Solved Examples

JEE Main/Boards

Example 1: Show that the points A, B & C with position vector $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively, are the vertices of a right angled triangle. Also find the remaining angles of the triangle.

Sol: We have,

$\overrightarrow{AB} = (\hat{i}-3\hat{j}-5\hat{k})-(2\hat{i}-\hat{j}+\hat{k}) = -\hat{i}-2\hat{j}-6\hat{k}$

$\overrightarrow{BC} = (3\hat{i}-4\hat{j}-4\hat{k})-(\hat{i}-3\hat{j}-5\hat{k}) = 2\hat{i}-\hat{j}+\hat{k}$ and

$\overrightarrow{CA} = (2\hat{i}-\hat{j}+\hat{k})-(3\hat{i}-4\hat{j}-4\hat{k}) = -\hat{i}+3\hat{j}+5\hat{k}$

Since $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}$

$= (-\hat{i}-2\hat{j}-6\hat{k})+(2\hat{i}-\hat{j}+\hat{k})+(-\hat{i}+3\hat{j}+5\hat{k}) = \vec{0}$

So, A, B and C are the vertices of a triangle.

Now, $\overrightarrow{BC}.\overrightarrow{CA}$

$= (2\hat{i}-\hat{j}+\hat{k})+(-\hat{i}+3\hat{j}+5\hat{k}) = -2-3+5=0$

$\overrightarrow{BC} \perp \overrightarrow{CA} \Rightarrow \angle BCA = \frac{\pi}{2}$

Hence, ABC is a right angled triangle. Since a is the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Therefore,

$$\text{CosA} = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|}$$

$$\frac{(-\hat{i}-2\vec{j}-6\vec{k}).(\hat{i}-3\hat{j}-5\hat{k})}{\sqrt{(-1)^2+(-2)^2+(-6)^2}\sqrt{1^2+(-3)^2+(-5)^2}}$$

$$= \frac{-1+6+30}{\sqrt{1+4+36}\sqrt{1+9+25}} = \frac{35}{\sqrt{41}\sqrt{35}} = \sqrt{\frac{35}{41}}$$

$$A = \cos^{-1}\sqrt{\frac{35}{41}}, \cos B = \frac{\overrightarrow{BA}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$$

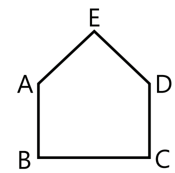
$$\frac{(\hat{i}+2\hat{j}+6\hat{k}).(2\hat{i}-\hat{j}+\hat{k})}{\sqrt{1^2+2^2+6^2}\sqrt{2^2+(-1)^2+(1)^2}}$$

$$\Rightarrow \cos B = \frac{2-2+6}{\sqrt{41}\sqrt{6}} = \sqrt{\frac{6}{41}} \Rightarrow B = \cos^{-1}\sqrt{\frac{6}{41}}$$

Example 2: If ABCDE is a pentagon, prove that the resultant of $\overrightarrow{AB}, \overrightarrow{AE}, \overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{ED}$ and $\overrightarrow{AC}$ is $3\overrightarrow{AC}$

Sol: By using resultant vector formula, we can obtain required result.

If R be the resultant vector then



$R = \overrightarrow{AB}+\overrightarrow{AE}+\overrightarrow{BC}+\overrightarrow{DC}+\overrightarrow{ED}+\overrightarrow{AC}$

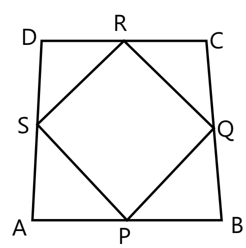
$= (\overrightarrow{AB}+\overrightarrow{BC})+(\overrightarrow{AE}+\overrightarrow{ED}+\overrightarrow{DC})+\overrightarrow{AC}$

$= \overrightarrow{AC}+\overrightarrow{AC}+\overrightarrow{AC} = 3\overrightarrow{AC}$

Example 3: Prove that the straight lines joining the mid points of the sides of a quadrilateral ABCD, taken in order, form a parallelogram.

Sol: Let the position vectors of A, B, C and D be $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$. Hence position vectors of P, Q, R and S (the mid points of AB, BC, CD & DA respectively) are

$\frac{\vec{a}+\vec{b}}{2}, \frac{\vec{b}+\vec{c}}{2}, \frac{\vec{c}+\vec{d}}{2}$ and $\frac{\vec{d}+\vec{a}}{2}$ respectively.



$\overrightarrow{PQ} = \frac{\vec{c}+\vec{b}}{2} - \frac{\vec{a}+\vec{b}}{2}$

$\overrightarrow{PQ} = PO + OQ = \frac{\vec{c}-\vec{a}}{2}$

$\overrightarrow{RS} = \frac{\vec{a}+\vec{d}}{2} - \frac{\vec{c}+\vec{d}}{2} = \frac{\vec{a}-\vec{c}}{2}$

$\Rightarrow \overrightarrow{SR} = \frac{\vec{c}-\vec{a}}{2}$

$\overrightarrow{PQ}$ is parallel and equal to $\overrightarrow{SR}$. Hence PQRS is a parallelogram.

Example 4: Write an equation for the plane that contains the points (2, 0, -3), (-4, -5, 2), and (0, 3, -4) in the form ax+by+cz = d.

Sol: Let $\vec{v} = (-4,-5,2)-(2,0,-3) = (-6,-5,5)$ and $\vec{\omega} = (0,3,-4)-(2,0,-3) = (-2,3,-1)$.

$\vec{v} \times \vec{\omega} = \vec{i}(5-15)-\vec{j}(6+10)+$

$\hat{k}(-18-10) = (-10,-16,-28)$

We can choose $\hat{n}$ to be any vector in the same direction as $\vec{v} \times \vec{\omega}$ so let $\hat{n}$ = (5, 8, 14). Then the plane has the form $5x+8y+14z=d$. Substituting the point (2,0,-3) for (x,y,z) and solving for d gives d=10+0+0(-42)=-32. So the plane has the equation $5x+8y+14z=-32$.

Example 5: Find a vector that is perpendicular to the vector (1, 2, 3) with the same length. Also, find a plane perpendicular to (1, 2, 3) that passes through the point (3, 2, 1).

Sol: By using formula of perpendicular vector we can obtain the result.

A vector $\vec{v}$ is perpendicular to (1, 2, 3) if $\vec{v}$.(1,2,3)=0. There are infinite number of possibilities to choose from, but one possible choice for $\vec{v}$ is (2,-1, 0). However, we want this vector to have length $\left\|(1,2,3)\right\| = \sqrt{14}$. Since $\left\|(2,-1,0)\right\| = \sqrt{5}$,we need to rescale our vector to be $\frac{\sqrt{14}}{\sqrt{5}}(2,-1,0)$.

If (1, 2, 3) is a normal vector to a plane then the plane will have the form x+2y+3z=d. since the plane passes through the point (3, 2, 1), we substitute these values for x, y, and z to get 3+4+3=10=d so our plane equation is $x+2y+3x=10$.

Example 6: Write an equation for the plane that contains the point (1, 0, 3) and the line (-3,-2,-2) + t(1, 2,-1) in the form ax+by+cz=d.

Sol: Since the plane contains the line (-3,-2,-2)+ t(1,2,-1) we know that one tangent vector to the plane is $\vec{v}$=(1,2,-1). We can get a second tangent vector by finding the vector between (-3,-2,-2) and (1, 0, 3).

So let $\vec{\omega}$=(4,2,5). Then

$\vec{v}\times\vec{\omega} = \vec{i}(10+2)-\vec{j}(5+4)+\vec{k}(2-8)$ =(12,-9,-6)

So we can choose $\vec{n}=(4,-3,-2)$ and our plane has the form 4x-3y-2z=d. Plugging in (1, 0, 3) for (x,y,z) and solving for d yields $4x-3y-2z=-2$

Example 7: Find the minimum distance between the point (3,-3,-3) and the plane 2x+y-z=3.

Sol: The point in the plane closest to (3,-3,-3) lies on a line that is perpendicular to the plane and passes through (3,-3,-3). Since (2, 1,-1) is a normal vector to the plane, we will use it as the direction of this line. Thus a parameterized form of the line is

$$c(t)=(3,-3,-3)+t(2,1,-1)=(3+2t,-3+t,,-3-t)$$

We substitute this into the plane equation to find its intersection with the plane and get:

$$2(3+2t)+1(-3+t)-(-3-t)=6+6t=3 \Rightarrow t=-\frac{1}{2}.$$

So the point in the plane closest to (3,-3,-3) is

$$c\left(-\frac{1}{2}\right)=\left(2,-\frac{7}{2},-\frac{5}{2}\right).$$

The distance between the point and the plane is thus

$$\sqrt{1^2+\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)^2}=\sqrt{\frac{3}{2}}.$$

Example 8: Determine if the three vectors $\vec{a}=(1,4,-7)$ $\vec{b}=(2,-1,4)$ and $\vec{c}=(0,-9,18)$ lie in the same plane or not.

Sol: Three vectors lies in the same plane if volume of the parallelepiped formed by these three vectors is zero.

So, as we noted prior to this example all we need to do is compute the volume of the parallelepiped formed by these three vectors. If the volume is zero, then they lie in the same plane and if the volume isn't zero they don't lie in the same plane.

$$\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)=\begin{vmatrix}1 & 4 & -7\\ 2 & -1 & 4\\ 0 & -9 & 18\end{vmatrix}=-18+126-144+36=0$$

So, the volume is zero and so they lie in the same plane.

JEE Advanced/Boards

Example 1: If O be the circumcenter; G, the centroid and H, the orthocenter of triangle ABC, prove that O, G, H are collinear and G divides OH in the ratio 1:2

Sol: Consider position vector of A, B, C be taken as $\vec{a}$, $\vec{b}$, $\vec{c}$. And then use geometry of triangle to solve this problem.

Let O, the circumcenter of the ΔABC be chosen as origin and position vector of A, B, C be taken $\vec{a}$, $\vec{b}$, $\vec{c}$. Hence position vector of G the centroid is

$$\overrightarrow{OG}=\frac{\vec{a}+\vec{b}+\vec{c}}{3} \quad \text{... (i)}$$

Since O is circumcenter

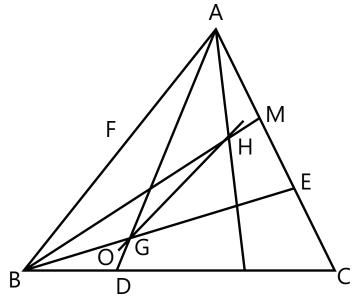
$$\therefore \overrightarrow{OA}=\overrightarrow{OB}=\overrightarrow{OC}=\overrightarrow{OA}^2=\overrightarrow{OB}^2=\overrightarrow{OC}^2 \text{ or } a^2=b^2=c^2$$

$$a^2-b^2=0,\ b^2-c^2=0,\ c^2-a^2=0$$

Or $(\vec{a}+\vec{b}).(\vec{a}-\vec{c})=0$

Or $\left(\vec{a}+\vec{b}+\vec{c}-\vec{c}\right).\left(\vec{a}-\vec{b}\right)=0$ ... (ii)

Let P be the point whose position vector is



$\vec{a}+\vec{b}+\vec{c}$ $\therefore \left(\overrightarrow{OP}-\overrightarrow{OC}\right).\left(\overrightarrow{OA}-\overrightarrow{OB}\right)=0$

Or $\overrightarrow{CP}\perp\overrightarrow{BA}$

In similar manner we can show that BP is perpendicular to AC and AP is perpendicular to CB.

Hence P is the orthocentre which is H.

$\overrightarrow{OP}=\overrightarrow{OH}=\vec{a}+\vec{b}+\vec{c}=3\overrightarrow{OG}$ ... (iii)

$\therefore \overrightarrow{OH}=3\overrightarrow{OG}$ or $\overrightarrow{GH}=2.\overrightarrow{OG}$ or $\dfrac{\overrightarrow{OG}}{\overrightarrow{GH}}=\dfrac{1}{2}$

Above show that O, G, H are collinear and G divides OH in the ratio 1:2

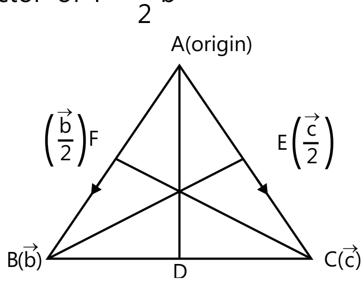
Example 2: Prove using vectors: If two medians of a triangle are equal, then it is isosceles.

Sol: Using mid – point formula of vector, we can solve this

Let ABC be a triangle and let BE and CF be two equal medians. Taking A as the origin, let the position vectors of B and C be $\vec{b}$ and $\vec{c}$ respectively. Then

Position vector of $E=\frac{1}{2}\vec{c}$ and

Position vector of $F=\frac{1}{2}\vec{b}$



$\therefore \overrightarrow{BE}=\frac{1}{2}\left(\vec{c}-2\vec{b}\right),\overrightarrow{CF}=\frac{1}{2}\left(\vec{b}-2\vec{c}\right)$

Now, $BE=CF=\left|\overrightarrow{BE}\right|=\left|\overrightarrow{CF}\right|$

$\Rightarrow \left|\overrightarrow{BE}\right|^2=\left|\overrightarrow{CF}\right|^2 \Rightarrow \left|\frac{1}{2}\left(\vec{c}-2\vec{b}\right)\right|^2=\left|\frac{1}{2}\left(\vec{b}-2\vec{c}\right)\right|^2$

$\Rightarrow \frac{1}{4}\left|\vec{c}-2\vec{b}\right|^2=\frac{1}{4}\left|\vec{b}-2\vec{c}\right|^2 \Rightarrow \left|\left(\vec{c}-2\vec{b}\right)\right|^2=\left|\left(\vec{b}-2\vec{c}\right)\right|^2$

$\Rightarrow \left(\vec{c}-2\vec{b}\right)\cdot\left(\vec{c}-2\vec{b}\right)=\left(\vec{b}-2\vec{c}\right)\cdot\left(\vec{b}-2\vec{c}\right)$

$\Rightarrow \left|\vec{c}\right|^2-4\vec{b}.\vec{c}+4\left|\vec{b}\right|^2=\left|\vec{b}\right|^2-4\vec{b}.\vec{c}+4\left|\vec{c}\right|^2$

$\Rightarrow 3\left|\vec{b}\right|^2=3\left|\vec{c}\right|^2 \Rightarrow \left|\vec{b}\right|^2=\left|\vec{c}\right|^2$

$\Rightarrow AB=AC$

Hence, triangle ABC is an isosceles triangle.

Example 3: D, E, F are points dividing side $\overrightarrow{BC}$, $\overrightarrow{CA}$, $\overrightarrow{AB}$ of a triangle ABC in the ratio 2:3, 1:2 and 3:1 respectively. Show that the lines $\overrightarrow{AD}$, $\overrightarrow{BE}$, $\overrightarrow{CF}$ are concurrent and hence find the position vector of their point of intersection.

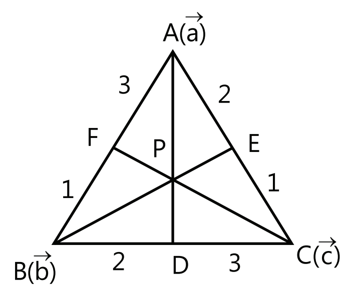
Sol: By using section formula we can obtain required result.

If $\vec{d}$, $\vec{e}$, $\vec{f}$ are position vector of points D, E & F respectively then, by section formula

$\vec{d}=\dfrac{2\vec{c}+3\vec{b}}{5}$ .... (i)

$\vec{e}=\dfrac{2\vec{c}+\vec{a}}{3}$ ....(ii)

$\vec{f}=\dfrac{3\vec{b}+\vec{a}}{4}$ ...(iii)



Equation of line AD is $\vec{r}=\vec{a}+t\left(\vec{d}-\vec{a}\right)$

Equation of line BE is $\vec{r}=\vec{b}+m\left(\vec{e}-\vec{b}\right)$

For intersection of $\overrightarrow{AD}$ and $\overrightarrow{BE}$ we need that

"$\vec{a}+t\left(\vec{d}-\vec{a}\right)=\vec{b}+m\left(\vec{e}-\vec{b}\right)$" be true for some 0<t, m<1.

$$\vec{a}+t\frac{\left(2\vec{c}+3\vec{b}-5\vec{a}\right)}{5}=\vec{b}+m\frac{\left(2\vec{c}+\vec{a}-3\vec{b}\right)}{3}$$

$\therefore\ 1-t=\frac{m}{3}\ ;\ \frac{3t}{5}=1-m\ ;\ \frac{2t}{5}=\frac{2m}{3}$

$\therefore\ t=\frac{5}{6},\ m=\frac{1}{2}$

The existence of t and m assures the intersection of $\overrightarrow{AD}$ and $\overrightarrow{BE}$.

The point of intersection is

$$\vec{r}=\vec{a}+\frac{5}{6}\left(\vec{d}-\vec{a}\right)=\frac{\left(\vec{a}+3\vec{b}+2\vec{c}\right)}{6}$$

Example 4: Find a parametric form for the line passing through the point (1,2) in the direction (3,4), which we will call $c_1(t)$. Set $c_1(t)$ equal to (x,y) and eliminate t to get the line into y = mx + b form. Now find a different parametrization $c_2(t)$ of the same line such that $c_2(0)$ =(-2,-2) and $c_2(2)$= (-5,-6).

Sol: $c_1(t)$ = (1,2) + t (3, 4) = (1 + 3t, 2 + 4t). Setting (x, y) = (1 + 3t, 2 + 4t) yields x = 1 + 3t and y = 2 + 4t.

Solving the former equation for t yields t = (x-1)/3. Substituting this into the second equation then gives us $y=\frac{4}{3}x+\frac{2}{3}$.

Let $c_2(t)=p+t\vec{v}$. c_2 will then be a parameterization of the same line given by c_1 if p is a point on the same line and $\vec{v}$ is in the same direction as (3,4) (i.e. some scalar multiple of (3,4)). Since $c_2(0)$ = (-2,-2) we will choose p= (-2,-2) (you can check that this point indeed lies on the line parameterized by c_1). Then

$c_2(2)=(-2,-2)+2\vec{v}=(-5,-6)$, so we get that

$\vec{v}=(-3/2,-2)$, which is indeed a scalar multiple of (3,4). So

$c_2(t)=(-2,-2)+t\left(\frac{-3}{2},-2\right)$ is a different parameterization of the line parameterized by c_1.

Example 5: Find the vector projection of (3, 2) onto (-1,-1). Then find the area of the triangle with one side vector (3, 2) and another side the result of this projection.

Sol: Use projection method to obtain vector projection of (3, 2) and area of triangle will be half of the area of parallelogram.

$$\text{proj}_{(-1,-1)}(3,2)=\frac{-5}{2}(-1,-1)=\left(\frac{5}{2},\frac{5}{2}\right).$$

Then the area of the triangle with sides (3, 2) and $\left(\frac{5}{2},\frac{5}{2}\right)$ is one half the area of the parallelogram with sides (3, 2) and $\left(\frac{5}{2},\frac{5}{2}\right)$. So, the area of the triangle is

$$\frac{1}{2}\left\|(3,2,0)X\left(\frac{5}{2},\frac{5}{2},0\right)\right\|=\frac{1}{2}\left\|\left(0,0,\frac{5}{2}\right)\right\|=\boxed{\frac{5}{4}}.$$

Example 6: Find the minimum distance between the point (4, 2,-3) and the line (1, 0, 2) + t (-1,-1, 2).

Sol: Let $\vec{v}(t)$ represents the vector from the point (4,2,-3) and line (1,0,2) + t(-1,-1,2) = (1-t,-t,2+2t) at any $t\in\mathbb{R}$.

So, $\overrightarrow{v(t)}$ = (4, 2,-3)-(1-t,-t, 2+2t) = (3+t, 2+t,-5-2t). We want to find the t such that $\vec{v}(t)$ is perpendicular to the line, which is when $\overrightarrow{v(t)}$. (-1,-1, 2) = 0.

(3 + t, 2 + t, -5 - 2t) . (-1, -1, 2) = -15 - 6t = 0

$\Rightarrow\ t=-\frac{5}{2}$. So the length of $\vec{v}\left(-\frac{5}{2}\right)$ should represent the minimum distance from (4, 2,-3) and the line.

$$\left\|\vec{v}\left(-5/2\right)\right\|=\left\|(1/2,-1/2,0)\right\|=\frac{1}{\sqrt{2}}.$$

Example 7: Prove that, in any triangle ABC

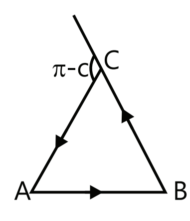
(i) $c^2=a^2+b^2-2ac\cos C$ (ii) $c=b\cos A+a\cos B$

Sol: By using simple scalar product method we can prove given relation.

(i) In ΔABC, $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0$

or, $\overrightarrow{BC}+\overrightarrow{CA}=-\overrightarrow{AB}$ ....(i)

Squaring both sides



$$\left(\overrightarrow{BC}\right)^2+\left(\overrightarrow{CA}\right)^2+2\left(\overrightarrow{BC}\right).\left(\overrightarrow{CA}\right)=\left(\overrightarrow{AB}\right)^2$$

$$\Rightarrow a^2+b^2+2\left(\overrightarrow{BC}.\overrightarrow{CA}\right)=c^2$$

$$\Rightarrow c^2=a^2+b^2=2ab\cos\left(\pi-C\right)$$

$$\Rightarrow c^2=a^2+b^2-2ab\ \cos C$$

(ii) $\left(\overrightarrow{BC}+\overrightarrow{CA}.\right).\overrightarrow{AB}=-\overrightarrow{AB}.\overrightarrow{AB}\Rightarrow\overrightarrow{BC}.\overrightarrow{AB}+\overrightarrow{CA}.\overrightarrow{AB}$

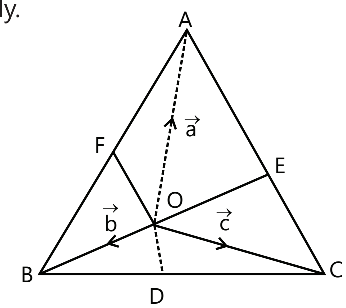
$=-c^2-ac\cos B-bc\cos A$

$\Rightarrow a\cos B+b\cos A=c$

Example 8: In any triangle, show that the perpendicular bisectors of the sides are concurrent.

Sol: By using formula of Dot product and Mid – point we can solve this problem.

Let ABC be the triangle and D, E and F are respectively middle points of sides $\overrightarrow{BC}$ $\overrightarrow{CA}$ and $\overrightarrow{AB}$. Let the perpendicular through D and E meet of O join $\overrightarrow{OF}$. We are required to prove that $\overrightarrow{OF}$ is $\perp$ to $\overrightarrow{AB}$. Let the position vectors of A, B, C with O as origin of reference be $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.



$\therefore \overrightarrow{OD} = \frac{1}{2}(\vec{b}+\vec{c})$,

$\overrightarrow{OE} = \frac{1}{2}(\vec{c}+\vec{a})$,

and $\overrightarrow{OF} = \frac{1}{2}(\vec{a}+\vec{b})$

Also

$\overrightarrow{BC} = \vec{c}-\vec{b}$, $\overrightarrow{CA} = \vec{a}-\vec{c}$ and $\overrightarrow{AB} = \vec{b}-\vec{a}$

Since, $\overrightarrow{OD} \perp \overrightarrow{BC}$, $1/2(\vec{b}+\vec{c}).(\vec{c}-\vec{b}) = 0$

$\Rightarrow b^2 = c^2$ ... (i)

Similarly, $\overrightarrow{OE} \perp \overrightarrow{CA}$, $1/2(\vec{a}+\vec{c}).(a-\vec{c}) = 0$

$\Rightarrow a^2 = c^2$ ... (ii)

From (i) and (ii) we have $b^2 - a^2 = 0$

$(\vec{a}+\vec{b}).(\vec{b}+\vec{a}) = 0 \Rightarrow 1/2(\vec{b}+\vec{a}).(\vec{b}-\vec{a}) = 0$

$\Rightarrow \overrightarrow{OF} \perp \overrightarrow{AB}$

Hence proved.

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Exercise 1

Q.1 The line L_1 passes through the points (2,-3, 1) and (-1,-2,-4). The line L_2 passes through the point (3, 2,-9) and is parallel to the vector $4\hat{i}-4\hat{j}+5\hat{k}$.

(i) Find an equation for L_1 in the form $\vec{r} = \vec{a}+t\vec{b}$

(ii) Prove that L_1 and L_2 are skew.

Q.2 Two lines have vector equations

$$\vec{r} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} + t\begin{pmatrix} -8 \\ 1 \\ -2 \end{pmatrix} \text{ and } \vec{r} = \begin{pmatrix} -2 \\ a \\ -2 \end{pmatrix} + s\begin{pmatrix} -9 \\ 2 \\ -5 \end{pmatrix},$$

Where 'a' is a constant.

(i) Calculate the acute angle between the lines.

(ii) Given that these two lines intersect, find the point of intersection.

Q.3 The points A and B have position vectors $\vec{a}$ and $\vec{b}$ relative to an origin O, where $\vec{a} = 4\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{b} = -7\hat{i}+5\hat{j}+4\hat{k}$

(i) Find the length of AB.

(ii) Use a scalar product to find angle OAB.

Q.4 The position vectors of the points P and Q with respect to an origin O are $5\hat{i}+2\hat{j}-9\hat{k}$ and $4\hat{i}+4\hat{j}-6\hat{k}$ respectively.

(i) Find the vector equation for the line PQ

The position vector of the point T is $\hat{i}+2\hat{j}-\hat{k}$

(ii) Write down a vector equation for the line OT and show that OT is perpendicular to PQ.

It is given that OT intersects PQ.

(iii) Find the position vector of the point of intersection of OT and PQ.

(iv) Hence find the perpendicular distance from O to PQ, giving your answer in an exact form.

Q.5 ABCD is a parallelogram. The position vectors of A, B and C are given respectively by

$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j}$, $\vec{c} = \hat{i} - \hat{j} - 2\hat{k}$

(i) Find the position vector of D.

(ii) Determine, to the nearest degree, the angle ABC.

Q.6 The position vectors of three points A, B and C relative to an origin O are given respectively by $\overrightarrow{OA} = 7\hat{i} + 3\hat{j} - 3\hat{k}$, $\overrightarrow{OB} = 4\hat{i} + 2\hat{j} - 4\hat{k}$, $\overrightarrow{OC} = 5\hat{i} + 4\hat{j} - 5\hat{k}$.

(i) Find the angle between AB and AC.

(ii) Find the area of triangle ABC.

Q.7 Two lines have vector equations

$\vec{r} = \hat{i} - 2\hat{j} + 4\hat{k} + \lambda\left(3\hat{i} + \hat{j} + a\hat{k}\right)$ and

$\vec{r} = -8\hat{i} + 2\hat{j} + 3\hat{k} + \mu(\hat{i} - 2\hat{j} - \hat{k})$,

Where 'a' is a constant

(i) Given that the lines are skew, find the value that a cannot take.

(ii) Given instead that the lines intersect, find the point of intersection.

Q.8 Lines, L_1, L_2 and L_3 have vector equations

$L_1 : \vec{r} = (5\hat{i} - \hat{j} - 2\hat{k}) + s(-6\hat{i} + 8\hat{j} - 2\hat{k})$,

$L_2 : \vec{r} = (3\hat{i} - 8\hat{j}) + t(\hat{i} + 3\hat{j} + 2\hat{k})$

$L_3 : \vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + u(3\hat{i} + c\hat{j} + \hat{k})$.

(i) Calculate the acute angle between L_1 and L_2.

(ii) Given that L_1 and L_3 are parallel, find the value of c.

(iii) Given that L_2 and L_3 intersect, find the value of c

Q.9 Given that $\vec{u} = \hat{i} - 2\hat{j} + 3\hat{k}$;

$\vec{v} = 2\hat{i} + \hat{j} + 4\hat{k}$; $\vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$ and

$(\vec{u}\cdot\vec{R} - 10)\,\hat{i} + (\vec{v}\vec{R} - 20)\,\hat{j} + (\vec{w}\cdot\vec{R} - 20)\hat{k} = 0$.

Find the unknown vector $\vec{R}$.

Q.10 The base vectors $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$ are given in terms of base vectors $\vec{b}_1$, $\vec{b}_2$, $\vec{b}_3$ as,

$\vec{a}_1 = 2\vec{b}_1 + 3\vec{b}_2 - \vec{b}_3$;

$\vec{a}_2 = \vec{b}_1 - 2\vec{b}_2 + 2\vec{b}_3$ & $\vec{a}_3 = -2\vec{b}_1 + \vec{b}_2 - 2\vec{b}_3$.

If $\vec{F} = 3\vec{b}_1 - \vec{b}_2 + 2\vec{b}_3$. then express $\vec{F}$ in terms of $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$.

Q.11 If $\vec{r}$ and $\vec{s}$ are non zero constant vectors and the scalar b is chosen such that $\left|\vec{r} + \overrightarrow{bs}\right|$ is minimum, then show that the value of $\left|b\vec{s}\right|^2 + \left|\vec{r} + \overrightarrow{bs}\right|$ is equal to $|r|^2$.

Exercise 2

Single Correct Choice Type

Q.1 Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors such that $\left|\vec{a}\right| = \left|\vec{c}\right| = 1$; $\left|\vec{b}\right| = 4$ and $\left|\vec{b} \times \vec{c}\right| = \sqrt{15}$. If $\vec{b} - 2\vec{c} = \lambda\vec{a}$ then a value of λ is

(A) 1 (B) -1 (C) 2 (D) -4

Q.2 Vector $\vec{r}$ which is equally inclined to coordinate axes such that $\left|\vec{r}\right| = 15\sqrt{3}$ is

(A) $\hat{i} + \hat{j} + \hat{k}$ (B) $15\left(\hat{i} + \hat{j} + \hat{k}\right)$

(C) $7\left(\hat{i} + \hat{j} + \hat{k}\right)$ (D) None of these

Q.3 For 3 vectors $\vec{u}, \vec{v}, \vec{w}$, which of the following expressions is ≠ to any remaining three.

(A) $\vec{u}.\left(\vec{v} \times \vec{w}\right)$ (B) $\left(\vec{v} \times \vec{w}\right).\vec{u}$

(C) $\vec{v}.\left(\vec{u} \times \vec{w}\right)$ (D) $\left(\vec{w} \times \vec{u}\right).\vec{v}$

Q.4 If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5$ & $\left|\vec{c}\right| = 7$ then $\angle\theta$ between $\vec{a}$ and $\vec{b}$ is

(A) 40° (B) 30° (C) 150° (D) None of these

Q.5 If 2 out of 3 vectors $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) + 3 = 0$, then third vector is length-

(A) 3 (B) 1 (C) 2 (D) None of these

Q.6 Let $\vec{a} + \vec{b}$ is orthogonal to $\vec{b}$ and $\vec{a} + 2\vec{b}$ is orthogonal to $\vec{a}$, then

(A) $\left|\vec{a}\right| = \sqrt{2}\left|\vec{b}\right|$ (B) $\left|\vec{a}\right| = 2\left|\vec{b}\right|$

(C) $\left|\vec{a}\right| = \left|\vec{b}\right|$ (D) $2\left|\vec{a}\right| = \left|\vec{b}\right|$

Q.7 Magnitude of projection of vector $\hat{i}+2\hat{j}+\hat{k}$ on vector $4\hat{i}+4\hat{j}+7\hat{k}$ is

(A) 3 (B) $3\sqrt{6}$ (C) $\sqrt{6}/3$ (D) None of these

Q.8 Magnitude of moment of force $-2\hat{i}+6\hat{j}-8\hat{k}$ acting at point $2\hat{i}-\hat{j}+3\hat{k}$ about point $\hat{i}+2\hat{j}-\hat{k}$

(A) $\sqrt{211}$ (B) 0 (C) $\sqrt{54}$ (D) None of these

Q.9 If $\hat{a}$ & $\hat{b}$ are unit vectors represented by $\vec{O}A$ and $\vec{O}B$, then unit vector along bisector of $\angle AOB$ is scalar multiple of

(A) $\hat{a}-\hat{b}$ (B) $\hat{a}\times\hat{b}$ (C) $\hat{b}\times\hat{a}$ (D) None of these

Q.10 If $\left[2\vec{a}+4\vec{b}\ \ \vec{c}\ \ \vec{d}\right]=\lambda\left[\vec{a}\ \ \vec{c}\ \ \vec{d}\right]+\mu\left[\vec{b}\ \ \vec{c}\ \ \vec{d}\right]$ then $\lambda+\mu$ =

(A) 6 (B) -6 (C) 10 (D) None of these

Previous Years' Questions

Q.1 The volume of the parallelepiped whose sides are given by $\overrightarrow{OA}=2\hat{i}-3\hat{j}$, $\overrightarrow{OB}=\hat{i}+\hat{j}-\hat{k}$, $\overrightarrow{OC}=3\hat{i}-\hat{k}$, is ***(1983)***

(A) $\frac{4}{13}$ (B) 4 (C) $\frac{2}{7}$ (D) None of these

Q.2 A vector $\vec{a}$ has components 2p and 1 with respect to a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to the new system, $\vec{a}$ has components p+1 and l, then ***(1986)***

(A) p = 0 (B) p=1 or p=$-\frac{1}{3}$

(C) p = -1 or p = $\frac{1}{3}$ (D) p = 1 or p = -1

Q.3 Let a ,b, c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is ***(1993)***

(A) The Arithmetic Mean of a and b.

(B) The Geometric Mean of a and b.

(C) The Harmonic Mean of a and b.

(D) Equal to zero.

Q.4 If $\vec{a},\vec{b},\vec{c}$ are non-coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{(\vec{b}+\vec{c})}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is ***(1995)***

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

Q.5 If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is ***(2002)***

(A) 45° (B) 60° (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

Q.6 Let $\vec{V}=2\vec{i}+\vec{j}-\vec{k}$ and $\vec{W}=\vec{i}+3\vec{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is ***(2002)***

(A) -1 (B) $\sqrt{10}+\sqrt{6}$ (C) $\sqrt{59}$ (D) $\sqrt{60}$

Q.7 The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is ***(2004)***

(A) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ (B) $\frac{2\vec{i}-3\vec{j}}{\sqrt{13}}$

(C) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (D) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

Q.8 Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by ***(2010)***

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Q.9 Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by ***(2011)***

Q.10 Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)-\frac{\sqrt{3}}{2}\left(\vec{b}+\vec{c}\right)$. If $\vec{b}$ is not parallel to $\vec{c}$ then the angle between $\vec{a}$ and $\vec{b}$ is: ***(2016)***

(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

Q.11 Let $\vec{a}.\ \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $\left(\vec{a}.\times\vec{b}\right)\times\vec{c}-\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$ If θ is the angle between vectors $\vec{b}$ and $\left|\vec{c}\right|$ then a value of $\sin\theta$ is ***(2015)***

(A) $\frac{-\sqrt{2}}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{2\sqrt{3}}{3}$ (D) $\frac{2\sqrt{3}}{3}$

Q.12 If $\left[\vec{a}\times\vec{b}\vec{b}\times\vec{c}\times\vec{a}\right]=\lambda\left[\vec{a}\vec{b}\vec{c}\right]^2$ then λ is equal to ***(2014)***

(A) 1 (B) 3 (C) 0 (D) 1

Q.13 If the vectors $\overrightarrow{AB}=3\hat{j}+4\hat{k}$ and $\overrightarrow{AC}=2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is ***(2013)***

(A) $\sqrt{72}$ (B) $\sqrt{33}$ (C) $\sqrt{45}$ (D) $\sqrt{18}$

Q.14 Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c}=\vec{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular to each other then the angle between $\hat{a}$ and $\hat{b}$ is ***(2012)***

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Q.15 If the vectors $\vec{a}=\hat{i}-\hat{j}+\hat{2}+\hat{4}j+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal then (λ,μ) = ***(2010)***

(A) (2, –3) (B) (–2, 3) (C) (3, –2) (D) (–3, 2)

Q.16 Let $\vec{a}=\hat{j}-\hat{k}$. Then vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c}=\vec{0}$ and $\vec{a}.\vec{b}$ is ***(2010)***

(A) $2\hat{i}-\hat{j}+2\hat{k}$ (B) $\hat{i}-\hat{j}+2\hat{k}$

(C) $\hat{i}+\hat{j}-2\hat{k}$ (D) $-\hat{i}+\hat{j}-2\hat{k}$

Q.17 If $\vec{a}=\frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $\vec{b}=\frac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$, then the value of $\left(2\vec{a}-\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right]$ is ***(2011)***

(A) -3 (B) 5 (C) 3 (D) -5

Q.18 The vector $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{a}$ and $\vec{b}$ are two vectors satisfying: $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}.\vec{d}=0$. Then the vector is equal to ***(2011)***

(A) $\vec{c}+\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (B) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

(C) $\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (D) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

JEE Advanced/Boards

Exercise 1

Q.1 What will be the value of $\frac{\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}\bullet\vec{b}\right)^2}{2\vec{a}^{2}\vec{b}^{2}}$?

Q.2 What will be the area of the triangle determined by the vectors 3i+4j and -5i+7j?

Q.3 What will be the value of a if points whose position vectors are $60\hat{i}+3\hat{j},\ 40\hat{i}-8\hat{j},\ a\hat{i}-52\hat{j}$ are collinear?

Q.4 What will be the angle between diagonals which adjacent sides of *ll*gm are along $\vec{a}=\hat{i}+2\hat{j}$ & $\vec{b}=2\hat{i}+\hat{j}$?

Q.5 What will be the angle between $\vec{a}$ and $\vec{b}$ if $\vec{a}$ & $\vec{b}$ are unit vectors such that $\vec{a}+3\vec{b}$ is $\perp$ to $7\vec{a}-5\vec{b}$?

Q. 6 If the unit vectors $\hat{A}$ and $\hat{B}$ are inclined at π then what will be the value of $\left|\hat{A}-\hat{B}\right|/2$?

Q.7 A particle acted upon by forces $3\hat{i}+2\hat{j}+5\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$ is displaced from a point P to a point Q whose respective position vectors are $2\hat{i}+\hat{j}+3\hat{k}$ and $4\hat{i}+3\hat{j}+7\hat{k}$. What will be the work done by the force?

Q.8 A force F = $6\hat{i}+\lambda\hat{j}+4\hat{k}$ acting on a particle displaces it from A (3,4,5) to B(1,1,1). If the work done is 2 units, then What will be the value of λ?

Q.9 What will be the length of longer diagonal of *ll*gm constructed on $5\vec{a}+2\vec{b}$ & $\vec{a}-3\vec{b}$. Given $|\vec{b}|=3$ & $|\vec{a}|=2\sqrt{2}$ and angle between $\vec{a}$ & $\vec{b}$ is $\pi/4$

Q.10 The vectors $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i}+(4x-2)\hat{j}+2\hat{k}$. Find x ?

Exercise 2

Single Correct Choice Type

Q.1 Moment of couple formed by forces $5\hat{i}+\hat{j}$ & $-5\hat{i}+\hat{j}$ acting at [9,-1, 2] and [3,-2, 1]

(A) $-\hat{i}+5\hat{j}+\hat{k}$ (B) $\hat{i}-\hat{j}-5\hat{k}$

(C) $2\hat{i}-2\hat{j}-10\hat{k}$ (D) $-2\hat{i}-2\hat{j}+10\hat{k}$

Q.2 Let $\vec{a},\vec{b},\vec{c}$ be vectors such that

$\vec{a}.(\vec{b}+\vec{c})+\vec{b}.(\vec{c}+\vec{a})+\vec{c}.(\vec{a}+\vec{b})=0$ and

$|a|=1, |b|=4, |c|=8$ then $|\vec{a}+\vec{b}+\vec{c}|$ equals

(A) 13 (B) 81 (C) 9 (D) None of these

Q.3 Position vectors of A and B are $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$. Length of internal bisector of $\angle BOA$ of ΔAOB is

(A) $\sqrt{\frac{136}{9}}$ (B) $\sqrt{\frac{136}{9}}$ (C) $\frac{20}{3}$ (D) None of these

Q.4 If $\vec{a}+2\vec{b}+3\vec{c}=0$, then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is equal to

(A) $6(\vec{b}\times\vec{c})$ (B) $6(\vec{a}\times\vec{b})$

(C) $6(\vec{c}\times\vec{a})$ (D) None of these

Q.5 Value of $|\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}|$ where $|\vec{a}|=1, |\vec{b}|=2$, and $|\vec{c}|=3$ is

(A) 1 (B) -6 (C) 0 (D) None of these

Q.6 If P and Q be two given points on the curve y=x+1/x such that OP.I = 1 and OQ.I = -1 where I is a unit vector along the x-axis, then the length of vector 2OP +3OQ is

(A) $5\sqrt{5}$ (B) $3\sqrt{5}$ (C) $2\sqrt{5}$ (D) $\sqrt{5}$

Q.7 Let A, B, C be three vectors such that A (B + C) + B. C = 0 and lAl = 1, lBl = 4, lCl =8, then lA+B+Cl equals

(A) 13 (B) 81 (C) 9 (D) 5

Q.8 If the unit vectors $\hat{A}$ and $\hat{B}$ are inclined at an angle 2θ and $|\hat{A}-\hat{B}|\le 1$, then for $\theta\in[0,\pi], \theta$, may lie in the interval

(A) $[\pi/6, \pi/3]$ (B) $[\pi/6, \pi/2]$

(C) $[5\pi/6, \pi]$ (D) $[\pi/2, 5\pi/6]$

Q.9 If unit vectors $\hat{A}$ and $\hat{B}$ such that STP $|\hat{A}\ \hat{B}\ \hat{A}\times\hat{B}|=1/4$ then $\hat{A}$ and $\hat{B}$ are inclined

(A) $\pi/6$ (B) $\pi/2$ (C) $\pi/3$ (D) $\pi/4$

Q.10 If $\hat{A}$ and $\hat{B}$ unit vectors then greatest value of $|\hat{A}-\hat{B}|+|\hat{A}-\hat{B}|$ is

(A) 2 (B) 4 (C) $2\sqrt{2}$ (D) $\sqrt{2}$

Previous Years' Questions

Q.1 (i) If C be a given non zero scalar and $\vec{A}$ and $\vec{B}$ be given non-zero vectors such that $\vec{A}\perp\vec{B}$, find the vector $\vec{X}$ which satisfies the equation $\vec{A}\bullet\vec{X}=c$ and $\vec{A}\times\vec{X}=\vec{B}$.

(ii) $\vec{A}$ vector A has components A_1, A_2, A_3 in a right-handed rectangular Cartesian coordinate system oxyz. The coordinate system is rotated about the x-axis through an angle $\frac{\pi}{2}$. Find the components of A in the new coordinate system, in terms of A_1, A_2, A_3. ***(1983)***

Q.2 If vectors $\vec{a},\vec{b},\vec{c}$ are coplanar, show that

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \end{vmatrix} = \vec{0}$$ ***(1989)***

Q.3 Let $\vec{A}=2\hat{i}+\hat{k}$, $\vec{B}=\hat{i}+\hat{j}+\hat{k}$, and $\vec{C}=4\hat{i}-3\hat{j}+7\hat{k}$. Determine a vector $\vec{R}$ satisfying $\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$ and $\vec{R}\cdot\vec{A}=0$. ***(1990)***

Q.4 If $\vec{a},\vec{b},\vec{c},\vec{d}$ are four distinct vectors satisfying the conditions $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ then prove that $\vec{a}\cdot\vec{b}+\vec{c}\cdot\vec{d}\neq\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{d}$. ***(2004)***

Q.5 Incident ray is along the unit vector $\hat{v}$ and the reflected ray is along the unit vector . The normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$. ***(2005)***

Q.6 Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is ***(2006)***

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

Q.7 The vector(s) which is /are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, are perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are ***(2011)***

(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

Q.8 If $\vec{a}$ and $\vec{b}$ are vectors in space by $\vec{a}=\frac{i-2j}{\sqrt{5}}$ and $\vec{b}=\frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a}+\vec{b}\right)\cdot\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$ is ***(2010)***

Q.9 Let $\vec{a}=-\hat{i}-\hat{k}$, $\vec{b}=-\hat{i}+\hat{j}$, and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$, $\vec{r}\cdot\vec{a}=0$, then the value of $\vec{r}\cdot\vec{b}$ is...... ***(2011)***

Q.10 Let $\hat{u}=u_1\hat{i}+u_2\hat{j}+u_3\hat{k}$ be a unit vector in R^2 and $\hat{w}=\frac{1}{\sqrt{6}}\left(\hat{i}+\hat{j}+2\hat{k}\right)$. Given that there exists a vector $\vec{v}$ in R^2 such that $\left|\hat{u}\times\vec{v}\right|=1$ and $\hat{w}=\left(\hat{u}\times\vec{v}\right)=1$. Which of the following statements (s)is (are) correct? ***(2016)***

(A) There is exactly one choice for such $\vec{v}$

(B) There are infinitely many choices for such $\vec{v}$

(C) If $\hat{u}$ lies in the xy-plane then $|u_1|=|u_2|$

(D) If $\hat{u}$ lies in the xz-plane then $2|u_1|=|u_3|$

Q.11 Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}$, $\vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12$, $|\vec{b}|=4\sqrt{3}$ and $\vec{b}\cdot\vec{c}=24$, then which of the following is (are) true ***(2015)***

(A) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=12$

(B) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=30$

(C) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$

(D) $\vec{a}\cdot\vec{b}=-72$

Q.12 Let $\vec{a}=-\hat{i}-\hat{k}$, $\vec{b}=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$ then the value of $\vec{r}.\vec{b}$ is ***(2011)***

Q.13 If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a}=\frac{\hat{i}-2\hat{j}}{\sqrt{5}}\vec{b}$ and $\vec{b}=\frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$ then the value of $\left(2\vec{a}+\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$ $\left(2\vec{a}+\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$ is ***(2010)***

Q.14 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$ If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$ where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is ***(2014)***

Q.15 Let $\overrightarrow{PR}=3\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{SQ}=\hat{i}-3\hat{j}-4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT}=\hat{i}+2\hat{j}-3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT},\overrightarrow{PQ}$ and $\overrightarrow{PS}$ is ***(2013)***

(A) 5 (B) 20 (C) 10 (D) 30

Q.16 A line I passing through the origin is perpendicular to the lines

$\ell_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k} \; -\infty < t < \infty$

$\ell_2 : (3+2x)\hat{j} + (3+2s)\hat{j} + (2+s)\hat{k}, -\infty < s < \infty$

Then, the coordinate (s) of the point(s) on ℓ_2 at a distance of $\sqrt{17}$ from the point of intersection of ℓ and ℓ_2 is (are) ***(2013)***

(A) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$ (B) $(-1, -1, 0)$

(C) $(1, 1, 1)$ (D) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

Q.17 Consider the set of eight vectors $V = \left\{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\right\}$. Three non-coplanar vectors can be chosen from V in 2^n ways. Then p is ***(2013)***

Q.18 Match list I with list II and select the correct answer using the code given below the lists : ***(2013)***

List I	List II
(i) Volume of parallelepiped by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c})$ and $(\vec{c} \times \vec{a})$ is	(p) 100
(ii) Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a} + \vec{b}), (\vec{b} \_ + \vec{c})$ and $2(\vec{c} + \vec{a})$ is	(q) 30
(iii) Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(\overrightarrow{2a} + \overrightarrow{3b})$ and $(\vec{a} - \vec{b})$ is	(r) 24
(iv) Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is	(s) 60

(A) i → s, ii → q, iii → r, iv → p

(B) i → q, ii → r, iii → p, iv → s

(C) i → r, ii → s, iii → p, iv → q

(D) i → p, ii → s, iii → r, iv → q

Q.19 If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a}, -\vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}^{2}\right| = 9$ then $\left|2\vec{a} + 5\vec{b} + 5\vec{c}\right|$ is ***(2012)***

Q.20 If $\vec{a}$ and $\vec{b}$ are vectors such that $\left|\vec{a} + \vec{b}\right| = \sqrt{29}$ and $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$ then a possible value of $(\vec{a} + \vec{b}).(-7\hat{i} + 2\hat{j} + 3\hat{k})$ is ***(2012)***

(A) 0 (B) 3 (C) 4 (D) 8

Q.21 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by ***(2011)***

(A) $\hat{i} + 3\hat{j} + 3\hat{k}$ (B) $-3\hat{i} - 3\hat{j} + \hat{k}$

(C) $3\hat{i} - \hat{j} + 3\hat{k}$ (D) $\hat{i} - 3\hat{j} - 3\hat{k}$

Q.22 The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is/are to ***(2011)***

(A) $\hat{j} - \hat{k}$ (B) $-\hat{i} + \hat{j}$ (C) $\hat{i} - \hat{j}$ (D) $-\hat{j} + \hat{k}$

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.2 Q.4 Q.7 Q.8
Q.10

Exercise 2

Q.3 Q.8 Q.10

Previous Years' Questions

Q.2 Q.8 Q.9

JEE Advanced/Boards

Exercise 1

Q.2 Q.9 Q.12 Q.15

Exercise 2

Q.1 Q.3

Previous Years' Questions

Q.1 Q.5 Q.6 Q.8
Q.10

Answer Key

JEE Main/Boards

Exercise 1

Q.1 (i) $r = \left(2\hat{i} - 3\hat{j} + \hat{k} \text{ or } -\hat{i} - 2\hat{j} - 4\hat{k}\right) + t\left(3\hat{i} - \hat{j} + 5\hat{k}\right)$

Q.2 (i) $15°$ (15.38.....), 0.268 rad (ii) a = 1 and intersection is (–20, 5, –12)

Q.3 (i) $\sqrt{161}$ (ii) 43°

Q.4 (i) r= (either point) + $t\left(\hat{i} - 2\hat{j} - 3\hat{k} \text{ or } -\hat{i} + 2\hat{j} + 3\hat{k}\right)$, (ii) $s\,(\hat{i} + 2\hat{j} - \hat{k})$ (iii) $3\hat{i} + 6\hat{j} - 3\hat{k}$ (iv) $\sqrt{54}$

Q.5 (i) $2\hat{j} + \hat{k}$ (ii) 86°

Q.6 (i) 45.3° (ii) 3.54

Q.7 (i) A cannot be 2. (ii) $-5\hat{i} - 4\hat{j}$

Q.8 (i) 68.5° (ii) c=-4 (iii) c=-3

Q.9 $-\hat{i} + 2\hat{j} + 5\hat{k}$

Q.10 $\vec{F} = 2\vec{a}_1 + 5\vec{a}_2 + 3\vec{a}_3$

Exercise 2

Single Correct Choice Type

Q.1 D **Q.2** B **Q.3** C **Q.4** B **Q.5** B **Q.6** A
Q.7 D **Q.8** B **Q.9** A **Q.10** A

Previous Years' Questions

Q.1 B	**Q.2** B	**Q.3** B	**Q.4** A	**Q.5** B	**Q.6** C
Q.7 C	**Q.8** A	**Q.9** $3\hat{i}-\hat{j}+3\hat{k}$	**Q.10** C	**Q.11** D	**Q.12** A
Q.13 B	**Q.14** C	**Q.15** D	**Q.16** D	**Q.17** D	**Q.18** C

JEE Advanced/Boards

Exercise 1

Q.1 $\frac{1}{2}$	**Q.2** $\frac{41}{2}$	**Q.3** –40	**Q.4** 90° and 90°	**Q.5** $\frac{\pi}{3}$	**Q.6** 1
Q.7 48 units	**Q.8** –10	**Q.9** $\sqrt{593}$	**Q.10** 2 or $\frac{-2}{3}$		

Exercise 2

Single Correct Choice Type

Q.1 A	**Q.2** C	**Q.3** A	**Q.4** A	**Q.5** C	**Q.6** D
Q.7 C	**Q.8** C	**Q.9** C	**Q.10** C		

Previous Years' Questions

Q.1 (i) $\vec{X}=\left(\frac{c}{|\vec{A}|^2}\right)\vec{A}-\left(\frac{1}{|\vec{A}|^2}\right)(\vec{A}\times\vec{B})$ (ii) $(A_2\hat{i}-A_1\hat{j}+A_3\hat{k})$ **Q.3** $-\hat{i}-8\hat{j}+2\hat{k}$

Q.5 $\hat{w}=\hat{v}-2(\hat{a}\cdot\hat{v})\hat{a}$		**Q.6** B, D	**Q.7** A, D	**Q.8** 5	**Q.9** 9
Q.10 B, C	**Q.11** A, B, C	**Q.12** 9	**Q.13** 5	**Q.14** 4	**Q.15** C
Q.16 B, D	**Q.17** 5	**Q.18** C	**Q.19** 3	**Q.20** C	**Q.21** C
Q.22 A, D					

Solutions

JEE Main/Boards

Exercise 1

Sol 1: (i) For (either point) + t(diff b/w vectors)

$r = (2i - 3j + k \;\; \text{or} \;\; -i - 2j - 4k) + t(3i - j + 5k)$

(ii) $L(2)(r) = 3i + 2j - 9k + s(4i - 4j + 5k)$

L(1) and L(2) must be of form r = a +tb

$2 + 3t = 3 + 4s, -3 - t = 2 - 4s, 1 + 5t = -9 + 5s$

$(t, s) = (+/-3, 2)$ or $(-/+1, 1)$ or $(-/+9, -7)$

Or (+/-4,2) or (0, 1) or (-/+8, -7)

Sol 2: (i) Angle between the lines

$$\cos\theta = \frac{-8\times 9 + 1\times 2 + (-2)\times(-5)}{\sqrt{64+1+4}\,\sqrt{81+4+25}} = \frac{84}{\sqrt{69}\,\sqrt{110}} = 0.9641$$

$\Rightarrow \theta = \cos^{-1}(0.9641) = 15.38$ degree

(ii) Let P be the point of intersection

Equation of lines are $\frac{x-4}{-8} = \frac{y-2}{1} = \frac{z+6}{-2} = r_1$

Point P be $(-8r_1 + 4, r_1 + 2, -2r_1 - 6)$ ....(i)

Similarly for second line $\frac{x+2}{-9} = \frac{y-a}{2} = \frac{z+2}{-5} = r_2$

The point P be $(-9r_2 - 2, 2r_2 - a, -5r_2 - 2)$ ...(ii)

From (i) and (ii), we get

$-8r_1 + 4 = -9r_2 - 2$

$r_1 + 2 = 2r_2 + a$

$-2r_1 - 6 = -5r_2 - 2$

On solving, we get $r_1 = 3, r_2 = 2$ and a = 1

The points of intersection is (-20, 5, -12)

Sol 3: (i) Find $\vec{a} - \vec{b}$ or $\vec{b} - \vec{a}$ irrespective of label (expect $11\hat{i} - 2\hat{j} - 6\hat{k}$ or $-11\hat{i} + 2\hat{j} + 6\hat{k}$)

Magnitude of vector = $\sqrt{161}$

(ii) Using $(\overrightarrow{AO}$ or $\overrightarrow{OA})$ and $(\overrightarrow{AB}$ or $\overrightarrow{BA})$

$$\cos\theta = \frac{\text{Scalar product of any two vector}}{\text{Product of their moduli}}$$

$$\frac{(4\hat{i} + 3\hat{j} - 2\hat{k}).(11\hat{i} - 2\hat{j} - 6\hat{k})}{\sqrt{4^2 + 3^2 + 2^2}\,\sqrt{11^2 + 2^2 + 6^2}}$$

$$= \frac{44 - 6 + 12}{\sqrt{29}\,\sqrt{161}} = \frac{50}{\sqrt{29}\,\sqrt{161}} = 43^\circ$$

Sol 4: (i) For (either point) + t(diff between position vectors)

(ii) r = s (i+2j-k) or (i + 2j – k) + s (i + 2j – k)

Evaluate scalar product of i + 2j – k and their dir vect in (i)

Show as $(1\times 1 \;\text{or}\; 1) + (2x - 2 \;\text{or}\; -4) + (-1x - 3 \;\text{or}\; 3)$ = 0 and

(iii) Obtain t = -2 or s = 3 (possibly – 3 or 2 or – 2)

Check if t = 2, 1 or -1

Subst. into eqn AB or OT and to produce 3i + 6j -3k

(iv) $|\overrightarrow{OC}|$ is to be found, where C is their point of intersection

$|\overrightarrow{OC}| = \sqrt{54}$

Sol 5: (i) OD = OA + AD or OB + BC + CD AEF

AD = BC or CD = BA

$\overrightarrow{OD} = 2\hat{i} + \hat{k}$

(ii) AB.CB = |AB||CB| $\cos\theta \Rightarrow \cos\theta = 86^\circ$

Sol 6: (i) Work out $\vec{b} - \vec{a}$ or $\vec{a} - \vec{b}$ or $\vec{c} - \vec{a}$ or $\vec{a} - \vec{c}$

$= \pm(-3i - j - k)$ or $\pm(-2i + j - 2k)$

Use cosine rule and find angle as 45.3^o

(ii) Use of $\frac{1}{2}|\overline{AB}| \times |\overline{AC}| \sin\theta$

$= \frac{1}{2}(\sqrt{11})(3)\sin 45.3^\circ = 3.54$

Sol 7: (i) Produce at least 2 of the 3 relevant eqns in λ and μ

Solving we get

1st solution: $\lambda = -2$ or $\mu = 3$

2nd solution: $\mu = 3$ or $\lambda = -2$

Substitute their λ and μ into 3rd eqn and find 'a'

We get a=2 but a cannot be 2

(ii) Subst their λ or μ (& pass a) into either line eqn

Point of intersection is $-5\hat{i} - 4\hat{j}$

Sol 8: (i) $\cos\theta = \dfrac{-6\times1+8\times3-2\times2}{\sqrt{36+64+4}\sqrt{1+9+4}}$

$= \dfrac{14}{\sqrt{104}\sqrt{14}} = 68.47^\circ$

(ii) Since, and are parallel

$$\cos\phi = \frac{-6\times3+8\times C-2\times1}{\sqrt{104}\sqrt{9+C^2+1}} = 1$$

$\Rightarrow 8C - 20 = \sqrt{104}\sqrt{10+C^2}$

$\Rightarrow (8C-20)^2 = 104(10+C^2)$

$\Rightarrow 64C^2 + 400 - 320C = 1040 + 104C^2$

$\Rightarrow 40C^2 + 320C + 640 = 0$

$\Rightarrow C^2 + 8C + 16 = 0$

$\Rightarrow (C+4)^2 = 0 \Rightarrow C = -4$

(iii) $L_2 \equiv \dfrac{x-3}{1} = \dfrac{y+3}{3} = \dfrac{z-0}{2} = m$

Any point $(m+3,\ 3m-8,\ 2m)$

$$L_3 \equiv \frac{x-2}{3} = \frac{y-1}{C} = \frac{z-3}{1} = n$$

Any point $(3n+2,\ Cn+1,\ n+3)$

If L_2 and L_3 intersect, then

$m+3 = 3n+2$ ...(i)

$3m-8 = Cn+1$ ...(ii)

$2m = n+3$ ...(iii)

On solution, we get C = -3

Sol 9: Let $\vec{R} = R_1\hat{i} + R_2\hat{j} + R_3\hat{k}$

$(\vec{u}.\vec{R} - 10)\hat{i} + (\vec{V}.\vec{R} - 20)\hat{j} + (\vec{W}.\vec{R} - 20)\hat{k} = 0$

$(R_1 - 2R_2 + 3R_3 - 10)\hat{i} + (2R_1 + R_2 + 4R_3 - 20)\hat{j}$

$+(R_1 + 3R_2 + 3R_3 - 20)\hat{k} = 0$

$\Rightarrow R_1 - 2R_2 + 3R_3 = 10$ ... (i)

$2R_1 + R_2 + 4R_3 = 20$ ... (ii)

$R_1 + 3R_2 + 3R_3 = 20$ ... (iii)

On solving, we get $R_1 = -1, R_2 = 2, R_3 = 5$

$\vec{R} = -\hat{i} + 2\hat{j} + 5\hat{k}$

Sol 10: $\vec{a}_1 = 2\vec{b}_1 + 3\vec{b}_2 - \vec{b}_3$

$\vec{a}_2 = \vec{b}_1 - 2\vec{b}_2 + 2\vec{b}_3$

$\vec{a}_3 = 3\vec{b}_1 - \vec{b}_2 + 2\vec{b}_3$

On solving, we get $\vec{F} = 2\vec{a}_1 + 5\vec{a}_2 + 3\vec{a}_3$

Sol 11: $|\vec{r} + b\vec{S}|^2 = |\vec{r}|^2 + b^2|\vec{s}|^2 + 2b\vec{r}.\vec{s}$

$|\vec{r} + b\vec{s}|$ is minimum when $\vec{r}.\vec{s} = -|\vec{r}|\,|\vec{s}|$ and $\vec{r} = -b\vec{s}$

Then, $|b\vec{s}|^2 + |\vec{r} + b\vec{s}|^2 = b^2|\vec{s}|^2 + |\vec{r}|^2 + b^2|\vec{s}|^2 - 2b|\vec{r}||\vec{s}|$

$= 2b^2|\vec{s}|^2 + |\vec{r}|^2 - 2b^2|\vec{s}|^2 = |\vec{r}|^2$

Exercise 2

Single Correct Choice Type

Sol 1: (D) $(\vec{b} - 2\vec{c}) = \lambda\vec{a}$ or $\dfrac{1}{\lambda}|\vec{b} - 2\vec{c}| = 1$

or $(\vec{b} - 2\vec{c})^2 = (\lambda)^2$ or $(\vec{b} - 2\vec{c}).(\vec{b} - 2\vec{c}) = \lambda^2$

or $\vec{b}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{c}.\vec{b} + 4\vec{c}.\vec{c}$

$\Rightarrow 16 - 4.|\vec{b}|.|\vec{c}|.\cos\theta + 4 = \lambda^2$

$\sin\theta = \dfrac{\sqrt{15}}{4} \Rightarrow \cos\theta = \dfrac{1}{4}$

$\Rightarrow 16 - 4\times\dfrac{1}{4}\times4\times1 + 4 = \lambda^2$

$\Rightarrow \lambda^2 = 16 \Rightarrow \lambda = \pm4$

Sol 2: (B) $l^2 + m^2 + n^2 = 1$

or $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

or $3\cos^2\theta = 1$ or $\cos\theta = \dfrac{1}{\sqrt{3}}$

$\therefore$ The desired vector is

$15\sqrt{3}\left(\dfrac{1}{\sqrt{3}}\hat{i} + \dfrac{1}{\sqrt{3}}\hat{j} + \dfrac{1}{\sqrt{3}}\hat{k}\right) = 15(\hat{i} + \hat{j} + \hat{k})$

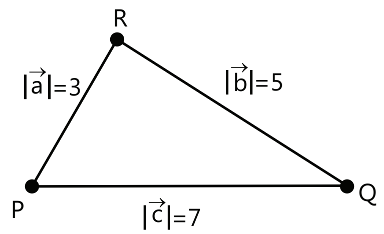
Sol 3: (C) Hint: Scalar Triple product

Sol 4: (B) $\vec{a}+\vec{b}+\vec{c}=0$

$|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$

$$\cos\theta=\frac{(5)^2+(3)^2-(7)^2}{2\times5\times3}=\frac{25+9-49}{30}$$

$$=\frac{-15}{30}=-\frac{1}{2}\Rightarrow\theta=150^\circ\text{ or }-30^\circ$$



Sol 5: (B) Let $|\vec{a}|=1=|\vec{b}|$

$2(\vec{a}\,\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})+3=0$

$(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$

$\Rightarrow 1+1+|\vec{c}|^2+2(\vec{a}\,\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\Rightarrow|\vec{c}|+2-3=0\Rightarrow|\vec{c}|=1$

Sol 6: (A) $(\vec{a}+\vec{b}).\vec{b}=0\Rightarrow 2\,\vec{a}.\vec{b}+2\,\vec{b}.\vec{b}=0$

Similarly $(\vec{a}+2\vec{b}).\vec{a}=0\Rightarrow\vec{a}.\vec{a}+2\vec{a}.\vec{b}=0$

$\Rightarrow|\vec{a}|^2=+2|\vec{b}|^2 \quad \Rightarrow|\vec{a}|=\sqrt{2}\,|\vec{b}|$

Sol 7: (D) Projection of $\vec{a}$ on $\vec{b}$ $=\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{b}=4\hat{i}+4\hat{j}+7\hat{k}$

$\therefore$ Projection $=\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$=\frac{(\hat{i}+2\hat{j}+\hat{k}).(4\hat{i}+4\hat{j}+7\hat{k})}{\sqrt{(4)^2+(4)^2+(7)^2}}=\frac{4+8+7}{\sqrt{16+16+49}}=\frac{19}{9}$$

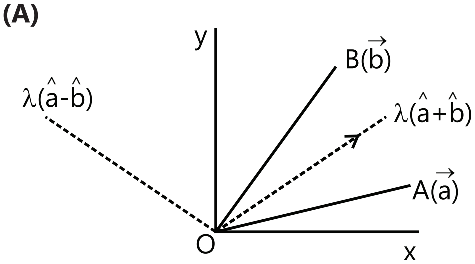
Hence, the correct option is d.

Sol 8: (B)

$$\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-2&6&-8\\1&-3&4\end{vmatrix}=\hat{i}(24-24)-\hat{j}(-8+8)+\hat{k}(6-6)=0$$

$\therefore$ Magnitude = 0

Sol 9: (A)



Sol 10: (A) $\left[2\vec{a}+4\vec{b}\,\vec{c}\,\vec{d}\right]=\lambda\left[\vec{a}\;\vec{c}\;\vec{d}\right]+\mu\left[\vec{b}\;\vec{c}\;\vec{d}\right]$

$\left[2\vec{a}+4\vec{b}.\vec{c}\,\vec{d}\right]=2\vec{a}+4\vec{b}.\left(\vec{c}\times\vec{d}\right)=2\vec{a}.\left(\vec{c}\times\vec{d}\right)+4\vec{b}.\left(\vec{c}\times\vec{d}\right)$

$=2\left[\vec{a}\;\vec{c}\;\vec{d}\right]+4\left[\vec{b}\;\vec{c}\;\vec{d}\right]\Rightarrow\lambda=2,\mu=4\Rightarrow\lambda+\mu=6$

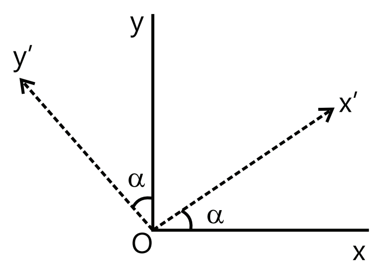
Previous Years' Questions

Sol 1: (B) The volume of parallelopiped

$$=\left[\vec{a}\;\vec{b}\;\vec{c}\right]=\begin{vmatrix}2&-3&0\\1&1&-1\\3&0&-1\end{vmatrix}$$

=2(–1) + 3(–1+3) = –2+6 = 4

Sol 2: (B) $\vec{a}=2p\hat{i}+\hat{j}$



New vector

$\vec{a}=\sqrt{4P^2+1}\cos\alpha i+\sqrt{4P^2+1}\sin\alpha j$

$\Rightarrow\sqrt{4p^2+1}\cos\alpha=p+1$

$\Rightarrow\cos\alpha=\dfrac{p+1}{\sqrt{4p^2+1}}$

And $\sqrt{4p^2+1}\sin\alpha=1$

$\Rightarrow\sin\alpha=\dfrac{1}{\sqrt{4p^2+1}}$

$\Rightarrow\sin^2\alpha+\cos^2\alpha=1=\dfrac{(p+1)^2}{4p^2+1}+\dfrac{1}{4p^2+1}$

$\Rightarrow 4p^2+1=p^2+1+2p+1$

$\Rightarrow 3p^2-2p-1=0 \quad \Rightarrow 3p^2-3p+p-1=0$

$\Rightarrow 3p(p-1)+1(p-1)=0$

$\Rightarrow (p-1)(3p+1)=0$

$\Rightarrow p = 1, \frac{-1}{3}$

Sol 3: (B) Since, three vectors are coplanar.

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_2$

$$\Rightarrow \begin{vmatrix} 0 & a & c \\ 1 & 0 & 1 \\ 0 & c & b \end{vmatrix} = 0$$

$\Rightarrow -1(ab - c^2) = 0 \Rightarrow ab = c^2$

Sol 4: (A) Since, $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = \frac{1}{\sqrt{2}}\vec{b} + \frac{1}{\sqrt{2}}\vec{c}$

On equating the coefficient of $\vec{c}$, we get

$\vec{a}\cdot\vec{b} = -\frac{1}{\sqrt{2}}$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = -\frac{1}{\sqrt{2}}$

$\therefore \cos\theta = -\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{3\pi}{4}$

Sol 5: (B) Since, $(\vec{a} + 2\vec{b})\cdot(5\vec{a} - 4\vec{b}) = 0$

$\Rightarrow 5|\vec{a}|^2 + 6\,\vec{a}\cdot\vec{b} - 8|\vec{b}|^2 = 0$

$\Rightarrow 6\,\vec{a}\cdot\vec{b} = 3\ [\because |\vec{a}| = |\vec{b}| = 1]$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

Sol 6: (C) Given, $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{w} = \hat{i} + 3\hat{k}$

$\therefore [\vec{u}\ \vec{v}\ \vec{w}] = \vec{u}\cdot[(2\hat{i} + \hat{j} - \hat{k}) \times (\hat{i} + 3\hat{k})]$

$= \vec{u}\cdot(3\hat{i} - 7\hat{j} - \hat{k}) = |\vec{u}||3\hat{i} - 7\hat{j} - \hat{k}|\cos\theta$

which is maximum, if angle between $\vec{u}$ and $3\hat{i} - 7\hat{j} - \hat{k}$ is 0 and maximum value= $|3\hat{i} - 7\hat{j} - \hat{k}| = \sqrt{59}$

Sol 7: (C) As we know, a vector coplanar to $\vec{a}, \vec{b}$ and orthogonal to $\vec{c}$ is $\lambda\{(\vec{a} \times \vec{b}) \times \vec{c}\}$

$\therefore$A vector coplanar to $(2\hat{i} + \hat{j} + \hat{k}), (\hat{i} - \hat{j} + \hat{k})$ and

Orthogonal to $3\hat{i} + 2\hat{j} + 6\hat{k}$

$= \lambda[\{(2\hat{i} + \hat{j} + \hat{k}) \times (\hat{i} - \hat{j} + \hat{k})\} \times (3\hat{i} + 2\hat{j} + 6\hat{k})]$

$= \lambda[(2\hat{i} - \hat{j} - 3\hat{k}) \times (3\hat{i} + 2\hat{j} + 6\hat{k})]$

$= \lambda(21\hat{j} - 7\hat{k})$

$\therefore$ Unit vector $= +\frac{(21\hat{j} - 7\hat{k})}{\sqrt{(21)^2 + (7)^2}} = +\frac{(3\hat{j} - \hat{k})}{\sqrt{10}}$

Sol 8: (A) $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$

$\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$

Angle 'θ' between $\overrightarrow{AB}$ and $\overrightarrow{AD}$ is

$\cos(\theta) = \left|\frac{\overrightarrow{AB}\cdot\overrightarrow{AD}}{|\overrightarrow{AB}||\overrightarrow{AD}|}\right| = \left|\frac{-2+20+22}{(15)(3)}\right| = \frac{8}{9}$

Sol 9: $\vec{V} = i + j + k + \lambda(i - j + k)$

$= (1+\lambda)i + (1-\lambda)j + (1+\lambda)k$

Projection on $\vec{C}$ is $\frac{1}{\sqrt{3}}$

$\frac{\vec{V}.\vec{C}}{|\vec{C}|} = \frac{1}{\sqrt{3}}$

$\frac{(1+\lambda) - (1-\lambda) - (1+\lambda)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow 1 + \lambda - 1 + \lambda - 1 - \lambda = 1 \Rightarrow \lambda = 2$

$\vec{V} = 3\hat{i} - \hat{j} + 3\hat{k}$

Sol 10: (C) $\vec{a} \times (\vec{b} \times \vec{c}) - \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$

$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$

$\Rightarrow \vec{a}.\vec{c} = \frac{\sqrt{3}}{2}$ and $\vec{a}.\vec{b} = -\frac{\sqrt{3}}{2}$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = -\frac{\sqrt{3}}{2}$

$\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2}$

$\Rightarrow \theta = \frac{5\pi}{6}$

Sol 11: (D) $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow (c.\ a)\vec{b}-\left(\vec{c}.\ \vec{b}\right)\vec{a}=\frac{1}{3}\left|\vec{b}\right|\ \left|\vec{c}\right|\ \vec{a}$

$\Rightarrow \vec{c}.\ \vec{b}=-\frac{1}{3}|b||c|\ \Rightarrow \cos\theta=-\frac{1}{3}$

$\Rightarrow \sin^2\theta=1-\cos^2\theta\ \ =1-\frac{1}{9}\ \ =\frac{8}{9}$

$\Rightarrow \sin\theta=\pm\frac{2\sqrt{3}}{3}$

But $\sin\theta=\frac{2\sqrt{3}}{3}$

Sol 12: (A) $\left[\vec{a}\times\vec{b}.\vec{b}\times\vec{c}\ \vec{c}\times\vec{a}\right]=\lambda\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

We know that

$\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}\right]=\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2$

$\Rightarrow \lambda=1$

Sol 13: (B) The length of median through A

$=\frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}\ =\frac{3\hat{i}+4\hat{k}+5\hat{i}-2\hat{j}+4\hat{k}}{2}=\frac{8\hat{i}-2\hat{j}+8\hat{k}}{2}$

$=4\hat{i}-\hat{j}+4\hat{k}$

Length $=\sqrt{16+1+16}=\sqrt{33}$

Sol 14: (C) $\vec{c}=\vec{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$

$\vec{c}\perp\vec{d}$

$\vec{c}.\ \vec{d}=0$

$\Rightarrow 5|a|^2-4\vec{a}.\ \vec{b}+10\vec{a}.\vec{b}-8\left|\vec{b}\right|^2\ =0$

$\Rightarrow 5+6\vec{a}.\vec{b}-8=0$

$\Rightarrow 6\ \vec{a}.\vec{b}=3$

$\Rightarrow \vec{a}.\vec{b}=\frac{1}{2}$

$\Rightarrow \cos\phi=\frac{1}{2}$

$\Rightarrow \phi=\frac{\pi}{3}$

Sol 15: (D) $\vec{a}.\vec{b}=0$ $\qquad \vec{b}.\vec{c}=0$ $\qquad \vec{c}.\vec{a}=0$

$\Rightarrow 2\lambda+4+\mu=0$ $\qquad \lambda-1+2\mu=0$

Solving we get : $\lambda=-3,\ \mu=2$

Sol 16: (D) $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$

Let $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\because\ \vec{a}\times\vec{b}+\vec{c}=0,\ \vec{a}\times\vec{b}=-\vec{c}$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & -1\\ b_1 & b_2 & b_3\end{vmatrix}=-\hat{i}+\hat{j}+\hat{k}$$

$\hat{i}(b_3+b_2)-\hat{j}(b_1)+k(-b_1)=-\hat{i}+\hat{j}+\hat{k}$

$b_3+b_2=-1$ ... (i)

$b_1=-1$ ... (ii)

$\vec{a}\cdot\vec{b}=3$

$b_2-b_3=3$ ... (iii)

Solve (i) and (iii)

$2b_2=2$ $\qquad b_1=2$ $\qquad b_3=-2$

$\therefore b_1=-1$ $\qquad b_2=1$ $\qquad b_3=-2$

Hence $\vec{b}=-\hat{i}+\hat{j}-2\hat{k}$

Sol 17: (D) $\left(2\vec{a}-\vec{b}\right)\cdot\left\{\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right\}$

$=\left(2\vec{a}-\vec{b}\right)\cdot\left\{\left[\vec{a}\cdot\left(\vec{a}+2\vec{b}\right)\right]\vec{b}-\left[\vec{b}\cdot\left(\vec{a}+2\vec{b}\right)\vec{a}\right]\right\}$

$=-5\left(\vec{a}\right)^2\left(\vec{b}\right)^2+5\left(\vec{a}\cdot\vec{b}\right)^2\ =-5$

Sol 18: (C) $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times\left(\vec{b}\times\vec{c}\right)=\vec{a}\times\left(\vec{b}\times\vec{d}\right)$

$\Rightarrow\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}=\left(\vec{a}.\vec{d}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{d}$

$\Rightarrow\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}=-\left(\vec{a}.\vec{b}\right)\vec{d}$

$\therefore \vec{d}=\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

JEE Advanced/Boards

Exercise 1

Sol 1: $\frac{\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}.\vec{b}\right)^2}{2\vec{a}^2\vec{b}^2}$

$$=\frac{(\vec a\times\vec b).(\vec a\times\vec b)+(\vec a.\vec b).(\vec a.\vec b)}{2|\vec a|^2.|\vec b|^2}=\frac{|\vec a\times\vec b|^2+|\vec a.\vec b|^2}{2|\vec a|^2.|\vec b|^2}$$

$$=\frac{|\vec a|^2.|\vec b|^2.\sin^2\theta+|\vec a|^2.|\vec b|^2.\cos^2\theta}{2|\vec a|^2.|\vec b|^2}$$

Sol 2: Area $=\frac{1}{2}\left|(3\hat i+4\hat j)\times(-5\hat i+7\hat j)\right|=\frac{1}{2}\left|21\hat K+20\hat k\right|=\frac{41}{2}$

Sol 3: $20\hat i+11\hat j=\lambda\{(40-a)\hat i+44\hat j\}$

$\therefore\lambda(40-a)=20$ and $\lambda(4)=11$

$\Rightarrow a=-40$

Sol 4: Note that since $|\vec a|=|\vec b|$ hence the parallelogram will be a rhombus.

Sol 5: $\because(\vec a+3\vec b)$ is perpendicular to $(7\vec a-5\vec b)$

$\therefore(\vec a+3\vec b).(7\vec a-5\vec b)=0$

$\Rightarrow 7.|\vec a|^2-5\,\vec a.\vec b+21\,\vec a.\vec b-15|\vec b|^2=0\Rightarrow 16\vec a.\vec b=8$

$\therefore\vec a.\vec b=\frac{1}{2}\Rightarrow|\vec a|.|\vec b|.\cos\theta=\frac{1}{2}\Rightarrow\cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$

Sol 6: $|\hat A-\hat B|^2=(\hat A-\hat B)\,(\hat A-\hat B)$

$=|\hat A|^2+|\hat B|^2-2\hat A.\hat B=1+1-2\,|1||1|\cos\pi=2+2=4$

$\Rightarrow|\hat A-\hat B|=2\ \Rightarrow\frac{|\hat A-\hat B|}{2}=1$

Sol 7: $\vec F_1=3\hat i+2\hat j+5\hat k$ and $\vec F_2=2\hat j+\hat j+3\hat k$

$\therefore\vec F=\vec F_1+\vec F_2=5\hat i+3\hat j+8\hat k$

$\Delta\vec x=(4\hat i+3\hat j+7\hat k)-(2\hat i+\hat j+3\hat k)=2\hat i+2\hat j+4\hat k$

$\therefore$ Work done $=\vec F.\Delta\vec x=10+6+32=48$ units

Sol 8: $\Delta\vec x=-2\hat i-3\hat j-4\hat k$

Work done $=\vec F.\Delta\vec x$

$\Rightarrow 2=-12-3\lambda-16\Rightarrow 30=-3\lambda\Rightarrow\lambda=-10$

Sol 9: Let $\vec b=3\hat i$ and $\vec a=2(\hat i+\hat j)$

The two diagonals will be $6\vec a-\vec b$ and $4\vec a+5\vec b$

Length of $6\vec a-\vec b=|9\hat i+12\hat j|=15$

Length of $4\vec a+5\vec b$

$=|8\hat j+23\hat i|=\sqrt{(23)^2+(8)^2}=\sqrt{593}$

Sol 10: $2\times|\hat i+x\hat j+3\hat k|=|4\hat i+(4x-2)\hat j+2\hat k|$

or, $2.\sqrt{1^2+x^2+9}=\sqrt{16+(4x-2)^2+4}$

$\therefore 4(10+x^2)=20+16x^2+4-16x$

or, $40+4x^2=24+16x^2-16x$

or, $12x^2-16x-16=0$ or, $3x^2-4x-4=0$

$\therefore x=\frac{4\pm\sqrt{16+4.4.3}}{6}=\frac{4\pm 8}{6}=2$ or $\frac{-2}{3}$

Exercise 2

Sol 1: (A) $\vec r=(9\hat i-\hat j+2\hat k)-(3\hat i-2\hat j+\hat k)=(6\hat i+\hat j+\hat k)$

$\therefore$ Moment of couple $=\vec r\times\vec F=(6\hat i+\hat j+\hat k)\times(5\hat i+\hat j)$

$$=\begin{vmatrix}\hat i&\hat j&\hat k\\6&1&1\\5&1&0\end{vmatrix}=\hat i(0-1)-\hat j(0-5)+\hat k(6-5)$$

$=-\hat i+5\hat j+\hat k$

Sol 2: (C) $(\vec a+\vec b+\vec c).(\vec a+\vec b+\vec c)$

$=|\vec a|^2+|\vec b|^2+|\vec c|^2+\vec a(\vec b+\vec c)+\vec b(\vec a+\vec c)+\vec c(\vec a+\vec b)$

$=(1)^2+(4)^2+(8)^2+0=81$

$\therefore|\vec a+\vec b+\vec c|=9$

Sol 3: (A) Let M be the point of intersection of internal bisector with AB.

$\therefore\frac{AM}{MB}=\frac{1}{2}$

$\therefore\overrightarrow{OM}=\frac{1(2\hat i+4\hat j+4\hat k)+2(2\hat i+2\hat j+\hat k)}{3}=\frac{6\hat i+8\hat j+6\hat k}{3}$

$\therefore|\overrightarrow{OM}|=\sqrt{4+\left(\frac{8}{3}\right)^2+4}=\sqrt{\frac{72+64}{9}}=\sqrt{\frac{136}{9}}$

Sol 4: (A) $\vec{a}+2\vec{b}+3\vec{c}=0$

$\Rightarrow \vec{a}\times\vec{b}+2\vec{b}\times\vec{b}+3\vec{c}\times\vec{b}=0$

$\Rightarrow \vec{a}\times\vec{b}+3\vec{c}\times\vec{b}=0 \Rightarrow \vec{a}\times\vec{b}=3\vec{b}\times\vec{c}$ ... (i)

Similarly, $\vec{a}+2\vec{b}+3\vec{c}=0$

$\vec{a}\times\vec{a}+2\vec{b}\times\vec{a}+3\vec{c}\times\vec{a}=0 \Rightarrow \vec{c}\times\vec{a}=\frac{2}{3}\vec{a}\times\vec{b}$

$=\frac{2}{3}\times 3\left(\vec{b}\times\vec{c}\right)=2\left(\vec{b}\times\vec{c}\right)$ ... (ii)

$\Rightarrow \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=6\left(\vec{b}\times\vec{c}\right)$

Sol 5: (C) $\left[\vec{a}-\vec{b}\quad \vec{b}-\vec{c}\quad \vec{c}-\vec{a}\right]=\left(\vec{a}-\vec{b}\right).\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)$

$=\left(\vec{a}-\vec{b}\right).\left\{\vec{b}\times\vec{c}-\vec{b}\times\vec{a}+\vec{c}\times\vec{a}\right\}=\vec{a}.\left(\vec{b}\times\vec{c}\right)-\vec{b}.\left(\vec{c}\times\vec{a}\right)=0$

Sol 6: (D) A general point on the curve will have vector

$x\hat{i}+y\hat{j}\ =x\hat{i}+\left(x+\frac{1}{x}\right)\hat{j}$

$\because\ \overrightarrow{OP}.\vec{I}=1$

$\therefore \left\{xi+\left(+\frac{1}{x}\right)\hat{j}\right\}.\hat{i}=1 \Rightarrow x=1$

$\therefore \overrightarrow{OP}=\hat{i}+2\hat{j}$

Again, $\overrightarrow{OQ}.\vec{I}=-1 \Rightarrow x=-1$

$\therefore \overrightarrow{OQ}=-\vec{i}-2\vec{j}$

$\therefore 2\overrightarrow{OP}+3\overrightarrow{OQ}=-\hat{i}-2\hat{j}$

$\therefore \left|2\overrightarrow{OP}+3\overrightarrow{OQ}\right|=\sqrt{(-1)^2+(2)^2}=\sqrt{5}$

Sol 7: (C) $\vec{A}\cdot\left(\vec{B}\cdot\vec{C}\right)+\vec{B}\vec{C}\left(\vec{A}+\vec{B}\right)=0$

$\Rightarrow A.B+A.C+B.C=0$

$\left|A+B+C\right|^2=\left|A\right|^2+\left|B\right|^2+\left|C\right|^2+2\left(A.B+B.C+C.A\right)$

$=1+16+64+0\ =81$

$\Rightarrow \left|A+B+C\right|=9$

Sol 8: (C) Given

$\left|A-B\right|\le 1$

$\therefore \sqrt{1^2+1^2-2\cos 2\theta}\le 1$

$\Rightarrow 2-2\cos 2\theta\le 1 \Rightarrow 2\cos 2\theta\ge 1 \Rightarrow \cos 2\theta\ge\frac{1}{2}$

$\therefore \frac{5\pi}{3}\le 2\theta\le 2\pi \Rightarrow \frac{5\pi}{6}\le\theta\le\pi$

Sol 9: (C) $\left|\vec{A}\ \vec{B}\ \ \vec{A}\times\vec{B}\right|=\frac{1}{4}$

$\Rightarrow \vec{A}\cdot\left(\vec{B}\times\vec{A}\times\vec{B}\right)=\frac{1}{4} \Rightarrow \vec{A}\ \left[\left(\vec{B}\cdot\vec{B}\right)\vec{A}-\left(\vec{A}\cdot\vec{B}\right)\vec{B}\right]=\frac{1}{4}$

$\Rightarrow \vec{A}\cdot\left[\vec{A}-\left(\vec{A}\cdot\vec{B}\right)\vec{B}\right]=\frac{1}{4} \Rightarrow \vec{A}\cdot\vec{A}-\left(\vec{A}\cdot\vec{B}\right)^2=\frac{1}{4}$

$\Rightarrow \left(\vec{A}\cdot\vec{B}\right)^2=1-\frac{1}{4}=\frac{3}{4} \Rightarrow \vec{A}\cdot\vec{B}=\frac{\sqrt{3}}{2}$

$\Rightarrow \cos\theta=\frac{\sqrt{3}}{2} \Rightarrow \theta=\frac{\pi}{3}$

Sol10:(C) $\left|\vec{A}-\vec{B}\right|+\left|\vec{A}+\vec{B}\right|=\sqrt{2-2\cos\theta}\ +\sqrt{2+2\cos\theta}$

$=\sqrt{2}\left(\sqrt{1-\cos\theta}+\sqrt{1+\cos\theta}\right)$

$=\sqrt{2}.\left\{\sqrt{2}\sin\frac{\theta}{2}+2\cos\frac{\theta}{2}\right\}=2\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)$

Greatest value is $2\sqrt{2}$

Previous Years' Questions

Sol 1: $\vec{A}.\ \vec{X}=C$ and $\vec{A}\times\vec{X}=\vec{B}$

Let $\vec{A}=A_1\hat{i}+A_2\hat{j}+A_3\hat{k}$

(i) $\vec{A}\times\vec{X}=\vec{B}$

$\vec{A}\times\left(\vec{A}\times\vec{X}\right)=\vec{A}\times\vec{B}\ \Rightarrow \left(\vec{A}.\ \vec{X}\right)\vec{A}-\left(\vec{A}.\ \vec{A}\right)\vec{X}=\vec{A}\times\vec{B}$

$\Rightarrow \left|\vec{A}\right|^2\vec{X}=C\vec{A}-\vec{A}\times\vec{B}\ \Rightarrow \vec{X}=\frac{C\vec{A}}{\left|\vec{A}\right|^2}-\frac{\vec{A}\times\vec{B}}{\left|\vec{A}\right|^2}$

(ii) If coordinate system is rotated about the x-axis through an angle $\frac{\pi}{2}$, then

x- component = A_2

y – component = A_1

z – component = A_3

$\vec{A}=A_2\hat{i}-A_1\hat{j}+A_3\hat{k}$

[New coordinates system]

Sol 2: $\begin{vmatrix}\vec{a} & \vec{b} & \vec{c}\\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c}\end{vmatrix}=\begin{vmatrix}\vec{a}+\vec{b}+\vec{c} & \vec{b} & \vec{c}\\ \vec{a}.\left(\vec{a}+\vec{b}+\vec{c}\right) & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{b}.\left(\vec{a}+\vec{b}+\vec{c}\right) & \vec{b}.\vec{b} & \vec{b}.\vec{c}\end{vmatrix}$

$$= \left(\vec{a}+\vec{b}+\vec{c}\right).\begin{vmatrix} 1 & \vec{b} & \vec{c} \\ \vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix}$$

$$= \left(\vec{a}+\vec{b}+\vec{c}\right).\begin{bmatrix} \left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{c}\right)-\left(\vec{b}.\vec{b}\right)\left(\vec{a}.\vec{c}\right)-\left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{c}\right)+ \\ \left(\vec{b}.\vec{b}\right)\left(\vec{a}.\vec{c}\right)+\left(\vec{a}.\vec{c}\right)\left(\vec{b}.\vec{b}\right)-\left(\vec{b}.\vec{c}\right)\left(\vec{a}.\vec{b}\right) \end{bmatrix}$$

$$= \left(\vec{a}+\vec{b}+\vec{c}\right)\left[\left(\vec{b}.\vec{b}\right)\left(\vec{a}.\vec{c}\right)-\left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{c}\right)\right]=0$$

$\Rightarrow$ if $\vec{a}+\vec{b}+\vec{c}=0$ [coplanar condition]

Sol 3: $\vec{R}\times\vec{B}=\vec{C}\times\vec{B} \;\Rightarrow\; \vec{A}\times\left(\vec{R}\times\vec{B}\right)=\vec{A}\times\left(\vec{C}\times\vec{B}\right)$

$\Rightarrow \left(\vec{A}.\vec{B}\right)\vec{R}-\left(\vec{A}.\vec{R}\right)\vec{B}=\left(\vec{A}.\vec{B}\right)\vec{C}-\left(\vec{A}.\vec{C}\right)\vec{B}$

$\left(2+0+1\right)\vec{R}-0=\left(2+0+1\right)\vec{C}-\left(8+0+7\right)\vec{B}$

$\Rightarrow 3\vec{R}=3\vec{C}-15\vec{B} \;\Rightarrow \vec{R}=\vec{C}-5\vec{B}$

$= 4\hat{i}-3\hat{j}+7\hat{k}-5\hat{i}-5\hat{j}-5\hat{k}=-\hat{i}-8\hat{j}+2\hat{k}$

Sol 4: Given, $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$

and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times\vec{b}-\vec{a}\times\vec{c}=\vec{c}\times\vec{d}-\vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})=(\vec{c}-\vec{b})\times\vec{d}$

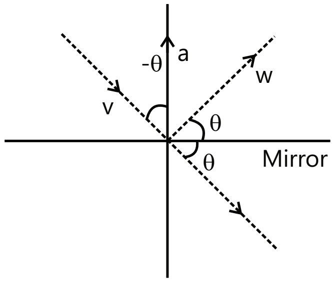
$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})-(\vec{c}-\vec{b})\times\vec{d}=0$

$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})-\vec{d}\times(\vec{b}-\vec{c})=0$

$\Rightarrow (\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=0 \;\Rightarrow\; (\vec{a}-\vec{d})\,||\,(\vec{b}-\vec{c})$

$\therefore (\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})\neq 0 \;\Rightarrow\; \vec{a}\cdot\vec{b}+\vec{d}\cdot\vec{c}\neq\vec{d}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

Sol 5: Since, $\hat{v}$ is unit vector along the incident ray and $\hat{w}$ is the unit vector along the reflected ray.



Hence, $\hat{a}$ is a unit vector along the external bisector of $\hat{v}$ and $\hat{w}$.

$\therefore\; \hat{w}-\hat{v} = \lambda\,\hat{a}$

On squaring both sides, we get

$\Rightarrow 1+1-\hat{w}\cdot\hat{v}=\lambda^2 \;\Rightarrow 2-2\cos 2\theta=\lambda^2 \;\Rightarrow \lambda=2\sin\theta$

where 2θ is the angle between $\hat{v}$ and $\hat{w}$.

Hence, $\hat{w}-\hat{v} = 2\sin\theta\cdot\hat{a}$

$= 2\cos(90^o-\theta)\,\hat{a} = -(2\,\hat{a}\cdot\hat{v})\,\hat{a}$

$\hat{w}=\hat{v}-2(\hat{a}\cdot\hat{v})\,\hat{a}$

Sol 6: (B, D) Let vector $\overrightarrow{AO}$ be parallel to line of intersection of planes P_1 and P_2 through origin

Normal to plane P_1 is

$\overrightarrow{n_1}=[(2\hat{j}+3\hat{k})\times(4\hat{j}-3\hat{k})]=-18\hat{i}$

Normal to plane P_2 is

$\overrightarrow{n_2}=(\hat{j}-\hat{k})\times(3\hat{i}+3\hat{j})=3\hat{i}-3\hat{j}-\hat{k}$

$\therefore\; \overrightarrow{OA}$ is parallel to $\pm\left(\overrightarrow{n_1}\times\overrightarrow{n_2}\right)=54\hat{j}-54\hat{k}$

$\therefore$ Angle between $54(\hat{j}-\hat{k})$ and $\left(2\hat{i}+\hat{j}-2\hat{k}\right)$ is

$$\cos\theta=\pm\left(\frac{54+108}{3\cdot 54\cdot\sqrt{2}}\right)=\pm\frac{1}{\sqrt{2}} \qquad \therefore\; \theta=\frac{\pi}{4},\frac{3\pi}{4}$$

Sol 7: (A, D) Let, $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}+\hat{k}$

$\therefore$ A vector coplanar to $\vec{a}$ and $\vec{b}$, and perpendicular to $\vec{c}$

$\vec{r}=\lambda(\vec{a}\times\vec{b})\times\vec{c} = \lambda\{(\vec{a}\cdot\vec{c})\vec{v}-(\vec{b}\cdot\vec{c})\vec{a}\}$

$=\lambda\{(1+1+4)(\hat{i}+2\hat{j}+\hat{k})-(1+2+1)(\hat{i}+\hat{j}+2\hat{k})\}$

$=\lambda\{6\hat{i}+12\hat{j}+6\hat{k}-6\hat{i}-6\hat{j}-12\hat{k}\}=\lambda\{6\hat{j}-6\hat{k}\}=6\lambda\{\hat{j}-\hat{k}\}$

For $\lambda=\dfrac{1}{6} \Rightarrow$ (a) is correct.

and $\lambda=-\dfrac{1}{6} \Rightarrow$ (d) is correct.

Sol 8: From the given information, it is clear that

$$\vec{a}=\frac{\hat{i}-2\hat{j}}{\sqrt{5}}$$

$\Rightarrow |\vec{a}| = 1,\ |\vec{b}|=1,\ |\vec{a}\cdot\vec{b}|=0$

Now, $(2\vec{a}+\vec{b})\cdot[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})]$

$= (2\vec{a}+\vec{b})\cdot[a^2\vec{b}-(\vec{a}\cdot\vec{b})\cdot\vec{a}+2\vec{b}^2\cdot\vec{a}-2(\vec{b}\cdot\vec{a})\cdot a]$

$=[2\vec{a}+\vec{b}]\cdot[\vec{b}+2\vec{a}]=4\vec{a}^2+\vec{b}^2 = 4\cdot1+1=5$ [as $\vec{a}.\vec{b}=0$]

Sol 9: $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$

$\Rightarrow (\vec{r}-\vec{c})\times\vec{b}=0 \quad \Rightarrow \vec{r}-\vec{c}=\lambda\vec{b}$

or $\vec{r}=\vec{c}+\lambda\vec{b}$ ...(i)

Given, $\vec{r}\cdot\vec{a}=0$, taking dot product with $\vec{a}$ for Eq. (i)

$\Rightarrow \vec{r}\cdot\vec{a}=\vec{a}\cdot\vec{c}+\lambda\vec{a}\cdot\vec{b}$

$\therefore \lambda = \dfrac{-\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}$ $(\because\ \vec{r}\cdot\vec{a}=0)$ ...(ii)

From Eqs. (i) and (ii), we get

$\vec{r}=\vec{c}-\dfrac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\vec{b}$, taking dot with $\vec{b}$, we get

$\vec{r}\cdot\vec{b}=\vec{c}\cdot\vec{b}-\dfrac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}(\vec{b}\cdot\vec{b})$

$= (-1+2)-\dfrac{(-1-3)}{(1)}(1+1)$ where,

$\begin{bmatrix}\vec{a}=-\hat{i}-\hat{k}\\ \vec{b}=-\hat{i}+\hat{j}\\ \vec{c}=\hat{i}+2\hat{j}+3\hat{k}\end{bmatrix} = 1+8 = 9$

Sol 10: (B, C) Given: $w.\left(\hat{u}\times\vec{v}\right)=1$

$\Rightarrow |w|\left|\left(\hat{u}\times\vec{v}\right)\right|\cos\theta=1 \Rightarrow \cos\theta=1$

$w\perp\hat{u}\times\hat{v}\Rightarrow w\perp\hat{u}$ and $w\perp\hat{v}$ and $|\hat{u}\times\hat{v}|=1$

Angle between $\hat{u}$ and $\hat{v}$ can change to have initially many of vectors $\hat{v}$ as $\hat{w}\perp\hat{v}$

If $\hat{u}$ lies in xy plane then $\hat{u} = u_1\hat{i}+u_2\hat{j}$

$\Rightarrow \hat{w}.\hat{u}=0 \Rightarrow u_1+u_2=0 \Rightarrow |u_1|=|u_2|$

Sol 11: (A, C, D) In ΔPQR

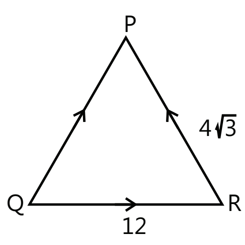
$-\vec{a}=\vec{b}+\vec{c}$

$\Rightarrow \vec{a}.\vec{a}=\left(\vec{b}+\vec{c}\right).\left(\vec{b}+\vec{c}\right)$

$\Rightarrow |\vec{a}|^2=|\vec{b}|^2+|\vec{c}|^2+2\vec{b}.\vec{c}$

$\Rightarrow |\vec{a}|^2=\left(4\sqrt{3}\right)^2+|c|^2+2\times 24$

$\Rightarrow (12)^2=\left(4\sqrt{3}\right)^2+|c|^2+2\times 24$



$\Rightarrow |c|^2=144-96$

$\Rightarrow \boxed{|c|=4\sqrt{3}}$

$\Rightarrow \dfrac{|c|^2}{2}-|\vec{a}|=\dfrac{48}{2}-12=24-12=12$

Given $\vec{b}.\vec{c}=24$

$-|\vec{b}||\vec{c}|\cos p=24$

$-4\sqrt{3}\times 4\sqrt{3}\cos p=24$

$\cos p=\dfrac{-1}{2}$

Since $|b|=|c|$

$\angle PQR=\angle PRQ \Rightarrow \angle QPR=120^0$

and $\angle PQR=\angle PRQ=30^0 \Rightarrow \left|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}\right|=48\sqrt{3}$

And $\vec{a}\times\vec{b}$ =-72

Sol 12: $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$

Taking cross with a

$\Rightarrow \vec{a}\times(\vec{r}\times\vec{b})=\vec{a}\times(\vec{c}\times\vec{b})$

$\Rightarrow \left(\vec{a}\,.\,\vec{b}\right)\vec{r}-\left(\vec{a}\,.\,\vec{r}\right)\vec{b}=\vec{a}\ x\left(\vec{c}\ x\ \vec{b}\right)$

$\Rightarrow \vec{r}=-3\hat{i}+6\hat{j}+3\hat{k}$

$\vec{r}.\vec{b}=3+6=9$

Sol 13: $\left(2\vec{a}+\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$

$\vec{a}=\dfrac{\hat{i}-2\hat{j}}{\sqrt{5}}\ \vec{b}=\dfrac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$

$\Rightarrow \left(2\vec{a}+\vec{b}\right)\left[\vec{a}\times\left(\vec{a}\times\vec{b}\right)-\left(\vec{a}\times\vec{b}\right)\times\vec{b}\right]$

$=\left(2\vec{a}\times\vec{b}\right)\left[\left(\vec{a}.\vec{a}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{a}+2\left(\vec{a}.\vec{b}\right)\vec{b}+2\left(\vec{b}\vec{b}\right)\vec{a}\right]$

$=\left(2\vec{a}+\vec{b}\right).\left[\vec{b}+2\vec{a}\right] \quad \left\{\vec{a}.\vec{b}=0\right\}$

$=2\vec{a}\ \vec{b}+4|\vec{a}|^2+|b|^2-2\vec{b}.\vec{a}$

$=4|a|^2+|b|^2$

$=4+1=5$

Sol 14: Given: $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$

$\Rightarrow \vec{a}.\left(\vec{a}\times\vec{b}\right)+\vec{a}.\left(\vec{b}\times\vec{c}\right)=p+q\left(\vec{a}.\vec{b}\right)+r\left(\vec{a}.\vec{c}\right)$

$\Rightarrow \vec{a}.\left(\vec{b}\times\vec{c}\right)=P+\dfrac{q}{2}+\dfrac{r}{2}$ ...(i)

Similarly, $\vec{b}.\left(\vec{a}\times\vec{b}\right)+\vec{b}.\left(\vec{b}\times\vec{c}\right)=\dfrac{p}{2}+q+\dfrac{r}{2}$

$\Rightarrow \dfrac{p}{2}+q+\dfrac{r}{2}=0$ ...(ii) and $\frac{p}{2}+\frac{q}{2}+r=a(b\times c)$ ...(iii)

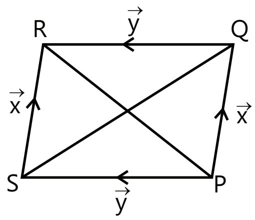
From (i), (ii) and (iii)

$P=-q=r \Rightarrow \frac{p^2+2q^2+r^2}{q^2}=4$

Sol 15: (C) $\Rightarrow \vec{x}+\vec{y}=3\hat{i}+\hat{j}-2\hat{k}$

and $\vec{x}-\vec{y}=\hat{i}-3\hat{j}-4\hat{k}$

On solving we get



$\vec{x}=2i-j-3k$

$\vec{y}=i+2j+k$

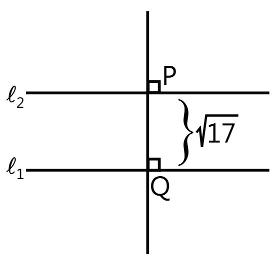
Volume of parallelopiped

$=\begin{vmatrix}2 & -1 & -3\\ 1 & 2 & 1\\ 1 & 2 & 3\end{vmatrix}$

$=2(6-2)+1(3-1)-3(2-0)$

$=8+2=10$

Sol 16: (B, D) Vector perpendicular to ℓ_1 and ℓ_2 is given by



$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & 1\\ 1 & 2 & 2\end{vmatrix}$

$=\hat{j}(4-2)-\hat{j}(4-1)+\hat{k}(4-2)$

$=2\hat{i}-3\hat{j}+2\hat{k}$

The eq. of line $\perp$ to ℓ_1 and ℓ_2

$\frac{x-0}{2}=\frac{y-0}{-3}=\frac{z-0}{2}=\gamma$

$\Rightarrow Q\equiv(2\gamma,-3\gamma,2\gamma)$

The point Q lies on ℓ_2, then $=\frac{2\gamma-3}{1}=\frac{-3\gamma+1}{2}=\frac{2\gamma-4}{2}$

$\Rightarrow \gamma=1$

$\Rightarrow Q\equiv(2,-3,2)$

Distance of P from Q in $\sqrt{17}$

$PQ^2=17=(2-3-2)^2+(-3-3-25)^2+(2-5-2)^2$

$\Rightarrow S=-2,\frac{-10}{9}$

$\Rightarrow P\equiv(-1,-1,0)$ and $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

Sol 17: V = $a\hat{i}+b\hat{j}+c\hat{k}:a,b,c\in\{-1,1\}$

Total number of selection $=8_{C_3}$

No. of coplanar vectors $=6\times 4=24$

Total number of non co-planar vet

$={}^8c_3-24=32=2^p$

$=P=5$

Sol 18: (C) (i) $\left[2\vec{a}\times b\ \ 3\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=6\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2$

$=6(2)^2=24\left[\vec{a}.\ \vec{b}.\ \vec{c}=2\right]$

(ii) $\left[3\left(\vec{a}+\vec{b}\right)\vec{b}+\vec{c}\ \ 2\vec{c}+\vec{a}\right]=6\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right]$

$=6\left(\vec{a}+\vec{b}.\left[\left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\right]\right)$

$=6\left(\vec{a}\times\vec{b}\right)\left[\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\right]=12a.(b\times c)$

$=12\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=12\times 5=60\quad\left(\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=5\right)$

(iii) $\frac{1}{2}\left|\left(2\vec{a}+3\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|$

$=\frac{1}{2}\left|\left(2\vec{a}\times\vec{a}-2\vec{a}\times\vec{b}+3\vec{b}\times\vec{a}-3\vec{b}\times b\right)\right|$

$=\frac{1}{2}\left|5\vec{a}\times\vec{b}\right|=\frac{1}{2}\times 5\times 40=100\left[\frac{1}{2}\left|\vec{a}\times\vec{b}\right|=20\right]$

(iii) $\left|\left(\vec{a}\times\vec{b}\right)\times\vec{a}\right|=\left|\vec{a}\times\vec{a}+\vec{b}\times\vec{a}\right|=\left|\vec{b}\times\vec{a}\right|$

$=30\left[\left|\vec{a}\times\vec{b}\right|=30\right]$

Sol 19: $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=9$

$\Rightarrow 3\left(\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2\right)-\left|\vec{a}+\vec{b}+\vec{c}\right|^2=9$

$\Rightarrow 3(1+1+1)-\left|\vec{a}+\vec{b}+\vec{c}\right|^2=9$

$\Rightarrow\left|\vec{a}+\vec{b}+\vec{c}\right|^2=0\Rightarrow\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow\vec{b}+\vec{c}=-\vec{a}$

Now, $\left|2\vec{a}+5\vec{b}+5\vec{c}\right|=\left|2\vec{a}+5\left(\vec{b}+\vec{c}\right)\right|$

$=\left|2\vec{a}-5\vec{a}\right|=3\left|\vec{a}\right|=3$

Sol 20: (C) Let $\vec{c}=2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{a}\times\vec{c}=\vec{c}\times b \Rightarrow (\vec{a}+\vec{b})\times\vec{c}=0$

$\Rightarrow (\vec{a}+\vec{b}) \parallel \vec{c}$

Let $(\vec{a}+\vec{b})=\lambda\vec{c}$

$\Rightarrow |\vec{a}+\vec{b}|=|\lambda||\vec{c}|$

$\Rightarrow \sqrt{29}=|\lambda|.\sqrt{29} \Rightarrow \lambda=\pm1$

$\therefore \vec{a}+\vec{b}=\pm(2\hat{i}+3\hat{j}+4\hat{k})$

Now, $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})=\pm(-14+6+12)=\pm4$

Sol 21: (C) Any vectors $\vec{v}$ coplanar with $\vec{a}$ and $\vec{b}$ is given by

$\vec{v}=m\vec{a}+n\vec{b}$

$=m(\hat{i}+\hat{j}+\hat{k})+n(\hat{i}+\hat{j}-\hat{k})$

$=(m+n)\hat{i}+(m-n)\hat{j}+(m+n)\hat{k}$ ... (i)

Projection to $\vec{v}$ on $\vec{c}$ is given by $\dfrac{\vec{v}.\vec{c}}{|\vec{c}|}=\dfrac{1}{\sqrt{3}}$

$\Rightarrow (m+n)-(m-n)-(m+n)=1$

$\Rightarrow m+n-m+n-m-n=1$

$\Rightarrow m+1=n$

$\Rightarrow m=n-1$

Substituting in (i)

$(2n-1)\hat{i}-\hat{j}+2n-1\hat{k}$

for $n=2$

$3\hat{i}-\hat{j}+3\hat{k}$

Sol 22: (A, D) Let $\vec{r}$ the vector coplanar with $i+j+2k$ and $i+2j+k$ then

$\vec{r}=m(i+j+2k)+n(i+2j+k)$

$=(m+n)i+(m+2n)j+(2m+n)k$

$\vec{r}\perp\vec{c}$, then

$m+n+m+2n+2m+n=0$

$\Rightarrow m+n=0$

$\Rightarrow \bar{r}=(0)i+(0+n)j+(n+0)k$

$=nj+mk$