) − cos (x − z)]

$= \sin (x - z) \left[2\sin\dfrac{(x-y)}{2}\sin\dfrac{(y-z)}{2} \right]$

= 2 sin (x − y) sin (y − z) sin (z − x)

Sol 22: (A, B, C, D) $\dfrac{-\pi}{4} < \theta < \dfrac{\pi}{2}, 0 \le A \le \pi/2$

$\begin{vmatrix} 1+\sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1+\cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$

$R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2 A & \cos^2 A & 1+\sin^2 \theta \end{vmatrix} = 0$

$\Rightarrow$ 1 + 2 sin4θ + cos²A − 1 [− sin² A]

$\Rightarrow$ 1 + 2 sin 4θ + sin²θ + cos² θ

$\Rightarrow$ 2 + 2 sin 4θ = 0 (only depend on θ)

$\Rightarrow$ sin 4θ = − 1 $\Rightarrow$ 4θ ∈ − π/2 + 2nπ, n ∈ I

θ ∈ − π/8 + nπ/2, n ∈ I

(A) θ → − π/8

(B) θ → 3π/8 sin 4θ = − 1

(C) θ = − π/8 sin 4θ = − 1

(D) θ = 3π/8

Sol 23: (A, D) $\begin{vmatrix} 1 & a & a^2 \\ 1 & x & x^2 \\ b^2 & ab & a^2 \end{vmatrix} = 0$

$\Rightarrow$ xa² − x²ab + a [x²b² − a²] + a² [ab − xb²]=0

$\Rightarrow$ x [a² − a²b²] + x² [ab² − ab] − a³ + a³b = 0

$\Rightarrow$ x²ab (b − 1) + a²x (1 − b²) + a³ (b − 1) = 0

$\Rightarrow$ x²ab − a²x (1 + b) + a³ (+1) = 0

$\Rightarrow$ x²ab − x (a² + a²b) + a³ = 0

$\Rightarrow$ x²ab − a²x − a² (bx − a) = 0

$\Rightarrow$ (bx − a) (ax − a²) = 0

$\Rightarrow$ bx − a = 0 or ax − a² = 0

$\Rightarrow x = \dfrac{a}{b}$ or $x = \dfrac{a^2}{a} = a$

Sol 24: (B, D) $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$

$R_3 \to R_3 - \alpha R_1 - R_2$

$\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ 0 & 0 & -(a\alpha^2 + b\alpha + b\alpha + c) \end{vmatrix}$

= (aα² + 2bα + c) (b² − ac) = 0

So (b² − ac) = 0

b² = ac → b is GM of a, c → ab, c are in GP

or (aα² + 2b α + c) = 0

$\Rightarrow$ x = α → (x − α)

$\Rightarrow$ ax² + 2bx + c, (x − α) is a factor of this)

Sol 25: (B, D) x − y + 3z = 2

2x − y + z = 4

x − 2y + αz = 3

$D = \begin{vmatrix} 1 & -1 & 3 \\ 2 & -1 & 1 \\ 1 & -2 & \alpha \end{vmatrix}$

= 1 [− α + 2] − 1 [1 − 2α] + 3 [− 4 + 1]

= − α + 2 − 1 + 2α − 9 = α − 8

D ≠ 0 → α ≠ 8

If α = 8, D = 0

$D_x = \begin{vmatrix} 2 & -1 & 3 \\ 4 & -1 & 1 \\ 3 & -2 & \alpha \end{vmatrix}$

= 2 [− α + 2] − 1 [3 − 4α] + 3 [− 8 + 3]

= − 2α + 4 − 3 + 4α − 15 = 2α − 14 = 2(α − 7)

= At α = 8 (0 = 0), $D_x \ne 0$

So, at α = 8, system has no solution.

Sol 26: (A,B,C,D) $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$

$= \dfrac{1}{abc} \begin{vmatrix} a & abc & abc(b+c) \\ b & abc & abc(c+a) \\ c & abc & abc(a+b) \end{vmatrix} = \dfrac{(abc)^2}{abc} \begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

$C_3 \to C_3 + C_1$

$= (abc) \begin{vmatrix} a & 1 & a+b+c \\ b & 1 & a+b+c \\ c & 1 & a+b+c \end{vmatrix} = (abc)\,(a+b+c) \begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix}$

$C_2 \to c_2 - c_3$

$= (a\,b\,c)\,(a+b+c) \begin{vmatrix} a & 0 & 1 \\ b & 0 & 1 \\ c & 0 & 1 \end{vmatrix} = 0$

(B) $\begin{vmatrix} 1 & ab & \dfrac{1}{a}+\dfrac{1}{b} \\ 1 & bc & \dfrac{1}{b}+\dfrac{1}{c} \\ 1 & ca & \dfrac{1}{c}+\dfrac{1}{a} \end{vmatrix} = \dfrac{1}{abc} \begin{vmatrix} c & abc & c\left(\dfrac{1}{a}+\dfrac{1}{b}\right) \\ a & abc & a\left(\dfrac{1}{b}+\dfrac{1}{c}\right) \\ b & abc & b\left(\dfrac{1}{c}+\dfrac{1}{a}\right) \end{vmatrix}$

$\dfrac{abc}{abc} \begin{vmatrix} c & \dfrac{c}{c} & \dfrac{c}{a}+\dfrac{c}{b} \\ a & \dfrac{a}{a} & \dfrac{a}{b}+\dfrac{a}{c} \\ b & \dfrac{b}{b} & \dfrac{b}{a}+\dfrac{b}{c} \end{vmatrix}$ $C_3 \to c_3 + c_2$

$\begin{vmatrix} c & 1 & c\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right) \\ a & 1 & a\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right) \\ b & 1 & b\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right) \end{vmatrix} = \left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right) \begin{vmatrix} c & 1 & C \\ a & 1 & a \\ b & 1 & b \end{vmatrix}$

$C_1 \to c_1 - c_3$

$= \left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right) \begin{vmatrix} 0 & 1 & c \\ 0 & 1 & a \\ 0 & 1 & b \end{vmatrix} = 0$

(C) $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix} = \begin{vmatrix} 0 & (a-b) & (a-c) \\ -(a-b) & 0 & (b-c) \\ -(a-c) & -(b-c) & 0 \end{vmatrix}$

This is stew symmetric matrix so value of determinate is zero.

(D) $\begin{vmatrix} \log_x xyz & \log_x y & \log_x z \\ \log_y xyz & 1 & \log_x z \\ \log_x xyz & \log_z y & 1 \end{vmatrix}$

$\begin{vmatrix} \dfrac{\log xyz}{\log x} & \dfrac{\log y}{\log x} & \dfrac{\log x}{\log x} \\ \dfrac{\log xyz}{\log y} & \dfrac{\log y}{\log y} & \dfrac{\log z}{\log y} \\ \dfrac{\log xyz}{\log z} & \dfrac{\log y}{\log z} & \dfrac{\log z}{\log z} \end{vmatrix}$

$= \dfrac{(\log xyz)(\log y)(\log z)}{(\log x)(\log y)(\log z)} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$(\because\ C_1 = c_2 = c_3)$

Sol 27: (A, B, C, D) $a^2x - by = a^2 - b^2$ posses an infinite no. of solution

$bx - b^2y = 2 + 4b$

So $D = 0 \Rightarrow \begin{vmatrix} a^2 & -b \\ b & -b^2 \end{vmatrix} = -b^2a^2 + b^2 = 0$ Þ

$\Rightarrow b^2(1 - a^2) = 0$

$\to b = 0$ or $a = \pm 1$ ... (i)

$D_x = 0 \Rightarrow \begin{vmatrix} a^2 - b & -b \\ 2+4b & -b^2 \end{vmatrix}$

$= -b^2\,(a^2 - b) + b\,(2 + 4b) = 0$

$-a^2b^2 + b^3 + 2b + 4b^2 = 0$

$b(b^2 + 4b + 2 - a^2b) = 0$

$\Rightarrow b = 0$ or $b^2 + 4b + 2 = a^2b$ ... (ii)

$D_x = 0 \Rightarrow \begin{vmatrix} a^2 & a^2 - b \\ b & 2+4b \end{vmatrix} = a^2(2 + 4b) + b(b - a^2) = 0$

$2a^2 + 4a^2b + b^2 - ba^2 = 0$ ...(iii)

All option are satisfied equation (i, ii, iii)

Sol 28: (A, C) p, q, r, s are in AP

$P = p\ q = p + d\ r = p + 2d, s = p + 3d$

D is common difference of the A.P.

$f(x) = \begin{vmatrix} p+\sin x & q+\sin x & p-r+\sin x \\ q+\sin x & r+\sin x & -1+\sin x \\ r+\sin x & s+\sin x & s-q+\sin x \end{vmatrix}$

$C_2 \to c_2 - c_1,\ C_3 \to c_3 - c_1$

$$f(x) = \begin{vmatrix} p+\sin x & q-p & -p+q-r \\ q+\sin x & r-q & -1-q \\ r+\sin x & s-r & s-q-r \end{vmatrix}$$

$$= \begin{vmatrix} p+\sin x & d & -p-2d \\ q+d+\sin x & d & -1-p-d \\ p+2d+\sin x & d & -p \end{vmatrix}$$

$R_3 \to R_3 - R_1, R_2 \to R_2 - R_1$

$$f(x) = \begin{vmatrix} p+\sin x & d & -p-2d \\ d & 0 & -1-p-d+p+2d \\ 2d & 0 & -p+p+2d \end{vmatrix}$$

$$= \begin{vmatrix} p+\sin x & d & -p-2d \\ d & 0 & d-1 \\ 2d & 0 & 2d \end{vmatrix}$$

$= d[2d(d-1) - 2d^2]$

$= d(2d^2 - 2d - 2d^2) = -2d^2$

$$\int_0^2 f(x)dx = \int_0^2 -2d^2 dx = \left[\frac{-2d^2}{d} x \right]_0^2 = -2d^2 (2) = -4$$

$\Rightarrow d^2 = 1 \Rightarrow d = \pm 1$

Sol 29: (A, C) $x + y = 3$ ... (i)

$(1 + k) x + (2 + k)y = 8$ ... (ii)

$x - (1 + k)y = -(2 + k)$ ... (iii)

$$D_{12} = \begin{vmatrix} 1 & 1 \\ 1+K & 2+K \end{vmatrix} = 2 + k - (1 + k) = 1$$

$D_{12} \neq 0$, So

$$D_{13} = \begin{vmatrix} 1 & 1 \\ 1 & -(1+k) \end{vmatrix} = -1 - k - 1 = -2 - k = -(2+k)$$

$$\text{So } D_x = \begin{vmatrix} 3 & 1 \\ -(2+k) & -(1+k) \end{vmatrix} = -3(1+k) + 2 + k$$

$= -3 - 3k + 2 + k = -2k - 1$

$$\therefore D_{23} = 0 \begin{vmatrix} 1+k & (2+k) \\ 1 & -(1+k) \end{vmatrix}$$

$= -1(1 + k)^2 - (2 + k) = -(k^2 + 3k + 3)$

From (ii) & (iii)

$$D_x = \begin{vmatrix} 3 & 1 \\ 8 & 2+k \end{vmatrix} = 3(2 + k) - 8 = 6 + 3k - 8 = 3k - 2$$

$$x = \frac{D_x}{D} = \frac{3k-2}{1}$$

$$D_y = \begin{vmatrix} 1 & 3 \\ 1+k & 8 \end{vmatrix} = 8 - 3(1 + k) = 5 - 3k$$

$y = 5 - 3k$

Which will satisfied other equation of (x, y)

$\Rightarrow (3k - 2) - (1 + k)(5 - 3k) + 2 + k = 0$

$\Rightarrow 3k - 2 - (5 - 3k^2 + 2k) + 2 + k = 0$

$\Rightarrow 3k - 2 - 5 + 3k^2 - 2k + 2 + k = 0$

$\Rightarrow 3k^2 + 2k - 5 = 0$

$\Rightarrow 3k^2 + 5k - 3k - 5 = 0$

$k(3k + 5) - 1(3k + 5) = 0$

$\Rightarrow (k - 1)(3k + 5) = 0$

$\Rightarrow k - 1 = 0$ or $3k + 5 = 0$

$k = 1$ or $k = -5/3$

Sol 30: (A, B, C)

$$D = \begin{vmatrix} \dfrac{1}{z} & \dfrac{1}{z} & \dfrac{(x+y)}{z^2} \\ \dfrac{-(y+z)}{x^2} & \dfrac{1}{x} & \dfrac{1}{x} \\ \dfrac{-y(y+2)}{x^2 z} & \dfrac{-(x+2y+z)}{xz} & \dfrac{-y(x+y)}{xz^2} \end{vmatrix}$$

$$D = \frac{1}{z^2} \cdot \frac{1}{x^2} \cdot \frac{1}{x^2 z^2}$$

$$\begin{vmatrix} z & z & -(x+y) \\ -(y+z) & x & x \\ -zy(y+z) & -(x+2y+z)(xz) & -yx(x+y) \end{vmatrix}$$

$C_1 \to C_2 - C_3$ and

$-zy(y + z) + yx(x + y) = -zy^2 - z^2y + yx^2 + y^2x$

$= y[y(x-z) + x^2 - z^2] = y(x-z)[y + x + z]$

$$D = \frac{1}{x^4 z^4}$$

$$\begin{vmatrix} (x+y+z) & (x+y+z) & -(x+y) \\ -(x+y+z) & 0 & x \\ y(x-z)(x+y+z) & x[(y-z)(x+y+z)-2yz] & -xy(x+y) \end{vmatrix}$$

$$D = \frac{(x+y+z)}{x^4 z^4}$$

$$\begin{vmatrix} 1 & x+y+z & -(x+y) \\ -1 & 0 & x \\ y(x-z) & x[(y-z)(x+y+z)-2yz] & -xy(x+y) \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_2$

$$D = \frac{(x+y+z)}{x^4 z^4}$$

$$\begin{vmatrix} 0 & x+y+z & -y \\ -1 & 0 & x \\ y(x-z) & x[(y-z)(x+y+z)(-xyz) & -xy(x+y) \end{vmatrix}$$

$$\frac{x+y+2}{x^4+4}$$

$$\left[(x+y+z)(xy+y) - xy(x-z) + yx(-xyz)(x+y+z)(y-z) \right]$$

Previous Years Questions

Sol 1: (B) Let $\Delta = \begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$

Applying $C_1 \rightarrow C_1 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2 & a & a^2 \\ \cos(p-d)x + \cos(p+d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x + \sin(p+d)x & \sin px & \sin(p+d)x \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2 & a & a^2 \\ 2\cos px \cos dx & \cos px & \cos(p+d)x \\ 2\sin px \cos dx & \sin px & \sin(p+d)x \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - 2\cos dx\, C_2$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2 - 2a\cos dx & a & a^2 \\ 0 & \cos px & \cos(p+d)x \\ 0 & \sin px & \sin(p+d)x \end{vmatrix}$$

$\Rightarrow \Delta = (1+a^2 - 2a\cos dx)$

$[\sin(p+d)x \cos px - \sin px \cos(p+d)x]$

$\Rightarrow \Delta = (1+a^2 - 2a\cos dx)\sin dx$

Which is independent of p.

Sol 2: Since, $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is linear equation in three

variables and that could have only unique, no solution or infinitely many solution.

$\therefore$ It is not possible to have two solutions.

Hence, number of matrices A is zero.

Sol 3: Given $\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$

$$\Rightarrow \frac{1}{a} \begin{vmatrix} a^2x-aby-ac & bx+ay & cx+a \\ abx+a^2y & -ax+by-c & cy+b \\ acx+a^2 & cy+b & -ax-by+c \end{vmatrix} = 0$$

Applying $C_1 \rightarrow C_1 + bC_2 + cC_3$

$$\Rightarrow \frac{1}{a} \begin{vmatrix} (a^2+b^2+c^2)x & ay+bx & cx+a \\ (a^2+b^2+c^2)y & by-c-ax & b+cy \\ a^2+b^2+c^2 & b+cy & c-ax-by \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{a} \begin{vmatrix} x & ay+bx & cx+a \\ y & by-c-ax & b+cy \\ 1 & b+cy & c-ax-by \end{vmatrix} = 0$$

$(\because a^2+b^2+c^2 = 1)$

Applying $C_2 \rightarrow C_2 - bC_1$ and $C_3 \rightarrow C_3 - cC_1$

$$\Rightarrow \frac{1}{a} \begin{vmatrix} x & ay & a \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{ax} \begin{vmatrix} x^2 & axy & ax \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 + yR_2 + R_3$

$$\Rightarrow \frac{1}{ax} \begin{vmatrix} x^2+y^2+1 & 0 & 0 \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{ax}[(x^2+y^2+1)\{(-c-ax)(-ax-by)-b(cy)\}] = 0$$

$$\Rightarrow \frac{1}{ax}[(x^2+y^2+1)(acx+bcy+a^2x^2+abxy-bcy)] = 0$$

$$\Rightarrow \frac{1}{ax}[(x^2+y^2+1)(acx+a^2x^2+abxy)] = 0$$

$$\Rightarrow \frac{1}{ax}[ax(x^2+y^2+1)(c+ax+by)] = 0$$

$\Rightarrow (x^2 + y^2 + 1)(ax + by + c) = 0 \Rightarrow ax + by + c = 0$

Which represents a straight line.

Sol 4: Since, the given system of equations posses non-trivial solution, if

$$\begin{vmatrix} 0 & 1 & -2 \\ 0 & -3 & 1 \\ k & -5 & 4 \end{vmatrix} = 0 \ k = 0$$

On solving the equations $x = y = z = \lambda$ (say)

$\therefore$ For k = 0, the system has infinite solutions for $\lambda \in R$.

Sol 5: Given system of equations are

$3x + my = m$ and $2x - 5y = 20$

Here, $\Delta = \begin{vmatrix} 3 & m \\ 2 & -5 \end{vmatrix} = -15 - 2m$

and $\Delta_x = \begin{vmatrix} m & m \\ 20 & -5 \end{vmatrix} = -25m$; $\Delta_y = \begin{vmatrix} 3 & m \\ 2 & 20 \end{vmatrix} = 60 - 2m$

If $\Delta = 0$, then system inconsistent i.e. it has no solution.

If $\Delta \neq 0$ i.e. $m \neq \dfrac{15}{2}$, then system has a unique solution

for any fixed value of m.

We have, $x = \dfrac{\Delta_x}{\Delta} = \dfrac{-25m}{-15-2m} = \dfrac{25m}{15+2m}$

and $y = \dfrac{\Delta_y}{\Delta} = \dfrac{60-2m}{-15-2m} = \dfrac{2m-60}{15+2m}$

For $x > 0$, $\dfrac{25m}{15+2m} > 0$

$\Rightarrow m > 0$ or $m < -\dfrac{15}{2}$ ... (i)

and $y > 0$, $\dfrac{2m-60}{2m+15} > 0$

$\Rightarrow m > 30$ or $m < -\dfrac{15}{2}$ ... (ii)

From equation (i) and (ii) we get

$m < -\dfrac{15}{2}$ or m > 30

Sol 6: Let $\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \dfrac{2\pi}{3}\right) & \cos\left(\theta + \dfrac{2\pi}{3}\right) & \sin\left(2\theta + \dfrac{4\pi}{3}\right) \\ \sin\left(\theta - \dfrac{2\pi}{3}\right) & \cos\left(\theta - \dfrac{2\pi}{3}\right) & \sin\left(2\theta - \dfrac{4\pi}{3}\right) \end{vmatrix}$

Applying $R_2 \rightarrow R_2 + R_3$

$$= \begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \dfrac{2\pi}{3}\right) + \sin\left(\theta - \dfrac{2\pi}{3}\right) & \cos\left(\theta + \dfrac{2\pi}{3}\right) + \cos\left(\theta - \dfrac{2\pi}{3}\right) & \sin\left(2\theta + \dfrac{4\pi}{3}\right) + \sin\left(2\theta - \dfrac{4\pi}{3}\right) \\ \sin\left(\theta - \dfrac{2\pi}{3}\right) & \cos\left(\theta - \dfrac{2\pi}{3}\right) & \sin\left(2\theta - \dfrac{4\pi}{3}\right) \end{vmatrix}$$

Now, $\sin\left(\theta + \dfrac{2\pi}{3}\right) + \sin\left(\theta - \dfrac{2\pi}{3}\right)$

$= 2\sin\left(\dfrac{\theta + \dfrac{2\pi}{3} + \theta - \dfrac{2\pi}{3}}{2}\right)\cos\left(\dfrac{\theta + \dfrac{2\pi}{3} - \theta + \dfrac{2\pi}{3}}{2}\right)$

$= 2\sin\theta\cos\dfrac{2\pi}{3} = 2\sin\theta\cos\left(\pi - \dfrac{\pi}{3}\right)$

$= -2\sin\theta\cos\dfrac{\pi}{3} = -\sin\theta$

and $\cos\left(\theta + \dfrac{2\pi}{3}\right) + \cos\left(\theta - \dfrac{2\pi}{3}\right)$

$= 2\cos\left(\dfrac{\theta + \dfrac{2\pi}{3} + \theta - \dfrac{2\pi}{3}}{2}\right)\cos\left(\dfrac{\theta + \dfrac{2\pi}{3} - \theta + \dfrac{2\pi}{3}}{2}\right)$

$= 2\cos\theta\cos\left(\dfrac{2\pi}{3}\right) = 2\cos\theta\left(-\dfrac{1}{2}\right) = -\cos\theta$

and $\sin\left(2\theta + \dfrac{4\pi}{3}\right) + \sin\left(2\theta - \dfrac{4\pi}{3}\right)$

$= 2\sin\left(\dfrac{2\theta + \dfrac{4\pi}{3} + 2\theta - \dfrac{4\pi}{3}}{2}\right)\cos\left(\dfrac{2\theta + \dfrac{4\pi}{3} - 2\theta + \dfrac{4\pi}{3}}{2}\right)$

$= 2\sin 2\theta\cos\dfrac{4\pi}{3} = 2\sin 2\theta\cos\left(\pi + \dfrac{\pi}{3}\right)$

$= -2\sin 2\theta\cos\dfrac{\pi}{3} = -\sin 2\theta$

$\therefore \Delta = \begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ -\sin\theta & -\cos\theta & -\sin 2\theta \\ \sin\left(\theta - \dfrac{2\pi}{3}\right) & \cos\left(\theta - \dfrac{2\pi}{3}\right) & \sin\left(2\theta - \dfrac{4\pi}{3}\right) \end{vmatrix}$

= 0 (Since, R_1 and R_2 are proportional).

Sol 7: (B) Method I

Total no. of ways $= 3^5 - {}^3C_1 3 - 1^5 + {}^3C_2 3 - 1^5$

$= 243 - 3 \times 32 + 3 = 246 - 96 = 150$

Alternative Method

System I

Boxes	I	II	III	
Balls	I	2	2	

For this system no. of ways

$= \left(\dfrac{5!}{2!2!1!} \times \dfrac{1}{2!} \right) \times \left(\dfrac{5 \times 4 \times 3 \times 2}{2 \times 2 \times 2} \right) \times 6 = 90$

System II

Boxes	I	II	III	
Balls	I	3	1	

For this system no. of ways

$= \left(\dfrac{5!}{3!1!1!} \times \dfrac{1}{2!} \right) \times 3! = 10 \times 6 = 60$

Total no. of ways $= 90 + 60 = 150$

Sol 8: (B, C)

$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$

$\begin{vmatrix} 1+\alpha^2+2\alpha & 1+4\alpha^2+4\alpha & 1+9\alpha^2+6\alpha \\ 4+\alpha^2+4\alpha & 4+4\alpha^2+8\alpha & 4+9\alpha^2+12 \\ 9+\alpha^2+6\alpha & 9+4\alpha^2+12\alpha & 9+9\alpha^2+18\alpha \end{vmatrix} R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_2 = -648\alpha$

$\begin{vmatrix} 1+\alpha^2+2\alpha & 1+4\alpha^2+4\alpha & 1+9\alpha^2+6\alpha \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix} C_2 \to C_2 - C_3$

$C_2 \to C_2 - C_2 = -648\alpha$

$\begin{vmatrix} \alpha^2-4 & 4\alpha^2-4 & 9\alpha^2-4 \\ -2 & -2 & -2 \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix} = -648\alpha$

$-2 \begin{vmatrix} \alpha^2-4 & 4\alpha^2-4 & 9\alpha^2-4 \\ 1 & 1 & 1 \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix} = 648\alpha$

$-2 \begin{vmatrix} \alpha^2-4 & 3\alpha^2 & 8\alpha^2 \\ 1 & 0 & 0 \\ 5+2\alpha & 2\alpha & 4\alpha \end{vmatrix} = -648\alpha$

$-2 \left(12\alpha^3 - 16\alpha^3 \right) = -648\alpha$

$\Rightarrow 2\left(-4\alpha^3 \right) = -648\alpha$

$\Rightarrow \alpha\left(\alpha^2 - 81 \right) = 0$

$\Rightarrow \alpha = 0, 9, -9$

Sol 9: (2) $\begin{vmatrix} 1 & 1 & 1+x^3 \\ 2 & 4 & 1+8x^3 \\ 3 & 9 & 1+27x^3 \end{vmatrix} = 10$

$x^3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^6 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 10$

$x^3 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 3 & 6 & -2 \end{vmatrix} + x^6 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 6 \\ 3 & 6 & 24 \end{vmatrix} = 10$

$6x^3 + x^3 - 5 = 0 \Rightarrow 6x^6 + 6x^3 - 5x^3 - 5 = 0$

$\left(6x^3 - 5 \right)\left(x^3 + 1 \right) = 0$

$x^3 = \dfrac{5}{6}$ or $x^3 = -1$ Two real distinct values of x.