PROBLEM SOLVING TACTICS

Think briefly about whether you could easily separate the variables or not. Remember that means getting all the x terms (including dx) on one side and all the y terms (including dy) on the other. Don't forget to convert y' to dy/dx or you might make a mistake.

If it's not easy to separate the variables (usually it isn't) then we can try putting our equation in the form y' + P(x)y = Q(x).

In other words, put the y' term and the y term on the left and then you may divide so that the coefficient of y' is 1.

Then we can use the trick of the integrating factor in which we multiply both sides by . $d\left(\frac{e^x}{x}\right)=\frac{xe^y dy-e^y dx}{x^2}$. This makes things much simpler, but it's best to see why from doing problems, not from memorizing formulas.

FORMULAE SHEET

(a) Order of differential equation: Order of the highest derivative occurring in the differential equation

(b) Degree of differential equation: Degree of the highest order derivative when differential coefficients are free from radicals and fractions.

(c) General equation : $\frac{dy}{dx}=f(x)\Rightarrow \quad y=\int f(x)dx+c$

(d) $\frac{dy}{dx}=f(ax+by+c)$, then put ax + by + c = v

(e) If $\frac{dy}{dx}=f(x)g(y) \;\Rightarrow\; g(y)^{-1}dy=f(x)dx$ then $\int(g(y))^{-1}dy=\int f(x)dx$

(f) Parametric forms

Case I: $x = r\cos\theta$, $y = r\sin\theta \Rightarrow x^2 + y^2 = r^2$; $\tan\theta = \frac{y}{x}$; $xdx + ydy = rdr$; $xdy - ydx = r^2d\theta$

Case II: $x = r\sec\theta$, $y = r\tan\theta \Rightarrow x^2 - y^2 = r^2$; $\frac{y}{x} = \sin\theta$; $xdx - ydy = rdr$; $xdy - ydx = r^2\sec\theta d\theta$

(g) If $\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}$, then substitute y = vx $\Rightarrow \int\frac{dx}{x}=\int\frac{dv}{f(v)-v}+c$

(h) If $\frac{dv}{dx}=\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$, then substitute x = X + h, y = Y + k

$$\Rightarrow \frac{dY}{dX}=\frac{a_1X+b_1Y+(a_1h+b_1k+c_1)}{a_2X+b_2Y+(a_2h+b_2k+c_2)}$$

choose h and k such that $a_1h + b_1k + c_1 = 0$ and $a_2h + b_2k + c_2 = 0$.

(i) If the equation is in the form of $\frac{dy}{dx}$ + Py = Q then $ye^{\int Pdx}=\int Qe^{\int Pdx}+c$