FORMULAE SHEET

(a) Distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

(b) Coordinates of the point dividing the distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio m:n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$$

(c) If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ are vertices of a triangle, then its centroid is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

(d) If $A(x_1, y_1, z_1)$ and B (x_2, y_2, z_2) are the two points, the point which divides the line segment AB in ratio λ:1 is

$$\left(\frac{\lambda x_2+x_1}{\lambda+1}, \frac{\lambda y_2+y_1}{\lambda+1}, \frac{\lambda z_2+z_1}{\lambda+1}\right)$$

(e) If (x_1, y_1, z_1) and (x_2, y_2, z_2) are the two points on the line with x_2-x_1, y_2-y_1, z_2-z_1 as direction ratios, then their d.c.s are

$$\pm\frac{x_2-x_1}{\sqrt{\Sigma(x_2-x_1)^2}}, \pm\frac{y_2-y_1}{\sqrt{\Sigma(x_2-x_1)^2}}, \pm\frac{z_2-z_1}{\sqrt{\Sigma(x_2-x_1)^2}}$$

(f) If ℓ, m, n are d.c.s of a line, then $l^2 + m^2 + n^2 = 1$. Thus, if a line makes angles α, β, γ with axes, then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ and $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

(g) If a, b, c are the d.r.s of a line, then the d.c.s of the line are $\pm\frac{a}{\sqrt{\Sigma a^2}}, \pm\frac{b}{\sqrt{\Sigma a^2}}, \pm\frac{c}{\sqrt{\Sigma a^2}}$

(h) If p(x, y, z) is a point in space such that $\overrightarrow{OP}=\vec{r}$ has d.c.s ℓ, m, n, then

(a) $\ell|\vec{r}|, m|\vec{r}|, n|\vec{r}|$ are the projections on x-axis, y-axis and z-axis, respectively.

(b) $x=\ell|\vec{r}|, y=m|\vec{r}|, z=n|\vec{r}|$

(c) $\vec{r}=|\vec{r}|(l\hat{i}+m\hat{j}+n\hat{k})$ and $\hat{r}=\ell\hat{i}+m\hat{j}+n\hat{k}$

Moreover, if a, b, c are d.r.s of a vector $\vec{r}$, then $\vec{r}=\frac{|\vec{r}|}{\sqrt{a^2+b^2+c^2}}(a\hat{i}+b\hat{j}+c\hat{k})$.

(i) Length of projection of the line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) on a line with d.c.s ℓ, m, n is $|\ell(x_2-x_1)+m(y_2-y_1)+n(z_2-z_1)|$

(j) If θ is the angle between two lines having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 then

$$\cos\theta=\pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{\Sigma a_1^2}\sqrt{\Sigma a_2^2}}$$

(k) Two lines are parallel if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ and two lines are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$

(l) Cartesian equations of a line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=t$$

(m) Vector equation of a line passing through the point $A(\vec{a})$ and parallel to vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$ for scalar λ.

(n) Cartesian equation of a line passing through two points having coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$.

(o) Vector equation of a line passing through two points having position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{\alpha} + \lambda(\vec{b} - \vec{a})$

(p) Distance between the parallel lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is $\frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|}$

(q) Shortest distance (S.D.) between two lines with equations; $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is $\frac{|(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|}$. If θ is the angle between the lines, then $\cos\theta = \frac{\vec{b}_1 . \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$

(r) The length of perpendicular from the point (α, β, γ) to the line $\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ (ℓ, m, n being d.cs)

is given by $\sqrt{(\alpha - x_1)^2 + (\beta - y_1)^2 + (\gamma - z_1)^2 - [\ell(\alpha - x_1) + m(\beta - y_1) + n(\gamma - z_1)]^2}$

(s) If $\vec{a}$ and $\vec{b}$ are the unit vectors along the sides of an angle, then $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are the vectors, respectively, along the internal and external bisector of the angle. In fact, the bisectors of the angles between the lines, $\vec{r} = x\vec{a}$ and $\vec{r} = y\vec{b}$ are given by $\vec{r} = \lambda\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}\right); \lambda \in R$

(t) Equation of plane passing through the point (x_1, y_1, z_1) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.

(u) Equation of plane passing through three points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

(v) Equation of a plane making intercepts a, b, c on axes is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

(w) Vector equation of a plane through the point $\vec{a}$ and perpendicular to the unit vector $\hat{n}$ is $(\vec{r} - \vec{a}).\hat{n} = 0$

(x) If θ is the angle between the two planes $\vec{r}.\hat{n}_1 = d_1$ and $\vec{r} \cdot \hat{n}_2 = d_2$, then $\cos\theta = \frac{\hat{n}_1 \cdot \hat{n}_2}{|\hat{n}_1||\hat{n}_2|}$

(y) Equation of a plane containing the line $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ and passing through the point (x_2, y_2, z_2) not on the line is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a & b & c \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \end{vmatrix} = 0$

(z) Equation of a plane through the line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and parallel to the line $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$