Sol: (c) Adding Au_1 and Au_2 we get $A(u_1 + u_2)$. Then using the invariance method we obtain $u_1 + u_2$.

By adding, we have $A(u_1 + u_2) = Au_1 + Au_2 = \begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix}$

We then solve the above equation for $u_1 + u_2$, if we consider the augmented matrix $(A|B) = \left(\begin{array}{ccc|c}1 & 0 & 0 & 1\\2 & 1 & 0 & 1\\3 & 2 & 1 & 0\end{array}\right)$

Applying $R_3 \to R_3 - 2R_2 + R_1$ and $R_2 \to R_2 - 2R_1$, we get $(A|B) \sim \left(\begin{array}{ccc|c}1 & 0 & 0 & 1\\0 & 1 & 0 & -1\\0 & 0 & 1 & -1\end{array}\right) \Rightarrow u_1 + u_2 = \begin{pmatrix}1\\-1\\-1\end{pmatrix}$

PROBLEM-SOLVING TACTICS

If A, B are square matrices of order n, and I_n is a corresponding unit matrix, then

(a) $A(\text{adj}.A) = |A| I_n = (\text{adj } A) A$

(b) $|\text{adj } A| = |A|^{n-1}$ (Thus A (adj A) is always a scalar matrix)

(c) $\text{adj}(\text{adj}.A) = |A|^{n-2} A$

(d) $|\text{adj}(\text{adj}.A)| = |A|^{(n-1)^2}$

(e) $\text{adj}(A^T) = (\text{adj } A)^T$

(f) $\text{adj}(AB) = (\text{adj } B)(\text{adj } A)$

(g) $\text{adj}(A^m) = (\text{adj } A)^m, m \in N$

(h) $\text{adj}(kA) = k^{n-1}(\text{adj}. A), k \in R$

(i) $\text{adj}(I_n) = I_n$

(j) $\text{adj } 0 = 0$

(k) A is symmetric $\Rightarrow$ adj A is also symmetric

(l) A is diagonal $\Rightarrow$ adj A is also diagonal

(m) A is triangular $\Rightarrow$ adj A is also triangular

(n) A is singular $\Rightarrow |\text{adj } A| = 0$

FORMULAE SHEET

(a) Types of matrix:

(i) Symmetric Matrix: A square matrix $A = [a_{ij}]$ is called a symmetric matrix if $a_{ij} = a_{ji}$, for all i,j.

(ii) Skew-Symmetric Matrix: when $a_{ij} = -a_{ji}$

(iii) Hermitian and skew – Hermitian Matrix: $A = A^{\theta}$ (Hermitian matrix)

$A^{\theta} = -A$ (skew-Hermitian matrix)

(iv) Orthogonal matrix: if $AA^T = I_n = A^TA$

(v) Idempotent matrix: if $A^2 = A$

(vi) Involuntary matrix: if $A^2 = I$ or $A^{-1} = A$

(vii) Nilpotent matrix: if $\exists\ p \in N$ such that $A^P = 0$

(b) Trace of matrix:

(i) $tr(\lambda A) = \lambda\, tr(A)$

(ii) $tr(A + B) = tr(A) + tr(B)$

(iii) $tr(AB) = tr(BA)$

(c) Transpose of matrix:

(i) $(A^T)^T = A$ **(ii)** $(A \pm B)^T = A^T \pm B^T$ **(iii)** $(AB)^T = B^TA^T$ **(iv)** $(kA)^T = k(A)^T$

(v) $(A_1A_2A_3 \ldots\ldots A_{n-1}A_n)^T = A_n^T A_{n-1}^T \ldots\ldots A_3^T A_2^T A_1^T$ **(vi)** $I^T = I$ **(vii)** $tr(A) = tr(A^T)$

(d) Properties of multiplication:

(i) $AB \neq BA$ **(ii)** $(AB)C = A(BC)$ **(iii)** $A.(B + C) = A.B + A.C$

(e) Adjoint of a Matrix:

(i) $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_n$ **(ii)** $|\text{adj } A| = |A|^{n-1}$

(iii) $(\text{adj } AB) = (\text{adj } B)(\text{adj } A)$ **(iv)** $\text{adj}(\text{adj } A) = |A|^{n-2}$

(v) $(\text{adj } KA) = K^{n-1}(\text{adj } A)$

(e) Inverse of a matrix: A^{-1} exists if A is non singular i.e. $|A| \neq 0$

(i) $A^{-1} = \frac{1}{|A|}(\text{Adj. } A)$ **(ii)** $A^{-1}A = I_n = AA^{-1}$

(iii) $(A^T)^{-1} = (A^{-1})^T$ **(iv)** $(A^{-1})^{-1} = A$

(v) $|A^{-1}| = |A|^{-1} = \frac{1}{|A|}$