(t) The perpendicular distance of a point P($\vec{r}$) from a plane passing through the points $\vec{a}$, $\vec{b}$ and $\vec{c}$ is given by

$$P = \frac{(\vec{r}-\vec{a}).(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b})}{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}|}$$

(u) **Angle between a line and the plane:** If θ is the angle between a line $\vec{r} = (\vec{a}+\lambda\vec{b})$ and the plane $\vec{r}.\vec{n} = d$, then $\sin\theta = \frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}$.

(v) The equation of sphere with center at C($\vec{c}$) and radius 'a' is $|\vec{r}-\vec{c}| = a$. If center is the origin then $|\vec{r}| = a$.

(w) The plane $\vec{r}.\vec{n} = d$ touches the sphere $|\vec{r}-\vec{a}| = R$, if $\frac{|\vec{a}.\vec{n}-d|}{|\vec{n}|} = R$, i.e. the condition of tangency.

(x) If $\vec{a}$ and $\vec{b}$ are the position vectors of the extremities of a diameter of a sphere, then its equation is given by $(\vec{r}-\vec{a}).(\vec{r}-\vec{b}) = 0$ or $|\vec{r}|^2 - \vec{r}.\ (\vec{a}+\vec{b}) + \vec{a}.\vec{b} = 0$ or $|\vec{r}-\vec{a}|^2 + |\vec{r}-\vec{b}|^2 = |\vec{a}-\vec{b}|^2$.

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(a) $\overrightarrow{OP} = x\hat{i} + y\hat{j}$

(b) $|\overrightarrow{OP}| = \sqrt{x^2+y^2}$ and direction is $\tan\theta = \frac{y}{x}$

(c) Unit vector $\hat{U} = \frac{\text{Vector}}{\text{Its modulus}} = \frac{\vec{a}}{|\vec{a}|}$

(d) Properties of vector addition:

i. $\vec{a}+\vec{b} = \vec{b}+\vec{a}$ commutative	(a) $\vec{a}+(\vec{b}+\vec{c}) = (\vec{a}+\vec{b})+\vec{c}$ Associative
ii. $\vec{a}+\vec{0} = \vec{a}$ Null vector is an additive identity	(b) $\vec{a}+(-\vec{a}) = \vec{0}$ Additive inverse
iii. $c(\vec{a}+\vec{b}) = c\vec{a}+c\vec{b}$	(c) $(c+d)\vec{a} = c\vec{a}+d\vec{a}$
iv. $(cd)\vec{a} = c(d\vec{a})$	(d) $1\times\vec{a} = \vec{a}$

(e) **Section formula:**

(i) If $\vec{a}$ and $\vec{b}$ are the position vectors of two points A and B, then the position vector of a point which divides AB in the ratio m:n is given by $\vec{r} = \frac{(n\vec{a}+m\vec{b})}{(m+n)}$.

(ii) Position vector of mid-point of $\overrightarrow{AB} = \frac{(\vec{a}+\vec{b})}{2}$.

(f) Collinearity of three points: If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are the position vectors (non-zero) of three points and given they are collinear then there exists λ, γ both not being 0 such that $\vec{a} + \lambda\vec{b} + \gamma\vec{c}$

(g) Coplanar vectors: Let $\vec{a}, \vec{b}$ be non-zero, non-collinear vectors. Then, any vector $\vec{r}$ coplanar with $\vec{a}, \vec{b}$ can be expressed uniquely as a linear combination of $\vec{a}, \vec{b}$ i.e. there exist some unique x, $y \in R$, such that $x\vec{a} + y\vec{b} = \vec{r}$

(h) Product of two vectors:

(i) Scalar Product (dot product)

If $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

Note : • $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

• $\vec{a}$ and $\vec{b}$ are perpendicular if $\theta = 90°$

(ii) Properties of scalar product:

i. $\vec{a}.\vec{b} = \vec{b}.\vec{a}$	ii. $m\vec{a}.n\vec{b} = mn\vec{a}.\vec{b} = \vec{a}(.mn\vec{b})$
iii. $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{c} + \vec{a}.\vec{b}$	iv. $(\vec{a} + \vec{b})^2 = \vec{a}^2 + 2.\vec{a}.\vec{b} + \vec{b}^2$
v. If $\hat{i} = (1,0,0), \hat{j} = (0,1,0), \hat{k} = (0,0,1)$ then $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$	

(iii) Vector (cross) Product of two vectors: Let $\vec{a} = (a_1, a_2, a_3), \vec{b} = (b_1, b_2, b_3)$ be two vectors then the cross product of $\vec{a} \times \vec{b}$ is devoted by $\vec{a} \times \vec{b}$ and defined by

$$\vec{a} \times \vec{b} = (a_1, a_2, a_3) \times (b_1, b_2, b_3) = \begin{pmatrix} a_2 & a_3 & a_1 & a_2 \\ b_2 & b_3 & b_1 & b_2 \end{pmatrix} = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$$

OR

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\hat{i} + (a_3b_1 - a_1b_3)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \times |\vec{b}| \sin\theta\, \hat{n}$$

Note: (i) θ being angle between $\vec{a}$ & $\vec{b}$

(ii) If $\theta = 0$, The $|\vec{a} \times \vec{b}| = 0$ i.e. $\vec{a} \times \vec{b} = 0$ and $\vec{a}$ & $\vec{b}$ are parallel if $\vec{a} \times \vec{b}$ =0.

(iv) Properties of cross product

i. $\vec{a}\times\vec{b}=0 \Rightarrow \vec{a}=0$ or $\vec{b}=0$ or $\vec{a}\parallel\vec{b}$	ii. $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$
iii. $\vec{a}\times\left(\vec{b}+\vec{c}\right)=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$	iv. $\left(n\vec{a}\right)\times\vec{b}=n\left(\vec{a}\times\vec{b}\right)$
v. $\vec{a}\times\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$	vi. $\left\|\vec{a}\times\vec{b}\right\|$ is a Area of parallelogram with sides $\vec{a}$ and $\vec{b}$.

(v) Scalar Triple Product: If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k},\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k},\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$.

Then $\vec{a}.\left(\vec{b}\times\vec{c}\right)=\vec{b}.\left(\vec{c}\times\vec{a}\right)=\vec{c}.\left(\vec{a}\times\vec{b}\right)$

$$\vec{a}.\left(\vec{b}\times\vec{c}\right)=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$\vec{a}.\left(\vec{b}\times\vec{c}\right)$ is also represented as $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\left[\vec{b}\ \vec{c}\ \vec{a}\right]=\left[\vec{c}\ \vec{a}\ \vec{b}\right]$$

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=-\left[\vec{a}\ \vec{c}\ \vec{b}\right]$$

- If any of the two vectors are parallel, then $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=0$
- $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]$ is the volume of the parallelepiped whose coterminous edges are formed by $\vec{a}\ \vec{b}\ \vec{c}$
- If $\vec{a}\ \vec{b}\ \vec{c}$ are coplanar, $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=0$
- $\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$ = area of triangle having $\vec{a},\vec{b},\vec{c}$ as position vectors of vertices of a triangle.

(vi) Vector Triple Product:

$$\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}$$
$$\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{b}.\vec{c}\right)\vec{a}$$

Unit vector coplanar with $\vec{a}$ and $\vec{b}$ perpendicular to $\vec{a}$ is $\pm\dfrac{\left(\vec{a}\times\vec{b}\right)\times\vec{a}}{\left|\left(\vec{a}\times\vec{b}\right)\times\vec{a}\right|}$.