16. GEOMETRICAL OPTICS

1. INTRODUCTION

Light is a form of radiant energy; that is, energy is emitted by the excited atoms or molecules that can cause the sensation of vision by a normal human eye.

The branch of physics that deals with the phenomena of light is called optics. There are two branches of optics: (a) ray optics and (b) wave optics.

SOME DEFINITIONS

(a) Ray: The 'path' along which the light travels is called a ray. The rays are represented by straight lines with arrows directed towards the direction of travel of light.

(b) Beam: A bundle of rays is called a beam. A beam is parallel when its rays are parallel, it is divergent when its rays spread out from a point, and it is convergent when its rays meet at a point.

Object and image

If the rays from a point on an object actually diverge from it and fall on the mirror, then the object is the real object of the mirror. If the rays incident on the mirror does not start from a point but appear to converge at a point, then that point is the virtual object of the mirror.

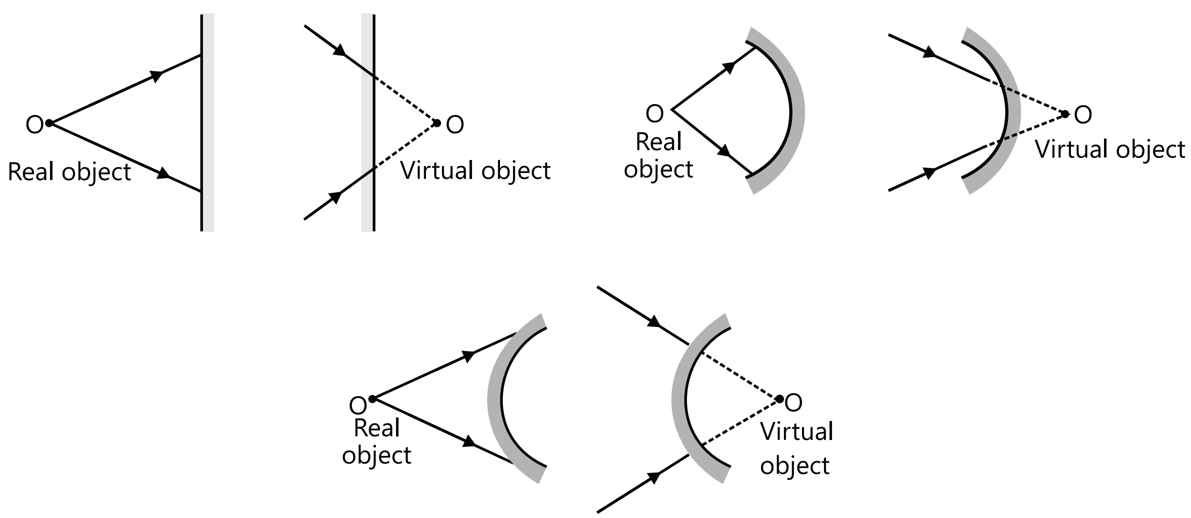


Figure 16.1

If the rays converge at a point after an interaction with a surface, then a real image will be formed, and if the rays diverge after an interaction with a surface, a virtual image will be formed.

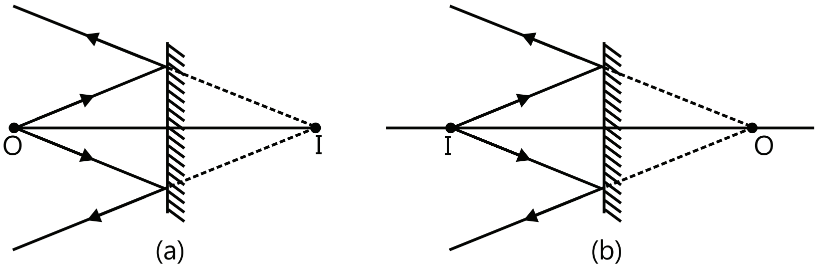


Figure 16.2

Real object, virtual object, real image, virtual image: In Fig. 16.2 (a), the object is real, while the image is virtual. In Fig. 16.2 (b), the object is virtual, while the image is real.

2. REFLECTION OF LIGHT

Definition

When the light falling on a surface turns back into the same medium, it means it is reflected. The angle made by the incident ray with the normal to the reflecting or refracting surface is called the angle of incidence, and the angle made by the reflected or refracted ray with normal is called the angle of reflection or refraction.

2.1 Laws of Reflection

(a) When the incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane, it is called the plane of incidence.

(b) The angle of incidence is equal to the angle of reflection $\boxed{\angle i = \angle r}$.

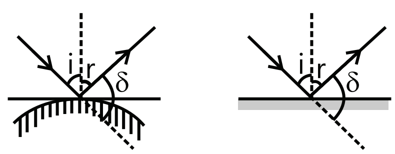


Figure 16.3

2.2 Deviation of Ray

The deviation is defined as the angle between the directions of the incident ray and the reflected ray (or the emergent ray). It is generally denoted by δ.

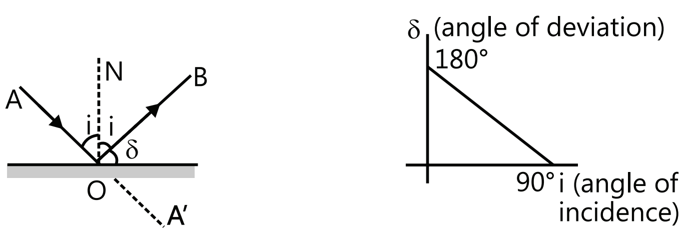


Figure 16.4

Here, $\angle A'OB = \delta = \angle AOA' - \angle AOB = 180^0 - 2i$

or $\delta = 180^0 - 2i$

MASTERJEE CONCEPTS

The above two laws of reflection can be applied to the reflecting surfaces that are not even horizontal. The following Fig. 16.5 illustrates this point.

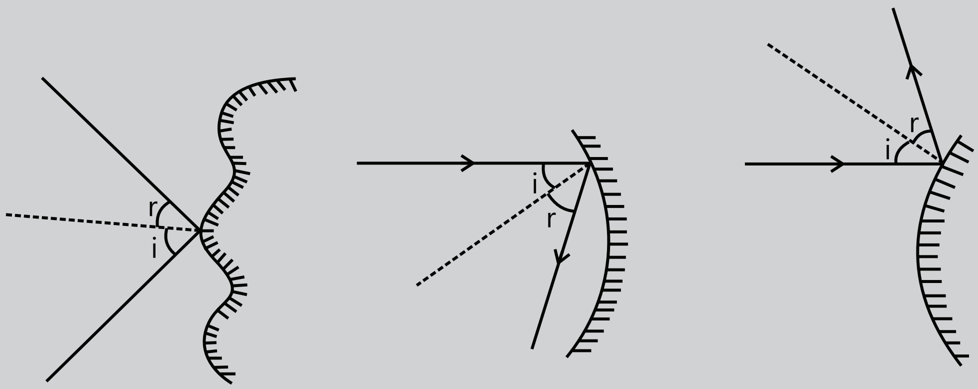


Figure 16.5

Vaibhav Gupta (JEE 2009 AIR 54)

3. REFLECTION FROM A PLANE SURFACE (OR PLANE MIRROR)

Almost everybody is familiar with the image formed by a plane mirror. If the object is real, the image formed by a plane mirror is virtual, erect, of same size of the original object and in the same distance from the mirror.

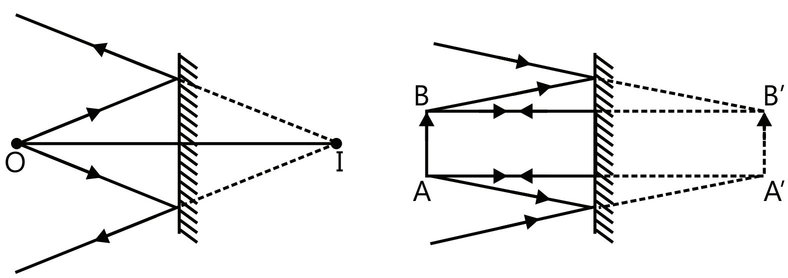


Figure 16.6

If an object is placed in front of a mirror as shown in Fig. 16.6, we get its image in the mirror due to the reflection of light.

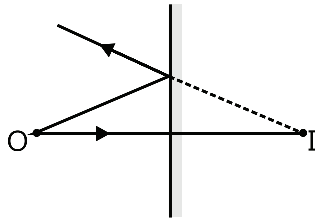


Figure 16.7

(a) The distance between the object and the mirror = the distance between the image and the mirror.

(b) The line joining the object point with its image is normal to the reflecting surface.

(c) The image is laterally inverted (left–right inversion).

(d) The size of the image is same as that of the object.

(e) For a real object, the image is virtual, and for a virtual object, the image is real.

(f) For a fixed incident light ray, if the mirror is rotated by an angle θ, the reflected ray turns through an angle 2θ. If plane mirror is rotated through about an axis perpendicular to plane of mirror, then the reflected ray image spot does not rotate.

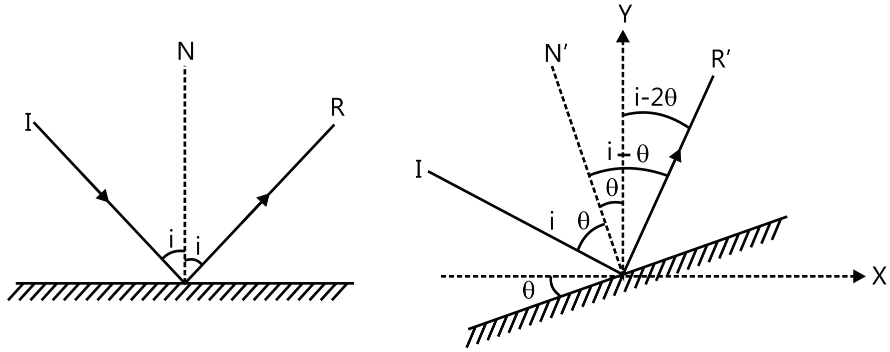


Figure 16.8

(g) The minimum size of a plane mirror required to see the full-size image of a person by himself is half the size of that person.

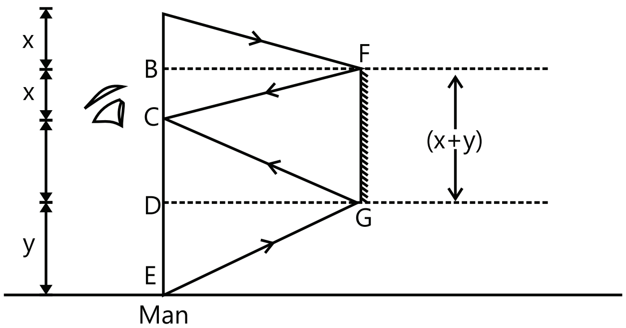


Figure 16.9

(h) A plane mirror behaves like a window to the virtual world.

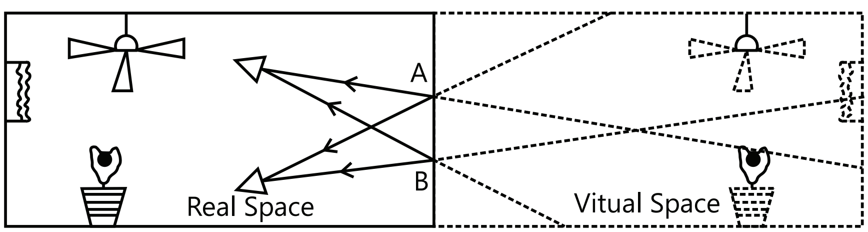


Figure 16.10

MASTERJEE CONCEPTS

To find the location of the image of an object from an inclined plane mirror, you have to see the perpendicular distance of that object from the mirror.

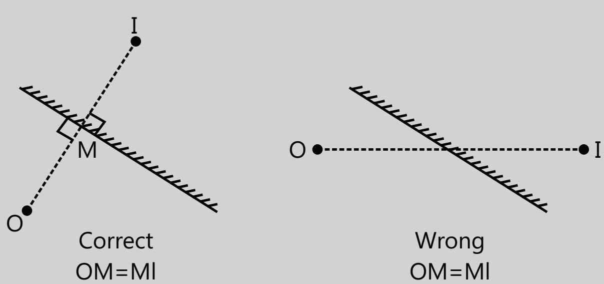


Figure 16.11

Vaibhav Krishnan (JEE 2009 AIR 22)

Illustration 1: A point source of light S, placed at a distance *L* in front of the center of a mirror of width *d*, hangs vertically on the wall. Assume that a man walks in front of the mirror along a line parallel to the mirror at a distance 2*L* from it as shown in the Fig. 16.12. The greatest distance over which he can see the image of the light source in the mirror is

(A) d/2 (B) d (C) 2d (D) 3d **(JEE MAIN)**

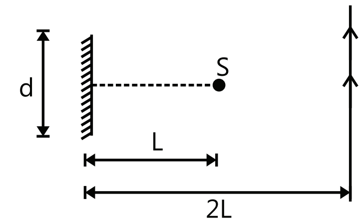


Figure 16.12

Sol: As the man is walking parallel to the mirror, the image of the point object S thus formed will also move relative to the man. We construct the ray diagram to obtain the position of the image from the man.

The ray diagram is shown in the Fig. 16.13.

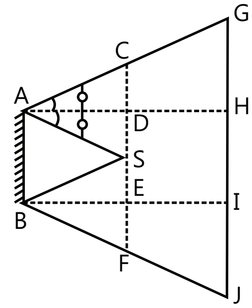


Figure 16.13

$HI = AB = d$; $DS = CD = \frac{d}{2}$

since $AH = 2AD$; $\therefore$ $GH = 2CD = 2\frac{d}{2} = d$

Similarly $IJ = d$; $\therefore$ $GJ = GH + HI + IJ$; $= d + d + d = 3d$

Illustration 2: Two plane mirrors M_1 and M_2 are inclined at an angle θ as shown in the Fig. 16.14. A ray of light 1, which is parallel to M_1, strikes M_2, and after two reflections, ray 2 becomes parallel to M_2. Find the angle θ.

(JEE MAIN)

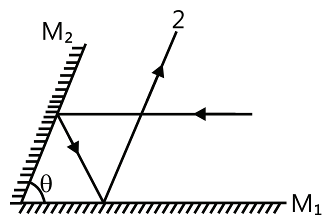


Figure 16.14

Sol: The angle of reflection is equal to angle of incidence about the normal. If ray makes angle α with the normal then the angle made with the surface is $\theta = 90 - \alpha$. Completing the ray diagram for multiple reflection we get the angle θ.

Different angles are as shown in the following Fig. 16.15. In triangle ABC,

$\theta + \theta + \theta = 180°$

$\theta = 60°$

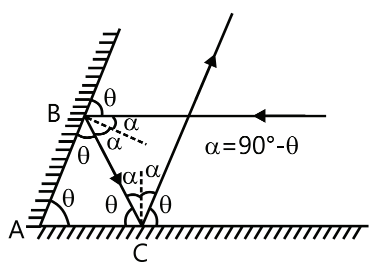


Figure 16.15

Illustration 3: A ray of light is travelling at an angle of 20° above the horizontal plane. At what angle with the horizontal plane must a plane mirror be placed in its path so that it becomes vertically upward after the reflection? **(JEE MAIN)**

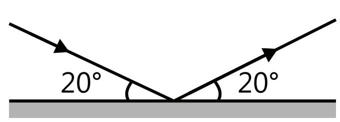


Figure 16.16

Sol: The angle of incidence and reflection are similar with respect to the normal. To make the ray reflect vertically upwards, we need to incline the mirror at an angle $\theta = \left(\frac{90 - i}{2}\right)$ where I is the angle of incidence.

$\Rightarrow \theta = \frac{90 - 20}{2} = 35°$

3.1 Velocity of Image Formed by a Plane Mirror

$X_{OM} \rightarrow$ *x* coordinate of the object relative to the mirror.

$X_{IM} \rightarrow$ *x* coordinate of the image relative to the mirror.

Differentiating, $\frac{dx_{IM}}{dt} = -\frac{dx_{OM}}{dt} \Rightarrow v_{IM} = -V_{OM}$

$\Rightarrow$ Velocity of the image relative to the mirror

= - velocity of the object relative to the mirror.

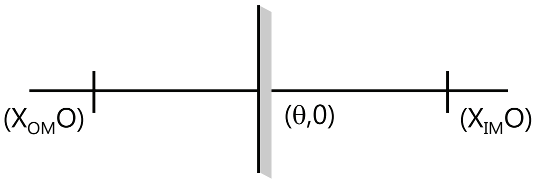


Figure 16.17

Illustration 4: Find the velocity of the image when the object and mirror both are moving toward each other with the velocities 2 m/s and 3 m/s, respectively. **(JEE MAIN)**

Sol: As both the object and mirrors are moving towards each other with a constant speed. The velocity of object with respect to the mirror and velocity of image with respect to the mirror are equal in magnitude but opposite in direction.

Here, $v_{OM} = -v_{IM}$

$$v_O - v_M = -(v_I - v_M)$$

$$\Rightarrow (+2m/s) - (-3m/s) = -v_I + (-3)$$

$$\Rightarrow v_I = -8m/s$$

3.2 Image Formation by Multiple Reflection

Case I: When the mirrors are parallel to each other Figure 16.18 shows an image formed by an object placed at a distance *y*, from M_1 and at a distance *x* from M_2. The number of images formed by the parallel plane mirrors is infinite.

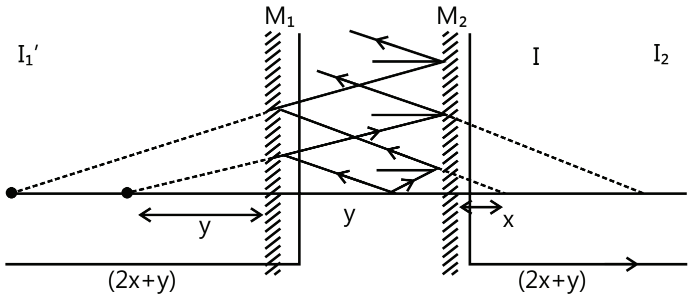


Figure 16.18

Case II: When the mirrors are inclined at an angle θ.

(a) All the images formed by the two mirrors lie on a circle with center C (an intersection point of the two mirrors). Here, if the angle between these two mirrors is θ, then an image will be formed on a circle at an angle $2\pi - \theta$. If the angle θ is less, the number of images formed is high.

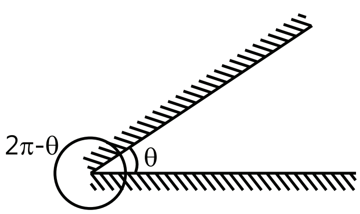


Figure 16.19

(b) If n is the number of image, then

If (i) $360^0/\theta$ is even
(ii) $360^0/\theta$ is odd and object is kept symmetrically } then $n = (360^0/\theta) - 1$

and $n = \left(\frac{360^0}{\theta}\right)$ for all other conditions.

MASTERJEE CONCEPTS

The number of images formed by two mutually perpendicular ($\theta=90^o$) mirrors is three. All these three images will lie on a circle with center at C, the point of intersection of mirrors M_1 and M_2, and whose radius is equal to the distance between C and object O.

In fact, whatever be the angle, all the images lie on a circle.

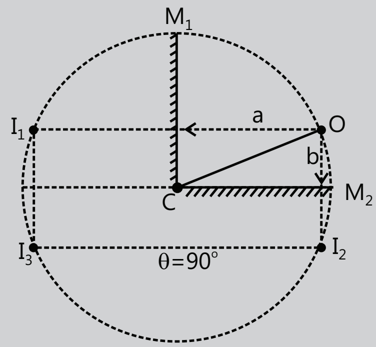
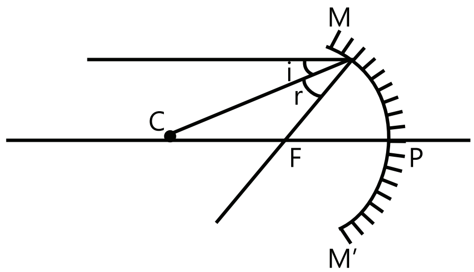


Figure 16.20

Vijay Senapathi (JEE 2011 AIR 71)

4. SPHERICAL MIRRORS

A spherical mirror is a smooth reflecting surface that forms a part of a spherical surface. If reflection takes place from the inner reflecting surface, then the mirror is called a concave mirror. If the reflection takes place from the outer surface, it is called a convex mirror. The reflection of a light from a concave and a convex mirror is shown in the Fig. 16.21.



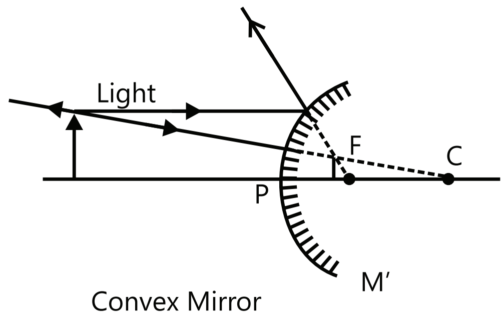


Figure 16.21

4.1 Parameters Associated with the Spherical Mirror

A pole (P) or vertex is the geometrical center of a reflecting surface. Center of curvature C is the center of the sphere, of which the mirror is a part. Radius of curvature R is equal to the distance between P and C of the mirror and is the radius of the sphere, of which the mirror is a part. The principal axis is the line CP that passes through P and C. If a ray of light is emitted from an object at infinity so that the beam of light is parallel to the principal axis, an image is formed at principal focus F after reflection. Focal length, f, is the distance between P and F along the principal axis. When a beam of light is incident parallel to the principal axis, the reflected rays converge on F in a concave mirror and diverge from F in a convex mirror after reflection. Aperture of a spherical mirror is the effective diameter MM′ of the light-reflecting area in the mirror. When the aperture of a mirror is small, the focal length is equal to half the radius of curvature.

4.2 Sign Convention

The following sign conventions based on coordinate geometry are used:

(a) The rays of light travel from the left to the right direction.

(b) All the distances measured from the pole and in the direction of light toward the right of the pole are positive. The distances measured in the opposite direction of light toward the left of the pole are negative.

(c) The transverse distances, above the principal axis, are positive and the principal axis are negative.

(d) If condition (1) in sign conventions is followed, this sign convention follows the right-hand Cartesian coordinate system.

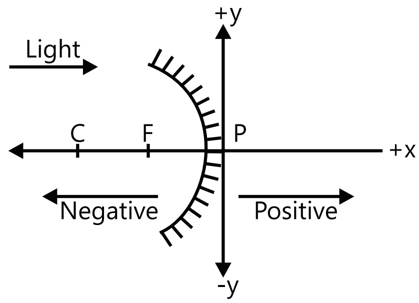


Figure 16.22

4.3 Rules for Image Formation

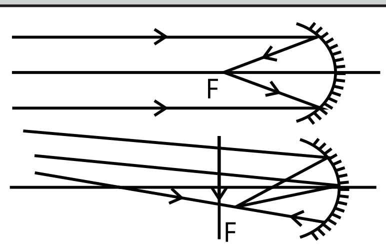
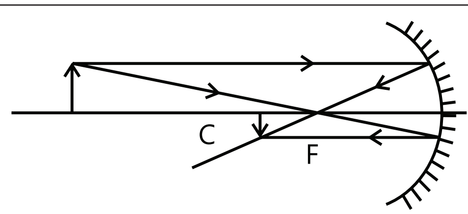
The following rules are used for locating the image of an object by considering the reflection of three types of rays based on laws of reflection:

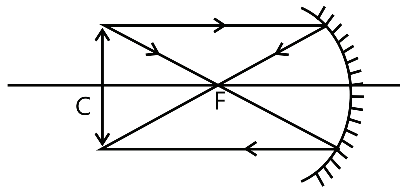
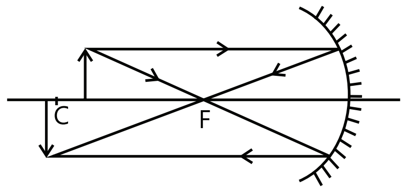
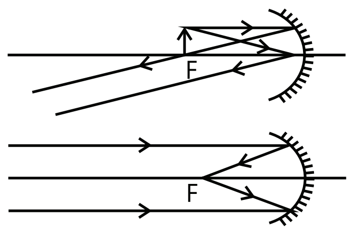
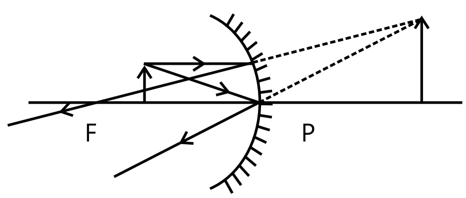
(a) A ray incident parallel to the principal axis will pass through the principal focus after reflection in the case of a concave mirror and will appear to originate from the focus in the case of a convex mirror.

(b) A ray that passes through the principal focus of a concave mirror, or that passes toward the principal focus of a convex mirror, travels parallel to the principal axis after reflection.

(c) A ray that passes through the center of curvature of a concave mirror or toward the center of curvature of a convex mirror is reflected from the mirror along the same path.

4.4 Image Formation by a Concave Mirror (for Real Object)

S. No.	Position of Object	Diagram	Position of Image	Nature of Image
1	Infinity	 **Figure 16.23**	At the principal focus F or in the focal plane Image problem	Real, inverted and diminished
2	Between infinity and C	 **Figure 16.24**	Between F and C	Real, inverted and diminished

S. No.	Position of Object	Diagram	Position of Image	Nature of Image
3	C	 **Figure 16.25**	C	Real, inverted and of same size as the object.
4	Between F and C	 **Figure 16.26**	Between C and infinity	Real, inverted and magnified
5	At F or in the focal plane	 **Figure 16.27**	At infinity	Real, inverted and highly magnified
6	Between F and P	 **Figure 16.28**	Behind the mirror	Virtual, erect and magnified.

4.5 Image Formation by a Convex Mirror

An image is formed between the pole and the focus for all the positions of the real object except when the objects are at infinity in which case the image is formed at F in the focal plane. The image formed is virtual, erect and diminished. The ray diagram for the formation of image I of object O after reflection from a convex mirror is shown in the Fig. 16.29.

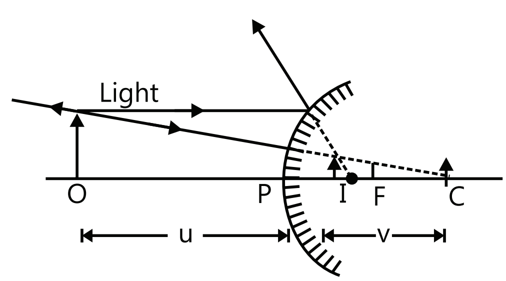


Figure 16.29

MASTERJEE CONCEPTS

Image formed by a convex mirror is always virtual, erect and diminished; no matter where the object is placed (except for virtual objects).

Anurag Saraf (JEE 2011 AIR 226)

4.6 Mirror Formula

If an object is placed at a distance *u* from the pole of a mirror, its image is formed at a distance *v* from the pole, and its focal length *f* is given by $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ where *f* = *R*/2 (only for paraxial rays).

According to the sign conventions, *f* and *R* are negative for a concave mirror and are positive for a convex mirror. The power of a mirror, *P*, measured in units of dioptres is given by

$$P=-\frac{1}{f}\text{ Where } f \text{ is in metres }=-\frac{100}{f}\text{ Where } f \text{ is in centimetres}$$

Illustration 5: A convex mirror has a radius of curvature of 20 cm. Find the position of the image of an object placed at a distance of 12 cm from the mirror. **(JEE MAIN)**

Sol: The position of the image is found using formula $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$.

According to sign convention, if the object is placed to the left side of the pole, object distance is considered to be negative.

The situation is shown in the Fig. 16.30. Here, *u* = −12 cm and *R* = +20 cm.

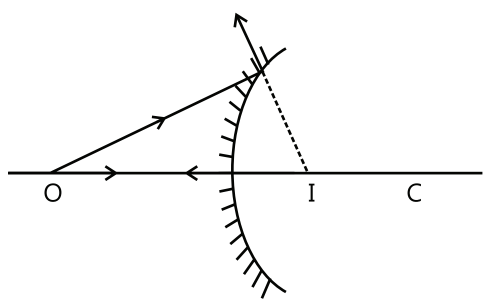


Figure 16.30

We have, $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$ or $\frac{1}{v}=\frac{2}{R}-\frac{1}{u}=\frac{2}{20\,cm}-\frac{1}{-12cm}=\frac{11}{60\,cm}$

$$\Rightarrow v=\frac{60}{11}cm.$$

The positive sign of *v* shows that the image is formed on the right side of the mirror and is a virtual image.

4.7 Magnification by Mirror

If an object of linear size *O* is placed vertically on the axis of a concave or convex mirror at a distance *u* from the pole and its image of size *I* is formed at a distance *v* from the pole, then the lateral or transverse magnification, *m*, is given by $m=\frac{I}{0}=-\left(\frac{v}{u}\right)$

A negative magnification indicates that the image is inverted with respect to the object, whereas a positive magnification implies that the image is erect with respect to the object.

Illustration 6: A concave and a convex mirror of focal lengths 10 cm and 15 cm, respectively, are placed at a distance of 70 cm from each other. An object AB of height 2 cm is placed at a distance of 30 cm from the concave mirror. First, a ray is incident on a concave mirror and then on the convex mirror. Find the size, position and nature of the image. **(JEE MAIN)**

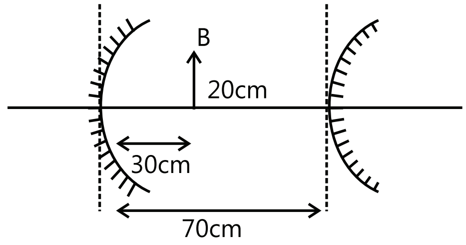


Figure 16.31

Sol: The position of the image is found using formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$.

The image formed by concave mirror is the object for the convex mirror.

For a concave mirror, $u=-30cm,\ \ f=-10cm$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ $\Rightarrow$ $\frac{1}{v}-\frac{1}{30}=\frac{-1}{10}$; $\Rightarrow$ $v=-15cm$

Now, $\frac{A'B'}{AB}=\frac{-v}{u}=\frac{-15}{-30}$; $\Rightarrow$ $A'B'=-1cm$

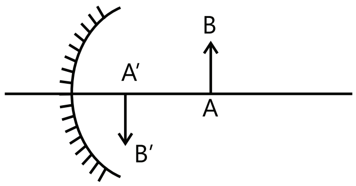


Figure 16.32

An image formed by the first reflection will be real, inverted and diminished.

For a convex mirror,

$u'=-55cm,\ f=+15cm$ Using $\frac{1}{v'}+\frac{1}{u'}=\frac{1}{f'}$

$\Rightarrow$ $\frac{1}{v'}-\frac{1}{55}=\frac{1}{15}$ $\Rightarrow$ $v'=165/14cm$

Now, $\frac{A''B''}{A'B'}=-\frac{v'}{u'}=-\frac{\left(\frac{165}{14}\right)}{(-55)}$ $\Rightarrow$ $A''B''=\left(\frac{3}{14}\right)(-1)=-0.2cm$

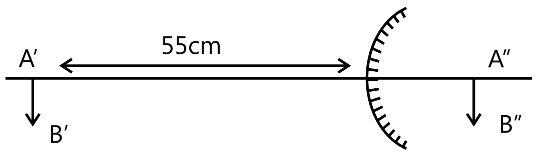


Figure 16.33

The final image will be virtual and diminished.

Illustration 7: An object ABED is placed in front of a concave mirror beyond the center of curvature C as shown in the Fig. 16.34. State the shape of the image. **(JEE ADVANCE)**

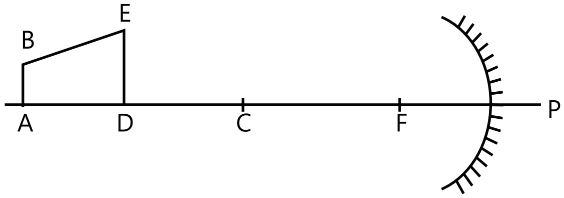


Figure 16.34

Sol: As the object is placed beyond center of the curvature, the image thus formed will lie between center of curvature and pole. The position of the image is found using formula

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

The object is placed beyond C. Hence, the image will be real and it will lie between C and F. Furthermore, *u*, *v* and *f* all are negative; hence, the mirror formula will become

$-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$ or $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{u-f}{uf}$ or $v=\frac{f}{1-\frac{f}{u}}$.

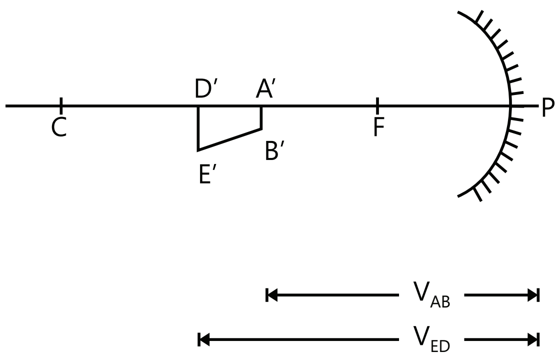


Figure 16.35

Now $u_{AB}>u_{ED}$; $\therefore$ $v_{AB}<v_{ED}$

and $|m_{AB}|<|m_{ED}|$ $\left(\text{as } m=\frac{-v}{u}\right)$.

Therefore, the shape of the image will be as shown in the Fig. 16.39. Also note that

$v_{AB}<u_{AB}$ and $v_{ED}<u_{ED}$, So, $|m_{AB}|<1$ and $|m_{ED}|<1$.

4.8 Relation between Object and Image Velocity

Differentiating equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow -\frac{1dv}{v^2dt}-\frac{1du}{u^2dt}=0$ $\Rightarrow -\frac{1}{v^2}V_{IM}-\frac{1}{u^2}V_{OM}=0$ $V_{IM}\Rightarrow$ velocity of image w.r.t. mirror

$\Rightarrow V_{IM}=-\frac{v^2}{u^2}V_{OM}$ $V_{OM}\Rightarrow$ velocity of object w.r.t. mirror

$\Rightarrow V_{IM}=-m^2V_{OM}$

Illustration 8: A mirror with a radius of curvature of 20 cm and an object that is placed at a distance 15 cm from the mirror both are moving with the velocities 1 m/s and 10 m/s as shown in the Fig. 16.36. Find the velocity of the image. **(JEE MAIN)**

Sol: As the object and the mirror are moving away from each other with different speed, the magnification of the image will also change. The velocity of image will be $V_{im} = -\frac{v^2}{u^2}V_{om}$.

The position of the image is found using formula $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$.

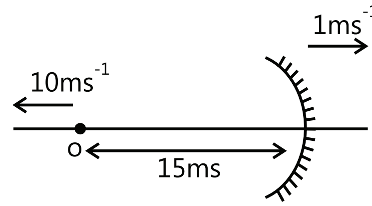


Figure 16.36

Using $\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$ $\Rightarrow$ $\frac{1}{v}-\frac{1}{15}=-\frac{1}{10}$ $\Rightarrow v=-30\text{cm}$

Now, using $V_{im}=-\frac{v^2}{u^2}V_{om}$ $\Rightarrow$ $\left(V_i-V_m\right)=-\frac{v^2}{u^2}\left(V_0-V_m\right)$

$$\Rightarrow \quad V_i-(1)=\frac{(-30)^2}{(-15)^2}\left[(-10)-(-1)\right] \Rightarrow V_i = 45\text{ cm/s}.$$

So the image will move with the velocity of 45 cm/s.

Illustration 9: A gun of mass m_1 fires a bullet of mass m_2 with a horizontal speed v_0. The gun is fitted with a concave mirror of focal length *f* facing toward a receding bullet. Find the speed of separations of the bullet and the image just after the gun was fired. **(JEE ADVANCED)**

Sol: The bullet when leave the gun it moves in direction opposite the motion of gun. As there are no external forces acting on the bullet, then the momentum of the system can be conserved. The velocity of image will be $V_{im}=-\frac{v^2}{u^2}V_{om}$.

Let v_1 be the speed of the gun (or the mirror) just after the firing of bullet. From conservation of linear momentum,

$m_2v_0 = m_1v_1 \Rightarrow v_1=\frac{m_2v_0}{m_1}$ ... (i)

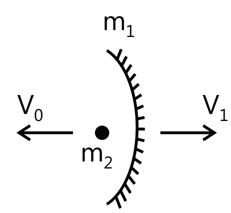


Figure 16.37

Now, $\frac{du}{dt}$ is the rate at which the distance between mirror and bullet is increasing

$=v_1+v_0$ ... (ii)

We know that ∴ $\frac{dv}{dt}=\left(\frac{v^2}{u^2}\right)\frac{du}{dt}$.

Here, $\frac{v^2}{u^2}=m^2=1$ (as at the time of firing bullet is at pole).

∴ $\frac{dv}{dt}=\frac{du}{dt}=v_1+v_0$. ... (iii)

Here, $\frac{dv}{dt}$ is the rate at which the distance between the image (of bullet) and the mirror is increasing. Hence, if v_2 is the absolute velocity of image (toward right), then

$v_2-v_1=\frac{dv}{dt}=v_1+v_0$ $\Rightarrow$ $v_2=2v_1+v_0$ ... (iv)

Therefore, the speed of separation of the bullet and the image will be

$v_r=v_2+v_0=2v_1+v_0+v_0$

$v_r=2\left(v_1+v_0\right)$.

Substituting the value of v_1 from Eq. (i), we have $v_r=2\left(1+\frac{m_2}{m_1}\right)v_0$.

5. REFRACTION OF LIGHT AND LAWS OF REFRACTION

(a) The deviation or bending of light rays from their original path while travelling from one medium to another is called refraction.

(b) If the refracted ray bends away from the normal, then the second medium is said to be RARER as compared to the first medium, and the speed increases.

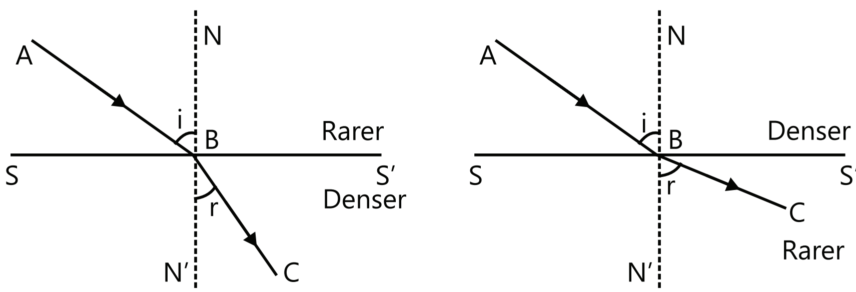


Figure 16.38

If the refracted ray bends toward the normal, then the second medium is said to be DENSER compared to the first, and the speed decreases.

Deviation due to refraction $\delta = i - r$.

MASTERJEE CONCEPTS

In general, light will travel in straight lines and the deviation occurs only when there is a change of medium (or refractive index (RI)).

B Rajiv Reddy (JEE 2012 AIR 11)

5.1 Laws of Refraction

The incident ray (AB), the normal (NN') to the refracting surface (SS') at the point of incidence (B), and the refracted ray (BC) all lie in the same plane called the plane of incidence or the plane of refraction.

5.2 Snell's Law

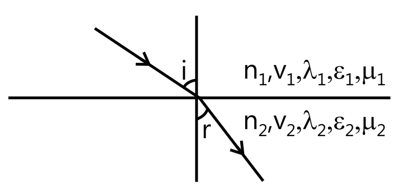


Figure 16.39

For any two given media and for light of given wavelength,

$$\frac{\sin i}{\sin r} = \text{constant}; \quad \frac{\sin i}{\sin r} = {}^1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}.$$

${}^1\mu_2$ = RI of the second medium with respect to the first medium;

μ_1 = RI of the first medium with respect to air or absolute

RI = c/v_1;

μ_2 = RI of the second medium with respect to air or absolute RI = c/v_2;

v_1, v_2 are the speeds of light in the first and the second medium, respectively;

λ_1, λ_2 are the wavelengths of light in the first and the second medium, respectively;

c=the speed of light in air (or vacuum) = $3\text{x}10^8$ m/s.

Note:

(i) The higher the value of RI, the denser (optically) the medium is.

(ii) The frequency of light does not change during refraction.

(iii) The refractive index of the medium relative to $\text{air} = \sqrt{\mu_r \varepsilon_r}$.

5.3 Refraction through a Transparent Sheet

Let the ray is incident at face AB.

Apply the Snell's law at faces AB and CD.

$\mu_1 \sin i_1 = \mu_2 \sin i_2$ ... (i)

$\mu_2 \sin i_2 = \mu_3 \sin i_3$ ... (ii)

(The angle of incidence for face CD is i_2)

From Eqs (i) & (ii), $\mu_1 \sin i_1 = \mu_3 \sin i_3$

or $\sin i_3 = \frac{\mu_1}{\mu_3} \sin i_1$ ... (iii)

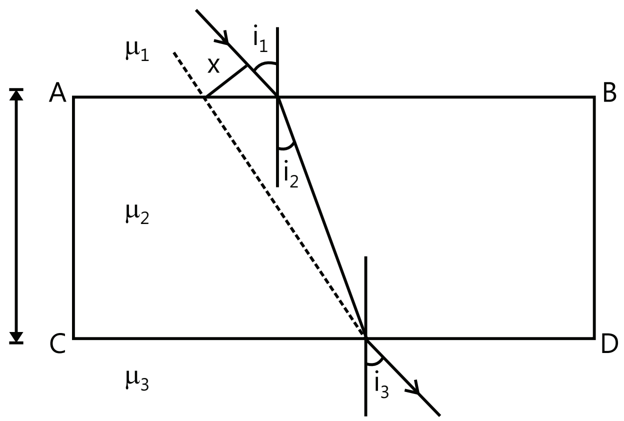


Figure 16.40

The incident ray and emerging ray are parallel. It shows that the deviation of ray is not affected by the refractive index of the sheet; it depends by μ_1 and μ_3. μ_2 only causes lateral displacement which is given by $x = \frac{t\sin(i_1 - i_2)}{\cos i_2}$.

At a glance:

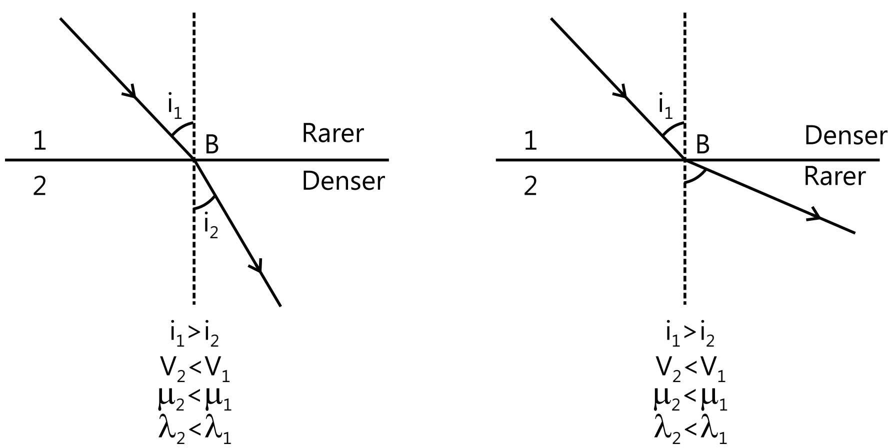


Figure 16.41

MASTERJEE CONCEPTS

In general, the speed of light in any medium is less than that in vacuum. The refractive index μ of a medium is defined as,

$$\mu = \frac{\text{Speed of light in vaccum}}{\text{Speed of light in medium}} = \frac{c}{v}.$$

Anand K (JEE 2011 AIR 47)

Illustration 10: A light beam passes from medium 1 to medium 2. Show that the emerging beam is parallel to the incident beam. **(JEE MAIN)**

Sol: When a ray of light enters from one medium to other medium of different refractive index, then according to Snell's law we get $\frac{\mu_1}{\mu_2} = \frac{\sin i}{\sin r}$ where i is the angle of incidence and r is the angle of refraction. The first refracted ray is incident on other surface. So the angle o incidence on the second surface is equal to the angle of refraction from first surface.

Applying the Snell's law at A and B, $\mu_1 \sin i_1 = \mu_2 \sin i_2$

Or $\frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_1}$ ... (i)

Similarly, $\mu_2 \sin i_2 = \mu_1 \sin i_3$

$\therefore$ $\frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_3}$ ... (ii)

From Eqs (i) and (ii) $i_3 = i_1$; i.e. the emergent ray is parallel to the incident ray.

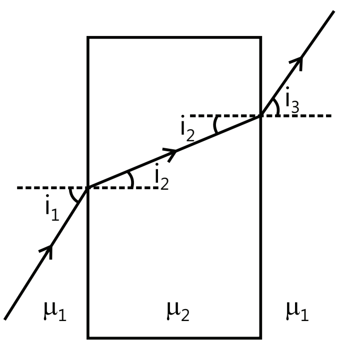


Figure 16.42

Illustration 11: The refractive index of glass with respect to water is (9/8) and the refractive index of glass with respect to air is (3/2). Find the refractive index of water with respect to air. **(JEE MAIN)**

Sol: The refractive index of medium one with respect to medium two is given by $_1\mu_2 = \frac{\mu_2}{\mu_1}$.

To find the refractive index of water with respect to air, we need to obtain the ratio between $_w\mu_g$ and $_a\mu_g$

Given $_w\mu_g = 9/8$ and $_a\mu_g = 3/2$.

As $_a\mu_g \times {_g\mu_w} \times {_w\mu_a} = 1$

$\therefore$ $\frac{1}{_w\mu_a} = {_a\mu_w} = {_a\mu_g} \times {_g\mu_w} = \frac{_a\mu_g}{_w\mu_g}$

$\therefore$ $_a\mu_w = \frac{3/2}{9/8} = \frac{4}{3}$

Illustration 12: (i) Find the speed of light of wavelength λ = 780 nm (in air) in a medium of refractive index μ = 1.55.
(ii) What is the wavelength of this light in the given medium? **(JEE MAIN)**

Sol: In a medium of refractive index μ the velocity of wave is given by $v=\frac{c}{\mu}$ and the wavelength of wave is given by $\lambda_{medium}=\frac{\lambda_{air}}{\mu}$

(i) $v=\frac{c}{\mu}=\frac{3.0\times10^8}{1.55}=1.94\times10^8 m/s$

(ii) $\lambda_{medium}=\frac{\lambda_{air}}{\mu}=\frac{780}{1.55}=503nm$.

5.4 Image Due to Refraction at a Plane Surface

Consider the situation given in the Fig. 16.43. A point object O is placed in a medium of refractive index μ_1. An another medium of refractive index μ_2 has its boundary at PA. Consider two rays OP and OA originating from O. Let OP fall perpendicularly on PA and OA fall on PA at a small angle *i* with the normal. OP enters the second medium without deviating, and OA enters by making an angle *r* with the normal. When produced backward, these rays meet at I that is the virtual image of O. If *i* and *r* are small,

$\sin i = \tan i = \frac{PA}{PO}$;

and $\sin r = \tan r = \frac{PA}{PI}$.

Thus, $\frac{\mu_2}{\mu_1}=\frac{\sin i}{\sin r}$

$=\left(\frac{PA}{PO}\right)\cdot\left(\frac{PI}{PA}\right)=\frac{PI}{PO}$. ... (i)

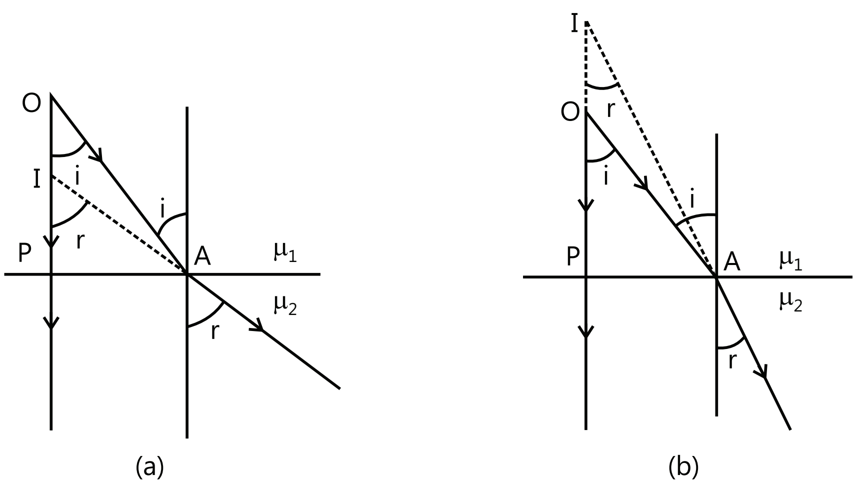


Figure 16.43

Suppose medium 2 is air and an observer looks at the image from this medium. The real depth of the object inside medium 1 is PO, whereas the depth as it appears to the observer is PI. Writing $\mu_2 = 1$ and $\mu_1=\mu$, Eq. (i) gives,

$$\frac{1}{\mu}=\frac{\text{apparent depth}}{\text{real depth}} \quad \text{or,} \quad \mu=\frac{\text{real depth}}{\text{apparent depth}}.$$

The image shifts closer to the observer's eye by an amount

$OI = PO - PI = \left(\frac{PO-PI}{PO}\right)PO=\left(1-\frac{PI}{PO}\right)PO$ or, $\Delta t=\left(1-\frac{1}{\mu}\right)t$, where *t* is the thickness of the medium over the object, and Δt is the apparent shift in its position toward the observer. Note that Δt is positive in Fig16.43 (a) and negative in Fig. 16.43 (b).

Different Scenarios at a Glance:

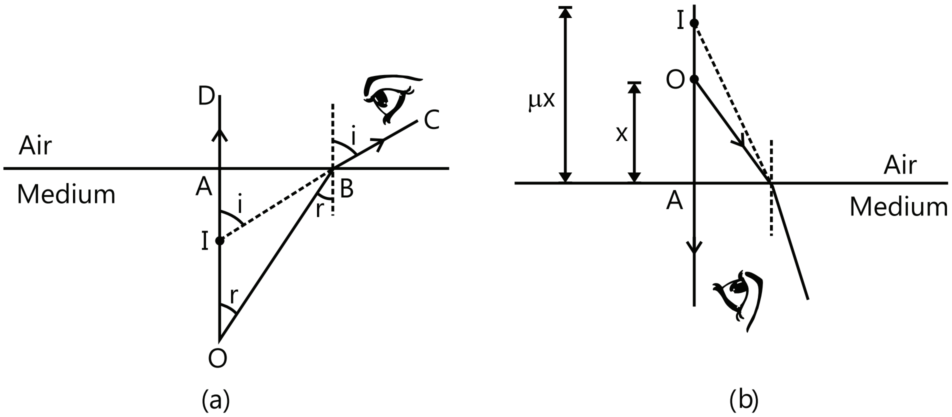


Figure 16.44

Illustration 13: A printed page is kept pressed by a glass cube $(\mu = 1.5)$ of edge 6.0 cm. By what amount will the printed letters appear to be shifted when viewed from the top? **(JEE MAIN)**

Sol: As the glass of thickness t = 6 cm is kept on the page, the image of letter appears to be shifted closer to eye by $\Delta t = \left(1 - \frac{1}{\mu}\right)t$

The thickness of the cube = *t* = 6.0 cm. The shift in its position of the printed letters is

$$\Delta t = \left(1 - \frac{1}{\mu}\right) \times t \quad = \left(1 - \frac{1}{1.5}\right) x 6.0\,\text{cm} = 2.0\,\text{cm}.$$

5.5 Shift Due to a Glass Slab (Double Refraction from Plane Surfaces)

(a) Normal Shift:

Refer Fig 16.45: An object is placed at O. Plane surface CD forms its image (virtual) at I_1. This image acts as an object for EF which initially forms the image (virtual) at I. Distance OI is called the normal shift, and its value is

$$OI = \left(1 - \frac{1}{\mu}\right)t \Rightarrow \left(\frac{\mu - 1}{\mu}\right)t$$

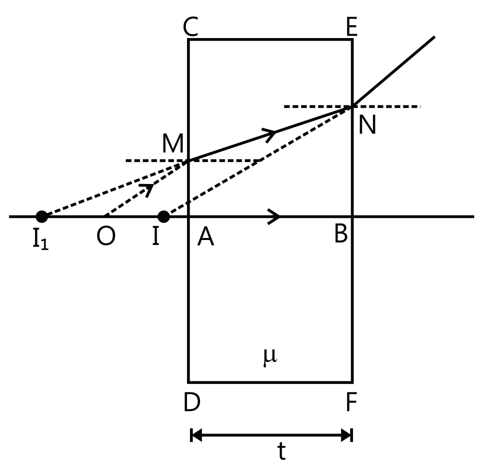


Figure 16.45

Proof: Let $OA = x$

$AI_1 = \mu x$ (Refraction from CD)

$BI_1 = \mu x + t$

$BI = \frac{BI_1}{\mu} = x + \frac{t}{\mu}$ (Refraction from EF)

$\therefore \quad OI = (AB + OA) - BI \quad = (t + x) - \left(x + \frac{t}{\mu}\right) = \left(1 - \frac{1}{\mu}\right)t.$

(b) Lateral Shift: We have already discussed that ray MA is parallel to ray BN, but the emergent ray is displaced laterally by a distance *d*, which depends on μ, *t* and *i*, and its value is given by the relation

$$d = t\left(1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right)\sin\ i.$$

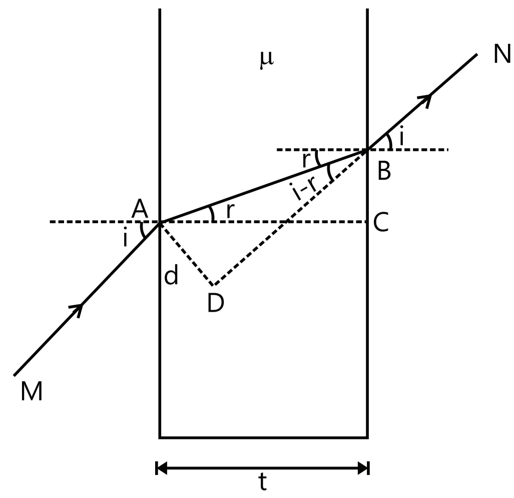


Figure 16.46

Proof:

$$AB = \frac{AC}{\cos\ r} = \frac{t}{\cos r} \qquad (\text{as } AC = t).$$

Now, $\quad d = AB\ \sin(i - r) = \frac{t}{\cos r}[\sin i \cos r - \cos i \sin r]$

or $\quad d = t[\sin i - \cos i\ \tan r] \qquad \text{... (i)}$

Further $\quad \mu = \frac{\sin i}{\text{sir } r} \quad$ or $\quad \sin r = \frac{\sin i}{\mu}$

$\therefore \quad \tan r = \frac{\sin i}{\sqrt{\mu^2 - \sin^2 i}}$

Substituting in Eq. (i), we get $d = t\left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right]\sin\ i.$

Illustration 14: A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab with a refractive index of $\mu = 3/2$ and thickness of 6 cm is inserted between an object and a mirror. Find the position of the final image when the distance *x* shown in the Fig. 16.47 is: (a) 5cm (b) 20 cm

(JEE MAIN)

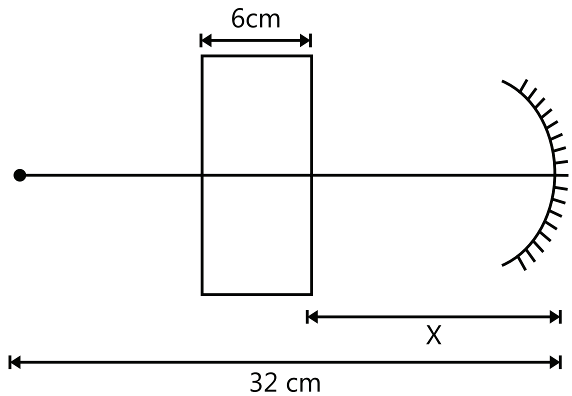


Figure 16.47

Sol: As the glass slab of thickness t = 10 cm is kept in front of the mirror, the image of object from the slab appears to be shifted closer to mirror by $\Delta t = \left(1 - \frac{1}{\mu}\right)t$. The position of the image is given by $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ where u is the distance of object, f is the focal length of the mirror and v is the distance of the image from mirror.

As we know that the normal shift produced by a glass slab is $\Delta x = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{2}{3}\right)(6) = 2\text{cm};$

i.e. for the mirror, the object is placed at a distance $(32 - \Delta x) = 30\text{cm}$ from it. Now apply the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \quad \frac{1}{v} - \frac{1}{30} = \frac{1}{10} \Rightarrow\ v = -15\text{cm}.$$

(i) **When x = 5cm:** The light falls on the slab on its return path as shown in the Fig. 16.48, but the slab will again shift it by a distance of $\Delta x = 2\text{cm}$. Hence, the final real image is formed at a distance of (15+2) =17 cm from the mirror.

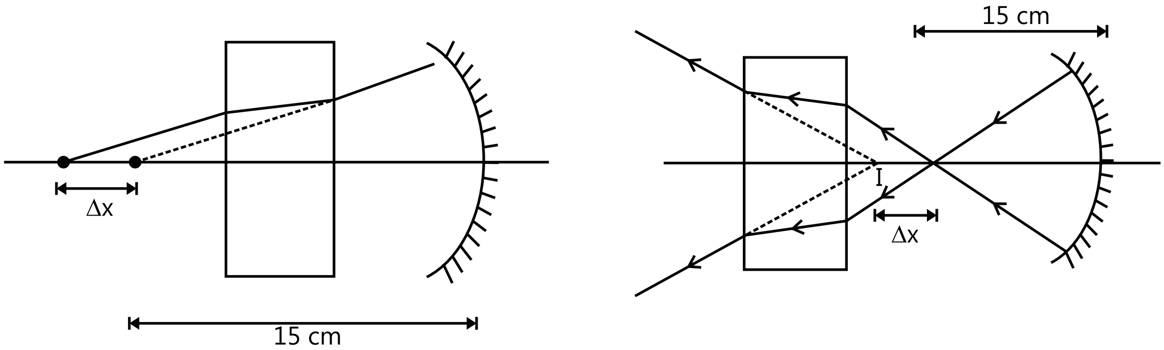


Figure 16.48

(ii) **When *x* = 20 cm:** The final image is at a distance of 17 cm from the mirror in this case also, but it is virtual.

6. CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

When a ray of light passes from an optically denser medium (a medium with larger μ) to an optically rarer medium (a medium with smaller μ), we have $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} < 1$.

When we gradually increase the value of *i*, the corresponding *r* value also increases, and at a certain point, *r* becomes 90°. Let the angle of incidence for this case be θ_c called the critical angle of the given pair of media. If *i* is increased further, there is no *r* that can satisfy the Snell's law. Then, all the light waves are reflected back into the first medium. This is called total internal reflection (TIR). Generally, the critical angle of a medium is quoted for light travelling from the denser to the rarer medium. In this case, $\mu_2 = 1$.

When substituting $\mu_1 = \mu$, then the Snell's law becomes $\frac{\sin\theta_c}{\sin 90^0} = \frac{1}{\mu}$

or $\sin\ \theta_c = (1/\mu)$ or $\theta_c = \sin^{-1}(1/\mu)$.

Illustration 16: A point source of light is placed at the bottom of a tank filled with water up to the level of 80 cm. Find the area of the surface of water through which light from the source emerges out. Assume that the refractive index is equal to 1.33. **(JEE ADVANCED)**

Sol: When light travels from a denser medium (water) to the rarer medium (air) for angle of incidence i greater than critical angle, than by Snell's law we get $\sin C = \frac{1}{\mu}$.

Let the light emerges out of a circular area of radius *r* as shown in the figure.

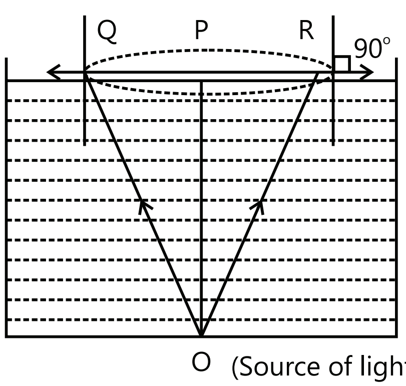


Figure 16.49

Step 1. Using $\sin C = \frac{1}{n}$, We get

$\sin C = \frac{1}{1.33} = 0.7513$

$\therefore$ C = sin^{-1}(0.7519) = 48.75°

Step 2. From ΔPOR,

$\tan\ C = \frac{PR}{OP} = \frac{r}{80}$

$\therefore$ r = 80tanC = 80tan(48.75°) = 80 × 1.14 = 91.2cm

Step 3. $\therefore$ Area through which light emerges out $= \pi r^2 = 3.14 \times (91.2)^2 = 26116.76\,cm^2 = 2.6 m^2$.

Illustration 17: The critical angle for water is 48.2°. Find its refractive index. **(JEE MAIN)**

Sol: By Snell's law we get $\mu = \frac{1}{\sin C}$.

$\mu = \frac{1}{\sin\theta_c} = \frac{1}{\sin 48.2^\circ} = 1.34$.

Day-to-day life: Due to heating of the earth, the refractive index of air near the surface of the earth is lesser than that above the earth. Light waves from a distant object reach the surface of earth at an angle of $i > \theta_c$, so that the TIR will take place, and the image of an object creates an illusion of water near the object.

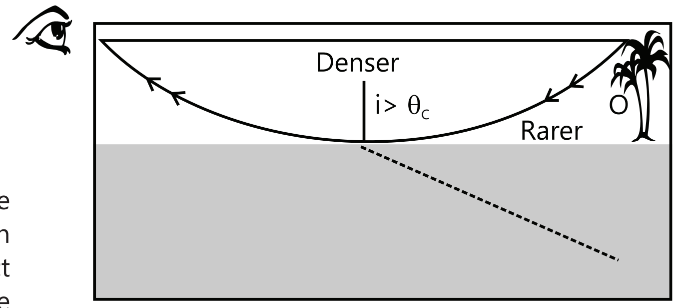


Figure 16.50

7. REFRACTION AT THE SPHERICAL SURFACES

Refraction at a single spherical surface

If the boundary between two transparent media is curved either as a convex or as a concave spherical surface, an object O in a medium of refractive index μ_1 forms an image I in a medium of refractive index μ_2 as shown in the Fig. 16.51. When the ray from O is incident at an angle *i* on the medium of refractive index μ_1, the refracted ray forms an image I after refraction in the medium of refractive index μ_2 at angle *r* according to the Snell's law.

$\mu_1 \sin i = \mu_2 \sin r$.

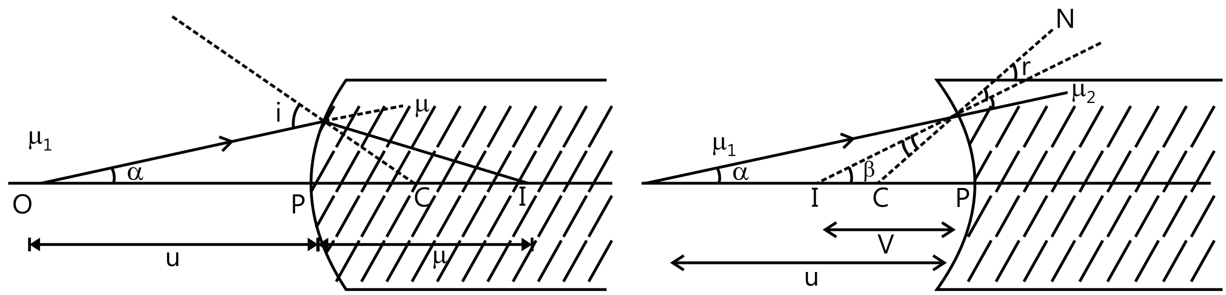


Figure 16.51

The relation between *u*=OP and *v*=IP is given by $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$.

MASTERJEE CONCEPTS

The relation is valid for a single spherical surface or plane refracting surfaces, and the sign convention for the spherical mirrors and spherical refracting surfaces are the same.

The refraction formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ can also be applied to plane refraction surfaces with $R = \infty$.

Let us derive $d_{app} = \frac{d_{actual}}{\mu}$ using this.

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with proper sign and values, we get

MASTERJEE CONCEPTS

$$\frac{1}{v}-\frac{\mu}{-d}=\frac{1-\mu}{\infty}=0 \text{ or } v=-\frac{d}{\mu}$$

i.e. the image of object O is formed at a distance $\frac{d}{\mu}$ on the same side.

or $d_{app}=\frac{d_{actual}}{\mu}$.

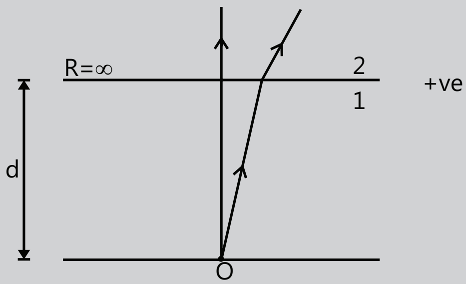


Figure 16.52

Anurag Saraf (JEE 2011 AIR 226)

Illustration 18: A sunshine recorder globe of 30 cm diameter is made of glass of refractive index *n* = 1.5. A ray of light enters the globe parallel to the axis. Find the position from the center of the sphere where the ray crosses the principle axis. **(JEE ADVANCED)**

Sol: As the light enters from air to the globe of refractive index *n* than for the refraction at the surface we have relation $\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$.

First refraction (from the rarer to the denser medium): Here, $u=-\infty, n_2=1.5, R=+15\text{cm}$.

Using the relation, $\frac{n_2}{v}+\frac{n_1}{u}=\frac{n_2-n_1}{R}$

i.e. $$\frac{n_2}{v}=\left(\frac{n_2-n_1}{R}\right)+\frac{n_1}{u}$$

$\Rightarrow$ $$\frac{1.5}{v}=\frac{1.5-1}{15}+\frac{1}{(-\infty)}=\frac{1}{30};$$

$\Rightarrow$ $$v=45\text{cm}.$$

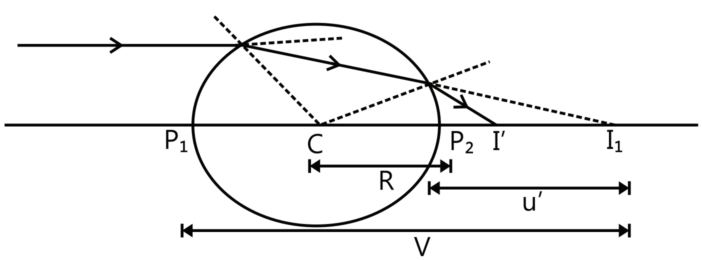


Figure 16.53

Second refraction (from the denser to the rarer medium):

Here, R=-15cm, u′= (45-30) =15cm.

Using the relation, $\frac{n_1}{v'}-\frac{n_2}{u'}=\frac{n_1-n_2}{R}$

i.e. $$\frac{1}{v'}-\frac{1.5}{5}=\frac{1-1.5}{-15}=\frac{1}{30}\Rightarrow \qquad v'=\frac{30}{7}=7.5\text{cm}.$$

$\therefore$ Distance at which the image is formed from the center of the globe is (15+7.5) =22.5 cm.

Illustration 19: Locate the image of a point object O in the situation shown in the figure. Point C denotes the center of curvature of the separating surface. **(JEE ADVANCED)**

Sol: According to the sign convention as the object is placed left to the pole, the distance of it is considered to be negative. Using the formula $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ we get the distance of image from the centre of curvature.

Here, u=-15cm, R=30cm, $\mu_1 = 1$ and $\mu_2 = 1.5$ We have

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad \frac{1.5}{v} - \frac{1.0}{-15cm} = \frac{1.5-1}{30cm}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{0.5}{30\ cm} - \frac{1}{15\ cm}; \qquad \Rightarrow \quad v=-30cm$$

The image is formed 30 cm left to the spherical surface and is virtual.

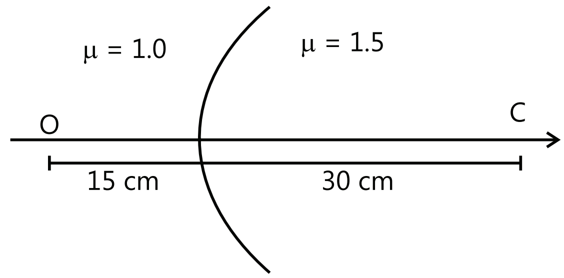


Figure 16.54

Illustration 20: A glass sphere of radius R=10 cm is kept inside water. A point object O is placed at 20 cm from A as shown in the figure. Find the position and nature of the image when seen from other side of the sphere and also draw the ray diagram. Assume μ_g=3/2 and μ_w= 4/3.

(JEE ADVANCED)

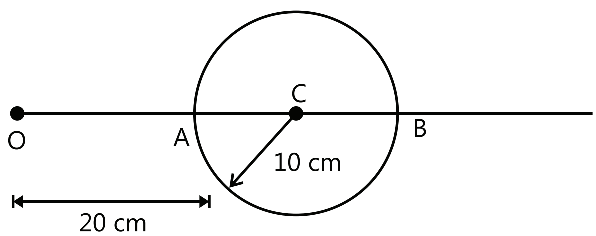


Figure 16.55

Sol: As the light passes through medium of different refractive indices at each refraction the position of image of point object is found by $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$.

A ray of light from object O gets refracted twice. The direction of this light ray is from the left to right. Hence, the distances measured in this direction are positive.

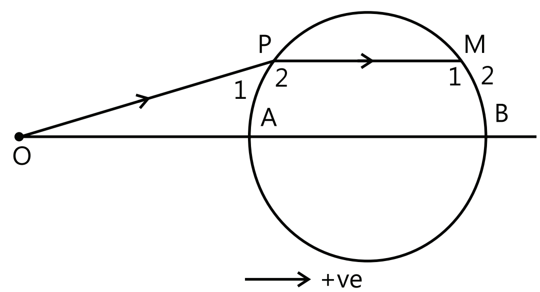


Figure 16.56

When applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice with proper signs,

we have $\frac{3/2}{AI_1} - \frac{4/3}{-20} = \frac{3/2 - 4/3}{10}$

or $\quad AI_1 = -30cm$.

Now, the first image I_1 acts an object for the second surface, where

$BI_1 = u = -(30 + 20) = -50cm$.

$$\therefore \frac{4/3}{BI_2} - \frac{3/2}{-50} = \frac{4/3 - 3/2}{-10}.$$

$\therefore BI_2 = -100$ cm, i.e. the final image I_2 is virtual and is formed at a distance of 100 cm (toward left) from B. The ray diagram is shown in the figure.

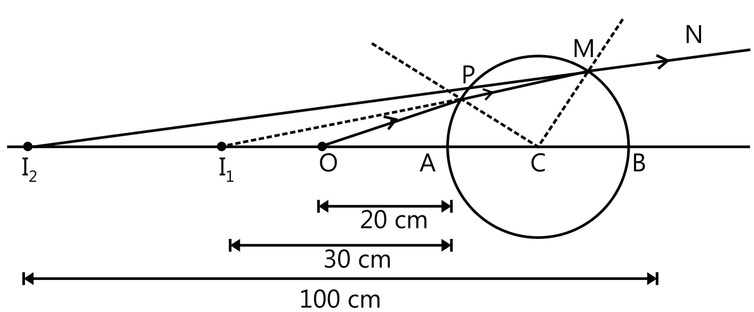


Figure 16.57

The following points should be taken into account while drawing the ray diagram.

(i) At P, the ray travels from a rarer to a denser medium. Hence, it will bend toward normal PC. At M, it travels from a denser to a rarer medium and hence moves away from normal MC.

(ii) PM ray when extended backward meets at I_1, and MN ray when extended meets at I_2.

7.1 Lateral Magnification

The lateral magnification may be obtained with the help of the adjacent Fig. 16.58, where two rays from the tip of an object of height h_0 meet at the corresponding point on an image of height h_1. One ray passes through the center of curvature of the spherical surface so that its direction is unchanged. The path of the second ray is obtained from the Snell's law. With the paraxial approximation,

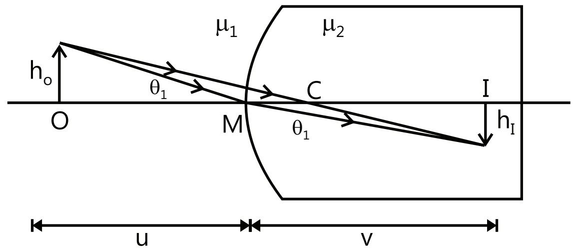


Figure 16.58

$$\sin\theta_1 \approx \frac{h_0}{u} \quad \text{and} \quad \sin\theta_2 \approx \frac{h_i}{v}.$$

Combining these equations with the Snell's law, then,

$$\mu_1\left(\frac{h_0}{u}\right) = \mu_2\left(\frac{h_i}{v}\right) \quad \text{or} \quad \frac{h_i}{h_0} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right).$$

The lateral magnification *m* is the ratio of the image height to the object height or $\frac{h_i}{h_0}$.

We, therefore, obtain $\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right)$ i.e. $\frac{h_i}{h_o} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right)$

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Here, *v* is +ve, *u* is −ve, h_i is −ve, h_0 is +ve (the distances measured above the axis are positive). So, if we put these sign conventions, in Eq. (vi), we obtain the same result $m = \frac{\mu_1}{\mu_2}\frac{v}{u}$.

Chinmay S Purandare (JEE 2012 AIR 698)

Illustration 21: Find the size of the image formed in the situation shown in the figure. **(JEE MAIN)**

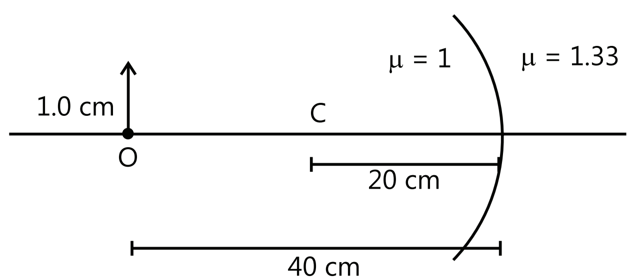


Figure 16.59

Sol: Here as the object is placed left to the pole beyond center of the curvature, then according to sign convention the distance of object is considered to be negative.

The distance of image is found by $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Here, u=−40cm, R=−20cm, $\mu_1 = 1$, $\mu_2 = 1.33$.

We have $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \frac{1.33}{v} - \frac{1}{-40\text{cm}} = \frac{1.33-1}{-20\text{cm}} \Rightarrow \frac{1.33}{v} = -\frac{1}{40\text{cm}} - \frac{0.33}{20\text{cm}}$$

$$\Rightarrow v = -32\text{cm}.$$

The magnification is $m = \frac{h_2}{h_1} = \frac{\mu_1 v}{\mu_2 u}$ or $\frac{h_2}{1.0\text{cm}} = \frac{-32\text{cm}}{1.33\text{x}(-40\text{cm})}$ $\Rightarrow h_2 = +6.0$ cm. .

The image is erect.

7.2 Refraction through Lenses

A lens is made of a transparent material with two refracting surfaces such that at least one of these is a curved one. The convex lens is thicker in the middle, and the concave lens is thinner in the middle.

The plano-convex and plano-concave lenses have one plane surface, and the other surfaces are convex and concave, respectively. Different types of typical lenses are shown in the Fig. 16.60.

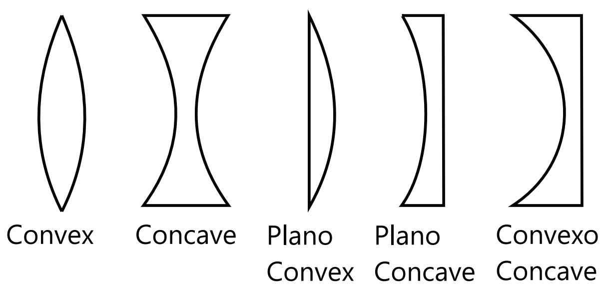


Figure 16.60

7.2.1 Parameters Associated with Lens

The center of curvature C is the center of the sphere, of which the curved surface of the lens is a part. The radius of curvature of either surface of the lens is the radius of the sphere, of which the curved surface is a part.

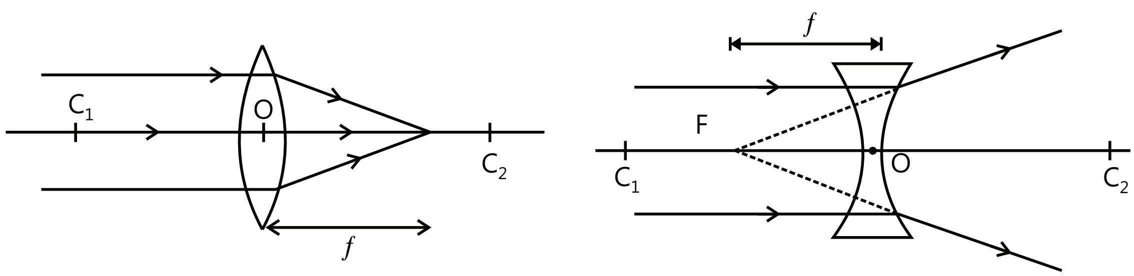


Figure 16.61

The optical center, O, of a lens is a point through which the ray does not get deviated. The principal axis is a line passing through the center (s) of curvature and the optical center. If the object is at infinity, the image is formed at the principal focus and vice versa. The focal length is the distance between the optical center of a lens and a point on which a parallel beam of light converges or appears to converge. The aperture of a lens is the effective diameter of its light-transmitting area. The intensity of the image is directly proportional to the square of the aperture.

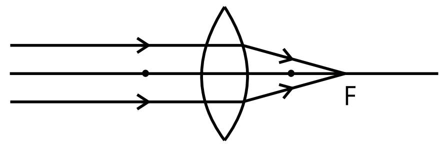
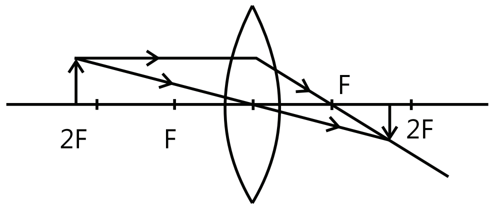
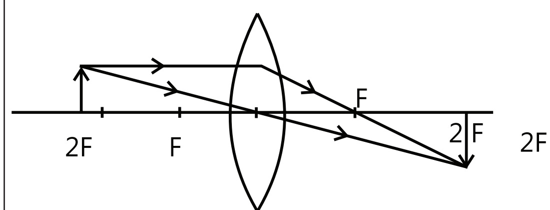
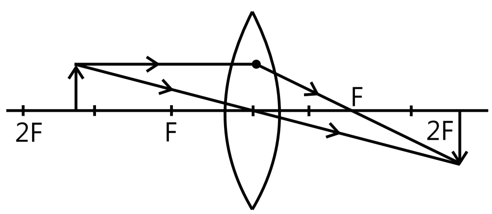
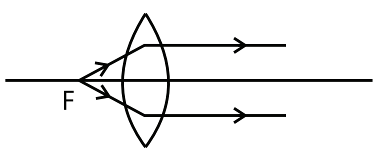
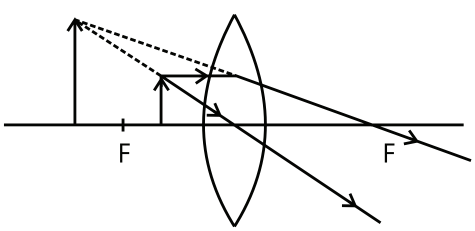
7.2.2 Rules of Image Formation

(a) A ray that passes through the optical center does not get deviated through the lens.

(b) A ray incident parallel to the principal axis after refraction through the lens either passes through the focus or appears to pass through the focus after extrapolation.

(c) A ray that passes through the focus or directed toward it becomes parallel to the principal axis after refraction. The rays of light from a point of object intersect or appear to intersect after refraction through the lens and form an image. If the rays actually intersect, then the image is real, and if the rays appear to intersect, then the image is virtual.

7.3 Image Formation by Convex Lens

S. No	Position of object	Diagram	Position of image	Nature of image
1	Infinity	 Figure 16.62	Principal focus	Real, inverted and diminished
2	Between 2F and ∞	 Figure 16.63	Between F and 2F	Real, inverted and diminished
3	2F	 Figure 16.64	2F	Real, inverted and of same size as the object
4	Between F and 2F	 Figure 16.65	Beyond 2F	Real, inverted and magnified
5	F	 Figure 16.66	Infinity	Real, inverted and highly magnified
6	Between F and optical center	 Figure 16.67	Same side as the optical center	Virtual, erect and magnified

A lens has two foci, which is not a case in a mirror.

First focus (F_1): If an object (real in case of a convex, virtual for concave) is placed at the first focus (F_1), the image of this object is formed at infinity, or we can say through F_1 it becomes parallel to the principal axis after refraction from the lens. The distance from the first focal length is f_1.

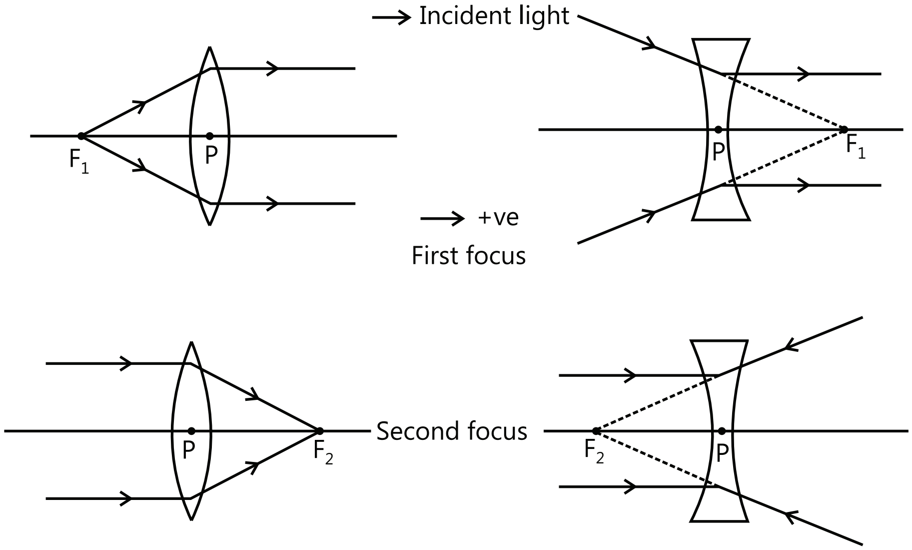


Figure 16.68

Second focus or principal focus (F_2): A narrow beam of light that travels parallel to the principal axis either converge (in case of a convex lens) or diverge (in case of a concave lens) at a refraction (*r*) from the lens. This point F_2 is called the second or principal focus. If the rays converge, the lens is a converging lens, and if the rays diverge, then the lens is a diverging lens. It can be seen from the Fig. 16.68 that f_1 is negative for a convex lens and positive for a concave lens. But f_2 is positive for a convex lens and negative for a concave lens.

$|f_1| = |f_2|$ if the media on the two sides of a thin lens have the same refractive index.

We mainly concern with the second focus f_2. Thus, wherever we write the focal length f, it means the second or principal focal length. Therefore, $f = f_2$ and, hence, f is positive for a convex lens and negative for a concave lens.

7.4 Image Formation by a Concave Lens

An image I formed by a concave lens of a real object O occurred beyond F and the optical center is virtual, erect and diminished. The image is formed at F if the object is at infinity.

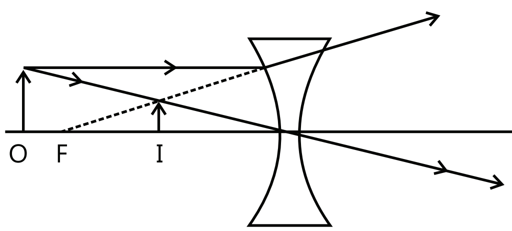


Figure 16.69

7.5 Lens Formulae

If an object is in a medium of refractive index μ_1 at a distance *u* from the optical center of a lens having radii of curvature R_1 and R_2 and of refractive index μ_2, its image is formed at a distance *v* from the optical center, then

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

If is the focal length of the lens, then $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

- If μ is the refractive index of the material the lens is made of with respect to the surrounding medium air, then $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.
- If I and O are the lateral or transverse size of the image and object, respectively, the magnification *m* is given by, $m=\frac{I}{O}=\frac{v}{u}$.
- The power *P* of the lens is given by $P=\frac{1}{f(m)}=\frac{100}{f(cm)}$ dioptre.

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Suppose *m* is positive, it implies *v* and *u* are of same sign, i.e. the object and its image are on the same side (left side), which implies that the image of a real object is virtual. Thus, *m*=+2; it implies that the image is virtual, erect and magnified to two times the actual size, and $|v|=2|u|$. Similarly, $m=-\frac{1}{2}$ implies that the image is real, inverted and diminished, and $|v|=\frac{1}{2}|u|$.

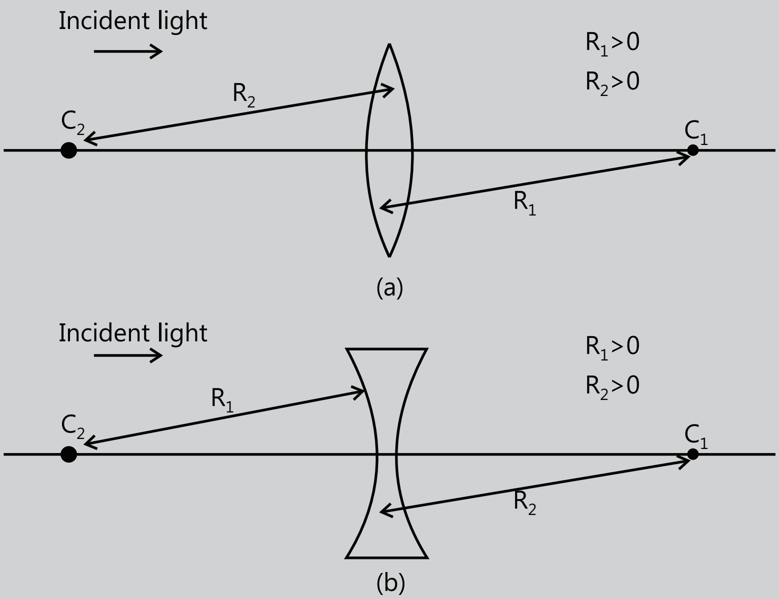

Figure 16.70

For a converging lens, R_1 is positive and R_2 is negative. Therefore, $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ is positive, and if the lens is placed in air, the value of $(\mu-1)$ is also positive. Hence, the focal length f of a converging lens is positive. For a diverging lens, however, R_1 is negative and R_2 is positive at the focal length f.

The focal length of a mirror $\left(f_M=\frac{R}{2}\right)$ depends only on the radius of curvature R, while that of a lens depends on μ_1,μ_2,R_1 and R_2. Thus, when a lens and a mirror both are immersed in a liquid, the focal length lens changes, whereas that of the mirror remains unchanged.

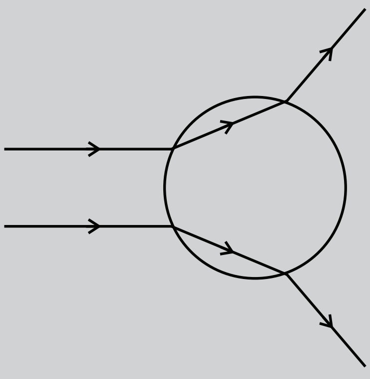

Figure 16.71

Suppose $\mu_2<\mu_1$, i.e. the refractive index of the medium (in which the lens is placed) is more than the refractive index of the material the lens is made of, then $\left(\frac{\mu_2}{\mu_1}-1\right)$ becomes negative, i.e. the lens' behavior changes. A converging lens behaves as a diverging lens and vice versa. An air bubble in water seems to be a convex lens, but behaves as a concave (diverging) lens.

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Illustration 22: The focal length of a convex lens in air is 10 cm. Find its focal length in water. Assume $\mu_g = 3/2$ and $\mu_w = 4/3$ **(JEE MAIN)**

Sol: The focal length of lens is given by $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \left({}_1\mu^2 - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

$$\frac{1}{f_{air}} = \left(\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{... (i)}$$

and $$\frac{1}{f_{water}} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{... (ii)}$$

Dividing Eq. (i) by Eq. (ii), we get $\frac{f_{water}}{f_{air}} = \frac{\left(\mu_g - 1\right)}{\left(\mu_g / \mu_w - 1\right)}$

Substituting the values, we get $f_{water} = \frac{\left(3/2 - 1\right)}{\left(\frac{3/2}{4/3} - 1\right)} f_{air}; \quad = 4f_{air} = 4\text{x}10 = 40\text{cm}$.

Illustration 23: An image I of point object O is formed by a lens whose optical axis is AB as shown in the Fig. 16.72.

(a) State whether it is a convex or a concave lens?

(b) Draw a ray diagram to locate the lens and its focus. **(JEE ADVANCED)**

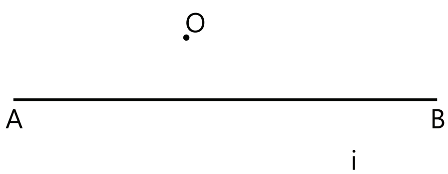


Figure 16.72

Sol: For the convex lens, the image of the object is real, formed on the opposite site and is always inverted while for concave lens the image of an object is always virtual, erect and on the same side of object.

(a) (i) A concave lens always forms an erect image. The given image I is on the other side of the optical axis. Hence, the lens is convex.

(ii) Join O with I. Line OI cuts the optical axis AB at pole P of the lens. The dotted line shows the position of the lens.

Then, draw a line parallel to AB from point O. It cuts the dotted line at M and join M with I. Line MI cuts the optical axis at focus (F) of the lens.

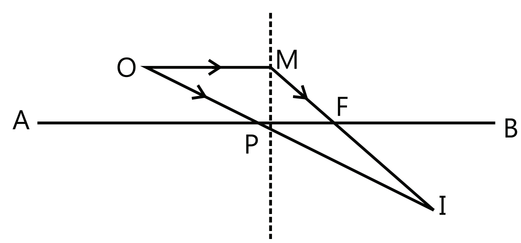


Figure 16.73

Illustration 24: Find the distance of an object from a convex lens if image is magnified two times the actual size. The focal length of the lens is 10 cm. **(JEE MAIN)**

Sol: For the convex lens, the position of the object from the lens is given by $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

A convex lens forms both type of images, i.e. real and virtual. Since the type of the image is not mentioned here, we have to consider both the cases.

When the image is real: In this case, *v* is positive and *u* is negative with $|v| = 2|u|$.

Thus, if $u = -x$ then $v = 2x$ and $f = 10$ cm.

Substituting in $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, We get $\frac{1}{2x}+\frac{1}{x}=\frac{1}{10}$ or $\frac{3}{2x}=\frac{1}{10}$

$\therefore$ x=15cm

x =15 cm; it implies that the object lies between F and 2F.

When the image is virtual: In this case, v and u both are negative. So let, $u=-y$ then $v=-2y$ and $f=10\text{cm}$.

Substituting in $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ We get, $\frac{1}{-2y}+\frac{1}{y}=\frac{1}{10}$

or $\frac{1}{2y}=\frac{1}{10}$; $\therefore$ y=5cm.

y=5 cm; it implies that the object lies between F and P.

7.6 Displacement Method to Determine the Focal Length of a Convex Lens

If the distance d between an object and a screen is greater than four times the focal length of a convex lens, then there are two possible positions of the lens between the object and the screen at which a sharp image of the object is formed on the screen. This method is called displacement method and is used in laboratory to determine the focal length of a convex lens.

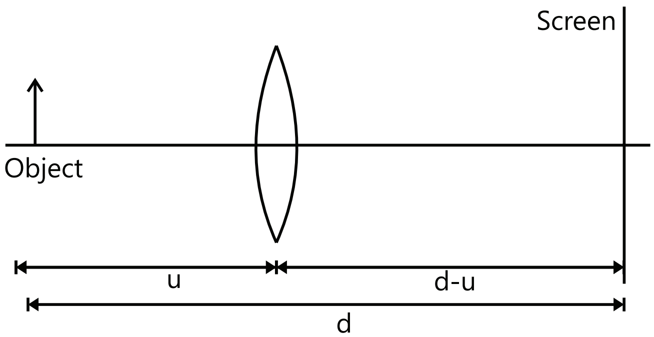


Figure 16.74

To prove this, let us take an object that is placed at a distance u from a convex lens of a focal length f. The distance between the image and the lens $v = (d-u)$.

From the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

We have $\frac{1}{d-u}-\frac{1}{-u}=\frac{1}{f} \Rightarrow u^2-du+df=0$

$$\therefore\ u=\frac{d\pm\sqrt{d(d-4f)}}{2}$$

Now, there are following possibilities:

(i) If $d<4f$, then u is imaginary.

(ii) If $d=4f$, then $u=\frac{d}{2}=2f$. Hence, there is only one possible position and the minimum distance between an object and its real image in case of a convex lens is $4f$.

(iii) If $d>4f$, there are two possible positions of lens at distances $\frac{d+\sqrt{d(d-4f)}}{2}$ and $\frac{d-\sqrt{d(d-4f)}}{2}$, for which an real image is formed on the screen.

(iv) If I_1 is the image length in the first position of the object and I_2 is the image length in the second position, then the object length O is given by $O=\sqrt{I_1I_2}$.

Proof:

$$|u_1|=\frac{d+\sqrt{d(d-4f)}}{2} \qquad \therefore |v_1|=d-|u_1|=\frac{d-\sqrt{d(d-4f)}}{2}$$

$$|u_2| = \frac{d - \sqrt{d(d-4f)}}{2} \qquad \therefore |v_1| = d - |u_2| = \frac{d + \sqrt{d(d-4f)}}{2}$$

Now $\quad |m_1 m_2| = \frac{I_1}{O} x \frac{I_2}{O} = \frac{|v_1|}{|u_1|} x \frac{|v_2|}{|u_2|}$

Substituting the values, we get $\quad \frac{I_1 I_2}{O_2} = 1 \quad$ or $\quad O = \sqrt{I_1 I_2}$

7.7 Lenses in Contact

If two thin lenses of focal lengths f_1 and f_2 are placed in contact, the equivalent focal length *F* of this combination is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$.

If one surface of the lens is coated with silver, the effective focal length *F* of the combination is given by

$\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$, where f_l is the focal length of the lens, and f_m is the focal length of the silvered surface.

When two lenses of focal lengths f_1 and f_2 are kept distance *d* apart from each other, the focal length of this combination is given by $\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_2} - \frac{d}{f_l f_2}$

If there is a medium of refractive index μ between the lenses, the equivalent focal length *F* is $\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_2} - \frac{d/\mu}{f_l f_2}$.

Note: When more than two lenses in contact, the equivalent focal length is given by the formula, $\frac{1}{F} = \sum_{i=1}^{n} \frac{1}{f_i}$

Here, f_1, f_2, ... should be substituted with their respective signs.

Important Observation: To find the position of an image when one face of a lens is coated with silver.

The given system finally behaves as mirror, whose focal length is given by

$1/v + 1/u = 1/f$.

$$\frac{1}{f} = \frac{2(\mu_2/\mu_1)}{R_2} - \frac{2(\mu_2/\mu_1 - 1)}{R_1}.$$

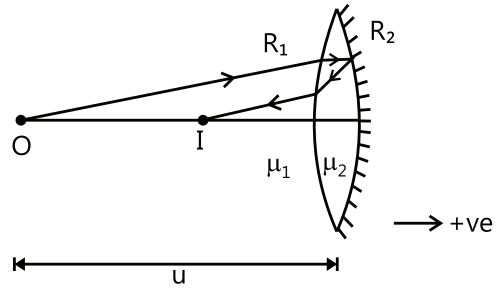


Figure 16.75

MASTERJEE CONCEPTS

A lens made of three different materials has three focal lengths. Thus, for a given object, there are three images.

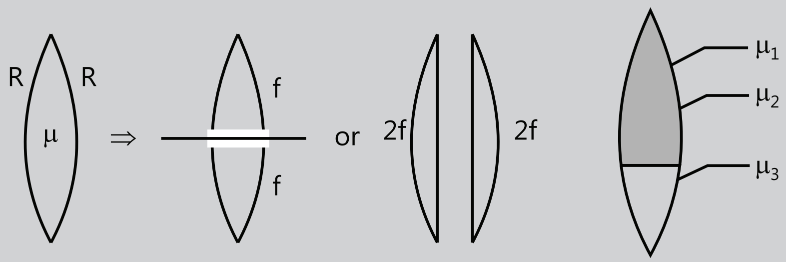


Figure 16.76

MASTERJEE CONCEPTS

Memory zone

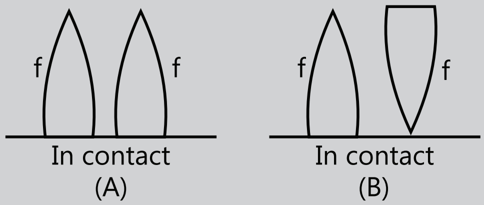


Figure 16.77

(A) The resultant focal length in this case is $\frac{f}{2}$.

(B) The resultant focal length in this case is ∞. This is because the optical axes of both parts have been inverted.

Anurag Saraf (JEE 2011 AIR 226)

Illustration 25: A double concave lens made up of glass of refractive index 1.6 has radii of curvature of 40 cm and 60 cm. Calculate its focal length in air. **(JEE MAIN)**

Sol: For double concave lens, the focal length is given by $\frac{1}{f}=\left({}^{\alpha}\mu_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ where ${}^{\alpha}\mu_g$ = n is refractive index of glass with respect to air.

Using the relation $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$. Here $R_1=-40\,cm$, $R_2=+60\,cm$ and $x=1.6$.

We get $\frac{1}{f}=(1.6-1)\left(\frac{1}{-40}-\frac{1}{60}\right)=-0.6\left(\frac{60+40}{60x40}\right)=-\frac{1}{40}$

i.e. $f=-40\,cm$.

Illustration 26: A biconvex lens has a focal length $\frac{2}{3}$ times the radius of curvature of either surface. Calculate refractive index of lens material. **(JEE MAIN)**

Sol: The biconvex or double convex lens, we can find the refractive index of material using $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.

Here, $f=\frac{2}{3}R$ $R_1=R$ and $R_2=R$.

Using $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$, we get $\frac{3}{2R}=(n-1)\left(\frac{1}{R}+\frac{1}{R}\right)=\frac{2(n-1)}{R}$

$\Rightarrow$ $(n-1)=\frac{3}{4}$ or $n=\frac{3}{4}+1=\frac{7}{4}=1.75$.

Illustration 27: A glass convex lens has a power of 10.0 D. When this lens is fully immersed in a liquid, it acts a concave lens of focal length 50 cm. Calculate the refractive index of the liquid (Assume ${}^{a}\mu_g=1.5$).
(JEE ADVANCED)

Sol: Power of convex lens is given by $P = \frac{100}{f}$ where $\frac{1}{f} = \left({}^{a}\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. When lens is immersed inside the liquid, it now behaves as concave lens thus the focal length of the lens is given as $\frac{1}{f} = \left(\frac{{}^{a}\mu_g}{{}^{a}\mu_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$P = \frac{100}{f(\text{in cm})}$ $\quad\quad \therefore f = \frac{100}{10} = 10\,\text{cm}$.

Now $\frac{1}{f} = \left({}^{a}\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$; $\quad\quad \frac{1}{10} = 0.5\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\therefore \quad \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{5} \quad \text{... (i)}$$

When fully immersed in a liquid, $f = -50\,\text{cm}$

$$\therefore \quad -\frac{1}{50} = \left(\frac{{}^{a}\mu_g}{{}^{a}\mu_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{{}^{a}\mu_g}{{}^{a}\mu_l} - 1\right) x \frac{1}{5}$$

$$\therefore \quad \frac{{}^{a}\mu_g}{{}^{a}\mu_l} - 1 = -\frac{1}{10}; \quad \text{or} \quad \frac{{}^{a}\mu_g}{{}^{a}\mu_l} = -\frac{1}{10} + 1 = \frac{9}{10} \quad \text{or} \quad {}^{a}\mu_l = \frac{10}{9} \times {}^{a}\mu_g = \frac{10}{9} \times 1.5 = 1.67$$

Illustration 28: A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis as shown in the Fig. 16.78. The separation between the object and image planes is 1.8 m. The magnification of the image formed by one of the half lens is 2. Find the focal length of the lens and the distance between the two halves. Draw the ray diagram of image formation. **(JEE ADVANCED)**

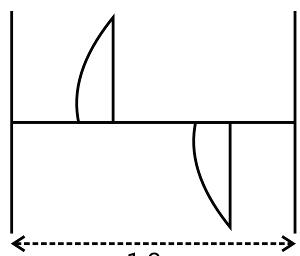


Figure 16.78

Sol: When the lens is cut in two halves along principle axis, the focal length of both the halves is equal as the original len.

For both the halves, the position of the object and the image is same. There is difference only in magnification. Magnification for one of the halves is given as 2 (>1). This is the first one, because $|v| > |u|$. So, for the first half,

$|v/u| = 2$ $\quad$ or $\quad$ $|v| = 2|u|$

Let $u = -x$, $\quad$ then $\quad$ $v = +2x$

and $|u| + |v| = 1.8\,\text{m}$.

i.e. $3x = 1.8\text{m}$ $\quad$ or $\quad$ $x = 0.6\text{m}$

Hence, $u = -0.6\text{m}$ $\quad$ and $\quad$ $v = +1.2\text{m}$

Using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{1.2} - \frac{1}{-0.6} = \frac{1}{0.4}$ $\quad$ $f = 0.4\text{m}$

For the second half, $\frac{1}{f} = \frac{1}{1.2 - d} - \frac{1}{-(0.6 + d)}$

or $\frac{1}{0.4} = \frac{1}{1.2 - d} + \frac{1}{(0.6 + d)}$.

By solving this, we get $\quad$ $d = 0.6\text{m}$

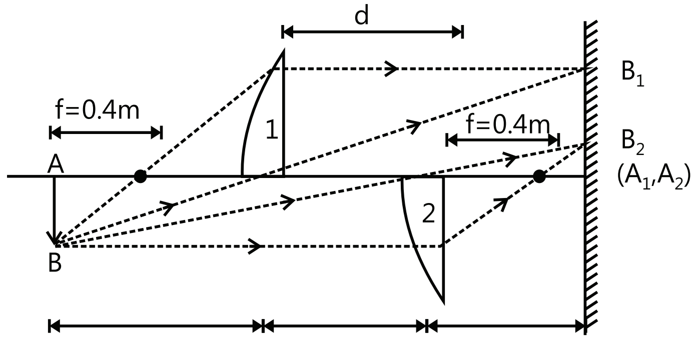


Figure 16.79

Magnification for the second half will be $m_2 = \frac{v}{u} = \frac{0.6}{-(1.2)} = -\frac{1}{2}$,

and for the first half is $m_1 = \frac{v}{u} = \frac{1.2}{-(0.6)} = -2$

The ray diagram is shown in the Fig. 16.79.

Illustration 29: A converging lens of focal length 5.0 cm is placed in contact with a diverging lens of focal length 10.0 cm. Find the combined focal length of the system. **(JEE MAIN)**

Sol: The focal length of the combination of lenses is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Here, $f_1 = +5.0\,\text{cm}$ and $f_2 = -10.0\,\text{cm}$

Therefore, the combined focal length *F* is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{5.0} - \frac{1}{10.0} = +\frac{1}{10.0}; \qquad \therefore \qquad F = +10.0\,\text{cm}$$

i.e. this combination behaves as a converging lens of focal length 10.0 cm.

7.8 Power of an Optical Instrument

By the optical power of an instrument (whether it is a lens, mirror or a refractive surface), we measure the ability of the instrument to deviate the path of rays passing through it. If the instrument converges the rays parallel to the principal axis, its power is positive, and if it diverges the rays, it is a negative power.

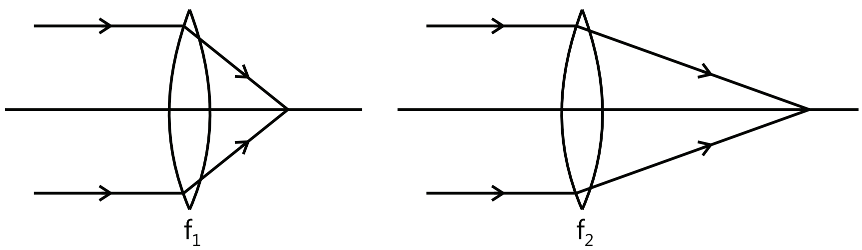
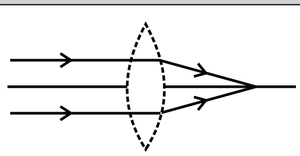


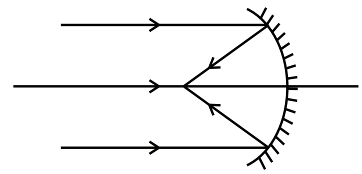
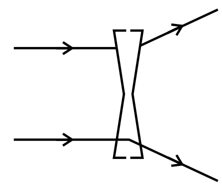
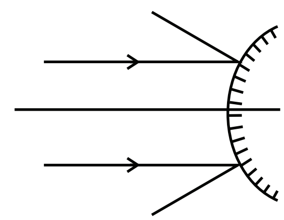
Figure 16.80

The shorter the focal length of a lens (or a mirror), the more it converges or diverges light. As shown in the Fig. 16.80, $f_1 < f_2$, and hence, the power $P_1 > P_2$, because bending of the light ray in case 1 is more than that in case 2. For a lens,

$P\text{ (in dioptre)} = \frac{1}{f\text{ (metre)}}$, and for a mirror, $P\text{ (in dioptre)} = \frac{-1}{f\text{ (metre)}}$

At a glance:

Nature of lens/ mirror	**Focal length (f)**	**Power** $P_L = \frac{1}{f}, P_M = -\frac{1}{f}$	**Converging/ diverging**	**Ray diagram**
Convex lens	+ve	+ve	Converging	 **Figure 16.81**

Nature of lens/ mirror	Focal length (f)	Power $P_L = \frac{1}{f}, P_M = -\frac{1}{f}$	Converging/ diverging	Ray diagram
Concave mirror	−ve	+ve	Converging	 Figure 16.82
Concave lens	−ve	−ve	Diverging	 Figure 16.83
Convex mirror	+ve	−ve	diverging	 Figure 16.84

MASTERJEE CONCEPTS

Both convex lens and concave mirror have positive power and are converging in nature, whereas both concave lens and convex mirror have negative power and are diverging in nature.

Vaibhav Krishnan (JEE 2009 AIR 22)

Illustration 30: A spherical convex surface separates an object and an image space of refractive index 1.0 and $\frac{4}{3}$. If the radius of curvature of the surface is 10 cm, find its power. **(JEE ADVANCED)**

Sol: For the convex lens power of magnification is given by $P = \frac{1}{f}$

where $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

We have to find where do the parallel rays converge (or diverge) on the principal axis and call it the focus and the corresponding length is the focal length.

Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$.

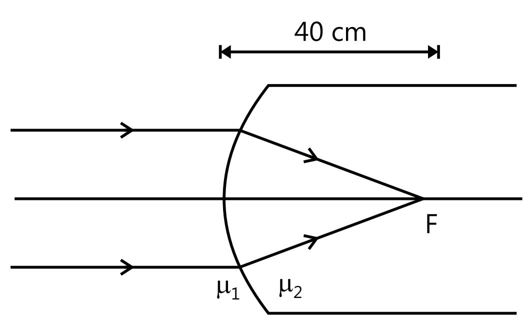


Figure 16.85

With proper values and signs, we have $\frac{4/3}{f}-\frac{1.0}{\infty}=\frac{4/3-1.0}{+10}$; or $f=40\text{cm}=0.4\text{m}$

Since, the rays are converging, its power should be positive. Hence,

$$P\text{ (in dioptre)}=\frac{+1}{f\text{ (metre)}}=\frac{1}{0.4};\qquad \text{or}\quad P=2.5\text{ dioptre}$$

Illustration 31: Two lenses of focal length 20 cm and −25cm are placed in contact. Find the total power of this combination. **(JEE MAIN)**

Sol: The power of combination of lenses is given by $P=P_1+P_2=\frac{1}{f_1}+\frac{1}{f_2}$.

$$f_1=20\text{cm},\quad f_2=-25\text{cm};\qquad \therefore\ P_1=\frac{100}{f_1}=\frac{100}{20}=5\text{D}$$

$$P_2=\frac{100}{-25}=-4\text{D};\qquad \therefore\ P=P_1+P_2=5-4=1.0\text{ D}$$

8. PRISM

The figure shows the cross section of a prism. AB and AC represent the refracting surfaces. The angle BAC is the angle of the prism. Assume that the prism is placed in air. A ray PQ, incident on a refracting surface AB, gets refracted along QR. The angle of incidence and the angle of refraction are *i* and *r*, respectively. The ray QR is incident on the surface AC. Here, the light travels from an optically denser medium to an optically rarer medium. If the angle of incidence *r′* is not greater than the critical angle, then the ray is refracted in air along RS. The angle of refraction is i′. The angle i′ is also called the angle of emergence. If the prism were not present, the incident ray would have passed un-deviated along PQTU. Due to the presence of the prism, the final ray travels along RS. The angle UTS= δ is called angle of deviation. From triangle TQR,

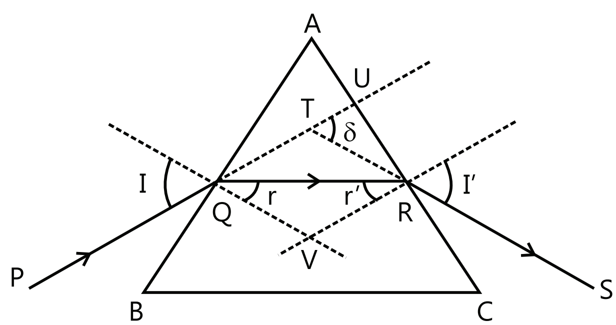


Figure 16.86

$$\angle UTS=\angle TQR+\angle TRQ$$

or $\quad \delta=(\angle TQV-\angle RQV)+(\angle TRV-\angle QRV)$

$$=(i-r)+(i'-r')\qquad =(i+i')-(r+r') \qquad \text{.... (i)}$$

Now, the sum of four angles of the quadrangle AQVR is 360°. The angles AQV and ARV both are 90°. Thus,

$A+\angle QVR=180°$.

Also, from the triangle QRV, $\quad r+r'+\angle QVR=180°$

Hence $\quad r+r'=A$

Substituting in Eq. (i) $\quad \delta=i+i'-A$

8.1 Angle of Minimum Deviation

The angle *i'* is determined by the angle of incidence *i*. Thus, the angle of deviation δ is also determined by *i*. For a particular value of *i*, δ is minimum. In this case, the ray passes symmetrically through the prism, so that *i=i'*.

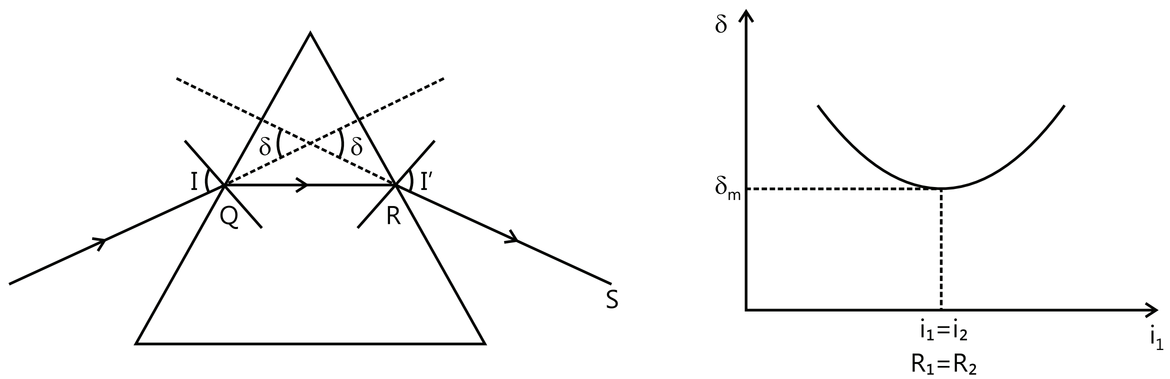


Figure 16.87

8.2 Relation between the Refractive index and the Angle of Minimum Deviation

Let the angle of minimum deviation be δ_m. For a minimum deviation, i=i' and r=r'.

We have $\delta_m = i + i' - A \quad = 2i - A$

or $i = \dfrac{A+\delta_m}{2}$ ... (i)

Also $r + r' = A$; or $r = A/2$ ... (ii)

The refractive index is $\mu = \dfrac{\sin i}{\sin r}$.

Using (i) and (ii) $\mu = \dfrac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}}$.

If the angle of prism A is small, δ_m is also small. Then, the equation becomes

$$\mu = \frac{\dfrac{A+\delta_m}{2}}{\dfrac{A}{2}} \Rightarrow \delta_m = (\mu - 1)A.$$

Illustration 32: The angle of minimum deviation from a prism is 37°. If the angle of prism is 53°, find the refractive index of the material of the prism. **(JEE MAIN)**

Sol: For prism, the refractive angle is given by $\mu = \dfrac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}}$

$$\mu = \frac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}} = \frac{\sin\dfrac{53^0+37^0}{2}}{\sin\dfrac{53^0}{2}} = \frac{\sin 45^0}{\sin 26.5^0} = 1.58.$$

Illustration 33: A ray of light passes through a glass prism such that the angle of incidence is equal to the angle of emergence. If the angle of emergence is ¾ times the angle of the prism, then calculate the angle of deviation when the angle of prism is 30°. **(JEE MAIN)**

Sol: For prism, we have relation $i+e=A+\delta$, where i and e are angle of incidence and emergence respectively. A is angle of prism and δ is angle of minimum deviation

Given that *i=e*, $e=\left(\frac{3}{4}\right)A$ and $A=30°$.

Using the relation $i+e=A+\delta$, we get $\delta=i+e-A=e+e-A=2e-A$

$=2\times\left(\frac{3}{4}\right)A-A=0.5A=0.5\times30^0=15^0$.

8.3 Condition of No Emergence

In this section, we want to find the condition such that a ray of light entering the face AB does not come out of the face AC for any angle i_I, i.e. TIR takes place on AC

$r_1+r_2=A$; $\therefore$ $r_2=A-r_1$

or $(r_2)_{min}=A-(r_1)_{max}$. ... (x)

Now, r_1 will be maximum when i_I is maximum, and the maximum value of i_I is 90°

Hence, $\mu=\frac{\sin(i_I)_{max}}{\sin(r_I)_{max}}=\frac{\sin90^0}{\sin(r_1)_{max}}$

$\text{sir}(r_I)_{max}=\frac{1}{\mu}=\sin\theta_c$; $\therefore$ $(r_I)_{max}=\theta_c$.

From Eq. (x) $(r_2)_{min}=A-\theta_c$ . ... (xi)

Now, if the minimum value of r_2 is greater than θ_C, then obviously all the values of r_2 will be greater than θ_c and TIR will take place under all the conditions. Thus, the condition of no emergence is,

$(r_2)_{min}>\theta_c$ or $A-\theta_c>\theta_c$ .... (xii)

or $A>2\theta_c$.

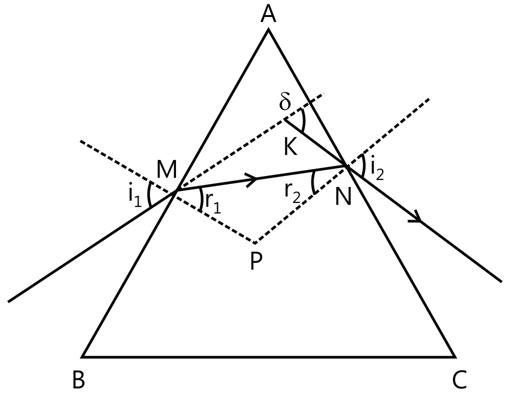


Figure 16.88

MASTERJEE CONCEPTS

Equation $r_1 + r_2 = A$ can be applied at any of the three vertices. For example in the Fig. 16.89, $r_1 + r_2 = B$.

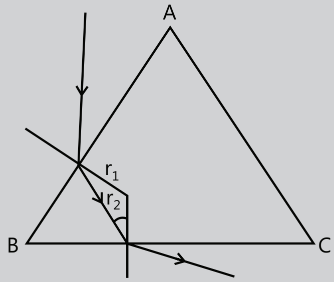


Figure 16.89

Anand K (JEE 2011 AIR 47)

9. DISPERSION OF LIGHT

When a beam of white light that consists of spectrum of various wavelengths ranging from long wavelengths in red color to short wavelengths in violet color passes through a prism, it is split into its constituent colors. This phenomenon is called dispersion. The dispersion of light takes place because the refractive index μ of a medium depends upon the wavelength λ of light according to the Cauchy's relation $\mu \simeq A + \frac{B}{\lambda^2}$, where μ is maximum for violet and minimum for red color.

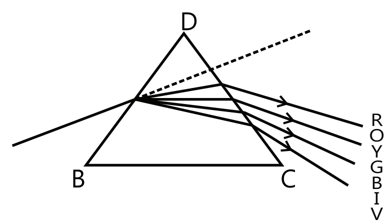


Figure 16.90

9.1 Angle of Dispersion

It is defined as an angle between emerging violet and red color rays, i.e. angle of dispersion is given by $\alpha = \delta v - \delta_R \; ; = A\left(\mu_V - \mu_R\right)$.

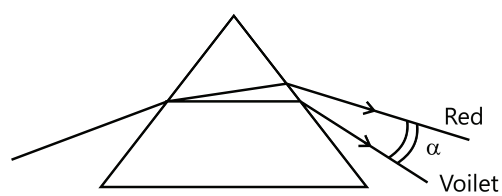


Figure 16.91

9.2 Dispersive Powers

The ratio of the angle of dispersion to the angle of deviation of the mean yellow color, δ, of the ray produced by any prism is called the dispersive power ω of the material of the prism

Dispersive power $= \omega = \frac{\delta_v - \delta_R}{\delta}$.

Also, $= \omega = \frac{\mu_v - \mu_R}{\mu - 1}$, where μ_v, μ_R and μ are the refractive indices of violet, red and mean yellow colors respectively.

9.3 Deviation without Dispersion

If two suitable prisms of small angle made up of two different transparent materials are combined and placed opposite to each other, then the net dispersion is equal to zero, and these prisms are called an achromatic pair of prisms that produce a deviation of ray of light, but without any dispersion.

For the two prisms of angle *A* and *A'* and refractive indices for violet and red colors μ_v, μR and $\mu_v{'}$ $\mu R{'}$, respectively, the net dispersion $= \left(\mu_v - \mu_R\right)A + \left(\mu_v{'} - \mu_R{'}\right)A' = 0$

$$A' = -\frac{(\mu_v - \mu_R)A}{(\mu'_v - \mu'_R)}$$

If ω and ω′ are dispersive powers of these prisms, and δ and δ' are their mean deviations, then

$$\frac{(\mu_v - \mu_R)}{(\mu - 1)}(\mu - 1)A + \frac{\mu'_v - \mu'_R}{\mu' - 1}(\mu' - 1)A' = 0$$

$\omega\delta + \omega'\delta' = 0.$

9.4 Dispersion without Deviation

If the deviation produced by the first prism for the mean ray is equal and opposite to that produced by the second prism, the combination of such two prisms produces dispersion without any deviation. For zone net deviation, $\delta + \delta' = 0$.

$(\mu - 1)A + (\mu' - 1)A' = 0$; $\quad A' = -\frac{(\mu - 1)A}{(\mu' - 1)}$, (− sign denotes that the second prism is inverted),

where μ and μ′ are refractive indices for the mean color for prisms of angles *A* and *A*′, respectively.

Note: Most of the problems of prisms can be easily solved by drawing proper ray diagram and then applying laws of geometry with the basic knowledge of prism formulae.

Illustration 34: The angle of a prism is *A*, and its one surface is coated with silver. A light ray falling at an angle of incidence 2*A* on the first surface returns back through the same path after suffering reflection at the second silvered surface. Find the refractive index of material. **(JEE MAIN)**

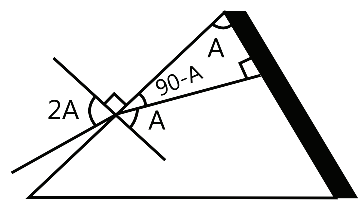


Figure 16.92

Sol: According to Snell's law we have $\mu = \frac{\sin i}{\sin r}$. And the angle of refraction r =90-i

Given i = 2A. From the figure, it can be obtained that *r* = *A*.

$$\therefore \mu = \frac{\sin i}{\sin r} = \frac{\sin 2A}{\sin A} = \frac{2\sin A \cos A}{\sin A} = 2\cos A.$$

Illustration 35: A crown glass prism of angle 5° is to be combined with a glass prism in such a way that the mean ray passes undeviated. Find (i) the angle of the flint glass prism needed and (ii) the angular dispersion produced by the combination when white light passes through it. Refractive indices for red, yellow and violet color light are 1.514, 1.523, respectively, for crown glass and 1.613, 1.620 and 1.632, respectively, for flint glass. **(JEE ADVANCED)**

Sol: For the angle of minimum deviation, we have the relation $\delta = (\mu - 1)A$. As prism of two different materials are joined together, the ratio $\frac{\delta'}{\delta}$ gives the angle of minimum deviation for flint glass is obtained.

The deviation produced by the crown prism is $\delta = (\mu - 1)A$, and by the flint prism is $\delta' = (\mu' - 1)A'$.

The prisms are placed at their inverted angles with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is

$$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A' \qquad \text{... (i)}$$

(i) If the net deviation for the mean ray is zero, $(\mu - 1)A = (\mu' - 1)A'$

or $\quad A' = \frac{(\mu - 1)}{(\mu' - 1)}A \quad = \frac{1.517 - 1}{1.620 - 1} x 5^0 = 4.2^0.$

(ii) The angular dispersion produced by the crown prism is $\delta_v - \delta_r = (\mu_v - \mu_r)A$, and that by the flint prism is $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A'$.

The net angular dispersion is

$$\delta = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A' = (1.523 - 1.514)x5^0 - (1.632 - 1.613)x4.2^0 = -0.0348^0.$$

The angular dispersion is 0.0348^0.

Illustration 36: An isosceles glass prism has one of its faces coated with silver. A ray of light is incident normally on the other face (which has an equal size to the silvered face). The ray of light is reflected twice on the same-sized faces and then emerges through the base of the prism perpendicularly. Find the angles of prism.

(JEE ADVANCED)

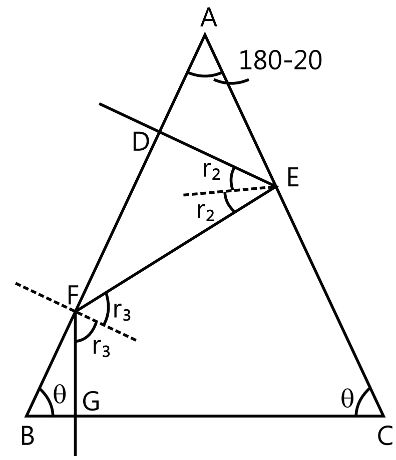


Figure 16.93

Sol: The angles of prism add up to 180°.

$r_1 = 0 \quad \therefore r_2 = A = 180^0 - 2\theta$ ... (i)

$\angle DFE = 180^0 - 90^0 - 2r_2 = 180^0 - 90^0 - 360^0 + 4\theta = 4\theta - 270^0$ ... (ii)

$\therefore \quad r_3 = 90^0 - \angle DFE = 360^0 - 4\theta$ ... (iii)

$\angle BFG = 90^0 - \theta = 90^0 - r_3$; or $r_3 = \theta$ ... (iv)

From Eqs (iii) and (iv),

$5\theta = 360^0$; $\quad \therefore \theta = 72^0$ and $180^0 - 2\theta = 36^0$.

$\therefore$ Angles of prism are 72°, 72° and 36°.

10. DEFECTS OF IMAGES

The relations developed regarding the formation of images by lenses and mirrors are approximate, which do not produce focused, perfect image of objects. The imperfections or defects in such images are called aberrations. If the defect in image is due to such approximations, these can produce spherical aberration apart from coma, distortion, astigmatism and so on. If the defect in image is due to the dispersion of white light into constituent colors similar to that in prism, it is called chromatic aberration producing blurred colored image of the object. The spherical and chromatic aberrations are briefly described in the following subsections:

10.1 Spherical Aberration

This aberration is produced due to the spherical nature of a lens or a mirror. The rays of light from point object when incident near the principal axis called paraxial rays are focused at a larger distance at point I_M, and the rays incident near the periphery or margin of the lens are focused near the lens at I_P as shown in the Fig. 16.94. It gives a rise to a defocused long image of a point object. The spherical aberration can be reduced by any one of the following methods:

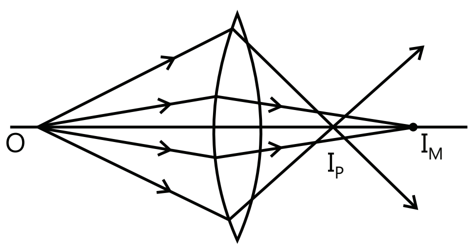


Figure 16.94

(a) Using two thin convex lenses of focal lengths f_1 and f_2 separated at a distance $d = f_1 - f_2$.

(b) Using a plano-convex lens with its convex surface facing the incident or emergent light, whichever is more parallel.

(c) Either using a lens of large focal length or using a specially designed aplanatic lens, crossed lens or parabolic reflectors.

10.2 Chromatic Aberration

As the refractive index for different wavelengths or colors comprising white or any composite light is different, the image of an object illuminated by white light is colored due to the chromatic aberration. Such a defect can by removed by using an achromatic doublet comprising a convex and a concave lens of focal lengths f_1 and f_2 and dispersive powers ω_1 and ω_2 such that the ratio of their focal lengths is equal to the ratio of their dispersive powers.

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0 \qquad \text{or} \qquad \frac{\omega_1}{\omega_2}=-\frac{f_1}{f_2},$$

where dispersive power $\omega=\frac{\mu_v-\mu_r}{\mu-1}$ and μ_v, μ_r and μ are respective indices for wavelengths of violet, red and mean yellow colors of white light.

11. OPTICAL INSTRUMENTS

Optical instruments are used to assist the eye in viewing an object. Let us first discuss about the human eye and the mechanism through which we see.

11.1 The Eye

The eye has a nearly spherical shape of diameter 1 inch each. Following are some of the terms related to the eye.

(a) **Cornea** – The front portion of the eye is more sharply curved and is covered by a transparent protective membrane called the cornea.

(b) **Aqueous humor** – Behind the cornea, there is a space filled with a liquid called aqueous humor.

(c) **Crystalline lens** – The part just behind aqueous humor is called crystalline lens.

(d) **Iris** – It is the muscular diaphragm between the aqueous humor and lens and is the colored part that we see in the eye.

(e) **Pupil** – The small hole in the iris is called the pupil. Varying aperture of the pupil controls the amount of light entering into the eye with the help of iris.

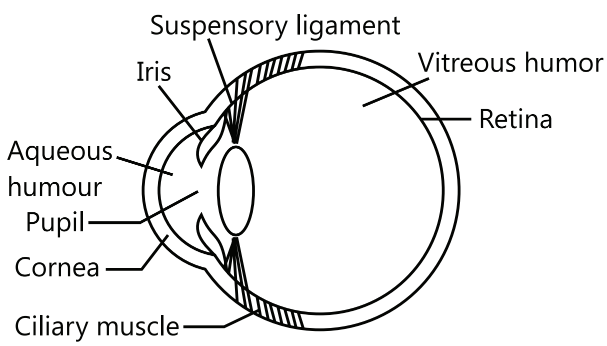


Figure 16.95

(f) **Retina** – This is a screen-like structure on which the eye forms an image. The retina contains rods and cones that receive light signal.

(g) **Accommodation –** When the eye is focused on a distant object, the ciliary muscles are relaxed so that the focal length of the eye lens has its maximum value that is equal to its distance from the retina. The parallel rays that enter into the eyes are focused on the retina, and we see the object clearly. When the eye is focused on a closer object, the ciliary muscles are strained and the focal length of the eye lens decreases. The ciliary muscles adjust the focal length in such a way that the image is again formed on the retina and we see the object clearly. This process of adjusting focal length is called accommodation. However, the muscles cannot be strained beyond a limit, and hence, if the object is brought too close to the eye, the focal length cannot be adjusted to form the image on the retina. Thus, there is a minimum distance for the clear vision of an object.

The nearest point at which the image can be formed on the retina is called the near point of the eye. The distance of the near point from the eye is called the least distance for clear vision. This varies from person to person and with age. At a young age (say below 10 years), the muscles are strong and flexible and can bear more strain. The near point may be as close as 7−8 cm at this age. In old age, the muscles cannot bear more strain and the near point shifts to large values, say 1 to 2 m or even more. We shall discuss about these defects of vision and use of glasses in a later section. The average value of the least distance for clear vision for a normal eye is generally 25 cm.

11.2 Apparent Size

The size of an object is related to the size of the image formed on the retina. A larger image on the retina activates larger number of rods and cones attached to it, and the object looks larger. As it is clear from the Fig. 16.96, if an object is taken away from the eye, the size of the image on the retina decreases, and hence, the same object looks smaller. Furthermore the size of the image on the retina is roughly proportional to the angle subtended by the object on the eye. This angle is called the visual angle, and optical instruments are used to increase this artificially in order to improve the clarity.

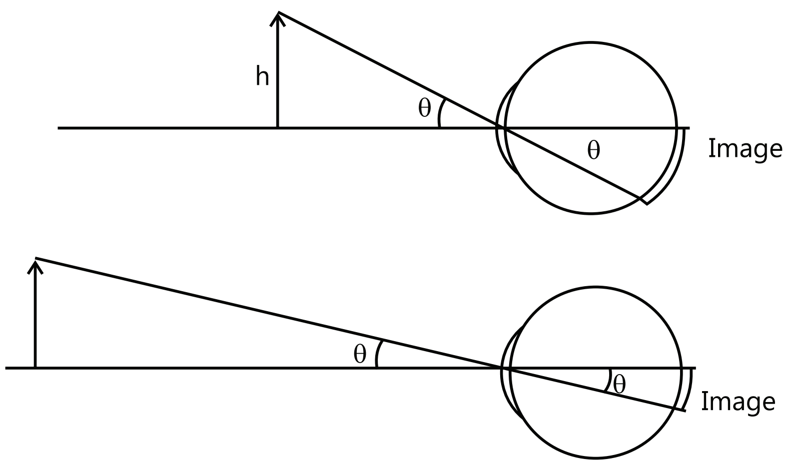


Figure 16.96

Illustration 37: Two boys, the one is 52 inches tall and the other 55 inches tall, are standing at distances 4.0 m and 5.0 m, respectively, from an human eye. Which boy will appear taller? **(JEE MAIN)**

Sol: The angle subtended by any object is given by $\alpha = \frac{\text{height of object}}{\text{Distance of object from observer}}$.

The boy which subtends the larger angle will appear taller.

The angle subtended by the first boy on the eye is $\alpha_1 = \frac{52\,\text{inch}}{4.0\text{m}} = 13\,\text{inch/m}$.

And the angle subtended by the second boy is $\alpha_2 = \frac{52\,\text{inch}}{5.0\text{m}} = 11\,\text{inch/m}$.

As $\alpha_1 > \alpha_2$, the first boy will look taller when seen through the eye.

12. SIMPLE MICROSCOPE

When we view an object with naked eyes, the object must be placed somewhere between infinity and the near point. The angle subtended on the eye is maximum when the object is placed at the near point. This angle is

$$\theta_0 = \frac{h}{D}, \qquad \text{... (i)}$$

where *h* is the size of the object, and *D* is the least distance for clear vision.

This angle can further be increased if a converging lens of short focal length that is called a simple microscope or a magnifier is placed just in front of the eye.

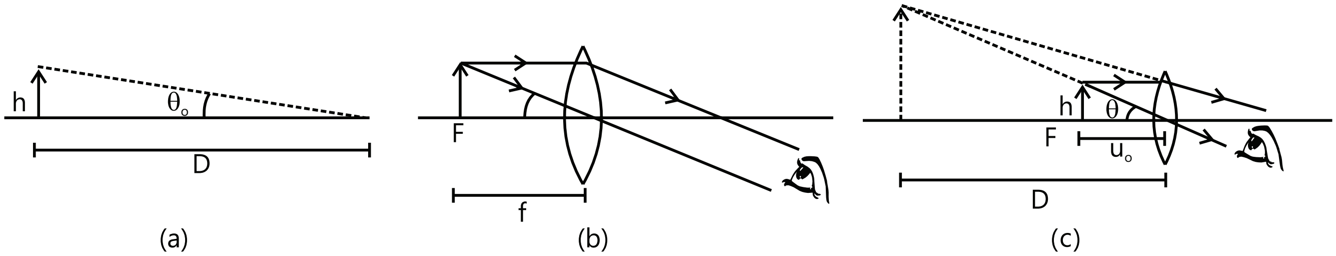


Figure 16.97

Suppose, the lens has a focal length f that is lesser than *D* and let us move the object to the first focal point F. The eye receives the rays that come from infinity. The actual size of the image is infinite, but the angle subtended on the lens (and hence on the eye) is

$$\theta = \frac{h}{f} \qquad \text{... (ii)}$$

As $f < D$, Eqs (i) and (ii) show that $\theta > \theta_0$. Hence, the eye perceives a larger image than it could have had without the microscope. Because the image is occurred at infinity, the ciliary muscles are least strained to focus the final image on the retina. This is called normal adjustment. The magnifying power of a microscope is θ / θ_0, where θ is the angle subtended by the image on the eye when the microscope is used, and θ_0 is the angle subtended on the naked eye when the object is placed at the near point. This is also known as the angular magnification. Thus, the magnifying power is a factor by which the image on the retina can be enlarged by using the microscope.

In the normal adjustment, the magnifying power of a simple microscope is by Eqs. (i) and (ii),

$$m = \frac{\theta}{\theta_0} = \frac{h/f}{h/D} \quad \text{or} \quad m = \frac{D}{f}$$

If $f < D$, the magnifying power is greater than 1.

The magnifying power can further be increased by moving the object more closer to the lens. Suppose we move the object to a distance u_0 from the lens such that the virtual, erect image is formed at the near point, although the eye is strained, it can still see the image clearly. The distance u_0 is calculated using the lens formula,

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}.$$

Here, $v = -D$ and $u = -u_0$, so that

$$\frac{1}{-u_0} = -\frac{1}{D} - \frac{1}{f}; \qquad \text{or,} \quad \frac{D}{u_0} = 1 + \frac{D}{f} \qquad \text{... (iii)}$$

The angle subtended by the image on the lens (and hence on the eye) is $\theta' = \frac{h}{u_0}$

In this case, the angular magnification or magnifying power is

$$m = \frac{\theta'}{\theta_0} = \frac{h/u_0}{h/D} = \frac{D}{u_0} = 1 + \frac{D}{f}$$

The above equations show that the magnification is high when the focal length f is small. However, due to several other aberrations, the image becomes too defective at a large magnification with simple microscope. Approximately, a magnification up to 4 is trouble-free.

The magnifying power is measured in a unit X; therefore, if a magnifier produces an angular magnification of 10, it is called as 10 X magnifier.

13. COMPOUND MICROSCOPE

The Fig. 16.98 shows a simplified version of a compound microscope and the ray diagram of image formation. It consists of two converging lenses set coaxially. The one that faces the object is called the objective, and the one that is close to the eye is called the eyepiece or ocular. The objective has a smaller aperture and smaller focal length than the eyepiece. The distance between the objective and the eyepiece can be varied by appropriate screws fixed on the panel of the microscope.

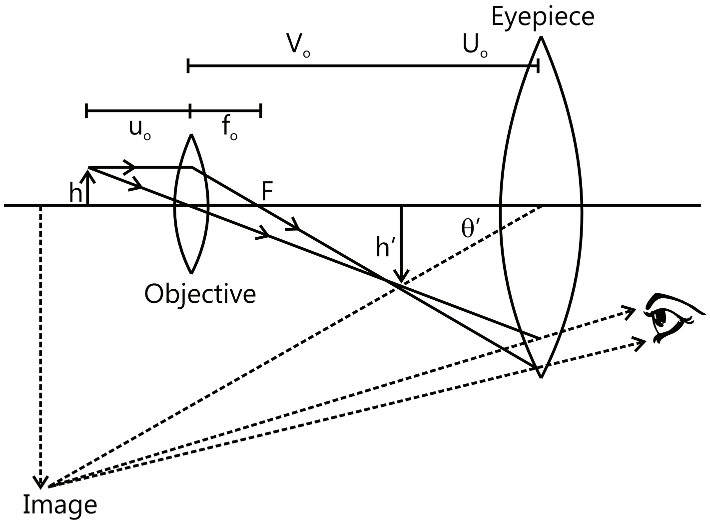


Figure 16.98

The object is placed at a distance u_0 from the objective which is slightly greater than its focal length f_0. A real image and an inverted image are formed at a distance v_0 on the other side of the objective. This image becomes the object for the eyepiece. For normal adjustment, the position of the eyepiece is adjusted such that the image formed by the objective falls in the focal plane of the eyepiece. Then, the final image is formed at infinity. It is erect with respect to the first image and, hence, inverted with respect to the object. The eye is least strained in this adjustment as it has to focus the parallel rays coming toward it. The position of the eyepiece can also be adjusted in such a way that the final virtual image is formed at the near point. The angular magnification is increased in this case. The ray diagram in the Fig. 16.98 refers to this case.

The eyepiece acts as a simple microscope effectively used to view the first image. Thus, the magnification by a compound microscope is a two-step process. In the first step, the objective produces a magnified image of the given object. In the second step, the eyepiece produces an angular magnification. The overall angular magnification is the product of the two.

Magnifying power

Refer to the figure, if an object of height *h* is seen by the naked eye and placed at the near point, the largest image is formed on the retina. The angle formed by the object on the eye in this situation is $\theta_0 = \frac{h}{D}$. ... (i)

When a compound microscope is used, the final image subtends an angle θ′ on the eyepiece (and hence on the eye) given by $\theta' = \frac{h'}{u_e'}$ (ii)

Where *h*′ is the height of the first image, and u_e is the distance between the first image and the eyepiece.

The magnifying power of the compound microscope is, therefore,

$$m = \frac{\theta'}{\theta_0} = \frac{h'}{u_e} x \frac{D}{h} = \left(\frac{h'}{h}\right)\left(\frac{D}{u_e}\right) \quad \text{... (iii)}$$

Also from the figure $\frac{h'}{h} = -\frac{v_0}{u_0} = \frac{v}{u}$ ... (iv)

Now, D/u_e is the magnifying power of the eyepiece that acts as a simple microscope. Using the equations given above, in normal adjustment, this value becomes D/f_e when the image is formed at infinity and $1 + D/f_e$ when the image is formed at the least distance for clear vision, i.e. at *D*. Thus, for the normal adjustment, the magnifying power of the compound microscope is, by Eq. (iii), $m = \frac{v}{u}\left(\frac{D}{f_e}\right)$ when the image is formed at infinity and is $m = \frac{v}{u}\left(1 + \frac{D}{f_e}\right)$ when the final image is formed at the least distance for clear vision.

Using lens equation for the objective,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \quad 1 - \frac{v}{u} = \frac{v}{f_0} \quad \Rightarrow \quad \frac{v}{u} = 1 - \frac{v}{f_0}$$

In general, the focal length of the objective is very small, so that $\frac{v}{f_0} >> 1$. Furthermore, the first image is close to the eyepiece, so that $v \approx l$, where *l* is the tube length (distance between the objective and the eyepiece). Thus, $\frac{v}{u} = 1 - \frac{v}{f_0} \approx -\frac{v}{f_0} \approx -\frac{l}{f_0}$.

If these conditions are satisfied, for the normal adjustment, the magnifying power of the compound microscope is $m = -\frac{l}{f_0}\frac{D}{f_0}$ when the image is formed at infinity and is $m = -\frac{l}{f_0}\left(1 + \frac{D}{f_0}\right)$ when the final image is formed at the least distance for clear vision.

In an actual compound microscope, all the objectives and the eyepieces consist of a combination of several lenses instead of a single lens assumed in the simplified version.

Illustration 38: A compound microscope has an objective of focal length 1 cm and an eyepiece of focal length 2.5 cm. An object has to be placed at a distance of 1.2 cm away from the objective for normal adjustment.

(i) Find the angular magnification.

(ii) Find the length of the microscope tube. **(JEE ADVANCED)**

Sol: As the objective lens of microscope is convex lens, the focal length is obtained as $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ and magnification is $m = \frac{v}{u}$. The length of microscopic tube is given as $L = v + f_e$ where v is the distance of image formed by objective.

(i) If the first image is formed at a distance *v* from the objective, we get

$$\frac{1}{v} - \frac{1}{(-1.2\,cm)} = \frac{1}{1\,cm} \qquad \text{or,} \quad v = 6\,cm.$$

The angular magnification in normal adjustment is $m = \frac{v}{u}\frac{D}{f_e} = -\frac{6\,cm}{1.2\,cm}.\frac{25\,cm}{2.5\,cm} = -50$.

(ii) For normal adjustment, the first image must be in the focal plane of the eyepiece. The length of the tube is, therefore, $L = v + f_e$ =6 cm + 2.5 cm = 8.5 cm.

14. TELESCOPES

A microscope is used to view the object placed close to it, i.e. within few centimeters. To look at the distant objects such as stars, planets and a distant tree, we use telescope. There are three types of telescopes that are used.

(A) Astronomical Telescope

The Fig. 16.99 shows the construction and working principle of a simplified version of an astronomical telescope.

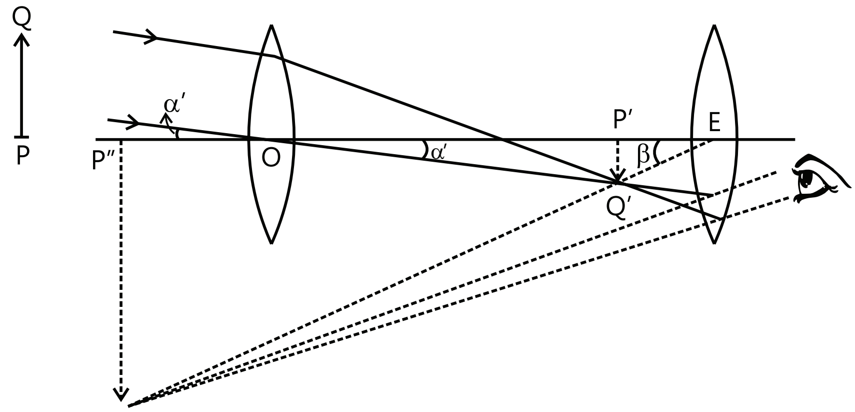


Figure 16.99

The telescope consists of two converging lenses placed coaxially. The one that faces the distant object is called the objective, and it has larger aperture and focal length. The other is called the eyepiece, as it is placed closer to the eye and has smaller aperture and focal length. The lenses are fixed in tubes. The eyepiece tube can slide within the objective tube so that the distance between the objective and the eyepiece can be changed.

When the telescope is directed toward a distant object PQ, the objective forms a real image of that object in its focal plane. If the point P is on the principal axis, the image point P′ is at the second focus of the objective. The rays from Q are focused at Q′. The eyepiece forms a magnified virtual image P″Q″ of P′Q′. This image is finally seen by the eye. In normal adjustment, the position is adjusted such that the final image is formed at infinity. In such a case, the first image P′Q′ is formed in the first focal plane of the eyepiece. The eye is least strained to focus this final image. The image can be brought closer by pushing the eyepiece closer to the first image. A maximum angular magnification is produced when the final image is formed at the near point.

Magnifying Power

Let the objective and the eyepiece have focal lengths f_0 and f_e, respectively, and the object is placed at a large distance u_0 from the objective. The object PQ in the Fig. 16.99 subtends an angle α on the objective. Since the object is at infinity, the angle it would subtend on the eye, if there were no telescope, is α′.

As u_0 is very large, the first image P′Q′ is formed in the focal plane of the objective.

From the figure $$|\alpha| = |\alpha'| \approx |\tan\alpha'| = \frac{P'Q'}{OP'} = \frac{P'Q'}{f_0} \quad \text{... (i)}$$

The final image P″Q″ subtends an angle β on the eyepiece (and hence on the eye). From the triangle P′Q′E,,

$$|\beta| \approx |\tan\beta| = \frac{P'Q'}{EP'} \Rightarrow \left|\frac{\beta}{\alpha}\right| = \frac{f_0}{EP'} \quad \text{... (ii)}$$

If the telescope is adjusted for normal adjustment so that the final image is formed at infinity, the first image P′Q′ must be in the focal plane of the eyepiece.

Then $EP' = f_e$.

Thus, Eq. (ii) becomes $$\left|\frac{\beta}{\alpha}\right| = \frac{f_0}{f_e}. \quad \text{... (iii)}$$

The angular magnification or the magnifying power of the telescope is

$$m = \frac{\text{Angle subtended by the final image on the eye}}{\text{Angle subtended by the object on the unaided eye}}.$$

The angles β and α are formed on the opposite sides of the axis. Hence, the signs of these angles are opposite, and β/α is negative. Hence, $m = \frac{\beta}{\alpha} = -\left|\frac{\beta}{\alpha}\right|$.

Using Eq. (iii), $m = -\frac{f_0}{f_e}$.

If the telescope is adjusted so that the final image is formed at the near point of the eye, the angular magnification is further increased. Let us apply the lens equation to the eyepiece in this case.

Here, $u = -EP'$ and $v = -EP'' = -D$.

The lens equation is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{-D} - \frac{1}{-EP'} = \frac{1}{f_e} \qquad \Rightarrow \qquad \frac{1}{-EP'} = \frac{1}{f_e} + \frac{1}{D} = \frac{f_e + D}{f_e D} \qquad \text{... (iv)}$$

By Eq. (ii), $\left|\frac{\beta}{\alpha}\right| = \frac{f_0(f_e + D)}{f_e D}$

The magnification is $m = \frac{\beta}{\alpha} = -\left|\frac{\beta}{\alpha}\right| = -\frac{f_0(f_e + D)}{f_e D} = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$

Length of the Telescope

From the Fig. 16.106, we see that the length of the telescope is $L = OP' + P'E' = f_0 + P'E$.

For normal adjustment, $P'E = f_e$ so that $L = f_0 + f_e$. For adjustment for the near-point vision, we get, by Eq. (iv),

$P'E = \frac{f_e}{f_e + D}$, so that the length is $L = f_0 + \frac{f_e D}{f_e + D}$.

14.1 Resolving Power of a Telescope

The resolving power of a microscope is defined as the reciprocal of the distance between two objects, which can be resolved when seen through the microscope. It depends on the wavelength λ of the light, the refractive index μ of the medium between the object and the objective of the microscope and the angle θ subtended by a radius of the objective on one of the object. It is given by

$$R = \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda}$$

To increase the resolving power, the objective and the object are kept immersed in oil. It increases μ and hence R.

The resolving power of a telescope is defined as the reciprocal of the angular separation between two distant objects which are just resolved when viewed through a telescope. It is given by

$$R = \frac{1}{\Delta\theta} = \frac{a}{1.22\,\lambda},$$

where *a* is the diameter of the objective of the telescope. The telescopes with larger objective aperture (1 m or more) are used in astronomical studies.

15. DEFECTS OF VISION

As described earlier, the ciliary muscles control the curvature of the lens in the eye and hence can change the effective focal length of the system. When the muscles are fully relaxed, the focal length is maximum. When the muscles are strained, the curvature of the lens increases and the focal length decreases. For clear vision, the image must be formed on the retina. The image distance is, therefore, fixed for clear vision, and it equals the distance of the retina from the eye lens. It is about 2.5 cm for a grown-up person. If we apply the lens formula to the eye, the magnitudes of the object distance, the image distance and the effective focal length satisfy

$$\frac{1}{v_0}+\frac{1}{u_0}=\frac{1}{f} \quad \text{or} \quad \frac{1}{u_0}=\frac{1}{f}-\frac{1}{v_0} \qquad \text{... (i)}$$

Here, v_0 is fixed, and hence by changing f, the eye is focused on the objects placed at different values of u_0. We see from Eq. (i) that when f increases, u_0 increases, and when f decreases, u_0 decreases. The maximum distance one can see is

$$\frac{1}{u_{max}}=\frac{1}{f_{max}}-\frac{1}{v_0}, \qquad \text{... (ii)}$$

where f_{max} is the maximum focal length possible for the eye lens.

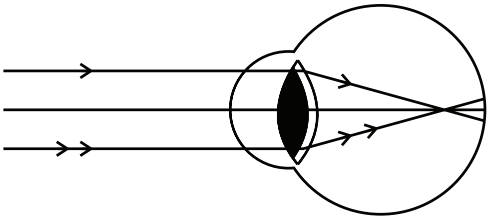
The focal length is maximum when the ciliary muscles are fully relaxed. In a normal eye, this focal length equals the distance v_0 from the lens to the retina. Thus,

$v_0 = f_{max}$ by (ii), $u_{max} = \infty$.

Theoretically, a person can have clear vision of the objects placed at any large distance from the eye. For the closer objects, u is smaller, and hence, f should be smaller. The smallest distance at which a person can have a clear vision is related to the minimum possible focal length f. The ciliary muscles are most strained in this position. By Eq. (ii), the closest distance for clear vision is given by

$$\frac{1}{u_{min}}=\frac{1}{f_{min}}-\frac{1}{v_0}. \qquad \text{... (iii)}$$

For an average grown-up person, u_{min} should be around 25 cm or less. This is a convenient distance at which one can hold an object in his/her hand and can see. Thus, a normal eye can clearly see objects placed in the range from about 25 cm from the eye to a large distance of the order of several kilometers. The nearest point and the farthest point up to which an eye can clearly see are called the near point and the far point. For a normal eye, the distance of the near point should be around 25 cm or less, and the far point should be at infinity. We now describe some common defects of vision.



(A) Defective-eye

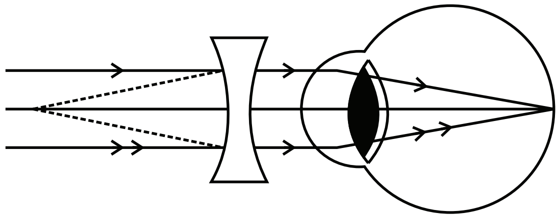
Figure 16.100

By the eye lens, real, inverted and diminished image is formed at retina.

The common defects of vision are as follows:

(a) Myopia or short sightedness: The distant objects are not clearly visible in this defect. The image of a distant object is formed before the retina.

The defect can be remedied by using a concave lens.



(B) Defective-eye

Figure 16.101

Illustration 39: A nearsighted man can clearly see the objects up to a distance of 1.5 m. Calculate the power of the lens of the spectacles necessary for the remedy of this defect. **(JEE MAIN)**

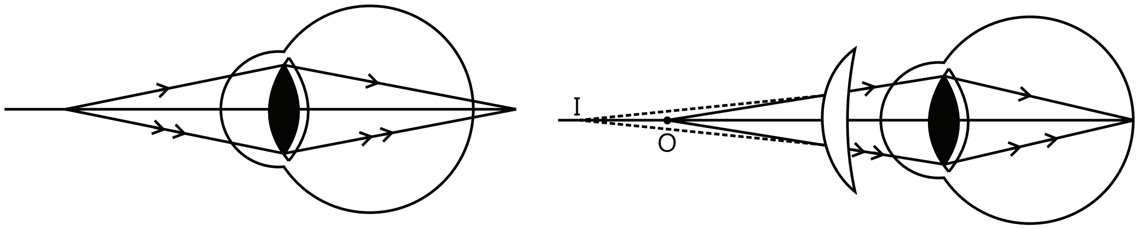
Sol: As the man has near sighted vision, he need to wear concave lens which can form virtual and erect images.

The power of magnification of lens is $P=\frac{1}{f}$.

The lens should form a virtual image of a distant object at 1.5 m from the lens. Thus, it should be a divergent lens, and its focal length should be −1.5 m. Hence,

$$f = -1.5\text{m} \quad \Rightarrow \quad P = \frac{1}{f} = -\frac{1}{1.5}\text{m}^{-1} = -0.67\text{D}.$$

(b) Hypermetropia or far sightedness: The near objects are not clearly visible in this defect. The image of a near object is formed behind the retina.



(A) Deflected-eye (B) Corrected-eye

Figure 16.102

This defect is remedied by using a convex lens.

(c) Presbyopia: In this defect, both near and far objects are not clearly visible. This is remedied either by using two separate lenses or by using a single spectacle having bifocal lenses.

(d) Astigmatism: In this defect, the eye cannot see objects in two orthogonal (perpendicular) directions clearly simultaneously. This defect is remedied by using a cylindrical lens.

MASTERJEE CONCEPTS

While testing your eye by reading a chart, if doctor finds it to 6/12, it implies that you can read a letter from 6 m which the normal eye can read from 12 m. Thus, 6/6 is the normal eye sight.

Worth Knowing: The persistence of vision is $\frac{1}{10}$ s, i.e. if the time interval between two consecutive light rays is less than 0.1 s, the eye cannot distinguish them separately. Hence, the fps (frames per second) of a video should be more than 10.

Anurag Saraf (JEE 2011 AIR 226)

PROBLEM-SOLVING TACTICS

1. Of *u*, *v* and *f*, any two values will be known to us and we will be asked to find the third. In such type of problems, two cases are possible.

Case 1: When signs of all the three will be known to us from the given information, substitute all the three with the known sign; then, we can get only the numerical value of the unknown (i.e. the third quantity) without sign.

Case 2: When the sign of the third unknown quantity is not known to us, substitute only the known quantities with sign. Then, the numerical value of the unknown with its respective sign can be obtained.

2. The experiments show that if the boundaries of the media are parallel, the emergent ray CD, although laterally displaced, is parallel to the incident ray AB if $\mu_1 = \mu_5$. We can also directly apply the Snell's law $\left(\mu \text{ sini} = \text{constant}\right)$ in media 1 and 5, i.e. $\mu_1 \text{ sini}_1 = \mu_5 \text{ sini}_5$ $\mu_1 \text{ sini}_1 = \mu_2 \text{ sini}_2 = \mu_3 \text{ sini}_3 = ... = \mu_i \text{ sini}_i$

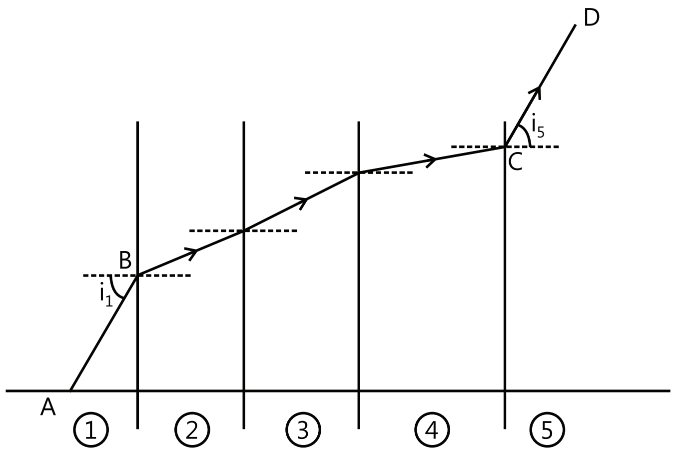


Figure 16.103

Notice that an apparent depth is multiple of either μ or $1/\mu$. It can be find out by knowing whether the medium through which light is entering is a denser or rarer medium.

3. Sometimes only a part of a prism will be given. To solve such problems, first complete the prism and then solve the problems.

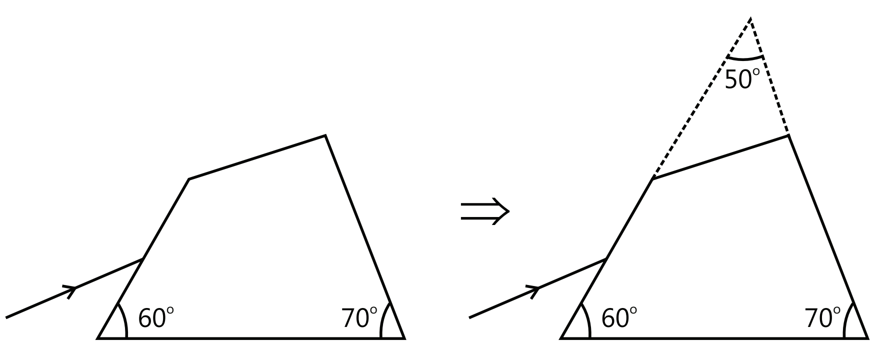
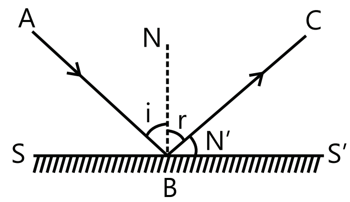
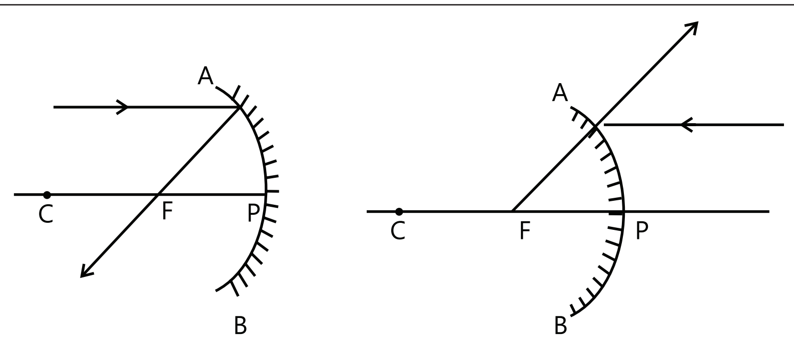
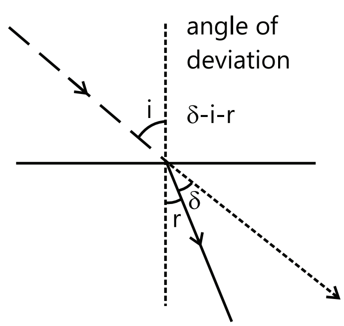


Figure 16.104

FORMULAE SHEET

S. No	KEY CONCEPTS	DESCRIPTIONS
1	**Law of reflection**	**(i)** The incident ray (AB), the reflected ray (BC) and normal (NN′) to the surface (SS′) of reflection at the point of incidence (B) lie in the same plane. This plane is called the plane of incidence (also the plane of reflection). **(ii)** The angle of incidence (the angle between the normal and the incident ray) and reflection angle (the angle between the reflected ray and the normal) are equal. $\angle i = \angle r$.  **Figure 16.105**
2	**Object**	**(a) Real:** Point from which rays actually diverge. **(b) Virtual:** Point toward which rays appear to converge.
3	**Image**	The image is decided by the reflected or refracted rays only. The point image for a mirror is that point (i) Toward which the rays reflected from the mirror actually converge (real image), OR (ii) From which the reflected rays appear to diverge (virtual image).
4	**Characteristics of reflection by a plane mirror**	(a) The size of the image is the same as that of the object. (b) For a real object, the image is virtual, and for a virtual object, the image is real. (c) For a fixed incident light ray, if the mirror is rotated through an angle θ, the reflected ray turns through an angle of 2θ.
5	**Spherical mirrors**	 Concave Convex **Figure 16.106**
6	**Paraxial rays**	Rays that form very small angle with principal axis are called paraxial rays.

S. No	KEY CONCEPTS	DESCRIPTIONS
7	**Sign convention**	We follow the Cartesian coordinate system convention according to which (a) The pole of the mirror is the origin. (b) The direction of the incident rays is a positive x-axis. (c) Vertically up is positive y-axis. **Note:** According to this, a convention radius of curvature and focus of concave mirror are negative and of convex mirror are positive.
8	**Mirror formula**	*f*=x-coordinate of focus; $\boxed{\frac{1}{f}=\frac{1}{v}+\frac{1}{u}}$, where *v*=x-coordinate of the image; *u*=x-coordinate of the object. **Note: Valid only for paraxial rays.**
9	**Transverse magnification**	$h_2 = y$ -coordinate of the image $\boxed{m=\frac{h_2}{h_1}=-\frac{v}{u}}$. $h_1 = y$ -coordinate of the object. (both are perpendicular to the principle axis of the mirror)
10	**Optical power**	Optical power of a mirror (in dioptres) = $-\frac{1}{f}$, where *f* is the focal length (in m) with a respective sign.
REFRACTION – PLANE SURFACE		
1	**Laws of refraction (at any refracting surface)**	**(i)** The incident ray (AB), the normal (NN') to the refracting surface (II') at the point of incidence (B) and the refracted ray (BC) all lie in the same plane called the plane of incidence or the plane of refraction. **(ii)** $\boxed{\frac{\sin i}{\sin r}=\text{Constant}:}$ for any two given media and light of a given wavelength. This is the Snell's law. $\boxed{\frac{\sin i}{\sin r}={}_1n_2=\frac{n_2}{n_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}}$ **Note:** The frequency of light does not change during refraction.
2	**Deviation of a ray due to refraction**	 **Figure 16.107**

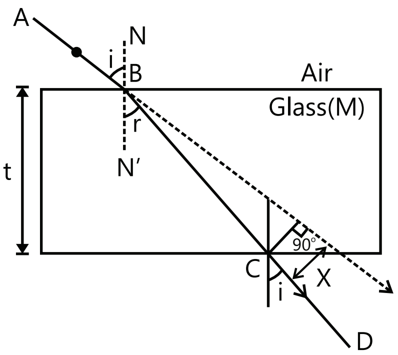
<table>
<tr><th>S. No</th><th>KEY CONCEPTS</th><th>DESCRIPTIONS</th></tr>
<tr><td>3</td><td>Refraction through a parallel slab</td><td>(i) Emerged ray is parallel to the incident ray, if medium is same on both sides.
(ii) Lateral shift $\boxed{x = \frac{t\sin(i-r)}{\cos r}}$,
where t= thickness of the slab.


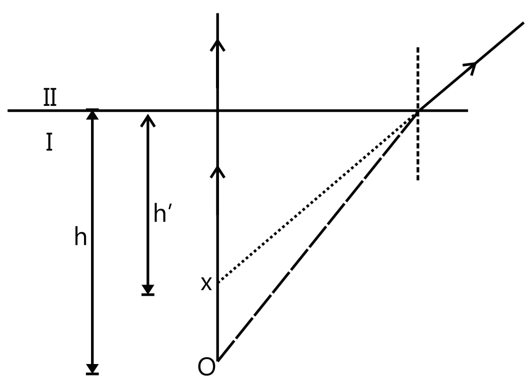
Figure 16.108
Note: Emerged ray is not parallel to the incident ray if the media on both the sides are different.</td></tr>
<tr><td>4</td><td>Apparent depth of a submerged object</td><td>At near normal incidence,
$\boxed{h' = \frac{\mu_2}{\mu_1}h}$


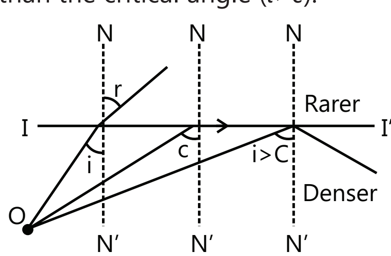
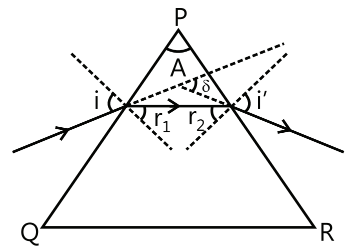
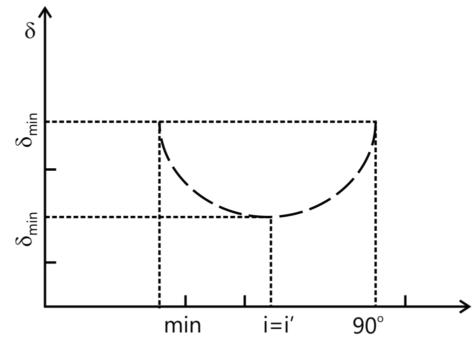
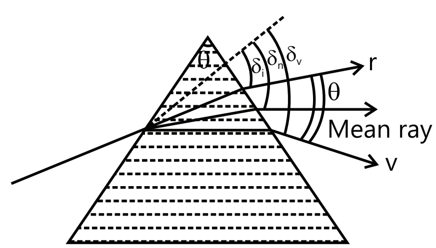
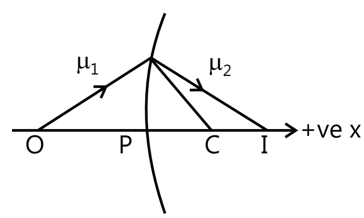
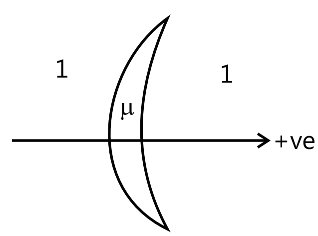
Figure 16.109
Note: h and h′ are always measured from the surface.</td></tr>
<tr><td>5</td><td>Critical angle & total internal reflection (TIR.)</td><td>(i) Ray travels from a denser to a rarer medium.
(ii) The angle of incidence should be greater than the critical angle (i>c).
Critical angle $\boxed{C = \sin^{-1}\frac{n_r}{n_i}}$


Figure 16.110</td></tr>
</table>

S. No	KEY CONCEPTS	DESCRIPTIONS
6	**Refraction through prism**	**1.** $\delta = (i + i') - (r + r')$ **.** **2.** $r + r' = A$. **3.** Variation in δ versus (shown in diagram). **4.** There is one and only one angle of incidence, for which the angle of deviation is minimum.  **Figure 16.111** When $\delta = \delta_m$ then $i = i'$ & $r = r'$, the ray passes symmetrically through the prism, and then (where *n*=absolute RI of glass), $n\frac{\sin\left[\frac{A+\delta m}{2}\right]}{\sin\left[\frac{A}{2}\right]}$.  **Figure 16.112** **Note:** When the prism is dipped in a medium, then (where *n*=RI of glass w.r.t. medium). **5.** For a thin prism, $\left(A < 10^0\right)$; $\delta = (n-1)A$. **6. Dispersion of light:** The angular splitting of a ray of white light into a number of components when it is refracted in a medium other than air is called dispersion of light. **7. Angle of dispersion:** An angle between the rays of the extreme colors in the refracted (dispersed) light is called angle of dispersion. $\theta = \delta_v - \delta_r$. **8. Dispersive power** (ω) of the medium of the material of prism. $(\omega) = \frac{\text{Angular dispersion}}{\text{Derivation of mean ray (yellow)}}$ For a small-angled prism, (A<10°) $\omega = \frac{\delta_v - \delta_R}{\delta_y} = \frac{n_v - n_R}{n-1}; n = \frac{n_v + n_R}{2}$,  **Figure 16.113** where n_v, n_R and *n* are RI of the material for violet, red and yellow colors, respectively.

S. No	KEY CONCEPTS	DESCRIPTIONS
		9. Combination of two prisms: **(i)** Achromatic combination: It is used for deviation without dispersion. Condition for this is $\left(n_v - n\right)A = \left(n'_v - n'_r\right)A'$. Net mean Deviation $= \left[\frac{n_v + n_R}{2} - 1\right] A - \left[\frac{n'_v + n'_R}{2} - 1\right] A'$. Or $\omega\delta + \omega'\delta' = 0$ where ω, ω' are dispersive powers for the two prisms and δ, δ' are the mean deviations. **(ii)** Direct vision combination: It is used to produce dispersion without deviation; condition for this is $\left[\frac{n_v + n_R}{2} - 1\right] A = \left[\frac{n'_v + n'_R}{2} - 1\right] A'$. Net angle of dispersion $\left(n_v - n\right)A - \left(n'_v - n'_r\right)A'$.
REFRACTION AT A SPERICAL SURFACE		
1		**(a)** $\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$; *v*, *u* a *R* are kept with sign. As *v* = PI *u* = −PO *R* = PC (**Note radius is with sign).** **(b)** $\boxed{m = \frac{\mu_1 v}{\mu_2 u}}$  **Figure 16.114**
2	**Lens formula**	**(a)** $\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$ **(b)** $\boxed{\frac{1}{f} = \left(\mu - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$ **(c)** $\boxed{m = \frac{v}{u}}$  **Figure 16.115**