Solved Examples

JEE Main/Boards

Example 1: Upon weighing 30 fishes a scientist finds that their mean weight is 30 gm with a standard deviation of 2 gm. Later, he realized that the measuring scale was misaligned and always underreported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are, respectively.

(A) 32,2 (B) 32,4 (C) 28,2 (D) 28,4

Sol: (A) Use Mean (X+b) = Mean (X)+b and Var (X+b) = Var X.

So we get correct mean as 30 + 2 = 32 gm and S.D., is 2 gm.

Example 2: If the variance of four numbers a, b, c, and d is 9, the variance of 5a, 5b, 5c, and 5d is

(A) 45 (B) 5/9 (C) 9/5 (D) 225

Sol: (D) $\sigma_x^2 = h^2\sigma_u^2$ if $u = \dfrac{x-A}{h}$;

here $h = 1/5$ so $\sigma_u^2 = 9 \times 25 = 225$.

Example 3: (C) What will be the standard deviation of the numbers 4, 6, 7, and 8 be if the standard deviation of 2, 4, 5, and 6 is a constant α?

(A) $\alpha + 2$ (B) 2α (C) α (D) $\sqrt{2\alpha}$

Example 4: What number should come next in the series 2, 1, (1/2), (1/4)?

(A) (1/3) (B) (1/8) (C) (2/8) (D) (1/16)

Sol: (B) In this series each number has been divided by 2, successively, to get the next result.

4/2 = 2 ; 2/2 = 1; ½ = 1/2 (1/2)/2 = 1/4

(1/4)/2 = 1/8 and so on.

Example 5: 42 40 38 35 33 31 28

(A) 25 22 (B) 26 23 (C) 26 24 (D) 25 23

Sol: (C) This is an alternating subtraction series. First 2 is subtracted twice, then 3 is subtracted once, then again 2 is subtracted twice, and so on.

Example 6: Vincent delivers 37 newspapers to the customers in his neighborhood. It takes him 50 minutes to deliver all the papers. Vincent's friend Thomas, who lives on the same street, sometimes delivers the papers for him when Vincent is indisposed.

(A) Vincent and Thomas live in the same neighborhood.

(B) It takes Thomas more than 50 minutes to deliver the papers.

(C) It is dark outside when Vincent begins his deliveries.

(D) Thomas would like to have his own paper route.

Sol: (A) Since Vincent and Thomas live on the same street, we can infer that they live in the same neighborhood. There is nothing else in the question that supports any of the other choices.

Example 7: A salesman made a monthly sale of Rs.12,000 for the first 11 months of the year, but due to his illness during the last month the average sales for the whole year came down to Rs.11,375. The value of the sale during the last month was

(A) RS. 4,500 (B) RS. 6,000

(C) RS. 10,000 (D) RS. 8,000

Sol: (A) Apply the formula for mean.

Let Rs x be the sale in the last month. Using the formula of combined mean, we have

$$\frac{12{,}000\times 11+x}{12}=11{,}375$$

$$\Rightarrow x=11{,}375\times 12-12{,}000\times 11$$

$$=1{,}36{,}500-1{,}32{,}000=4500.$$

Example 8: In any discrete series (when all the values are not same) the relationship between M.D. about mean and S.D. is

(A) M.D. = S.D (B) M.D. > S.D.

(C) M.D. < S.D. (D) M.D. ≤ S.D.

Sol: (C) Use formula for the M.D. and S.D. to establish the inequality.

$$\text{M.D.}=\frac{1}{n}\sum_{i=1}^{n}\left|x_i-\bar{x}\right| \text{ and } \text{S.D.}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$\text{Let } \alpha_i=\left|x_i-\bar{x}\right| \text{ so} \left(\text{M.D.}\right)^2=\left(\frac{1}{n}\sum_{i=1}^{n}a_i\right)^2$$

$$\text{and } \left(\text{S.D.}\right)^2=\frac{1}{n}\sum_{i=1}^{n}a_i^{\;2}$$

By Cauchy Schwarz inequality

$$\left(\sum_{i=1}^{n}a_i\right)^2<n\sum_{i=1}^{n}a_i^2\Rightarrow\left(\frac{1}{n}\sum a_i\right)^2<\frac{1}{n}\sum a_i^2\;\Rightarrow \text{M.D.}<\text{S.D}.$$

Example 9: What will be the effect on the standard deviation (20) and median (4) of a distribution if each item is increased by 2?

(A) Median will increase and S.D. will also increase

(B) Median will go up by 2 but S.D. will remain same

(C) Median will increase but S.D. will decrease

(D) None of these

Sol: (B) Median will go up by 2 and S.D. will remain the same.

Example 10: Read the following statements carefully a mark the correct option out of the options given below:

(A) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I

(B) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Statement-I: The variance of first n even natural numbers is $\frac{n^2-1}{4}$

Statement-II: The sum of first n natural numbers is $\frac{n\left(n+1\right)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$

Sol: (D) It is the fundamental concept.

Example 11: Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance

Statement-I: Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$

Statement-II: Arithmetic mean $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$

Sol: (C) A.M. of $2x_1, 2x_2, ..., 2x_n$ is $\frac{2x_1 + 2x_2 + ... + 2x_n}{n}$

$$= 2\left(\frac{x_1 + x_2 + ... + x_n}{n}\right) = 2\bar{x}$$

So statement-II is false. Variance $(2x_i) = 2^2$ variance $(x_i) = 4\sigma^2$. So statement-I is true.

Example 12: Determine the truth value of each of the following statements:

(i) Kolkata is in India and 2 + 2 = 4

(ii) Kolkata is in England and 12 + 4 = 16

(iii) Kolkata is in India and 2 + 21 = 5

(iv) Kolkata is in England and 2 + 2 = 5

Sol: We know that "p and q" is true only when both sub-statements are true. Therefore, true values are:

(i) T, because both sub-statements are true.

(ii) F, because first sub-statement "Kolkata is in England" is false.

(iii) F, because second sub-statement "2 + 21 = 5" is false.

(iv) F, because both sub-statements are false.

Example 13: Find the mean deviation from the mean for the following data:

Classes	Frequencies
0-10	6
10-20	8
20-30	14
30-40	16
40-50	4
50-60	2

Sol: Use direct method or the short-cut formula.

First Method: We construct the following table:

Classes	x_i	f_i	$f_i x_i$	$\|x_i - \bar{x}\|$	$f_i\|x_i - \bar{x}\|$
0-10	5	6	30	$\|5-27\| = 22$	132
10-20	15	8	120	$\|15-27\| = 12$	96
20-30	25	14	350	$\|25-27\| = 2$	28
30-40	35	16	560	$\|35-27\| = 8$	128
40-50	45	4	180	$\|45-27\| = 72$	72
50-60	55	2	110	$\|55-27\| = 28$	56
Total		50	1350		512

Therefore, $\bar{X} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1350}{50} = 27$; Mean $= \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{512}{50} = 10.24$

Second Method: We could have avoided calculations of computing $\bar{x}$ by following step deviation method. Let the assumed mean of the data be 25.

Classes	Mid Values x_i	$d_i = \frac{x_i - 25}{10}$	Frequencies	$f_i d_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
0-10	5	-2	6	-12	22	132
10-20	15	-1	8	-8	12	96
20-30	25	0	14	0	2	28
30-40	35	1	16	16	8	128
40-50	45	2	4	8	18	72
50-60	55	3	2	6	28	56
Total			50	10		512

Mean, $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} \times \text{class size}$

Mean, $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} \times \text{class size} = 25 + \frac{10}{50} \times 10 = 25 + 2 = 27$

Mean deviation from the mean $= \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{512}{50} = 10.24$

JEE Main/Boards

Exercise 1

Q.1 In a moderately skewed distribution the values of mean and median are 5 and 6 respectively. What will be the value of mode in such a situation?

Q.2 In a class of 100 students, the average amount of pocket money is Rs.35 per student. If the average is Rs.25 for girls and Rs.50 for boys, then what will be the number of girls in the class?

Q.3 A group of 10 items has arithmetic mean 6. If the arithmetic mean of 4 of these items is 7.5, then what will be the mean of the remaining items?

Q.4 If $\bar{x}$ is the arithmetic mean of n independent variates $x_1, x_2, x_3 ...., x_n$ each of the standard deviation σ, then find the variance $(\bar{x})$.

Q.5 If the standard deviation of the observations -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, is $\sqrt{10}$. What will be the standard deviation of observations 15, 16, 17, 19, 20, 21, 22, 23, 24, 25?

Q.6 The mean of the distribution in which the values of X are 1,2,3,.....n. What is the frequency of each being unity?

Q.7 If the mean of a set of observations $x_1, x_2, ..., x_n$ is $\bar{x}$ then find the mean of the observations $x_i + 2i; i = 1, 2, ..., n$.

Exercise 2

Single Correct Choice Type

Q.1 If SD of X is x, then SD of the variable $\mu = \frac{\alpha X + b}{c}$, where a,b,c are constants, is;

(A) $\left|\frac{c}{a}\right|\sigma$ (B) $\left|\frac{a}{c}\right|\sigma$ (C) $\left|\frac{b}{c}\right|\sigma$ (D) $\frac{c^2}{a^2}\sigma$

Q.2 The 7th percentile is equal to:

(A) 7th decile (B) Q_3

(C) 6th decile (D) None of these

Q.3 The algebraic sum of the deviation of 20 observations measured from 30 is 2. Then mean of observations is:

(A) 28.5 (B) 30.1 (C) 30.5 (D) 29.6

Q.4 The mean deviation of any series is 15, then value of quartile deviation is:

(A) 10.5 (B) 11.5 (C) 12.5 (D) 13.5

Q.5 If the mean deviation about the median of the number a, 2a ...50a is 50, then $|a|$ equals:

(A) 2 (B) 3 (C) 4 (D) 6

Q.6 The mean of the number a, b, 8,5,10 is 6 and the variance is 6.80. Then, which one of the following gives possible values of a and b?

(A) a = 0, b = 7 (B) a = 5, b = 2

(C) a = 1, b = 6 (D) a = 3, b = 4

Q.7 For two datasets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined dataset is

(A) $\frac{11}{2}$ (B) 6 (C) $\frac{13}{2}$ (D) $\frac{5}{2}$

Q.8 If the mean deviation of number 1, 1+d, 1+2d,..., 1+100d from their mean is 255, then the d is equal to

(A) 10 (B) 20 (C) 10.1 (D) 20.2

Assertion Reasoning Type

Q.9 Statement-I: The variance of first n even natural numbers is $\frac{n^2-1}{4}$

Statement-II: The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$

(A) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I

(B) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Previous Years' Questions

Q.1 Consider any set of 201 observations x_1, x_2....., x_{200}, x_{201}. It is given that $x_1 < x_2 < ..... < x_{200} < x_{201}$. Then the mean deviation of this set of observations about a point k equals ***(1981)***

(A) $(x_1 + x_2 + ... + x_{200} + x_{201})$ (B) x_1

(C) x_{101} (D) x_{201}

Q.2 If $(x_1, x_2, ..., x_n)$ are any real numbers and n is any positive integer, then ***(1985)***

(A) $n\sum_{i=1}^{n} X_i^2 < \left(\sum_{i=1}^{n} X_i\right)^2$ (B) $\sum_{i=1}^{n} X_i^2 \geq \left(\sum_{i=1}^{n} X_i\right)^2$

(C) $\sum_{i=1}^{n} X_i^2 \geq n\left(\sum_{i=1}^{n} X_i\right)^2$ (D) None of these

Q.3 In a college of 300 students every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspaper is ***(1998)***

(A) At least 30 (B) At most 20

(C) Exactly 25 (D) None of the above

Q.4 The mean square deviations of a set of observations $x_1, x_2, ..., x_n$ about a point c is defined to be $\frac{1}{n}\sum_{i=1}^{n}(x_i - c^2)$.

The mean square deviations about -1 and +1 of a set of observations are 7 and 3 respectively. Find the standard deviation of this set of observations. ***(1981)***

Q.5 The marks obtained by 40 students are grouped in a frequency table in class intervals of 10 marks each. The mean and the variance obtained from this distribution are found to be 40 and 49 respectively. It was later discovered that two observations belonging to the class interval (21-30) were included in the class interval (31-40) by mistake. Find the mean and the variance after correcting the error. ***(1982)***

Q.6 The mean of the numbers a,b,8,5,10 is 6 and the variance is 6.80. Then which one of the following gives possible values of and b? ***(2008)***

(A) a = 0,b = 7 (B) a = 5,b = 2

(C) a = 1,b = 6 (D) a = 3,b = 4

Q.7 Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the Statement-I : r is equivalent to either q or p ***(2008)***

(A) Statement-I is false, statement to $\sim(p \leftrightarrow \sim q)$.

(B) Statement-I is true, statement-II is a correct explanation for Statement-I

(C) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I.

(D) Statement-I is true, statement-II false

Q.8 The statement $p \to (q \to p)$ is equivalent to ***(2008)***

(A) $p \to (p \to q)$ (B) $p \to (p \vee q)$

(C) $p \to (p \wedge q)$ (D) $p \to (p \leftrightarrow q)$

Q.9 If the mean deviation of number $1, 1+d, 1+2d, ..., 1+100d$ from their mean is 255, then the d is equal to

(A) 10.0 (B) 20.0 (C) 10.1 (D) 20.2

Q.10 Statement-II: The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$

(A) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I

(B) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Q.11 Let S be a non-empty subset of R. Consider the following statement; P: There is a rational number $x \in s$ such that x > 0.

Which of the following statements is the negation of the statement P? ***(2010)***

(A) There is no rational number $x \in S$ such that $x \leq 0$

(B) Every rational number $x \in S$ satisfies $x \leq 0$

(C) $x \in S$ and $x \leq \Rightarrow x$ is not rational

(D) There is a rational number $x \in S$ such that $x \leq 0$

Q.12 For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is ***(2009)***

(A) $\frac{11}{2}$ (B) 6 (C) $\frac{13}{2}$ (D) $\frac{5}{2}$

Q.13 Consider the following statements

P : Suman is brilliant

Q : Suman is rich

R : Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as ***(2011)***

(A) $\sim(Q \leftrightarrow (p \wedge \sim R))$ (B) $\sim Q \leftrightarrow \sim p \wedge R$

(C) $\sim(p \wedge \sim R) \leftrightarrow Q$ (D) $\sim P \wedge (Q \leftrightarrow \sim R)$

Q.14 If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then $|a|$ equals

(A) 3 (B) 4 (C) 5 (D) 2

Q.15 The negation of the statement "If I become a teacher, then I will open a school" is ***(2012)***

(A) I will become a teacher and I will not open a school

(B) Either I will not become a teacher or I will not open a school

(C) Neither I will become a teacher nor I will open a school

(D) I will not become a teacher or I will open a school

Q.16 Let $x_1, x_2 ...., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.

Statement-I: Variance of $2x_1, 2x_2 ...., 2x_2, ...2x_n$ is 4 σ^2.

Statement-II: Arithmetic mean of $2x_1, 2x_2 ..., 2x_n$ is $4\bar{x}$

(A) Statement-I is false, statement-II is true

(B) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I

(C) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I

(D) Statement-I is true, statement-II is false

Q.17 All the students of a class performed poorly in Mathematics. The teacher decided to give gracemarks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given? ***(2013)***

(A) Mean (B) Median

(C) Mode (D) Variance

Q.18 The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to ***(2015)***

(A) $s \vee \sim r$ (B) $s \wedge (r \wedge \sim s)$

(C) $s \vee (r \wedge \sim s)$ (D) $s \wedge r$

Q.19 If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true? ***(2016)***

(A) $3a^2 - 32a + 84 = 0$ (B) $3a^2 - 32a + 91 = 0$

(C) $3a^2 - 23a + 44 = 0$ (D) $3a^2 - 26a + 55 = 0$

Q.20 The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to: ***(2016)***

(A) $p \wedge q$ (B) $p \vee q$

(C) $p \vee \sim q$ (D) $\sim p \wedge q$

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.1 Q.5 Q.6 Q.8

Exercise 2

Q.1 Q.4 Q.6 Q.7 Q.9

Previous Years' Questions

Q.1 Q.2 Q.4

Answer Key

JEE Main/Boards

Exercise 1

Q.1 A **Q.2** C **Q.3** D **Q.4** A **Q.5** C **Q.6** C **Q.7** C

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** D **Q.3** B **Q.4** C **Q.5** C **Q.6** D **Q.7** A
Q.8 C **Q.9** D

Previous Year Questions

Q.1 C **Q.2** D **Q.3** C **Q.4** $\sqrt{3}$ **Q.5** Mean = 39.5 and Variance = 49.2
Q.6 D **Q.7** D **Q.8** B **Q.9** C **Q.10** D **Q.11** B **Q.12** A
Q.13 A **Q.14** B **Q.15** A **Q.16** D **Q.17** D **Q.18** D **Q.19** A
Q.20 B

Solutions

JEE Main/Boards

Exercise 1

Sol 1: Given that mean =5, median =6

For a moderately skewed distribution, we have

Mode =3 median – 2 mean

$\Rightarrow$ Mode = 3(6) – 2(5) = 8

Sol 2: Let the number of girls in the class = y

$\therefore$ Number of boys in the class = 100 – y

$$\therefore 35 = \frac{25\times y + 50\times(100-y)}{100}$$
$$\Rightarrow 3500 = 25y + 5000 - 50y$$
$$\Rightarrow 25y = 1500 \Rightarrow y = 60$$

$\therefore$ Number of girls in the class = 60

Sol 3: Given that, $n_1 = 4, \bar{x}_1 = 7.5, n_1 + n_2 = 10, \bar{x} = 6$

$$\therefore 6 = \frac{4\times 7.5 + 6\times \bar{x}_2}{10} \Rightarrow 60 = 30 + 6\bar{x}_2$$

$$\Rightarrow \bar{x}_2 = \frac{30}{6} = 5$$

Sol 4: We have, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$$\therefore \text{var}(\bar{x}) = \frac{1}{n^2}\left[\sum_{i=1}^{n}\text{var}(x_i) + \sum_{i\neq j}^{n}\text{cov}(x_i, x_j)\right]$$

$$= \frac{1}{n^2}[n\sigma^2] = \frac{\sigma^2}{n}$$

$\therefore x_i$ and x_j are independent variable, therefore cov $(x_i, x_j) = 0$

Sol 5: The new observations are obtained by adding 20 to each. Hence, σ does not change.

Sol 6: Since, the frequency of each observation is unity.

$$\therefore \bar{x} = \frac{1(1)+2(1)+.......+n(1)}{n} = \frac{n(n+1)}{2n} = \frac{(n+1)}{2}$$

Sol 7: $\therefore\ \bar{x} = \frac{x_1 + x_2 ..... + x_n}{n}$

$\therefore\ n\bar{x} = x_1 + x_2 + ... + x_n$ .....(i)

Let $\bar{y}$ be the mean of observations $x_i + 2i, i = 1,2,3,......., n$.

Then,

$$\bar{y} = \frac{[(x_1+2)+(x_2+2.2)+(x_3+2.3)+...+(x_n+2.n)]}{n}$$

$$= \frac{(x_1+x_2+....+x_n)+2(1+2+....+n)}{n}$$

$$= \frac{x_1+x_2+....+x_n}{n} + \frac{2n(n+1)}{2n}$$

$\Rightarrow \bar{y} = \bar{x} + (n+1)$ [from Eq. (i)]

Exercise 2

Single Correct Choice Type

Sol 1: (B) We know that

$\text{var}\ (aX+b) = a^2\ \text{var}(X)$

$$\therefore \text{var}\left(\frac{aX+b}{c}\right) = \left(\frac{a}{c}\right)^2 \text{var}(X) = \frac{a^2}{c^2}\sigma^2$$

$$\therefore \text{SD} = \sqrt{\text{var}\left(\frac{aX+b}{c}\right)} = \left|\frac{a}{c}\right|\sigma$$

Sol 2: (D) None of these

7^{th} decile $D_7 = \frac{7n}{10}$ ....(i)

And 7^{th} percentile, $P_{70} = \frac{7n}{100}$ .....(ii)

From Eqs. (i) and (ii), we get

$D_7 \neq P_{70}$

Sol 3: (B) Given that

$$\Sigma_{i=1}^{20}(x_i - 30) = 2 \Rightarrow \Sigma_{i=1}^{20} x_i - \Sigma_{i=1}^{20}(30) = 2$$

$$\Rightarrow \bar{x} = \frac{20.30}{20} + \frac{2}{20} = 30 + 0.1 = 30.1$$

Sol 4: (C) $\text{SD} = \frac{5}{4}(\text{mean deviation}) = \frac{5}{4}(15) = \frac{75}{4}$

$$\therefore \text{Quartile deviation} = \frac{2}{3}(\text{SD}) = \frac{2}{3}\left(\frac{75}{4}\right) = 12.5$$

Sol 5: (C) Median $= \frac{25a+26a}{2} = \frac{51}{2}a$

$$\frac{\Sigma|(X-M)|}{50} = 50 = \left|a - \frac{51}{2}a\right| + \left|2a\frac{51}{2}a\right| + ... + \left|25a - \frac{51}{2}a\right|$$

$$+\left|26a - \frac{51}{2}a\right| + ... + \left|50a - \frac{51}{2}a\right| = 2500$$

$$\Rightarrow 2\left(\frac{49a}{2} + \frac{47}{2}a + \frac{45a}{2} + ... + \frac{a}{2}\right) = 2500$$

$\Rightarrow (1+3+....+4a) = 2500$

$\Rightarrow \frac{25}{2}(1+49)a = 2500 \Rightarrow 25 \times 25a = 2500$

$\Rightarrow a = 4$

Sol 6: (D) According to the given condition

$$6.80 = \frac{(6-a)^2+(6-b)^2+(6-8)^2+(6-5)^2+(6-10)^2}{5}$$

$\Rightarrow 34 = (6-a)^2 + (6+b)^2 + 4 + 1 + 16$

$\Rightarrow (6-a)^2 + (6-b)^2 = 13 = 9 + 4 = 3^2 + 2^{\ 2} \Rightarrow a = 3, b = 4$

Sol 7: (A) $\sigma_x^2 = 4;\ \sigma_y^2 = 5;\ \bar{x} = 2;\quad \bar{y} = 4$

$\frac{\Sigma x_i}{5} = 2 \Rightarrow \Sigma x_i = 10;\qquad \Sigma y_i = 20$

$\sigma_x^2 = \left(\frac{1}{5}\Sigma x_i^2\right) - (\bar{x})^2;\qquad \sigma_y^2 = \frac{1}{5}(\Sigma y_i^2) - (\bar{y})^2$

$\Sigma x_i^2 = 40;\qquad \Sigma y_i^2 = 105$

$$\sigma_z^2 = \frac{1}{10}(\Sigma x_i^2 + \Sigma y_i^2) - \left(\frac{\bar{x}+\bar{y}}{2}\right)^2 = \frac{11}{2}$$

Sol 8: (C)

$$\text{Mean}(\bar{x}) = \frac{\text{sum of quantities}}{n} = \frac{\frac{\cancel{n}}{2}(a+l)}{\cancel{n}} = 1 + 50d$$

$$\text{M.D.} = \frac{1}{n}\Sigma|x_i - \bar{x}|$$

$$\Rightarrow 225=\frac{1}{101}[50d+49d+\ldots+d+0+d+\ldots+50d]=\frac{\cancel{2}d}{101}$$

$$\Rightarrow d=\frac{255\times101}{50\times51}=10.1$$

Sol 9: (D) Statement-I:

Sum of n even natural number =n(n+1)

$$\text{Mean}(\bar{x})=\frac{n(n+1)}{n}=n+1$$

$$\text{Variance}=\left[\frac{1}{n}\Sigma(x_i)^2\right]-(\bar{x})^2$$

$$=\frac{1}{n}[2^2+4^2+\ldots..+(2n)^2]-(n+1)^2$$

$$=\frac{1}{n}2^2[1^2+2^2+\ldots..+n^2]-(n+1)^2$$

$$=\frac{4}{n}\frac{n(n+1)(2n+1)}{6}-(n+1)^2$$

$$=\frac{(n+1)[2(2n+1)-3(n+1)]}{3}$$

$$=\frac{(n+1)[4n+2-3n-3]}{3}=\frac{(n+1)(n-1)}{3}=\frac{n^2-1}{3}$$

$\therefore$ Statement-I is false.

Previous Years' Questions

Sol 1: (C) Given that, $x_1 < x_2 < x_3 < \ldots < x_{201}$

$\therefore$ Median of the given observation = $\left(\frac{201+1}{2}\right)$th item= x_{101}

Now, deviations will be minimum, if we taken from the median.

$\therefore$ Mean deviation will be minimum, if k = x_{101}

Sol 2: (D) Since, $x_1, x_2, \ldots, x_n$ are any real numbers.

$$\frac{x_1^2+x_2^2+\ldots+x_n^2}{n}\geq\left(\frac{x_1+x_2+\ldots+x_n}{n}\right)^2$$

$$\Rightarrow n\sum_{i=1}^{n}x_i^2\geq\left(\sum_{i=1}^{n}x_i\right)^2$$

Sol 3: (C) Let n be the number of newspapers which are read by the students. Then,

60n = (300) × 5 $\Rightarrow$ n =25

Sol 4: Mean square deviations = $\frac{1}{n}\sum_{i=1}^{n}(x_i-c)^2$, about c.

Also, given that mean square deviation about – 1 and +1 are 7 and 3 respectively.

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}(x_i+1)^2=7 \text{ and } \frac{1}{n}\sum_{i=1}^{n}(x_i-1)^2=3$$

$$\Rightarrow \sum_{i=1}^{n}x_i^2+2\sum_{i=1}^{n}x_i+n=7n$$

$$\text{and } \sum_{i=1}^{n}x_i^2-2\sum_{i=1}^{n}x_i+n=3n$$

$$\Rightarrow \sum_{i=1}^{n}x_i^2+2\sum_{i=1}^{n}x_i=6n \text{ and } \sum_{i=1}^{n}x_i^2-2\sum_{i=1}^{n}x_i=2n$$

$$\Rightarrow \sum_{i=1}^{n}x_i=n$$

$$\therefore \bar{x}=\frac{\sum_{i=1}^{n}x_i}{n}=1$$

$\therefore$ Standard deviation

$$=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-1)^2}=\sqrt{3}$$

Sol 5: Given, n = 40, $\bar{x}$ = 40, var (x) = 49

$$\Rightarrow \bar{x}=\frac{\Sigma f_i x_i}{40}=40 \Rightarrow \Sigma f_i x_i=1600$$

Also, var (x) = 49

$$\Rightarrow \frac{1}{40}\Sigma f_i(x_i-40)^2=49$$

$$\therefore 49=\frac{1}{40}(\Sigma x_i^2 f_i)-2\Sigma x_i f_i+40\Sigma f_i$$

$$\Rightarrow 49=\frac{1}{40}(\Sigma x_i^2 f_i)-2(1600)+40\times40$$

$$\therefore \Sigma x_i^2 f_i=1649\times40$$

Let (21 –30) and (31 –40) denote the kth and (k+1)th class intervals respectively.

Then, if before correction f_k and f_{k+1} are frequencies of those intervals then after correction (2 observations are shifted from (31-40) to (21-30), frequency of k th interval becomes f_{k+2} and frequency of (k+1)th interval becomes $f_{k+1}-2$

$$\therefore \bar{x}_{new}=\frac{1}{40}\left\{\sum_{\substack{i=1\\ i\neq k,k+1}}^{40}f_i x_i+(f_k+2)x_k+(f_{k+1}-2)x_{k+1}\right\}$$

$$\Rightarrow \bar{x}_{new} = \frac{1}{40}\left\{\sum_{i=1}^{40} f_i x_i\right\} + \frac{2}{40}\{(x_k - x_{k+1})\}$$

$$= \frac{1}{40}\sum_{i=1}^{40} f_i x_i + \frac{1}{20}(-10) = 39.5$$

$$(\text{var})_{new} = \frac{1}{40}\left\{\begin{matrix}\sum_{i=1}^{40} f_i(x_i - 39.5)^2 + f_k(x_k - 39.5)^2 \\ i \neq k, k+1 \quad + f_{k+1}(x_{k+1} - 39.5)^2\end{matrix}\right\}$$

$$= \frac{1}{40}\left[\Sigma\left(f_i x_i^2 - 79\, f_i x_i + (39.5)^2 f_i\right)\right]$$

$$= \frac{1}{40}\sum_{i=1}^{40} f_i \cdot x_i^2 - 79\left(\frac{1}{40}\right)\sum_{i=1}^{40} f_i x_i + (39.5)^2 \cdot \frac{1}{40}\sum_{i=1}^{40} f_i$$

$= 1649 - 3160 + 1560.25 = 49.25$

Sol 6: (D) Mean of a, b, 8, 5, 10 is 6

$$\Rightarrow \frac{a+b+8+5+10}{5} = 6$$

$\Rightarrow a + b = 7$ ...(i)

Given that Variance is 6.8

$$\therefore \text{ Variance } = \frac{\sum (x_i - A)^2}{n}$$

$$= \frac{(a-6)^2 + (b-6)^2 + 4 + 1 + 16}{5} = 6.8$$

$\Rightarrow a^2 + b^2 = 25$

$a^2 + (7-a)^2 = 25$ (Using (i))

$\Rightarrow a^2 - 7a + 12 = 0$

$\therefore a = 4,3$ and $b = 3,4$.

Sol 7: (D) Give statement $r = \sim p \leftrightarrow q$ Statement-I $r_1 = (p \wedge \sim q) \vee (\sim p \wedge q)$

Statement-$r_2 = \sim (p \leftrightarrow \sim q) = (p \wedge q) \vee (\sim q \wedge \sim p)$

From the truth table of r, r_1 and r_2,

$r = r_1$.

Hence, statement-I is true and Statement-II is false.

Sol 8: $p \to (q \to p) = \sim p \vee (q \to p)$

$= \sim p \vee (\sim q \vee p)$ since $p \vee \sim p$ is always true

$= \sim p \vee p \vee q = p \to (p \vee q)$

Sol 9: Mean $(\bar{X}) = \frac{\text{sum of quantities}}{n} = \frac{\frac{\cancel{n}}{2}(a+1)}{\cancel{n}}$

$\frac{1}{2}[1 + 1 + 100d] = 1 + 50d$

M.D $\frac{1}{2}\sum |x_i - \bar{x}| \Rightarrow 225 = \frac{1}{101}[50d + 49d + 48d +$

$\ldots + d + 0 + d] = \frac{\cancel{2}d}{101}\left[\frac{50 \times 51}{\cancel{2}}\right]$

$$\Rightarrow d = \frac{255 \times 101}{50 \times 51} = 10.1$$

Sol 10: Statement-II is true

Statement-I: Sum of n even natural numbers = n(n+1)

$$\text{Mean } (\bar{x}) = \frac{n(n+1)}{n} = n + 1$$

$$\text{Variance } \left[\frac{1}{n}\sum (x_i)^2\right]$$

$$(\bar{x})^2 = \frac{1}{n}\left[2^2 + 4^2 + \ldots + (2n)^2\right] - \geq\leq (n+1)^2$$

$$\frac{1}{n}2^2\left[1^2 + 2^2 + \ldots + n^2\right] - (n+1)^2$$

$$= \frac{4}{n}\frac{n(n+1)(2n+1)}{6} - (n+1)^2$$

$$\frac{(n+1)[2(2n+1) - 3(n+1)]}{3} = \frac{(n+1)[4n + 2 - 3n - 3}{3}$$

$$\frac{(n+1)(n-1)}{3} = \frac{n^2 - 1}{3}$$

$\therefore$ Statement-I is false.

Sol 11: (B) P: there is a rational number $x \in S$ such that $x > 0$

$\sim$ P: Every rational number $x \in S$ satisfies $x \leq 0$

Sol 12: (A) $\sigma_x{}^2 = 4$

$\sigma_y{}^2 = 5$

$\bar{x} = 2$

$\bar{y} = 4$

$$x \leq 0 \;\; \frac{\sum x_i}{5} = 2 \;\; \sum x_i = 10; \sum y_i = 20$$

$$\sigma_x{}^2 = \left(\frac{1}{2}\sum x_i{}^2\right) - (\bar{x})^2 = \frac{1}{5}\left(\sum y^2{}_i\right) - 16$$

$\sum x_i^2 = 40$

$\sum y_i^2 = 105$

$\sigma_z^2 = \frac{1}{10}\left(\sum x_i^2 + \sum y_i^2\right) - \left(\frac{\bar{x}+\bar{y}}{2}\right) = \frac{1}{10}$

$(40+105) - 9 = \frac{145-90}{10} = \frac{55}{10} = \frac{11}{2}$

Sol 13: (A) $P \wedge -R$ stands for Suman is brilliant and dishonest. Thus, $P \wedge \sim R \leftrightarrow Q$ stands for suman is brilliant and dishonest if and only if suman is rich.

Its negation is $\sim(P \wedge \sim R \leftrightarrow Q)$ or $\sim(Q \leftrightarrow P \wedge \sim R)$

Sol 14: (B) The median of this list $= \frac{25a+26a}{2} = 25.5a$

Mean deviation about the median

$= \frac{|a-25.5a| + |2a-25.5a| ... |50a-25.5a|}{50} = 50$

$50(50) = 25.5a(25) - (a + 2a ... 25a) + (26a + 27a ... 50a) - 25.5a(25)$

$50(50) = \frac{50(51)a}{2} - \frac{25(26)a}{2} - \frac{25(26)a}{2}$

$50(50) = 25(51)a - 25(26)a$

$50(2) = 51a - 26a$

$50(2) = 25a \ \therefore a = 4$

Sol 15: (A) The negation of the statement if I become a teacher, then I will open a school is

Option A - I will become a teacher and I will not open a school

Let p: I will become a teacher

q: I will open a school

The given statement is $P \to q \equiv (-P)Vq$

Its negations is $-(-P)Vq \equiv p \wedge (-q)$

Thus negation is I will become a teacher and I will not open a school.

Sol 16 :(D) $\sigma^2 = \sum \frac{x_i^2}{n} - \left(\sum \frac{x_i}{n}\right)^2$

Variance of

$2x_1, 2x_2, ..., 2x_n = \sum \frac{(2x_i)^2}{n} - \left(\sum \frac{2x_i}{n}\right)^2 = 4\left[\sum \frac{x_i^2}{n} - \left(\sum \frac{x_i}{n}\right)^2\right]$

$= 4\sigma^2$

Statement-I is true.

A.M. of

$2x_1, 2x_2 ... 2x_n = \frac{2x_1 + 2x_2 + 2x_n}{n} = 2\left(\frac{x_1 + x_2 + ... + x_n}{n}\right)$

$= 2\bar{x}$

Statement-II is false.

Sol.17: (D) If initially all marks were x_i then

$\sigma_1^2 = \frac{\sum (x_i - \bar{x})^2}{N}$

Now each is increased by 10

$\sigma_2^2 = \frac{\sum [(x_i + 10) - (\bar{x} + 10)^2]}{N} = \sigma_1^2$

$\sigma_2^2 = \frac{\sum [(x_i + 10) - (\bar{x} + 10)^2]}{N} = \sigma_1^2$

So variance will not change whereas mean, median and mode will increase by 10.

Sol 18: (D) $\sim s \vee (\sim r \wedge s) = (\sim s \vee \sim r) \wedge (\sim s \vee s)$

$= \ \sim(s \wedge r) \wedge t \ = \ \sim(s \wedge r)$

So negation is $s \wedge r$.

Sol 19: (A) Standard deviation of numbers 2, 3, a and 11 is 3.5

$\therefore (3.5)^2 = \frac{\sum x_i^2}{4} - (\bar{x})^2$

$\Rightarrow \ (3.5)^2 = \frac{4 + 9 + a^2 + 121}{4} - \left(\frac{2 + 3 + a + 11}{4}\right)^2$

On solving, we get $3a^2 - 32a + 84 = 0$

Sol 20: (B) $[(p \wedge \sim q) \vee q] \vee (\sim p \wedge q)$

$= (p \vee q) \wedge (\sim q \vee q) \vee (\sim p \wedge q)$

$= (p \vee q) \wedge [t \vee (\sim p \wedge q)]$

$= (p \vee q) \wedge t$

$= p \vee q$