

Mathematical Reasoning and Statistics

PROBLEM-SOLVING TACTICS

- Formulates and solves a variety of meaningful problems.
- Extracts pertinent information from situations and figures out what additional information is needed.
- Formulates conjectures and argues why they must be or seem true.
- Makes sensible, reasonable estimates.
- Makes justified, logical statements.
- Employs forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures, and using counterexamples and indirect proof.
- Differentiates clearly between giving examples that support a conjecture and giving a proof of the conjecture.

FORMULAE SHEET

Arithmetic mean: (a) For ungrouped data (individual series) $\bar{x} = \frac{X_1 + X_2 + \dots + X_n}{n(\text{no. of terms})} = \frac{\sum_{i=1}^n f_i X_i}{\sum_{i=1}^n f_i}$

(b) For grouped data (continuous series)

(i) Direct method $\bar{X} = \frac{\sum_{i=1}^n f_i X_i}{\sum_{i=1}^n f_i}$, where $x_i, i = 1, \dots, n$, (n = observations, f_i = corresponding frequencies)

(ii) Shortcut method : $\bar{X} = A + \frac{\sum_{i=1}^n f_i d_i}{\sum_{i=1}^n f_i}$, Where A = assumed mean, $d_i = x_i - A$ = deviation for each term

Median: (a) Individual series (ungrouped data): If data is raw, arrange in ascending or descending order. n denotes the number of observations. If n is odd, median = value of $\left(\frac{n+1}{2}\right)^{\text{th}}$ observations

If n is even, median = $\frac{1}{2}$ [value of $\left(\frac{n}{2}\right)^{\text{th}}$ + value of $\left(\frac{n}{2} + 1\right)^{\text{th}}$] observation

(b) Discrete series: First find cumulative frequencies of the variables arranged in ascending or descending order and Median = $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation, where n is the cumulative frequency.

(c) Continuous distribution (grouped data)

(i) For a series in the ascending order, median = $\ell + \frac{((N/2) - C)}{f} \times i$

where ℓ = Lower limit of the median class.

F = Frequency of the median class.

N = Sum of all frequencies.

i = The width of the median class.

C = Cumulative frequency of the class preceding to median class.

(ii) For a series in descending order, median = $u - \frac{((N/2) - C)}{f} \times i$

Where u = upper limit of median class.

Mode: (a) For individual series: In the case of individual series, the value which is repeated maximum number of times is the mode of the series.

(b) For discrete frequency distribution series: In the case of discrete frequency distribution, mode is the value of the variate corresponding to the maximum frequency.

(c) For continuous frequency distribution: First find the modal class i.e., the class which has maximum frequency.

For continuous series, mode = $\ell_1 + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times i$

Where, ℓ_1 = Lower limit of the modal class; f_1 = Frequency of the modal class

f_0 = Frequency of the class preceding modal class; f_2 = Frequency of the class succeeding modal class ; i = Size of the modal class

Relation between mean, mode and median:

(i) In symmetrical distribution: Mean = mode = median

(ii) In moderately symmetrical distribution: Mode = 3median - 2mean

Measure of Dispersion: The degree to which data points diverge from the average or mean value is called variation or dispersion. Popular methods of measure of dispersion:

(a) Mean Deviation: The arithmetic average of deviations from the mean, median, or mode is known as mean deviation.

(i) Individual series (ungrouped data). Mean deviation = $\frac{\sum |x - S|}{n}$

Where n = number of terms, S = deviation of variate from mean, mode, and median.

(ii) Continuous series (grouped data). Mean deviation = $\frac{\sum f|x-s|}{\sum f} = \frac{\sum f|x-s|}{N}$

Note: Mean deviation is the least when measured from the median.

(b) Standard Deviation: S.D. (σ) is the square root of the arithmetic mean of the squares of the deviations of the terms from their arithmetic mean (A.M.).

(i) For individual series (ungrouped data)

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}}$$

where \bar{x} = arithmetic mean of the series. N = Total frequency

(ii) For continuous series (grouped data)

- Direct method $\sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$

where \bar{x} = Arithmetic mean of series

x_i = Mid-value of the class

f_i = Frequency of the corresponding x_i . $N = \sum f =$ Total frequency

- Shortcut method $\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$ or $\sigma = \sqrt{\frac{\sum d^2}{N} - \left(\frac{\sum d}{N}\right)^2}$

where $d = x - A =$ Deviation from assumed mean A

$f =$ Frequency of item (term), $N = \sum f =$ Total frequency.

Variance – Square of standard deviation i.e., variance = $(S.D.)^2 = (\sigma)^2$

Coefficient of variance = coefficient of S.D. X 100 = $\frac{\sigma}{x} \times 100$