(a) Standard Areas:

(i) Area bounded by two parabolas $y^2 = 4ax$ and $x^2 = 4by$; $a > 0$, $b > 0$: Area $= \frac{16ab}{3}$

(ii) Area bounded by Parabola $y^2 = 4ax$ and Line $y = mx$: Area $= \frac{8a^2}{3m^3}$

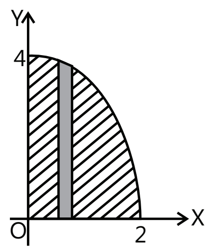
(iii) Area of an Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$: Area $= \pi ab$

Solved Examples

JEE Main/Boards

Example 1: Find area bounded by $y = 4 - x^2$, x-axis and the lines $x = 0$ and $x = 2$.

Sol: By using the formula of Area Bounded by the x – axis, we can obtain



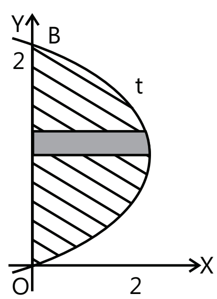
Required Area.

$$= \int_0^2 y\,dx = \int_0^2 (4 - x^2)\,dx$$

$$= \left(4x - \frac{x^3}{3}\right)_0^2 = 8 - \frac{8}{3} = \frac{16}{3} \text{ sq. units}$$

Example 2: Find the area bounded by the curve $y^2 = 2y - x$ and the y-axis.

Sol: Here given equation is the equation of parabola with vertex (1, 1) and curve passes through the origin.



Curve is $y^2 - 2y = -x$ or $(y - 1)^2 = -(x - 1)$

It is a parabola with

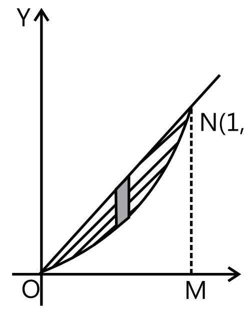
Vertex at (1, 1) and the curve passes through the origin. At B, $x = 0$ and $y = 2$

Area

$$= \int_0^2 x\,dy = \int_0^2 (2y - y^2)\,dy = \left(y^2 - \frac{y^3}{3}\right)_0^2 = \frac{4}{3} \text{ sq. units}$$

Example 3: Find the area of the region $\{(x, y): x^2 \le y \le x\}$

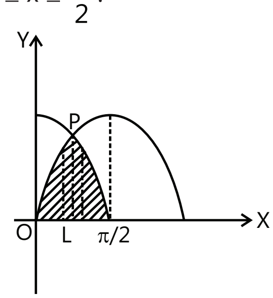
Sol: Consider the function $y = x^2$ and $y = x$ Solving them, we get $x = 0$, $y = 0$ and $x = 1$, $y = 1$; $x^2 \le y \Rightarrow$ area is above the curve $y = x^2$ $y \le x \Rightarrow$ area is below the line $y = x$



$$\text{Area} = \int_0^1 (x - x^2)\,dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 = \frac{1}{6} \text{ sq. units}$$

Example 4: Find the area of the region enclosed by $y = \sin x$, $y = \cos x$ and x-axis, $0 \le x \le \frac{\pi}{2}$.

Sol: Find point of intersection is P. Therefore after obtaining the co-ordinates of P and then integrating with appropriate limits, we can obtain required Area.



At point of intersection P, $x = \frac{\pi}{4}$ as ordinates of $y = \sin x$ and ; $y = \cos x$ are equal

Hence, P is $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ Required area

$$= \int_0^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \cos x\,dx = \left(-\cos x\right)_0^{\pi/4} + \left(\sin x\right)_{\pi/4}^{\pi/2}$$

$$= \left(-\frac{1}{\sqrt{2}} + 1\right) + \left(1 - \frac{1}{\sqrt{2}}\right) = 2 - \sqrt{2} \text{ sq. units}$$

Example 5: The area bounded by the continuous curve y = f(x), (lying above the x-axis), x-axis and the ordinates x = 1 and x = b is (b – 1) sin (3b + 4). Find f(x)

Sol: Using Leibniz rule, we can solve given problem.

$$\int_1^b f(x)\,dx = (b-1)\sin(3b+4)$$

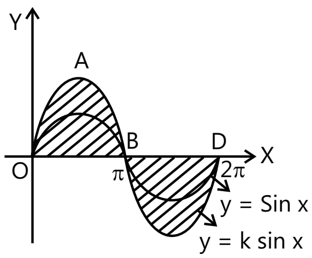
Apply Leibniz Rule: differentiate both sides w.r.t. "b",

f(b) = sin (3b + 4) + 3(b – 1) cos (3b + 4)

⇒ f(x) = sin (3x + 4) + 3(x – 1) cos (3x + 4)

Example 6: Find the area bounded by the curve y = k sin x and y = 0 from x = 0 to x = 2π.

Sol: Here the area of OAB is above the x-axis (y = 0) and thus it is positive while the area BCD is below x-axis (y = 0) and in negative but equal in quantity.



Area OAB = $\int_0^{\pi} y\,dx = \int_0^{\pi} k\sin x\,dx = k[-\cos x]_0^{\pi}$

= k[– cos π] – k [– cos 0]

= k[– (–1)] – k [–(1)] = k + k = 2k

∴ Total area = 4k sq. units.

Example 7: Find the area bounded by the curve

x = a (θ – sin θ), y = a (1 – cos θ), 0 ≤ θ ≤ 2π, with x-axis.

Sol: Substitute the value of y and dx and integrate.

Area = $\int_{\theta=0}^{\theta=2\pi} y\,dx = \int_{\theta=0}^{\theta=2\pi} y\frac{dx}{d\theta}d\theta$

$= \int_0^{2\pi} a(1-\cos\theta)a(1-\cos\theta)d\theta$

$= a^2\int_0^{2\pi}(1-2\cos\theta+\cos^2\theta)d\theta$

$= a^2\int_0^{2\pi}\left[1-2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\right]$

$= a^2\left[\frac{3}{2}\theta-2\sin\theta+\frac{1}{4}\sin 2\theta\right]_0^{2\pi}$ = $3\pi a^2$ sq. units.

Example 8: Find the area bounded by the curves {(x, y) : y ≥ x^2, y ≤ |x|}

Sol: Here the region is symmetric about y-axis, the required area is 2[area of shaded region in first quadrant].

The curves intersect each other at x = 0 and x = ±1 as shown in figure. The points of intersection are (–1, 1), (0, 0) and (1, 1).

Since, the region is symmetric about y-axis, the required area is 2[area of shaded region]

Hence, Area = $2\int_0^1 (x-x^2)dx = 2\left[\frac{1}{2}x^2-\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$ sq. units.

Example 9: Draw a rough sketch of the curve y = $\sin^2 x$, $x \in \left[0, \frac{\pi}{2}\right]$. Find the area enclosed between the curve, x-axis and the line $x = \frac{\pi}{2}$.

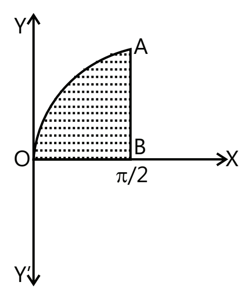
Sol: Here by substituting x = 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ we will get respective values of y. hence by plotting these values we can draw the given curve.

Some points on the $\sin^2 x$ graph are :

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	0	0.25	0.5	0.75	1

By plotting points and joining them, we trace the curve.

Area bounded by curve y = $\sin^2 x$ between x = 0 and $x = \frac{\pi}{2}$



$= \int_0^{\frac{\pi}{2}} y\,dx = \int_0^{\frac{\pi}{2}} \sin^2 x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{2}}(1-\cos 2x)dx$

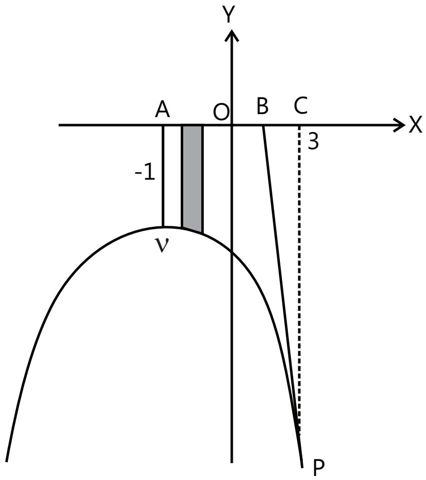
$= \frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}} = \frac{1}{2}\left[\left(\frac{\pi}{2}-0\right)-(0-0)\right]$

$= \frac{\pi}{4}$ sq.units

JEE Advanced/Boards

Example 1: A tangent is drawn to $x^2 + 2x - 4ky + 3 = 0$ at a point whose abscissa is 3. The tangent is perpendicular to $x + 3 = 2y$. Find the area bounded by the curve, this tangent, x-axis and line $x = -1$

Sol: As we know multiplication of slopes of two perpendicular line is – 1, by using this, we can obtain the value of k and will get standard equation. After that using integration with respective limit, we will be get required area.



$x^2 + 2x - 4ky + 3 = 0$; $\frac{dy}{dx} = \frac{x+1}{2k}$ Tangent is perpendicular to $x + 3 = 2y$

$\therefore \frac{x+1}{2k}\left(\frac{1}{2}\right) = -1$ at $x = 3$

$\Rightarrow 1/k = -1 \Rightarrow k = -1$

$\therefore$ Curve becomes $(x + 1)^2 = -4(y + 1/2)$ which is a parabola with vertex at $V(-1, -1/2)$.

Coordinates of P are $(3, -9/2)$.

Equation of tangent at P is $y + 9/2 = -2(x - 3)$

B is $(3/4, 0)$, C is $(3, 0)$

Required Area = Area (ACPV) – Area of triangle BPC.

$$= \left|\int_{-1}^{3} \frac{x^2+2x+3}{-4}dx\right| - \frac{1}{2}(BC)(CP)$$

$$= \frac{1}{4}\left|\left(\frac{x^3}{3}+x^2+3x\right)_{-1}^{3}\right| - \frac{1}{2}\left(3-\frac{3}{4}\right)\left(\frac{9}{2}\right)$$

$$= \frac{1}{4}\left(27+\frac{1}{3}-(1-3)\right) - \frac{81}{16} = \frac{109}{48} \text{ sq. units.}$$

Example 2: Let A_n be the area bounded by the curve $y = (\tan x)^n : n \in N$ and the lines $x = 0$, $y = 0$ and $x = \frac{\pi}{4}$. Prove that for $n > 2$, $A_n + A_{n-2} = \frac{1}{n-1}$ and deduce $\frac{1}{2n+2} < A_n < \frac{1}{2n-2}$.

Sol: We can write $(\tan x)^n$ as $\tan^{n-2} x\,(\sec^2 x - 1)$. Therefore by solving $A_n = \int_0^{\pi/4} \tan^{n-2} x\,(\sec^2 x - 1)dx$ we can prove given equation.

$$A_n = \int_0^{\pi/4} \tan^n x\, dx : n > 2$$

$$= \int_0^{\pi/4} \tan^{n-2} x\,(\sec^2 x - 1)dx$$

$$\text{or } A_n = \left[\frac{\tan^{n-1} x}{n-1}\right]_0^{\pi/2} - A_{n-2}$$

$$\therefore A_n + A_{n-2} = \frac{1}{n-1} \quad \text{... (i)}$$

$\tan^n x \le \tan^{n-2} x$

(as $0 \le \tan x \le 1$ for $0 \le x \le \frac{\pi}{4}$)

$\Rightarrow A_n < A_{n-2}$

$\therefore A_n + A_n < A_n + A_{n-2} = \frac{1}{n-1}$ by (1)

$$\therefore A_n < \frac{1}{2(n-1)} \quad \text{... (ii)}$$

Similarly $A_{n+2} < A_n$

$\Rightarrow A_{n+2} + A_n < A_n + A_n$

or $\frac{1}{(n+2)-1} < 2A_n$ by (1)

$$\Rightarrow \frac{1}{2n+2} < A_n \quad \text{... (iii)}$$

$$\Rightarrow \frac{1}{2n+2} < A_n < \frac{1}{2n-2}$$

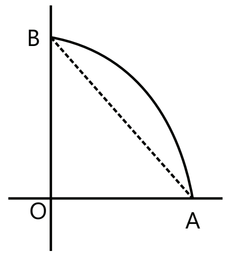
Example 3: $A(a, 0)$ and $B(0, b)$ are points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Show that the area between the arc AB and chord AB of the ellipse is $\frac{1}{4}$ ab $(\pi - 2)$.

Sol: Area between the chord and ellipse = Area bounded by curve AB - Area of Δ OAB.

Equation of line AB is : $y = -\frac{b}{a}(x-a)$

Equation of curve AB is $y = \frac{b}{a}\sqrt{a^2 - x^2}$

Area of bounded region is



$$\int_0^a \left[\frac{b}{a}\sqrt{a^2 - x^2} - \left(-\frac{b}{a}(x-a)\right)\right] dx$$

$$= \frac{b}{a}\left[0 + \frac{a^2\pi}{4} - \frac{a^2}{2}\right] = \frac{(\pi - 2)}{4}ab$$

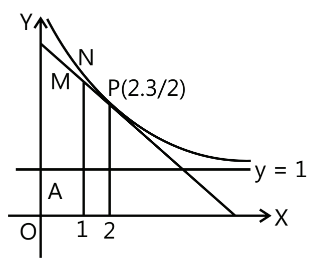
Alternate method:

Area between the chord and ellipse = Area bounded by curve AB - Area of Δ OAB

$$= \frac{1}{4}\pi ab - \frac{1}{2}ab = \frac{(\pi - 2)ab}{4} \text{ sq. units}$$

Example 4: Find the area of the region bounded $y = \frac{1}{x} + 1, x = 1$ and tangent drawn at the point P(2, 3/2) to the curve y = $\frac{1}{x}$ + 1.

Sol: Here first obtain equation of tangent and then use the formula for area.



Equation of tangent at P(2, 3/2) to y = $\frac{1}{x} + 1$ is $y - \frac{3}{2} = -\frac{1}{4}(x - 2)$ or x + 4y = 8.

Required area is area of region PMN

$$\text{Area} = \int_1^2 \left(\left(\frac{1}{x} + 1\right) - \frac{8 - x}{4}\right) dx$$

$$= \left(\ln x - x + \frac{1}{4}\frac{x^2}{2}\right)_1^2 = \ln 2 - \frac{5}{8} \text{ sq. units}$$

Example 5: Find the area of the region bounded by the x-axis and the curve $y = \frac{1}{2}(2 - 3x - 2x^2)$.

Sol: Here the curve will intersect the x-axis when y = 0, therefore by substituting y = 0 in the above equation we will get the points of intersection of curve and x – axis.

$\Rightarrow 2 - 3x - 2x^2 = 0$ or $(2 + x)(1 - 2x) = 0$ or $x = -2$, $x = \frac{1}{2}$

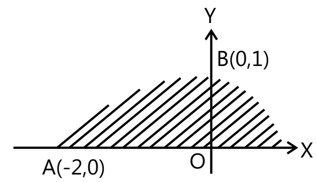
Thus, the curve passes through the points (–2, 0) and $\left(\frac{1}{2}, 0\right)$ on the x-axis.

It will have a turning points where $\frac{dy}{dx} = 0$

$$\therefore \frac{dy}{dx} = \frac{1}{2}(-3 - 4x) = 0 \ \text{Þ} x = -\frac{3}{4}$$

Also $\frac{d^2y}{dx^2} = -4$. That is, it is a max. at $x = \frac{3}{4}$

Also it cuts y-axis where x = 0, then y = 1. Thus the shape of the curve is as shown in the figure.



The required area is ABC. It is given by

$$\int_{-2}^{1/2} y\, dx = \int_{-2}^{1/2} \frac{1}{2}(2 - 3x - 2x^2)\, dx$$

$$= \frac{1}{2}\left[2x - \frac{3}{2}x^2 - \frac{2x^3}{3}\right]_{-2}^{1/2}$$

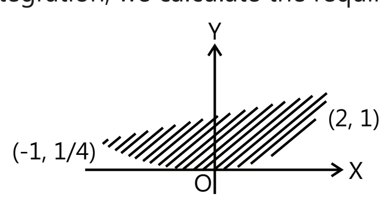
$$= \frac{1}{2}\left[2\left(\frac{1}{2}\right) - \frac{3}{2}\left(\frac{1}{2}\right)^2 - \frac{2}{3}\left(\frac{1}{2}\right)^3\right] -$$

$$\frac{1}{2}\left[2(-2) - \frac{3}{2}(-2)^2 - \frac{2}{3}(-2^3)\right]$$

$$= \frac{1}{2}\left(\frac{13}{24}\right) - \frac{1}{2}\left(-\frac{14}{3}\right) = \frac{125}{48} \text{ sq. units.}$$

Example 6: Find the area of the region bounded by the curve $x^2 = 4y$ and $x = 4y - 2$.

Sol: Solving given equation simultaneously, we will get the point of intersection. Using these points as the limits of integration, we calculate the required area.



The curve intersect each other, where $\frac{x^2}{4} = \frac{x+2}{4}$, or $x^2 - x - 2 = 0$, or $x = -1, 2$

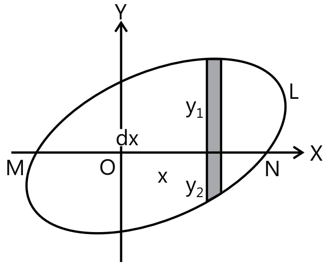
Hence, the points of intersection are (–1, 1/4) and (2, 1). The region is plotted in figure. Since, the straight line $x = 4y - 2$ is always above the parabola $x^2 = 4y$ in the interval [–1, 2], the required area is given by

$$\text{Area} = \int_{-1}^{2} [f(x) - g(x)]\, dx$$

$$\text{Area} = \int_{-1}^{2}\left[\frac{x+2}{4}-\frac{x^2}{4}\right]dx = \frac{1}{4}\left[\frac{1}{2}x^2+2x-\frac{1}{3}x^3\right]_{-1}^{2}$$

$$= \frac{1}{4}\left[\left(2+4-\frac{8}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)\right] = \frac{9}{8} \text{ sq. units.}$$

Example 7: Find by using integration, the area of the ellipse $ax^2 + 2hxy + by^2 = 1$.



Sol: The equation can be put in the form $by^2 + 2hxy + (ax^2 - 1) = 0$

Cut an elementary strip.

Let the thickness of strip = dx

If y_1, y_2 be the values of y corresponding to any value at x.

Length of strip = $y_1 - y_2$

$$= \frac{2}{b}\sqrt{h^2x^2 - b(ax^2-1)} = \frac{2}{b}\sqrt{b-(ab-h^2)x^2}$$

$ab - h^2$ being positive here, since the conic is an ellipse.

The extreme values of x, are given by

$$y_1 - y_2 = 0, \text{ i.e., } x = \pm\sqrt{\frac{b}{ab-h^2}}$$

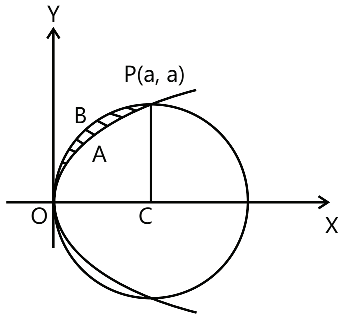
$$\text{Hence, the area required} = \int_{-\sqrt{b/ab-h^2)}}^{\sqrt{b/ab-h^2)}} (y_1 - y_2)dx$$

$$= \int_{-\sqrt{b/(ab-h^2)}}^{\sqrt{b/(ab-h^2)}} \frac{2}{b}\sqrt{b-(ab-h^2)x^2}\,dx$$

and putting $\sqrt{(ab-h^2)}\,x = \sqrt{b}\sin\theta$, this becomes

$$\frac{2}{\sqrt{(ab-h^2)}}\int_{-\pi/2}^{\pi/2}\cos^2\theta\,d\theta = \frac{\pi}{(ab-h^2)} \text{ sq. units.}$$

Example 8: Find the area of region lying above x-axis, and included between the circle $x^2 + y^2 = 2ax$ and the parabola $y^2 = ax$.



Sol: By solving these two equation simultaneously, we can obtain their intersection points and then by subtracting area of parabola from area of circle we will be get the result.

Solving the two equation, simultaneously we see that the two curves intersect at (0, 0), (a, a) and (a, –a). We have to find the area of the region OAPBO, where P is the point of intersection (a, a)

$$\text{Required area} = \int_0^a [f(x)-g(x)]dx$$

$$= \int_0^a \sqrt{(2ax-x^2)}\,dx - \int_0^a \sqrt{ax}\,dx$$

$$\text{Now, } \int_0^a \sqrt{(2ax-x^2)}\,dx = \int_0^a \sqrt{[a^2-(a-x)^2]}\,dx$$

To evaluate this integral, we substitute $a - x = a\sin\theta$ and obtain

$$\int_0^a \sqrt{(2ax-x^2)}dx = \int_{\pi/2}^{0}(a\cos\theta)(-a\cos\theta)d\theta$$

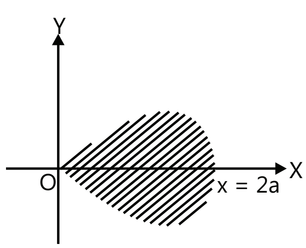
$$= \int_0^{\pi/2} a^2\cos^2\theta\,d\theta = a^2\frac{1}{2}\frac{\pi}{2} = \frac{\pi a^2}{4}$$

$$\text{Also } \int_0^a \sqrt{(ax)}\,dx = \left|\sqrt{a}\frac{2}{3}x^{3/2}\right|_0^a = \frac{2a^2}{3}$$

$$\therefore \text{ Required area} = a^2\left(\frac{\pi}{4}-\frac{2}{3}\right) \text{sq. units}$$

Example 9: Prove that the area of the region bounded by the curve $a^4y^2 = x^5(2a - x)$, is $\frac{5}{4}$ times to that of the circle whose radius is a.

Sol: The curve is a loop lying between the line x = 0 and x = 2a and is symmetrical about the x-axis. Therefore the required area



$$= 2\int_0^{2a} y\,dx$$

$$= \frac{2}{a^2}\int_0^{2a} x^{5/2}\sqrt{2a-x}dx$$

To evaluate this integral, we put x = 2a $\sin^2 q$. When, x = 0, θ = 0 and when x = 2a, $\theta = \frac{1}{2}\pi$ ∴ Required area

$$= \frac{2}{a^2}\int_0^{\pi/2}(2a)^{5/2}\sin^5\theta.\sqrt{2a}\cos\theta.4a\sin\theta\cos\theta\,d\theta$$

$$= 64a^2\int_0^{\pi/2}\sin^6\theta\cos^2\theta\,d\theta = 64a^2\,\frac{5.3.1.1}{8.6.4.2}.\frac{\pi}{2} = \frac{5a^2\pi}{4}$$

$= \frac{5}{4}$x area of the circle whose radius is a.

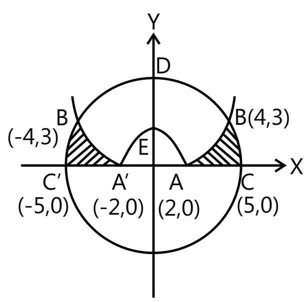
Example 10: Find the area bounded by the curves $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and y = 0.

Sol: Here $x^2 + y^2 = 25$ represent circle with centre at origin and radius 5 unit. Therefore the required area = 2 area ABC

$$= 2\left[\int_2^4 \frac{1}{4}(4-x^2)dx + \int_4^5\sqrt{25^2 - x^2}\,dx\right]$$

Note: Here the portion is also bounded by two curves but we do not apply $A = \int[f(x) - g(x)]dx$ rule.

Reason: Range of integration of both the curves is not same.



$$= 2\left[\int_2^4 \frac{1}{4}(x^2 - 4)dx + \int_4^5\sqrt{(5^2 - x^2)}\,dx\right]$$

$$= \frac{2}{4}\left[\left(\frac{x^3}{3} - 4x\right)\right]_2^4 + 2\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_4^5$$

$$= \frac{1}{2}\left[\left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 8\right)\right] +$$

$$2\left[\left(0 + \frac{25}{2}\sin^{-1}1\right) - \left(6 + \frac{25}{2}\sin^{-1}\frac{4}{5}\right)\right]$$

$$= \frac{1}{2}\left[\frac{32}{3}\right] + 25\sin^{-1}1 - 12 - 25\sin^{-1}\frac{4}{5}$$

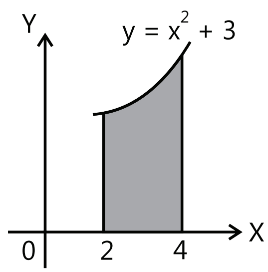
$$= \left(\frac{16}{3} - 12\right) + 25\frac{\pi}{2} - 25\sin^{-1}\frac{4}{5}$$

$$= -\frac{20}{3} + 25\left(\frac{\pi}{2} - \sin^{-1}\frac{4}{5}\right) = \left(25\cos^{-1}\frac{4}{5} - \frac{20}{3}\right)\text{ sq. units.}$$

JEE Main/Boards

Exercise 1

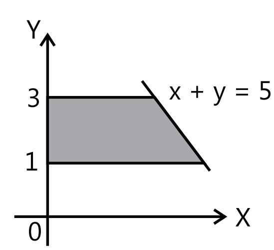
Q.1 Write an expression for finding the area of the shaded portion.



Q.2 Find the area bounded by the curve y = cos x, x-axis and between x = 0, x = π.

Q.3 Find the area bounded by the curve y = sinx, x-axis and between x = 0, x = π.

Q.4 Write an expression for finding the area of the shaded portion.



Q.5 Write an expression for finding the area bounded by the curve $x^2 = y$ and the line y = 2.

Q.6 Write an expression for finding the area of a circle $x^2 + y^2 = a^2$, above x-axis.

Q.7 On sketching the graph of y = |x – 2| and evaluating $\int_{-1}^{3}|x-2|\,dx$, what does $\int_{-1}^{3}|x-2|\,dx$, represent on the graph?

Q.8 Draw the rough sketch of the curve $y=\sqrt{3x+4}$ and find the area under the curve above x-axis and between x = 0 and x = 4.

Q.9 Find the area under the curve $y=\frac{3}{(1-2x)^3}$ above x-axis and between x = –4 an x = –1.

Q.10 Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$.

Q.11 Draw a rough graph of $f(x)=\sqrt{x}+1$ in the interval [0, 4] and find the area of the region enclosed by the curve, x-axis and the lines x = 0 and x = 4.

Q.12 Find the area of the region bounded by the curve xy – 3x – 2y – 10 = 0; x-axis and the lines x = 3, x = 4.

Q.13 Find the area bounded by the curve $y = x \sin x^2$, x-axis and between x = 0 and $x=\sqrt{\frac{\pi}{2}}$.

Q.14 Using integration, find the area of the region bounded by the following curves, after making a rough sketch:

y = |x + 1| + 1, x = –2, x = 3, y = 0.

Q.15 Draw the rough sketch of y = sin 2x and determine the area enclosed by the curve, the x-axis and the lines x = π/4 and x = 3π/4.

Q.16 Find the area of the following region: $\{(x, y) : x^2 + y^2 \le 2ax, y^2 \ge ax, x \ge 0, y \ge 0\}$.

Q.17 Find the area bounded by the curve $y^2 = 4a^2(x - 3)$ and the lines x = 3, y = 4a.

Q.18 Make a rough sketch of the region given below and find its area using integration. $\{(x,y): 0 \le y \le x^2 + 3 ; 0 \le y \le 2x + 3, 0 \le x \le 3\}$.

Q.19 Determine the area enclosed between the curve $y = 4x - x^2$ and the x-axis.

Exercise 2

Single Correct Choice Type

Q.1 The area of the figure bounded by the curve $y = e^x$, $y = e^{-x}$ and the straight line x = 1 is

(A) $e+\frac{1}{e}$ (B) $e-\frac{1}{e}$

(C) $e+\frac{1}{e}-2$ (D) None of these

Q.2 The area bounded in the first quadrant by the normal at (1, 2) on the curve y = 4x, x-axis & the curve is given by

(A) $\frac{10}{3}$ (B) $\frac{7}{3}$ (C) $\frac{4}{3}$ (D) $\frac{9}{2}$

Q.3 The area of the figure bounded by the curves y = lnx and $y = (\ln x)^2$ is

(A) e + 1 (B) e – 1 (C) 3 – e (D) 1

Q.4 The area bounded by the curves $y = x^2 + 1$ & the tangents to it drawn from the origin is:

(A) 2/3 (B) 4/3 (C) 1/3 (D) 1

Q.5 The area bounded by $x^2 + y^2 - 2x = 0$ & $y = \sin\frac{\pi x}{2}$ in the upper half of the circle is

(A) $\frac{\pi}{2}-\frac{4}{\pi}$ (B) $\frac{\pi}{4}-\frac{2}{\pi}$ (C) $\pi-\frac{8}{\pi}$ (D) $\frac{\pi}{2}-\frac{2}{\pi}$

Q.6 Consider the region formed by the lines x = 0, y = 0, x = 2, y = 2. Area enclosed by the curves $y = e^x$ and y = lnx, within this region is removed, then the area of the remaining region is

(A) $2(1 + 2\ell n^2)$ (B) $2(2\ell n2 - 1)$

(C) $(2\ell n2 - 1)$ (D) $1 + 2\ell n^2$

Q.7 The area bounded by the curves y = x(1 – lnx); $x = e^{-1}$ and positive x-axis between $x = e^{-1}$ and x = e is

(A) $\left(\frac{e^2-4e^{-2}}{5}\right)$ (B) $\left(\frac{e^2-5e^{-2}}{4}\right)$

(C) $\left(\frac{4e^2-e^{-2}}{5}\right)$ (D) $\left(\frac{5e^2-e^{-2}}{4}\right)$

Q.8 The positive values of the parameter 'a' for which the area of the figure bounded by the curve y = cos ax, $y = 0,\ x = \frac{\pi}{6a},\ x = \frac{x\pi}{2a}$ is greater than 3 are

(A) f (B) (0, 1/3)

(C) (3, ∞) (D) None of these

Q.9 The value of 'a' (a > 0) for which the area bounded by the curves $y = \frac{x}{6} + \frac{1}{x^2}, y = 0,\ x = a$ and $x = 2a$ has the least value, is

(A) 2 (B) $\sqrt{2}$ (C) $2^{1/3}$ (D) 1

Q.10 The ratio in which the area enclosed by the curve $y = \cos x \left(0 \le 0 \le \frac{\pi}{2}\right)$ in the first quadrant is divided by the curve y = sin x, is

(A) $(\sqrt{2}-1):1$ (B) $(\sqrt{2}+1):1$

(C) $\sqrt{2}:1$ (D) $\sqrt{2}+1:\sqrt{2}$

Q.11 The area bounded by the curve y = f(x), the co-ordinate axes & the line $x = x_1$ is given by $x_1.e^{x_1}$. Therefore f(x) equals

(A) e^x (B) $x\,e^x$ (C) $xe^x - e^x$ (D) $xe^x + e^x$

Q.12 The area bounded by the curves $y = -\sqrt{-x}$ and $x = -\sqrt{-y}$ where x, y ≤ 0

(A) Cannot be determined

(B) Is 1/3

(C) Is 2/3

(D) Is same as that of the figure bounded by the curves $y= \sqrt{-x}$; x ≤ 0 and $x = \sqrt{-y}$; y ≤ 0

Q.13 The area from 1 to x under a certain graph is given by $A = (1 + 3x)^{1/2} - 1, x \ge 0$.. The average value of y w.r.t. x as x increases from 1 to 8 is

(A) 3/7 (B) 1/2 (C) 3/8 (D) 4/7

Q.14 The slope of the tangent to a curve y = f(x) at (x, f(x)) is 2x + 1. If the curve passes through the point (1, 2) then the area of the region by the curve, the x-axis and the line x = 1 is

(A) 5/6 (B) 6/5 (C) 1/6 (D) 1

Q.15 The area of the region for which $0 < y < 3 - 2x - x^2$ & x > 0 is

(A) $\int_1^3 (3-2x-x^2)dx$ (B) $\int_0^3 (3-2x-x^2)dx$

(C) $\int_0^1 (3-2x-x^2)dx$ (D) None of these

Q.16 The graphs of $f(x) = x^2$ and $g(x) = cx^3$ (c > 0) intersect at the points (0, 0) & $\left(\frac{1}{c}, \frac{1}{c^2}\right)$. If the region which lies between these graphs & over the interval [0, 1/c] has the area equal to 2/3 then the value of c is

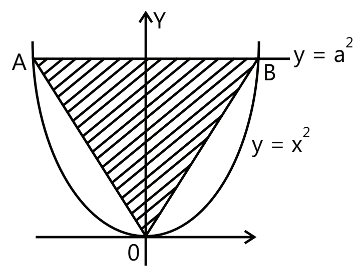
(A) 1 (B) 1/3 (C) 1/2 (D) 2

Q.17 The curvilinear trapezoid is bounded by the curve $y = x^2 + 1$ and the straight lines x = 1 and x = 2. The co-ordinates of the point (on the given curve) with abscissa xÎ[1, 2] where tangent drawn cut off from the curvilinear trapezoid are ordinary trapezium of the greatest area, is

(A) (1, 2) (B) (2, 5)

(C) $\left(\frac{3}{2}, \frac{13}{4}\right)$ (D) None of these

Q.18 In the given figure, if A_1 is the area of the ΔAOB and A_2 is the area of the parabolic region AOB then the ratio $\frac{A_1}{A_2}$ as a → 0 is



(A) 1 (B) 8/9 (C) 3/4 (D) 2/3

Previous Years' questions

Q.1 The area of the quadrilateral formed by the tangents at the end points of latus rectum to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is ***(2003)***

(A) 27/4 sq. unit (B) 9 sq. unit

(C) 27/2 sq. unit (D) 27 sq. unit

Q.2 The area bounded by the curves $y=\sqrt{x}, 2y+3=x$ and x-axis in the 1st quadrant is ***(2003)***

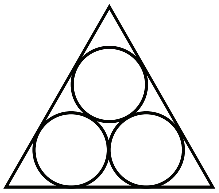
(A) 9 sq. unit (B) 27/4 sq. unit

(C) 36 sq. unit (D) 18 sq. unit

Q.3 The area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 sq unit. Then, the value of a is ***(2004)***

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{1}{3}$

Q.4 The area of the equilateral triangle, in which three coins of radius 1 cm are placed, as shown in the figure, is ***(2005)***



(A) $(6+4\sqrt{3})$ sq. cm

(B) $(4\sqrt{3}-6)$ sq. cm

(C) $(7+4\sqrt{3})$ sq. cm

(D) $4\sqrt{3}$ sq. cm

Q.5 The area enclosed within the curve $|x| + |y| = 1$ is …….. ***(1981)***

Q.6 The area of the triangle formed by the positive x-axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1,\sqrt{3})$ is …….. **(1989)**

Q.7 The area bounded by the curves $y = (x - 1)^2$, $y = (x + 1)^2$ and $y=\frac{1}{4}$ is ***(2005)***

(A) $\frac{1}{3}$ sq. unit (B) $\frac{2}{3}$ sq. unit

(C) $\frac{1}{4}$ sq. unit (D) $\frac{1}{5}$ sq. unit

Q.8 The area (in sq. units) of the region $\left\{(x,y): y^2 \ge 2x \text{ and } x^2+y^2 \le 4x, x\ge 0, y \ge 0\right\}$ is: ***(2016)***

(A) $\pi-\frac{8}{3}$ (B) $\pi-\frac{4\sqrt{2}}{3}$

(C) $\frac{\pi}{2}-\frac{2\sqrt{2}}{3}$ (D) $\pi-\frac{4}{3}$

Q.9 The area (in sq. units) of the region described by $\left\{(x,y): y^2 \le 2x \text{ and } y \ge 4x-1\right\}$ is : ***(2015)***

(A) $\frac{5}{64}$ (B) $\frac{15}{64}$ (C) $\frac{9}{32}$ (D) $\frac{7}{32}$

Q.10 The area (in square units) bounded by the curves $y=\sqrt{x}, 2y-x+3=0$, x-axis, and lying in the first quadrant is ***(2013)***

(A) 36 (B) 18 (C) $\frac{27}{4}$ (D) 9

Q.11 The area of the region described by A $=\left\{(x,y): x^2+y^2 \le 1 \text{ and } y^2 \le 1-x\right\}$ is ***(2014)***

(A) $\frac{\pi}{2}+\frac{4}{3}$ (B) $\frac{\pi}{2}-\frac{4}{3}$

(C) $\frac{\pi}{2}-\frac{2}{3}$ (D) $\frac{\pi}{2}+\frac{2}{3}$

Q.12 If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to ***(2012)***

(A) -1 (B) $\frac{2}{9}$ (C) $\frac{9}{2}$ (D) 0

Q.13 The area bounded between the parabolas $x^2=\frac{y}{4}$ and $x^2=9y$ and the straight line $y=2$ is **(2012)**

(A) $20\sqrt{2}$ (B) $\frac{10\sqrt{2}}{3}$ (C) $\frac{20\sqrt{2}}{3}$ (D) $10\sqrt{2}$

Q.14 The area bounded by the curves $y=\cos x$ and $y=\sin x$ between the ordinates $x=0$ and $x=\frac{3\pi}{2}$ ***(2010)***

(A) $4\sqrt{2}+2$ (B) $4\sqrt{2}-1$

(C) $4\sqrt{2}+1$ (D) $4\sqrt{2}-2$

Q.15 The area of the region enclosed by the curves $y=x, x=e, y=\frac{1}{x}$ and the positive x-axis is ***(2011)***

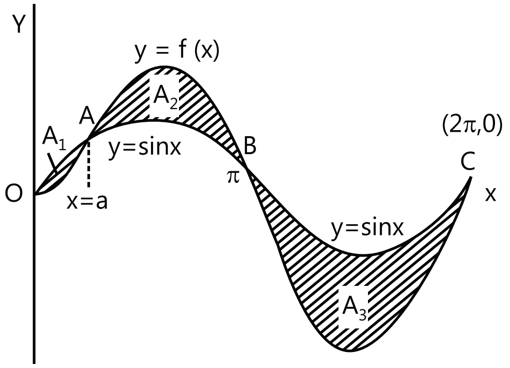
(A) 1 sq. unit (B) $\frac{3}{2}$ sq. units

(C) $\frac{5}{2}$ sq. units (D) $\frac{1}{2}$ sq. units

JEE Advanced/Boards

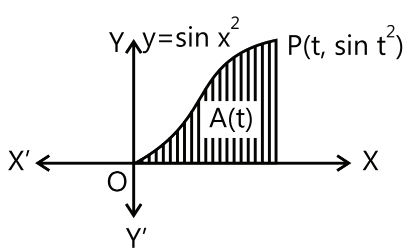
Exercise 1

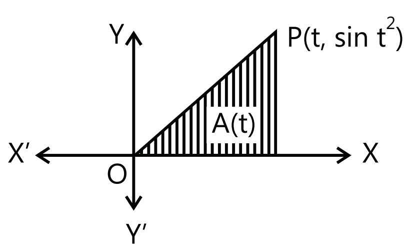
Q.1 In the adjacent figure, graphs of two functions y = f(x) and y = sinx are given. y = sinx intersects, y = f(x) at A(a, f(a)); B(π, 0) and C(2π, 0). A_i (i = 1, 2, 3) is the area bounded by the curve y = f(x) and y = sin x between x = 0 and x = a; i = 1, between x = a and x = π; i = 2, between x = π and x = 2π; i = 3.



If A_i = 1 – sin a + (a – 1) cos a, determine the function f(x). Hence determine 'a' and A_1. Also calculate A_2 and A_3.

Q.2 The figure shows two regions in the first quadrant.





A(t) is the area under the curve y = sin x^2 from 0 to t and B(t) is the area of the triangle with vertices O, P and M(t, 0). Find $\lim\limits_{t\to 0}\frac{A(t)}{B(t)}$.

Q.3 A polynomial function f(x) satisfies the condition f(x + 1) = f(x) + 2x+ 1. Find f(x) if f(0) = 1. Find also the equations of the pair of tangents from the origin on the curve y = f(x) and compute the area enclosed by the curve and the pair of tangents.

Q.4 Show that the area bounded by the curve $y=\frac{\log x - c}{x}$, the x-axis and the vertical line through the maxima point of the curve is independent of the constant c.

Q.5 Consider the curve y = x^n where n > 1 in the 1st quadrant. If the area bounded by the curve, the x-axis and the tangent line to the graph of y = x^n at the point (1, 1) is maximum then find the value of n.

Q.6 For what value of 'a' is the area of the figure bounded by the lines $y=\frac{1}{x}$, $y=\frac{1}{2x-1}$, x = 2 & x = a equal to ln $\frac{4}{\sqrt{5}}$?

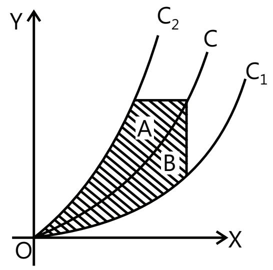
Q.7 For the curve $f(x)\frac{1}{1+x^2}$ let two points on it are $A(\alpha, f(\alpha)), B\left(-\frac{1}{\alpha}, f'\left(-\frac{1}{\alpha}\right)\right)$ (α > 0). Find the minimum area bounded by the line segments OA, OB and f(x), where 'O' is the origin.

Q.8 Find the area bounded by the curve $y=\sin^{-1}x$ and the lines x = 0, $|y|=\frac{\pi}{2}$.

Q.9 If f(x) is monotonic in (a, b) then prove that the area bounded by the ordinates at x = a ; x = b ; y = f(x) and y = f(c), c ∈ (a, b) is minimum when $c=\frac{a+b}{2}$. Hence if the area bounded by the graph of f(x) = $\frac{x^3}{3}-x^2+a$, the straight lines x = 0, x = 2 and the x-axis is minimum then find the value of 'a'.

Q.10 Let 'c' be the constant number such that c > 1. If the least area of the figure given by the line passing through the point (1, c) with gradient 'm' and the parabola y = x^2 is 36 sq. units find the value of (c^2 + m^2).

Q.11 Let C_1 & C_2 be two curves passing through the origin as shown in the figure. A curve C is said to "bisect the area" the region between C_1 & C_2, if for each point P of C, the two shaded regions A & B shown in the figure have equal area. Determine the upper curve C_2, given that the bisecting curve C has the equation $y = x^2$ & that the lower curve C_1 has the equation $y = x^2/2$.



Q.12 Consider one side AB of a square ABCD, (read in order) on the line $y = 2x - 17$ and the other two vertices C, D on the parabola $y = x^2$.

(i) Find the minimum intercept of the line CD on y-axis.

(ii) Find the maximum possible area of the square ABCD.

(iii) Find the area enclosed by the line CD with minimum y-intercept and the parabola $y = x^2$. Consider the two curves $C_1 : y = 1 + \cos x$ & $C_2 : y = 1 + \cos(x - \alpha)$ for $\alpha \in (0, \pi/2)$; $x \in [0, p]$. Find the value of α, for which the area of the figure bounded by the curves C_1, C_2 & $x = 0$ is same as that of the figure bounded by C_2, $y = 1$ & $x = \pi$. For this value of α, find the ratio in which the line $y = 1$ divides the area of the figure by the curves C_1, C_2 & $x = \pi$.

Q.13 Draw the rough sketch of $y^2 = x + 1$ and $y^2 = -x + 1$ and determine area enclosed by the two curves.

Q.14 Let $f : (0, \infty) \to R$ be a continuous and strictly increasing function such that $f^3(x) = \int_0^x t f^2(t)dt$, $\forall$ x > 0. Find the area enclosed by $y = f(x)$, the x-axis and the ordinate at $x = 3$.

Q.15 For what values of a Î[0, 1] does the area of the figure bounded by the graph of the function $y = f(x)$ and the straight lines $x = 0$, $x = 1$ & $y = f(a)$ is at a minimum & for what values it is at a maximum if $f(x) = \sqrt{1 - x^2}$. Find also the maximum & the minimum area.

Q.16 Let $f(x) = \sin x \ \forall \ x \in \left[0, \frac{\pi}{2}\right]$

$f(x) + f(\pi - x) = 2 \ \forall \ x \in \left(\frac{\pi}{2}, \pi\right]$ and $f(x) = f(2\pi - x) \ \forall \ x \in (\pi, 2\pi)$.

If the area enclosed by $y = f(x)$ and x-axis is $a\pi + b$, then find the value of $(a^2 + b^2)$.

Q.17 Find the values of $m (m > 0)$ for which the area bounded by the line $y = mx + 2$ and $x = 2y - y^2$ is, (i) 9/2 square units & (ii) minimum. Also find the minimum area.

Q.18 Find the area bounded by the curve $y = x e^{-x}$; $xy = 0$ and $x = c$ where c is the x-coordinate of the curve's inflection point.

Q.19 Find the area of the region

$$\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$$

Q.20 For what value of 'a' is the area bounded by the curve $y = a^2x^2 + ax + 1$ and the straight line $y = 0$, $x = 0$ & $x = 1$ the least?

Q.21 Consider two curves C_1: $y = \frac{1}{x}$ and C_2: $y = \ln x$ on the xy plane. Let D_1 denotes the region surrounded by C_1, C_2 and the line $x = 1$ and D_2 denotes the region surrounded by C_1, C_2 and the line $x = a$. If $D_1 = D_2$. Find the value of 'a'.

Exercise 2

Single Correct Choice Type

Q.1 The area bounded by the curve $y = x^2 - 1$ & the straight line $x + y = 3$ is

(A) $\frac{9}{2}$ (B) 4 (C) $\frac{7\sqrt{17}}{2}$ (D) $\frac{17\sqrt{17}}{6}$

Q.2 The area bounded by the curve $y = e^{-x}$ & the lines $y = e^{-4}$ & $x = 1$ is given by

(A) $\frac{e^3 - 4}{e^4}$ (B) $\frac{e^3 + 4}{e^4}$

(C) $\frac{e^3 + 1}{e^4}$ (D) None of these

Q.3 Area common to the curve $y = \sqrt{9 - x^2}$ & $x^2 + y^2 = 6x$ is

(A) $\frac{\pi + \sqrt{3}}{4}$ (B) $\frac{\pi - \sqrt{3}}{4}$

(C) $3\left(\pi + \frac{\sqrt{3}}{4}\right)$ (D) $3\left(\pi + \frac{3\sqrt{3}}{4}\right)$

Q.4 The area bounded by y = 2 – |2 – x| & $y = \frac{3}{|x|}$ is

(A) $\frac{4+3\ell n3}{2}$ (B) $\frac{4-3\ell n3}{2}$

(C) $\frac{3}{2}+\ell n3$ (D) $\frac{1}{2}+\ell n3$

Q.5 The area bounded by the curves y = sin x & y = cos x between x = 0 & x = 2 π is

(A) $\int_0^{2\pi}(\sin x-\cos x)dx$ (B) $2\sqrt{2}$ sq. unit

(C) 0 (D) $4\sqrt{2}$ sq. unit

Q.6 If f(x) = –1 + |x – 2|, 0 ≤ x ≤ 4 ; g(x) = 2 – |x|, –1 ≤ x ≤ 3.

Then the area bounded by y = gof(x); x = 1, x = 4 and x-axis is

(A) 7/2 sq. units (B) 9/4 sq. units

(C) 9/2 sq. units (D) None of these

Q.7 Area enclosed between the curve y = $\sec^{-1}x$, y = $\operatorname{cosec}^{-1}x$ and the line x = 1 is

(A) $\ln(3+2\sqrt{2})$ (B) $\ln(3+2\sqrt{2})-1$

(C) $\ln(3+2\sqrt{2})-\pi/2$ (D) None of these

Q.8 The area of the closed figure bounded by y = x, y = –x the tangent to the curve $y=\sqrt{x^2-5}$ at the point (3, 2) is

(A) 5 (B) $\frac{15}{2}$

(C) 10 (D) $\frac{35}{2}$

Q.9 The line y = mx bisects the area enclosed by the curve y = 1 + 4x – x^2 & the lines x = 0, $x=\frac{3}{2}$ & y = 0, then m is equal to

(A) $\frac{13}{6}$ (B) $\frac{6}{13}$ (C) $\frac{3}{2}$ (D) 4

Q.10 The area common to $y\ge\sqrt{x}$ & $x>-\sqrt{y}$ and the curve $x^2 + y^2 = 2$ is

(A) $\frac{\pi}{4}+\frac{1}{3}$ (B) $\frac{3\pi}{2}$ (C) $\frac{1}{3}$ (D) $\frac{\pi}{2}$

Q.11 The ratio in which the curve y = x^2 divides the region bounded by the curve ; y = $\sin\left(\frac{\pi x}{2}\right)$ & the x-axis as x varies from 0 to 1, is

(A) 2 : π (B) 1 : 3

(C) 3 : π (D) (6 – π) : π

Q.12 Area of the region enclosed between the curves x = y^2 – 1 and $x=|y|\sqrt{1-y^2}$ is

(A) 1 (B) 4/3 (C) 2/3 (D) 2

Q.13 Let y = g(x) be the inverse of a bijective mapping f : R → R, f(x) = $3x^3$ + 2x. The area bounded by the graph of g(x). The x-axis and the ordinate at x = 5 is

(A) 5/4 (B) 7/4 (C) 9/4 (D) 13/4

Previous Years' Questions

Q.1 The area of the region between the curves $y=\sqrt{\frac{1+\sin x}{\cos x}}$ and $y=\sqrt{\frac{1-\sin x}{\cos x}}$ and bounded by the lines x = 0 and x = $\frac{\pi}{4}$ is ***(2008)***

(A) $\int_0^{\sqrt{2}-1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

(B) $\int_0^{\sqrt{2}-1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(C) $\int_0^{\sqrt{2}+1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(D) $\int_0^{\sqrt{2}+1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

Q.2 Let the straight line x = b divide the area enclosed by y = $(1 - x)^2$, y = 0 and x = 0 into two parts R_1(0 ≤ x ≤ b) and R_2(b ≤ x ≤ 1) such $R_1 - R_2 = \frac{1}{4}$. Then, b equals ***(2011)***

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Q.3 Let $f : [-1, 2] \rightarrow [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all xÎ[–1, 2], Let $R_1 = \int_{-1}^{2} x f(x)dx$ and R_2 be the area of the region bounded by y = f(x), x = –1, x = 2 and the x-axis. Then, ***(2011)***

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$

(C) $2R_1 = R_2$ (D) $3R_1 = R_2$

Q.4 Find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$. ***(1981)***

Q.5 Find the area bounded by the x-axis, part of the curve $y = \left(1 + \frac{8}{x^2}\right)$ and the ordinates at x = 2 and x = 4. If the ordinate at x = a divides the area into two equal parts, find a. ***(1983)***

Q.6 Find the area of the region bounded by the x-axis and the curves defined by y = tan x, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$ and y = cot x, $\frac{\pi}{6} \le x \le \frac{\pi}{2}$ ***(1984)***

Q.7 Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area. ***(1985)***

Q.8 Find the area bounded by the curves $x^2 + y^2 = 4$, $x^2 = -\sqrt{2}y$ and x = y. **(1986)**

Q.9 Find the area of the region bounded by the curve C : y = tan x, tangent drawn to C at $x = \frac{\pi}{4}$ and the x-axis. ***(1988)***

Q.10 Find all maxima and minima of the function $y = x(x - 1)^2$, $0 \le x \le 2$.

Also, determine the area bounded by the curve $y=x(x - 1)^2$, the y-axis and the line x = 2. ***(1989)***

Q.11 Compute the area of the region bounded by the curves y = ex log x and y = $\frac{\log x}{ex}$ where log e = 1 ***(1990)***

Q.12 If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$,

f(x) is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of y = f(x) with x-axis and point B is such that chord AB subtends a right angled at V. Find the area enclosed by f(x) and chord AB. ***(2005)***

Q.13 A curve passes through (2, 0) and the slope of tangent at point P(x, y) equals $\frac{(x+1)^2 + y - 3}{(x+1)}$. Find the equation of the curve and area enclosed by the curve and the x-axis in the fourth quadrant. ***(2004)***

Q.14 Area of the region $\left\{(x, y) \in R^2 : y \ge \sqrt{|x+3|},\ 5y \le x + 9 \le 15\right\}$ is equal to ***(2016)***

(A) $\frac{1}{6}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{3}$

Q.15 Let S be the area of the region enclosed by $y = e^{-x^2}, y = 0, x = 0$, and $x = 1$. Then ***(2012)***

(A) $S \ge \frac{1}{e}$ (B) $S \ge -\frac{1}{e}$

(C) $S \le \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$ (D) $S \ge \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.4 Q.10 Q.17

Exercise 2

Q.2 Q.6 Q.10
Q.12 Q.15 Q.17
Q.18

Previous Years' Questions

Q.2 Q.4 Q.7

JEE Advanced/Boards

Exercise 1

Q.1 Q.5 Q.12
Q.14 Q.16 Q.20
Q.21

Exercise 2

Q.2 Q.3 Q.7
Q.11 Q.13

Previous Years' Questions

Q.1 Q.3 Q.7
Q.10 Q.11 `Q.13

Answer Key

JEE Main/Boards

Exercise 1

Q.1 $\frac{74}{3}$ sq.units **Q.2** 2 sq. units **Q.3** 2 sq. units **Q.4** $\int_1^3 (5-y)\,dy$

Q.5 $2\int_0^2 \sqrt{y}\,dy$ **Q.6** $\int_{-2}^{a} \sqrt{a^2 - x^2}\,dx$ **Q.7** 5 sq. units **Q.8** $\frac{112}{9}$ sq. units

Q.9 $\frac{2}{27}$ sq. units **Q.10** $\frac{64}{3}$ sq. units **Q.11** $\frac{28}{3}$ sq. units

Q.12 (3 + 16 log 2) sq. units **Q.13** $\frac{1}{2}$ sq. units **Q.14** 13.5 sq. units **Q.15** 1 sq. units

Q.16 $\frac{a^2}{12}(3\pi - 8)$ sq. units **Q.17** $\frac{16a}{3}$ sq. units **Q.18** $\frac{50}{3}$ sq. units **Q.19** $\frac{32}{3}$ sq. units

Exercise 2

Single Correct Choice Type

Q.1 C	**Q.2** A	**Q.3** C	**Q.4** A	**Q.5** A	**Q.6** B
Q.7 B	**Q.8** B	**Q.9** D	**Q.10** C	**Q.11** D	**Q.12** B
Q.13 A	**Q.14** A	**Q.15** C	**Q.16** C	**Q.17** C	**Q.18** C

Previous Years' Questions

Q.1 D	**Q.2** A	**Q.3** A	**Q.4** A	**Q.5** 2 sq. units	
Q.6 $2\sqrt{3}$ sq. units	**Q.7** A	**Q.8** A	**Q.9** C	**Q.10** D	**Q.11** A
Q.12 C	**Q.13** C	**Q.14** D	**Q.15** B		

JEE Advanced/Boards

Exercise 1

Q.1 f(x) = x sin x, a = 1 ; A_1 = 1 – sina ; A_2 = π – 1 – sina ; A_3 = (3π – 2) sq. units **Q.2** 2/3

Q.3 f(x) = x^2 + 1;y = ± 2x ; A = $\frac{2}{3}$sq. units **Q.4** $\frac{1}{2}$ **Q.5** $\sqrt{2}+1$ **Q.6** a = 8

Q.7 $\frac{(\pi-1)}{2}$ **Q.8** 2 **Q.9** $a=\frac{2}{3}$ **Q.10** 104 **Q.11** $(16/9)x^2$

Q.12 (i) 3 ; (ii) 1280 sq. units ; (iii) $\frac{32}{3}$ sq. units **Q.13** $\sqrt{3}$ **Q.14** $\frac{3}{2}$ **Q.15** $\frac{8}{3}$ sq. units

Q.16 4 **Q.17** A $\frac{3m+2m^2+\frac{7}{6}}{m^3}$ **Q.18** $1-3e^{-2}$ **Q.19** $\left(\frac{\sqrt{2}}{6}+\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}\right)$ sq. units

Q.20 a = –3/4 **Q.21** e

Exercise 2

Single Correct Choice Type

Q.1 D	**Q.2** A	**Q.3** D	**Q.4** B	**Q.5** D	**Q.6** C	**Q.7** C
Q.8 A	**Q.9** A	**Q.10** A	**Q.11** D	**Q.12** D	**Q.13** B	

Previous Years' Questions

Q.1 B **Q.2** B **Q.3** C **Q.4** 9/8 sq. units **Q.5** $2\sqrt{2}$

Q.6 $\frac{1}{2}\log_e 3$ sq. units **Q.7** $\frac{5\pi}{4}-\frac{1}{2}$ sq. units **Q.8** $\frac{1}{3}-\pi$ sq. units **Q.9** $\left(\log\sqrt{2}-\frac{1}{4}\right)$sq. units

Q.10 10/3 sq. units **Q.11** $\frac{e^2-5}{4e}$ sq. units **Q.12** 125/3 sq. units **Q.13** 4/3 sq. units

Q.14 3/2 sq. units **Q.15** A, B, D

Solutions

JEE Main/Boards

Exercise 1

Sol 1: Area = $\int_2^4 x^2 + 3 = \left.\frac{x^3}{3} + 3x\right|_2^4$

$= \frac{64}{3} + 12 - \frac{8}{3} - 6 \Rightarrow$ Area = $\frac{56}{3} + 6 = \frac{74}{3}$ sq. units

Sol 2: $2\int_0^{\pi/2} \cos x dx = 2\left[\sin x\right]_0^{\pi/2}$ = 2[1 – 0] = 2 sq. units

Sol 3: $\int_0^{\pi} \sin x dx = \left[-\cos x\right]_0^{\pi} = -\cos\pi + \cos 0$ = 1 + 1

= 2 sq. units

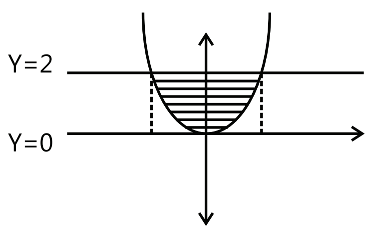
Sol 4: x + y = 5 ⇒ x = –y + 5

$\Rightarrow \int_4^2 x = \int_1^3 (-y+5)dy$ = $\left[-\frac{y^2}{2} + 5y\right]_1^3$ = $\left|\frac{9}{2} - 15 - \frac{1}{2} + 5\right|$

= 6 sq. units

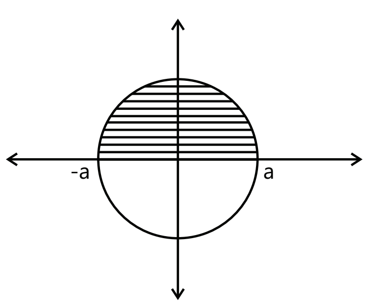
Expression = $\left|\int_1^3 (5-y)\right| dy$

Sol 5: y = x^2;



x = $\sqrt{y}$ $A = 2\int_0^2 \sqrt{y} dy$

Sol 6:



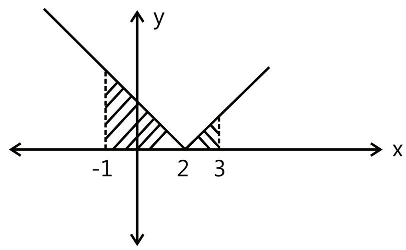
Area = $\int_{-a}^{a} \sqrt{a^2 - x^2} dx$

Sol 7: $\int_{-1}^{3} |x-2| dx$ is the area under curve |x – 2|

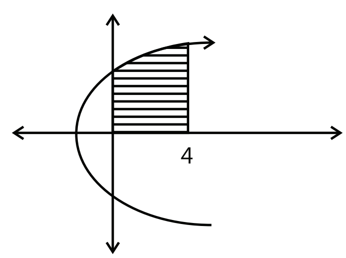
where x ∈ [–1, 3]

A = $\int_{-1}^{2} 2 - x + \int_2^3 x - 2$ = $\left.2x - \frac{x^2}{2}\right|_{-1}^{2} + \left.\frac{x^2}{2} - 2x\right|_2^3$

⇒ A = 2 + 2 + $\frac{1}{2} + \frac{9}{2}$ – 6 – 2 + 4 = 5 sq. units



Sol 8: y = $\sqrt{3x+4}$



A = $\int_0^4 y\, dx = \int_0^4 \sqrt{3x+4}\, dx$ = $\frac{2\left[(3x+4)^{3/2}\right]_0^4}{9x^3}$

= $\frac{2}{9}\left[16^{3/2} - 4^{3/2}\right]$ = $\frac{2}{9}\left[4^3 - 2^3\right]$

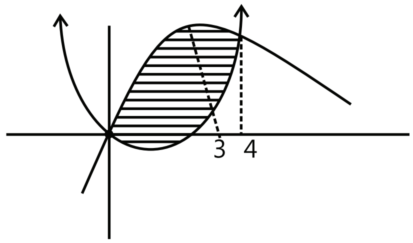
= $\frac{2}{9}\left[64 - 8\right] = \frac{56 \times 2}{9} = \frac{112}{9}$ sq. units

Sol 9: y = $\frac{3}{(1-2x)^3}$ above x axis & x ∈ [–4, –1]

$\int_{-4}^{-1} \frac{3dx}{(1-2x)^3}$ = $\left[\frac{3}{(1-2x)^{+2}} \times \frac{1}{(-2)(-2)}\right]_{-4}^{-1}$

= $\frac{3}{4}\left[\frac{1}{(1-2x)^2}\right]_{-4}^{-1}$ = $\frac{3}{4}\left[\frac{1}{3^2} - \frac{1}{9^2}\right]$ = $\frac{3 \times 8}{4 \times 81} = \frac{2}{27}$ sq. units

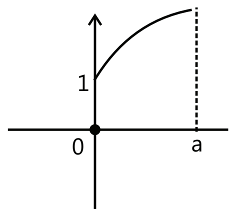
Sol 10:



Area $= \int_0^4 \left(6x - x^2 - x^2 + 2x\right)dx$

$= \left[8x - 2x^2\right]_0^4 = \left[4x^2 - \frac{2x^3}{3}\right]_0^4 = 64 - \frac{2}{3} \times 64 = \frac{64}{3}$ sq. units

Sol 11: $f(x) = 1 + \sqrt{x}$

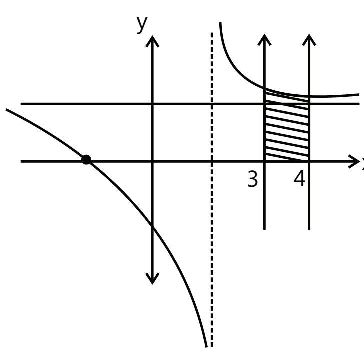


$\int_0^4 y\,dx = \int_0^4 \sqrt{x} + 1 = \left[\frac{2x^{3/2}}{3} + x\right]_0^4$

$= \frac{2}{3} \times 2^3 + 4 = \frac{16}{3} + 4 = \frac{28}{3}$ sq. units

Sol 12: $xy - 3x - 2y - 10 = 0$

$y = \frac{3x + 10}{x - 2}$



$A = \int_3^4 \frac{3x + 10}{x - 2}dx$ $A = \frac{3x - 6 + 16}{x - 2}$ $= \int_3^4 \left(3 + \frac{16}{x - 2}\right)dx$

$= \left[3x + 16\ell n(x - 2)\right]_3^4$

$\left[12 + 16\ \ell\ n2 - 9 - \ln 1\right]$

$= (3 + 16\log 2)$ sq units.

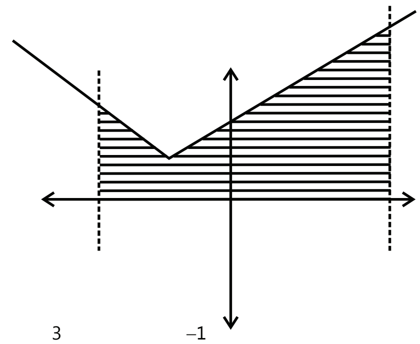
Sol 13: $\int y\,dx = \int_0^{\sqrt{\pi/2}} x \sin x^2 dx$

Substituting $x^2 = t$

$xdx = \frac{dt}{2}$

$\int_0^{\pi/2} \sin t \frac{dt}{2} = \left[-\frac{\cos t}{2}\right]_0^{\pi/2} = \frac{-0 + 1}{2} = \frac{1}{2}$ sq. units

Sol 14:

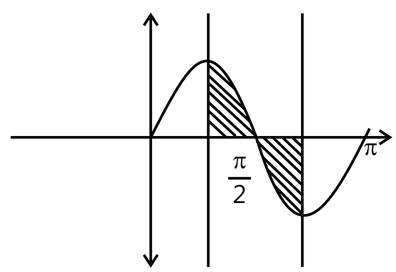


$\int_{-2}^{3} |x + 1| + 1 = \int_{-1}^{3} x + 1 + 1 + \int_{-2}^{-1} -1 - x + 1$

$= \left.\frac{x^2}{2} + 2x\right|_{-1}^{3} - \left[\frac{x^2}{2}\right]_{-2}^{-1}$

$= \frac{9}{2} + 6 - \frac{1}{2} + 2 - \frac{1}{2} + 2 = \frac{7}{2} + 10 = \frac{27}{2}$ sq. units

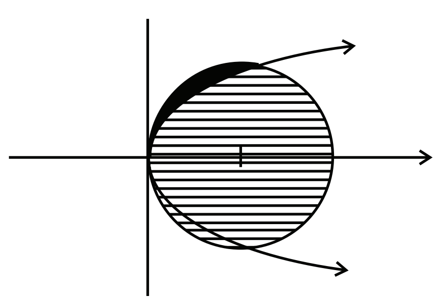
Sol 15:



Area $= 2\int_{\pi/4}^{\pi/2} \sin 2x\,dx = \frac{2}{2}\left[-\cos 2x\right]_{\pi/4}^{\pi/2}$

$= |+1 - 2| = 1$ sq. units

Sol 16:



$x^2 - 2ax + y^2 \le 0$

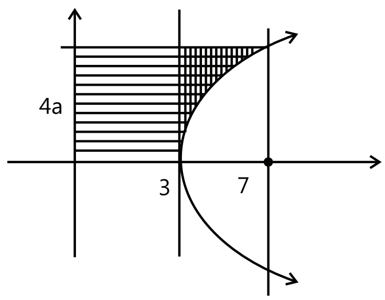
$y^2 - ax \ge 0$

$$A=\int_0^a\left(\sqrt{2ax-x^2}-\sqrt{ax}\right)dx \Rightarrow A=\int_0^a\sqrt{2ax-x^2}\,dx-\int_0^a\sqrt{ax}\,dx$$

$$=\pi\frac{a^2}{4}-\frac{\sqrt{a}2x^{3/2}}{3}\Bigg|_0^a$$

$$=\frac{\pi a^2}{4}-\frac{2\sqrt{a}}{3}a^{3/2}=\frac{\pi a^2}{4}-\frac{2a^2}{3}=\frac{a^2}{12}\left(3\pi-8\right) \text{ sq. units}$$

Sol 17:



$y = 4a \Rightarrow 16a^2 = 4a^2(x-3) \Rightarrow x = 7$

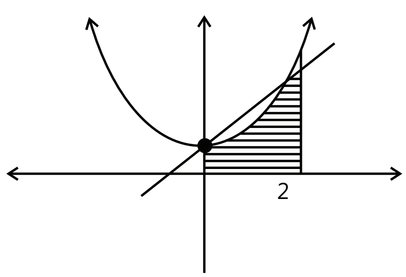
$$=\int_0^{4a}x\,dy=\int_0^{4a}\left(\frac{y^2}{4a^2}+3\right)dy=\left[\frac{y^3}{12a^2}+3y\right]_0^{4a}$$

$$=\frac{(4a)^3}{12a^2}+12a=\frac{64}{12}a+12a$$

$$\Rightarrow\left(\frac{16+36}{3}\right)a=\frac{52a}{3}$$

$$A=\frac{52a}{3}-3\times 4a=\frac{16a}{3} \text{ sq. units}$$

Sol 18: The points of intersection of $y=x^2+3$ and $y=2x+3$ are (0, 3) and (2, 7).

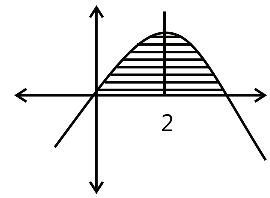


$$A_1=\int_2^3 2x+3=x^2+3x\Big|_2^3=\frac{27}{3}+9-4-6=18-10=8$$

$$A_2=\int_0^2 x^2+3=\frac{x^3}{3}+3x\Bigg|_0^2=\frac{8}{3}+6$$

$$A_1+A_2=\frac{8}{3}+6+8=\frac{50}{3} \text{ sq. units}$$

Sol 19: $y = 4x - x^2$



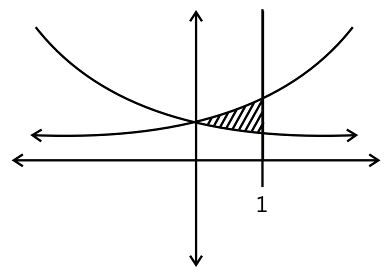
$$\int\left(4x-x^2\right)dx=\left[2x^2-\frac{x^3}{3}\right]_0^4$$

$$=32-\frac{64}{3}=\frac{32}{3} \text{ sq. units}$$

Exercise 2

Single Correct Choice Type

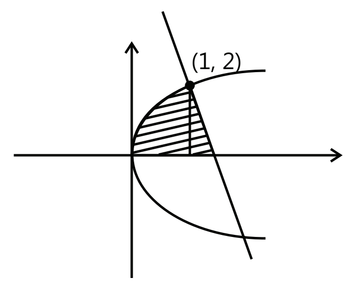
Sol 1: (C)



$$A=\int_0^1\left(e^x-e^{-x}\right)dx=\left[e^x+e^{-x}\right]_0^1$$

$$=e+\frac{1}{e}-1-1=e+\frac{1}{e}-2$$

Sol 2: (A)



Area = $A_1 + A_2$

$$A_1=\int_0^1\sqrt{4x}\,dx=2\frac{\left[x^{3/2}\right]_0^1}{3/2}=\frac{4}{3}$$

Equation normal $\dfrac{y-2}{x-1}=-1$

$\Rightarrow y - 2 = 1 - x$

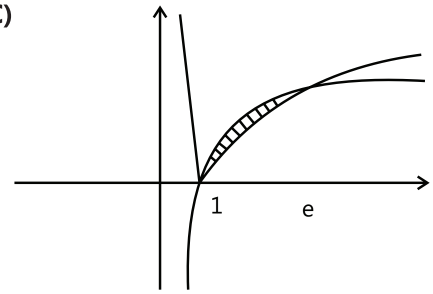
$x + y = 3$ ...... (i)

$$A_2=\int_1^3(3-x)dx=\left[3x-\frac{x^2}{2}\right]_1^3=9-\frac{9}{2}-3+\frac{1}{2}$$

$= 6 - 4 = 2$

Area $= 2 + \dfrac{4}{3} = \dfrac{10}{3}$

Sol 3: (C)



A = $\int_1^e \left(\ln x - (\ln x)^2 \right) dx \quad \int \ln dx = x \ln x - x$

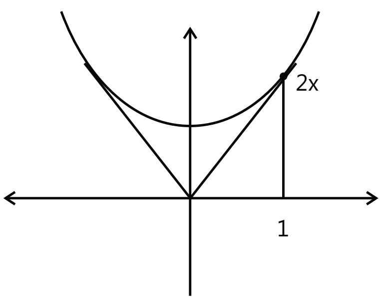
A = $(x\ln x - x) - \left[\ln x (x\ln x - x) - \int (\ln x - 1) \right]$

= $x\ln x - x - x(\ln x)^2 + x\ln x + x\ln x - x - x$

$\left[-x(\ln x)^2 + 3x\ln x - 3x \right]_1^e$

= $-e + 3e - 3e + 3 = 3 - e$

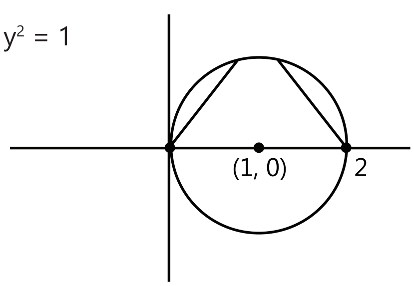
Sol 4: (A)



$A = 2\int_0^1 \left(x^2 + 1 - 2x \right) dx = 2\int_0^1 (x-1)^2 dx = 2 \Rightarrow \left. \frac{(x-1)^3}{3} \right|_0^1 = \frac{2}{3}$

Sol 5: (A) $(x - 1)^2 + y^2 = 1$

If $\sin\left(\frac{\pi x}{2} \right)$



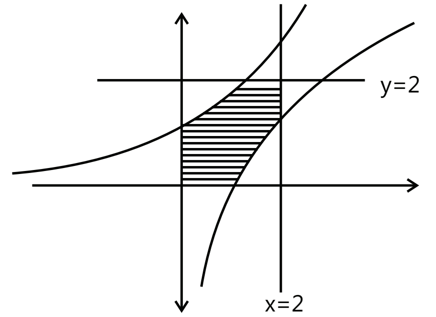
A = $\int_0^2 \left(\sqrt{1 - (x-1)^2} - \sin\frac{\pi x}{2} \right) dx$

= $\left[\sqrt{2x - x^2} - \sin\frac{\pi x}{2} \right]_0^2$

= $\left[\frac{\sin^{-1}(x-1) + (x-1)\sqrt{2x - x^2}}{2} + \frac{2}{\pi}\cos\frac{\pi x}{2} \right]_0^2$

= $\frac{\pi}{2 \times 2} + \frac{2}{\pi}(-1) + \frac{\pi}{2.2} - \frac{2}{\pi} = \frac{\pi}{2} - \frac{4}{\pi}$

Sol 6: (B)



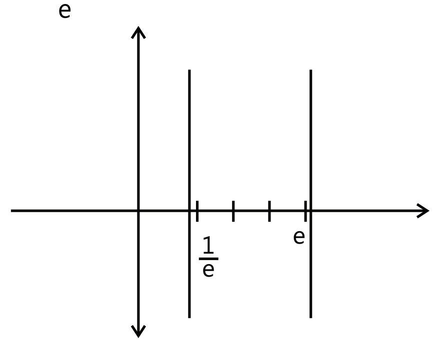
Area = $\int_1^2 \ln x\, dx + \int_1^2 \ln y\, dy$

= $2\int_1^2 \ln x = 2\left[x\ln x - x \right]_1^2 = 2[2\ln 2 - 2 + 1]$

= $4\ln 2 - 2 = 2(2\ln 2 - 1)$

Sol 7: (B) $y = x(1 - \ln x)$ & $x = \frac{1}{e}$

Between $x = \frac{1}{e}$ & e



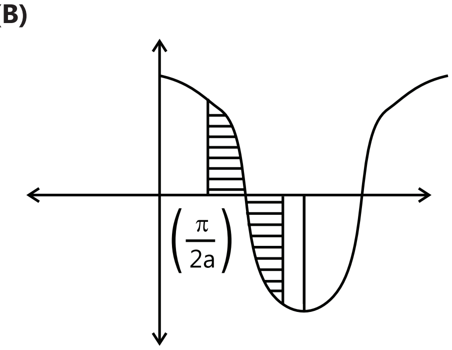
I = $\int \left(x - x\ln x \right) dx = \frac{x^2}{2} - \frac{1}{2}\left[x^2 \ln x - \frac{x^2}{2} \right]$

I = $\frac{3x^2}{2} - \frac{x^2}{2}\ln x$

$\Rightarrow A = \left[I \right]_{\frac{1}{e}}^{e}$

A = $\frac{3e^2}{4} - \frac{e^2}{2} - \left(\frac{3}{4e^2} + \frac{1}{2e^2} \right) = \frac{1}{4}e^2 - \frac{5}{4e^2}$

Sol 8: (B)



$$\text{Area} = \int_{\pi/6a}^{\pi/2a} (\cos ax)\,dx = \frac{2}{a}\left[\sin ax\right]_{\pi/6a}^{\pi/2a}$$

$$= \frac{2}{a}\left[1-\frac{1}{2}\right] = \frac{1}{a} > 3 \ = a \in \left(0, \frac{1}{3}\right)$$

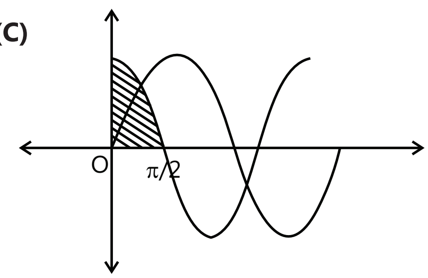
Sol 9: (D) $y = \frac{x}{6}+\frac{1}{x^2}; y = 0$

$$A = \int_a^{2a}\left(\frac{x}{6}+\frac{1}{x^2}\right)dx = \left[\frac{x^2}{12}-\frac{1}{x}\right]_a^{2a}$$

$$= \frac{a^2}{3}-\frac{1}{2a}-\frac{a^2}{12}+\frac{1}{a} \ = \frac{a^2}{4}+\frac{1}{2a}$$

A_{least} when $A' = \frac{a}{2}-\frac{1}{2a^2} = 0 \Rightarrow a = 1$

Sol 10: (C)



$$\text{Area}_1 = \left[\int_0^{\pi/4} \sin x\right]2 \ = \ 2\left[-\cos x\right]_0^{\pi/4}$$

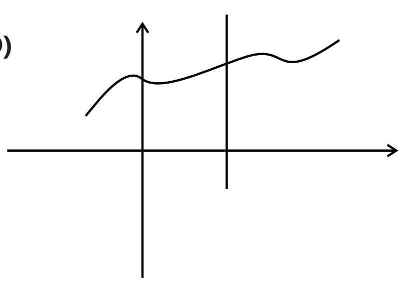
$$= 2\left[-\frac{1}{\sqrt{2}}+1\right] \ = \left(\sqrt{2}-1\right)\sqrt{2} = 2-\sqrt{2}$$

$$\text{Area}_2 = \int_0^{\pi/2} \cos x - \text{area}_1 = \left[\sin x\right]_0^{\pi/2} - 2+\sqrt{2}$$

$= 1 - 2 + \sqrt{2} \ = \sqrt{2} - 1$

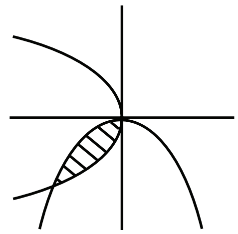
ratio is $\frac{\sqrt{2}\left(\sqrt{2}-1\right)}{\sqrt{2}-1} = \sqrt{2}$

Sol 11: (D)



$$\int_0^{x_1} f(x) = x_1 e^{x_1} \Rightarrow f(x) = xe^x + e^x$$

Sol 12: (B)



$$\text{Area} = \int_{-1}^{0} -\sqrt{-x} - \left(-x^2\right) \ = \int_{-1}^{0} -\sqrt{-x} + x^2$$

$$= \left[\frac{x^3}{3}+\frac{2\left(-x\right)^{3/2}}{3}\right]_{-1}^{0} \ = \left|\frac{1}{3}-\frac{2}{3}\right| = \frac{1}{3}$$

Sol 13: (A) $\int_1^x f(x)\,dx = \sqrt{1+3x}-2$

$$f(x) = \frac{3}{2\sqrt{1+3x}}$$

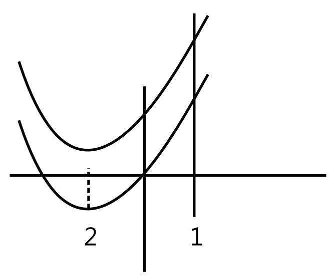
$x = 1, y = \frac{3}{4}$ and $x = 8, y = \frac{3}{10}$

$$A = \frac{\int_1^8 y\,dx}{\Delta x} = \frac{\sqrt{1+3.8}-2}{8-1} = \frac{3}{7}$$

Sol 14: (A) $\frac{dy}{dx} = 2x+1$

$y = x^2 + x + c$

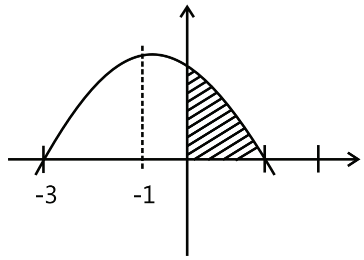
(1, 2); c = 0



$y = x^2 + x + 1$

$$A = \int_0^1 x^2 + x = \left[\frac{x^3}{3}+\frac{x^2}{2}\right]_0^1 = \frac{5}{6}$$

Sol 15: (C)

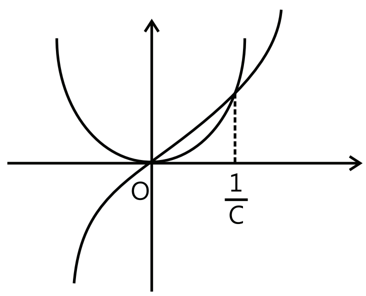


$0 \le y < 3 - 2x - x^2$

$0 < y < 4 - (x + 1)^2$

$$= \int_0^1 (3 - 2x - x^2)\,dx$$

Sol 16: (C)

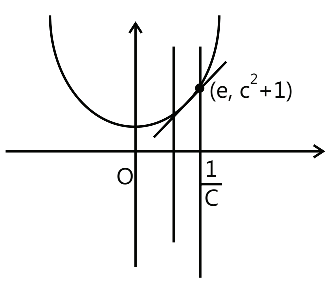


$$\int_0^{\frac{1}{c}} \left(x^2 - cx^3\right)dx = \frac{2}{3}$$

$$\Rightarrow \left[\frac{x^3}{3} - \frac{cx^4}{4}\right]_0^{1/c} = \frac{1}{3c^3} - \frac{1}{4c^3} = \frac{2}{3}$$

$$\Rightarrow \frac{1}{12c^3} = \frac{2}{3} \Rightarrow c = \frac{1}{2}$$

Sol 17: (C)



$$\frac{y - c^2 - 1}{x - c} = 2c$$

$\Rightarrow y = 2cx - c^2 + 1$

$x = 1,\qquad y = 2c - c^2 + 1$

$x = 2,\qquad y = 4c - c^2 + 1$

$\Rightarrow$ Area of trapezoid $= \frac{1}{2}\left[6c - 2c^2 + 2\right] \times 1 = 3c - c^2 + 1$

For $\text{area}_{max} = A' = 3 - 2c = 0 \Rightarrow C = \frac{3}{2}$

$$\left(\frac{3}{2}, \frac{13}{4}\right)$$

Sol 18: (C) $A_1 = \frac{1}{2} \times a^2 \times 2a = a^3$

$$\Rightarrow = \int_{-a}^{a} x^2 = \left[\frac{x^3}{3}\right]_{-a}^{a} = \frac{2a^3}{3}$$

$$\Rightarrow_2 = 2a \times a^2 - \frac{2a^3}{3} = \frac{4a^3}{3}$$

$$\Rightarrow \frac{A_1}{A_2} = \frac{3}{4}$$

Previous Years' Questions

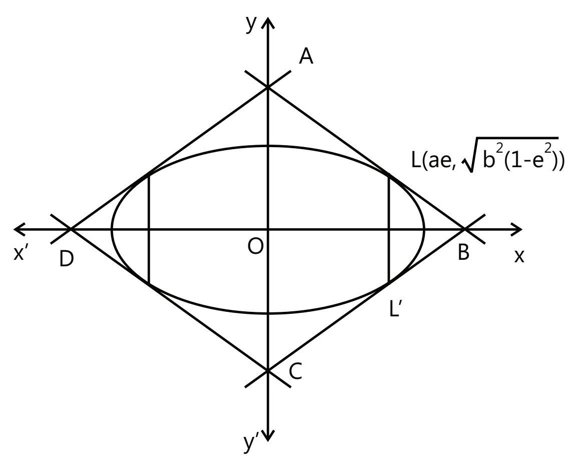
Sol 1: (D) Given, $\frac{x^2}{9} + \frac{y^2}{5} = 1$

To find tangents at the end points of latus rectum, we find ae.

i.e. $ae = \sqrt{a^2 - b^2} = \sqrt{4} = 2$

and $\sqrt{b^2(1 - e^2)} = \sqrt{5\left(1 - \frac{4}{9}\right)} = \frac{5}{3}$

By symmetry, the quadrilateral is a rhombus.



So, area is four times the area of the right angled triangle formed by the tangent and axes in the 1st quadrant.

$\therefore$ Equation of tangent at $\left(2, \frac{5}{3}\right)$ is

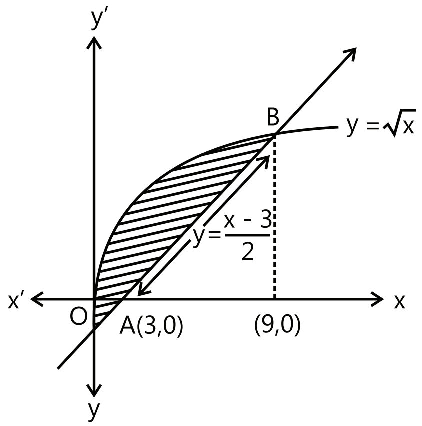
$$\frac{2}{9}x + \frac{5}{3}.\frac{y}{5} = 1 \Rightarrow \frac{x}{9/2} + \frac{y}{3} = 1$$

$\therefore \quad \frac{x}{9/2} + \frac{y}{3} = 1$

$\therefore$ Area of quadrilateral ABCD

$= 4$ (area of ΔAOB)$= 4\left(\frac{1}{2}.\frac{9}{2}.3\right)$ = 27 sq. units

Sol 2: (A) To find the area between the curves, $y = \sqrt{x}, 2y + 3 = x$ and x-axis in the 1st quadrant (we can plot the above condition as) ;



Area of shaded portion OABO

$= \int_0^9 \sqrt{x}\,dx - \int_3^9 \left(\frac{x-3}{2}\right)dx$

$= \left(\frac{x^{3/2}}{3/2}\right)_0^9 - \frac{1}{2}\left(\frac{x^2}{2} - 3x\right)_3^9$

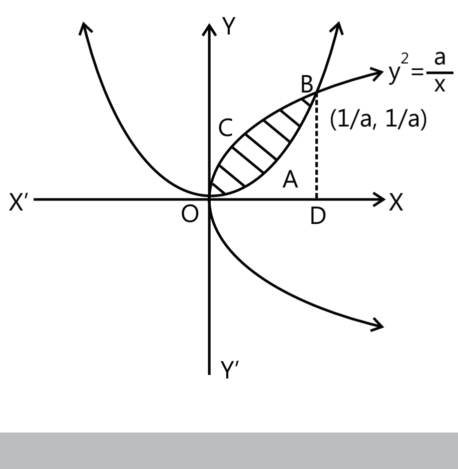
$= \left(\frac{2}{3}.27\right) - \frac{1}{2}\left\{\left(\frac{81}{2} - 27\right) - \left(\frac{9}{2} - 9\right)\right\}$

$= 18 - \frac{1}{2}(18)$ = 9 sq. unit

Sol 3: (A) As from the figure, area enclosed between the curves is OABCO.

Thus, the point of intersection of

$y = ax^2$ and $x = ay^2$



$\Rightarrow x = a\,(ax^2)^2$

$\Rightarrow x = 0, \frac{1}{a} \Rightarrow y = 0, \frac{1}{a}$

$\therefore$ Point of intersection are (0, 0) and $\left(\frac{1}{a}, \frac{1}{a}\right)$

Thus, required area OABCO = Area of curve OCBDO – area of curve OABDO

$\Rightarrow \int_0^{1/a}\left(\sqrt{\frac{x}{a}} - x^2\right)dx = 1$ (given)

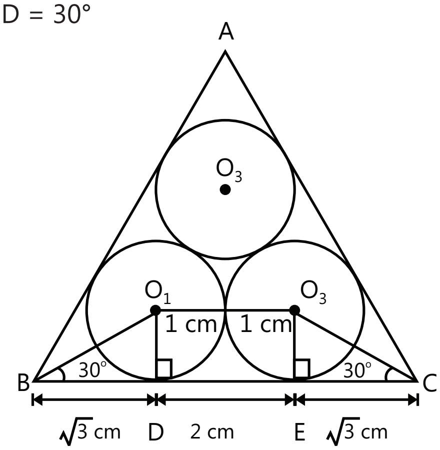
$\Rightarrow \left[\frac{1}{\sqrt{a}}.\frac{x^{3/2}}{3/2} - \frac{ax^3}{3}\right]_0^{1/a} = 1$

$\Rightarrow \frac{2}{3a^2} - \frac{1}{3a^2} = 1$

$\Rightarrow a^2 = \frac{1}{3} \Rightarrow d = \frac{1}{\sqrt{3}}$ ($\because$ a > 0)

Sol 4: (A) Since, tangents drawn from external point to the circle subtends equal angle at the centre

$\therefore \angle O_1BD = 30°$



In ΔO_1BD, tan 30° = $\frac{O_1D}{BD}$

$\Rightarrow$ BD = $\sqrt{3}$ cm

Also, DE = O_1O_2 = 2 cm and EC = $\sqrt{3}$ cm

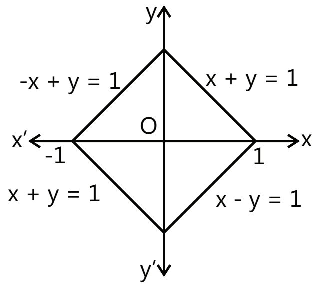
Now, BC = BD + DE + EC = $2 + 2\sqrt{3}$

$\Rightarrow$ Area of ΔABC

$= \frac{\sqrt{3}}{4}(BC)^2 = \frac{\sqrt{3}}{4} . 4(1+\sqrt{3})^2$

$= (6 + 4\sqrt{3})$ sq. cm.

Sol 5: The area formed by |x| + |y| = 1 is square shown as below,



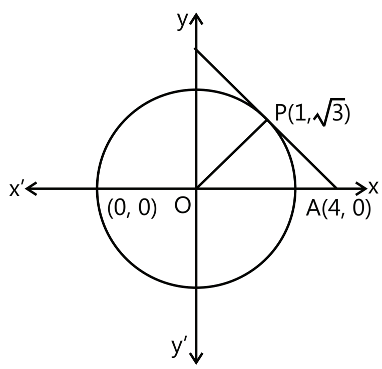
$\therefore$ Area of square = $(\sqrt{2})^2$ = 2 sq. units

Sol 6: Equation of tangent at the point $(1, \sqrt{3})$ to the curve

$x^2 + y^2 = 4$

is $x + \sqrt{3}y = 4$,

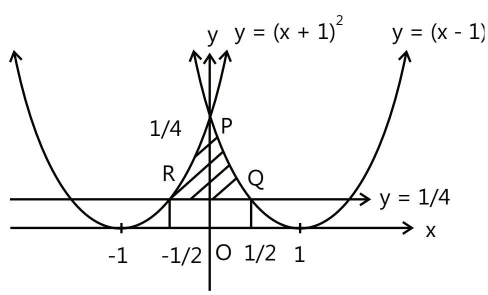
whose x-axis intercept (4, 0)



Thus, area of Δ formed by (0, 0) $(1, \sqrt{3})$ and (4, 0)

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & \sqrt{3} & 1 \\ 4 & 0 & 1 \end{vmatrix} = \frac{1}{2}|(0 - 4\sqrt{3})| = 2\sqrt{3} \text{ sq. units}$$

Sol 7: (A) The curves y = $(x - 1)^2$, y = $(x + 1)^2$ and y = 1/4 are shown are



Where points of intersection are

$(x - 1)^2 = \frac{1}{4} \Rightarrow x = \frac{1}{2}$

and $(x + 1)^2 = \frac{1}{4} \Rightarrow -\frac{1}{2}$

$\therefore$ $Q\left(\frac{1}{2}, \frac{1}{4}\right)$ and $R\left(-\frac{1}{2}, \frac{1}{4}\right)$

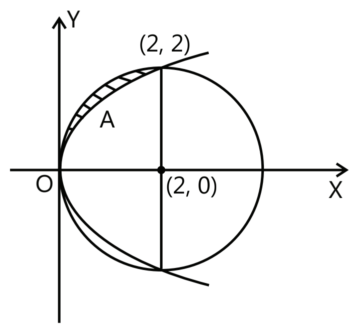
$\therefore$ Required area

$$= 2\int_0^{1/2}\left[(x-1)^2 - \frac{1}{4}\right]dx = 2\left[\frac{(x-1)^3}{3} - \frac{1}{4}x\right]_0^{1/2}$$

$$= 2\left[-\frac{1}{8.3} - \frac{1}{8} - \left(-\frac{1}{3} - 0\right)\right] = \frac{8}{24} = \frac{1}{3} \text{ sq. unit}$$

Sol 8: (A) Region $(x, y) : y^2 \geq 2x$

$x^2 + y^2 < 4x$ and $x \geq 0,\ y \geq 0$



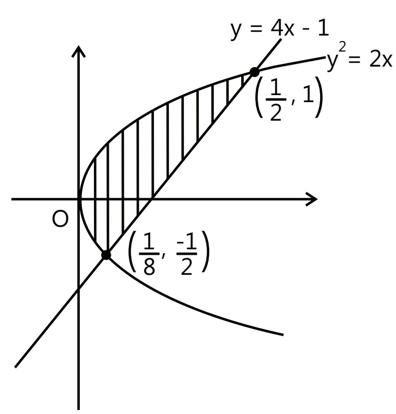
$$\text{Area } = \frac{1}{4}\pi(2)^2 - \int_0^2 \sqrt{2x}\, dx$$

$$= \pi - \left[\sqrt{2}\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 = \pi - \frac{2\sqrt{2}}{3}\left[2^{\frac{3}{2}}\right]$$

$$= \pi - \frac{2\sqrt{2}}{3} \times 2\sqrt{2}$$

$$= \pi - \frac{8}{3} \text{ sq. units}$$

Sol 9: (C) Region $(x, y) : y^2 < 2x$ and $y \geq 4x - 1$



Area $= \int_{-1/2}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy$

$= \left[\frac{y^2}{8}+\frac{y}{4}-\frac{y^3}{6}\right]_{-1/2}^{1} = \left[\frac{1}{3}+\frac{1}{4}-\frac{1}{6}-\frac{1}{32}+\frac{1}{8}-\frac{1}{48}\right] = \frac{9}{32}$

Sol 10: (D) The point of intersection $x-3=2\sqrt{x}$

$\Rightarrow (x-3)^2 = 4x$

$\Rightarrow x^2-10x+9=0$

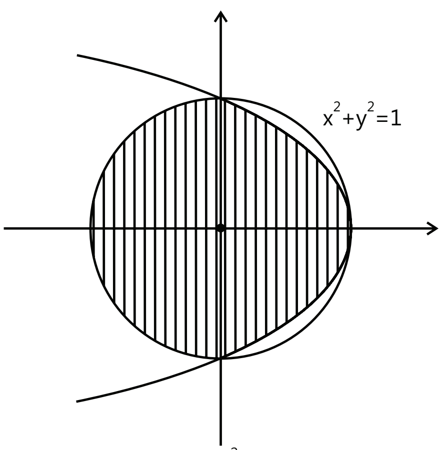
$\Rightarrow x=1,9$

$\Rightarrow y=1,3$

Area $\int_0^3\left[2y+3-y^2\right]dy = \left[y^2+3y-\frac{y^3}{3}\right]_0^3$

$= \left[9+9-9\right] = 9$ sq. units

Sol 11: (A)



Area = Area of half circle

$+2\int_0^1\sqrt{1-x}\,dx = \frac{1}{2}\pi(1)^2+2\left[-\frac{(1-x)^{3/2}}{3/2}\right]_0^1$

$= \frac{\pi}{2}+2\left[\frac{2}{3}\right] = \frac{\pi}{2}+\frac{4}{3}$

Sol 12: (C) $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=m$

$\Rightarrow$ Any point on this line is $(2m+1, 3m-1, 4m+1)$

Similarly for $\frac{x-3}{1}=\frac{y-k}{2}=\frac{2}{1}=n$

$(n+3, 2n+k, n)$

It they intersect, then

$2m+1=n+3 \Rightarrow 2m-n=2$

$3m-1=2n+k \Rightarrow 3m-2n=k+1$

$4m+1=n \Rightarrow 4m-n=-1$

On solving these equations, we get

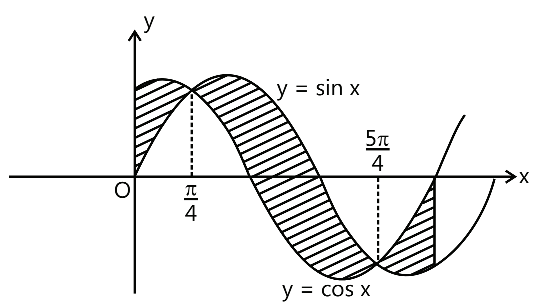
$m=-\frac{3}{2}, n=-5 \Rightarrow K=\frac{9}{2}$

Sol 13: (C) The required area

$= 2\left[\int_0^2\left(3\sqrt{y}-\frac{\sqrt{y}}{2}\right)dy\right] = 2\left[\int_0^2\frac{5\sqrt{y}}{2}dy\right] = 5\int_0^2\sqrt{y}\,dy$

$= 5\left[\frac{y^{3/2}}{3/2}\right]_0^2 = \frac{10}{3}\left[2^{3/2}\right] = \frac{10\times 2\sqrt{2}}{3} = \frac{20\sqrt{2}}{3}$

Sol 14: (D)



Area $= \int_0^{\pi/4}(\cos x-\sin x)dx+\int_{\pi/4}^{5/4}(\sin x-\cos x)dx$

$+\int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}(\cos x-\sin x)dx$

$= \left[\sin x+\cos x\right]_0^{\pi/4}+\left[-\cos x-\sin x\right]_{\pi/4}^{5\pi/4}$

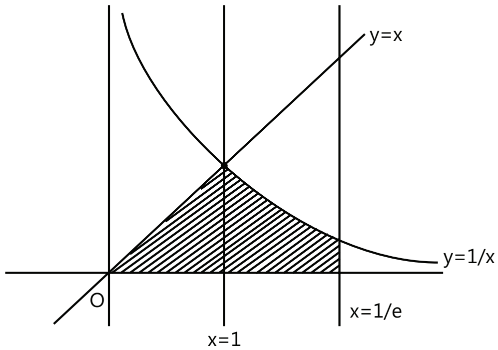
$+\left[\sin x+\cos x\right]_{5\pi/4}^{3\pi/2}$

$= \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1+\left[\frac{-1}{\sqrt{2}}-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right]$

$+\left[-1+0+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right]$

$= 4\sqrt{2}-2$

Sol 15: (B)



Area $\left|\int_0^1 x dx\right| + \left|\int_1^{1/e} \frac{1}{x} dx\right|$

$= \left[\frac{x^2}{2}\right]_0^1 + \left|\left[\ln x\right]_1^{1/e}\right| = \frac{1}{2} + \left|\ln 1/e\right|$

$= \frac{1}{2} + 1 = \frac{3}{2}$ sq. units

JEE Advanced/Boards

Exercise 1

Sol 1: $A_1 = \int_0^a \sin x - f(x)$

$= -\cos x\Big|_0^a - \int_0^a f(x) = 1 - \cos a - \int_0^a f(x)$

$\Rightarrow A_2 = \int_a^\pi f(x) - \sin x = \int_a^\pi fx + \cos x\Big|_a^\pi = \int_a^\pi f(x) - 1 - \cos a$

$\Rightarrow A_3 = \int_\pi^{2\pi} \sin x - f(x) = -\cos x\Big|_\pi^{2\pi} - \int_\pi^{2\pi} f(x) = -2 - \int_\pi^{2\pi} f(x)$

$\Rightarrow A_1$ = 1 – sina + acosa – cosa

= 1 – cosa – $\int_0^a f(x)\,dx$

$\Rightarrow$ –acosa + sina = $\int_0^a f(x)\,dx$ = sina – acosa

$\Rightarrow$ f(x) = xsinx

$\int_a^\pi f(x)dx = \int_a^\pi x\sin x\,dx = \left[-x\cos x + \sin x\right]_a^\pi$

= π – [–acosa + sina] = π + acosa – sina

$\Rightarrow A_2$ = π + acosa – sina – 1 – cosa

$\int_\pi^{2\pi} f(x)dx = \int_a^\pi x\sin x\,dx = \left[-x\cos x + \sin x\right]_\pi^{2\pi}$

= –2π – (π) = –3p

$\Rightarrow A_3$ = (3π – 2) sq units.

xsinx = sinx $\Rightarrow$ x = 1

$\Rightarrow$ a = 1 $\Rightarrow A_2$ = π – 1 – sina

$\Rightarrow A_1$ = 1 – sina

Sol 2: A = $\int_0^t \sin x^2 dx$, B = $\frac{1}{2}\times t\sin t^2$

Now, $\lim_{t\to 0}\frac{A}{B} = \frac{\int_0^t \sin x^2 dx}{\frac{1}{2}\times t\sin t^2}$

$\frac{0}{0}$ form $\Rightarrow \lim_{t\to 0} = \frac{\sin t^2}{\frac{t}{2}(2t)\cos t^2 + \frac{1}{2}\sin t^2}$

[L' Hospital's rule]

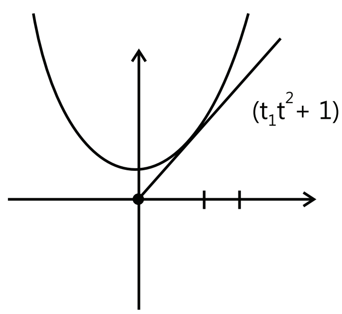
$= \frac{2\sin t^2}{2t^2\cos t^2 + \sin t^2} = \frac{2\tan t^2}{2t^2 + \tan t^2}$

$= \frac{\frac{2\tan t^2}{t^2}}{2 + \frac{\tan t^2}{t^2}} = \frac{2}{2+1} = \frac{2}{3}$

Sol 3: f(x + 1) = f(x) + 2x + 1

f(0) = 1

f(1) = 2$\Rightarrow$ f(x) = x^2 + 1



f(–1) = 2;f(2) = 5;f(–2) = 5

y = x^2 + 1

Equation of tangent at $P(t, t^2+1)$ y – (t^2 + 1) = 2t(x – t)

Since, it passes through (0, 0)

–t^2 – 1 = 2t(–t)

$\Rightarrow$ t^2 = t $\Rightarrow$ t = ±1

$\Rightarrow$ y = ±2x

$$\text{Area} = 2\int_0^1 \left(x^2+1-2x\right) dx$$

$$= 2\int_0^1 (x-1)^2 dx = \frac{2}{3}(x-1)^3\Big|_0^1 = \frac{2}{3}[0+1] = \frac{2}{3} \text{ sq. units}$$

Sol 4: $y = \dfrac{\ln x - c}{x} = 0 \Rightarrow x = e^c$

$$f'(x) = \frac{x\left(\frac{1}{x}\right) - (\ln x - c)}{x^2} = 0$$

$\Rightarrow 1 - \ln x + c = 0$

$\Rightarrow \ln x = c + 1$

$\Rightarrow x = e^{c+1}$

$$\Rightarrow \int_{e^c}^{e^{c+1}} \frac{\ln x - c}{x}dx = \int_{e^c}^{e^{c+1}} \frac{\ln x}{x}dx - \int_{e^c}^{e^{c+1}} \frac{c}{x}\, dx\sqrt{b^2-4ac}$$

$$= \left[\frac{(\ln x)^2}{2}\right]_{e^c}^{e^{c+1}} - c\left[\ln x\right]_{e^c}^{e^{c+1}}$$

$$= \frac{(c+1)^2 - c^2}{2} - c[c+1-c] = \frac{2c+1}{2} - [c] = \frac{1}{2}$$

Sol 5: $y = x^n$

$y - 1 = n(x - 1)$

$$A = \int_0^1 x^n dx - \int_{1-\frac{1}{n}}^{1} [n(x-1)+1]dx$$

$$= \frac{\left[x^{n+1}\right]_0^1}{n+1} - \left[n\frac{(x-1)^2}{2} + x\right]_{1-\frac{1}{n}}^{1}$$

$$= \frac{1}{n+1} - \left[1 - \frac{n}{2n^2} - 1 + \frac{1}{n}\right] = \frac{1}{n+1} - \frac{1}{2n}$$

$$\Rightarrow = \frac{n-1}{2n(n+1)}$$

$\dfrac{dA}{dn} = 0 \Rightarrow 2n(n + 1) - (n - 1)(4n + 2) = 0$

$\Rightarrow 2n^2 + 2n - 4n^2 - 2n + 4n + 2 = 0$

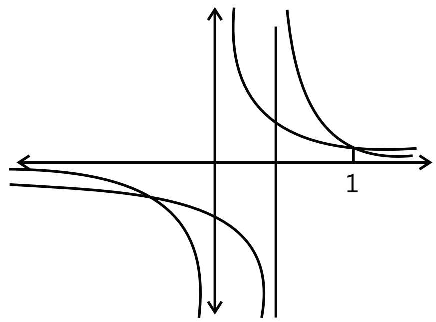
$\Rightarrow -2n^2 + 4n + 2 = 0$

$\Rightarrow n^2 - 2n - 1 = 0$

$$n = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

$\Rightarrow > 1$ i. e. $n = \sqrt{2} + 1$

Sol 6:



$$A = \int_2^a \frac{1}{x} - \frac{1}{2x-1} = \left[\ln x - \frac{\ln(2x-1)}{2}\right]_2^a$$

$$= \ln a - \frac{\ln(2a-1)}{2} - \ln 2 + \frac{\ln 3}{2}$$

$$= \ln\frac{3a}{2(2)(2a-1)} \times 2 = \ln\frac{3a}{4a-2} - \ln\frac{4}{5}$$

$\Rightarrow 15a = 16a - 8 \qquad \Rightarrow a = 8$

Sol 7: $f(x) = \dfrac{1}{1+x^2}$

We have, $A(\alpha, f(\alpha))$ and $B\left(-\dfrac{1}{\alpha}, f\left(-\dfrac{1}{\alpha}\right)\right)$

$OA \to y = \dfrac{x}{\alpha(1+\alpha^2)}$ .... (i)

$OB \to y = \dfrac{-\alpha^3}{1+\alpha^2}(x)$

$$\int_0^\alpha \left(\frac{1}{1+x^2} - \frac{x}{\alpha(1+\alpha^2)}\right)dx + \int_{-\frac{1}{\alpha}}^{0} \left(\frac{1}{1+x^2} + \frac{\alpha^3}{1+\alpha^2}x\right)dx$$

$$= \left[\tan^{-1}x - \frac{x^2}{2\alpha(1+\alpha^2)}\right]_0^\alpha + \left[\tan^{-1}x + \frac{\alpha^3 x^2}{2(1+\alpha^2)}\right]_{-\frac{1}{\alpha}}^0$$

$$= \tan^{-1}\alpha - \frac{\alpha}{2(1+\alpha^2)}$$

$$+ \left[0 + 0 - \tan^{-1}\left(-\frac{1}{\alpha}\right) - \frac{\alpha}{2(1+\alpha^2)}\right]$$

$$= \tan^{-1}\alpha - \tan^{-1}\left(-\frac{1}{\alpha}\right) - \frac{(\alpha+\alpha)}{2(1+\alpha^2)}$$

$$= \tan^{-1}\frac{\alpha^2-1}{1-1} - \frac{(\alpha+\alpha)}{2(1+\alpha^2)}$$

$$= \frac{\pi}{2} - \frac{(\alpha+\alpha)}{2(1+\alpha^2)} = \frac{\pi}{2} - \frac{2\alpha}{2(1+\alpha^2)}$$

$A' = 0 \Rightarrow \alpha = 1$

$A = \frac{\pi}{2} - \frac{1}{2}$

Sol 8: The curve is $y = \sin^{-1} x$, i.e., x = sin y. This is a standard curve. Lines x = 0, $y = \frac{\pi}{2}$ and $y = -\frac{\pi}{2}$ are the y-axis and two lines parallel to the x-axis at a distance $\frac{\pi}{2}$, one above and the other below the x-axis respectively.

Hence, the shaded part is the required area Δ. By symmetry of the curve and the lines,

ar (OABO) = ar (OCDO)

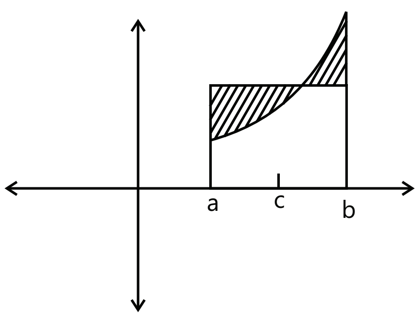
$\therefore \Delta = 2 \times \text{ar}(\text{OABO})$

$= 2\int_0^{\frac{\pi}{2}} (x)_{\text{curve}} dy = 2\int_0^{\frac{\pi}{2}} \sin y \, dy,$

($\because$ The equation of the curve is x = sin y)

$\therefore \Delta = 2\left[-\cos y\right]_0^{\frac{\pi}{2}} = 2\left[0+1\right] = 2$

Sol 9:



$A = \left(\int_a^c f(c) - f(x)\right) dx + \int_c^b \left(f(x) - f(c)\right) dx \; A$

$= f(c)(c-a) - \int_a^c f(x)\, dx + \int_c^b f(x)\, dx - f(c)(b-c)$

$A = f(c)\,[c - a - b + c] - \int_a^c f(x)\, dx + \int_c^b \left(f(x)\right) dx$

This is minimum when 2c – a – b = 0

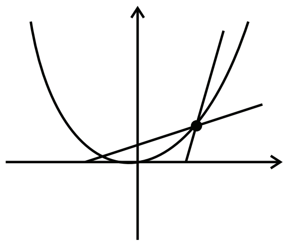
$\Rightarrow c = \frac{a+b}{2}$

$\int f(x) = \int_0^2 \left(\frac{x^3}{3} - x^2 + a\right) dx = \left[\frac{x^4}{12} - \frac{x^3}{3} + ax\right]_0^2$

$= \frac{16}{12} - \frac{8}{3} + 2a = \left(2a - \frac{4}{3}\right)$

Is minimum $\Rightarrow 2a = \frac{4}{3} \Rightarrow a = \frac{2}{3}$

Sol 10:



$y - c = m(x - 1)$

$A = \int \left(x^2 - m(x-1) - c\right) dx$

$= \left[\frac{x^3}{3} - \frac{m(x-1)^2}{2} - cx\right]_a^b$

$= \left[\frac{b^3 - a^3}{3} - \frac{m}{2}\left[(b-1)^2 - (a-1)^2\right] - c(b-a)\right]$

$= \frac{(b-a)(b^2 + a^2 + ab)}{3}$

$-\frac{m}{2}\left[b^2 - a^2 - 2b + 2a\right] - c(b-a)$

$c + mx - m = x^2$

$\Rightarrow x^2 - mx + m - c = 0$

$x_1 + x_2 = m$

$x_1 x_2 = m - c$

$\Rightarrow x_1 - x_2 = \sqrt{m^2 - 4m + 4c}$

$\Rightarrow (b - a)\left[\frac{m^2 - m + c}{3} - \frac{m}{2}[m-2] - c\right]$

$\Rightarrow (b - a)\left[\frac{m^2 - m + c}{3} - \frac{m^2}{2} - \frac{3c}{3} + \frac{3m}{3}\right]$

$\Rightarrow (b - a)\left[-\frac{m^2}{6} - \frac{2c}{3} + \frac{2m}{3}\right]$

$A = +\frac{1}{6}[m^2 + 4c - 4m]^{3/2}$

$\boxed{m^2 - 4m + 4c = 36}$

$B^2 - 4AC = 0$

$16 - 4(4c - 36) = 0$

$\Rightarrow c = 10 \Rightarrow m = 2$

$\Rightarrow c^2 + m^2 = 104$

Sol 11: $A_1 + A_2$ for any point $P(t, t^2)$

$$\int_0^k C_2 dx - C + \int_k^t t^2 dx - C = \int_0^t (C - C_1) dx$$

$$\int_0^k C_2 dx - \left[\frac{x^3}{3}\right]_0^k + \left[\frac{-x^3}{3}\right]_k^t + t^2(t-k) = \frac{t^3}{6}$$

$$\int_0^k C_2 dx - \frac{k^3}{3} + \frac{k^3}{3} - \frac{t^3}{3} + t^3 - t^2k = \frac{t^3}{6}$$

$$\int_0^k C_2 dx = -\frac{t^3}{2} + t^2k$$

Let C_2 = f(x) l^2x^2

$$\int_0^{t/\lambda} (\lambda^2 x^2) dx = \frac{t^3}{2} + t^2k$$

$$\frac{\lambda^2\left[x^3\right]_0^{t/\lambda}}{3} = -\frac{t^3}{2} + \frac{t^3}{\lambda} \Rightarrow \frac{\lambda^2}{3}\frac{t^3}{\lambda^3} = -\frac{t^3}{2} + \frac{t^3}{\lambda}$$

$$\Rightarrow \frac{10}{2} = \frac{1}{\lambda} - \frac{1}{3\lambda} = \frac{2}{3\lambda} \Rightarrow \lambda = \frac{4}{3}$$

$$\Rightarrow c_2 = \frac{16x^2}{9}$$

Sol 12: (i) Let equation of CD be y = 2x+c

For intersection with $y = x^2$

$\Rightarrow x^2 = 2x + c$; $x^2 - 2x - c = 0$

$\Rightarrow x_1 + x_2 = 2$ and $x_1x_2 = -c$

Length of CD $= |x_1 + x_2|\sqrt{5}$

$= 2\sqrt{5}\sqrt{1+c}$

Length of $AC = BD = \frac{c+17}{\sqrt{5}}$

Given ABCD is square than,

$$2\sqrt{5}\sqrt{1+c} = \frac{c+17}{\sqrt{5}}$$

$\Rightarrow c^2 - 66c + 189 = 0$

$\Rightarrow c = 3, 63$

Therefore, least value of c is 3.

(ii) For maximum Area of sq. ABCD

Length $2\sqrt{5}\sqrt{1+c}$ must be maximum

For c = 63 (From Previous questions)

Area = $\left(2\sqrt{5}\sqrt{1+c}\right)^2$

$= \left(2\sqrt{5}\sqrt{1+63}\right)^2 = 4 \times 5 \times 64$

= 1280 sq. units.

(iii) Area bounded by y = 2x + 3 and

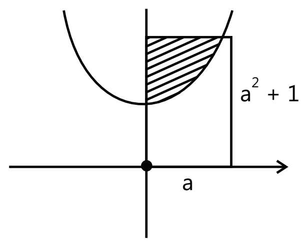
Area = $\int_{-1}^{3} (2x + 3 - x^2) dx$

$= \left[x^2 + 3x - \frac{x^3}{3}\right]_{-1}^{3}$

$= 9 + \cancel{9} - \cancel{9} - 1 + 3 - \frac{1}{3}$

$= 11 - \frac{1}{3} = \frac{32}{3}$ sq. units

Sol 13:



Area of rectangle = $a(a^2 + 1)$

$$A = \int_1^{a^2+1} \sqrt{y-1}\, dy = \left[\frac{2(y-1)^{3/2}}{3}\right]_1^{a^2+1}$$

$= \frac{2}{3}a^3 - 0 = \frac{2}{3}a^3$

$= \frac{2}{3}a^3 = \frac{1}{2}a(a^2+1) = \frac{4}{3}a^3 = a^3 + a$

$a^3 = 3a \Rightarrow a = \sqrt{3}, -\sqrt{3}, 0$

$\Rightarrow a = \sqrt{3}$

Sol 14: $f^3(x) = \int_0^x t f^2(t)\, dt$

$\Rightarrow f'(x)\, 3f^2(x) = x f^2(x)$

$\Rightarrow f^2(x)[3f'(x) - x] = 0$

$\Rightarrow f'(x) = \frac{x}{3} \Rightarrow f(x) = \frac{x^2}{6} + c$

$\Rightarrow f(x) = \frac{x^2}{6} + c$

$$\left[\frac{x^2}{6}+c\right]^3 = \int_0^x t\left(\frac{t^2}{6}+c\right)^2 dx$$

$$\frac{x^6}{216}+c^3+\frac{3cx^4}{36}+\frac{3x^2c^2}{6}$$

$$= \int_0^x \left[\frac{t^4}{36}+c^2+\frac{ct^2}{3}\right] t\ dx = \int_0^x \left[\frac{t^5}{36}+c^2t+\frac{ct^3}{3}\right] dx$$

$$= \left[\frac{t^6}{216}+\frac{c^2t^2}{2}+\frac{ct^4}{12}\right]_0^x = \frac{x^6}{216}+\frac{c^2x^2}{2}+\frac{cx^4}{12} \Rightarrow c = 0$$

$$\Rightarrow f(x) = \frac{x^2}{6}$$

Hence, $\int_0^3 f(x)dx = \left[\frac{x^3}{18}\right]_0^3 = \frac{3}{2}$

Sol 15: Given curves are $y^2 = x+1$ .... (i)

and $y^2 = -x+1$ or $y^2 = -(x-1)$ .... (ii)

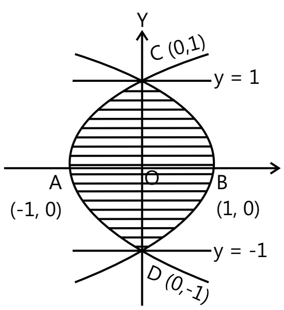
Curve (i) is the parabola having axis y = 0 and vertex (-1, 0).

Curve (ii) is the parabola having axis y = 0 and vertex (1, 0)

(1) - (2) 2x = 0 x = 0

From (i), x = 0 $\Rightarrow y = \pm 1$

Required area = area ACBDA



$$= \int_{-1}^{1} (x_1 - x_2)\ dy$$

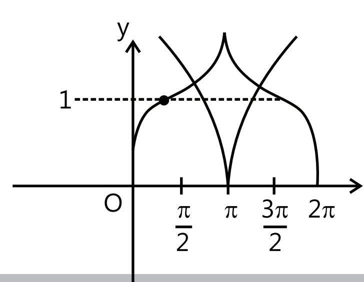
$$= \int_{-1}^{1} \left[\left(1-y^2\right)-\left(y^2-1\right)\right] dy$$

$$=2\int_{-1}^{1} \left(1-y^2\right) dy$$

$$= 2\left[y-\frac{y^3}{3}\right]_{-1}^{1} = 2\left[\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)\right]$$

$$= \frac{8}{3} \text{ sq. units}$$

Sol 16:



$$A = 2\int_0^{\pi/2} \sin x + 2\int_{\pi/2}^{\pi} 2-\sin x$$

$$= 2\left[-\cos x\right]_0^{\pi/2} + 2\left[2x+\cos x\right]_{\pi/2}^{\pi}$$

$= 2[+1] + 2[2\pi - 1 - p] = 2\pi = a\pi + b$

a = 2, b = 0

$a^2 + b^2 = 4$

Sol 17: $-1 + x = 2y - y^2 - 1$

$x - 1 = -(y - 1)^2$

y = $\sqrt{1-x} + 1 = mx + 2$

$1 - x = (mx + 1)^2$

$1 - x = m^2x^2 + 1 + 2mx$

$\Rightarrow m^2x^2 + (2m + 1)x = 0$

$$\Rightarrow x_1 = 0,\ x_2 = \frac{-(2m+1)}{m^2}$$

$$x_1x_2 = 0,\ x_1 + x_2 = \frac{-(2m+1)}{m^2}$$

$$A = \int_{x_1}^{x_2} \left(1+\sqrt{1-x}-mx-2\right) dx$$

$$A = \left[x-\frac{(1-x)^{3/2}}{3/2}-\frac{mx^2}{2}-2x\right]_{x_1}^{x_2}$$

$$= \left[\frac{-2(1-x)\sqrt{1-x}}{3}-\frac{mx^2}{2}-x\right]_{x_1}^{x_2}$$

$$= -\frac{2}{3}\left[\left(1-x_2\right)^{3/2}-\left(1-x_1\right)^{3/2}\right] - \left[\frac{m}{2}\left(x_2^2-x_1^2\right)\right]-\left[\left(x_2-x_1\right)\right]$$

$$= -\frac{2}{3}\left[\left(1-x\right)^{3/2}-1\right]-\frac{m}{2}x^2-x$$

$$= \frac{2}{3}-\frac{2}{3}(1-x)^{3/2}-\frac{mx^2}{2}-x$$

$$= \frac{2}{3}-\frac{2}{3}\left(1+\frac{(2m+1)}{m^2}\right)^{3/2} - \frac{m}{2}\left(\frac{2m+1}{m^2}\right)^2+\left(\frac{2m+1}{m^2}\right)$$

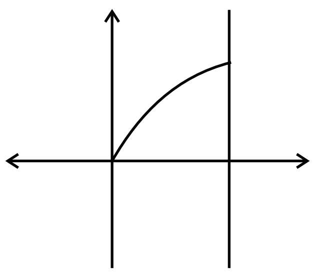
$$= \frac{2}{3}-\frac{2}{3m^3}(m+1)^3+\frac{2}{m}+\frac{1}{m^2}$$

$$-\frac{m}{2}\left(\frac{4}{m^2}+\frac{1}{m^4}+\frac{4}{m^3}\right) = \frac{2}{3}+\frac{2}{m}+\frac{1}{m^2}-\frac{2}{m}-\frac{1}{2m^3}$$

$$-\frac{2}{m^2}-\frac{2}{3}-\frac{2}{3m^3}-\frac{2}{m}-\frac{2}{m^2} = -\frac{3}{m^2}-\frac{2}{m}-\frac{7}{6m^3}$$

$$A = \frac{3m+2m^2+\frac{7}{6}}{m^3}$$

Sol 18:



$f'(x) = -xe^{-x} + e^{-x}$

$f''(x) = xe^{-x} - e^{-x} - e^{-x}$

$= (x-2)e^{-x} = 0$

$\Rightarrow x = 2$ inflection point

$$\int_0^2 xe^{-x} = \left[-xe^{-x} - e^{-x}\right]_0^2$$

$= -2e^{-2} - e^{-2} + 1 = 1 - 3e^{-2}$

Sol 19: Let $R = \{(x,y): y^2 \le 4x,\ 4x^2+4y^2 \le 9\}$

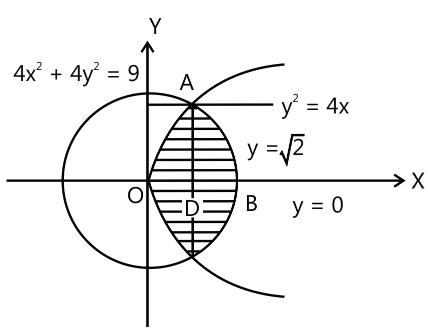
$= \{(x,y): y^2 \le 4x\} \cap \{(x,y): 4x^2+4y^2 \le 9\} = R_1 \cap R_1$

Where $R_1 = \{(x,y): y^2 \le 4x\}$ and $R_2 = \{(x,y): 4x^2+4y^2 \le 9\}$

Equation of the given curves are

$y^2 = 4x$ ... (i)

and $4x^2+4y^2 = 9$ ... (ii)



Curve (i) is a parabola having axis y = 0 and vertex (0, 0).

Curve (ii) is a circle having centre at (0, 0) and radius $\frac{3}{2}$.

Clearly region R_1 is the interior of the parabola (i) and region R_2 is the interior of the circle (ii). Therefore, $R_1 \cap R_2$ is the shaded region.

Putting the value of y^2 from (i) in (ii), we get

$$4x^2+16x-9 = 0 \Rightarrow x = \frac{1}{2} - \frac{9}{2}$$

From (i), $x = \frac{1}{2} \Rightarrow y = \pm\sqrt{2}$

And $x = -\frac{9}{2}$ is not possible

Thus, $A = \left(\frac{1}{2}, \sqrt{2}\right)$ and $C = \left(\frac{1}{2}, -\sqrt{2}\right)$

Required area = 2 area OABD

$$= 2\int_0^{\sqrt{2}} (x_1 - x_2)dy$$

$$= 2\int_0^{\sqrt{2}} \frac{1}{2}\sqrt{3^2-(2y)^2}\,dy - 2\int_0^{\sqrt{2}} \frac{y^2}{4}\,dy$$

$$= \frac{1}{2}\int_0^{2\sqrt{2}} \sqrt{3^2-z^2}\,dz - \frac{1}{2}\left[\frac{y^3}{3}\right]_0^{\sqrt{2}}$$

[Putting z = 2y in first integral]

$$= \frac{1}{2}\left[\frac{2\sqrt{9-z^2}}{2} + \frac{9}{2}\sin^{-1}\frac{z}{3}\right]_0^{2\sqrt{2}} - \frac{1}{6}.2\sqrt{2}$$

$$= \frac{1}{2}\left[\sqrt{2} + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right] - \frac{2\sqrt{2}}{6}$$

$$= \frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3} = \frac{\sqrt{2}}{6} + \frac{9}{4}\cos^{-1}\frac{1}{3}$$

$\sin^{-1}\frac{2\sqrt{3}}{3} = \theta$, then $\sin\theta = \frac{2\sqrt{2}}{3}$ and

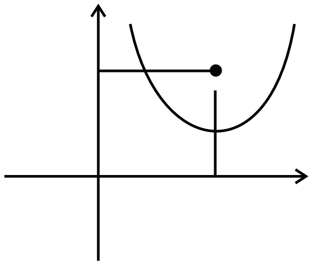
$$\cos\theta = \sqrt{1-\sin^2\theta} = \frac{1}{3}$$

Required area $= \frac{\sqrt{2}}{6} + \frac{9}{4}\left[\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right]$

$$\left[\because \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x\right]$$

$$= \left(\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}\right) \text{ sq. units.}$$

Sol 20:



$y = a^2x^2 + ax + 1$

$$y = \left(ax + \frac{1}{2}\right)^2 + \frac{3}{4}$$

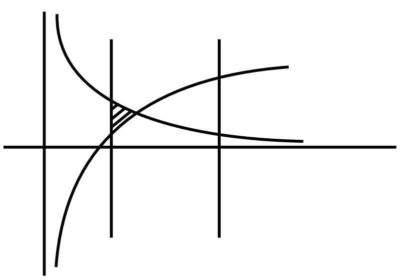
$$\Rightarrow A = \int_0^1 \left[\left(ax + \frac{1}{2}\right)^2 + \frac{3}{4}\right] dx$$

$$\Rightarrow A = \left[\frac{a^2x^3}{3} + \frac{ax^2}{2} + x\right]_0^1 = -\frac{a^2}{3} + \frac{a}{2} + 1$$

A least if A′=0 ie $\frac{2a}{3} + \frac{1}{2} = 0$

$$a = -\frac{3}{4}$$

Sol 21:



$$\int_1^K \left(\frac{1}{x} - \ell nx\right) dx = \int_K^a \left(\ell nx - \frac{1}{x}\right) dx$$

$1 = K\ell nK$

$K^K = e$

$$\left[\ell nx - x\ell nx + x\right]_1^K = \left[x\ell nx - x - \ell nx\right]_K^a$$

$\ell nK - 1 + K - 1$

$= a\ell na - a - \ell na - 1 + K + \ell nK$

$(a - 1) = (a - 1)(\ell na)$

$(a - 1)[1 - \ell na] = 0$

Either a = 1 or a = e

$\therefore a = e$

Exercise 2

Single Correct Choice Type

Sol 1: (D)

$$A = \int_a^b \left(x^2 - 1 - 3 + x\right) dx = \int_a^b \left(x^2 + x - 4\right) dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} - 4x\right]_a^b$$

$$= \frac{b^3 - a^3}{3} + \frac{b^2 - a^2}{2} - 4(b - a)$$

$$= \sqrt{17}\left|\frac{1+4}{3} + \frac{(-1)}{2} - 4\right| = \sqrt{17}\left|\frac{5}{3} - \frac{9}{2}\right| = \frac{17\sqrt{17}}{6}$$

For point of intersection

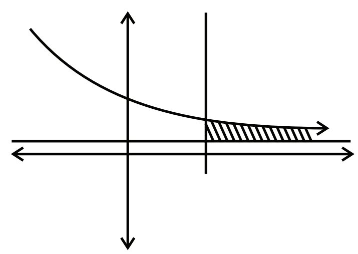
$x^2 - 1 = 3 - x$

$\Rightarrow x^2 + x - 4 = 0$

$\Rightarrow b + a = -1 \Rightarrow ab = -4$

$\Rightarrow b - a = \sqrt{1+16} \Rightarrow b - a = \sqrt{17}$

Sol 2: (A) e^{-x} and linearly $= e^{-4}$ & $x = 1$



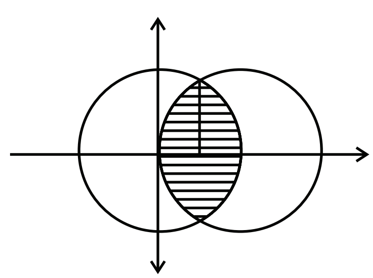
$$\int_1^4 \left[e^{-x} - e^{-4}\right] dx$$

$= -e^{-4} - 4e^{-4} - (-e^{-1} - e^{-4})$

$= -5e^{-4} + e^{-4} + e^{-1}$

$$= e^{-1} - 4e^{-4} = \frac{1}{e} - \frac{4}{e^4} = \frac{e^3 - 4}{e^4}$$

Sol 3: (D) $y = \sqrt{9 - x^2}$ $x^2 - 6x + y^2 = 0$



$$\text{Area} = 2\int_0^{3/2}\left(\sqrt{9-x^2}-\sqrt{6x-x^2}\right)dx$$

$$= 2\int_0^{3/2}\left[\sqrt{9-x^2}-\sqrt{9-(x-3)^2}\right]dx$$

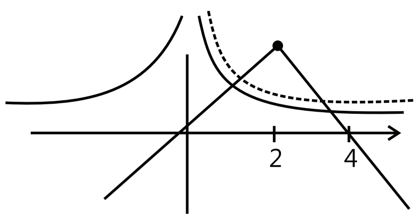
$$= 2\left[\frac{9\sin^{-1}\frac{x}{3}+x\sqrt{9-x^2}}{2}\right]_0^{3/2} -$$

$$2\left[\frac{9\sin^{-1}\frac{x-3}{3}+(x-3)\sqrt{6x-x^2}}{2}\right]_0^{3/2}$$

$$= 9.\frac{\pi}{6}+\frac{3}{2}\sqrt{9-\frac{9}{4}}+9.\frac{\pi}{6}+\frac{3}{2}\sqrt{9-\frac{9}{4}}$$

$$= 9.\frac{\pi}{3}+3\times 3\frac{x\sqrt{3}}{2} = 3\frac{\pi}{6}+\frac{9\sqrt{3}}{2} = 3\left[\pi+\frac{3\sqrt{3}}{2}\right]$$

Sol 4: (B)



$$\text{Area} = \int_{\sqrt{3}}^{2}\left(x-\frac{3}{x}\right)dx+\int_{2}^{3}\left(4-x-\frac{3}{x}\right)dx$$

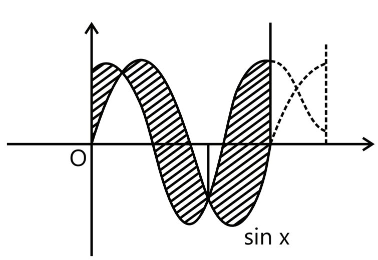
$$= \left[\frac{x^2}{2}-3\ell nx\right]_{\sqrt{3}}^{2}+\left[4x-\frac{x^2}{2}-3\ell nx\right]_2^3$$

$$= 2-3\,\ell\,n2-\frac{3}{2}+3\,\ell\,n\sqrt{3}+12-\frac{9}{2}-3\,\ell\,n3-8$$

$$+2+3\,\ell\,n2$$

$$= \frac{1}{2}+\frac{15}{2}-6+3\ell n\frac{1}{\sqrt{3}} = 2-\frac{3}{2}\ell n3 = \frac{4-3\ell n3}{2}$$

Sol 5: (D)



Area =

$$\int_0^{\pi/4}(\cos x-\sin x)\,dx+\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)\,dx$$

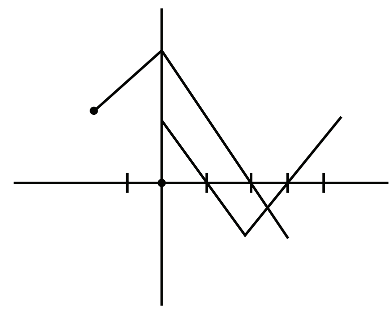
$$+\int_{5\pi/4}^{2\pi}(\cos x-\sin x)\,dx$$

$$= [\sin+\cos]_0^{\pi/4}+[\sin+\cos]_{5\pi/4}^{2\pi}+$$

$$[-\sin-\cos]_{\pi/4}^{5\pi/4}$$

$= \sqrt{2}-1+1+\sqrt{2}-[-\sqrt{2}-\sqrt{2}] = 4\sqrt{2}$ sq. units

Sol 6: (C)



$f(x) = -1 + |x-2|$

$g(x) = 2 - |x|$

$f(x) = 1 - x \in (0, 2)$

$x - 3 \in (2, 4)$

$g(x) = 2 - x \in (0, 3)$

$2 + x \in (-1, 0)$

$g(f(x)) = 2 - f(x)\ f(x) \in (0, 3)$

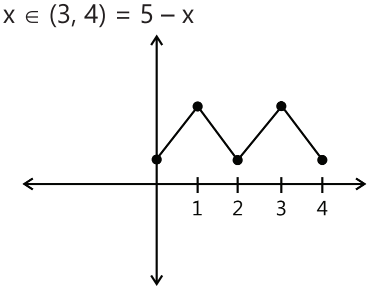
$= 2 + f(x) f(x) \in (-1, 0)$

$= 2 - (1-x),\ x \in (0, 1) = 1 + x$

$= 2 + (1-x),\ x \in (1, 2) = 3 - x$

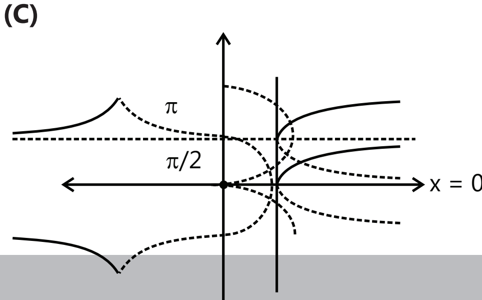
$= 2 + (x-3),\ x \in (2, 3) = x - 1$

$= 2 - (x-3),\ x \in (3, 4) = 5 - x$



$$\text{Area} = \frac{3}{2}[1+2]\times 1 = \frac{9}{2}$$

Sol 7: (C)

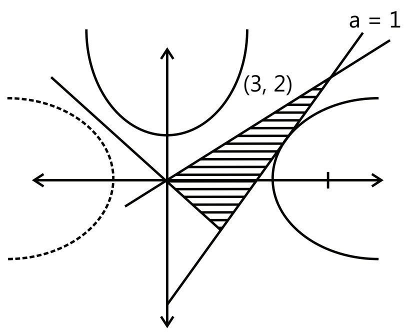


$$\text{Area} = 2\int_0^{\pi/4}(\sec x - 1)\,dx$$

$$= 2\left[\ell n(\sec x + \tan x)\right]_0^{\pi/4} - \frac{\pi}{2}$$

$$= 2\ln\left(\sqrt{2}+1\right) - \frac{\pi}{2} \quad = \ln\left(3+2\sqrt{2}\right) - \frac{\pi}{2}$$

Sol 8: (A)



Eq. of tangent

$$\frac{y-2}{x-3} = \frac{2\times 3}{2\sqrt{4}} = \frac{3}{2} \Rightarrow 2y - 4 = 3x - 9$$

$$3x - 2y = 5 \qquad \ldots \text{(i)}$$

$$A = \frac{1}{2}\times 2\times 1 + \int_1^5\left(x - \left(\frac{3x-5}{2}\right)\right)dx$$

$$= 1 + \left[\frac{-x^2+10x}{4}\right]_1^5 = 1 + \frac{1}{4}(16) = 5$$

Sol 9: (A) $y = 1 + 4x - x^2$

$\Rightarrow y - 5 = -(x-2)^2$

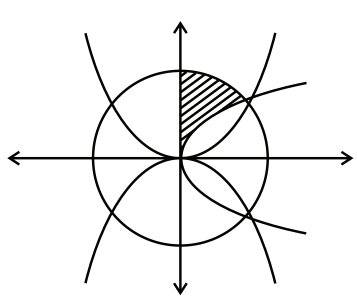
$$\text{Area} = \int_0^{3/2} 1 + 4x - x^2$$

$$= \left[x + 2x^2 - \frac{x^3}{3}\right]_0^{3/2} = \frac{3}{2} + 2\times\frac{9}{4} - \frac{27}{24} = 6 - \frac{9}{8} = \frac{39}{8}$$

$$\text{Area of } \Delta = \frac{1}{2}\times\frac{3}{2}\times\frac{3m}{2} = \frac{1}{2}\times\frac{39}{8}$$

$$9m = \frac{39}{2} \Rightarrow m = \frac{13}{6}$$

Sol 10: (A)



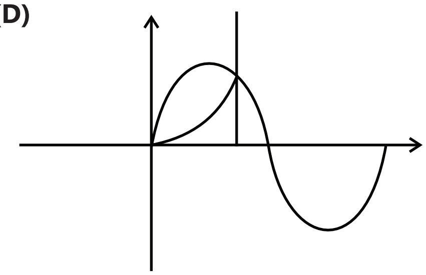
$$\text{Area}_1 = \int_0^1 \sqrt{x}\,dx = \left[\frac{2x^{3/2}}{3}\right]_0^1 = \frac{2}{3}$$

$$\text{Area}_2 = \int_0^1\left(\sqrt{2-x^2}\right)dx$$

$$= 2\left[\frac{\sin^{-1}\frac{x}{\sqrt{2}} + x\sqrt{2-x^2}}{2}\right]_0^1 = \frac{\pi}{4} + 1$$

$$\text{Area} = \frac{\pi}{4} + 1 - \frac{2}{3} = \left(\frac{\pi}{4} + \frac{1}{3}\right)$$

Sol 11: (D)



$$A_1 = \int_0^1 \sin\frac{\pi x}{2}\,dx = -\frac{2}{\pi}\left[\cos\frac{\pi x}{2}\right]_0^1 = -\frac{2}{\pi}(-1) = \frac{2}{\pi}$$

$$A_2 = \int_0^1 x^2\,dx = \frac{1}{3}$$

$$\text{Ratio} = \frac{\frac{1}{3}}{\frac{2}{\pi} - \frac{1}{3}} = \frac{6-\pi}{\pi}$$

Sol 12: (D) $A = \int\left(y^2 - 1 - |y|\sqrt{1-y^2}\right)dy$

$$= \int_0^1\left(y^2 - 1 - y\sqrt{1-y^2}\right)dy + \int_{-1}^0\left(y^2 - 1 + y\sqrt{1-y^2}\right)dy$$

$$= \left[\frac{y^3}{3} - y + \frac{\left(\sqrt{1-y^2}\right)^{3/2}}{2\left(\frac{3}{2}\right)}\right]_0^1 + \left[\frac{y^3}{3} - y - \frac{\left(\sqrt{1-y^2}\right)^{3/2}}{3}\right]_{-1}^0$$

$$= \frac{1}{3} - 1 - \left(0 - 0 + \frac{1}{3}\right) + \left(0 - 0 - \frac{1}{3} - \left(-\frac{1}{3} + 1\right)\right)$$

$$= -1 + \frac{1}{3} - \frac{1}{3} - 1 = -2; \quad \text{Area} = 2$$

Sol 13: (B) $f(x) = 3x^3 + 2x$

$g(x) = f^{-1}(x)$

$$\int_0^5 g(x) = \int_0^1 f(x) = \left[\frac{3x^4}{4} + x^2\right]_0^1 = \left[\frac{3}{4} + 1\right] = \frac{7}{4}$$

Previous Years' Questions

Sol 1: (B) Required area $= \int_0^{\pi/4}\left(\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}}\right)dx$

$= \left(\because \frac{1+\sin x}{\cos x} > \frac{1-\sin x}{\cos x} > 0\right)$

$$= \int_0^{\pi/4}\left(\sqrt{\frac{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}} - \sqrt{\frac{1-\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}}\right)dx$$

$$= \int_0^{\pi/4}\left(\sqrt{\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}} - \sqrt{\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}}\right)dx$$

$$= \int_0^{\pi/4}\frac{1+\tan\frac{x}{2} - 1 + \tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}dx = \int_0^{\pi/4}\frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}dx$$

Substitute $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$$= \int_0^{\tan\frac{\pi}{8}}\frac{4t\,dt}{(1+t^2)\sqrt{1-t^2}}$$

As $\tan\frac{\pi}{8} = \sqrt{2} - 1$

So, $\int_0^{\sqrt{2}-1}\frac{4t\,dt}{(1+t^2)\sqrt{1-t^2}}$

Sol 2: (B) Here, area between 0 to b is R_1 and b to 1 to R_2.

$$\therefore \int_0^b (1-x)^2 dx - \int_b^1 (1-x)^2 dx = \frac{1}{4}$$

$$\Rightarrow \left(\frac{(1-x)^3}{-3}\right)_0^b - \left(\frac{(1-x)^3}{-3}\right)_b^1 = \frac{1}{4}$$

$$\Rightarrow -\frac{1}{3}\{(1-b)^3 - 1\} + \frac{1}{3}\{0 - (1-b)^3\} = \frac{1}{4}$$

$$\Rightarrow -\frac{2}{3}(1-b)^3 = -\frac{1}{3} + \frac{1}{4} = -\frac{1}{12}$$

$$\Rightarrow (1-b)^3 = \frac{1}{8} \Rightarrow (1-b) = \frac{1}{2} \Rightarrow b = \frac{1}{2}$$

Sol 3: (C) $R_1 = \int_{-1}^2 x\, f(x)dx$ ... (i)

Using, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$R_1 = \int_{-1}^2 (1-x)f(1-x)dx,$

[given, f(x) = f(1 – x)]

$\therefore R_1 = \int_{-1}^2 (1-x)f(x)dx$ ... (ii)

Given, R_2 is area bounded by

f(x), x = –1 and x = 2

$\therefore R_2 = \int_{-1}^2 f(x)dx$ ... (iii)

Adding Eqs. (i) and (ii), we get

$2R_1 = \int_{-1}^2 f(x)dx$ ... (iv)

$\therefore$ From Eqs. (iii) and (iv), we get

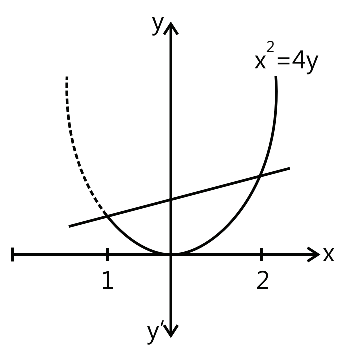
$2R_1 = R_2$

Sol 4: The point of intersection of the curves $x^2 = 4y$ and $x = 4y - 2$ could be sketched as, are x = –1 and x = 2.

$\therefore$ Required area

$$= \int_{-1}^2\left\{\left(\frac{x+2}{4}\right) - \left(\frac{x^2}{4}\right)\right\}dx = \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^2$$

$$= \frac{1}{4}\left[\left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)\right]$$

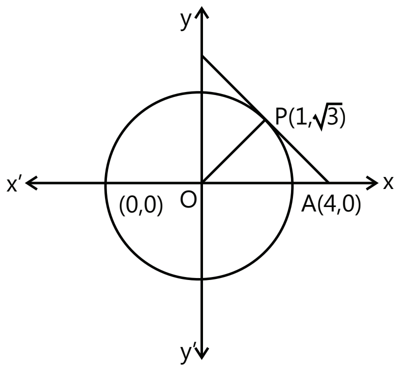


$$= \frac{1}{4}\left[\frac{10}{3} - \left(\frac{-7}{6}\right)\right] = \frac{1}{4}.\frac{9}{2} = \frac{9}{8} \text{ sq unit.}$$

Sol 5: Here, $\int_2^a \left(1 + \frac{8}{x^2}\right)dx = \int_a^4 \left(1 + \frac{8}{x^2}\right)dx$

$$\Rightarrow \left[x - \frac{8}{x}\right]_2^a = \left[x - \frac{8}{x}\right]_a^4$$

$$\Rightarrow \left(a - \frac{8}{a}\right) - (2-4) = (4-2) - \left(a - \frac{8}{a}\right)$$



Thus, area of Δ formed by (0, 0) $(1, \sqrt{3})$ and (4, 0)

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & \sqrt{3} & 1 \\ 4 & 0 & 1 \end{vmatrix} = \frac{1}{2}(0 - 4\sqrt{3}) = 2\sqrt{3} \text{ sq unit}$$

$$\Rightarrow a - \frac{8}{a} + 2 = 2 - a + \frac{8}{a}$$

$$\Rightarrow 2a - \frac{16}{a} = 0$$

$\Rightarrow 2(a^2 - 8) = 0$

$\therefore$ a = $\pm 2\sqrt{2}$, (neglecting –ve sign)

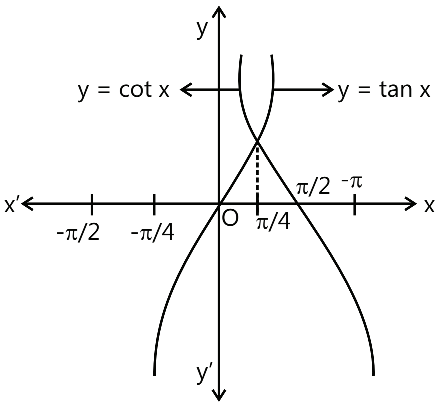
$a = 2\sqrt{2}$.

Sol 6: Given, $y = \begin{cases} \tan x, & -\frac{\pi}{3} \le x \le \frac{\pi}{3} \\ \cot x, & \frac{\pi}{6} \le x \le \frac{\pi}{2} \end{cases}$

which could be plotted as, y-axis.

$\therefore$ Required area $= \int_0^{\pi/4} (\tan x)dx + \int_{\pi/4}^{\pi/3} (\cot x)dx$

$$= \left[-\log|\cos x|\right]_0^{\pi/4} + \left[\log \sin x\right]_{\pi/4}^{\pi/3}$$

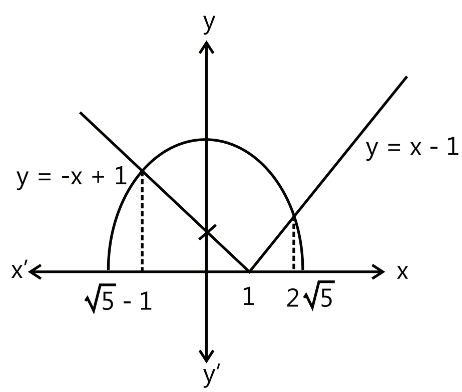


$$= \left(\log\frac{1}{\sqrt{2}} - 0\right) + \left(\log\frac{\sqrt{3}}{2} - \log\frac{1}{\sqrt{2}}\right)$$

$$= \log\frac{\sqrt{3}}{2} - 2\log\frac{1}{\sqrt{2}} = \log\frac{\sqrt{3}}{2}$$

$$\Rightarrow -\log\frac{1}{2} = \frac{1}{2}\log_e 3 \text{ sq. units}$$

Sol 7: Given curves $y = \sqrt{5 - x^2}$ and y = |x – 1| could be sketche as shown whose point of intersection are.



$5 - x^2 = (x-1)^2$

$\Rightarrow 5 - x^2 = x^2 - 2x + 1$

$\Rightarrow 2x^2 - 2x - 4 = 0$

$\Rightarrow x = 2, -1$

$\therefore$ Required area

$$= \int_{-1}^{2} \sqrt{5 - x^2}\,dx - \int_{-1}^{1} (-x+1)dx - \int_1^2 (x-1)dx$$

$$= \left[\frac{x}{2}\sqrt{5 - x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right]_{-1}^{2} - \left[\frac{-x^2}{2} + x\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_1^2$$

$$= \left(1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right) - \left(-1 + \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right)$$

$$-\left(-\frac{1}{2}+1+\frac{1}{2}+1\right)-\left(2-2-\frac{1}{2}+1\right)$$

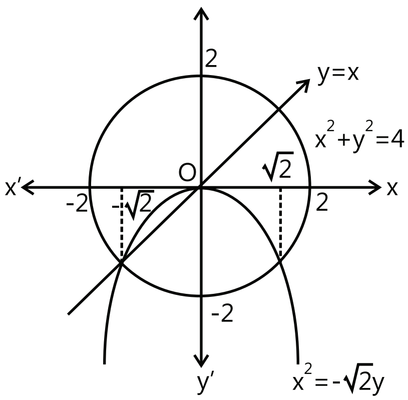
$$= \frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}}\right)-\frac{1}{2}$$

$$= \frac{5}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\sqrt{1-\frac{1}{5}}+\frac{1}{\sqrt{5}}\sqrt{1-\frac{4}{5}}\right)-\frac{1}{2}$$

$$=\frac{5}{2}\sin^{-1}(1)-\frac{1}{2}=\frac{5\pi}{4}-\frac{1}{2} \text{ sq. units}$$

Sol 8: Given curves are $x^2 + y^2 = 4$, $x^2 = -\sqrt{2}y$

Thus, the required area



$$= \left|\int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{4-x^2}\,dx\right|-\left|\int_{-\sqrt{2}}^{0}x\,dx\right|-\left|\int_{0}^{\sqrt{2}}\frac{-x^2}{\sqrt{2}}dx\right|$$

$$= 2\int_0^{\sqrt{2}}\sqrt{4-x^2}dx-\left|\left(\frac{x^2}{2}\right)_{-\sqrt{2}}^{0}\right|-\left|\frac{x^3}{3\sqrt{2}}\right|_0^{\sqrt{2}}$$

$$= 2\left\{\frac{x}{2}\sqrt{4-x^2}-\frac{4}{2}\sin^{-1}\frac{x}{2}\right\}_0^{\sqrt{2}}-1-\frac{2}{3}$$

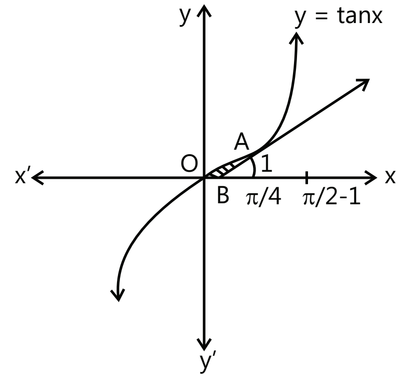
$$= (2-\pi)-\frac{5}{3}=\frac{1}{3}-\pi \text{ sq. units}$$

Sol 9: $y = \tan x \Rightarrow \frac{dy}{dx}=\sec^2 x$

$$\therefore \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}}=2$$

Hence, equation of tangent at $A\left(\frac{\pi}{4},1\right)$ is

$$\frac{y-1}{x-\pi/4}=2 \Rightarrow y-1=2x-\frac{\pi}{2}$$



$$\Rightarrow (2x-y) = \left(\frac{\pi}{2}-1\right)$$

$\therefore$ Required area is OABO

$$= \int_0^{\pi/4}(\tan x)dx - \text{area of } \Delta ALM$$

$$= \left[\log|\sec x|\right]_0^{\pi/4}-\frac{1}{2}BL\,.\,AL$$

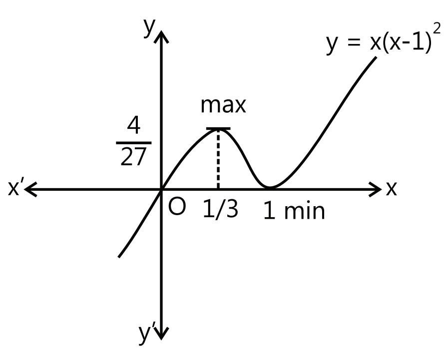
$$= \log\sqrt{2}-\frac{1}{2}\left(\frac{\pi}{2}-\frac{\pi-2}{4}\right).1$$

$$= \left(\log\sqrt{2}-\frac{1}{4}\right) \text{ sq. unit}$$

Sol 10: $y = x(x-1)^2 \Rightarrow \frac{dy}{dx}=x\,.\,2(x-1)+(x-1)^2$

$\therefore$ Maximum at $x = 1/3 \Rightarrow y_{max}=\frac{1}{3}\left(-\frac{2}{3}\right)^2=\frac{4}{27}$

Minimum at x = 1

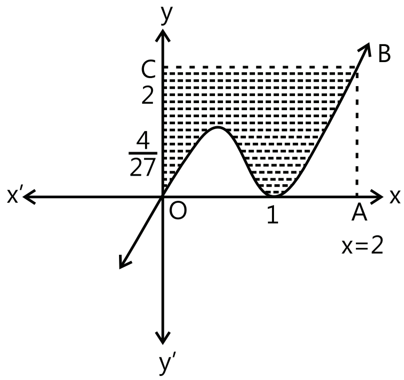


$$= (x-1)(2x+x-1)$$
$$= (x-1)(3x-1)$$

+	–	+
	1/3	1

$$\Rightarrow \qquad y_{min} = 0$$

Now, to find the area bounded by the curve $y = x(x - 1)^2$, the y-axis and line x = 2



$\therefore$ Required area

$= \text{Area of square OABC} - \int_0^2 y\, dx$

$= 2 \times 2 - \int_0^2 x(x-1)^2 dx = 4 - \int_0^2 \left(x^3 - 2x^2 + x\right) dx$

$= 4 - \left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right]_0^2 = 4 - \left[\frac{16}{4} - \frac{16}{3} + \frac{y}{2}\right]$

$= \frac{10}{3}$ sq. units

Sol 11: Both the curves are defined for x > 0. Both are positive when x > 1 and negative when 0 < x < 1

We know, $\lim_{x \to 0^+} (\log x) \to -\infty$

Hence, $\lim_{x \to 0^+} \frac{\log x}{ex} \to -\infty$, This, y-axis is asymptote of second curve.

And $\lim_{x \to 0^+}$ ex log x [(0) x ∞ form]

$= \lim_{x \to 0^+} \frac{e \log x}{1/x}\left(-\frac{\infty}{\infty} \text{form}\right) = \lim_{x \to 0^+} \frac{e\left(\frac{1}{x}\right)}{\left(-\frac{1}{x^2}\right)} = 0$

(using L'Hospital's rule)

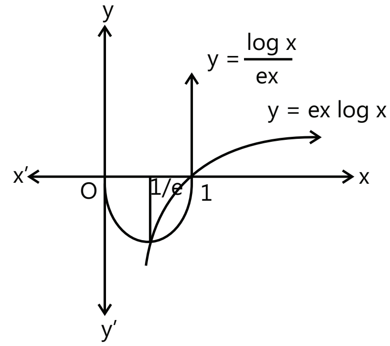
Thus, the first curve starts from (0, 0) but does not include (0, 0).

Now, the given curves intersect, therefore

$ex \log x = \frac{\log x}{ex}$

i.e. $(e^2x^2 - 1) \log x = 0$

$\Rightarrow x = 1, \frac{1}{e}$ (since x > 0)



$\therefore$ The required area $= \int_{1/e}^{1} \left(\frac{\log x}{ex} - ex \log x\right) dx$

$= \frac{1}{e}\left[\frac{(\log x)^2}{2}\right]_{1/e}^{1} - e\left[\frac{x^2}{4}(2\log x - 1)\right]_{1/e}^{1} = \frac{e^2 - 5}{4e}$ sq. units

Sol 12: Given, $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$

$\Rightarrow 4a^2 f(-1) + 4a f(1) + f(2) = 3a^2 + 3a,$ ... (i)

$4b^2 f(-1) + 4b f(1) + f(2) = 3b^2 + 3b$ ... (ii)

and $4c^2 f(-1) + 4cf(1) + f(2) = 3c^2 + 3c$ ... (iii)

Where f(x) is quadratic expression given by,

$f(x) = ax^2 + bc + c$ and (i), (ii) and (iii)

$\Rightarrow 4x^2 f(-1) + 4x f(1) + f(2) = 3x^2 + 3x$

or $\{4 f(-1) - 3\}x^2 + \{4 f(1) - 3\} x + f\{2\} = 0$ ... (iv)

As above equation has 3 roots a, b an c

$\therefore$ above equation is identity in x.

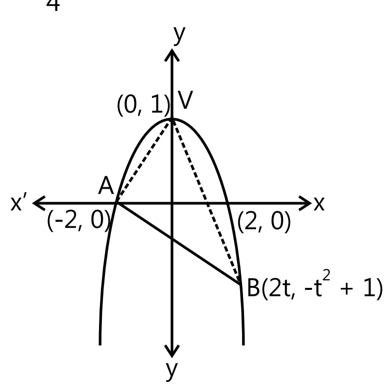
i.e., Coefficients must be zero.

$\Rightarrow f(-1) = 3/4, f(1) = 3/4, f(2) = 0$ ... (v)

$\because f(x) = ax^2 + bx + c$

$\therefore a = -1/4, b = 0$ and $c = 1$, using Eq. (v)

Thus, $f(x) = \frac{4 - x^2}{4}$ shown as,



Let $A(-2,0), B=(2t,-t^2+1)$ v

Since, AB subtends right angle at vertex V(0, 1)

$$\Rightarrow \frac{1}{2}.\frac{-t^2}{2t} = -1 \Rightarrow t = 4$$

$\therefore B(8, -15)$

Equation of chord AB is

$$y = \frac{-(3x+6)}{2}$$

$\therefore$ Required area

$$= \left|\int_{-2}^{8}\left(\frac{4-x^2}{4}+\frac{3x+6}{2}\right)dx\right|$$

$$= \left|\left(x-\frac{x^3}{12}+\frac{3x^2}{4}+3x\right)_{-2}^{8}\right|$$

$$= \left|\left[8-\frac{128}{3}+48+24-\left(-2+\frac{2}{3}+3-6\right)\right]\right|$$

$$= \frac{125}{3} \text{ sq. units}$$

Sol 13: Here, slope of tangent

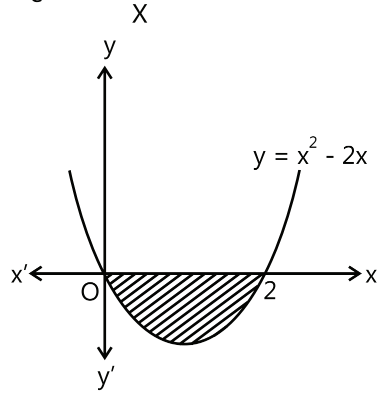
$$\frac{dy}{dx} = \frac{(x+1)^2+y-3}{(x+1)} \Rightarrow \frac{dy}{dx} = (x+1)+\frac{(y-3)}{(x+1)}, X$$

Substitute x + 1 = X and y – 3 = Y

$$\Rightarrow \frac{dy}{dx} = \frac{dY}{dX}$$

$$\therefore \frac{dY}{dX} = X+\frac{Y}{X} \Rightarrow \frac{dY}{dX}-\frac{1}{X}Y = X$$

$$\text{I.F} = e^{\int -\frac{1}{x}dX} = e^{-\log X} = \frac{1}{X}$$



$\therefore$ Solution is,

$$Y.\frac{1}{X} = \int X.\frac{1}{X}dX + c \Rightarrow \frac{Y}{X} = X + c$$

$y - 3 = (x + 1)^2 + c(x + 1)$, which passes through (2, 0).

$\Rightarrow -3 = (3)^2 + 3c$

$\Rightarrow c = -4$

$\therefore$ Required curve $y = (x + 1)^2 - 4(x + 1) + 3$

$\Rightarrow y = x^2 - 2x$

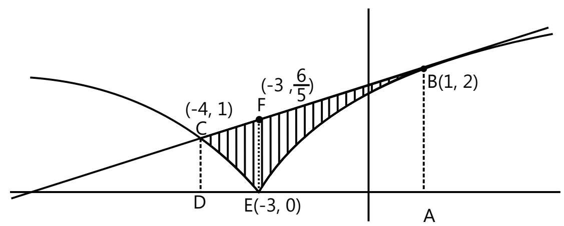
$$\therefore \text{Required area} = \left|\int_0^2 (x^2-2x)dx\right| = \left|\left(\frac{x^3}{3}-x^2\right)_0^2\right|$$

$$= \frac{8}{3} - 4 = \frac{4}{3} \text{ sq. units}$$

Sol 14:

$$\text{Region} = \left\{(x,y) \in R^2 : y \ge \sqrt{|x+3|}, 5y \le x+9 \le 15\right\}$$

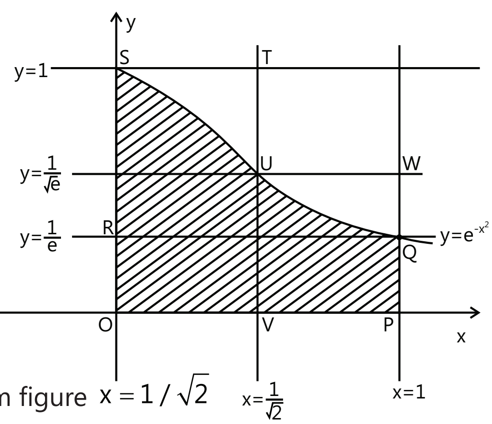
Plotting all the curves



$$\text{Area} = \text{Area}(ABFE) - \text{Area}(AEB)$$
$$+ \text{Area}(DEFC) - \text{Area}(DEC)$$

$$= \frac{32}{5} - \int_{-3}^{1}\left(\sqrt{-x-3}\right)dx + \frac{11}{10} - \int_{-4}^{-3}\left(\sqrt{x+3}\right)dx = \frac{3}{2} \text{ sq. units}$$

Sol 15: (A, B, D)



Form figure $x = 1/\sqrt{2}$

S > Area $(OPQR)$

$$\Rightarrow S > 1 \times \frac{1}{e} \Rightarrow S > \frac{1}{e}$$

S > Area $(PVUW)$ + Area $(OSTV)$

$< \left(1-\frac{1}{12}\right)\frac{1}{\sqrt{e}} + 1\times\frac{1}{\sqrt{2}}$

$< \frac{1}{12} + \frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$ D is correct

Now, $e^{-x} \le e^{-x^2}$ if $x \in (0, 1)$

$\int_0^1 e^{-x}dx \le \int_0^1 e^{-x^2}dx \Rightarrow \left(1-\frac{1}{e}\right) \le S$ B is correct

$1-\frac{1}{e} > \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ and $S > 1-\frac{1}{e}$

$\Rightarrow S > \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$

(B) and (D) Correct