

FORMULAE SHEET

Table: Domain and range of some standard functions-

Functions	Domain	Range
Polynomial function	\mathbb{R}	\mathbb{R}
Identity function x	\mathbb{R}	\mathbb{R}
Constant function K	\mathbb{R}	$\{K\}$
Reciprocal function $\frac{1}{x}$	\mathbb{R}_0	\mathbb{R}_0
$x^2, x $ (modulus function)	\mathbb{R}	$\mathbb{R}^+ \cup \{x\}$
$x^3, x x $	\mathbb{R}	\mathbb{R}
Signum function $\frac{ x }{x}$	\mathbb{R}	$\{-1, 0, 1\}$
$x + x $	\mathbb{R}	$\mathbb{R}^+ \cup \{x\}$
$x - x $	\mathbb{R}	$\mathbb{R}^- \cup \{x\}$
$[x]$ (greatest integer function)	\mathbb{R}	\mathbb{Z}
$x - \{x\}$	\mathbb{R}	$[0, 1]$
\sqrt{x}	$(0, \infty)$	$[0, \infty]$
a^x (exponential function)	\mathbb{R}	\mathbb{R}^+
$\log x$ (logarithmic function)	\mathbb{R}^+	\mathbb{R}

Inverse Trigo Functions	Domain	Range
$\sin^{-1}x$	$(-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1,1]$	$[0, \pi]$
$\tan^{-1}x$	\mathbb{R}	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	\mathbb{R}	$(0, \pi)$
$\sec^{-1}x$	$\mathbb{R} - (-1,1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\operatorname{cosec}^{-1}x$	$\mathbb{R} - (-1,1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Inverse function: f^{-1} exists iff f is both one-one and onto.

$$f^{-1}:B \rightarrow A, f^{-1}(b)=a \Rightarrow f(a)=b$$

Even and odd function: A function is said to be

- (a) Even function if $f(x)=f(-x)$ and
- (b) Odd function if $f(-x)=-f(x)$

Properties of even & odd function:

- (a) The graph of an even function is always symmetric about y-axis.
- (b) The graph of an odd function is always symmetric about origin.
- (c) Product of two even or odd function is an even function.
- (d) Sum & difference of two even (odd) function is an even (odd) function.
- (e) Product of an even or odd function is an odd function.
- (f) Sum of even and odd function is neither even nor odd function.
- (g) Zero function, i.e. $f(x) = 0$, is the only function which is both even and odd.
- (h) If $f(x)$ is an odd (even) function, then $f'(x)$ is even (odd) function provided $f(x)$ is differentiable on \mathbb{R} .
- (i) A given function can be expressed as sum of even and odd function.

$$\text{i.e. } f(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)] = \text{even function} + \text{odd function.}$$

Increasing function: A function $f(x)$ is an increasing function in the domain, D if the value of the function does not decrease by increasing the value of x .

Decreasing function: A function $f(x)$ is a decreasing function in the domain, D if the value of function does not increase by increasing the value of x .

Periodic function: Function $f(x)$ will be periodic if a +ve real number T exists such that

$$f(x + T) = f(x), \forall x \in \text{Domain.}$$

There may be infinitely many such real number T which satisfies the above equality. Such a least +ve number T is called period of $f(x)$.

(i) If a function $f(x)$ has period T , then period of $f(x/n+a)=T/n$ and period of $f(x/n+a)=nT$.

(ii) If the period of $f(x)$ is T_1 & $g(x)$ has T_2 then the period of $f(x) \pm g(x)$ will be L.C.M. of T_1 & T_2 provided it satisfies definition of periodic function.

(iii) If period of $f(x)$ & $g(x)$ are same T , then the period of $af(x)+bg(x)$ will also be T .

Function	Period
$\sin x, \cos x$	2π
$\sec x, \operatorname{cosec} x$	
$\tan x, \cot x$	π
$\sin(x/3)$	6π
$\tan 4x$	$\pi/4$
$\cos 2\pi x$	1
$ \cos x $	π
$\sin^4 x + \cos^4 x$	$\pi/2$
$2 \cos\left(\frac{x-\pi}{3}\right)$	6π
$\sin 3x + \cos^3 x$	$2\pi/3$
$\sin^3 x + \cos^4 x$	2π
$\frac{\sin x}{\sin 5x}$	2π
$\tan^2 x - \cot^2 x$	π
$x - [x]$	1
$[x]$	1