JEE Main/Boards

Exercise 1

Sets and Relations

Q.1 If R be a relation and N defined by $x + 2y = 8$ then find the domain of R.

Q.2 R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y = x - 3$ then find R^{-1}.

Q.3 Let A be the set of all students of a boy's school. Let relation R in set A is given by R = {(a, b) $\in$ A × A is a sister of b}. Can we say that R is an empty relation? Give reason.

Q.4 Let A = {1, 2} and B = {1, 3} and R be a relation from set A to set B defined as R = {(1, 1), (1, 3), (2, 1), (2, 3)}. Is R a universal relation? Explain.

Q.5 Let R be a relation in the set of natural numbers N, defined by R = {(a, b) $\in$ N × N : a < b}. Is relation R reflexive? Explain.

Q.6 Let A be any non-empty set and P(A) be the power set of A. A relation R defined on P(A) by $X R Y \Leftrightarrow X \cap Y = X$, Y $\in$ P(A). Examine whether R is symmetric.

Q.7 Let A = {a, b, c} and R is a relation in A given by R = {(a, a), (a, b), (a, c), (b, a), (c, c)}. Is R symmetric? Explain.

Q.8 Given a relation R = {(yellow, black), (cat, dog), (red, green)}. White R^{-1}.

Q.9 Let A = {1, 3, 5}, B = {9, 11} and let R = {(a, b)$\in$ A × B: a – b is odd}. Write the relation r.

Q.10 Let A = {a, b, c} and relation R in the set A be given by R = {(a, c), (c, a)}. Is relation R symmetric? Explain.

Functions

Q.1 If $f(x) = \dfrac{2\tan x}{1+\tan^2 x}$, then find $f\left(\dfrac{\pi}{4}\right)$.

Q.2 If $f(x) = \dfrac{|x|}{x}$, $x \neq 0$, prove that $|f(\alpha) - f(-\alpha)| = 2\alpha$, $a \neq 0$.

Q.3 If $f(x) = \log\left(\dfrac{1+x}{1-x}\right)$, prove that $f\left(\dfrac{2x}{1+x^2}\right) = 2f(x)$.

Q.4 Find the domain of the function $f(x) = \dfrac{x^2+2x+1}{x^2-8x+12}$

Q.5 Find the domain of definition of the function $y = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

Q. 6 Find the range of the function $y = \dfrac{x}{1+x^2}$.

Q.7 Find the domain and range of the function $\left\{\left(x, \dfrac{1}{1-x^2} : x \in R, x \neq \pm 1\right)\right\}$.

Q.8 Find the domain and range of the function $y = \dfrac{1}{2-\sin 3x}$.

Q.9 If f: R $\to$ R is defined by $f(x) = x^3 + 1$ and g : R $\to$ R is defined by $g(x) = x + 1$, then find f + g, f – g, f · g, $\dfrac{f}{g}$ and α f(a $\in$ R).

Q.10 Let f: R $\to$ R is defined by f(x) = x and g : R $\to$ R is defined by g(x) = | x |. Find

(i) f + g (ii) f – g

(iii) f · g (iv) a f, a $\in$ R

(v) f/g

Q.11 Let f be the exponential function and g be the logarithmic function defined by $f(x) = e^x$ and $g(x) = \log_e x$. Find

(i) (f + g) (1) (ii) (f – g) (1) (iii) (f · g) (1)

Q.12 If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where [x] denotes the integral part of x, write the value of $f\left(\dfrac{\pi}{2}\right)$.

Q.13 ({x}, represents fractional part function)

(i) Domain of the function

$f(x) = \ln(1 - \{x\}) + \sqrt{\sin x + \frac{1}{2}} + \sqrt{4 - x^2}$ is ______.

(ii) Range of the function cos(2 sin x) is ______.

(iii) Period of the function

$f(x) = \sin\left(\frac{\pi x}{3}\right) + \{x\} + \tan^2(\pi x)$ is ______.

Q.14 Let A = {1, 2, 3}, B = {3, 6, 9, 10}. Which of the following relations are functions from A to B? Also find their range if they are function.

f = {(1, 9), (2, 3), (3, 10)}

g = {(1, 6), (2, 10), (3, 9), (1, 3)}

h = {(2, 6), (3, 9)}

u = {(x, y): y = 3x, x ∈ A}

Q.15 Let A = {a, b, c, d}. Examine which of the following relation is a function on A?

(i) f = {(a, a), (b, c), (c, d), (d, c)}

(ii) g = {(a, c), (b, d), (b, c)}

(iii) h = {(b, c), (d, a), (a, a)}

Q.16 (i) Let f = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a function from Z to Z defined by f(x) = ax + b for some integers a, b determine a and b.

(ii) Let f = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a linear function from Z to Z, find f(x).

Q.17 Function f is given by f = {(4, 2), (9, 1), (6, 1), (10, 3)}. Find the domain and range of f.

Q.18 If A = {(–3, –2, –1, 0, 1, 2, 3)} and $f(x) = x^2 - 1$ defines f: A → R. Then find range of f.

Q.19 Find the domain and range of the following functions.

(i) f(x) = x (ii) f(x) = 2 – 3x

(iii) $f(x) = x^2 - 1$ (iv) $f(x) = x^2 + 2$ (v) $f(x) = \sqrt{x - 1}$

Q.20 Find the domain of the function $f(x) = \frac{x^2 + 3x + 5}{x^2 - 5x + 4}$

Q.21 Find the domain of the definition and range of the function defined by the rules:

(i) $f(x) = x^2$ (ii) $g(x) = |x|$

(iii) $h(x) = \frac{1}{3 - x^2}$ (iv) $u(x) = \sqrt{4 - x^2}$

Q.22 Consider the following rules:

(i) f: R → R : $f(x) = \log_e x$

(ii) g: R → R : $g(x) = \sqrt{x}$

(iii) h: A → R : $h(x) = \frac{1}{x^2 - 4}$, where A = R – {–2, 2}

Which of them are functions? Also find their range, if they are function.

Q.23 Let f: R – {2} → R be defined by $f(x) = \frac{x^2 - 4}{x - 2}$ and g: R → R be defined by g(x) = x + 2. Find whether f = g or not.

Q.24 Let f = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a linear function from Z into Z and g(x) = x. Find f + g.

Q.25 Find f + g, f – g, f · g, f/g and a f (a ∈ R) if

(i) $f(x) = \frac{1}{x + 4}$, $x \neq -4$ and $g(x) = (x + 4)^3$

(ii) f(x) = cos x, $g(x) = e^x$.

Q.26 If f(x) = x, g(x) = | x |, find (f+g)(–2), (f – g) (2), (f·g)(2), $\left(\frac{f}{g}\right)(-2)$, and 5f(2).

Q.27 Define the function f: R → R by $y = f(x) = x^2$. Complete the table given below:

x	–4	–3	–2	–1	0	1	2	3	4
$y=f(x)=x^2$									

Q.28 Define the real valued function on

f: R – {0} → R as f(x) = 1/x

Complete the figure given below :

x	–2	–1.5	–1	–0.5	0	1	1	2	2
$y=f(x)=\frac{1}{x_1}$									

Find the domain and range of f.

Q.29 If $f(x + 3) = x^2 - 1$, write the expression for f(x).

Exercise 2

Sets and Relations

Single Correct Choice Type

Q.1 Let A = {1, 2, 3, 4}, and let R = {(2, 2),(3, 3),(4, 4), (1, 2)} be a relation on A. Then R is

(A) Reflexive (B) Symmetric

(C) Transitive (D) None of these

Q.2 The void relation on a set A is

(A) Reflexive

(B) Symmetric and transitive

(C) Reflexive and symmetric

(D) Reflexive and transitive

Q.3 For real number x and y, we write x R y ⇔ x – y + is an irrational number. Then the relation R is

(A) Reflexive (B) Symmetric

(C) Transitive (D) None of these

Q.4 Let R be a relation in N defined by

$R = \{(1 + x, 1 + x^2) : x \le 5, x \in N\}$.

Which of the following is false

(A) R = {(2, 2), (3, 5), (4, 10), (5, 17), (6, 25)}

(B) Domain of R = {2, 3, 4, 5, 6}

(C) Range of R = {2, 5, 10, 17, 26}

(D) None of these

Q.5 The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on the set A = {1, 2, 3} is

(A) Reflexive but not symmetric

(B) Reflexive but not transitive

(C) Symmetric and transitive

(D) Neither symmetric nor transitive

Q.6 Let A = {2, 3, 4, 5} and let R = {(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)} be a relation in A. Then R is

(A) Reflexive and transitive

(B) Reflexive and symmetric

(C) Reflexive and anti-symmetric

(D) None of these

Q.7 If A = {2, 3} and B = {1, 2}, then A × B is equal to

(A) {(2, 1), (2, 2), (3, 1), (3, 2)}

(B) {(1, 2), (1, 3), (2, 2), (2, 3)}

(C) {(2, 1), (3, 2)}

(D) {(1, 2), (2, 3)}

Q.8 If A = {1, 2, 3} and B = {1, 4, 6, 9} and R is a relation from A to B defined by ‘x is greater than y’. The range of R is

(A) {1, 4, 6, 9} (B) {4, 6, 9}

(C) {1} (D) None of these

Q.9 Let R = {(1, 3), (4, 2), (2, 4),(2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is

(A) Transitive (B) Not symmetric

(C) Reflexive (D) A function

Q.10 If A, B and C are these sets such that A ∩B = A ∩ C and A ∪ B = A ∪ C, then

(A) A = B (B) A = C

(C) B = C (D) A ∩ B = f

Q.11 The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on the set A = {1, 2, 3} is

(A) Reflexive but not symmetric

(B) Reflexive but not transitive

(C) Symmetric and transitive

(D) Neither symmetric nor transitive

Q.12 Let A be the set of all children in the world and R be a relation in A defined by x R y if x and y have same sex. Then R is

(A) Not reflexive (B) Not symmetric

(C) Not transitive (D) An equivalence relation

Q.13 Let A = {2, 3, 4, 5} and R = {(2 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)} be a relation on A. Then R is

(A) Reflexive and transitive

(B) Reflexive and symmetric

(C) An equivalence relation

(D) None of these

Q.14 Let L be the set of all straight lines in the xy-plane. Two lines l_1 and l_2 are said to be related by the relation R if l_1 is parallel to l_2. Then the relation R is

(A) Reflexive (B) Symmetric

(C) Transitive (D) Equivalence

Q.15 Given the relation R = {(2, 3), (3, 4)} on the set {2, 3, 4}. The number of minimum number of ordered pair to be added to R so that R is reflexive and symmetric

(A) 4 (B) 5 (C) 7 (D) 6

Q.16 The minimum number of elements that must be added to the relation R = {(1, 2),(2, 3)} on the set {1, 2, 3}, so that it is equivalence is

(A) 4 (B) 7 (C) 6 (D) 5

Functions

Single Correct Choice Type

Q.1 If f(x+ay, x–ay) = ay then f(x, y) is equal to:

(A) $\frac{x^2-y^2}{4}$ (B) $\frac{x^2+y^2}{4}$

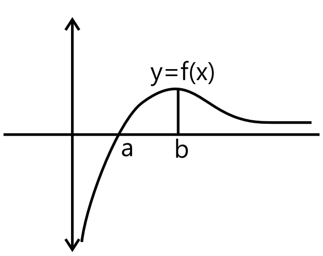
(C) 4xy (D) None of these

Q.2 The set of values of 'a' for which f: R → R f(x) = ax + cos x is bijective is

(A) [–1, 1] (B) R – {–1, 1}

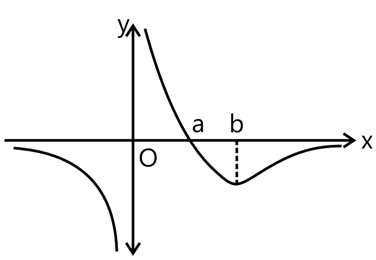
(C) R – (–1, 1) (D) R – {0}

Q.3 The graph of function f(x) is as shown, adjacently

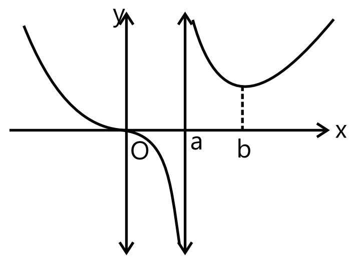


Then the graph of $\frac{1}{f(|x|)}$ is

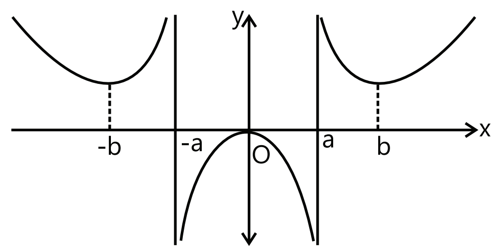
(A)



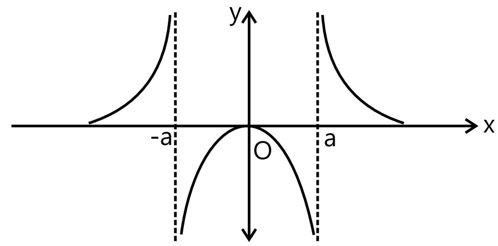
(B)



(C)



(D)



Q.4 Period of the function f(x) = $\frac{1}{2}\left(\frac{|\sin x|}{\cos x}+\frac{\sin x}{|\cos x|}\right)$ is

(A)π /2 (B) p

(C) 2 p (D) 4p

Q.5 Let f : R → R be a function defined by

$f(x) = \frac{2x^2-x+5}{7x^2+2x+10}$ then f is

(A) One–one but not onto

(B) Onto but not one–one

(C) Onto as well as one–one

(D) Neither onto nor one–one

Q.6 If f(x) = $\cos\left[\frac{1}{2}\pi^2\right]x + \sin\left[\frac{1}{2}\pi^2\right]x$, [x] denoting the greatest integer function, then

(A) f(0) = 0 (B) $f\left(\frac{\pi}{3}\right)=\frac{1}{4}$

(C) $f\left(\frac{\pi}{2}\right)=1$ (D) f(π) = 0

Q.7 Let f(x)= ln x and g(x) = $\frac{x^4 - x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$.

The domain of the composite function fog(x) is

(A) $(-\infty, \infty)$ (B) $(0, \infty)$

(C) $(0, \infty)$ (D) $(1, \infty)$

Previous Years' Questions

Q.1 Let f(x) = | x – 1 |. Then, ***(1983)***

(A) $f(x^2) = [f(x)]^2$

(B) f(x + y) = f(x) + f(y)

(C) f(| x |) = | f(x) |

(D) None of the above

Q.2 If f(x) = cos(log x), then $f(x).f(y) - \frac{1}{2}\frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$ has the value ***(1983)***

(A) –1 (B) $\frac{1}{2}$

(C) –2 (D) None of these

Q.3 The domain of definition of the function ***(1983)***

$y = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ is

(A) (–3,–2) excluding –2.5

(B) [0, 1] excluding 0.5

(C) (–2, 1) excluding 0

(D) None of the above

Q.4 Which of the following functions is periodic? ***(1983)***

(A) f(x) = x – [x] where [x] denotes the greatest integer less than or equal to the real number x

(B) f(x) = sin $\frac{1}{x}$ for x 0, f(0) = 0

(C)f(x) = x cos x

(D) None of the above

Q.5 For real x, the function $\frac{(x-a)(x-b)}{(x-c)}$ will assume all real values provided ***(1984)***

(A) a > b > c (B) a < b < c

(C) a > c < b (D) $a \le c \le b$

Q.6 If g{f(x)} = | sin x| and f{g(x)} = $(\sin \sqrt{x})^2$, then ***(1998)***

(A) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$

(B) f(x) = sinx, g(x) = | x |

(C) $f(x) = x^2$, $g(x) = \sin \sqrt{x}$

(D) f and g cannot be determined

Q.7 If f(x) = 3x – 5, then $f^{-1}(x)$ ***(1998)***

(A) Is given by $\frac{1}{3x-5}$

(B) Is given by $\frac{x+5}{3}$

(C) Does not exist because f is not one–one

(D) Does not exist because f is not onto

Q.8 If the function f : $[1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is ***(1999)***

(A) $\left(\frac{1}{2}\right)^{x(x-1)}$ (B) $\frac{1}{2}(1+\sqrt{1+4\log_2 x})$

(C) $\frac{1}{2}(1-\sqrt{1+4\log_2 x})$ (D) Not defined

Q.9 Let f(θ) = sin θ (sin θ + sin 3θ). Then f(θ) ***(2000)***

(A) ≥ 0 only when $q \ge 0$ (B) ≤ 0 for all real q

(C) ≥ 0 for all real q (D) ≤ 0 only when $\theta \le 0$

Q.10 The domain of definition of the function y(x) is given by the equation $2^x + 2^y = 2$, is ***(2000)***

(A) $0 < x \le 1$ (B) $0 \le x \le 1$

(C) $-\infty < x \le 0$ (D) $-\infty < x < 1$

Q.11 Let $f: N \to Y$ be a function defined as f (x) = 4x + 3, where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$. Show that f is invertible and its inverse is ***(2008)***

(A) $g(y) = \frac{3y+4}{3}$ (B) $g(y) = 4 + \frac{y+3}{4}$

(C) $g(y) = \frac{y+3}{4}$ (D) $g(y) = \frac{y-3}{4}$

Q.12 For real x, let $f(x) = x^3 + 5x + 1$, then ***(2009)***

(A) f is one-one but not onto R

(B) f is onto R but not one-one

(C) f is one-one and onto R

(D) f is neither one-one nor onto R

Statement-I (assertion) and statement-II (reason).

Each of these questions also have four alternative choices, only one of which is the correct answer. You have to select the correct choice

Q.13 Let $f(x)=(x+1)^2-1,\ x\geq -1$

Statement-I : The set

$\{x:f(x)=f^{-1}(x)\}=\{0,-1\}$

Statement-2 : f is a bijection. ***(2009)***

(A) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I

(B) Statement-I is true, statement-II is true; statement-II is nota correct explanation for statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Q.14 Let $f(x)=x\,|x|$ and $g(x)=\sin x$ ***(2009)***

Statement-I : gof is differentiable at x = 0 and its derivative is continuous at that point. Statement-II : gof is twice differentiable at x = 0.

(A) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I

(B) Statement-I is true, statement-II is true; statement-II is nota correct explanation for statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Q.15 Consider the following relations:

$R=\left\{\begin{matrix}(x,y)\ |\ x,\text{ are real numbers and }x\\ =wy\text{ for some rational number }w\end{matrix}\right\}$

$S=\left\{\left(\frac{m}{n},\frac{p}{q}\right)\middle|\begin{matrix}m,n,p\text{ and }q\text{ are integers such that }n,\\ q\neq 0\text{ and }qm=pn\end{matrix}\right\}$

Then ***(2010)***

(A) Neither R nor S is an equivalence relation

(B) S is an equivalence relation but R is not an equivalence relation

(C) R and S both are equivalence relations

(D) R is an equivalence relation but S is not an equivalence relation

Q.16 The domain of the function

$f(x)=\frac{1}{\sqrt{|x|-x}}$ is ***(2011)***

(A) $(0,\infty)$ (B) $(-\infty,0)$

(C) $(-\infty,\infty)-\{0\}$ (D) $(-\infty,\infty)$

Q.17 If $f:R\to R$ is a function defined by

$f(x)=[x]\cos\left(\frac{2x-1}{2}\right)\pi,$

where [x] denotes the greatest integer function, then f is ***(2012)***

(A) Continuous for every real x

(B) Discontinuous only at x = 0

(C) Discontinuous only at non-zero integral values of x

(D) Continuous only at x = 0

Q.18 Consider the function

$f(x)=|x-2|+|x-5|,\ x\in R$. ***(2012)***

Statement-I: f′(4) = 0

Statement-II: f is continuous in [2, 5], differentiable in (2, 5) and f(2) = f(5).

(A) Statement-I is false, statement-II is true

(B) Statement-I is true, statement-II is true; statement-II is a correct explanation for statement-I

(C) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I

(D) Statement-I is true, statement-II is false

Q.19 If $a\in R$ and the equation

$-3(x-[x])^2+2(x-[x])+a^2=0$

(where [x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval ***(2014)***

(A) (-2, -1) (B) $(-\infty,-2)\cup(2,\infty)$

(C) $(-1,0)\cup(0,1)$ (D) (1, 2)

Q.20 Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals. ***(2014)***

(A) $\frac{1}{4}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$

Q.21 If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$, and $S = \left\{x \in R : f(x) = f(-x)\right\}$; then S: ***(2016)***

(A) Contains exactly one element

(B) Contains exactly two elements.

(C) Contains more than two elements.

(D) Is an empty set.

JEE Advanced/Boards

Exercise 1

Sets and Relations

Q.1 Is set a collection of objects or a collection of well-defined objects which are distinct and distinguishable?

Q.2 Is the set { x : x ∈ N, x is prime and 3 < x < 5} is void or non-void?

Q.3 A = {a, e, i, o, u} and B = {i, o} then $A \subset B$ or $B \subset A$?

Q.4 A set is defined as A = {x : x is irrational and 0.1 < x < 0.101} then comment on whether A is null set or A is finite set or A is infinite set.

Q.5 Which of these: f, {}, {2, 3} and {ϕ} is a singleton Set?

Q.6 Two points A and B in a plane are related if OA = OB, where O is a fixed point. Then comment whether this relation is reflexive, symmetric, transitive or equivalence?

Q.7 If A = {2, 3} and B = {-2, 3}, then what is the value of A ∪ B?

Q.8 Given the sets A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, then what is the value of A ∪(B ∩ C)?

Q.9 If N_a = {an : n ∈ N}, then what is the value of $N_6 \cap N_8$?

Q.10 Is it true that both I = { x : x ∈ R and $x^2 + x + 1 = 0$}, II = { x : x ∈ R and $x^2 - x + 1 = 0$} are empty sets?

Q.11 Two finite sets have m and n elements respectively. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Calculate the values of m and n

Q.12 If A = {x | x/2 ∈ Z, 0 ≤ x ≤ 10}. B = { x | x is one digit prime} C= {x | x/3 ∈ N, x ≤ 12}, Then what is the value of A ∩ (B ∪ C) ?

Q.13 If n(A) = 10, n(B) = 15 and n(A ∪ B) = x, then what is the range of x?

Q.14 Among 1000 families of a city, 40% read newspaper A, 20% read newspaper B, 10% read newspaper C, 5% read both A and B, 3% read both B and C, 4% read A and C and 2% read all three newspapers. What is the number of families which read only newspaper A?

Q.15 If for three disjoint sets A, B, C; n(A) = 10, n(B) = 6 and n(C) = 5, then what is the value of n(A ∪ B ∪ C) ?

Q.16 If A and B are disjoint, then what is the value of n(A ∪ B)?

Q.17 If X and Y are two sets, then what is the value of $X \cap (Y \cup X)^c$?

Q.18 Let n(U) = 700, n(A) = 200, n(B) = 300 and n(A ∩ B) = 100, then what is the value of $n(A^c \cap B^c)$?

Q.19 What is the value of set $(A \cap B^c)^c \cup (B \cap C)$?

Q.20 Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in A ∪ B?

Q.21 In a class of 100 students, 55 students have passed in Mathematics and 67 students have passed in physics, no student fails. Calculate the number of student who have passed in Physics only?

Q.22 Let X = {1, 2, 3, 4, 5, 6} be an universal set. Sets A, B, C in the universal set X be defined by A = {1, 2, 3}, B = {2, 4,5} and C = {3, 4, 5, 6}, then what is the value of (A – B) ∪ (B – A), (A – B) – C and A ∩ C′?

Q.23 If A, B and C are any three sets, then is A × (B ∪ C) equal to (A × B) ∪ (A × C) or (A × B) ∩(A × C)?

Q.24 If A, B and C are any three sets, then is A × (B ∩ C) equal to (A × B) ∩ (A ×C) or (A ∩ B) × (A ∩C)?

Q.25 Let A = {a, b, c, d}, B = {b, c, d, e}. Then what is the value of [(A × B) ∩ (B × A)]?

Q.26 In the set A= {1, 2, 3, 4, 5}, a relation R is defined by R = {(x, y) | x, y ∈ A and x < y}. Then is R reflexive or transitive or symmetric?

Q.27 Let R be a relation on the set N of natural numbers defined by nRm ⇔ is a factor of m (i.e., n | m). Then is R symmetric?

Q.28 If R is a relation from a finite set A having m elements to a finite set B having n elements, then what will be the number of relations from A to B ?

Q.29 Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha \, R \, \beta \Leftrightarrow \alpha \perp \beta$, $\alpha, \beta \in L$. Then is R symmetric?

Functions

Q.1 Find the domain of the definitions of the following functions: (Read the symbol [*] and {*} as greatest integers and fractional part functions respectively.)

(i) $y = \sqrt{\dfrac{x-2}{x+2}} + \sqrt{\dfrac{1-x}{1+x}}$

(ii) $y = \sqrt{x^2-3x+2} + \sqrt{\dfrac{1}{3+2x-x^2}}$

(iii) $y = \sqrt{x} + \sqrt[3]{\dfrac{1}{x-2}} - \log_{10}(2x-3)$

(iv) $f(x) = \sqrt{\dfrac{1-5^x}{7^{-x}-7}}$

(v) $y = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

(vi) $f(x) = \log_{100x}\left(\dfrac{2\log_{10}x+1}{-x}\right)$

(vii) $f(x) = y = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

(viii) $y = \sqrt{\log_{10}\left(\dfrac{5x-x^2}{4}\right)}$

(ix) $f(x) = \sqrt{x^2-|x|} + \dfrac{1}{\sqrt{9-x^2}}$

(x) $f(x) = \sqrt{(x^2-3x-10)\,\ell n^2(x-3)}$

(xi) $f(x) = \sqrt{(\sin x + \cos x)^2 - 1}$

(xii) $f(x) = \sqrt{\dfrac{\cos x-(1/2)}{6+35x-6x^2}}$

(xiii) $f(x) = \sqrt{\log_{1/3}(\log_4([x]^2-5))}$

(xiv) $f(x) = \dfrac{1}{[x]} + \log_{(2|x|-5)}(x^2-3x+10) + \dfrac{1}{\sqrt{1-|x|}}$

(xv) $f(x) = \log_7\log_5\log_3\log_2(2x^3+5x^2-14x)$

(xvi) $f(x) = \sqrt{\cos 2x} + \sqrt{16-x^2}$

(xvii) $f(x) = \ln(\sqrt{x^2-5x-24} - x - 2)$

(xvii) $y = \log10\,(1-\log_{10}(x^2-5x+16))$

(xix) $f(x) = \log4\left(2-\sqrt[4]{x}-\dfrac{2\sqrt{x}+1}{\sqrt{x}+2}\right)$

Q.2 Find the domain and range of the following functions. (Read the symbols [*] and {*} as greatest integers and fractional part function respectively)

(i) $y = \log_{\sqrt{5}}\left(\sqrt{2}(\sin x - \cos x) + 3\right)$

(ii) $y = \dfrac{2x}{1+x^2}$

(iii) $f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}$

(iv) $f(x) = \frac{x}{1 + |x|}$

(v) $y = \sqrt{2 - x} + \sqrt{1 + x}$

(vi) $f(x) = \log_{(\operatorname{cosec} x - 1)}(2 - [\sin x] - [\sin x]^2)$

(vii) $f(x) = \frac{x + 1}{x - 2}$

Q.3 Classify the following functions f(x) defined in $R \to R$ as injective, surjective, both or none.

(a) $f(x) = \frac{x^2 + x + 1}{x^2 + 2x + 3}$

(b) $f(x) = (x^2 + 5x + 9)(x^2 + 5x + 1)$

Q.4 Let $f(x) = \frac{1}{1 - x}$. Let $f_2(x)$ denote f[f(x)] and $f_3(x)$ denotes f[f{f(x)}]. Find $f_{3n}(x)$ where n is a natural number. Also state the domain of this composite function.

Q.5 The function f(x) is defined as follows : on each of the intervals $n \le x < n + 1$, where n is a positive integer, f(x) varies linearly, and $f(n) = -1$, $f\left(n + \frac{1}{2}\right) = 0$. Draw the graph of the function.

Q.6 (a) For what values of x is the inequality $|f(x) + \varphi(x)| < |f(x)| + |\phi(x)|$ true if, $f(x) = x - 3$, and $\varphi(x) = 4 - x$.

(b) For what values of x is the inequality $|f(x) - \varphi(x)| > |f(x)| - |\phi(x)|$ true if, $f(x) = x$, and $\varphi(x) = x - 2$.

Q.7 Find whether the following functions are even or odd or none:

(a) $f(x) = \log(x + \sqrt{1 + x^2})$

(b) $f(x) = \frac{(a^x + 1)}{a^x - 1}$

(c) $f(x) = x^4 - 2x^2$

(d) $f(x) = x^2 - |x|$

(e) $f(x) = x \sin^2 x - x^3$

(f) f(x) = K, where K is constant

(g) $f(x) = \ln\left(\frac{1 - x}{1 + x}\right)$

(h) $f(x) = \frac{(1 + 2^x)^2}{2^x}$

(i) $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$

(j) $f(x) = [(x + 1)^2]^{1/3} + [(x - 1)^2]^{1/3}$

Q.8 Find the period for each of the following functions:

(a) $f(x) = \sin^4 x + \cos^4 x$

(b) $f(x) = |\sin x| + |\cos x|$

(c) $f(x) = \cos\frac{3}{5}x - \sin\frac{2}{7}x$

Q.9 Write explicitly, function of y defined by the following equations and also find the domains of definition of the given implicit functions:

(a) $10^x + 10^y = 10$ (b) $x + |y| = 2y$

Q.10 Find out for what integral values of n the number 3π is a period of the functions: $f(x) = \cos nx \cdot \sin(5/n)x$.

Q.11 Compute the inverse of the functions:

(a) $f(x) = \ln(x + \sqrt{x^2 + 1})$ (b) $f(x) = 2^{\frac{x}{x-1}}$

(c) $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$

Q.12 Show if $f(x) = \sqrt[n]{a - x^n}$, $x > 0$, $n \ge 2$, $n \in N$, then $(f \circ f)(x) = x$. Find also the inverse of f(x).

Q.13 $f: \left[-\frac{1}{2}, \infty\right) \to \left[-\frac{9}{4}, \infty\right)$,

Defined as $f(x) = x^2 + x - 2$. Find $f^{-1}(x)$ and solve the equation $f(x) = f^{-1}(x)$.

Q.14 f(x) is defined for x <0 as

$$f(x) = \begin{cases} x^2 + 1 & x < -1 \\ -x^3 & -1 \le x < 0 \end{cases}$$

Define f(x) for $x \ge 0$, if f is

(a) Odd (b) Even

Q.15 If $f(x) = \max\left(x, \frac{1}{x}\right)$ for $x > 0$ where $\max(a, b)$ denotes the greater of the two real numbers a and b. Define the function $g(x) = f(x)\, f\left(\frac{1}{x}\right)$ and plot its graph.

Q.16 Show that the function $f(x) = \frac{x^2 + 2x + c}{x^2 + 4x + 3c}$ attains any real value if $0 < c \le 1$.

Q.17 (a) Find the domain and range of the function

$f(x) = \sqrt{\log_2(x^2 - 2x + 2)}$

(b) $f : R - \{2\} \to R - \{2\}$; $f(x) = \frac{2x+3}{x-2}$, find whether f(x) is bijective or not.

Q.18 Prove that function $f(x) = 1 + 2^{\sqrt{\frac{x(x-2)+1}{4}}}$ is many one.

Exercise 2

Sets and Relations

Single Correct Choice Type

Q.1 Let R be a relation on the set N of natural numbers defined by nRm⇔ n is a factor of m (i.e. n | m). Then R is

(A) Reflexive and symmetric

(B) Transitive and symmetric

(C) Equivalence

(D) Reflexive, transitive but not symmetric

Q.2 Let R be a relation defined in the set of real numbers by $a\,R\,b \Leftrightarrow 1 + ab > 0$. Then R is

(A) Equivalence relation

(B) Transitive

(C) Symmetric

(D) Anti–symmetric

Q.3 Which one of the following relations on R is equivalence relation

(A) $x\,R_1\,y \Leftrightarrow |x| = |y|$

(B) $x\,R_2\,y \Leftrightarrow x \ge y$

(C) $x\,R_3\,y \Leftrightarrow x \mid y$

(D) $x\,R_4\,y \Leftrightarrow x < y$

Q.4 The relation R defined in A = {1, 2, 3} by aRb if $|a^2 - b^2| \le 5$. Which of the following is false

(A) R = {(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)}

(B) $R^{-1} = R$

(C) Domain of R = {1, 2, 3}

(D) Range of R = {5}

Q.5 Let a relation R in the set N of natural numbers be defined as (x, y) ∈ R if and only if $x^2 - 4xy + 3y^2 = 0$ for all x, y ∈N. The relation R is

(A) Reflexive

(B) Symmetric

(C) Transitive

(D) An equivalence relation

Q.6 Let R {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}. The relation is

(A) An equivalence relation

(B) Reflexive and symmetric only

(C) Reflexive and transitive only

(D) Reflexive only

Q.7 Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is

(A) Reflexive (B) Transitive

(C) Not symmetric (D) A function

Q.8 Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if $ad(b + c) = bc(a + d)$, then R is

(A) Symmetric only

(B) Reflexive only

(C) Transitive only

(D) An equivalence relation

Q.9 Let W denote the words in the English dictionary. Define the relation R by R = {(x, y) ∈ W × W | the words x and y have at least one letter in common}. Then R is

(A) Reflexive, symmetric and not transitive

(B) Reflexive, symmetric and transitive

(C) Reflexive, not symmetric and transitive

(D) Not reflexive, symmetric and transitive

Q.10 Let R be the real line. Consider the following subsets of the plane R × R :

S = {(x, y) : y = x + 1 and 0 < x < 2}

T = {(x, y) : x – y is an integer}

Which one of the following is true?

(A) Both Sand T are equivalence relations on R

(B) S is an equivalence relation on R but T is not

(C) T is an equivalence relation on R but S is not

(D) Neither S nor T is an equivalence relation on R

Multiple Correct Choice Type

Q.11 Let X = {1, 2, 3, 4, 5} and Y = {1, 3, 5, 7, 9}. Which of the following is/are relations from X to Y

(A) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$

(B) R_2 = {(1, 1),(2, 1), (3, 3), (4, 3), (5, 5)}

(C) R_3 = {(1, 1),(1, 3), (3, 5), (3, 7), (5, 7)}

(D) R_4 = {(1, 3),(2, 5), (2, 4), (7, 9)}

Functions

Q.1 Domain of the function,

$f(x) = \sin^{-1}\sqrt{x - x^2} + \sec^{-1}\left(\frac{1}{x}\right) + \ln x$ is

(A) [0, 1) (B) (0, 1] (C) (0, 1) (D) None of these

Q.2 In the square ABCD with side AB = 2, two points M and N are on the adjacent sides of the square such that MN is parallel to the diagonal BD. If x is the distance of MN from the vertex A and f(x) = Area (ΔAMN) then range of f(x) is

(A) $(0, \sqrt{2}]$ (B) (0, 2] (C) $(0, 2\sqrt{2}]$ (D) $(0, 2\sqrt{3}]$

Q.3 If ‘f’ and ‘g’ are bijective functions and gof is defined the, gofis :

(A) Injective (B) Surjective

(C) Bijective (D) Into only

Q.4 If y = 5[x] + 1 = 6[x – 1] – 10, where [.] denotes the greatest integer function, then [x + 2y] is equal to

(A) 76 (B) 61 (C) 107 (D) 189

Q.5 f : R → R, $f(x) = ax^3 + e^x$ is one–one onto, then ‘a’ belongs to the interval

(A) (–∞, 0) (B) (–∞, 0] (C) [0, ∞) (D) (0, ∞)

Q.6 The value of x in [–2π, 2π], for which the graph of the function $y = \sqrt{\frac{1+\sin x}{1-\sin x}} - \sec x$ and $y = -\sqrt{\frac{1-\sin x}{1+\sin x}} + \sec x$, coincide are

(A) $\left[-2\pi, -\frac{3\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$

(B) $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, -\frac{3\pi}{2}\right)$

(C) [–2 π, 2π]

(D) $[-2\pi, 2\pi] - \left\{\pm\frac{\pi}{2}, \pm\pi, \pm\frac{3\pi}{2}\right\}$

Q.7 The period of the function

$f(x) = \sin\left(\cos\frac{x}{2}\right) + \cos(\sin x)$ equal

(A) $\frac{\pi}{2}$ (B) 2π (C) p (D) 4p

Q.8 Let f : R → R $f(x) = \frac{x}{1+|x|}$. Then f(x) is

(A) Injective but not surjective

(B) Surjective but not injective

(C) Injective as well as surjective

(D) Neither injective nor surjective

Multiple Correct Choice Type

Q.9 Let f : I → R (where I is the set of positive integers) be a function defined by, $f(x) = \sqrt{x}$, then f is

(A) One–one (B) Many one

(C) Onto (D) Into

Q.10 The function $f(x) = \sqrt{\log_{x^2} x}$ is defined for x belonging to

(A) (–∞, 0) (B) (0, 1) (C) (1, ∞) (D) (0, ∞)

Q.11 If $f(x) = \frac{x\sqrt{x - 2\sqrt{x-1}}}{\sqrt{x-1} - 1}$, then

(A) f(x) = –x if x < 2

(B) 2f(1.5) + f(3) is non negative integer

(C) f(x) = x if x > 2

(D) None

Q.12 Which of the following function(s) is(are) bounded on the intervals as indicated

(A) $f(x) = 2^{\frac{1}{x-1}}$ on (0, 1)

(B) $g(x) = x\cos\frac{1}{x}$ on $(-\infty, \infty)$

(C) $h(x) = xe^{-x}$ on $(0, \infty)$

(D) $\ell(x) = \arctan 2^x$ on $(-\infty, \infty)$

Q.13 Which of the following function(s) is/are periodic?

(A) f(x) – x – [x]

(B) $g(x) = \sin(1/x)$, $x \neq 0$ and $g(0) = 0$

(C) $h(x) = x\cos x$

(D) $w(x) = (\sin x)$

Q.14 On the interval [0, 1], f(x) is defined as,

$$f(x) = \begin{cases} x & \text{if } x \in Q \\ 1-x & \text{if } x \notin Q \end{cases}.$$

Then for all $x \in R$ the composite function f[f(x)] is

(A) A constant function

(B) An identity function

(C) An odd linear polynomial

(D) 1 + x

Q.15 Identify the pair(x) of functions which are identical.

(A) $y = \tan(\cos^{-1} x) : y = \frac{\sqrt{1-x^2}}{x}$

(B) $y = \tan(\cos^{-1}) : y = 1/x$

(C) $y = \sin(\arctan x) : y = \frac{x}{\sqrt{1+x^2}}$

(D) $y = \cos(\arctan x) : y = \sin(\text{arc}\cot x)$

Previous Years' Questions

Q.1 If $y = f(x) = \frac{x+2}{x-1}$, then ***(1984)***

(A) $x = f(y)$

(B) $f(1) = 3$

(C) y increases with x for x < 1

(D) f is a rational function of x

Q.2 If S is the set of all real x such that $\frac{2x-1}{2x^3+3x^2+x}$ is positive, then S contains ***(1986)***

(A) $\left(-\infty, -\frac{3}{2}\right)$ (B) $\left(-\frac{3}{2}, -\frac{1}{4}\right)$

(C) $\left(-\frac{1}{4}, \frac{1}{2}\right)$ (D) $\left(\frac{1}{2}, 3\right)$

Q.3 Let g(x) be a function defined on [–1, 1]. If the area of the equilateral triangle with two of its vertices at (0, 0) and [x, g(x)] is $\sqrt{3}/4$, then the function g(x) is ***(1989)***

(A) $g(x) = \pm\sqrt{1-x^2}$ (B) $g(x) = \sqrt{1-x^2}$

(C) $g(x) = -\sqrt{1-x^2}$ (D) $g(x) = \sqrt{1+x^2}$

Q.4 If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where [x] stands for the greatest integer function, then ***(1991)***

(A) $f\left(\frac{\pi}{2}\right) = -1$ (B) $f(\pi) = 1$

(C) $f(-\pi) = 0$ (D) $f\left(\frac{\pi}{4}\right) = 1$

Q.5 Let $f : (0, 1) \to R$ be defined by $f(x) = \frac{b-x}{1-bx}$, where b is a constant such that 0 < b < 1. Then, ***(2014)***

(A) f is not invertible on (0, 1)

(B) $f \neq f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$

(C) $f = f^{-1}$ on (0, 1)and $f'(b) = \frac{1}{f'(0)}$

(D) f^{-1} is differentiable on (0, 1)

Match the Columns

Q.6 Match the condition/expression in column I with statement in column II

Let the functions defined in column I have domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and range $(-\infty, \infty)$ ***(1992)***

Column I	Column II
(A) 1 + 2x	(p) Onto but not one–one
(B) tan x	(q) One–one but not onto
	(r) One–one and onto
	(s) Neither one–one nor onto

Q.7 Let $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$ ***(2007)***

Column I	**Column II**
(A) If $-1 < x < 1$, then f(x) satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then f(x) satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then f(x) satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then f(x) satisfies	(s) $f(x) < 1$

Q.8 Match the statements/expressions in column I with the values given in column II. ***(2009)***

Column I	**Column II**
(A) The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p) 1
(B) Value(s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q) 2
(C) Value(s) of k for which $\lvert x - 1\rvert + \lvert x - 2\rvert + \lvert x + 1\rvert + \lvert x + 2\rvert = 4k$ has integer solution(s)	(r) 3
(D) If $y' = y + 1$ and $y(0) = 1$ then value(s) of $y(\ln 2)$	(s) 4
	(t) 5

Q.9 Match the statements/expressions in column I with the values given in column II. **(2009)**

Column I	**Column II**
(A) Root(s) of the expression $2\sin^2\theta + \sin^2 \theta - 2$	(p) $\frac{\pi}{6}$
(B) Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where [y] denotes the largest integer less than or equal to y	(q) $\frac{\pi}{4}$
(C) Volume of the parallelopiped with its edges represented by the vectors $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}$ and $\hat{i} + \hat{j} + \pi\hat{k}$	(r) $\frac{\pi}{3}$
(D) Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a} + \vec{b} + \sqrt{3}\,\vec{c} = \vec{0}$	(s) $\frac{\pi}{2}$
	(t) π

Q.10 If the function $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is ***(2009)***

Q.11 Let f be a real-valued function defined on the interval (−1, 1) such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4 + 1}\,dt$, for all $x \in (-1, 1)$ and f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to ***(2010)***

(A) 1 (B) 1/3 (C) 1/2 (D) 1/e

Q.12 For any real number, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [-10, 10] by

$$f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd,} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is ***(2010)***

Q.13 Let $\vec{a} = -\hat{i} - \hat{k}$, $\hat{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is ***(2011)***

Q.14 Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in$ R and $g(x) = \frac{\pi}{2}\sin x$ for all $x \in$ R. Let (f o g)(x) denote f(g(x)) and (g o f)(x) denote g(f(x)). Then which of the following is (are) true? ***(2015)***

(A) Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(B) Range of f o g is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$

(D) There is an $x \in$ R such that (g o f)(x) = 1

Q.15 Match the statements given in column I with the intervals/union of intervals given in column II ***(2011)***

	Column I		**Column II**
(A)	The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right): z \text{ is a complex number}, \lvert z\rvert = 1, z \neq 1\right\}$ is	(p)	$(-\infty, -1) \cup (1, \infty)$
(B)	The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q)	$(-\infty, 0) \cup (0, \infty)$
(C)	If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta): 0 \le \theta < \frac{\pi}{2}\right\}$ is	(r)	$[2, \infty)$
(D)	If $f(x) = x^{3/2}(3x-10), x \ge 0$, then f (x) is increasing in	(s)	$(-\infty, -1] \cup [1, \infty)$
		(t)	$(-\infty, 0] \cup [2, \infty)$

Q.16 The function $f:[0,3] \to [1,29]$, defined by $f(x) = 2x^3 - 15x^2 + 36 + 1$, is **(2012)**

(A) One–one and onto (B) Onto but not one–one

(C) One–one but not onto (D) Neither one–one nor onto

Q.17 Consider the statements:

P: There exists some $x \in IR$ such that $f(x) + 2x = 2(1+x^2)$

Q: There exists some $x \in IR$ such that $2f(x) + 1 = 2x(1+x)$

Then ***(2012)***

(A) Both P and Q ate true (B) P is true and Q is true

(C) P is false and Q is true (D) Both P and Q are false

Q.18 Let f : (-1 , 1) $\to$ IR be such that

$\to f(\cos 4\theta) = \frac{2}{2-\sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are) ***(2012)***

Q.19 If the function $e^{-1}f(x)$ assumes its minimum in the interval [0, 1] at $x = \frac{1}{4}$, which of the following is true ? ***(2013)***

(A) $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$ (B) $f'(x) > f(x), 0 < x < \frac{1}{4}$

(C) $f'(x) < f(x), 0 < x < \frac{1}{4}$ (D) $f'(x) < f(x), \frac{3}{4} < x < 1$

Q.20 Let $f_1 : R \to R, f_2 : [0,\infty) \to R, f_3 : R \to R$ and $f_4 : R \to [0,\infty)$ be defined by ***(2014)***

$f_1(x) = \begin{cases} |x| \text{ if } x < 0 \\ e^x \text{ if } x \ge 0 \end{cases}$; $f_2(x) = x^2$; $f_3(x) = \begin{cases} \sin x \text{ if } x < 0 \\ x \quad \text{ if } x \ge 0 \end{cases}$

and

$f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(f_1(x))) - 1 & \text{if } x \ge 0 \end{cases}$

List I	**List II**
(1) f_4 is	(p) Onto but not one-one
(2) f_3 is	(q) Neither continuous nor one-one
(3) f_2 o f_1 is	(r) Differentiable but not one-one
(4) f_2 is	(s) Continuous and one-one

Codes:

	1	2	3	4
(A)	r	p	s	q
(B)	p	r	s	q
(C)	r	p	q	s
(D)	p	r	q	s

Q.21 let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$ be given by

$f(x)-(\log(\sec x+\tan x))^3$. Then **(2014)**

(A) f (x) is an odd function

(B) f (x) is a one-one function

(C) f (x) is an onto function

(D) f (x) is an even function

Q.22 Let f : R → R be a continuous odd function, which vanishes exactly at one point and $f(1)=\frac{1}{2}$. Suppose that $F(x)=\int_{-1}^{x} f(t)dt$ for all $x\in[-1,2]$ and $G(x)=\int_{-1}^{x} t\left|f(f(t))\right|dt$ for all $x\in[-1,2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is **(2015)**

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Sets and Relations

Q.4 Q.11 Q.14 Q.16

Functions

Q.5 Q.13 Q.19 Q.22

Q.26 Q.28 Q.31

Exercise 2

Sets and Relations

Q.2 Q.4 Q.8

Functions

Q.3

Previous Years' Questions

Q.4 Q.10

JEE Advanced/Boards

Exercise 1

Sets and Relations

Q.6 Q.11 Q.14 Q.21

Q.28 Q.30

Functions

Q.9 Q.11 Q.15 Q.19

Exercise 2

Sets and Relations

Q.1 Q.5 Q.9

Functions

Q.6 Q.8 Q.11 Q.15

Previous Years' Questions

Q.5 Q.7

Answer Key

JEE Main/Boards

Exercise 1

Sets and Relations

Q.1 {2, 4, 6}

Q.2 {(8, 11), (10, 13)}

Q.3 Empty relation

Q.4 yes

Q.5 No

Q.6 yes

Q.7 No

Q.8 R^{-1} = {(black, yellow), (dog, cat), (green, red)}

Q.9 Empty relation

Q.10 Yes

Functions

Q.1 1 **Q.2** $2\alpha, a \neq 0$ **Q.3** 2f(x)

Q.4 f = R– {–2, –6}

Q.5 $[-2, 0) \cup (0, 1)$

Q.6 $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Q.7 $x \in R - \{-1, 1\}$; $y \in (-\infty, 0) \cup [1, \infty)$

Q.8 $\left[\frac{1}{3}, 1\right]$

Q.9 $\alpha x^2 + \alpha$

Q.10 (i) $\begin{cases} 0, & x \leq 0 \\ 2x, & x \geq 0 \end{cases}$ (ii) $\begin{cases} 2x, & x \leq 0 \\ 0, & x \geq 0 \end{cases}$

(iii) $\begin{cases} -x^2, & x \leq 0 \\ x^2, & x \geq 0 \end{cases}$ (iv) ax

(v) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x}{|x|} = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \end{cases}$

Q.11 (i) e (ii) e (iii) 0

Q.12 –1

Q13 (i) $x \in \left[\frac{-\pi}{6}, 2\right]$ (ii) [cos 2, 1] (iii) 6

Q.14 f, u are function g, h are not function. Range f = {3, 9, 10}, Range u = (3, 6, 9)

Q.15 Only f is a function from A to A

Q.16 (i) a = 2, b = –1 (ii) f(x) = 2x – 1

Q.17 Domain = {4, 6, 9, 10}. Range = {1, 2, 3}

Q.18 Range f = {–1, 0, 3, 8}

Q.19 (i) Dom. f = R; Range of f = R

(ii) Dom. f = R; Range of f = R

(iii) Dom f = R; Range of f = $[1, \infty) = \{x : x \geq -1\}$

(iv) Dom f = R; Range of f = $[2, \infty) = \{x : x \geq 2\}$

(v) Dom. f = $[1, \infty)$; Range f = $[0, \infty) = \{x : x \geq 0\}$

Q.20 Domain of f = R – {1, 4}

Q.21 (i) Dom. f = R, Range f = $[0, \infty)$, (ii) Dom. g = R, Range g = $[0, \infty)$

(iii) Dom h = R – $\{-\sqrt{3}, \sqrt{3}\}$, Range h = $(-\infty, 0) \cup [1/3, \infty)$, (iv) Dom u = [–2, 2], Range u = [0, 2]

Q.22 f and g are not functions as they are not defined form negative values of x. h is function. Range

$h = \left(-\infty, -\frac{1}{4}\right] \cup [0, \infty)$.

Q.23 $f \neq g$ as dom. $f \neq$ dom. g.

Q.24 f + g = {(1, 2), (2, 5), (0, –1), (–1, –4)}

Q.25 (i) $\frac{1+(x+4)^4}{x+4}$; $\frac{1-(x+4)^4}{x+4}$; $(x + 4)^2$; $\frac{1}{(x+4)^4}$; $x \neq -4$; $\frac{\alpha}{x+4}$ (ii) $\cos x + e^x$; $\cos x - e^x$; $e^x \cos x$; $e^x \cos x$; $a \cos x$

Q.26 0; 4; 4; –1; 10

Q.27

x	–4	–3	–2	–1	0	1	2	3	4
$y=f(x)=x^2$	16	9	4	1	0	1	4	9	16

Domain of f = $\{x : x \in R\}$ = R

Range of f = $\{x : x \geq 0, x \in R\} = [0, \infty)$

Graph of y = f(x) i.e., $y = x^2$ is as shown in the following figure.

Q.28 Domain f = R – {0}, Range f = R – {0}

Q.29 $f(x) = (x - 3)^2 - 1$

Exercise 2

Sets and Relation

Q.1 C **Q.2** B **Q.3** B **Q.4** A **Q.5** A **Q.6** B

Q.7 A **Q.8** C **Q.9** B **Q.10** C **Q.11** A **Q.12** D

Q.13 B **Q.14** D **Q.15** B **Q.16** B

Functions

Q.1 A **Q.2** C **Q.3** C **Q.4** C **Q.5** D **Q.6** C **Q.7** A

Previous Years' Questions

Q.1 D **Q.2** D **Q.3** C **Q.4** A **Q.5** D **Q.6** A

Q.7 B **Q.8** B **Q.9** C **Q.10** D **Q.11** D **Q.12** C

Q.13 C **Q.14** C **Q.15** B **Q.16** B **Q.17** A **Q.18** B

Q.19 C **Q.20** B **Q.21** B

JEE Advanced/Boards

Exercise 1

Sets and relations

Q.8 {1, 2, 3, 4} **Q.11** 6, 3 **Q.14** 330 families **Q.16** n $(A \cup B) = n(A) + n(B)$

Q.25 9

Functions

Q.1 (i) defined no where (ii) $-1 < x \le 1$ and $2 \le x < 3$

(iii) $\frac{3}{2} < x < 2$ and $2 < x <$¥ (iv) $(-\infty, -1) \cup [0, \infty)$

(v) $-2 \le x < 0$ and $0 < x < 1$ (vi) $\left(0, \frac{1}{10}\right) \cup \left(\frac{1}{10}, \frac{1}{\sqrt{10}}\right)$

(vii) $(-2 \le x < 1)$-{0} (viii) $1 \le x \le 4$

(ix) $(-3, -1] \cup \{0\} \cup [1, 3)$ (x) $\{4\} \cup [5, \infty)$

(xi) $\left[n\pi, \left(n + \frac{1}{2}\right)\pi\right]$, $n \in I$

(xii) $\left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right) \cup \left[2K\pi + \frac{\pi}{3}, 2K\pi + \frac{5\pi}{3}\right]$, $k \in I - \{0\}$

(xiii) $[-3, -2) \cup [3, 4)$ (xiv) ϕ (xv) $\left(-4, -\frac{1}{2}\right) \cup (2, \infty)$

(xvi) $\left[-\frac{5\pi}{4}, \frac{-3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$

(xvii) $(-\infty, -3]$ (xviii) $2 < x < 3$ (xix) $[0, 1)$

Q.2 (i) D : $x \in R$, R : [0, 2]

(ii) D = R; range [–1, 1]

(iii) D: $\{x \mid x \in R;\ x \ne -3;\ x \ne 2\}$, R: $\{f(x) \mid f(x) \in R,\ f(x) \ne 1/5:\ f(x) \ne 1\}$

(iv) D:R;R: (–1, 1)

(v) D: $-1 \le x \le 2$ R: $[\sqrt{3}, \sqrt{6}]$

(vi) D : $x \in (2n\pi, (2n + 1)\pi) - \left\{2n\pi + \frac{\pi}{6}, 2n\pi + \frac{\pi}{2}, 2n\pi + \frac{5\pi}{6}, n \in I\right\}$ and R : $\log_a 2 : a \in (0, \infty) - \{1\} \Rightarrow$ Range is $(-\infty, \infty) - \{0\}$

(vii) $x \in R - \{2\}$, $f(x) \in R - \{1\}$

Q.3 (a) Neither surjective nor injective (b) Neither injective nor surjective

Q.4 $f_{3n}(x) = x$; Domain = R – {0, 1}

Q.5 2 **Q.6** (a) x < 3 or x > 4 (b) x < 2

Q.7 (a) Odd (b) Odd (c) Even (d) Even (e) Odd (f) Even (g) Odd (h) Even

(i) Neither odd nor even (j) Even

Q.8 (a) $\pi/2$ (b) $\pi/2$ (c) 70p

Q.9 (a) $y = \log(10 - 10^x), -\infty < x < 1$ (b) $y = x/3$ when $-\infty < x < 0$ and $y = x$ when $0 \le x < +\infty$

Q.10 ±1, ±3, ±5, ±15

Q.11 (a) $\frac{e^x - e^{-x}}{2}$ (b) $\frac{\log_2 x}{\log_2 x - 1}$ (c) $\frac{1}{2}\log\frac{1+x}{1-x}$

Q.12 $f^{-1}(x) = (a - x^n)^{1/n}$

Q.13 $x = \pm\sqrt{2}$

Q.14 (a) $f(x) = \begin{cases} -(x^2+1) & x > 1 \\ x^3 & x \in (0, 1] \end{cases}$ (b) $\begin{cases} (x^2+1) & x > 1 \\ -x^3 & x \in (0, 1] \end{cases}$

Q.15 $g(0) = \begin{cases} \frac{1}{x^2} & \text{if } 0 < x \le 1 \\ x^2 & \text{if } x > 1 \end{cases}$

Q.17 (a) Domain $x \in R$, Range $[0, \infty)$ (b) yes

Exercise 2

Sets and Relations

Single Correct Choice Type

Q.1 D **Q.2** C **Q.3** A **Q.4** D **Q.5** A **Q.6** C **Q.7** C

Q.8 D **Q.9** A **Q.10** C

Multiple Correct Choice Type

Q.11 A, B, C

Functions

Single Correct Choice Type

Q.1 C **Q.2** B **Q.3** A **Q.4** D **Q.5** D **Q.6** A **Q.7** D

Q.8 A

Multiple Correct Choice Type

Q.9 A, D **Q.10** B, C **Q.11** B, C **Q.12** A, C, D **Q.13** A, D **Q.14** B, C **Q.15** A, C, D

Previous Years' Questions

Q.1 A, D **Q.2** A, D **Q.3** B, C **Q.4** A, C **Q.5** A **Q.6** A → q; B → r

Q.7 A → p; B → q; C → q; D → p

Q.8 A→ p; B → q, s; C→ q, r, s, t; D → r

Q.9 A→ q, s; B→ p, r, s, t; C→ t; D→ r

Q.11 B **Q.12** D **Q.14** A, B, C

Q.15 A → s; B → t; C→r; D → r

Q.16 B **Q.17** C **Q.18** A, B

Solutions

JEE Main/Boards

Exercise 1

Sets and Relations

Sol 1: Relation R: x + 2y = 8 (defined in N)

x = 8 – 2y

x, y ∈ N

so for

$$\left.\begin{array}{ll} y=1 & x=8-2(1)=6 \\ y=2 & x=8-2(2)=4 \\ y=3 & x=8-2(3)=2 \end{array}\right\} \text{naturalnumbers}$$

y = 4 x = 8 – 2(4) = 0

so domain of R = {2, 4, 6}

Sol 2: Given relation R

R: {11, 12, 13} → {8, 10, 12}

y = x – 3

for R^{-1} x = y + 3

{8, 10, 12} → {11, 12, 13}

$$\begin{array}{l} \text{for } y=8\, x=10+3=11\ ✓ \\ \quad y=10\, x=10+3=13 ✓ \\ \quad y=12\, x=12+3=15 \end{array}$$ } Element of {11, 12, 13}

R^{-1} → {(8, 11), (10, 13)}

Sol 3: R = {(a, b) ∈ A × H: a is sister of b}

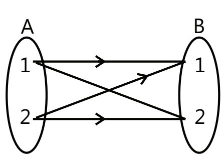
Domain and range both (a, b) are set {A (a, b) ∈ A × A}

And it is given that A is only boy's school a and b both are boys. So It is not possible that a is sister of b.

Sol 4: Given A = {1, 2}, B = {1, 3}

R: A → B {(1, 1),(1, 3),(2, 1),(2, 3)}

Universal relation: Each element of A is related to every element of B.



So R is universal relation.

Sol 5: R = {(a, b) ∈ N × N: a < b}

Reflective relation → R: A → B is said to be reflective iff a R a ∀ a ∈ A (here A, B ⇒ N)

R = {(a, a) ∈ N × N: a < a}

we know a < a is universal false

so R is not a reflexive relation.

Sol 6: P(A) ⇒ Power set of A

X R Y ⇒ X ∩ Y = X , Y ∈ P(A)

X R Y ↔ – X ∩ Y = X, Y

YRX ↔ Y ∩ X

We know X ∩ Y = Y ∩ X

so XRY → YRX

so R is symmetric.

Sol 7: A = {a, b, c}

and R = {(a, a),(a, b), (a,c),(b,a),(c, c)}

R^{-1} = {(a, a), (a, b),(c, a),(b, a),(c, c)}

$R \neq R^{-1}$

so R is not symmetric relation.

Sol 8: R = {(yellow, black), (cat, dog), (red, green)}

R → x → y

R–1 → y → x

So R^{-1} = {(black, yellow), (dog, cat), (green, red)}

Sol 9: A = {1, 3, 5}, B = {9, 11}

R = {(a, b) ∈ A × B: a – b is odd}

A – b will be odd when a, b both are not even and not odd.

In A, B all elements are odd

so (a – b) ∈ π

R is empty relation.

Sol 10: A = {a, b, c}

R = {(a, c), (c, a)}

R^{-1} = {c, a},(a, c)

$R \equiv R^{-1}$ relation R and R^{-1} both are same so R is symmetric.

Functions

Sol 1: $f(x) = \dfrac{2\tan x}{1+\tan^2 x} \Rightarrow f\left(\dfrac{\pi}{4}\right) = \dfrac{2\tan \pi/4}{1+\tan^2(\pi/4)}$

We know that tan (π/4) = 1

$$f\left(\frac{\pi}{4}\right) = \frac{2(1)}{1+(1)^2} = 1$$

Sol 2: $f(x) = \dfrac{|x|}{x}$

If $x > 0$ $f(x) = 1$

If x < 0; $f(x) = \dfrac{-x}{x} = -1$

Case I $\alpha > 0$

$f(\alpha) = 1$ $f(-\alpha) = -1$

$|1-(-1)| = 2$

Hence proved

Sol 3: $f(x) = \log\left(\dfrac{1+x}{1-x}\right)$

$$= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right) = \log\left(\frac{1+x}{1-x}\right)^2$$

$$f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right) = 2\log\left(\frac{1+x}{1-x}\right) = 2f(x)$$

Sol 4: $f(x) = \dfrac{x^2+2x+1}{x^2+8x+12}$

$$f(x) = \frac{x^2+2x+1}{\left(x^2+8x+16\right)-16+12} = \frac{(x+1)^2}{(x+4)^2-4}$$

$(x+4)^2 - 4 \neq 0$

$(x+4) \neq \pm 2$

$x \neq -2$ and $x \neq -6$

Domain $x \in R - \{-2, -6\}$

Sol 5: $y = \dfrac{1}{\log(1-x)} + \sqrt{x+2}$

Logarithm is not defined for $(1-x) \le 0,\ x \ge 1$

$\sqrt{x+2}$ is not defined for $x+2 < 0,\ x < -2$

$\log(1-x) \neq 0 \Rightarrow x \neq 0$

So domain will be $x \in (-2, 0) \cup (0, 1)$

Sol 6: Function $y = \dfrac{x}{1+x^2}$

$$y = \frac{1}{\left(\frac{1}{x}+x\right)}$$

$\left(x+\dfrac{1}{x}\right)$ is always greater than 2

From arithmetic mean > G.M

For $x > 0$; $\dfrac{x+\frac{1}{x}}{2} > \sqrt{x\left(\frac{1}{x}\right)}$

For $x < 0$; $x+\dfrac{1}{x} < -2,\ x+\dfrac{1}{x} > 2$

So range will be $\dfrac{-1}{2} \le y \le \dfrac{1}{2}$

$$y \in \left[\frac{-1}{2}, \frac{1}{2}\right]$$

Sol 7: $f(x) = \dfrac{1}{1-x^2}$ domain $x \in R - \{-1, 1\}$

$x^2 \ge 0 \Rightarrow$ So the range $y \in (-\infty, 0) \cup [1, \infty)$

Sol 8: $y = \dfrac{1}{2-\sin 3x} \Rightarrow 2 - \sin 3x \neq 0$

$\sin 3x \neq 2$

So the domain $x \in R$

$-1 \le \sin 3x \le 1$

Range $y \in \left[\dfrac{1}{3}, 1\right]$

Sol 9: $f+g = x^3+1+x+1 = x^3+x+2$ $R \to R$

$f-g = x^3+1-x-1 = x^3-x$ $R \to R$

$f.g = (x^3+1) = x^4+1+x+x^3$

$\dfrac{f}{g} = \dfrac{x^3+1}{x+1}$ $x \neq -1$

$\alpha f = ax^2 + \alpha$

Sol 10: $f: R \to R$ $f(x) = x$

$g: R \to R$ $g(x) = |x|$

if $x \geq 0$ $g(x) = x$

if $x \leq 0$ $g(x) = -x$

$$f + g = \begin{cases} 2x, x > 0 \\ 0, x \leq 0 \end{cases}; \quad f - g = \begin{cases} 0, x \geq 0 \\ 2x, x \leq 0 \end{cases}$$

$$f \cdot g = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x \leq 0 \end{cases}$$

$$\frac{f}{g} = \begin{cases} 1 & x \geq 0 \\ -1 & x \leq 0 \\ \text{not defined} & x \neq 0 \end{cases}$$

$af = ax$

Sol 11: $f(x) = e^x$

$g(x) = \log_e x$

$(f + g)(1) = e^{(1)} + \log_e^{(1)} = e$

$(f - g)(1) = e^{(1)} - \log_e^{(1)} = e$

$f.g(1) = e^{(1)} \log_e^{(1)} = 0$

Sol 12: $\pi^2 = 9.8 \Rightarrow [\pi^2] = 9$ and $[-\pi^2] = -10$

$f(x) = \cos 9x + \cos(-10)x = \cos 9x + \cos 10x$

$$f\left(\frac{\pi}{2}\right) = \cos 9\left(\frac{\pi}{2}\right) + \cos 10\frac{\pi}{2} = -1$$

Sol 13: (i) $f(x) = \ell n(1 - \{x\}) + \sqrt{\sin x + \frac{1}{2}} + \sqrt{4 - x^2}$

$1 - \{x\} > 0$ (Always true)

AND $\sin x + \frac{1}{2} \geq 0$

$\sin x \geq -\frac{1}{2}$

$$x \in \left[2n\pi - \frac{\pi}{6}, 2n\pi + \frac{7\pi}{6}\right]$$

And $4 - x^2 \geq 0$

$x^2 \leq 4$

$x \in [-2, 2]$

So domain is $x \in \left[\frac{-\pi}{6}, 2\right]$

(ii) Range cos(2sinx)

$-1 \leq \sin \leq 1$

$-2 \leq 2\sin x \leq 2$

$y = \cos(2\sin x) \in [\cos 2, 1]$

(iii) $f(x) = \sin\left(\frac{\pi x}{3}\right) + \{x\} + \tan^2 \pi x$

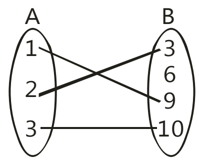
$\sin\frac{\pi x}{3}$ has time period = 6

{ x } has time period = 1

$\tan^2 \pi x$ has time period = 1

LCM (6,1,1) = 6

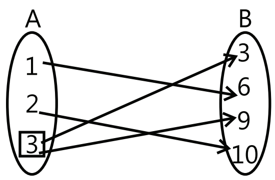
Sol 14: f



Function

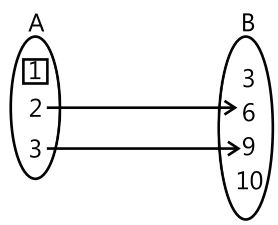
Range {3, 9, 10}

g



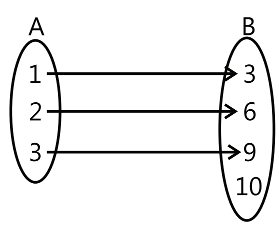
not a function as 3 has two values in Range.

h



not a function as 1 has no value

u



putting x =1 y = 3

x = 2y = 6

x = 3y = 3

So, it is a function and Range $\in \{3, 6, 9\}$

Sol 15: (i) Yes, it is because it cover all A and each corresponds to one value.

(ii) No, as it gives two value at $x = b$

(iii) No, as x=c has no image.

Sol 16: (i) f(x) = ax + b

at $x = 1,\ f(1) = 1$

$1 = a + b$ ... (i)

at $x = 2,\ f(2) = 3$

3 = 2a + b ... (ii)

Solving 1 & 2

$a = 2$

$b = -1$

(ii) Let $f(x) = a\times + b$

Some question as part (i)

So the answer will be $f(x) = 2x - 1$

Sol 17: Domain $= \{4, 9, 6, 10\}$

Range $= \{2, 1, 3\}$

Sol 18: $f(x) = x^2 - 1$

$x^2 \geq 0$

$f(0) = -1$

$f(1) = 0 = f(-1)$

$f(2) = 3 = f(-2)$

$f(3) = 8 = f(-3)$

So the range will be $\{-1, 0, 3, 8\}$

Sol 19: (i) Domain $x \in R$

Range $y \in R$

(ii) Domain $x \in R$

Range $y \in R$

(iii) Domain $x \in R$

$x^2 \geq 0$

So the range $y \in [-1, \infty)$

(iv) Domain $x \in R$

$x^2 \geq 0$

So the range $y \in [2, \infty)$

(v) Domain $x - 1 \geq 0$

$x \geq 1$

$x \in [1, \infty)$

Range $y \geq 0$ (Because value of under root is always non neg.) $y \in [0, \infty)$

Sol 20: $f(x) = \dfrac{x^2 + 3x + 5}{x^2 - 5x + 4} = \dfrac{x^2 + 3x + 5}{(x-4)(x-1)}$

$(x-4)(x-1) \neq 0$

$x \neq 4\ \ x \neq 1$

Domain $x \in R - \{1, 4\}$

Sol 21: (i) Domain =R

Range $y \in [0, \infty)$

(ii) Domain =R

Range $y \in [0, \infty)$

(iii) Domain $3 - x^2 \neq 0$

$x \neq \pm\sqrt{3}$

$x \in R - \{\sqrt{3}, -\sqrt{3}\}$

Range $x^2 \geq 0$

$3 - x^2 \leq +3$

$y \in (-\infty, 0) \cup \left[\dfrac{1}{3}, \infty\right)$

(iv)Domain $4 - x^2 \geq 0$

$4 \geq x^2$

$x^2 \leq 4$

$x \in [-2, 2]$

Range $u(2) = 0$

$u(0) = 2$

$y \in [0, 2]$

Sol 22: (i) $f : R \to R :\ f(x) = \log x$

Not a function

Because f(x) is not defined for negative values of x.

(ii) $g : R \to R :\ g(x) = \sqrt{x}$

Not a function

As f(x) is not defined for negative values.

(iii) $h : A \to R :\ h(x)\dfrac{1}{x^2 - 4}; A = R - \{-2, 2\}$

h(x) is defined for all values of x in A set.

So it is a function.

Sol 23: $f: R-\{2\} \to R$

$f(x)=\frac{x^2-4}{x-2}=\frac{(x-2)(x+2)}{(x-2)}=(x+2)$ for $x \neq 2$

$g: R \to R$ $g(x)=x+2$ at $x \in R$

$f \neq g$

As their domain are not same.

Sol 24: $f=a\times+b$ (Suppose)

$f(1)=1=a+b$ ...(i)

$f(2)=3=2a+b$ ...(ii)

$a=2 \quad b=-1$

$f(x)=2x-1$

$g(x)=\times$

$f+g=3x-1$

at $x=1$ $(f+g)(1)=2$

$(f+g)(2)=5$

$(f+g)(0)=-1$

$(f+g)(-1)=4$

So $f+g=[(1,2)(2,5)(0,-1)(-1,-4)\}$

Sol 25: (i) $f+g=\frac{1}{x+4}+(x+4)^3 \quad x\neq-4$

$f-g=\frac{1}{x+4}-(x+4)^3 \quad x\neq-4$

$f.g=(x+4)^2;\ x\neq-4$

$\frac{f}{g}=\frac{1}{(x+4)^4};\ x\neq-4$

(ii) $f(x)=\cos x \quad g(x)=e^x$

$f+g=\cos x+e^x$

$f-g=\cos x-e^x$

$f.g=e^x(\cos x)$

$f/g=\frac{\cos x}{e^x}$

$\alpha f=\alpha\cos x$

Sol 26: $f(x)=x \quad g(x)=|x|$

If $x\geq 0$ $g(x)=x$

If $x<0$ $g(x)=-x$

$(f+g)(-2)=x-x=0$

$(f-g)(2)=2\ (2)=4$

$f.g\ (2)=(2)^2=4$

$\frac{f}{g}(-2)=-1$

$5f(2)=5(2)=10$

Sol 27:

x	–4	–3	–2	–1	0	1	2	3	4
$f(x)=x^2$	16	9	4	1	0	1	4	9	16

Sol 28:

x	–2	–15	–1	–0.5	0	0.5	1	1.5	2
f(x) = 1/x	–1/2	–2/3	–1	–2	X	2	1	2/3	1/2

Domain R–{0}

Range $(-\infty,0)\cup(0,\infty)$

Sol 29: $f(x+3)=x^2-1=(x+3)^2-9-6x=$

$(x+3)^2-6(x+3)+8-10$

Exercise 2

Sets and Relations

Single Correct Choice Type

Sol 1: (C) A = {1, 2, 3, 4}

R = {(2, 2), (3, 3), (4, 4), (1, 2)}

$\to$ A = {1, 2, 3, 4}

and (1, 1) $\notin$ R so R is not reflexive

$\to$ (1, 2) $\in$ R but (2, 1) $\notin$ R

So R is not symmetric

$\to$ (1,2) $\in$ R, (2, 2) $\in$ R

(1, 2) $\in$ R

So R is transitive.

Sol 2: (B) Void relation: A $\to$ A

It is also called empty relation

A relation R is void relation, if no element of set A is related to any element of A.

(x, y) $\Rightarrow$ x is not related to y

$\therefore$ (y, x) $\Rightarrow$ y is not related to x

$\therefore$ Symmetric

(x, x) $\Rightarrow$ x is any how will be related to itself

$\therefore$ So it can't be reflexive

(x, y)(y, z)

$\Rightarrow$ x is not related to y

y is not related to z

$\therefore$ This is transitive.

Sol 3: (B) x R y $\leftrightarrow$ x − y is an irrational number if 0 = x – x is an irrational no. (say z)

so R is not reflexive

$\rightarrow$ if x – y is a irrational number

than y – x is also an irrational number R is symmetric

$\rightarrow$ (x, y) $\in$ R and (y, z) $\in$ R

say(z_1) and say(z_2)

$z_1 + z_2 = x - y + y - z = x - z$

Its not necessary that $z_1 + z_2$ will be an irrational no. R is not transitive.

Sol 4: (A) R = {(1 + x, 1 + x^2): x $\le$ 5, x $\in$ N}

(A)R = {(2, 2), (3, 5), (4, 10), (5, 17), (6, 25)}

$$(2, 2)\rightarrow \left.\begin{matrix} 1+x=2 \\ 1+x^2=2 \end{matrix}\right\} x = 1\ x \le 5$$

$$(3, 5)\rightarrow \left.\begin{matrix} 1+x=3 \\ 1+x^2=5 \end{matrix}\right\} x = 2, x \le 5$$

$$(4, 10)\rightarrow \left.\begin{matrix} 1+x=4 \\ 1+x^2=10 \end{matrix}\right\} x = 3, x \le 5$$

$$(5, 17)\rightarrow \left.\begin{matrix} 1+x=5 \\ 1+x^2=17 \end{matrix}\right\} x = 4, x \le 5$$

$$(6, 25)\rightarrow \left.\begin{matrix} 1+x=6 \\ 1+x^2=25 \end{matrix}\right\} \text{no solution}$$

Option A is false.

Sol 5: (A) R = {(1, 1), (2, 2), (3, 3),(1, 2), (2, 3), (1, 3)}

on the set A = {1, 2, 3}

$\rightarrow$ (1, 1), (2, 2), (3, 3) $\in$ R

R is reflexive

$\rightarrow$ (1, 2) $\in$ R but (2, 1) $\notin$ R

R is not symmetric

$\rightarrow$ (1, 2) $\in$ R and (2, 3) and also (1, 3) $\in$ R

So R is transitive.

Sol 6: (B) R = {(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)}

in set A = {2, 3, 4, 5}

$\rightarrow$ for set A = {2, 3, 4, 5}

(2, 2), (3, 3), (4, 4), (5, 5) $\in$ R

R is reflexive

$\rightarrow$ (2, 3) (3, 2) $\in$ R

and (3, 5) (5, 3) $\in$ R

(x, x) $\in$ R

So R is symmetric

$\rightarrow$ (3, 5) $\in$ R, (5, 3) $\in$ R and also (3, 3) $\in$ R

[(a, b) $\in$ R , (b, c) $\in$ R for transitive (a, c) $\in$ R]

so R is not transitive as (2, 3), (3, 5) $\in$ R

but (2, 5) $\notin$ R

Sol 7: (A) A = {2, 3}B = {1, 2}

then A × B = {(2, 1) (2, 2), (3, 1)(3, 2)}

Sol 8: (C) A = {1, 2, 3}, B = {1, 4, 6, 9}

(x, y) $\in$ R $\therefore$ x > y

R: A $\rightarrow$ B

$\Rightarrow$ 2 > 1, 3 > 1 $\Rightarrow$ R = {(2, 1), (3, 1)}

So range = {1}

Sol 9: (B) R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)}

on the set A = {1, 2, 3, 4}

$\rightarrow$ for set A

(1, 1), (2, 2), (3, 3), (4, 4) $\notin$ R

R is not reflexive

$\rightarrow$ (2, 3) $\in$ R but (3, 2) $\notin$ R

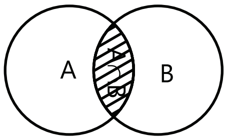
R is not symmetric

$\rightarrow$ (1, 3) $\in$ R, (3,1) $\in$ R but

(1, 1) $\notin$ R so

R is not transitive.

Sol 10: (C)



Given $A \cap B = A \cap C$ ... (i)

and $A \cup B = A \cup C$ ... (ii)

we know that $A \cup B = A + B - A \cap B$

from (i) and (ii)

$A \cup C = A + B - A \cap C$ ... (iii)

but $A \cup C = A + C - A \cap C$ ... (iv)

From (iii) – (iv)

O = B – C

B = C

Sol 11: (A) R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

Domain → {1, 2, 3}

→ there are pairs such that

(a, a) → (1, 1),(2, 2), (3,3)

So R is reflexive → $R^{-1} \neq R$

So R is not symmetric

→ transitive if (x, y) ∈ R, (y, z) ∈ R

⇒ (x, z) ∈ R here (1, 2) ∈ R, (2, 3) ∈ R and (1, 3) is also solution of R.

so R is transitive.

Sol 12: (D) A → set of all children in the world x R y if x and y have same sex.

So → x and x have same sex → Reflexive if x and y have same sex. So y and x also have same sex → symmetric

x and y have same sex, y and z have same sex so it is clear that x, y, z have same sex transitive

So R is an equivalence relation.

Sol 13: (B) A set A = {2, 3, 4, 5}

Given relation R={(2, 2), (3, 3), (4, 4), (5, 5), (2, 3),(3, 2), (3, 5) (5, 3)}

⇒ if (x, y) ∈ R and x = y

(x, y) ∈ R

Here x ∈ A

x = 2, 3, 4, 5 and (2, 2), (3, 3), (4, 4), (5, 5) are in solution of R

R is reflexive ⇒ (x, y) ∈ R

and (y, x) ∈ R

⇒ (2, 3) ∈ R ⇒ (3, 2) ∈ R

(5, 3) ∈ R ⇒ (3, 5) ∈ R

or say $R^{-1} = R$

so R is symmetric.

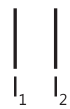
Sol 14: (D) L = get of all times in x-y plane

R = {l_1, l_2 = l_1 is parallel to l_2}

⇒ if (l_1, l_2) ∈ R

so (l_1, l_2) ∈ R (l_2 is also parallel to l_1)

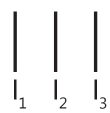
symmetric



⇒ (l_1, l_2) ∈ R because a line is parallel to itself reflexive

⇒ (l_1, l_2) ∈ R, (l_2, l_3) ∈ R ⇒ (l_1, l_3) ∈ R

l_1, l_2 are parallel and (l_2 and l_3) are parallel, so l_1 and l_3 also are parallel R transitive



∴ R is equivalent

Sol 15: (B) Relation R = {(2, 3), (3, 4)}

on the set assume A = {2, 3, 4}

⇒ For reflexive → (x, x) ∈ R, x ∈ A

$\underbrace{(2, 2), (3, 3), (4, 4)}_{3\text{pairs}}$ should be pair of R

⇒ For symmetric → given (2, 3), (3, 4) ∈ R to be R → symmetric (3, 2) and (4, 3) should be pair of R

Total added pair is 5{(2, 2),(3, 3),(4, 4),(3, 2),(4,3)}

Sol 16: (B) R = {(1, 2), (2, 3)} on the set A (assume)

So A = {1, 2, 3}

→ Reflexive (x, x) ∈ R, x ∈ A

$\underbrace{(1, 1), (2, 2), (3, 3)}$ must be added

→ symmetric (x, y) ∈ R for (y, x) ∈ R

$\underbrace{(2, 1), (3, 2)}$ must be added

$\rightarrow$ Transitive (1, 2) $\in$ R and (2, 3) $\in$ R

so $\underbrace{(1, 3)}$ must be added for transitive relation

Now $\rightarrow$ R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3),(2, 1), (3, 2)}

Now for symmetric pair (1, 3), pair (3, 1) must be added

Total added pairs $\rightarrow$

7 {(1, 1)(2, 2)(3, 3)(2, 1)(3, 2)(1, 3) (3, 1)}

Functions

Single Correct Choice Type

Sol 1: (A) $f(x+ay, x-ay) = a\times y$

Put $x+ay=b$

$x-ay=c$

$2x=b+c$

$2ay=b-c$

$y=\frac{b-c}{2a}$

$f(b,c)=a\left(\frac{b+c}{2}\right)\left(\frac{b-c}{2a}\right)$

$f(b,c)=\frac{(b+c)(b-c)}{4}=\frac{b^2-c^2}{4}$

$f(x,y)=\frac{x^2-y^2}{4}$

Sol 2: (C) $f(x)=ax+\cos x$

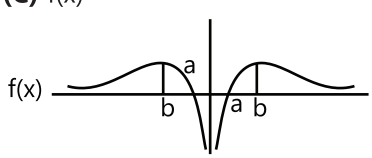
It is bijective so it must be strictly increasing or decreasing

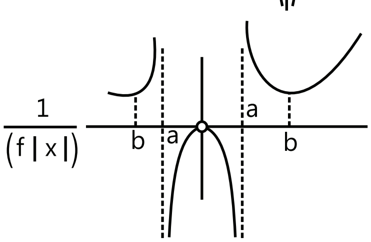
$f'(x)=a-\sin x$

$a\le -1$ or $a\ge 1$

$x\in R-(-1, 1)$

Sol 3: (C) f(x)





Sol 4: (C) $f(x)=\frac{1}{2}\left(\frac{|\sin x|}{\cos x}+\frac{\sin x}{|\cos x|}\right)$

If $\sin x>0, \cos>0$

F(x)=tanx

If sinx>0, cosx<0

F(x) =0

If sinx<0, cosx<0

F(x) = –tanx

If sinx<0, cosx>0

F(x) = 0

} Time period = 2π

Sol 5: (D) $d(x)=\frac{2x^2-x+5}{7x^2+2x+10}$

Not onto as range is not R. the function

Does not goes to infinite at any x.

$f(x)=\frac{\frac{2}{7}\left(7x^2-\frac{7}{2}x+\frac{25}{2}\right)}{7x^2+2x+10}$

$=\frac{\frac{2}{7}\left(7x^2+2x+10-\frac{11}{2}x+\frac{15}{2}\right)}{7x^2+2x+10}$

$=\frac{2}{7}-\frac{11x-15}{49x^2+14x+70}$

Quadratic cannot be one-one function

So f(x) is not one-one function

Sol 6: (C) $f(x)=\cos\left[\frac{1}{2}\pi^2\right]x+\sin\left[\frac{1}{2}\pi^2\right]x$

$\left[\frac{\pi^2}{2}\right]=4$

$f(x)=\cos 4x+\sin 4x$

f(0)=1

$f\left(\frac{\pi}{3}\right)=\frac{1}{2}+\frac{\sqrt{3}}{2}\ne 4$

$f\left(\frac{\pi}{2}\right)=1$

$f(\pi)=1$

Sol 7: (A) $f(x)=\ell nx$ & $g(x)=\dfrac{x^4-x^3+3x^2-2x+2}{2x^2-2x+1}$

$$fog(x)=\ell n\left(\frac{x^4-x^3+3x^2-2x+2}{2x^2-2x+1}\right)$$

$$=\ell n\left(\frac{x^4-x^3+3x^2-2x+2}{2x^2-2x+1}\right)$$

$$2x^2-2x+1\neq 0$$

$$\Rightarrow 2\left(x-\frac{1}{2}\right)^2+\frac{1}{2}>0$$

Let say $h(x)=x^4-x^3+3x^2-2x+2$

By hit and trial method, h(x) has no root. It is always ≥ 0 so domain $x\in(-\infty,\infty)$

Previous Years' Questions

Sol 1: (D) Given, f(x) = | x – 1 |

$\therefore$ $f(x^2)$ = | x^2 – 1 | and $\{f(x)\}^2 = (x-1)^2$

$\Rightarrow f(x^2)\neq\{f(x)\}^2$, hence (a) is false

Also, f(x + y) = | x + y – 1 | and f(x) = | x – 1 |, f(y) = | y – 1| Also,

$\Rightarrow$ f(x + y) ≠ f(x) = f(y), hence (b) is false.

f(| x |) = || x | – 1 | = | f(x) | = || x – 1|| = | x – 1 |

$\therefore$ f(| x |) ≠ | f(x) |, hence c is false.

Sol 2: (D) Given, f(x) = cos(log x)

$$\therefore f(x)\,.\,f(y)-\frac{1}{2}\left[f\left(\frac{x}{y}\right)+f(xy)\right]$$

= cos(log x). cos (log y) – $\frac{1}{2}$[cos (log x – log y) + cos (log x + log y)]

= cos (log x) . cos (log y) – $\frac{1}{2}$[(2cos(log x) . cos (log y)]

= cos(log x) . cos (log y) – cos (log x) . cos (log y) = 0

Sol 3: (C) For domain of y,

1– x >0, 1 – x ≠ 1and x + 2 > 0

$\Rightarrow$ x < 1, x ≠ 0 and x > –2

$\Rightarrow$ –2 < x < 1 excluding 0 $\Rightarrow$ x ∈ (–2, 1) – {0}

Sol 4: (A) Clearly, f(x) = x – [x] = {x} which has period 1 and $\sin\frac{1}{x}$, x cos x are non–periodic function.

Sol 5: (D) Let y = $\dfrac{x^2-(a+b)x+ab}{x-c}$

$\Rightarrow yx - cy = x^2 - (a + b)x + ab$

$\Rightarrow x^2 - (a + b + y)x + (ab + cy) = 0$

for real roots, D ≥ 0

$\Rightarrow (a + b + y)^2 - 4(ab + cy) \geq 0$

$\Rightarrow (a + b)^2 + y^2 + 2(a + b) y - 4ab - 4cy \geq 0$

$\Rightarrow y^2 + 2(a + b - 2c) y + (a - b)^2 \geq 0$

which is true for all real value of y.

$\Rightarrow$ D ≤ 0

$\Rightarrow 4(a + b - 2c)^2 - 4(a - b)^2 \leq 0$

$\Rightarrow$ (a + b – 2c + a – b)(a + b – 2c – a + b) ≤ 0

$\Rightarrow$ (2a – 2c) (2b – 2c) ≤ 0

$\Rightarrow$ (a – c) (b – c) ≤ 0

$\Rightarrow$ (c – a) (c – b) ≤ 0

$\Rightarrow$ c must lie between a and b

i.e. a ≤ c ≤ b or b ≤ c ≤ a

Sol 6: (A) Let $f(x) = \sin^2x$ and $g(x) = \sqrt{x}$

Now, fog(x) = f[g(x)] = $f(\sqrt{x}) = \sin^2\sqrt{x}$

and gof(x) = g[f(x)] = $g(\sin^2 x) = \sqrt{\sin^2 x}$ = | sin x |

Again let f(x) = sin x, g(x) = | x |

fog(x) = f[g(x)] = f(| x |) = sin | x | ≠ $(\sin\sqrt{x})^2$

When $f(x) = x^2$, $g(x) = \sin\sqrt{x}$

fog(x) = f[g(x)] = $f(\sin^{\sqrt{x-1}\sqrt{x+1}}) = (\sin\sqrt{x})^2$

and (gof) (x) = g[f(x)] = $g(x^2) = \sin\sqrt{x^2}$

= sin | x | ≠ | sin x |

Therefore, (a) is the answer.

Sol 7: (B) Given, f(x) = 3x – 5(given)

Let y = f(x) = 3x – 5 $\Rightarrow$ y + 5 = 3x

$$\Rightarrow x = \frac{y+5}{3}$$

$$f^{-1}(y) = \frac{y+5}{3} \Rightarrow f^{-1}(x) = \frac{x+5}{3}$$

Sol 8: (B) Let y $=2^{x(x-1)}$, where y ≥ 1 as x ≥ 1

Taking $\log_2$ on both sides, we get

$\log_2 y = \log_2 2^{x(x-1)}$

$\Rightarrow \log_2 y = x(x-1)$

$\Rightarrow x^2 - x - \log_2 y = 0$

$\Rightarrow x = \dfrac{1 \pm \sqrt{1+4\log_2 y}}{2}$

For y ≥ 1, $\log_2 y \ge 0 \Rightarrow 4\log_2 y \ge 0 \Rightarrow 1 + 4\log_2 y \ge 1$

$\Rightarrow \sqrt{1+4\log_2 y} \ge 1$

$\Rightarrow -\sqrt{1+4\log_2 y} \le -1$

$\Rightarrow 1 - \sqrt{1+4\log_2 y} \le 0$

But x ≥ 1

So, $x = 1 - \sqrt{1+4\log_2 y}$ is not possible.

Therefore, we take $x = \frac{1}{2}(1+\sqrt{1+4\log_2 y})$

$\Rightarrow f^{-1}(y) = \frac{1}{2}(1+\sqrt{1+4\log_2 y})$

$\Rightarrow f^{-1}(x) = \frac{1}{2}(1+\sqrt{1+4\log_2 x})$

Sol 9: (C) It is given,

$f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$= (\sin\theta + 3\sin\theta - 4\sin^3\theta)\sin q$

$= (4\sin\theta - 4\sin^3\theta)\sin q$

$= \sin^2\theta(4 - 4\sin^2\theta)$

$= 4\sin^2\theta\cos^2\theta = (2\sin\theta\cos\theta)^2$

$= (\sin 2\theta)^2 \ge 0$

Which is true for all θ.

Sol 10: (D) Given that $2^x + 2^y = 2\ \forall x, y \in R$

But 2^x, 2y > 0, $\forall$ x, y ∈ R

Therefore, $2^x = 2 - 2^y < 2$

$\Rightarrow 0 < 2x < 2$

Taking log on both sides with base 2, we get

$\log_2 0 < \log_2 2^x < \log_2 2$

$\Rightarrow -\infty < x < 1$

Sol 11: (D) Function is increasing $x = \dfrac{y-3}{4} = g(y)$

Sol 12: (C) Given $f(x) = x^3 + 5x + 1$

Now $f'(x) = 3x^2 + > 0, \forall x \in R$

$\therefore f(x)$ is strictly increasing function

$\therefore$ It is one-one

Clearly, f(x) is a continuous function and also increasing on R,

$\underset{x\to-\infty}{\text{Lt}} f(x) = -\infty$ and $\underset{x\to\infty}{\text{Lt}} f(x) = \infty$

$\therefore f(x)$ takes every value between $-\infty$ and ∞.

Thus, f(x) is onto function.

Sol 13: (C) There is no information about co-domain therefore f(x) is not necessarily onto.

Sol 14: (C) $f(x) = x|x|$ and $g(x) = \sin x$

$gof(x) = \sin(x|x|) = \begin{cases} -\sin x^2 & ,x<0 \\ \sin x^2 & ,x\ge 0 \end{cases}$

$\therefore (gof)'(x) = \begin{cases} -2x\cos x^2 & ,x<0 \\ 2x\cos x^2 & ,x\ge 0 \end{cases}$

Clearly, $L(gof)'(0) = 0 = R(gof)'(0)$

$\therefore$ g of is differentiable at x = 0 and also its derivative is continuous at x = 0

Now, $(gof)''(x) = \begin{cases} -2\cos x^2 + 4x^2\sin x^2 & , x<0 \\ 2\cos x^2 - 4x^2\sin x^2 & , x \ge 0 \end{cases}$

$\therefore L(gof)''(0) = -2$ and $R(gof)''(0) = 2$

$\therefore L(gof)''(0) \ne R(gof)''(0)$

$\therefore gof(x)$ is not twice differentiable at x = 0.

Sol 15: (B)

xRy need not implies yRx

$S: \frac{m}{s} S \frac{p}{q} \Leftrightarrow qm = pn$

$\frac{m}{n} S \frac{m}{n}$ reflexive

$\frac{m}{n} S \frac{p}{q} \Rightarrow \frac{p}{q} S \frac{m}{n}$ symmetric

$\frac{m}{n} S \frac{p}{q}, \frac{p}{q} S \frac{r}{s}$

$\Rightarrow qm = pn, ps = rq$

$\Rightarrow ms = rn$ transitive.

S is an equivalence relation.

Sol 16: (B)

$\frac{1}{\sqrt{|x| - x}} \Rightarrow |x| - x > 0 \Rightarrow |x| > x \Rightarrow x$ is negative

$x \in (-\infty, 0)$

Sol 17: (A)

$f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi = [x] \cos\left(x - \frac{1}{2}\right)\pi$

$= [x] \sin \pi x$ is continuous for every real x.

Sol 18: (B) $f(x) = 7 - 2x;\ x < 2$

$= 3;\quad 2 \le x \le 5$

$= 2x - 7;\ x > 5$

f(x) is constant function in [2, 5]

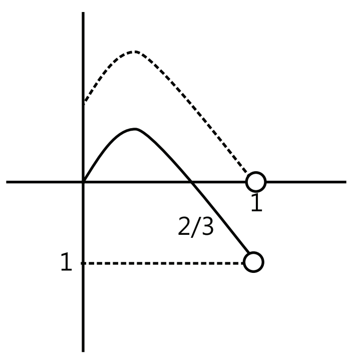
f is continuous in [2, 5] and differentiable in (2, 5) and f(2) = f(5)

by Rolle's theorem f′ (4) = 0

∴ Statement-II and statement-I both are true and statement-II is correct explanation for statement-I.

Sol 19: (C) $-\{x\}^2 + 2\{x\} + a^2 = 0$

Now, $-3\{x\}^2 + 2\{x\}$



to have no integral roots $0 < a^2 < 1$

$\therefore a \in (-1, 0) \cup (0, 1)$

Sol 20: (B) $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$

$f_4(x) - f_6(x) = \frac{1}{4}\left(\sin^4 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$

$= \frac{1}{4}\left[1 - 2\sin^2 x \cos^2 x\right] - \frac{1}{6}\left[1 - 3\sin^2 x \cos^2 x\right]$

$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

Sol 21: (B) $f(x) + 2f\left(\frac{1}{x}\right) = 3x$

S : f(x) = f(–x)

$\because f(x) + 2f\left(\frac{1}{x}\right) = 3x$ ... (i)

$x \to \frac{1}{x}\quad f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$ ... (ii)

(1) – 2 × (2) $\quad -3f(x) = 3x - \frac{3}{x}\qquad f(x) = \frac{2}{x} - x$

Now, f (x) = f (-x)

$\therefore \frac{2}{x} - x = -\frac{2}{x} + x\qquad \frac{4}{x} = 2x$

$\frac{2}{x} = x \Rightarrow x = \pm\sqrt{2}$

Exactly two elements.

JEE Advanced/Boards

Exercise 1

Sets and Relations

Sol 1: Sets → a collection of well-defined objects which are distinct and distinguishable

Sol 2: Set {x: x ∈ N, x is prime and 3 < x < 5} there is only one natural number 4 which follow 3 < x < 5, but 4 is not a prime no. therefore the set is void.

Sol 3: A = {a, e, i, Q, 4}, B {i,Q}

So B is subset of A

Sol 4: A = {x: x is irrational and 0. 1 < x < 0. 101 between 0.1 and 0. 101 there is infinite number which are irrational so A is infinite set}

Sol 5: Singleton set $\rightarrow$ also known as a unit set which is with exactly on element (i.e., {0}) {π} is singleton set

Sol 6: In a plane there are two points A and B such that OA = OB, O is a fixed point

$\rightarrow$ if OA = OB (assume it is a relation R = {(A, B): OA = OB})

(B, A) $\in$ R R is symmetric

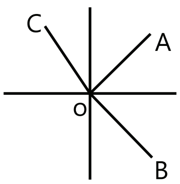
$\rightarrow$ OA = OA (always true)

so (A, A) $\in$ R R is reflexive

$\rightarrow$ if (A, B) $\in$ R$\Rightarrow$ OA = OB ...... (i)

and (B, C) $\in$ R$\Rightarrow$ OB = OC ...... (ii)

from equation (i) and (ii)



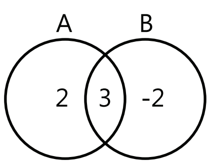
OA = OB = OC

$\Rightarrow$ OA = OC

(A, C) $\in$ R $\Rightarrow$ R is transitive

$\Rightarrow$ R is an equivalence relation

Sol 7:



A = {2, 3}; B = {– 2, 3}

So A $\cup$ B = {–2, 2, 3}

Sol 8: A = {1, 2, 3}, B = {3, 4}

C = {4, 3, 6}

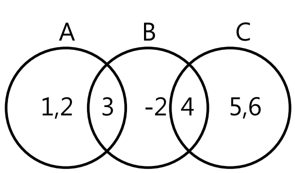
{4 is common is both set B and C}

B $\cap$ C = {4}

and A $\cup$ (B $\cap$ C)

= {1, 2, 3} $\cup$ ({4})

= {1, 2, 3, 4}



Sol 9: Na = {an: n $\in$ N}

$N_6 \cap N_8 \rightarrow$ those element which are common in both sets N_6 and N_8

It means elements which are divisible by 6 and 8 both.

$\Rightarrow$ LCM (6, 8) = 24

so $N_{24} = N_6 \cap N_8$

which are divisible by 24 (6 and 8 both)

Sol 10: (A){x: x $\in$ R and $x^2 + x + 1 = 0$}

$x^2 + x + 1 = 0 \Rightarrow (x^2 + 2x + 1) - x = 0$

$(x + 1)^2 - x = 0$

$(x + 1)^2 = x$

no solution

(A) is empty set

(B){x: x $\in$ R and $x^2 - x + 1 = 0$}

$x^2 - x + 1 = 0$

$x = \dfrac{+1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$ $ax^2 + bx + c = 0$

$x = \dfrac{1 \pm \sqrt{-3}}{2}$ $x = -\dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

no solution

(B) is empty set

(C) {x: x $\in$ R and $x^2 + 2x + 1 \le 0$}

$x^2 + 2x + 1$

$(x + 1)^2$

Which is zero at (x + 1) = 0

$\Rightarrow x = -1$

So (C) is not empty set

(D) {x: x $\in$ R and $x^2 - 2x + 1 \ge 0$

$x^2 - 2x + 1 \ge 0$

$(x - 2)^2 \ge 0$

$(x - 2)^2$ is always greater than or equal to '0'

$(x - 2)^2 \ge 0$ always true for any $x \in R$

Sol 11: Total no. of subsets of M is 56 more than the total no. of subsets of N so we know no. of total subsets of any set A which have n element = 2^n

so $2^m - 2^n = 56$

m and n are integer

64 is the just bigger no. in 2^n terms after 56

so assume

$2^m = 64 = 2^6$

so $64 - 2^n = 56$

$2^n = 64 - 56 = 8$

$8 = 2^3$

n = 3 which is integer

(m, n) = (6, 3)

Sol 12: Given $A = \{x \mid x/2 \in z, 0 \le x \le 10\}$

$B = \{x \mid x \text{ is one digit prime}\}$

$C = \{x \mid x/3 \in N, x \le 12\}$

(A) $\Rightarrow \frac{x}{2} \in z, 0 \le x \le 10$

x = 2z, z is integer, $0 \le x \le 10$

x = 0, 2, 4, 6, 8, 10

(B) x is one digit prime no.

X = 2, 3, 5, 7

(C) $\frac{x}{3} \in N, x \le 12$

$x = 3N, x \le 12$

x = 3, 6, 9, 12

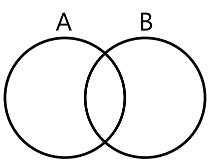
$B \cup C = \{3, 6, 9, 12\} \cup \{2, 3, 5, 7\}$

$= \{2, 3, 5, 6, 7, 9, 12\}$

$A \cap (B \cap C) = \{0, 2, 4, 6, 8, 10\}$

$\cap \{2, 3, 5, 6, 7, 9, 12\} = \{2, 6\}$

Sol 13:



n(A) = 10

n(B) = 15

Then $A \cup B$ will have

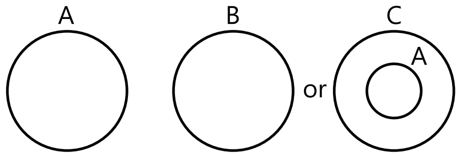
$\rightarrow A \cup B = A + B - A \cap B$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ '

$= 10 + 15 - n(A \cap B)$

$= 25 - n(A \cap B)$

$n(A \cap B)$ can be zero to n(A)



$n(A \cap B) = 0$ $\qquad$ $n(A \cap B) = n(A)$

so $15 \le n(A \cup B) \le 25$

Sol 14: Total set = 1000 families of a city

40% read newspaper A

20% read newspaper B

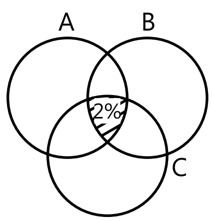
5% read newspaper both A and B($A \cap B$)

10% read newspaper C

3% read newspaper both B and C($B \cap C$)

4% read newspaper both A and C($A \cap C$)

2% read newspaper both all ($A \cap B \cap C$)



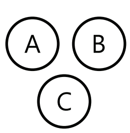
Families which read only news paper A

$= A - A \cap B - A \cap C + A \cap B \cap C$

40 – 5 – 4 + 2 = 33 %

33% of 1000 = 330 families

Sol 15:



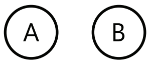
$\left.\begin{array}{l} n(A) = 10 \\ n(B) = 6 \\ n(C) = 5 \end{array}\right\}$ all are disjoint

So $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

$= 10 + 6 + 5 = 21$

Other terms terns zero because there is no joints between any two sets

Sol 16: A and B are disjoint

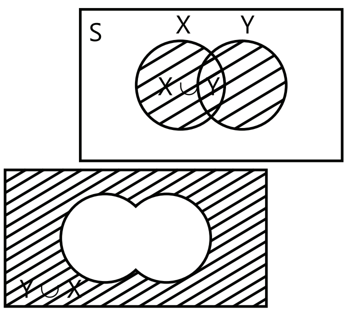


so $A \cap B = f$

$n(A \cap B) = 0$

$\therefore\ n(A \cup B) = n(A) + n(B)$

Sol 17:

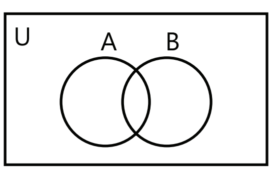


$X \cap (Y \cup X)^C$

so in $(Y \cup X)^C$, there is no elements of X

so $X \cap (Y \cup X)^C$ is π

Sol 18:



$n(U) = 700$

$n(A) = 200$

$n(B) = 300$, $n(A \cap B) = 100$

$A^c \cap B^c$

We know

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

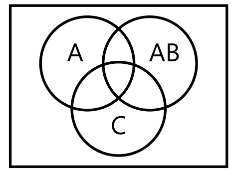
$\overline{(A \cup B)} = A^C \cap B^C$

So $n\overline{(A \cup B)} = n(A^C \cap B^C) = n(U) - n(A \cup B)$

$n(A \cup B) = 200 + 300 - 100 = 400$

$n(A^C \cap B^C) = 700 - 400 = 300$

Sol 19:



Let $(A \cap B) \cup (B \cap C)$

$(A \cap B^C)^C = (A^C \cup B)$

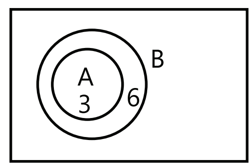
$\Rightarrow (A^C \cup B) \cup (B \cap C)$

We know that

$(B \cap C) \subset (B)$

so $(A^C \cup B) \cup (B \cap C) = A^C \cup B$

Sol 20:



Given $n(A) = 3$

$n(B) = 6$

we know

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

for minimize $n(A \cup B)$, $n(A \cap B)$ should be maximum which can be $n(A)$ (lower in both)

so $n(A \cup B) = 3 + 6 - 3 = 6$

Sol 21: Let $\rightarrow$ class of 100 students

55 students have passed in mathematics (M)

67 students have passed in physics (P)

no student fail

so $n(M \cap P) = -n(M \cup P) + n(M) + n(P)$

$= -100 + 55 + 67 = 22$

no of students which are passed in physics only

$= n(P) - n(M \cap P) = 67 - 22 = 45$

Sol 22: $X = \{1, 2, 3, 4, 5, 6\}$ universal set

$A = \{1, 2, 3\}$

$B = \{2, 4, 5\}$

$C = \{3, 4, 5, 6\}$

So $A - B = \{1, 2, 3\} - \{2, 4, 5\} = \{1, 3\}$

$B - A = \{2, 4, 5\} - \{1, 2, 3\} = \{4, 5\}$

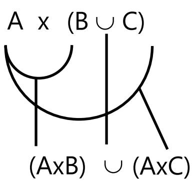
$(A - B) \cup (B - A) = \{1, 3) \cup \{4, 5\} = \{1, 3, 4, 5\}$

$(A - B) - C = \{1, 3\} - \{3, 4, 5, 6\} = \{1\}$

$C' = X - C = \{1, 2\}$

$A \cap C' = \{1, 2, 3\} \cap \{1, 2\} = \{1, 2\}$

Sol 23:



$A \times (B \cup C) \quad \Rightarrow (A \times B) \cup (A \times C)$

Sol 24: $A \times (A \cap C)$

$(A \times B) \cap (A \times C)$

Sol 25: $A = \{a, b, c, d\}$

$B = \{b, c, d , e\}$

for $\rightarrow (A \times B) \cap (B \times A)$

$A \times B = \{(a_i, b_i)\}$

= {(a, b)(a, c)(a, d)(a, e), (b, b)(b, c)(b, d) (b, e)(c, b)(c, c) (c, d)(c, e) (d, b)(d, c)(d, d) (d, e)}

B × A = {(b,a) (b,b) (b,c) (b,d) (c,a) (c,b) (c,c) (c,d) (d,a) (d,b) (d,c) (d,d) (e,a) (e,b) (e,c) (e,d)}

(A × B) ∩ (B × A) = {(b,b) (b,c) (b,d) (c,b) (c,c) (c,d) (d,b) (d,c) (d,d)}

total elements = 9

Sol 26: Given $A = \{1, 2, 3, 4, 5\}$

$R = \{x, y\}|x, y \in A$ and $x < y|$

$X < x$ always false

so $(x, x) \notin R$

R is not reflexive $\rightarrow$ if $(x, y) \in R \Rightarrow x < y$

so $y < x$ always false

$(y, x) \notin R$

R is not symmetric

$\Rightarrow$ if $(x, y) \in R \Rightarrow x < y$ .... (i)

and $(y, z) \in R \Rightarrow y < z$ .... (ii)

so from equation (i) and (ii)

$x < y < z$

so $x < z$

$\therefore (x, z) \in R$ R is transitive

Sol 27: $n R m \leftrightarrow n$ is a factor or of m (i. e. n/m)

Same as exercise –III question III

Sol 28: $n(A) = m$

$n(B) = n$ and $R: A \rightarrow B$

then total no. of relations form A to B is

$(2^m)^n = 2^{mn}$

Sol 29: $L \Rightarrow$ set of all straight lines in a plane

Relation $R \rightarrow \alpha R \beta \leftrightarrow \alpha \ \hat{} \beta, \alpha, \beta \in L$ any line never perpendicular to itself

so $(\alpha, \alpha) \notin R \ \alpha \perp \alpha$ (false)

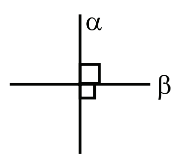
R is not reflexive

$\rightarrow$ if $(\alpha, \beta) \in R \Rightarrow \alpha \perp b$

so $\beta \perp a$

$\rightarrow (\beta, \alpha) \in R$

R is symmetric

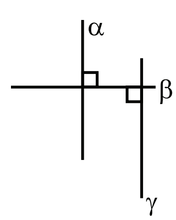


$\rightarrow$ if $(\alpha, \beta) \in R \Rightarrow \alpha \perp b$

and $(\beta, \gamma) \in R \Rightarrow \beta \perp g$

so $a || \gamma \Rightarrow (\alpha \perp \gamma) \rightarrow$ false

$\Rightarrow (\alpha, \gamma) \notin R$ R is not transitive.



Functions

Sol 1: (i) $y=\sqrt{\frac{x-2}{x+2}}+\sqrt{\frac{1-x}{1+x}}$

$x+2\neq 0\ x\neq -2$

$x+1\neq 0\ x\neq -1$

$\frac{x-2}{x+2}\geq 0$

Case I $x-2\geq 0$ & $x+2>0$

$x\geq 2$ & $x>-2\ \ x\in[2,\infty]$

Or Case II $x-2\leq 0$ & $X+2<0$

$x\leq 2$ & $x<-2$

$x\in(-\infty,-2)$

And $\frac{1-x}{1+x}\geq 0$

Case II $1-x\geq 0$ & $1+x>0$

$x\leq 1$ & $x>-1\Rightarrow x\in(-1,1]$

Or Case II $(1-x)\leq 0$ & $1+x<0$

$x\geq 1$ & $x>-1$

So the range domain will be $x\in(-1,1]\cap[2,\infty)$

$x\in\phi$

(ii) $y=\sqrt{x^2-3x+2}+\frac{1}{\sqrt{3+2x-x^2}}$

$y=\sqrt{(x-2)(x-1)}+\frac{1}{\sqrt{(3+x)(x+1)}}$

$(x-2)(x-1)\geq 0\Rightarrow x\in(-\infty,1]\cup[2,\infty)$

And $(3-x)(x+1)>0\Rightarrow x\in(-1,3)$

The domain $x\in(-1,1],\cup[2,3)$

(iii) $y=\sqrt{x}+\frac{1}{(x-2)^{1/3}}-\log_{10}(2x-3)$

$x\geq 0$

And $x-2\neq 0$

$x\neq 2$

And $2x-3>0$

$x>\frac{3}{2}$

So the domain $x\in\left(\frac{3}{2},\infty\right)-\{2\}$

(iv) $y=\sqrt{\frac{1-5^x}{7^{-x}-7}}$

$\frac{1-5^x}{7^{-x}-7}\geq 0$

5^x & 7^x are always greater than zero.

Case I $1-5^x\geq 0$ & $7^{-x}-7<0$

$5^x\leq 1$ & $7^{-x}>7$

$x\leq 0$ & $x<-1$

$x\in(-\infty,-1)$

Or Case II $1-5^x\leq 0$ & $7^{-x}-7<0$

$5^x\geq 1$ & $7^{-x}<7$

$x\geq 0$ & $x\geq -1$

$x\in[0,\infty)$

So the answer is $x\in(-\infty,-1)\cup[0,\infty)$

(v) $y=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2}$

$(1-x)>0$ & $1-x\neq 1$

$x<1\ \ x\neq 0$

And $x+2>0$

$x\geq -2\ \ x\in[-2,\infty)$

So the domain $x\in[-2,1)-\{0\}$

(vi) $f(x)=\log_{100x}\left(\frac{2\log_{10}x+1}{-x}\right)$

$100x\neq 0,1\ 100\ x>0$

$x\neq 0,\frac{1}{100}\ \ x>0$

And $\frac{2\log_{10}x+1}{-x}>0$

$\frac{2\log_{10}x+1}{x}<0$

$x>0$ So

$2\log_{10}x+1<0$

$\log_{10}x<-\frac{1}{2}\ \Rightarrow 0<x<\frac{1}{\sqrt{10}}$

So domain $x\in\left(0,\frac{1}{\sqrt{10}}\right)-\left(\frac{1}{10}\right)$

(vii) $f(x)=\dfrac{1}{\log(1-x)}+\sqrt{x+2}$

$x+2 \ge 0$ & $x \ge -2$

And $(1-x)>0$ & $1-x \ne 1$

$x<1$ & $x \ne 0$

$x \in(-\infty,1)-\{0\}$

So domain $[-2,1)-\{0\}$

(viii) $y=\sqrt{\log\left(\dfrac{5x-x^2}{4}\right)}$

$\log\left(\dfrac{5x-x^2}{4}\right) \ge 0$

$\dfrac{5x-x^2}{4} \ge 1$

$5x-x^2-4 \ge 0$

$x^2-5x+4 \le 0$

$(x-4)(x-1) \le 0$

$x \in [1,4]$

(ix) $f(x)=\sqrt{x^2=|x|}+\dfrac{1}{\sqrt{9-x^2}}$

$9-x^2>0$

$x^2<9$

$x \in(-3,3)$

And $x^2-|x| \ge 0$

$\sqrt{1+x+x^2}-\sqrt{1+x+x^2}\ \dfrac{1+2^{Kx}}{1-2^{Kx}}$

$x \in [1,\infty)$

if $x<0$ $x(x+1) \ge 0$

$x \in (-\infty,-1]$

So the domain $x \in (-3,-1] \cup [1,3)$

(x) $\sqrt{(x^2-3x-10)\ell n^2(x-3)}$

$x-3>0$ at $x=4$

$x>3$ $f(x)=0$

And $x^2-3x-10 \ge 0$ $(x-5)(x+2) \ge 0$

$x \in (-\infty,-2] \cup [5,\infty)$

So domain $x \in (5,\infty) \cup \{4\}$

(xi) $f(x)=\sqrt{(\sin x)^2+\cos^2 x+2\sin x\cos x-1}$

$=\sqrt{2\sin x\cos x}=\sqrt{\sin 2x}$

$\sin 2x \ge 0$

$2x \in [2n\pi,(2n+1)\pi]$

$x \in \left[n\pi,\left(n+\dfrac{1}{2}\right)\pi\right]$

(xii) $f(x)=\sqrt{\dfrac{\cos x-1/2}{6+35x-6x^2}}$

$=\sqrt{\dfrac{\cos x-1/2}{(6-x)(6x+1)}}$ $x \ne 6, \dfrac{-1}{6}$

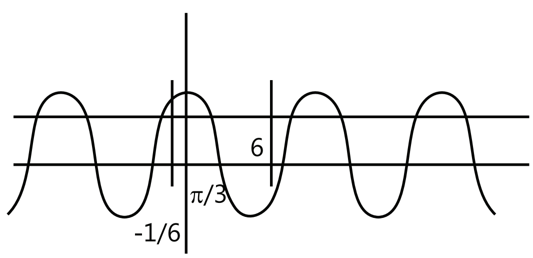
Case I: Let's say $\dfrac{-1}{6}<x<6$

So $\cos x-\dfrac{1}{2}>0$

Case II: $x<\dfrac{-1}{6}$

So $\cos x\dfrac{-1}{2}<0$

$\cos x<\dfrac{1}{2}$



Case III: $x>6$

$\cos x \le \dfrac{1}{2}$

From group analysing solution

$\left(\dfrac{-1}{6},\dfrac{\pi}{3}\right] \cup \left[\dfrac{5\pi}{3},6\right) \cup$

$\left[2k\pi+\dfrac{\pi}{3},2k\pi+\dfrac{5\pi}{3}\right], k \in I-\{0\}$

(xiii) $f(x)=\sqrt{\log_{\frac{1}{3}}(\log_4[x]^2-5)}$

$\log_3(\log_4[x]^2-5) \ge 0$

$\log_4[x]^2-5) \le 1$

$[x]^2 \le 9$

$[x] \in [-3, 3]$

And $[x]^2 - 5 > 0$

$[x]^2 > 5 \Rightarrow$

And $[x]^2 - 5 \ne 1$ $[x]^2 - 5 > 1$

$[x]^2 \ne 6$ $[x]^2 > 6$ $[x] = 3, -3$

And $[x]^2 - 5 \ne 4$ $[x]^2 \ne 9$

$[x] \ne \pm 3$

So domain $x \in [-3, 2) \cup [3, 4)$

(xiv) $f(x) = \frac{1}{[x]} + \log_{(2\{x\}-5)}(x^2 - 3x + 10) + \frac{1}{\sqrt{1-|x|}}$

$[x] \ne 0$ $x \in [0, 1)$

$2\{x\} - 5 < 0$ So $\log_{2\{x\}-5}(x^2 - 3x + 10)$ is not defined. So $x \in \phi$

(xv) $f(x) = \log_7 \log_5 \log_3 \log_2 (2x^2 + 5x^2 - 14x)$

$\log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x) > 0$

$\log_3 \log_2 (2x^3 + 5x^2 - 14x) > 1$

$\log_2 (2x^3 + 5x^2 - 14x) > 3$

$2x^3 + 5x^2 - 14x > 8$

$2x^3 + 5x^2 - 14x - 8 > 0$

$(x-2)(2x^2 + 9x + 4) > 0$

$(x-2)(2x+4)(2x+1) > 0$

$x \in \left(-4, \frac{-1}{2}\right) \cup (2, \infty)$

(xvi) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

$\cos 2x > 0$ and $16 - x^2 \ge 0$

$2x \in \left[2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right]$ & $x \in [-4, 4]$

$x \in \left[n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{4}\right]$ & $x \in [-4, 4]$

So domain $\in \left[-\frac{5\pi}{4}, -\frac{3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$

(xvii) $f(x) = \ell n(\sqrt{x^2 - 5x - 24 - x - 2})$

$\sqrt{x^2 - 5x - 24} - x - 2 > 0$

And $x^2 - 5x - 24 > 0$

$x^2 - 3x - 8x - 24 > 0$

$(x+3)(x-8) > 0$

$x \in [-\infty, -3] > 0$

$x \in [-\infty, -3] \cup [8, \infty)$

Case I if $x + 2 > 0 \Rightarrow x > -2$

The $x^2 - 5x - 24 > (x+2)^2$

$-5x - 24 > 4 + 4x$

$9x + 28 < 0$

$x < \frac{-28}{9}$

So no answer

Case II if $x + 2 < 0$ then always true in interval

$\sqrt{x^2 - 5x - 24} > x + 2$

$x < -2$

So the domain $x \in (-\infty, -3]$

(xvii) $y = \log(1 - \log_{10}(x^2 - 5x + 16)$

$1 - \log_{10}(x^2 - 5x + 16) > 0$ AND $x^2 - 5x + 16 > 0$

$\log_{10}(x^2 - 5x + 16) < 1$ AND is the always positive as its minimum value of $x = \frac{-b}{2a} = \frac{5}{2}$ is positive.

$x^2 - 5x + 6 < 0$

$(x-3)(x-2) < 0$

$x \in (2, 3)$

So the domain $2 < x < 3$

(xix) $f(x) = \log_4 (2 - (x)^{1/4} - \frac{2\sqrt{x+1}}{\sqrt{x+2}}$

$2 - (x)^{1/4} - \frac{2\sqrt{x+1}}{\sqrt{x+2}} > 0$

$(x)^{1/4} - \frac{2\sqrt{x+1}}{\sqrt{x+2}} > 2$

$(x)^{3/4} + 2(x)^{1/4} + 2(x)^{2/4} + 1 < 2(x)^{2/4} + 4$

$x \ge 0$ so put $x = t^4$

$t^3 + 2t + 2t^2 + 1 < 2t^2 + 4$

$t^3 + 2t - 3 < 0$

$(t-1)(t^2+t+3)<0$

t^2+t+3

Is always greater then zero.

So $(t-1)<0$

$t<1$

$x^{1/4}<1 \Rightarrow 0\le x<1$

Sol 2: (i) $y=\log_{\sqrt{5}}\left(\sqrt{2}(\sin x-\cos x)+3\right)$

$\sqrt{2}(\sin x-\cos x)+3>0$

$\sqrt{2}\sqrt{2}(\sin x\cos 45^0-\cos x\sin 45^0)>-3$

$2\sin(x-45^0)>-3$

$\sin(x-45^0)>\frac{-3}{2}$ so the domain $x\in R$

Range $f(x)=\log_{\sqrt{5}}\left[2\sin\left(x-\frac{\pi}{4}\right)+3\right]$

$+1\le 2\sin\left(x-\frac{\pi}{4}\right)+3\le 5$

$\log_{\sqrt{5}}(1)\le\log_{\sqrt{5}}\left(2\sin\left(x-\frac{\pi}{4}\right)+3\right)\le\log_{\sqrt{5}}(5)$

$0\le f(x)\le 2$

(ii) $y=\frac{2x}{1+x^2}$

Domain $x\in R$

Range $y=\frac{2}{x+\frac{1}{x}}$

$x+\frac{1}{x}\ge 2$ for $x\ge 0$ from arithmetic mean > geometric mean

$x+\frac{1}{x}\le -2$ for $x\le 0$ from A.M >G.M

So [-1,1]

(iii) $f(x)=\frac{x^2-3x+2}{x^2+x-6}$

Domain $x^2+x-6\ne 0$

$x^2+3x-2x-6\ne 0$

$(x+3)\,(x-2)\ne 0$

$x\ne 2,-3$

Range $f(x)=\frac{(x-2)(x-1)}{(x-2)(x+3)}$

For $x\ne 2\, f(x)=1-\frac{4}{x+3}$

$x=2\, f(x)$ not defined

Range $f(x)\in R-\{\frac{1}{5}\}-\{1\}$

at $x=2\Rightarrow\left(\frac{1}{5}\right)$ at $x=\infty$ [1]

(iv) $f(x)=\frac{x}{1+|x|}$ domain $x\in R$

for $x>0$ $f(x)=\frac{x}{1+x}=1-\frac{1}{1+x}$

$0<f(x)<1$

for $x<0$ $f(x)=\frac{x}{1-x}=\frac{1}{\frac{1}{x}-1}$

$-1<f(x)<0$

So $f(x)\in(-1,1)$

(v) $y=\sqrt{2-x}+\sqrt{1+x}$

$z-x\ge 0$ and $1+x\ge 0$

$x\le 2$ and $x\ge -1$

Domain $x\in[-1,2]$

Range $\sqrt{2-x}$ decreases with increment in x $\sqrt{1+x}$ Increase with increment in x

Since both are linear function root, so we can say that it's maximum will be at middle point of boundary defined.

So $f(x)\le\sqrt{2-\left(\frac{1}{2}\right)}+\sqrt{1+\left(\frac{1}{2}\right)}$

$f(x)\le\sqrt{\frac{3}{2}}+\sqrt{\frac{3}{2}}=\sqrt{6}$

$f(x)\ge\sqrt{3}$ (at boundary point)

$\sqrt{3}\le f(x)\le\sqrt{6}$

(vi) $f(x)=\log_{(\operatorname{cosec} x-1)}(2-[\sin x]-[\sin x]^2)$

$[\sin x]=0$ or -1

$2-[\sin x]-[\sin x]^2=2$

$\text{cosec}x-1>0$ & $\text{cosec}x-1\neq 1$

$\sqrt{x-1}+\sqrt{6-x}$

$\dfrac{1}{\sqrt{|x|-x}}$

$x\neq 2n\pi+\dfrac{\pi}{6},(2n+1)-\dfrac{\pi}{6}$

$x\in(2n\pi(2n+1)\pi)-\{2n\pi+\dfrac{\pi}{2}\}$

So domain

$\left[\dfrac{3}{2},\infty\right)\rightarrow\left[\dfrac{7}{4},\infty\right)$

Range $\log_a 2$;

$a\in(0,\infty)-\{1\}\ \Rightarrow$ Range $(-\infty,\infty)-\{0\}$

(vii) $f(x)=\dfrac{x+1}{x-2}$

Domain $x\in R-\{2\}$

Range $f(x)=1+\dfrac{3}{x-2}$

$f(x)\in(-\infty,\infty)-\{1\}$

Sol 3: Injective = one to one mapping

Surjective = onto function

(a) $f(x)=\dfrac{x^2+x+1}{x^2+2x+3}\ =1-\dfrac{(x+2)}{x^2+2x+3}$

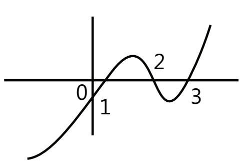
$=1-\dfrac{(x+2)}{(x+1)^2+1}$

It's range is not R. So not surjective not injective

$f(x)=x^3-6x^2+11x-6$

Range $\in R$ onto function $\Rightarrow$ surjective

$f(x)=(x-1)(x^2-5x+6)$



$=(x-1)(x-3)(x-2)$

Not one-one as at x = 1, 2, 3 f(x) = 0

Not injective

(b) $f(x)=(x^2+5x+9)(x^2+5x+1)$

$=\left[\left(x+\dfrac{5}{2}\right)^2+9-\dfrac{25}{4}\right]\left[\left(x+\dfrac{5}{2}\right)^2+1-\dfrac{25}{4}\right]$

$=\left[\left(x+\dfrac{5}{2}\right)^2-\dfrac{11}{4}\right]\left[\left(x+\dfrac{5}{2}\right)^2-\dfrac{25}{4}\right]$

It has two roots so not injective nor Surjective (as the Ranges not R)

Sol 4: $f(x)=\dfrac{1}{1-x}$

$f_2(x)=f[f(x)]\ =f\left[\dfrac{1}{1-x}\right]\ =\dfrac{1}{1-\left[\dfrac{1}{1-x}\right]}$

$f_3(x)=f\left[f\{f(x)\}\right]=f\left[f\left\{\dfrac{1}{1-x}\right\}\right]$

$=\dfrac{1}{1-\dfrac{1}{1-\left\{\dfrac{1}{1-x}\right\}}}=\dfrac{1-\left\{\dfrac{1}{1-x}\right\}}{-\left\{\dfrac{1}{1-x}\right\}}$

Sol 5: $f(n)=-1$, $\qquad f\left(n+\dfrac{1}{2}\right)=0$

$f(n+x)=a(n+x)+b$ where $0\leq x<1$

$x=0,\ f(n)=-1=ax+b$ ....(i)

$x=\dfrac{1}{2},\ f\left(n+\dfrac{1}{2}\right)=0=a\left(n+\dfrac{1}{2}\right)+b$

$-\dfrac{a}{2}=ax+b$ ...(ii)

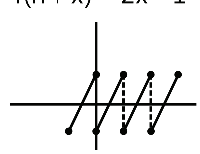
$-\dfrac{a}{2}=-1$

a = 2

$b=-1-ax=-1-2x$

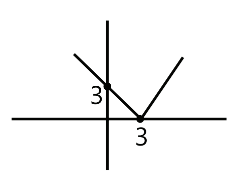
So $f(n+x)=2(n+x)-1-2x$

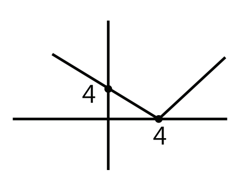
$f(n+x)=2x-1$ where $0\leq x<1,\ n\in R^+$



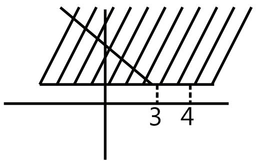
Sol 6: (a) $|(x-3)+(4-x)|<|x-3|+|4-x|$

$\Rightarrow 1<|x+3|+|4-x|$





$|x+3|+|4-x|>1$



$x \in (-\infty, 3) \cup (4, \infty)$

(b) $|x-(x-2)| > |x| - |x-2|$

$2 > |x| - |x-2|$

$|x| - |x-2| < 2$

If $x > 2$ not true

If $0 \le x \le 2$ $2x - 2 < 2$

$x \le 2$ $x \in [0,2]$

If $x < 0$ $-2 < 2$ always true

$(-\infty, 2)$

Sol 7: (a) $f(-x) = \log\left(-x + \sqrt{1+x^2}\right)$

$f(-x) + f(x) = \log\left(\sqrt{1+x^2} - x\right)\left(\sqrt{1+x^2} + x\right) = 0$

$f(x) = -f(x)$

So odd function

(b) $f(x) = \dfrac{a^x + 1}{a^x - 1}$

$f(-x) = \dfrac{\frac{1}{a^x} + 1}{\frac{1}{a^x} - 1} = \dfrac{1 + a^x}{a^x - 1} = -f(x)$

Odd function

(c) $f(x) = x^4 - 2x^2$

$f(-x) = x^4 - 2x^2 = f(x)$

Even

(d) $f(x) = x^2 - |x|$

$f(-x) = x^2 - |x|$

Even function

(e) $f(-x) = -x\sin^2 x + x^3 = -f(x)$

Odd function

(f) $f(x) = k = f(-x)$

Even

(g) $\ell n\left(\dfrac{1+x}{1-x}\right) = f(-x)$

$f(-x) = -\ell n\left(-\dfrac{1-x}{1+x}\right) = -f(x)$

Odd function

(h) $f(x) = \dfrac{(1+2^x)^2}{2^x}$

$f(-x) = \dfrac{(2^x+1)^2}{2^x} = f(x)$

Even

(i) $f(-x) = \dfrac{-x}{\frac{1}{e^x} - 1} - \dfrac{x}{2} + 1 \ne f(x) \ne -f(x)$

So neither odd nor even

(j) $f(x) = [(x+1)^2]^{1/3} + [(x-1)^2]^{1/3}$

$f(-x) = \left[(1-x)^2\right]^{1/3} + \left[(-x-1)^2\right]^{1/3}$

$= \left[(x-1)^2\right]^{1/3} + \left[(1+x)^2\right]^{1/3} = f(x)$

Even function

Sol 8: (a) $f(x) = \sin^4 x + \cos^4 x$

$f(x) = (\sin^2 x)^2 + ((\cos^2)^3 x)^2 + 2\sin^2 x$

$\cos^2 x - 2\sin^2 \cos^2 x$

$f(x) = \left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x \cos^2 x$

$f(x) = 1 - \dfrac{(\sin 2x)^2}{2}$

$\sin 2x \Rightarrow$ Time period $\Rightarrow \pi$

$(\sin 2x)^2 \Rightarrow$ Timeperiod$\Rightarrow \pi/2$

So f(x) has $\dfrac{\pi}{2}$ Time period

(b) $f(x) = |\sin x| + |\cos x|$

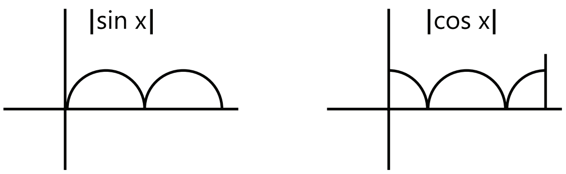
Case-I $0 < x < \dfrac{\pi}{2}$

$f(x) = \sin x + \cos x$

Case-II $\dfrac{\pi}{2} < x < \pi$ $f(x) = \sin x - \cos x$

Case-III $\pi < x < \dfrac{3\pi}{2}$ $f(x) = -(\sin x + \cos x)$

Case-IV $\frac{3\pi}{2} < x < 2\pi \quad f(x) - \sin x + \cos x$



Since combining graphs, we can see $|\sin x| + |\cos x|$ has $\frac{\pi}{2}$ Time period.

(c) $f(x) = \cos\frac{3}{5}x - \sin\frac{2x}{7}$

$\cos 3x$ has time period $\frac{2\pi}{3}$

$\cos\frac{3x}{5}$ has time period $\frac{10\pi}{3}$

$g(x) = \sin x$

has time period $\frac{2\pi}{2} \times (7)$

$$\text{L.C.M.}\left(\frac{10\pi}{3}, 7\right) = \frac{\text{L.C.M.}(10\pi, 7\pi)}{\text{H.C.F.}(3,1)} = 70\pi$$

Sol 9: (a) $10^x + 10^y = 10$

$y = \log(10 - 10^x)$

Domain $x \le 1$

(b) $(-\infty, \infty) - \{0\}$

If $y > 0 \;\; y = x$

If $y < 0 \;\; y = \frac{x}{3}$

Sol 10: $f(x) = \cos nx \cdot \sin\left(\frac{5}{n}x\right)$

$$= \left[2\cos nx \cdot \sin\frac{5}{n}x\right]\frac{1}{2}$$

$$= \left[\sin\left(\frac{5}{n} - x\right)x + \sin\left(\frac{5}{x} + n\right)x\right]\frac{1}{2}$$

$$\text{L.C.M.}\left[\frac{2x}{\frac{5}{n} - n}, \frac{2\pi}{\frac{5}{n} + n}\right] = 3\pi$$

$$\text{L.C.M.}\left(\frac{2n\pi}{5 - n^2}, \frac{2n\pi}{5 + n^2}\right) = 3\pi$$

$$\frac{2n\pi}{\text{H.C.F.}(5 - n^2, 5 + n^2)} = 3\pi$$

Sol 11: $f(x) = \ell n\left(x + \sqrt{x^2 + 1}\right)$

$y = \ell n\left(x + \sqrt{x^2 + 1}\right)$

$e^y - x = \sqrt{x^2 + 1}$

$e^{2y} + x^2 - 2e^y(x) = x^2 + 1$

$x = -\frac{(1 - e^{2y})}{2e^y} = \frac{e^{2y} - 1}{2e^y}$

$\frac{1}{2}\left(e^y - \frac{1}{e^y}\right)$

$f^{-1}(x) = \frac{1}{2}\left(e^x - \frac{1}{e^x}\right)$

(b) $f(x) = 2^{\frac{x}{x-1}}$

$\log_2 y = \frac{x}{x-1}$

$\log_2 y = 1 + \frac{1}{x-1}$

$(x - 1) = \frac{1}{\log_2 y - 1}$

$x = 1 + \frac{1}{\log_2 y - 1}$

$x = \frac{\log_2 y}{\log_2 y - 1}$

$f^{-1}(x) = \frac{\log_2 y}{\log_2 x - 1}$

(c) $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$

$y = \frac{10^{2x} - 1}{10^{2x} + 1}$

$\frac{y+1}{y-1} = \frac{2(10^{2x})}{-2}$

$\frac{y+1}{y-1} = 10^{2x} \Rightarrow x = \frac{\log_{10} - \left(\frac{y+1}{y-1}\right)}{2}$

$\Rightarrow f^{-1}(x) = \frac{1}{2}\log\left(\frac{x+1}{1-x}\right)$

Sol 12: $f(x)=(a-x^n)^{1/n}$

$$fof(x)=\left[a-\left[(a-x)^n\right]^n\right]^{1/n}$$

$$=\left[a-\left[(a-x^n)\right]^n\right]^{1/n}$$

to $f(x)=x$

If g is inverse of f then fog(x)=x from

Above we can say $f^{-1}(x)=f(x)$

Sol 13: $f(x)=x^2+x-2$

$$y=\left(x+\frac{1}{2}\right)^2-\frac{9}{4}$$

$$x=-\frac{1}{2}+\sqrt{y-\frac{9}{4}}$$

$$f^{-1}(x)=y=-\frac{1}{2}+\sqrt{x-\frac{9}{4}}$$

$$f(x)=f^{-1}(x)$$

$$\Rightarrow fof(x)=x$$

$$(x^2+x-2)^2+(x^2+x-2)-2=x$$

Solving this equation we get

$x=\pm\sqrt{2}$

Sol 14: $f(x)\begin{cases}x^2+1, & x<-1\\ x^3-1, & x\le x<0\end{cases}$

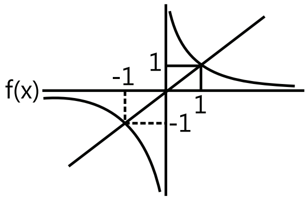
(a) If f(x) is odd

$$f(x)\begin{cases}x^2+1, & x<-1\\ +x^3, & -1\le x<0\\ +x^3, & 1\ge x\ge 0\\ -(x^2+1), & x>1\end{cases}$$

(b) Even function

$$f(x)\begin{cases}x^2+1, & x<-1\\ +x^3, & -1\le x<0\\ -x^3, & 1\ge x\ge 0\\ x^2+1, & x>1\end{cases}$$

Sol 15: $f(x)=\max\left(x,\frac{1}{x}\right)$

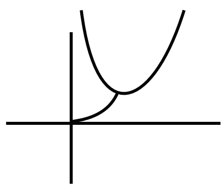


$$f\left(\frac{1}{x}\right)=f(x)=\max\left(\frac{1}{x},x\right)$$

$$f(x)\begin{cases}\frac{1}{x}, & 0<x<1\\ x, & x>1\\ \frac{1}{x}, & x<-1\\ x, & 0>x>-1\end{cases}$$

$$f\left(\frac{1}{x}\right)=f(x)$$

$$g(x)\begin{cases}\frac{1}{x^2} & \text{if} & 0<x\le 1\\ x^2 & \text{if} & x>1\end{cases}$$



Sol 16: $f(x)=\dfrac{x^2+2x+c}{x^2+4x+3c}$

$$f(x)=\frac{(x+1)^2+c-1}{(x+2)^2+3c-4}$$

$(c-1)\le 0$

$(36-4)\le 0$

So the value of f(x) is always F if $x\in R$

Sol 17: (a) $f(x)=\sqrt{\log_2(x^2-2x+2)}$

$$\sqrt{\log_2(x-1)^2+1)}$$

$$\log_2\left[(x-1)^2+1\right]\ge 0$$

$$(x-1)^2\ge 0$$

That is always true so domain is R

$(x-1)^2 + 1 \geq 1$

$\log_2[(x-1)^2 + 1] \geq 0$

Range $f(x) \geq 0$

(b) $f(x) = \dfrac{2x+3}{x-2}$

$= \dfrac{2(x-2)+7}{x-2} = 2 + \dfrac{7}{12}$

$f(x) \neq 7$ $f(x) \in R - \{2\}$

Bijective function

Sol 18: $-1 \leq x(x-2) < \infty$

$1 \leq \dfrac{x(x-2)+1}{4} < \infty$

$1 \leq 2^{\sqrt{\frac{x(x-2)+1}{4}}} < \infty$

$2 \leq f(x) < \infty$

At x=0 & x=2 value of f(x) is same.

So many one function.

Exercise 2

Sets and Relations

Single Correct Choice Type

Sol 1: (D) n R m $\leftrightarrow$ n is a factor of m

$\rightarrow$ every natural no. is a factor of itself

R is reflexive

$\rightarrow$ if n is factor of m, its not necessary that m is also factor of n

R is not symmetric

$\rightarrow$ if n is factor Q m

and m is factor Q ℓ

so n is also a factor of ℓ

(i.e., 3 R 6 and 6 R 18 and 3 R 18 is true)

Sol 2: (C) a R b $\leftrightarrow$ 1 + ab > 0

for a R b $\Rightarrow$ 1 + ab > 0

$\Rightarrow$ ab > – 1

$\rightarrow$ a R a $\Rightarrow$ 1 + a^2 > a

it is always true

so R is reflexive

$\rightarrow$ if a R b $\rightarrow$ 1 + ab > 0

1 + ab = 1 + ba

so 1 + ba also greater than zero

so (b, a) $\in$ R

R is symmetric

$\rightarrow$ if (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow$ 1 + ab > 0,1 + bc > 0

Its not necessary to 1 + ac > 0

R is not transitive.

Sol 3: (A) (A)x R, y $\leftrightarrow$ |x| = |y|

|x| = |x| reflexive

|x| = |y| $\Rightarrow$ |y| = |x| symmetric

|x| = |y| and |y| = |z|

equivalence relation

so |x| = |y| = |z| $\Rightarrow$ |x| = |z| transitive

Sol 4: (D) Relation R defined in A = {1, 2, 3}

such that a R b $\Rightarrow$ $|a^2 - b^2| \leq 5$

(1, 2) $\Rightarrow$ $|(1^2 - 2^2)| = 3 \leq 5$

(1, 3) $\Rightarrow$ $|1^2 - 3^2| = 8 \leq 5$

(2, 3) $\Rightarrow$ $|2^2 - 3^2| = 5 \leq 5$

and (a, a) = $|a^2 - a^2| = 0 \leq 5$

and (a, b) = (b, a) $\Rightarrow$ $|a^2 - b^2| = |b^2 - a^2|$

so R = {(1, 2), (2, 1), (2, 3), (3,2), (1, 1), (2, 2), (3, 3)}

so R^{-1} = R

Domain = {1, 2, 3}

Range = {1, 2, 3} , so

Option D is false (Range = {5})

Sol 5: (A) (x, y) $\in$ R $\leftrightarrow$ $x^2 - 4xy + 3y^2 = 0$

for all x, y $\in$ N

$\rightarrow$ for (x, x) $\in$ R $\leftrightarrow$ $x^2 - 4x^2 + 3x^2 = 0$

= 0

R is reflexive

$\rightarrow$ if (x, y) $\in$ R $\Rightarrow$ $x^2 - 4xy + 3y^2 = 0$

then for (y, x) $\in$ R $\Rightarrow$ $y^2 - 4xy + 3x^2$ should be zero

but $3x^2 - 4xy + y^2$

$= x^2 - 4xy + 3y^2 + 2x^2 - 2y^2$

$= 0 + 2x^2 - 2y^2$

$\rightarrow (y, x) \in R \Rightarrow x^2 - y^2 = 0$

its not solution for all $(x, y) \in N$

so R is not symmetric

$\rightarrow (x, y) \in R\ x^2 - 4xy + 3y^2 = 0$

$(y, z) \in R \rightarrow y^2 - 4zy + 3z^2 = 0$

but we cannot find $x^2 - 4zx + 3z^2 = 0$ from above equations, so R is not transitive.

Sol 6: (C) R = {(3, 3), (6, 6), (9, 9),(12, 12), (6, 12), (3, 9), (3, 12), (3, 6)}

on the set A = {3, 6, 9, 12}

$\rightarrow$ for set A

(3, 3), (6, 6), (9, 9), (12, 12) $\in$ R

R is reflexive

$\rightarrow (6, 12) \in R$ but $(12, 6) \notin R$

So R is not symmetric

$\rightarrow (3, 6) \in R$ and $(6, 12) \in R$ and also $(3, 12) \in R$

R is transitive.

Sol 7: (C) R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)}

on the set A = {1, 2, 3, 4}

$\rightarrow$ for set A

(1, 1), (2, 2), (3, 3), (4, 4) $\notin$ A

R is not reflexive

$\rightarrow (2, 3) \in R$, $(3, 2) \notin R$

So R is not symmetric $(1,3)\in R$, $(3,1) \in R$

But $(1, 1) \notin R$

so R is not transitive.

Sol 8: (D) R: N × N

(a, b) R (c, d) if ad(b + c) = bc(a + d)

$\Rightarrow$ for (a, b)R(a, b) $\Rightarrow$ ab(b+a) = ba(a+b)

Which is true so R is reflexive

$\rightarrow$ for symmetric

(a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b)

ad(b + c) = bc(a + d)

cb(d + a) = da(c + b) ... (ii)

ad(b + c) = bc(a + d)

$\Rightarrow$ cb(d + a) = da(c + b)

Which is equation (ii)

$\Rightarrow$ (a, b) R (c, d) Þ (c, d) R (a, b)

so R is symmetric relation

$\rightarrow$ assume (a, b) R (c, d) and (c, d) R (e, f)

ad(b + c) = bc(a + d) ... (i)

cf(d + e) = de(c + f) ... (ii)

for (a, b), (e, f)

$\Rightarrow$ af(b + e) = be(a + f)

If (e, f) = (c, d)

then (c, d) R (e, f) is always true

(R is reflexive)

so in equation (i) (c, d) $\rightarrow$ (e, f)

af (b + e) = be (a + f)

so ((a, b), (e, f)) $\in$ R

$\therefore$ R is transitive.

Sol 9: (A) W = all wards in the English dictionary

R = {(x, y) $\in$ w × w the words x and y have at least one letter in common}

$\rightarrow$ a word have all letter common to itself R is reflexive

$\rightarrow$ if x and y have one letter common soy and x is same condition

$(x, y) \in R \rightarrow (y, x) \in R$

R is symmetric

$\rightarrow$ if $(x, y) \in R$, $(y, z) \in R$

x and y have one letter common

y and z have one letter common

its not mean that it is necessary to x and z have one letter common

R is not transitive.

Sol 10: (C) R $\rightarrow$ real line

Given subset S = {(x, y): y = x + 1 and 0 < x < 2}

and T = {(x, y): x – y is an integer}

for S y = x + 1

but $x \neq x + 1$

s is not reflexive

so s is not equivalence

for T (x, y) $\in$ T $\Rightarrow$ x – y is an integer x – x = 0 is an integer

T is reflexive

$\rightarrow$ if (x, y) $\in$ T $\rightarrow$ x – y is an integer

so + (y – x) also an integer

so (y, x) $\in$ T

T is symmetric

$\rightarrow$ if (x, y) $\in$ T and (y, z) $\in$ T

$x - y = z_1$ ... (i);

$y - z = z_2$ ... (ii)

(assume z_1 and z_2 are integer)

(i) + (ii)

$x - y + y - z = z_1 + z_2$

$x - z = z_1 + z_2$

$\therefore$ sum of two integer is also an integer

so x – z is an integer

(x, z) $\in$ T

T is transitive

$\Rightarrow$ T is an equivalence relation

Multiple Correct Choice Type

Sol 11: (A, B, C) x = {1, 2, 3, 4, 5}, y= {1, 3, 5, 7, 9}

(A) R_1 = {(x, y) | y = 2 + x , $x \in X$, $y \in Y$ }

R = {(1, 3) (3, 5)(5, 7)}

(B) R_2 = {(1, 1), (2, 1)(3, 3), (4, 3), (5, 5)}

is satisfied R: x $\rightarrow$ y

$$\begin{pmatrix}1\\2\\3\\4\\5\end{pmatrix}\begin{pmatrix}1\\1\\3\\3\\5\end{pmatrix} \rightarrow \text{ a subset of y}$$

(C) R_3 = {(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)}

All elements are from x

$$\begin{pmatrix}1\\1\\3\\3\\5\end{pmatrix}\begin{pmatrix}1\\3\\5\\7\\7\end{pmatrix}$$

all element are from y

(D) R_4 = {(1, 3),(2, 5), (2, 4), (7, 2)}

in (2, 4), 4 is not belongs to y

So R is not relation from x to y.

Functions

Sol 1: (C)

$$f(x) = \sin^{-1}\sqrt{(x - x^2)} + \sec^{-1}\left(\frac{1}{x}\right) + \ell n|x - 1|$$

$|x - 1| \neq \Rightarrow x \neq 1$

AND $\sqrt{(x - x^2)} \leq 1$

$x - x^2 \leq 0$

$x(x - 1) \leq 0 \Rightarrow x \in [0,1]$

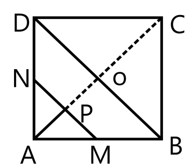
$\sqrt{x(x-1) \leq 1}$ always true in internal [0,1]

And $\frac{1}{x} \geq 1$ or $\frac{1}{x} \leq -1$ $x \neq 0$

$0 \leq x \leq 1$ or $1 \leq x \pi 0$

So domain $x \in (0,1)$

Sol 2: (B)



$AC = 2\sqrt{2}$

$BD = 2\sqrt{2}$

$AP = x$

$PC = 2\sqrt{2} - x \Rightarrow MN = 2x$

$(\Delta AMN)\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times (AP) \times (MN)$

$= \frac{1}{2} \times (2x)(x) = x^2$ (for $x < \sqrt{2}$)

$f(-1) = -1$ $f(-2) = -8$ ΔAMN is maximum

f(x) =maximum= $(\sqrt{2})^2 = 2$

So the range is (0,2)

Sol 3: (A) Bijective

gof

$f \rightarrow$ one-one function $g \rightarrow$ one-one

function so gof will be one-one function

f & $g \rightarrow$ onto function

gof(x) will be onto only of if domain of g = range of f

Sol 4: (D) $y = 5[x]+1 = 6[x-1]-10$

$5I+1 = 6I-6-10$

$I = 17$

$x \in [17,18)$

$y = 5(17)+1 = 86$

$[x+2y] = 189$

Sol 5: (D) $f(x) = ax^3 + e^x$

$f'(x) = 2ax^2 + e^x$

For being a one-one $f'(x) \geq 0$ or $f'(x) \leq 0$

f(x) is always greater for any x. if $a \geq 0$

$a \in (0,\infty)$ if $a \neq 0$ then f is not onto function.

Sol 6: (A)

$$\sqrt{\frac{1+\sin x}{1-\sin x}} - \sec x = -\sqrt{\frac{1-\sin x}{1+\sin x}} + \sec x$$

$$\sqrt{\frac{1+\sin x}{1-\sin x}} + \sqrt{\frac{1-\sin x}{1+\sin x}} = 2\sec x$$

$$\frac{2}{\sqrt{1-\sin^2 x}} = 2\sec x$$

$$\frac{2}{\sqrt{\cos^2 x}} = 2\sec x$$

$\sec x = |\sec x|$ if $\cos x \neq 0$

$$x \in \left[-2\pi, \frac{-3\pi}{2}\right) \cup \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$$

Sol 7: (D) $f(x) = \sin\left(\cos\frac{x}{2}\right) + \cos(\sin x)$

$\cos\frac{x}{2}$ has time period = 4π

sin x has time period= 2π

So the common time period = 4π

Sol 8: (A) $f(x) = \frac{x}{1+|x|}$

If $x>0$

$0 \leq f(x) = 1 - \frac{1}{1+x} < 1$ one-one is the interval.

if $x<0$

$$f(x) = \frac{x}{1-x}$$

$$= -\left(\frac{-x}{1-x}\right) = -1\left(1 - \frac{-x}{1-x}\right)$$

$$0 \geq f(x) = \frac{1}{1-x} - 1 > -1$$

So f(x) is injective (one-one)

Multiple Correct Choice Type

Sol 9: (A, D) $f(x) = \sqrt{x}$ $f: I \rightarrow R$

F is not onto function.

But f is one-one function.

Sol 10: (B, C) $f(x) = \sqrt{\log_{x^2} x}$

$x \neq 0, x > 0$ $x \neq 1$

$$\log_{(x)^2} = \frac{1}{2}\log_{x^x} = \frac{1}{2}$$

$x \in (0,\infty) - \{1\}$

x is defined for (0, 1) and (1, ∞)

Sol 11: (B, C) $y = \frac{x\sqrt{x-2\sqrt{x-1}}}{\sqrt{x-1}-1}$

$x-1 \geq 0 \Rightarrow x \geq 1$

And $\sqrt{x-1} \neq 1$ $\Rightarrow x \neq 2$

And $x - 2\sqrt{x-1} \geq 0$

$x \geq 2\sqrt{x-1}$

Case I if $x \geq 0$ $x^2 \geq 4(x-1)$

$x^2 - 4x + 4 \geq 0$

$(x-2)^2 \geq 0$ which is always true.

Case II if $x \leq 0$ then not true

So the domain is $x \in [1,\infty) - \{2\}$

$$2f(1.5)+f(3)=2\left[\frac{2/3\sqrt{\frac{3}{2}-2\sqrt{\frac{1}{2}}}}{\sqrt{\frac{1}{2}}-1}\right]+\frac{3\sqrt{3-2\sqrt{2}}}{\sqrt{2}-1}$$

$$=2\left[\frac{3\sqrt{3-2\sqrt{2}}}{2\sqrt{2}}\right]+\frac{(3\sqrt{3-2\sqrt{2})}}{\sqrt{2}-1}$$

= 0 = non negative

Putting $x-1=t^2$ for x>2

$$f(x)=\frac{(t^2+1)\sqrt{t^2+1-2t}}{t-1}=t^2+1=x$$

Putting for x<2, it is not always defined.

Sol 12: (A, C, D) (A) $f(x)=2^{\frac{1}{x-1}}$

Decreasing function

So at boundary conditions

$f(x)=2^{1/x-1}$

At $f(x)=2^{1/x-1}$ $f(x)=\frac{1}{2}$

At $x=1-h$ $f(x)=0$

Bounded

(B) $g(x)=x\cos\frac{1}{x}$

$\cos\frac{1}{x}$ will oscillate between [-1,1] for any is not bounded so g(x) is also not

(C) $h(x)^{-x}\ge 0$ in $(0,\infty)$

$h'(x) = e^{-x} - xe^{-x}$

If x > 1 then h (x) is decreasing

x < 1 then h (x) is increasing

at x = 0 ,h (x) = 0

at x = 1 , h (x) = $\frac{1}{e}$

at x = ∞ h (x) = 0 (so it is bounded)

(D) $\ell(x)=\tan^{-1}(2^x)$

2^x is strictly increasing and it is positive.

$\ell(x)$ is bounded as $=x-\infty,$ and $\ell(x)=0$

$=x^{-}\infty,\ \ell(x)=\pi/2$

Sol 13: (A, D) (A) $f(x)=x-[x]=\{x\}$

Periodic

(B) $g(x)=\sin\left(\frac{1}{x}\right),\ x\ne 0\ \&\ g(0)=0$

$\frac{1}{x}$ is not periodic so $\sin\frac{1}{x}$ is also not.

(C) $h(x)=x\cos x$

Not periodic C

(D) $w(x)=\sin x$

Periodic

Sol 14: (B, C) $f(x)=\begin{cases}x & \text{if } x\in Q\\ 1-x & \text{if } x\notin Q\end{cases}$

$x\in[0,1]$

$0\le f(x)<1\quad [x\in[0,1]-\{1\}]$

At x=1, f(x) = 1

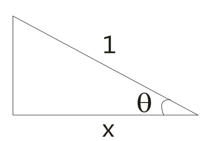
$[f(x)]=0$ for $x\in[0,1)$

at $n=1;\ [f(x)]=1$

$f([f(x)])=1$,for $x\in[0,1)$

Sol 15: (A, C, D) (A) $y=\tan(\cos^{-1}x)=\frac{\sqrt{1-x^2}}{x}$ (identical)

$\sqrt{1-x^2}$



Domain of both function, are not same at x=0

$y=\tan^{-1}(\cos x)$ is defined while the order is not.

(B) $y=\tan(\cos^{-1}x)$ (is not identical) $y=\frac{1}{x}$

(C) $y=\sin(\tan^{-1}(x))=\frac{x}{\sqrt{1+x^2}}$ (identical)

(D) $y=\cos(\tan^{-1}x)\,;\,y=\sin(\cot^{-1}x)$ (identical)

$$y=\frac{1}{\sqrt{1+x^2}}$$

Previous Years' Questions

Sol 1: (A, D) Given, $y = f(x) = \frac{x+2}{x-1}$

$\Rightarrow yx - y = x + 2 \Rightarrow x(y - 1) = y + 2$

$\Rightarrow x = \frac{y+2}{y-1} \Rightarrow x = f(y)$

Here, f(1) does not exist, so domain $\in R - \{1\}$.

$\frac{dy}{dx} = \frac{(x-1)\cdot 1(x+2)\cdot 1}{(x-1)^2}$

$= -\frac{3}{(x-1)^2}$

$\Rightarrow$ f(x) is decreasing for all $x \in R - \{1\}$

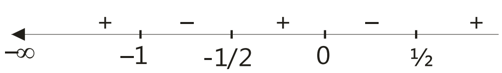
Also, f is rational function of x.

Hence, (a) and (d) are correct options.

Sol 2: (A, D) Since, $\frac{2x-1}{2x^3+3x^2+x} > 0$

$\Rightarrow \frac{(2x-1)}{x(2x^2+3x+1)} > 0$

$\Rightarrow \frac{(2x-1)}{x(2x+1)(x+1)} > 0$

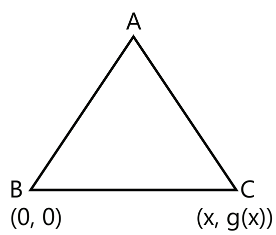


Hence, the solution set is,

$x \in (-\infty, -1) \cup (-1/2, 0) \cup (1/2, \infty)$

Sol 3: (B, C) Since, area of equilateral triangle = $\frac{\sqrt{3}}{4}(BC)^2$

$\Rightarrow \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4}.[x^2 + g^2(x)] \Rightarrow g^2(x) = 1 - x^2$



$\Rightarrow g(x) = \sqrt{1-x^2}$ or $-\sqrt{1-x^2}$

Hence, (b) and (c) are the correct options.

Sol 4: (A, C) Since, $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$

$\Rightarrow f(x) = \cos(9)x + \cos(-10)x$,

(using $[\pi^2] = 9$ and $[-\pi^2] = -10$)

$\therefore f\left(\frac{\pi}{2}\right) = \cos\frac{9\pi}{2} + \cos 5\pi = -1$

$f(\pi) = \cos 9\pi + \cos 10\pi = -1 + 1 = 0$

$f(-\pi) = \cos 9\pi + \cos 10\pi = -1 + 1 = 0$

$f\left(\frac{\pi}{4}\right) = \cos\frac{9\pi}{4} + \cos\frac{10\pi}{4} = \frac{1}{\sqrt{2}} + 0 = \frac{1}{\sqrt{2}}$

Hence, (a) and (c) are correct options.

Sol 5: (A) Here, $f(x) = \frac{b-x}{1-bx}$, where

$0 < b < 1, 0 < x < 1$

For function to be invertible it should be one–one onto.

$\therefore$ Check Range:

Let $f(x) = y \Rightarrow y = \frac{b-x}{1-bx}$

$\Rightarrow y - bxy = b - x$

$\Rightarrow x(1 - by) = b - y$

$\Rightarrow x = \frac{b-y}{1-by}$, where $0 < x < 1$

$\therefore 0 < \frac{b-y}{1-by} < 1$

$\frac{b-y}{1-by} > 0$ and $\frac{b-y}{1-by} < 1$

$\Rightarrow y < b$ or $y > \frac{1}{b}$ ... (i)

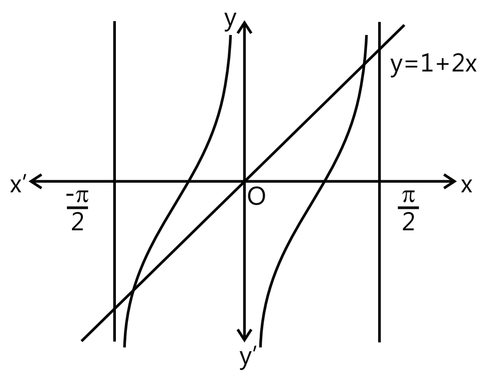
$\frac{(b-1)(y+1)}{1-by} < 0 \; -1 < y < \frac{1}{b}$ ... (ii)

From Eqs. (i) and (ii), we get $y \in \left(-1, \frac{1}{b}\right) \subset$ co–domain

Thus, f(x) is not invertible.

Sol 6: A $\to$ q; B $\to$ r

y = 1 + 2x is linear function therefore, it is one–one and its range is $(-\pi + 1, \pi + 1)$. Therefore, (1 + 2x) is one–one but not onto so (A) $\to$ (q) Again, see the figure.

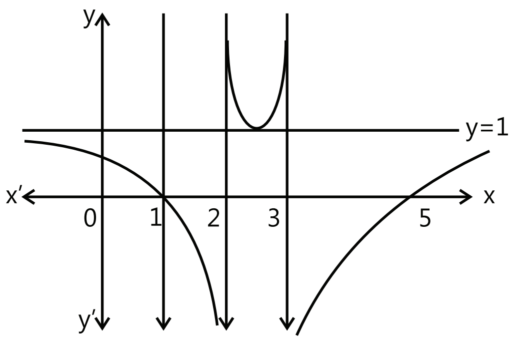


It is clear from the graph that y = tan x is one–one and onto, therefore (B) $\to$ (r)

Sol 7: A$\to$ p; B $\to$ q; C$\to$ q; D $\to$ p

Given, $f(x) = \dfrac{(x-1)(x-5)}{(x-2)(x-3)}$

The graph of f(x) is shown



(A) If $-1 < x < 1 \Rightarrow 0 < f(x) < 1$

(B) If $1 < x < 2 \Rightarrow f(x) < 0$

(C) If $3 < x < 5 \Rightarrow f(x) > 0$

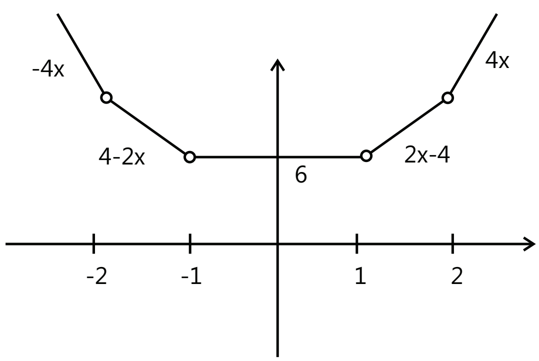
(D) If $x > 5 \Rightarrow f(x) < 1$

Sol 8: A$\to$ p; B $\to$ q, s; C$\to$ (q, r, s, t); D $\to$ r

(A) $f'(x) > 0, \forall x \in (0, \pi/2)$

f (0) < 0 and $f(\pi/2) > 0$

so one solution.



(B) Let (a, b, c) is direction ratio of the intersected line, then

ak + 4b + c = 0

4a + kb + 2c = 0

$$\frac{a}{8-k} = \frac{b}{4-2k} = \frac{c}{k^2-16}$$

We must have

$$2(8-k) + 2(4-2k) + (k^2-16) = 0$$

$\Rightarrow k = 2, 4$.

(C) Let f(x) = |x + 2| + |x + 1| + |x −1| + |x −2|

$\Rightarrow$ k can take value 2, 3, 4, 5.

(D) $\int \frac{dy}{y+1} = \int dx$

$\Rightarrow f(x) = 2e^x - 1 \;\Rightarrow f(\ln 2) = 3$

Sol 9: A$\to$ q, s; B$\to$ p, r, s, t; C$\to$ t; D$\to$ r

$$2\sin^2\theta + 4\sin^2\theta\cos^2\theta = 2$$

$$\sin^2\theta + 2\sin^2\theta(1-\sin^2\theta) = 1$$

$$3\sin^2\theta + 2\sin^2\theta - 1 = 0 \Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}, \pm 1$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}.$$

(B) Let $y = \dfrac{3x}{\pi}$

$\Rightarrow \frac{1}{2} \le y \le 3 \;\forall\, x \in \left[\frac{\pi}{6}, \pi\right]$

Now f(y) = [2y] cos[y]

Critical points are $y = \frac{1}{2}, y = 1, y = \frac{3}{2}, y = 3$

$\Rightarrow$ points of discontinuity $\left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi\right\}$.

(C) $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi \Rightarrow$ volume of parallelepiped = π

(D) $|\vec{a}+\vec{b}| = \sqrt{3}$

$$\Rightarrow \sqrt{2 + 2\cos\alpha} = \sqrt{3}$$

$$\Rightarrow 2 + 2\cos\alpha = 3$$

$$\Rightarrow \alpha = \frac{\pi}{3}.$$

Sol 10: $f(0) = 1, f'(x) = 3x^2 + \frac{1}{2}e^{x/2}$

$$\Rightarrow f'(g(x))\, g'(x) = 1$$

Put $x = 0 \Rightarrow g'(1) = \dfrac{1}{f'(0)} = 2$.

Sol 11: (B) $e^{-x}f(x)=2+\int_0^x \sqrt{t^4+1}\,dt$ ... (i)

$f\left(f^{-1}(x)\right)=x$

$\Rightarrow f\left(f^{-1}(x)\right)'\left(f^{-1}(x)\right)'=1 \Rightarrow \left(f^{-1}(2)\right)'=\dfrac{1}{f'\left(f^{-1}(2)\right)}$

$\Rightarrow f(0)=2 \Rightarrow f^{-1}(2)=0$

$\left(f^{-1}(2)\right)'=\dfrac{1}{f'(0)}$

$e^{-x}\left(f'(x)-f(x)\right)=\sqrt{x^4+1}$

Put x = 0

$\Rightarrow f'(0)-2=1 \Rightarrow f'(0)=3$

$\left(f^{-1}(2)\right)'=1/3$

Sol 12: (D) $f(x)=\begin{cases}\{x\} & ,2n-1\le x<2n\\ 1-\{x\} & ,2n\le x<2n+1\end{cases}$

Clearly f(x) is a periodic function with period = 2

Hence f(x). $\cos \pi x$ is also periodic with period = 2

$\dfrac{\pi^2}{10}\int_{-10}^{10} f(x)\cos(\pi x)dx=\pi^2\int_0^2 f(x)\cos(\pi x)dx$

$=\pi^2\int_0^1\left(\left(1-\{x\}\right)+\{x\}\right)\cos(\pi x)\,dx$

$=2\pi^2\int_0^1(-x\cos \pi x)dx=-2\pi^2\left[\dfrac{\pi \sin \pi x}{\pi}+\dfrac{\cos \pi x}{\pi^2}\right]_0^1$

$=-2\pi^2\left(-\dfrac{2}{\pi^2}\right)=4$

Sol 13: $\vec{r}.\times\vec{b}=\vec{c}\times\vec{b}$

taking cross with a

$\vec{a}\times\left(\vec{r}.\times\vec{b}\right)=\vec{a}\left(\vec{c}\times\vec{b}\right)$

$\left(\vec{a}.\vec{b}\right)\vec{r}-\left(\vec{a}.\vec{r}\right)\vec{b}=\vec{a}\times\left(\vec{c}\times\vec{b}\right)$

$\Rightarrow \vec{r}=-3\hat{i}+6\hat{j}+3\hat{k}$

$\vec{r}\,.\,\vec{b}=3+6=9$

Sol 14: (A, B, C) Given $g(x)=\dfrac{\pi}{2}\sin x\,\forall x\in R$

$f(x)=\sin\left(\dfrac{1}{3}g\left(g(x)\right)\right)$

$\Rightarrow g\left(g(x)\right)\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]\forall x\in R$

Also, $g\left(g\left(g(x)\right)\right)\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]\forall x\in R$

Hence, f (x) and $f\left(g(x)\right)\in\left[-\dfrac{1}{2},\dfrac{1}{2}\right]$

$\lim\limits_{x\to 0}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to 0}\dfrac{\sin\left(\frac{1}{3}g\left(g(x)\right)\right)}{\frac{1}{3}g\left(g(x)\right)}\cdot\dfrac{\frac{1}{3}g\left(g(x)\right)}{g(x)}$

$\Rightarrow \lim\limits_{x\to 0}\dfrac{\pi}{6}\cdot\dfrac{\sin\left(\frac{\pi}{2}\sin x\right)}{\frac{\pi}{2}\sin x}=\dfrac{\pi}{6}$

Range of $g\left(f(x)\right)\in\left[-\dfrac{\pi}{2}\sin\left(\dfrac{1}{2}\right),\dfrac{\pi}{2}\sin\left(\dfrac{1}{2}\right)\right]$

$\Rightarrow g\left(f(x)\right)\ne 1$.

Sol 15: A → s; B → t; C→r; D → r

(A) $z=\dfrac{2i(x+iy)}{1-(x+iy)^2}=\dfrac{2i(x+iy)}{1-\left(x^2-y^2+2ixy\right)}$

Using $1-x^2=y^2$

$Z=\dfrac{2ix-2y}{2y^2-2ixy}=-\dfrac{1}{y}$.

$\because -1\le y\le 1 \Rightarrow -\dfrac{1}{y}\le -1$ or $-\dfrac{1}{y}\ge 1$.

(B) For domain

$-1\le\dfrac{8.3^{x-2}}{1-3^{2(x-1)}}\le 1$

$\Rightarrow -1\le\dfrac{3^x-3^{x-2}}{1-3^{2x-2}}-\le 1$

Case I: $\dfrac{3^x-3^{x-2}}{1-3^{2x-2}}-1\le 0$

$\Rightarrow \dfrac{\left(3^x-1\right)\left(3^{x-2}-1\right)}{\left(3^{2x-2}-1\right)}\ge 0$

$\Rightarrow x\in(-\infty,0]\cup(1,\infty)$

Case – II: $\frac{3^x - 3^{x-2}}{1-3^{2x}-2}+1 \geq 0$

$\Rightarrow \frac{\left(3^{x-2}-1\right)\left(3^x+1\right)}{\left(3^x.3^{x-2}-1\right)} \geq 0$

$\Rightarrow x \in \left(-\infty,1\right) \cup \left[2,\infty\right)$

So, $x \in \left(-\infty,0\right) \cup \left[2,\infty\right)$.

(C) $R_1 \to R_1 + R_3$

$$f\left(\theta\right) = \begin{vmatrix} 0 & 0 & 2 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$$

$= 2\left(\tan^2\theta + 1\right) = 2\sec^2\theta$.

(D) $f'\left(x\right) = \frac{3}{2}\left(x\right)^{1/2}\left(3x-10\right)+\left(x\right)^{3/2}\times 3$
$= \frac{15}{2}\left(x\right)^{1/2}\left(x-2\right)$

Increasing, when $x \geq 2$.

Sol 16: (B) $f\left(x\right) = 2x^3 - 15x^2 + 36x + 1$

$f'\left(x\right) = 6x^2 - 30x + 36$

$= 6\left(x^2 - 5x + 6\right)$

$= 6\left(x-2\right)\left(x-3\right)$

f (x) is increasing in [0, 2] and decreasing in [2, 3]

f (x) is many one

f (0) = 1

f (2) = 29

f (3) = 28

Range is [1, 29]

Hence, f (x) is many-one-onto.

Sol 17: (C)

$f\left(x\right) + 2x = 1\left(1-x\right)^2 \sin^2 x + x^2 + 2x$

$\because f\left(x\right) + 2x = 2\left(1+x^2\right)$

$\Rightarrow \left(1-x\right)^2 \sin^2 x + x^2 + 2x = 2 + 2x^2$

$\left(1-x\right)^2 \sin^2 x = x^2 - 2x + 1 + 1$

$= \left(1-x\right)^2 + 1$

$\Rightarrow \left(1-x\right)^2 \cos^2 x = -1$

Which can never be possible

P is not true

$\Rightarrow$ Let $H\left(x\right) = 2f\left(x\right) + 1 - 2x\left(1+x\right)$

H (0) = 2 f (0) + 1 – 0 = 1

H (1) = 2 f (1) + 1 – 4 = -3

$\Rightarrow$ So H (x) has a solution

So Q is true.

Sol 18: (A, B)

$\cos 4\theta = \frac{1}{3} \Rightarrow 2\cos^2 2\theta - 1 = \frac{1}{3}$

$\Rightarrow \cos^2 2\theta = \frac{2}{3} \Rightarrow \cos 2\theta = \pm\sqrt{\frac{2}{3}}$

Now $f\left(\cos 4\theta\right) = \frac{2}{2-\sec^2\theta} = \frac{1+\cos 2\theta}{\cos 2\theta} = 1 + \frac{1}{\cos 2\theta}$

$\Rightarrow f\left(\frac{1}{3}\right) = 1 \pm \sqrt{\frac{3}{2}}$

Sol 19: (C) Let, $g\left(x\right) = e^{-x} f\left(x\right)$

As $g''\left(x\right) > 0$ so g′ (x) is increasing.

So, for $x < 1/4,\ g'\left(x\right) < g'\left(1/4\right) = 0$

$\Rightarrow \left(f'\left(x\right) - f\left(x\right)\right)e^{-x} < 0$

$\Rightarrow f'\left(x\right) < f\left(x\right)$ in $\left(0, 1/4\right)$

Sol 20: (D) $f_2\left(f_1\right) = \begin{cases} x^2 & , x < 0 \\ e^{2x} & x \geq 0 \end{cases}$

$f_4 R \to \left[0,\infty\right)$

$$f_4\left(x\right) = \begin{cases} f_2\left(f_1\left(x\right)\right) & , x < 0 \\ f_2\left(f_1\left(x\right)\right) - 1 & , x \geq 0 \end{cases}$$

$$= \begin{cases} x^2 & , x < 0 \\ e^{2x} - 1 & , x \geq 0 \end{cases}$$

Sol 21: (A, B, C)

$f(x) = \left(\log(\sec x + \tan x)\right)^3 \ \forall\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f(-x) = -f(x)$, hence f (x) is odd function

Let g (x) = sec x + tan x $\forall\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow g'(x) = \sec x(\sec x + \tan x) > 0 \ \forall\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow g(x)$ is one-one function

Hence $\left(\log_e(g(x))\right)^3$ is one-one function.

And $g(x) \in (0, \infty) \ \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow \log(g(x)) \in R$. Hence f (x) is an onto function.

Sol 22: $G(1) = \int_{-1}^{1} t\left|f(f(t))\right| dt = 0$

f (- x) = - f (x)

Given $f(1) = \frac{1}{2}$

$$\lim_{x\to1}\frac{F(x)}{G(x)} = \lim_{x\to1}\frac{\frac{F(x)-F(1)}{x-1}}{\frac{G(x)-G(1)}{x-1}} = \frac{f(1)}{\left|f(f(1))\right|} = \frac{1}{14}$$

$$\Rightarrow \frac{1/2}{\left|f(1/2)\right|} = \frac{1}{14}$$

$$\Rightarrow f\left(\frac{1}{2}\right) = 7.$$