Solved Examples

JEE Main/Boards

Example 1: If $\begin{bmatrix} x-y & 2x+z \\ 3x+y & 3z+4w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 5 & 25 \end{bmatrix}$,

find x, y, z, w.

Sol. We know that in equal matrices the corresponding elements are equal. Therefore, by equating the elements of these two matrices which have the same number of rows and columns, we get the value of x, y, z and w.

Given $\begin{bmatrix} x-y & 2x+z \\ 3x+y & 3z+4w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 5 & 25 \end{bmatrix}$

$x - y = -1$,

$2x + z = 5$;

$3x + y = 5$,

$3z + 4w = 25$

By solving these equations, we get

$x = 1, y = 2, z = 3, w = 4$

Example 2: Show that the matrix

$\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is a nilpotent matrix of index 3

Sol: Value of the index at which all elements of the matrix become 0, i.e. null matrix, is called the nilpotent matrix of that index. Here we calculate the n^{th} power of the matrix, where n =1, 2, 3, ... The value of n at which the matrix becomes null matrix is the index value.

Given $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$

$\Rightarrow A^2 = A \times A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} =$

$\begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$

Similarly,

$\therefore A^3 = A^2. A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} =$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow A^3 = 0$.

i.e. $A^k = 0$

Here $k = 3$

Hence, A is nilpotent matrix of index 3

Example 3: Solve the following system of homogeneous equations:

$2x + 3y - z = 0$, $x - y - 2z = 0$ and

$3x + y + 3z = 0$

Sol: In this problem we can write the given homogeneous equations in a matrix form, i.e. [A][X] = [O] and then by calculating the determinant of matrix A we can find if that given system has a trivial solution or not.

The given system can be written as

$\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ or AX = O

Where, $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{vmatrix}$

$= 2(-3 + 2) - 3(3 + 6) - 1(1 + 3)$

$= -2 - 27 - 4 = -33 \neq 0$

Thus $|A| \neq 0$.

So the given system has only the trivial solution given by $x = y = z = 0$

Example 4: Find x, y, z and a for which

$$\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$$

Sol: We know that, in equal matrices the corresponding elements are equal. Therefore by equating the elements of these two matrices which have the same number of rows and columns we get the values of x, y, z and w.

Given, $\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$

We know, that for equal matrices the corresponding elements are equal, therefore

$$x+3=0;$$
$$2y+x=-7;$$
$$z-1=3;$$
$$4a-6=2a;$$

By solving these equations, we get

$\therefore$ x = – 3, z = 4, y = – 2, a = 3.

Example 5: Compute the adjoint of the matrix

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 3 & 2 & 6 \\ 0 & 1 & 0 \end{bmatrix}$$

Sol: For this problem, we use the formula to get the co-factors of all the elements of matrix A. Then by taking the transpose of the co-factor matrix we can get the adjoint of matrix A.

Consider C_{ij} be a co-factor of a_{ij} in matrix A.

Then the co-factors of the elements of A are given by

$$C_{11} = \begin{vmatrix} 2 & 6 \\ 1 & 0 \end{vmatrix} = 0-6 = -6,$$

$$C_{12} = -\begin{vmatrix} 3 & 6 \\ 0 & 0 \end{vmatrix} = 0,$$

$$C_{13} = \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} = 3-0 = 3,$$

$$C_{21} = -\begin{vmatrix} 4 & 5 \\ 1 & 0 \end{vmatrix} = -(0-5) = 5$$

$$C_{22} = \begin{vmatrix} 1 & 5 \\ 0 & 0 \end{vmatrix} = 0,$$

$$C_{23} = -\begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} = -(1-0) = -1,$$

$$C_{31} = \begin{vmatrix} 4 & 5 \\ 2 & 6 \end{vmatrix} = (24-10) = 14,$$

$$C_{32} = -\begin{vmatrix} 1 & 5 \\ 3 & 6 \end{vmatrix} = -(6-15) = 9,$$

$$C_{33} = \begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix} = (2-12) = -10,$$

$$\therefore \text{adj } A = \begin{bmatrix} -6 & 0 & 3 \\ 5 & 0 & -1 \\ 14 & 9 & -10 \end{bmatrix}^T = \begin{bmatrix} -6 & 5 & 14 \\ 0 & 0 & 9 \\ 3 & -1 & -10 \end{bmatrix}$$

Example 6: If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, Then find $\lim_{n\to\infty} \frac{1}{n} A^n$.

Sol: For this problem, we first have to calculate the n^{th} power of matrix A, i.e. A^n, and multiply the matrix A^n by $\frac{1}{n}$.

Then, by with the given limit we can find the solution of this problem.

Given $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$$A^n = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$$

$$\Rightarrow \frac{1}{n} A^n = \begin{bmatrix} \frac{\cos n\theta}{n} & \frac{\sin n\theta}{n} \\ -\frac{\sin n\theta}{n} & \frac{\cos n\theta}{n} \end{bmatrix}$$

But $-1 \le \cos n\theta, \sin n\theta \le 1$;

$$\therefore \lim_{n\to\infty} \frac{\cos n\theta}{n} = 0,$$

$$\lim_{n\to\infty} \frac{\sin n\theta}{n} = 0,$$

$$\therefore \lim_{n\to\infty} \frac{1}{n} A^n = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Example 7: A trust fund has Rs. 50,000 that is to be invested into two types of bonds. The first bond pays 5% interest per year and the second bond pays 6% interest per year. Using matrix multiplication determine how to divide by Rs, 50,000 among the two types of bonds so as to obtain an annual total interest of Rs. 2,780.

Sol: In this problem, investment amounts can be written in the form of a row matrix and interest amounts can be written in the form of column matrix. By multiplying these two matrix we will get the equation for annual interest rates. By equating this to the given annual interest value we will get the required answer.

Consider investment of first type of bond = Rs. x

And second type of bond = Rs. 50,000 – x

These amounts can be written in the form of a row matrix A which is given by

$$A = \begin{bmatrix} x & 50000-x \end{bmatrix}_{1\times 2}$$

The interest amounts per rupee, per year from the two bonds are Rs. $\frac{5}{100}$ and $\frac{6}{100}$ which can be written in the form of a column matrix B which is given by

$$B = \begin{bmatrix} \frac{5}{100} \\ \frac{6}{100} \end{bmatrix}_{2\times 1}$$

∴ The total interest per year is given by

$$A.B = [x\ 50,000 - x] \times \begin{bmatrix} \frac{5}{100} \\ \frac{6}{100} \end{bmatrix}$$

= [x. 5/100 + (50,000 – x). 6/100]

= [3000 – x/100]

Since the required total annual interest is

= Rs.2,780. ∴ [3000 – x/100] = [2780]

⇒ 3000 – x/100 = 2780

⇒ x = 100(3000 – 2780) = 22,000

Hence the required amounts to be invested in the two bonds are Rs. 22,000 and Rs. (50,000 – 22,000), i.e. Rs. 22,000 and Rs. 28,000 respectively.

Example 8: If $f(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and if α, β, γ are the angles of a triangle, then prove that f(α). f(β) , $f(\gamma) = -I_2$

Sol: In this problem, by the methods of substitution and multiplication of matrices we can easily prove the given equation.

Given that $f(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

∴ $f(\beta) = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$ and

$$f(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{bmatrix}$$

$$f(\alpha)f(\beta) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & \cos\alpha\sin\beta + \sin\alpha\cos\beta \\ -\sin\alpha\cos\beta - \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$$

Similarly f(α) f(β) f(γ)

$$= \begin{bmatrix} \cos(\alpha+\beta+\gamma) & \sin(\alpha+\beta+\gamma) \\ -\sin(\alpha+\beta+\gamma) & \cos(\alpha+\beta+\gamma) \end{bmatrix}$$

$= \begin{bmatrix} \cos\pi & \sin\pi \\ -\sin\pi & \cos\pi \end{bmatrix}$ and as α + β + γ = p

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -I_2$$

Example 9: If $M(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$M(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

then prove that $[M(\alpha)\,M(\beta)]^{-1} = M(-\beta)\,M(-\alpha)$

Sol: In this problem, by finding the inverse of the matrix we can easily get the required answer.

$[M(\alpha)\,M(\beta)]^{-1} = M(\beta)^{-1}\,M(\alpha)^{-1}$

Given $M(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$M(\alpha)^{-1} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can also write this in the form

$$\begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} = M(-\alpha)$$

Similarly,

$$M(\beta)^{-1} = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$

$$= \begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) \\ 0 & 1 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix} = M(-\beta)$$

$\therefore$ $[M(\alpha)\, M(\beta)]^{-1} = M(-\beta)\, M(-\alpha)$

Example 10: Show that the homogeneous system of equations $x - 2y + z = 0$, $x + y - z = 0$, $3x + 6y - 5z = 0$ has a non-trivial solution, Also, find the solution.

Sol: In this problem we can write the given homogeneous equations in a matrix form, i.e. [A][X] = [O] and then by calculating the determinant of matrix A we can find if that given system has a non- trivial solution or not.

The given equations are

$x - 2y + z = 0,$

$x + y - z = 0,$

$3x + 6y - 5z = 0,$

We can write these equations in the form of matrices as shown below

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & -1 \\ 3 & 6 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ or } AX = O, \text{ where}$$

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & -1 \\ 3 & 6 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & -2 & 1 \\ 1 & 1 & -1 \\ 3 & 6 & -5 \end{vmatrix}$$

$= 1(-5 + 6) + 2(-5 + 3) + 1(6 - 3) = 0$

Thus, $|A| = 0$

Hence, the given system of equations has a non-trivial solution.

To find the solution, we take $z = k$ in the first two equations and write them as follows:

$x - 2y = -k$ and $x + y = k$

$$\text{or } \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -k \\ k \end{bmatrix} \quad \text{or } AX = B,$$

$$\text{where } A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\text{and } B = \begin{bmatrix} -k \\ k \end{bmatrix} \quad \text{Now, } |A| = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} = 3 \neq 0.$$

$$\text{So } A^{-1} \text{ exists;} \quad \text{We have, adj } A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$\text{So, } A^{-1} = \frac{1}{|A|} \text{adj } A \Rightarrow A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

Now $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -k \\ k \end{bmatrix} = \begin{bmatrix} k/3 \\ 2k/3 \end{bmatrix}$$

$\Rightarrow x = k/3,\ y = 2k/3$

These values of x, y and z also satisfy the third equation. Hence $x = k/3$, $y = 2k/3$ and $z = k$, where k is any real number and which satisfy the given system of equations.

JEE Advanced/Boards

Example 1: Let A and B be symmetric matrices of the same order. Then show that

(i) A + B is symmetric

(ii) AB – BA is skew-symmetric

(iii) AB + BA is symmetric

Sol: In this problem, by using the conditions for symmetric and skew-symmetric matrices we can get the required result.

As given, A and B are symmetric.

$\therefore$ $A' = A$ and $B' = B$

(i) $(A + B)' = A' + B' = A + B$

$\therefore$ A + B is symmetric

(ii) $(AB - BA)' = (A'B)' - (BA)'$

$= B'A' - A'B'$ [by reversal law]

$= BA - AB$ [$A' = A$, $B' = B$]

$\therefore$ AB – BA is skew-symmetric

(iii) $(AB + BA)' = (AB)' + (BA)'$

$= B'A' + A'B' = BA + AB = AB + BA$

$\therefore$ AB + BA is symmetric.

Example 2: Solve the following equations:

2x – 3y + z = 9 x + y + z = 6 x – y + z = 2

Sol: In this problem, we can write the given homogeneous equations in a matrix form, i.e. [A][X] = [O]. Then, by calculating the determinant of matrix A and adjoint of A, we get an inverse of matrix A, i.e. A^{-1}. By multiplying this into [A][X] = [O] we get the required values of x, y, and z.

We can also find if that given system has a trivial solution or not.

As given, 2x – 3y + z = 9

x + y + z = 6

x – y + z = 2

This system can be written as AX = B,

Where, $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $B = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$

|A|=2(2)+3(0)+1(-2) = 2

$$\text{Adj } A = \begin{bmatrix} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \\ -\begin{vmatrix} -3 & 1 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix} \\ \begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 2 & -4 \\ 0 & 1 & -1 \\ -2 & -1 & 5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{Adj } A$$

Now, $X = A^{-1}B$

$$= \frac{1}{2}\begin{bmatrix} 2 & 2 & -4 \\ 0 & 1 & -1 \\ -2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 22 \\ 4 \\ -14 \end{bmatrix}$$

∴ x =11, y=2, z=-7 is the solution.

Example 3: Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$,

prove that $A^2 - 4A - 5I = 0$, hence obtain A^{-1}:

Sol: In this problem, by using a simple multiplication method we can get the matrix A^2, then by substituting these in the given equation we will easily obtain the required result.

$$A^2 = A.\,A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} = \begin{pmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{pmatrix}$$

Now $A^2 - 4A - 5I$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

= 0 [Here 0 is the zero matrix]

Thus $A^2 - 4A - 5I = O$

$\therefore A^{-1}A^2 - 4A^{-1}A - 5A^{-1}I = A^{-1}O = O$

or $(A^{-1}A)A - 4(A^{-1}A) - 5A^{-1}I = O$;

or $IA - 4I - 5A^{-1} = O$; $\therefore 5A^{-1} = A - 4I$

$$= \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} -3/5 & 2/5 & 2/5 \\ 2/5 & -3/5 & 2/5 \\ 2/5 & 2/5 & -3/5 \end{bmatrix}$$

Example 4: Find the product of two matrices

A and B where $A = \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix}$

$B = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$ and use it for solving the equations

x + y + 2z =1, 3x + 2y + z = 7 and 2x + y + 3z = 2

Sol: As the given system of equations is in the form BX = C, multiplying it by B^{-1}, which is obtained by the multiplication of AB, we can get the required result.

$$AB = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-5 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Also the given system of equations in matrix form is BX = C ... (ii)

Where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

From (ii), $X = B^{-1}C$

[Multiplying both sides of (ii) by B^{-1}

$\therefore B^{-1} B = I$

From (1), $AB = 4I_3 \therefore \frac{A}{4}. B = I_3$

$$\therefore B^{-1} = \frac{A}{4} = \begin{bmatrix} -5/4 & 1/4 & 3/4 \\ 7/4 & 1/4 & -5/4 \\ 1/4 & -1/4 & 1/4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = X = B^{-1}C$$

$$= \begin{bmatrix} -5/4 & 1/4 & 3/4 \\ 7/4 & 1/4 & -5/4 \\ 1/4 & -1/4 & 1/4 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \begin{pmatrix} -\frac{5}{4}+\frac{7}{4}+\frac{6}{4} \\ \frac{7}{4}+\frac{7}{4}-\frac{10}{4} \\ \frac{1}{4}-\frac{7}{4}+\frac{2}{4} \end{pmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \therefore x = 2, y = 1, z = -1$$

Example 5: Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

Find P such that $BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Sol: Pre-multiplying both sides by B^{-1} and Post-multiplying both sides by A^{-1} in

$BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ we can find P.

Given $BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$B^{-1}BPA\,A^{-1} = B^{-1}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}A^{-1}$$

$$\Rightarrow IPI = B^{-1}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}A^{-1}$$

$$\Rightarrow P = B^{-1}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}A^{-1} \quad ...(i)$$

To find B^{-1}, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

$$|B| = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 8 - 9 = -1 \neq 0$$

Let C be the matrix of co-factors of elements in | B |;

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$\therefore C_{11} = 4\ C_{12} = -3\ C_{21} = -3\ C_{22} = 2$

$$\therefore C = \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{adjB}{|B|} = \frac{C'}{-1} = -C'$$

$$= -\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} \quad ... (ii)$$

To Find A^{-1}, Since $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

$\therefore |A| = 1(4-3) - 1(2-2) + 1(6-8)$

$= 1 - 0 - 2 = -1 \neq 0$

Let C be the matrix of co-factors of elements in | A |;

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{vmatrix} 4 & 1 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 4 \\ 2 & 3 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 2 & -1 & -1 \\ -3 & 1 & 2 \end{bmatrix}; \quad \therefore C' = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 1 \\ -2 & -1 & 2 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 1 \\ -2 & -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj} A}{|A|} = -\text{ Adj, } A$$

$$= \begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -1 \\ 2 & 1 & -2 \end{bmatrix} \quad \text{... (iii)}$$

Substituting eq. (ii) and (iii) in eq. (i), we get

$$P = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$$

$$P = \begin{bmatrix} -4 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix} \times \begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$$

$$P = \begin{bmatrix} 4+0-8 & 8+3-4 & -12-3+8 \\ -3-0+6 & -6-2+3 & 9+2-6 \end{bmatrix}$$

$$P = \begin{bmatrix} -4 & 7 & -7 \\ 3 & -5 & 5 \end{bmatrix}$$

Example 6: If $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that F(x). F(y) = F(x+ y).

Hence, prove that $[F(x)]^{-1} = F(-x)$.

Sol: By substituting x and y in place of α in given matrices we will get F(x) and F(y) respectively and then by multiplying them we will get the required result.

$$F(x)\,F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y & -\cos x\sin y - \sin x\cos y & 0 \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y)$$

i.e. F(x).F(y) = F(x +y) ... (i)

2nd part.

As we know that $F(x)\,[F(x)]^{-1} = I$ ... (ii)

Replacing y by –x in (i),

we get F(x). F(–x) = F(x – x) = F(0)

$$= \begin{bmatrix} \cos 0 & -\sin 0 & 0 \\ \sin 0 & \cos 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e. F(x) F(–x) = I ... (iii)

therefore from (ii) and (iii)

$\Rightarrow [F(x)]^{-1} = F(-x)$.

Example 7: Show that every square matrix A can be uniquely expressed as P + iQ where P and Q are Hermitian matrices.

Sol: By considering $P = \frac{1}{2}(A + A^{\theta})$

And $Q = \frac{1}{2i}(A - A^{\theta})$ we get A = P + iQ

Then, using the property of a Hermitian matrix we can prove the above problem.

Now $P^{\theta} = \left\{\frac{1}{2}(A + A^{\theta})\right\}^{\theta} = \frac{1}{2}(A + A^{\theta})^{q}$

$$= \frac{1}{2}\{A^{\theta} + (A^{\theta})^{q}\} = \frac{1}{2}(A^{\theta} + A) = \frac{1}{2}(A + A^{\theta}) = P$$

$\therefore P = P^{\theta}$, hence P is a Hermitian matrix.

Similarly

$$Q^{\theta} = \left\{\frac{1}{2i}(A - A^{\theta})\right\}^{\theta} = \left\{\frac{1}{2i}\right\}(A - A^{\theta})^{q}$$

$$= -\frac{1}{2i}\{A^{\theta} - (A^{\theta})^{q}\} = -\frac{1}{2i}(A^{\theta} - A) = \frac{1}{2i}(A - A^{\theta}) = Q$$

$\therefore$ Q is also Hermitian matrix, Therefore A can be expressed as P + iQ ,where P and Q are Hermitian matrices.

Let A = R + iS where R and S are both Hermitian matrices

We have $A^\theta = (R + iS)^\theta = R^\theta + (iS)^q$

$= R^\theta + iS^\theta = R^\theta - iS^\theta = R - iS$

(since R and S are both Hermitian)

$\therefore A + A^\theta = (R + iS) + (R - iS) = 2R$

$\Rightarrow R = \frac{1}{2}(A + A^\theta) = P$

Also $A - A^\theta = (R + iS) - (R - iS) = 2iS$

$\Rightarrow S = \frac{1}{2i}(A - A^\theta) = Q$

Hence expression (1) for A is unique

Example 8: If A is Hermitian such that $A^2 = 0$, show that A = 0,

Sol: As A is a Hermitian matrix therefore $A^\theta = A$. By considering $A = [a_{ij}]_{n\times n}$ to be a Hermitian matrix of order n and as given $A^2 = 0$, we can solve given problem as follows.

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2} & \ldots\ldots & a_{nn} \end{bmatrix} \text{ and}$$

$$A^\theta = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{21} & \ldots\ldots & \bar{a}_{n1} \\ \bar{a}_{12} & \bar{a}_{22} & \ldots\ldots & \bar{a}_{n2} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \bar{a}_{1n} & \bar{a}_{2n} & \ldots\ldots & \bar{a}_{nn} \end{bmatrix}$$

Since $A^2 = 0$;

Let $AA^\theta = [b_{ij}]_{n\times n} \Rightarrow AA^\theta = 0$

Then each element of AA^θ is zero and so all the principal diagonal elements of AA^θ are zero

$\therefore b_{ii} = 0$ for all i = 1, 2, ..., n

Now, $b_{ii} = a_{i1}\,\bar{a}_{i1} + a_{i2}\,\bar{a}_{i2} + \ldots + a_{in}\,\bar{a}_{in}$

$= |a_{i1}|^2 + |a_{i2}|^2 + \ldots\ldots |a_{in}|^2 \quad \therefore b_{ii} = 0$

$\Rightarrow |a_{i1}|^2 + |a_{i2}|^2 + \ldots\ldots + |a_{in}|^2 = 0$

$\Rightarrow |a_{i1}| = |a_{i2}| = \ldots\ldots = |a_{in}| = 0$

$\Rightarrow a_{i1} = a_{i2} = \ldots\ldots = a_{in} = 0$

$\Rightarrow$ each element of the i^{th} row of A is zero, but $b_{ii} = 0\ \forall$ i = 1, ... n

$\therefore$ Each element of each row of A is zero. Hence, A = O

Example 9: If the non-singular matrix A is symmetric, then prove that A^{-1} is also symmetric.

Sol: By using the conditions of non-singular and symmetric matrix we can easily find the required result.

As given matrix A is a non-singular symmetric matrix.

$\therefore |A| \neq 0$ and $A^T = A$,

So, A^{-1} exists

Now, $AA^{-1} = I = A^{-1}A$

$\Rightarrow (AA^{-1})^T = (I)^T = (A^{-1}A)^T$

$\Rightarrow (A^{-1})^T A^T = I = A^T(A^{-1})^T$

$\Rightarrow (A^{-1})^T A = I = A(A^{-1})^T \quad [A^T = A]$

$\Rightarrow A^{-1} = (A^{-1})^T$

$\Rightarrow A^{-1}$ is symmetric.

Example 10: Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Sol: given

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$$\begin{bmatrix} R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1 \\ \text{and } R_4 \to R_4 - 6R_1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

[Applying $R_4 \to R_4 - R_2 - R_3$]

$$= \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

[Applying $R_2 \to R_2 - R_3$]

$$= \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

[Applying $R_3 \to R_3 - 4R_2$]

$$= \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

[Applying $R_3 \to 1/11\ R_3$]

$$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the equivalent matrix is in echelon form having three non-zero rows. Hence, r(A) = 3

JEE Main/Boards

Exercise 1

Q.1 Find x and y, if $\begin{pmatrix} 2x-1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ x+y \end{pmatrix}$

Q.2 A matrix has 2 rows and 3 columns. How many elements a matrix has? Find the number of elements of a matrix if it has 3 rows and 2 columns.

Q.3 Order of matrix A is 2 × 2 and order of matrix B is 2 × 3. Find the order of AB and BA, if defined.

Q.4 Given a matrix $A = [a_{ij}]$, $1 \le i \le 3$ and $1 \le j \le 3$, where $a_{ij} = i + 2j$. Write the element

(i) a_{11} (ii) a_{32} (iii) a_{23} (iv) a_{34}

Q.5 A matrix has 18 elements. Write the possible orders of matrix.

Q.6 Give an example of a diagonal matrix, which is not a scalar matrix. Also give an example of a scalar matrix.

Q.7 For the matrix A, show that $A + A^T$ is a symmetric matrix.

Q.8 For the matrix A, Show that $A - A^T$ is a skew-symmetric matrix.

Q.9 The total number of elements in a matrix represents a prime number. How many possible orders a matrix can have ?

Q.10 Find x and y, if $\begin{pmatrix} x \\ 2y \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

Q.11 If $f(x) = 3x^2 - 9x + 7$, then for a square matrix A, write f(A).

Q.12 If A, B and AB are symmetric matrices, then what is the relation between AB and BA?

Q.13 If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ B, [2 –2 4], find AB.

Q.14 Are the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ equal? Give reasons.

Q.15 Given a matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$.

Find matrix kA, where $k = -\frac{1}{2}$

Q.16 Simplify: $\tan\theta \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & -\sec\theta \end{bmatrix}$

$+ \sec\theta \begin{bmatrix} -\tan\theta & -\sec\theta \\ -\sec\theta & \tan\theta \end{bmatrix}$.

Q.17 If $X_{m\times 3} Y_{P\times 4} = Z_{2\times b}$, for three matrices X, Y, Z, find the values of m, p and b.

Q.18 Is matrix $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$ symmetric or skew-symmetric? Give reasons.

Q.19 If $R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$, write (i) $R\left(\frac{\pi}{2}\right)$, (ii) R(x+y)

Q.20 For a skew-symmetric matrix A = $[a_{ij}]$, what is the nature of elements a_{ij}, if i = j.

Q.21 If A = $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$, find A^{16}.

Q.22 Find x, if $[x\ 1] \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$

Q.23 Find the sum of matrix A = $\begin{pmatrix} 2 & -1 \\ 4 & 6 \end{pmatrix}$ and its additive inverse.

Q.24 Find X, if $X + \begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & 0 \end{bmatrix}$

Q.25 Evaluate, $\begin{bmatrix} \sin^2\theta & 1 \\ \cot^2\theta & 0 \end{bmatrix} + \begin{bmatrix} \cos^2\theta & 0 \\ -\operatorname{cosec}^2\theta & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Q.26 If A and B are symmetric matrices, show that AB is symmetric.

Q.27 If a matrix has 8 elements, what the possible orders it can have ? What if it has 5 elements?

Q.28 Evaluate the following:

$$[a, b] \begin{bmatrix} c \\ d \end{bmatrix} + [a\ b\ c\ d] \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Q.29 If A = $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, find A^2. Hence find A^6

Q.30 Show that the element of the main diagonal of a skew-symmetric matrix are all zeros.

Q.31 Find AB, if A = $\begin{bmatrix} 0 & -4 \\ 0 & -3 \end{bmatrix}$ and B = $\begin{bmatrix} 5 & -7 \\ 0 & 0 \end{bmatrix}$

Q.32 If A = $\begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$, find values of x and y such that $A^2 - xA + yI = O$ where I is a 2×2 unit matrix and O is a 2 × 2 zero matrix.

Q.33 If A = $\begin{pmatrix} 1 & 3 & 5 \\ -2 & 5 & 7 \end{pmatrix}$ and A – 3B = $\begin{pmatrix} 4 & 5 & -9 \\ 1 & 2 & 3 \end{pmatrix}$, find B.

Q.34 If A = $\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, then show that

$A^2 = \begin{pmatrix} \cos2\alpha & \sin2\alpha \\ -\sin2\alpha & \cos2\alpha \end{pmatrix}$

Q.35 If A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

prove that $(aI + bA)^3 = a^3I + 3a^2bA$.

Q.36 If A = $\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$, find the values of

p and q such that $(pI + qA)^2 = A$.

Q.37 If A = $\begin{bmatrix} 2 & 3 & -4 \\ 1 & 0 & 6 \\ -2 & 1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 5 & 1 & 2 \\ 6 & -1 & 4 \\ 5 & 3 & -4 \end{bmatrix}$

find 2A – 3B.

Q.38 Construct a 3 × 3 matrix $[a_{ij}]$, whose elements are given by $a_{ij} = 2i - 3j$.

Q.39 If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find x, y, z, w.

Q.40 Find matrices X and Y, if

$X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

Q.41 If A = $\begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}$;

B = $\begin{pmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{pmatrix}$, then show that AB is zero

matrix, provided $(\theta - \phi)$ is an odd multiple of $\pi/2$.

Q.42 If A = $\begin{pmatrix} -1 & 1 & -1 \\ 1 & -3 & 3 \\ 5 & -5 & 5 \end{pmatrix}$ and B = $\begin{pmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{pmatrix}$,

compute A^2B^2.

Q.43 Find the matrix X such that,

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X + \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$$

Exercise 2

Single Correct Choice Type

Q.1 If number of elements in a matrix is 60 then how many dimensions of matrix are possible

(A) 12 (B) 6 (C) 24 (D) None of these

Q.2 Matrix A has x rows and x + 5 columns. Matrix B has y rows and 11 – y columns. Both AB and BA exist, then

(A) x = 3, y = 4 (B) x = 4, y = 3

(C) x = 3, y = 8 (D) x = 8, y = 3

Q.3 If A is square invertible matrix such that $A^2 = A$, then det.$(A^2 - I)$ is

(A) 1 (B) 2 (C) 3 (D) None of these

Q.4 Number of distinct matrices that can be formed using all the 143 distinct elements is

(A) 4! (B) 4(143)! (C) 2(143)! (D) (143)!

Q.5 If $A^2 = A$, then $(I + A)^4$ is equal to

(A) I + A (B) I + 4A

(C) I + 15A (D) None of these

Q.6 If $A = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$ is an orthogonal matrix, where α, β and the roots other than the common root of the equations $x^2 - px + q = 0$ & $x^2 + px - q = 0$, then

(A) $p=\pm\frac{1}{\sqrt{2}}, q=\pm\frac{1}{\sqrt{2}}$ (B) $p = 0, q = \pm\frac{1}{\sqrt{2}}$

(C) $p = \pm\frac{1}{\sqrt{2}}, q = 0$ (D) None of these

Q.7 A is a square matrix of order n and (det A) = 3. If det $(\lambda A) = 81$; where $\lambda \in N$, then possible value of n is

(A) 3 (B) 5 (C) 2 (D) 7

Q.8 If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = \frac{1+x}{1-x}$, then f(A)is

(A) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ (D) None of these

Q.9 If A is a skew symmetric matrix such that $A^TA = I$, then $A^{4n-1} (n \in N)$ is equal to

(A) $-A^T$ (B) I (C) – I (D) A^T

Q.10 A and B are 2 × 2 matrices satisfying det A = det B and tr(A) = tr(B), further $A^2 - 3A + 14I = 0$ and $B^2 - \lambda B + \mu I = 0$, then μ is equal to

(A) 3 (B) 11 (C) –11 (D) 14

Q.11 The false statement is -

(A) The adjoint of a scalar matrix is scalar matrix.

(B) The adjoint of upper triangular matrix is lower triangular matrix.

(C) The adjoint of upper triangular matrix is upper triangular matrix.

(D) adj(adj A) = A, A is a square matrix of order 2.

Q.12 If the matrices A, B, (A + B) are non-singular, then $[A(A + B)^{-1}B]^{-1}$ is equal to

(A) A + B (B) $A^{-1} + B^{-1}$

(C) $(A + B)^{-1}$ (D) None of these

Q.13 If A is an orthogonal matrix | A | = – 1, then A^T is equal to

(A) – A (B) A

(C) – (adj A) (D) (adj A)

Q.14 If A and B are square matrices of order 3, then

(A) adj(AB) = adj A+adj B

(B) $(A + B)^{-1} = A^{-1} + B^{-1}$

(C) AB = 0 ⇒ | A | = 0 or | B | = 0

(D) AB = 0 ⇒ | A | = 0 and B = 0

Q.15 If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then | A | | adj A | is equal to

(A) a^{25} (B) a^{27}

(C) a^{81} (D) None of these

Q.16 If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then the value of $|A^TA^{-1}|$ is

(A) cos 4x (B) $\sec^2 x$

(C) – cos 4x (D) 1

Q.17 If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then $19A^{-1}$ is equal to

(A) A^T (B) 2A (C) $\frac{1}{2}A$ (D) A

Q.18 If P is a two-rowed matrix satisfying $P^T = P^{-1}$, then P is

(A) $\begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ (B) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(C) $\begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ (D) None of these

Q.19 If A and B are two non-singular matrices of the same order such that $B^r = I$, for some positive integer r > 1, then $A^{-1}B^{r-1}A\ A^{-1}B^{-1}A$ is equal to.

(A) 0 (B) I

(C) A^{-1} (D) None of these

Q.20 If A and B are orthogonal matrices of same order, then:

(A) A + B is also orthogonal.

(B) A – B is also orthogonal.

(C) AB is also orthogonal.

(D) AB + BA is also orthogonal.

Q.21 If C is an orthogonal matrix and A is a square matrix of same order then, trace of C^TAC is equal to

(A) Trace of C (B) Trace of AC

(C) Trace of A (D) None of these matrix

Q.22 Let A and B are idempotent matrices such that A.B = BA and A – B is non singular then | A + B | is equal to

(A) 0 (B) –1 (C) 1 (D) ±1

Q.23 If A and B are square matrices of same order and $AA^T = I$, then $(A^TBA)^{10}$ is equal to

(A) $AB^{10}A^T$ (B) $A^TB^{10}A$

(C) $A^{10}B^{10}(A^T)^{10}$ (D) $10A^TBA$

Q.24 If A = $\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and

B = $\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$ then AB is equal to

(A) A^3 (B) B^2 (C) O (D) I

Q.25 If A, B, C are square matrices of same order & AB = BA, $C^2 = B$, then $(A^{-1}CA)^2$ is equal to

(A) B^2 (B) A^2 (C) C^2 (D) C

Q.26 A is a diagonal matrix of order 3, and tr(A) = 12. If all diagonal entries are positive then maximum value of det (A) is

(A) 8 (B) 16 (C) 32 (D) 64

Q.27 If A and B are two matrix such that AB = B and BA = A, then $A^2 + B^2$ is equal to

(A) 2AB (B) 2BA (C) A + B (D) AB

Q.28 A = $\begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$ and B is column matrix such that $(A^8 + A^6 + A^4 + A^2 + I)$, B = $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ where I is a unit matrix of order 2 ×2, then B is equal to

(A) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 \\ \frac{2}{11} \end{bmatrix}$ (C) $\begin{bmatrix} 0 \\ \frac{1}{|2|} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{11} \\ \frac{1}{11} \end{bmatrix}$

Q.29 If A and B are square matrices of same order such that AB = BA and $A^2 = I$, then ABA is equal to

(A) $(AB)^2$ (B) I (C) B (D) B^2

Previous Years' Questions

Q.1 The parameter, on which the value of the

determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$

does not depend upon, is ***(1997)***

(A) a (B) p (C) d (D) x

Q.2 If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$

then f(100) is equal to ***(1999)***

(A) 0 (B) 1 (C) 100 (D) – 100

Q.3 The number of distinct real roots of

$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval

$-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is ***(2001)***

(A) 0 (B) 2 (C) 1 (D) 3

Q.4 The number of values of k for which the system of equations ***(2004)***

$(k + 1)x + 8y = 4k, kx + (k + 3)y = 3k - 1$

(A) 0 (B) 1 (C) 2 (D) –1

Q.5 Given, $2x - y + 2z = 2$, $x - 2y + z = -4$, $x + y + lz = 4$, then the value of λ such that the given system of equations has no solution is ***(2004)***

(A) 3 (B) 1 (C) 0 (D) –3

Q.6 If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and

$Q = PAP^T$, then $P^TQ^{2005}P$ is ***(2005)***

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 2005 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q.7 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $6A^{-1} = A^2 + cA + dI$, then

(c, d) is ***(2005)***

(A) (–6, 11) (B) (–11, 6) (C) (11, 6) (D) (6, 11)

Q.8 Let $\alpha_1, \alpha_2, \beta_1, \beta_2$ be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-trivial

solution. Then prove that $\frac{b^2}{q^2} = \frac{ac}{pr}$ ***(1987)***

Q.9 Find the value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$

where a, b, and c are respectively the p^{th}, q^{th} and r^{th} terms of a harmonic progression ***(1987)***

Q.10 Suppose, f(x) is a function satisfying the following conditions: ***(1998)***

(a) f(0) = 2, f(1) =1

(b) f has a minimum value at $x = \frac{5}{2}$, and

(c) For all x,

$f'(x) = \begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$

where a, b are some constants. Determine the constants a, b and the function f(x)

Q.11 Prove that for all values of θ, ***(2000)***

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix} = 0$$

Q.12 If A is an 3 × 3 non – singular matrix such that $AA' = A'A$ and $B = A^{-1}A$ then BB′ equals: ***(2014)***

(A) $I+B$ (B) I (C) B^{-1} (D) (B^{-1})

Q.13 If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the

equation $AA^T = I$ where I is 3 3×3 identity matrix, then the ordered pair (a, b) is equal to: ***(2015)***

(A) $(2,-1)$ (B) $(-2,1)$

(C) $(2,1)$ (D) $(-2,-1)$

Q.14 If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A\,\text{adj}\,A = AA^T$, then 5a + b is

equal to ***(2016)***

(A) -1 (B) 5 (C) 4 (D) 13

JEE Advanced/Boards

Exercise 1

Q.1 (a) $A_{3\times3}$ is a matrix such that | A | = a, B = (adj A) such that | B | = b. Find the value of

$(ab^2 + a^2b + I)$ S Where $\frac{1}{2}S = \frac{a}{b} + \frac{a^2}{b^3} + \frac{a^3}{b^5} +$

...... up to ∞, and a = 3

(b) If A and B are square matrices of order 3, where | A | = –2 and | B | = I, then find (A^{-1}) adj (B^{-1}) adj$(2A^{-1})$

Q.2 Let A be the 2 × 2 matrices given by A = $[a_{ij}]$ where $a_{ij} \in \{0, 1, 2, 3, 4\}$ such that $a_{11} + a_{12} + a_{21} + a_{22} = 4$

(i) Find the number of matrices A Such that the trace of A is equal to 4.

(ii) Find the number of matrices A such that A is invertible.

(iii) Find the absolute value of the difference between maximum value and minimum value of det (A).

(iv) Find the number of matrices A such that A is either symmetric or skew-symmetric or both and det (A) is divisible by 2.

Q.3 For the matrix A = $\begin{bmatrix} 4 & -4 & 5 \\ -2 & 3 & -3 \\ 3 & -3 & 4 \end{bmatrix}$ find A^2.

Q.4 (a) Given A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$,

Find P such that BPA = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(b) Find the matrix A satisfying the matrix

equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} . A . \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix}$

Q.5 Let S be the set which contains all possible values of I, m, n, p, q, r for which

A = $\begin{bmatrix} l^2-3 & p & 0 \\ 0 & m^2-8 & q \\ r & 0 & n^2-15 \end{bmatrix}$ Be a non singular idempotent matrix. Find the absolute value of sum of the products of elements of the set S taken two at a time.

Q.6 If F(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that F(x). F(y) = F(x + y).

Hence, Prove that $[F(x)]^{-1} = F(-x)$.

Q.7 Let A_n and B_n be square matrices of order 3, which are defind as $A_n = [a_{ij}]$ and $B_n = [b_{ij}]$

where $a_{ij} = \frac{2i+j}{3^{2n}}$ and $b_{ij} = \frac{3i-j}{2^{2n}}$ for all i and j, $1 \le i.,j \le 3$.

If $l = \lim_{n\to\infty} \text{Tr}\,(3A_1 + 3^2A_2 + 3^3A_3 + ... 3^nA_n)$ and

$m = \lim_{n\to\infty} \text{Tr}(2B_1 + 2^2B_2 + 2^3B_3 + ... 2^nB_n)$

Then find the value of (l + m).

[Note: Tr (P) denotes the trace of matrix P]

Q.8 Let A be a 3 × 3 matrix such that $a_{11} = a_{33} = 2$ and all the other $a_{ij} = 1$.

Let $A^{-1} = xA^2 + yA + zI$ then find the value of (x + y + z) where I is a unit matrix of order 3.

Q.9 Given that A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & 3 \end{bmatrix}$,

C = $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, D = $\begin{bmatrix} 10 \\ 13 \\ 9 \end{bmatrix}$ and that Cb = D.

Solve the matrix equation Ax = b.

Q.10 Let A = $\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$ are two matrices such that $AB = (AB)^{-1}$ and AB ≠ 1 (where I is an identity matrix of order 3 × 3).

Find the value of Tr $(AB + (AB)^2 + (AB)^3 + ... + (AB)^{100})$ where Tr.(A) denotes the trace of matrix A.

Q.11 Let $M_n = [m_{ij}]$ denotes a square matrix of order n with entries as follows.

For $1 \le i \le n$, $m_{ii} = 10$; For $1 \le i \le n-1$, $m_{i+1, i} = m_{i, i+1} = 3$;

And all other entries in M_n are zero. Let D_n be the determinant of matrix M_n, then find the value of $(D_3 - 9D_2)$.

Q.12 Find the product of two matrices A & B,

where $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the following system of linear equations

$x + y + 2z = 1$; $3x + 2y + z = 7$; $2x + y + 3z = 2$

Q.13 Determine the values of a and b for which the system $\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$

(i) Has a unique solution;

(ii) Has no solution and

(iii) Has infinitely many solutions.

Q.14 If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

and $X = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ then solve the following matrix equations.

(a) $AX = B - 1$ (b) $(B - 1)X = IC$

(c) $CX = A$

Q.15 If A is an orthogonal matrix and B = AP where P is a non singular matrix, then show that the matrix PB^{-1} is also orthogonal.

Q.16 Let M be a 2 × 2 matrix such that $M\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $M^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. If x_1 and x_2 $(x_1 > x_2)$ are the two values x for which $\det(M - xI) = 0$, where I is an identity matrix of order 2, then find the value of $(5x_1 + 2x_2)$.

Q.17 The set of natural numbers is divided into arrays of rows and columns in the form of matrices as $A_1 = (1)$,

$A_2 = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, $A_3 = \begin{pmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{pmatrix}$ .... So on.

Find the value of $T_r(A_{10})$.

[Note: $T_r(A)$ denotes trace of A]

Q.18 Consider $I_{n.m} = \int_0^1 \frac{x^n}{x^m - 1} dx$ and $J_{n.m} = \int_0^1 \frac{x^n}{x^m + 1} dx \; \forall$ n > m and n, m ∈ N.

(a) Consider a matrix $A = [a_{ij}]_{3\times3}$,

where $a_{ij} = \begin{cases} I_{6+i,3} - I_{i+3,3}, & i = j \\ 0, & i \ne j \end{cases}$. Then find trace (A^{-1}).

[Note: Trace of a square matrix is sum of the diagonal elements.]

(b) Let $A = \begin{bmatrix} J_{6,5} & 72 & J_{11,5} \\ J_{7,5} & 63 & J_{12,5} \\ J_{8,5} & 56 & J_{13,5} \end{bmatrix}$ and $B = \begin{bmatrix} I_{6,5} & 72 & I_{11,5} \\ I_{7,5} & 63 & I_{12,5} \\ I_{8,5} & 56 & I_{13,5} \end{bmatrix}$

then find the value of det (A) – det (B)

Q.19 Consider the matrices $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and

$B = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ and let P be any orthogonal matrix and

$Q = PAP^T$ and $R = P^TQ^KP$ also $S = PBP^T$ and $T = P^TS^KP$

Column I	**Column II**
(A) If we vary K from 1 to n then the first row first column elements of R will form	(p) G.P. with common ratio a
(B) If we vary K from 1 to n then the 2nd row 2nd column elements of R will form	(q) A.P. with volume difference 2
(C) If we vary K from 1 to n then the first row first column elements of T will form	(r) G.P. with common ratio b
(D) If we vary K from 3 to n then the first row 2nd column elements of T will represent the sum of	(s) A.P. with volume difference –2

Q.20 Consider a square matrix A of order 2 which has its elements as 0, 1, 2 and 4. Let N denote the number of such matrices, all elements of which are distinct.

Column I	Column II
(A) Possible non-negative value of det (A) is	(p) 2
(B) Sum of values of determinants corresponding to N matrices is	(q) 4
(C) If absolute value of (det(A)) is least, then possible value of |adj(adj(adj A))|	(r) – 2
(D) If det (A) is algebraically least, then possible value of det $(4A^{-1})$ is	(s) –2
	(t) 8

Exercise 2

Single Correct Choice Type

Q.1 Let A, B be two square matrices of the same dimension and let [A, B] = AB – BA, then for three 2 × 2 matrices

A, B, C, [[A, B],C] + [[B, C],A] + [[C, A],B] =

(A) 1 (B) 0

(C) ABC – CBA (D) None of these

Q.2 $A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 2 & 3 & \alpha \end{vmatrix}$ & $f(x) = x^3 - 8x^2 + bx + \gamma$. If A

satisfies f(x) = 0, then ordered pair (α, γ) is

(A) (2, –7) (B) (–2, 7)

(C) (2, 7) (D) (–2, –7)

Q.3 If $\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ is a square root of the two rowed unit matrix, then δ is equal to

(A) α (B) β

(C) γ (D) None of these

Q.4 For $A = \begin{vmatrix} 4 & 2i \\ i & 1 \end{vmatrix}$, (A – 2I) (A – 3I) is a

(A) Null-matrix (B) Hermitian matrix

(C) Unit matrix (D) None of these

Q.5 If α, β, γ are the real numbers and

$$A = \begin{pmatrix} 1 & \cos(\alpha-\beta) & \cos(\alpha-\gamma) \\ \cos(\beta-\alpha) & 1 & \cos(\beta-\gamma) \\ \cos(\gamma-\alpha) & \cos(\gamma-\beta) & 1 \end{pmatrix} \text{ then}$$

(A) A is skew symmetric

(B) A is invertible

(C) A is non singular

(D) | A | = 0

Q.6 The values of x for which the matrix

$$\begin{bmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{bmatrix} \text{ is non-singular are}$$

(A) R – {0}

(B) R – { – (a + b + c)}

(C) R – { 0, – (a + b + c)}

(D) None of these

Q.7 Let A is a skew symmetric matrix such $A^2 = A$, and B is a square matrix such that $B^TB = B$; | B | ≠0. If X = (A + B) (A – B), then X^TX is

(A) A – I (B) I – A

(C) A (D) None of these

Q.8 For two uni-modular complex numbers z_1 and z_2,

$$\begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_2 & z_1 \end{bmatrix}^{-1} \begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}^{-1} \text{ equal to}$$

(A) $\begin{bmatrix} z_1 & z_2 \\ \bar{z}_1 & \bar{z}_2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$ (D) None of these

Q.9 If $\begin{bmatrix} 1/25 & 0 \\ x & 1/25 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix}^{-2}$,

then the value of x is

(A) $\frac{a}{125}$ (B) $\frac{2a}{25}$

(C) $\frac{2a}{125}$ (D) None of these

Q.10 If A is square matrix such that $A^2 = I$, $|A| = 1$ and $B = (\text{adj } A)^{-1}$ then incorrect statement is

(A) AB = BA (B) AB = I

(C) A = B (D) B = I

Q.11 If A and B are square matrices of order 3 and adj A = B, then adj (3AB) is equal to

(A) $3|B|^2 I_3$ (B) $9|B|I_3$

(C) $3|A|^2 I_3$ (D) $9|A|I_3$

Q.12 Let A and B are square matrices of order n such that $A^T + B = O$, O is a null matrix, A = adj B, tr (A) = –1 and $A^2 = A$ then tr {adj(A^TB)} is equal to

(A) $(-1)^{n-1}$ (B) 1

(C) $(-1)^n$ (D) None of these

Q.13 If A is a non-singular matrix such that C = A + B, $|C|^2 = |A|^2 |I - (A^{-1}B)^2|$ and AB = BA, then

(A) B is null matrix (B) A is null matrix

(C) $|C| = |A - B|$ (D) $|A| = |B|$

Previous Years' Questions

Q.1 Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the

form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b or c is

either ω and w^2. Then, the number of distinct matrices in the set S is ***(2011)***

(A) 2 (B) 6 (C) 4 (D) 8

Q.2 Let M and N be two 3 × 3 non-singular skew-symmetric matrices such that MN = NM. If PT denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to ***(2011)***

(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

Q.3 Without expanding a determinant at any

stage, show that $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

= Ax + B Where A and B are determinants of order 3 not involving x. ***(1982)***

Q.4 Show that the system of equations 3x – y + 4z = 3, x + 2y – 3z = – 2, 6x + 5y + lz = – 3 has at least one solution for any real number $\lambda \neq -5$. Find the set of solutions, if $\lambda = -5$. ***(1983)***

Q.5 Consider the system of linear equations in x, y, z (sin 3θ) x – y + z = 0, (cos 2θ) x + 4y + 3z = 0 and 2x + 7y + 7z = 0. Find the values of θ for which this system has non-trivial solution. ***(1986)***

Q.6 Let $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^8 & 3n^2-3n \end{vmatrix}$

Show that $\sum_{a=1}^{n} \Delta_a = c \in$ constant. ***(1989)***

Q.7 If $a \neq p$, $b \neq q$, $c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$

Then, find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ ***(1991)***

Q.8 For a fixed positive integer n, if

$D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$, then show that

$\left[\frac{D}{(n!)^3} - 4\right]$ is divisible by n. ***(1992)***

Q.9 Let λ and α be real. Find the set of all values of λ for which the system of linear equations λx + (sin α)y + (cos α)z = 0 x + (cos α)y + (sin α)z = 0 and –x + (sin α) y – (cos α)z = 0 has a non-trivial solution for λ = 1, find all values of α. ***(1993)***

Q.10 Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that the equation

$\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$

represents a straight line. ***(2001)***

Q.11 Let $\omega \neq 1$ be a cube root of unity and S be the set of all non–singular matrices of the form

$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$ where each of a, b and c is either ω or ω^2.

Then the number of distinct matrices in the set S is ***(2011)***

(A) 2 (B) 6 (C) 8 (D) 4

Q.12 Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \text{ and } M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

Then the sum of the diagonal entries of M is ***(2011)***

(A) 5 (B) 6 (C) 9 (D) 8

Q.13 If P is 3 × 3 matrix such that $P^T = 2P + I$ where P^T is he transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that **(2012)**

(A) $PX \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) PX = X (C) PX = 2X (D) PX = – X

Q.14 Let $P = [a_{ij}]$ is 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is ***(2012)***

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

Q.15 Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $P_{ij} = \omega^{i+j}$ Then $P^2 \neq 0$, when n = ***(2013)***

(A) 57 (B) 55 (C) 58 (D) 56

Q.16 Let M and N be two 3×3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N_4$ then ***(2014)***

(A) determinant of (M^2+MN^2) is 0.

(B) there is a 3×3 non–zero matrix U such that (M^2+MN^2) U is the zero matrix.

(C) determinant of $(M^2+MN^2) \ge 1$.

(D) for a 3×3 matrix U, if (M^2+MN^2) U equals the zero matrix the U is the zero matrix.

Q.17 The quadratic equation p(x)=0 with real coefficients has purely imaginary roots. Then the equation p(p(x))=0 has

(A) Only purely imaginary roots.

(B) All real roots.

(C) Two real and two purely imaginary roots.

(D) Neither real nor purely imaginary roots.

Q.18 Let $z = \frac{-1+\sqrt{3}i}{2}$, where $\sqrt{-1}$, and $r, s \in \{1, 2, 3\}$.

Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of or dered pairs (r, s) for which $P^2 = -I$ is

Q.19 Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & 5 & 0 \end{bmatrix}$ where $\alpha \in R$.

Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$ where $R, k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then

(A) $\alpha = 0, k = 8$ (B) $4\alpha - k + 8 = 0$

(C) det $(P\,\text{adj}(Q)) = 2^9$ (D) det $(P\,\text{adj}(P)) = 2^{13}$

Q.20 Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that $p^{50} - Q = I$ then $\frac{q_{31} + q_{32}}{q_{21}}$ equals **(2016)**

(A) 52 (B) 103 (C) 201 (D) 205

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.7 Q.8 Q.17
Q.23 Q.32 Q.35
Q.38 Q.41 Q.44

Exercise 2

Q.4 Q.11 Q.14
Q.19 Q.22 Q.26

Previous Years' Questions

Q.1 Q.2 Q.6
Q.10 Q.13

JEE Advanced/Boards

Exercise 1

Q.7 Q.10 Q.13
Q.18 Q.19 Q.20
Q.17

Exercise 2

Q.2 Q.5 Q.8
Q.12

Previous Years' Questions

Q.2 Q.4 Q.11
Q.12

Answer Key

JEE Main/Boards

Exercise 1

Q.1 $x = 2, y = 3$

Q.2 6; 6

Q.3 Order of AB is 2×3; order of BA is not defined

Q.4 (i) 3 (ii) 7 (iii) 8 (iv) 11

Q.5 $1 \times 18, 2 \times 9, 3 \times 6, 6 \times 3, 9 \times 2, 18\times 1$

Q.6 $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Q.9 Two

Q.10 $x = 1, y = -\frac{1}{2}$

Q.11 $f(A) = 3A^2 - 9S + 7I$

Q.12 AB = BA

Q.13 $\begin{bmatrix} 2 & -3 & 4 \\ 4 & -6 & 8 \\ 6 & -9 & 12 \end{bmatrix}$

Q.14 No

Q.15 $\begin{bmatrix} -1 & 1/2 \\ -2 & -1 \end{bmatrix}$

Q.16 $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Q.17 m = 2, p = 3, b = 4

Q.18 Skew-symmetric

Q.19 (i) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} \cos(x+y) & \sin(x+y) \\ \sin(x+y) & -\cos(x+y) \end{bmatrix}$

Q.20 Each element is zero

Q.21 $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Q.22 $1 \pm \sqrt{10}$

Q.23 $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Q.24 $\begin{bmatrix} 0 & 5 \\ 2 & 1 \end{bmatrix}$

Q.25 $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

Q.27 1 × 8, 2 × 4, 4 × 2, 8 × 1; 1 × 1, 5 × 1

Q.28 $[ac + bd + a^2 + b^2 + c^2 + d^2]$

Q.29 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $A^6 = A^2$

Q.31 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Q.32 x = 9m, y = 14

Q.33 $B = \frac{1}{3}\begin{bmatrix} -2 & 1 & 19 \\ -5 & 8 & 11 \end{bmatrix}$

Q.36 $P \pm \frac{1}{\sqrt{3}}, q \pm \frac{1}{\sqrt{3}}$

Q.37 $\begin{bmatrix} -11 & 3 & -14 \\ -16 & 3 & 0 \\ -19 & -7 & 22 \end{bmatrix}$

Q.38 $\begin{bmatrix} -1 & -4 & -7 \\ 1 & -2 & -5 \\ 3 & 0 & -3 \end{bmatrix}$

Q.39 x = 3, y = 7, z = –2, w = 14

Q.40 $X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}, Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

Q.42 $A^2 = A, B^2 = I; A^2B^2 = AI = A$

Q.43 $X = \begin{bmatrix} 1 & -1 & -5 \\ 3 & 4 & 0 \end{bmatrix}$

Exercise 2

Single Correct Choice Type

Q.1 A	**Q.2** C	**Q.3** D	**Q.4** B	**Q.5** C	**Q.6** C
Q.7 A	**Q.8** B	**Q.9** D	**Q.10** D	**Q.11** B	**Q.12** B
Q.13 C	**Q.14** C	**Q.15** D	**Q.16** D	**Q.17** D	**Q.18** B
Q.19 A	**Q.20** C	**Q.21** C	**Q.22** C	**Q.23** B	**Q.24** C
Q.25 C	**Q.26** D	**Q.27** C	**Q.28** C	**Q.29** C	

Previous Years' Questions

Q.1 B	**Q.2** A	**Q.3** C	**Q.4** B	**Q.5** B
Q.6 A	**Q.7** A	**Q.9** 0	**Q.10** $a = \frac{1}{4}, b = \frac{5}{4}$ $f(x) = \frac{1}{4}x^2 - \frac{5}{4}x + 2$	
Q.12 B	**Q.13** D	**Q.14** B		

JEE Advanced/Boards

Exercise 1

Q.1 (a) 225(b) –8

Q.2 (i) 5 (ii) 18 (iii) 8 (iv) 5

Q.3 $\begin{bmatrix} 17 & 4 & -19 \\ -10 & 0 & 13 \\ -21 & -3 & 25 \end{bmatrix}$

Q.4 (a) $\begin{bmatrix} -4 & 7 & -7 \\ 3 & -5 & 5 \end{bmatrix}$ (b) $\frac{1}{19}\begin{bmatrix} 48 & -25 \\ -70 & 42 \end{bmatrix}$

Q.5 29

Q.7 21

Q.8 1

Q.9 $x_1 = 1, x_2 = -1, x_3 = 1$

Q.10 100

Q.11 1

Q.12 $x = 2, y = 1, z = -1$

Q.13 (i) $a \neq -3, b \in R$ (ii) $a = -3$ and $b \neq 1/3$ (iii) $a = -3, b = 1/3$

Q.14 (a) $X = \begin{bmatrix} -3 & -3 \\ \frac{5}{2} & 2 \end{bmatrix}$ (b) $X = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ (c) No solution

Q.16 8`

Q.17 3355

Q.18 (a) 18 (b) 0

Q.19 A→q; B → s; C→ p; D→ p

Q.20 A → p, q, t; B → s; C→ p, r; D → r

Exercise 2

Single Correct Choice Type

Q.1 B	**Q.2** A	**Q.3** A	**Q.4** A	**Q.5** D	**Q.6** C
Q.7 B	**Q.8** C	**Q.9** C	**Q.10** D	**Q.11** B	**Q.12** C
Q.13 C					

Previous Years' Questions

Q.1 A

Q.2 C

Q.4 $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

Q.5 $\theta = n\pi, n\pi + (-1)^n \frac{\pi}{6}, n \in Z$

Q6. $\sum_{a=1}^{n} \Delta_a = c$

Q.7 2

Q.8 $2n(n^2 + 4n + 5)$

Q.9 $\alpha = n\pi$ or $n\pi + \pi/4$

Q.10 0	**Q.11** A	**Q.12** 9	**Q.13** D	**Q.14 D**	**Q.15** B, C, D
Q.16 A, B	**Q.17** D	**Q.18** A	**Q.19** B, C	**Q.20** B	

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $\begin{bmatrix} 2x-1 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ x+y \end{bmatrix}$

$2x - 1 = 3$

$2x = 4 \Rightarrow x = 4/2 = 2$

$5 = x + y = 2 + y$

$y = 5 - 2 = 3$

$(x, y) = (2, 3)$

Sol.2 row = n

Column = m

Then total elements = mn

if $(n, m) = (2, 3) \Rightarrow nm = 2.3 = 6$

if $(n, m) = (3, 2) \Rightarrow mn = 3.2 = 6$

Sol 3: $A_{2\times2}$ and $B_{2\times3}$

For AB $\Rightarrow$ order will be $\Rightarrow 2 \times 3$

For BA $\Rightarrow$ row of B $\neq$ column of A

So, BA does not exist

Sol 4: $A = [a_{ij}]$, $1 \le i \le 3$, $i \le j \le 3$

$a_{ij} = i + 2j$

(i) $a_{11} = 1 + 2 = 3$ (ii) $a_{32} = 3 + 2(2) = 3 + 4 = 7$

(iii) $a_{23} = 2 + 3(2) = 8$ (iv) $a_{34} \Rightarrow$ not a element $= i \le j \le 3$ but here $4 > 3$

Sol 5: Total element = 18

Assume no of row = n

And no. of column = m

so $n \times m = 18 = 1 \times 18 = 2 \times 9 = 6 \times 3 = 3 \times 6 = 9 \times 2 \times 18 \times 1$

Sol 6: Diagonal matrix = $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

scalar matrix = $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Sol 7: Matrix $A = [a_{ij}]$ assume

$A^T = [a_{ji}]$

So $A + A^T = [a_{ij} + a_{ji}] = [b_{ij}]$ assume

Here $b_{ij} = a_{ij} = a_{ji}$

$b_{ji} = a_{ij} + a_{ji}$

Here $b_{ij} = b_{ji}$

Matrix is symmetric.

Sol 8: $A - A^T$

$\Rightarrow [a_{ij}] - [a_{ji}] = [b_{ij}]$ assume

$b_{ij} = a_{ij} - a_{ji}$

$b_{ji} = a_{ji} - a_{ij}$

$\Rightarrow b_{ij} = -b_{ij}$

This matrix is known as symmetric matrix.

Sol 9: A matrix $\rightarrow$ row n, column = m

Total element = mn

mn is prime no.

so mn could be $\rightarrow$ 2, 5, 7, 11

factor of $2 = 1 \times 2$ or 2×1

So for any prime no. of only 2 order

$\Rightarrow 1 \times n$ and $n \times 1$($n \in$ prime no.)

Sol 10: $\begin{bmatrix} x \\ 2y \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x-1 \\ 2y+4 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$x - 1 = 0 \Rightarrow x = 1$

$2y + 4 = 3 \Rightarrow 2y = 3 - 4 = -1$

$Y = -\frac{1}{2}$

Sol 11: $f(x) = 3x^2 - 9x + 7$

f(A) $\Rightarrow$ if A is a matrix

$f(A) = 3A^2 - 9A + 7I$

A is a square matrix so A^2 is possible.

Sol 12: A, B and AB are symmetric matrices

$A = a_{ij}$

$B = b_{ij}$

$AB = A_{ij}\,B_{ji} = C_{ij}$

$BA = B_{ij}\,.\,A_{ji} = d_{ij}$

$\left.\begin{array}{l}\text{but } B_{ij} = B_{ji}\\ \text{and } A_{ij} = A_{ji}\end{array}\right\}$ symmetric matrix's property

$\therefore\ AB = A_{ij}\,B_{ij} = A_{ij}\,.\,B_{ij} = BA$

$AB = BA$

Sol 13: $A = \begin{bmatrix}1\\2\\3\end{bmatrix}_{3\times1}$ $B = \begin{bmatrix}2 & -2 & 4\end{bmatrix}_{1\times3}$

$$AB = \begin{bmatrix}1\\2\\3\end{bmatrix}_{3\times1}\begin{bmatrix}2 & -2 & 4\end{bmatrix}_{1\times3} = \begin{bmatrix}2 & -2 & 4\\4 & -4 & 8\\6 & -6 & 12\end{bmatrix}$$

Sol 14: $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}_{2\times2}$ and $\begin{bmatrix}1 & 2 & 0\\3 & 4 & 0\\0 & 0 & 0\end{bmatrix}_{3\times3}$

Both have different orders. So they are not same.

Sol 15: $A = \begin{bmatrix}2 & -1\\4 & 2\end{bmatrix}$, $K = -\dfrac{1}{2}$

$$KA = K\begin{bmatrix}2 & -1\\4 & 2\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}2 & -1\\4 & 2\end{bmatrix}$$

$$= \begin{bmatrix}-\dfrac{2}{2} & -\dfrac{1}{2}(-1)\\ -\dfrac{1}{2}(4) & -\dfrac{1}{2}(2)\end{bmatrix} = \begin{bmatrix}-1 & \dfrac{1}{2}\\ -2 & -1\end{bmatrix}$$

Sol 16: $\tan\theta\begin{bmatrix}\sec\theta & \tan\theta\\ \tan\theta & -\sec\theta\end{bmatrix} + \sec\theta\begin{bmatrix}-\tan\theta & -\sec\theta\\ -\sec\theta & \text{tam}\theta\end{bmatrix}$

$$= \begin{bmatrix}\tan\theta\sec\theta - \tan\theta\sec\theta & \tan^2\theta - \sec^2\theta\\ \tan^2\theta - \sec^2\theta & -\tan\theta\sec\theta + \cos\theta\sec\theta\end{bmatrix}$$

$$= \begin{bmatrix}0 & \dfrac{\sin^2\theta - 1}{\cos^2\theta}\\ \dfrac{\sin^2\theta - 1}{\cos^2\theta} & 0\end{bmatrix} = \begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$$

Sol 17: $X_{m\times3}\,Y_{p\times4} = Z_{2\times b}$

Column of x = row of y $\Rightarrow$ 3 = p and 2 × b = (m × 4)

So m = 2; b = 4

Sol18: $A = \begin{bmatrix}0 & -1 & 2\\1 & 0 & -3\\-2 & 3 & 0\end{bmatrix}$

$a_{12} = -a_{21},\ a_{13} = -a_{31}$

$a_{23} = -a_{32},$

so A is skew symmetric.

Sol 19: $R(\theta) = \begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}$

$$R\left(\frac{\pi}{2}\right) = \begin{bmatrix}\cos\dfrac{\pi}{2} & \sin\dfrac{\pi}{2}\\ \sin\dfrac{\pi}{12} & -\cos\dfrac{\pi}{2}\end{bmatrix} = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

$$R(x+y) = \begin{bmatrix}\cos(x+y) & \sin(x+y)\\ \sin(x+y) & -\cos(x+y)\end{bmatrix}$$

Sol 20: Skew symmetric $A = [a_{ij}]$

For all skew symmetric Matrix dia. l element (a_{ij}) are zero so $a_{ij} = 0$ & when i = j

Sol 21: $A = \begin{bmatrix}a & 0\\0 & 0\end{bmatrix}$

$$A^2 = \begin{bmatrix}a & 0\\0 & 0\end{bmatrix}\begin{bmatrix}a & 0\\0 & 0\end{bmatrix} \Rightarrow A^2 = \begin{bmatrix}a^2 & 0\\0 & 0\end{bmatrix}$$

$$A^2 \times A^2 = \begin{bmatrix}a^2 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}a^2 & 0\\0 & 0\end{bmatrix} \Rightarrow A^4 = \begin{bmatrix}a^4 & 0\\0 & 0\end{bmatrix}$$

$$\Rightarrow A^{16} = \begin{bmatrix}a^{16} & 0\\0 & 0\end{bmatrix}$$

Sol 22: $[X\ \ 1]_{1\times2}\begin{bmatrix}1 & 0\\-2 & -3\end{bmatrix}_{2\times2}\begin{bmatrix}X\\3\end{bmatrix}_{2\times1} = 0$

$$\Rightarrow [x-2\ \ 0-3]_{1\times2}\begin{bmatrix}X\\3\end{bmatrix}_{2\times1}\ [x-2\ -3]\begin{bmatrix}X\\3\end{bmatrix} = 0$$

$[(x-2)x - 3(3)] = 0 \Rightarrow x^2 - 2x - 9 = 0$

$x = \dfrac{2 \pm \sqrt{2^2 - 4(-9)}}{2} = 1 \pm \sqrt{10}$

Sol 23: $A = \begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}_{2\times 2}$

Additive inverse B which is – A

So, A + B =A – A = 0

Sol 24: $x + \begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & 0 \end{bmatrix}$

Assume $x = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x_1 + 2 & x_2 - 1 \\ x_3 + 3 & x_4 - 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & 0 \end{bmatrix}$

$\Rightarrow x_1 + 2 = 2 \Rightarrow x_1 = 0$

$\Rightarrow x_2 - 1 = 4 \Rightarrow x_2 = 1 + 4 = 5$

$\Rightarrow x_3 + 3 = 5 \Rightarrow x_3 = 5 - 3 = 2$

$\Rightarrow x_4 - 1 = 0 \Rightarrow x_4 = 1$

$x = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 2 & 1 \end{bmatrix}$

Sol 25:

$\begin{bmatrix} \sin^2\theta & 1 \\ \cot^2\theta & 0 \end{bmatrix} + \begin{bmatrix} \cos^2\theta & 0 \\ -\mathrm{cosec}^2\theta & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} \sin^2\theta + \cos^2\theta + 0 & 1 + 0 - 1 \\ \cot^2\theta - \mathrm{cosec}^2\theta - 1 & 0 + 1 + 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ -1-1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

Sol 27: Matrix has 8 element

m × n = 8 = 1 × 8 = 8 × 1 = 2 × 4 = 4 × 2

if m × n = 5 = 1 × 5 = 5 × 1 (only 2 possible order)

Sol 28: $\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} \times \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$[ac + bd] \times [a^2 + b^2 + c^2 + d^2]$

$\Rightarrow [a^2 + b^2 + c^2 + d^2 + ac + bd]$

Sol 29: $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$A^6 = [A^2]^3 = [I]^3 = I$

$A^6 = I = A^2$

Sol 30: Properties of skew – symmetric matrix [a, j]

$\Rightarrow$ All diagonal element are zero

$\Rightarrow a_{ij} = -a_{ji}$

Sol 31: $A = \begin{bmatrix} 0 & -4 \\ 0 & -3 \end{bmatrix}$. $B = \begin{bmatrix} 5 & -7 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & -4 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0.5 - 4.0 & 0(-7) \\ 0(5) & 0(-3) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Sol 32: $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

$A^2 - XA + YI = 0$

$A^2 = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 4^2 + 3.2 & 4.3 + 3.5 \\ 2.4 + 5.2 & 2.3 + 5.2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$

$A^2 - XA + YI = 0$

$\Rightarrow \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} \times \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} + \dfrac{1}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 22 - 4x + y & 27 - 3x \\ 18 - 2x + x & 31 - 5x + y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow$ Compare elements

$27 - 3x = 0$

$3x = 27 \Rightarrow x = \dfrac{27}{3} = 9 \Rightarrow y = 45 - 31 = 14$

(x, y) = (9, 14)

Sol 33: $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 5 & 7 \end{bmatrix}$

$2A - 3B = \begin{bmatrix} 4 & 5 & -9 \\ 1 & 2 & 3 \end{bmatrix}$

Assume $B = \begin{bmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \end{bmatrix}$

$\Rightarrow 2A - 3B = \begin{bmatrix} 2.1 - 3b_1 & 2.3 - 3b_2 & 2 \times 5.3b_3 \\ -4 - 3b_4 & 2.5 - 3b_5 & 2.7 - 3b_6 \end{bmatrix}$

$= \begin{bmatrix} 4 & 5 & -9 \\ 1 & 2 & 3 \end{bmatrix}$

$2 - 3b_1 = 4 \rightarrow b_1 = \dfrac{4-2}{-3} = -\dfrac{2}{3}$

$\Rightarrow 6 - 3b_2 = 1$

$\Rightarrow 3b_2 = 6 - 5 = 1$

$\Rightarrow b_2 = \dfrac{1}{3}$

Same as $b_3 = \dfrac{19}{3}$

$b_4 = -\dfrac{5}{3}, b_5 = \dfrac{8}{3}, b_6 = \dfrac{11}{3}$

So $B = \dfrac{1}{3}\begin{bmatrix} -2 & 1 & 19 \\ -5 & 8 & 11 \end{bmatrix}$

Sol 34: $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & \cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha - \sin\alpha\cos\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$

we know $\rightarrow \cos^2\alpha - \sin^2\alpha = \cos^2\alpha$

and $2\cos\alpha \sin\alpha = \sin 2\alpha$

so $A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2a \end{bmatrix}$

Sol 35: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

For $(aI + bA)^3$; $aI = a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

$bA = b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

$aI + bA = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} + \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$

$(aI + bA)^3 = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$

$= \begin{bmatrix} a^2 & ab + ba \\ 0 & a^2 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a^3 & a^2b + 2a^2b \\ 0 & a^3 \end{bmatrix}$

$= \begin{bmatrix} a^3 & 3a^2b \\ 0 & a^3 \end{bmatrix}$

and R. H. S. $= a^3I + 3a^2bA$

$= a^3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 3a^2b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} a^3 & 0 \\ 0 & a^3 \end{bmatrix} + \begin{bmatrix} 0 & 3a^2b \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a^3 & 3a^2b \\ 0 & a^3 \end{bmatrix}$

L. H. S. = R. H. S

Sol 36: $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$

$(pI + qA)^2 = A$

$pI = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix}, qA = q\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & q \\ -q & q \end{bmatrix}$

$pI + qA = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix} + \begin{bmatrix} 0 & q \\ -q & q \end{bmatrix} = \begin{bmatrix} p & q \\ -q & p+q \end{bmatrix}$

$(pI + qA)^2 = \begin{bmatrix} p & q \\ -q & p+q \end{bmatrix}\begin{bmatrix} p & q \\ -q & p+q \end{bmatrix}$

$= \begin{bmatrix} p^2 - q^2 & pq + q(p+q) \\ -pq - q(p+q) & -q^2 + (q+q)^2 \end{bmatrix} = A \text{ (given)}$

$A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$

So $p^2 - q^2 = 0 \Rightarrow p^2 = q^2 \Rightarrow p = \pm q$

$pq + qp + q^2 = 1$

$q^2 + 2qp = q^2 \pm 2q^2 = 1$

–ve $\rightarrow q^2 - 2q^2 = 1 \Rightarrow q^2 = 1$ not possible

+ve $\rightarrow q^2 + 2q^2 = 3q^2 = 1 \Rightarrow q^2 = 1/3$

So $p = q = \pm \dfrac{1}{\sqrt{3}}$

Sol 37: $A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & 0 & 6 \\ -2 & 1 & 5 \end{bmatrix}$

$B = \begin{bmatrix} 5 & 1 & 2 \\ 6 & -1 & 4 \\ 5 & 3 & -4 \end{bmatrix}$

$2A - 3B = 2\begin{bmatrix} 2 & 3 & -4 \\ 1 & 0 & 6 \\ -2 & 1 & 5 \end{bmatrix} - 3\begin{bmatrix} 5 & 1 & 2 \\ 6 & -1 & 4 \\ 5 & 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} -11 & 3 & -14 \\ -16 & 3 & 0 \\ -19 & -7 & 22 \end{bmatrix}$

Sol 38: $A_{3\times3} = [a_{ij}]$

$a_{ij} = 2i - 3j$

$\therefore a_{11} = 2(1) - 3(1) = -1,\ a_{12} = 2(1) - 3(2) = -4$

$a_{13} = 2(1) - 3(3) = -7,\ a_{21} = 2(2) - 3(1) = 1$

$a_{22} = 2(3) - 3(2) = -2,\ a_{23} = 2(2) - 3(3) = -5$

$a_{31} = 2(3) - 3(1) = 3,\ a_{32} = 2(3) - 3(2) = 0$

$a_{33} = 2(3) - 3(3) = -3$

So $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} -1 & -4 & -7 \\ 1 & -2 & -5 \\ 3 & 0 & -3 \end{bmatrix}$

Sol 39: $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$

Compare elements

$x = 3$

$3x - y = 3(3) - y = 9 - y = 2$

$y = 9 - 2 = 7$

$2x + z = 2(3) + 7 = 6 + 7 = 4 \Rightarrow 7 = 4 - 6 = -2$

$3y - w = 3(7) - w = 7 \Rightarrow w = 21 - 7 = 14$

$(x, y, z, w) = (3, 7, -2, 14)$

Sol 40: $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix},\ X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

sum of X + Y, X – Y

$\Rightarrow X + Y + X - Y = 2X$

$= \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 5+3 & 2+6 \\ 0 & 9-1 \end{bmatrix}$

$2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix} = 2\begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$

$X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$

$Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - X = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$

$Y = \begin{bmatrix} 5-4 & 2-4 \\ 0 & 9-4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

Sol 41: $A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$

$B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\theta \end{bmatrix}$

$AB = (C_{ij})$

$C_{11} = \cos^2\theta\cos^2\phi + \cos\theta\sin\theta\cos\phi\sin\phi$

$C_{11} = \cos\theta\cos\phi(\cos\theta\cos\phi + \sin\theta\sin\phi)$

$C_{11} = \cos\theta\cos\phi\cos(\theta - \phi) = 0$

Similarly C_{12}, C_{21} and C_{22} will also be zero

So $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Sol 42: $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 3 \end{bmatrix},\ B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+3-5 & -1-3+5 & 1+3-5 \\ -3-9+15 & 3+9+15 & -3-9+15 \\ -5-15+25 & 5+15-25 & -5-15+25 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix} = A$$

$$B^2 = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 4-3 & -12+12 & -12+12 \\ -3+3 & 4+9-12 & 3+9-12 \\ 4-4 & -4-12+16 & -3-12+16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^2B^2 = A^2.I = A. I = A$

Sol 43: $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} -1 & 8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}_{3\times3}$

X 's number of row = column of $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix}$

order of X = 2 × 3

assume $X = \begin{bmatrix} X_1 & X_2 & X_3 \\ X_4 & X_5 & X_6 \end{bmatrix}$

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} X_1 & X_2 & X_3 \\ X_4 & X_5 & X_6 \end{bmatrix}$$

$$= \begin{bmatrix} 2x_1 - x_4 & 2x_2 - x_5 & 2x_3 - x_6 \\ x_4 & x_5 & x_6 \\ -2x_1 + 4x_4 & -2x_2 + 4x_5 & -2x_3 + 4x_6 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$$

$x_4 = 3, x_5 = 4, x_6 = 0$

$-2x_1 + 4x_4 = 10 \Rightarrow -2x_1 + 4(3) = 10$

$2x_1 = 12 - 10 = 2 \Rightarrow x_1 = 1$

$-2x_2 + 4x_5 = -2x_2 + 4(4) = 20$

$-2x_2 + 16 = 20$

$2x_2 = 16 - 20 = -4 \Rightarrow x_2 = -\frac{4}{2} = -2$

$-2x_3 + 4x_6 = -2x_3 + 4(0) = 10 = -2x_3$

$x_3 = \frac{10}{-2} = -5$

$$X = \begin{bmatrix} X_1 & X_2 & X_3 \\ X_4 & X_5 & X_6 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

Exercise 2

Single Correct Choice Type

Sol 1: (A) Number of elements in a matrix = 60

$60 = 2^2\, 5^1\, 3^1$

Number of order matrix can have = (2 + 1) (1 + 1) (1 + 1)

= 3 × 2 × 2 = 12

Sol 2: (C) $A_{(x)\times(x+5)}$ $B_{y\times(11-y)}$

AB and BA both exist

$\Rightarrow$ for AB $x + 5 = y$ ...(i)

$\Rightarrow$ for BA $11 - y = x$ ...(ii)

$\Rightarrow y = 8; x = 3$

Sol 3: (D) A is a square invertible matrix

$A^2 = A$

Multiply A^{-1} both sides

$A^{-1} A^2 = A^{-1}A = I$

$A = I$

So $A^2 = I$

$A^2 - I = 0$ (zero matrix)

Sol 4: (B) Total 143 elements all are different.

$143 = 1 \times 143 = 143 \times 1 = 11\times13 = 13 \times 11$

Total Number of order that exist = 4

Number of way to arrange 143 elements = 143!

Total not of matrix = 4×1431

Sol 5: (C) $A^2 = A$

$(I + A)^4 = (I^2 + A^2 + 2A)^2$

$= [I + A + 2A]^2 = [I + 3A]^2$ $(\because A^2 = A)$

$= I^2 + 9A^2 + 6A = I + 9A + 6A = I + 15A$

Sol 6: (C) $A = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$

Since, A is orthogonal matrix

So, $AA' = A'A = I_n$

$$AA^{-1} = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}\begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} \alpha^2+\beta^2 & -\alpha\beta+\alpha\beta \\ -\alpha\beta+\alpha\beta & \alpha^2+\beta^2 \end{bmatrix}$$

$$\begin{bmatrix} \alpha^2+\beta^2 & 0 \\ 0 & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\alpha^2 + \beta^2 = 1$ ...(i)

$x^2 - px + q = 0$ & $x^2 + px - q = 0$

Sum of both equation for common roots

$x^2 - px + q + x^2 + px - q = 0$

$2x^2 = 0 \Rightarrow x = 0$

So if α is roots of $x^2 - px + q = 0$ and β is roots of $x^2 + px - q = 0$

$\Rightarrow \alpha + 0 = p$ and $\alpha(0) = q = 0$

$\beta + 0 = -p$ and $\beta(0) = -q = 0$

$\Rightarrow \alpha = p$ and $\beta = -p$

In (i) equation $a^2 + b^2 = 1$

$\Rightarrow p^2 + (-p)^2 - 1$

$\Rightarrow 2p^2 = 1$

$\Rightarrow p = \pm \dfrac{1}{\sqrt{2}}, q = 0$

Sol 7: (A) (det A) = 3

(det λA) = 81

if A's order = $n \times n$

then (det λA) = λ^n(det A) = $\lambda^x 3 = 81$

$\lambda^n = \dfrac{81}{3} = 27$

$\lambda^n = 27 = 3^n \quad \because \lambda \in N$

So, n = 3

Sol 8: (B) $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $f(x) = \dfrac{1+x}{1-x}$

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix} = -2\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$(I-A)^{-1} = \frac{1}{\det(I-A)}\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$\det(I-A) = \begin{vmatrix} 0 & -2 \\ -2 & 0 \end{vmatrix} = 0 - (-2)(-2) = -4$$

$$f(A) = \frac{(I+A)}{(I-A)} = \frac{2}{-4}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 0+2 & 2+0 \\ 0+2 & 2+0 \end{bmatrix} = -\frac{2}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

Sol 9: (D) A is skew symmetric matrix

$\Rightarrow A^TA = I$

$\Rightarrow A^T = -A \Rightarrow (AT)^2 = (A)^2$

$\Rightarrow A^TA = -A^2 = I$

Taking square of both sides

$A^4 = I$

$\Rightarrow A^{4n} = I$ ($\because -A^2 = I$ are $A = -A^T$) \

$A^3A^{4n} = A^3I$

$A^{4n+3} = -A^2(-A)\,I$

$A^{4(n+1)-1} = (-A)I = A^T I = A^I$

$4n - 1 \in N$

$\Rightarrow A^{4n-1} = A^T$

Sol 10: (D) $|A| = |B| A_{2\times 2}, B_{2\times 2}$

Tr (A) = Tr(B)

$A^2 - 3A + 14I = 0$ and $B^2 - \lambda B + \mu I = 0$

if A = B, |A| = |B|

Tr |A| = Tr(B) satisfied so $A^2 - \lambda A + \mu I = 0$

$\mu = 14$ ($\because$ A's order = 2×2)

Sol 11: (B) The adjoint of upper triangular matrix is false.

$\because$ That is equal to upper triangular not lower triangular matrix.

Sol 12: (B) A, B, (A + B) are non-singular

$[A(A+B)^{-1}B]^{-1}$

$= [A^{-1}((A+B)^{-1})^{-1}B^{-1}] = (A^{-1}(A+B)\,B^{-1})$

$= [(A^{-1}A + A^{-1}B)B^{-1}) = [(I + A^{-1}B)B^{-1})$

$= [B^{-1} + A^{-1}BB^{-1}] = A^{-1} + B^{-1}$

Sol 13: (C) A is an orthogonal matrix

$|A| = -1$

$\Rightarrow AA^T = A^TA = I_n$

$\Rightarrow |A| = |A^T| = -1$

$\Rightarrow A^T = +A^{-1}$($\because$ A is an orthogonal matrix)

$\Rightarrow A^T = \dfrac{1}{\det(A)}$ (adjA) = – (adjA)

Sol 14: (C) A and B are square matrices of order 3

(A) adj(AB) = adj(A) + adj(B) is not necessary

$\Rightarrow$ option (A) is wrong.

(B) $(A + B)^{-1} \neq A^{-1} + B^{-1}$

(C)AB = 0

A, B are square matrix

So if AB = 0

$\Rightarrow |A|\,|B| = 0$

$\Rightarrow |A| = 0$ or $|B| = 0$

Sol 15: (D) $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \Rightarrow |A| = a^3|I| = a^3$

$$\text{adj}(A) = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$$

$|\text{adj } |A|| = (a^2)^3 = a^6$

$\therefore |A|\,|\text{adj }(A)| = a^3\, a^6 = a^9$

Sol 16: (D) $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

$|A| = 1 + \tan^2 x$

$$A^{-1} = \frac{1}{1+\tan^2 x}\begin{bmatrix} 1 & \tan x \\ \tan x & 1 \end{bmatrix} = \frac{A}{(1+\tan^2 x)}$$

$$AT = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^TA^{-1} = \frac{1}{(1+\tan^2 x)}\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan x \\ -\tan x & 1 \end{bmatrix}$$

$$= \frac{1}{(1+\tan^2 x)}\begin{bmatrix} 1+\tan^2 x & \tan x - \tan x \\ \tan x - \tan x & \tan^2 x + 1 \end{bmatrix}$$

$$= \frac{1}{(1+\tan^2 x)}\begin{bmatrix} 1+\tan^2 x & 0 \\ 0 & 1+\tan^2 x \end{bmatrix}$$

$$= \frac{1+\tan^2 x}{1+\tan^2 x}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|A^T A^{-1}| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

Sol 17: (D) $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$\Rightarrow |A| = 2(-2) - 3(5) = -15 - 4 = -19$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix}^T = \frac{-1}{-19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19}A$$

Sol 18: (B) $P^T = P^{-1}$

Assume $P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}$, $P^T = \begin{bmatrix} P_1 & P_3 \\ P_2 & P_4 \end{bmatrix}$

$\Rightarrow |P| = P_1P_4 - P_2P_3$

$$P^{-1} = \frac{1}{|P|}(\text{adj } P) = \frac{1}{|P|}\begin{bmatrix} P_4 & -P_3 \\ -P_2 & P_1 \end{bmatrix}^T = \frac{1}{|P|}\begin{bmatrix} P_4 & -P_2 \\ -P_3 & P_1 \end{bmatrix}$$

$|P| = P_1P_4 - P_2P_3$

$$\begin{bmatrix} P_1 & P_3 \\ P_2 & P_4 \end{bmatrix} = \frac{1}{|P|}\begin{bmatrix} P_4 & -P_2 \\ -P_3 & P_1 \end{bmatrix}$$

$|P| = 1$

$P_2 = -P_3$

Only option (B) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is correct.

Sol 19: (A) $B^r = I$ r > 1

$A^{-1} B^{r-1} A - A^{-1}B^{-1}A = A^{-1}B^r B^{-1}A - A^{-1}B^{-1}A$

$= A^{-1}B^{-1}A - A^{-1}B^{-1}A = 0$

Sol 20: (C) A & B are orthogonal matrices

$\Rightarrow AA^T = A^TA = I_n$ and $BB^T = B^TB = I_n$

$AB \Rightarrow (AB)\,(AB)^T$

$\Rightarrow (AB)\,(B^TA)^T$

$\Rightarrow A(BB^T)A^t = AIA^T$

$\Rightarrow AA^T = I$

$(AB)^T (AB) = B^TA^TAB = B^TIB = B^TB = I_n$

So $(AB)^T(AB)$

$gE = AB(AB)^T = I_n$

So, AB also satisfying property of orthogonal

Sol 21: (C) C is an orthogonal matrix

$\Rightarrow CC^T = C^TC = I_n$

Tr (C^TAC) = Tr $[(C^TA)C]$

= Tr $[C(C^TA)]$ = Tr (CC^TA) = Tr (IA)= Tr (A)

Sol 22: (C) A and B are idempotent matrices

so, $A^2 = A$ and $B^2 = B$

|A|, |B| = or 1

AB = BA

A – B is non-singular

$\Rightarrow$ |A| = and |B| = 1 or |A| = 1 and |B| = 0

Sol 23: (B) $AA^T = I$

$(A^T BA)^{10}$

$= (A^TBA)(A^TBA)(A^TBA)\ldots\ldots\ldots.$

$\Rightarrow A^TBIBIBI\ldots\ldots\ldots BA = A^TB^{10}A$

Sol 24: (C) $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$

$$B = \begin{bmatrix} a^2 & ab & ca \\ ab & b^2 & cb \\ ac & bc & c^2 \end{bmatrix}$$

$$AB = \begin{bmatrix} abc - abc & b^2c - b^2c & c^2b - bc^2 \\ -a^2bc + ab^2c & -abc + abc & -ac^2 + ac^2 \\ a^2b - a^2b^{\infty} & ab^2 - ab^2 & abc - abc \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Sol 25: (C) AB = BA, $C^2 = B$

$(A^{-1}CA)^2 = (A^{-1}CA)(A^{-1}CA) = A^{-1}C\,ICA = A^{-1}C^2A$

$= A^{-1}BA = A^{-1}(AB) = IB = B = C^2$

Sol 26: (D) Tr(A) = 12

Assume $A = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$ ($\because$ A is diagonal matrix)

Tr(A) = $a_1 + a_2 + a_3 = 12$ and det |A| = $a_1\,a_2\,a_3$ for maximum of det (A) = $a_1\,a_2\,a_3$

$a_1 + a_2 + a_3 = 3a_1 = 3a_2 = 3a_3 = 12$

$a_1 = \frac{12}{3} = 4$

det |A| = $4 \times 4 \times 4 = 64$

Sol 27: (C) AB = B

BA = A

$(A + B)^2 = (AB + BA)^2$

$A^2 + B^2 + AB + BA = (AB)^2 + (BA)^2 + (AB)(BA) + (BA)(AB)$

$A^2 + B^2 + AB + BA = ABAB + BABA + AB + BA$

$A^2 + B^2 = AAB + BBA = AB + BA$

$A^2 + B^2 = A + B$

Sol 28: (C) $A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$, B is column matrix

$(A^8 + A^6 + A^4 + A^2 + I)\,B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2\times1}$

$A^2 = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 3I$

$A^4 = A^2A^2 = 3^2I$

$A^6 = 3^3I,\ A^8 = 3^4I$

$A^8 + A^6 + A^4 + A^2 + I = I(1 + 3 + 3^2 + 3^3 + 3^4) = 121\,I$

$|2|\,I\,B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 0 \\ \frac{1}{|2|} \end{bmatrix}$

Sol 29: (C) AB = BA and $A^2 = I$

$ABA = A(AB) = A^2B = IB = B$

Previous Years' Questions

Sol 1: (B)

$$\Rightarrow \Delta = \begin{vmatrix} 1 + a^2 & a & a^2 \\ \cos(p-d)x + \cos(p+d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x + \sin(p+d)x & \sin px & \sin(p+d)x \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 + a^2 & a & a^2 \\ 2\cos px \cos dx & \cos px & \cos(p+d)x \\ 2\sin px \cos dx & \sin px & \sin(p+d)x \end{vmatrix}$$

Applying $C_1 \to C_2 - 2\cos dxC_2$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2-2a\cos dx & a & a^2 \\ 0 & \cos px & \cos(p+d)x \\ 0 & \sin px & \sin(p+d)x \end{vmatrix}$$

$\Rightarrow \Delta = (1 + a^2 - 2a \cos dx)$ [sin (p+ d) x cos px

– sin px cos (p + d) x]

$\Rightarrow \Delta = (1 + a^2 - 2a \cos dx) \sin dx$

Which is independent of p

Sol 2: (A)

$$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$

Applying $C_3 \to C_3 - (C_1 + C_2)$

$$= \begin{vmatrix} 1 & x & 0 \\ 2x & x(x-1) & 0 \\ 3x(x-1) & x(x-1)(x-2) & 0 \end{vmatrix} = 0$$

$\therefore f(x) = 0$

$\Rightarrow (100) = 0$

Sol 3: (C) Given, $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} \sin x+2\cos x & \cos x & \cos x \\ \sin x+2\cos x & \sin x & \cos x \\ \sin x+2\cos x & \cos x & \sin x \end{vmatrix}$$

$$= (2\cos x+ \sin x) \begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$\Rightarrow (2\cos x + \sin x)$

$$\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x-\cos x & 0 \\ 0 & 0 & \sin x-\cos x \end{vmatrix} = 0$$

$\Rightarrow (2\cos x + \sin x) (\sin x - \cos x)^2 = 0$

$\Rightarrow 2\cos x + \sin x = 0$ or $\sin x - \cos x = 0$

$\Rightarrow 2\cos x = -\sin x$ or $\sin x = \cos x$

$\Rightarrow \cot x = -1/2$ gives no solution in $-\dfrac{\pi}{4} \le x \le \dfrac{\pi}{4}$

and $\sin x = \cos x \Rightarrow \tan x = 1$

$\Rightarrow x = \pi/4$

Sol 4: (B) For infinitely many solutions, we must have

$$\frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1} \Rightarrow k = 1$$

Sol 5: (B) Since, given system has no solution

$\therefore \Delta = 0$ and any one amongst D_x, D_y, D_z is non-zero.

$$\text{Let} = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 1$$

Sol 6: (A) Now,

$$P^TP = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$\Rightarrow P^TP = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow P^TP = I \Rightarrow P^T = P^{-1}$

Since, $Q = PAP^T$

$\therefore P^TQ^{2005}P$ ... (i)

$= P^T(PAP^T)(PAP^T)$ .... 2005 times]P

$$= \underbrace{(P^TP)A(P^TP)A(P^TP)........(P^TP)A(P^TP)}_{2005 \text{ times}}$$

$= IA^{2005} = A^{2005}$[from eq.(i)]

$$\therefore A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

....

....

$$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

$$\therefore P^TQ^{2005} P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

Sol 7: (A) Every square matrix satisfied its characteristic equation

i.e. $| A - \lambda I | = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & -2 & 4-\lambda \end{vmatrix} = 0$$

$\Rightarrow (1 - \lambda)\{(1 - \lambda)(4 - \lambda) + 2\} = 0$

$\Rightarrow 1^3 - 61^2 + 11\lambda - 6 = 0$

$\Rightarrow A^3 - 6A^2 + 11A - 6I = O$ ... (i)

$\Rightarrow A^2 6A + 11I = 6A^{-1}$

Sol 8: Since, a_1, a_2 are the roots of $ax^2 + bx + c = 0$

$\Rightarrow a_1 + a_2 = -\dfrac{b}{a}$ and $a_1a_2 = \dfrac{c}{a}$ ... (i)

Also, b_1, b_2 are the roots of

$px^2 + qx + r = 0$

$\Rightarrow b_1 + b_2 = -\dfrac{q}{p}$ and $b_1b_2 = \dfrac{r}{p}$ ... (ii)

Given system of equations

$a_1y + a_2z = 0$

And $b_1y + b_2z = 0$, has non-trivial solution

$$\therefore \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0 \qquad \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{\beta_1}{\beta_2}$$

Applying componendo-dividendo

$$\frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} = \frac{\beta_1 + \beta_2}{\beta_1 - \beta_2}$$

$\Rightarrow (\alpha_1 + \alpha_2)(\beta_1 - \beta_2) = (\alpha_1 - \alpha_2)(\beta_1 + \beta_2)$

$\Rightarrow (\alpha_1 + \alpha_2)^2 \{(\beta_1 - \beta_2)^2 - 4\beta_2\beta_2\}$

$= (\beta_1 + \beta_2)^2 \{(\alpha_1 + \alpha_2)^2 - 4\alpha_1\alpha_2\}$

From equation (i) and (ii), we get

$$\frac{b^2}{a^2}\left(\frac{q^2}{p^2} - \frac{4r}{p}\right) = \frac{q^2}{p^2}\left(\frac{b^2}{a^2} - \frac{4c}{a}\right)$$

$$\Rightarrow \frac{b^2q^2}{a^2p^2} - \frac{4b^2r}{a^2p} = \frac{b^2q^2}{a^2p^2} - \frac{4q^2c}{ap^2}$$

$$\Rightarrow \frac{b^2r}{a} = \frac{q^2c}{p} \Rightarrow \frac{b^2}{q^2} = \frac{ac}{pr}$$

Sol 9: $\left.\begin{array}{l} \dfrac{1}{a} = A + (p-1)D \\ \dfrac{1}{b} = A + (q-1)D \\ \dfrac{1}{c} = A + (r-1)D \end{array}\right\}$ ... (i)

$$\text{Let } \Delta = \begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = abc \begin{vmatrix} \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix},$$

[From equation (i)]

$$= abc \begin{vmatrix} A+(p-1)D & A+(q-1)D & A+(r-1)D \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 - (A - D)R_3 - DR_2$

$$= abc \begin{vmatrix} 0 & 0 & 0 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Sol 10: Given,

$$f'(x) = \begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1 - 2R_2$, We get

$$f'(x) = \begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 2ax & 2ax-1 \\ b & b+1 \end{vmatrix} = \begin{vmatrix} 2ax & -1 \\ b & 1 \end{vmatrix} (C_2 \to C_2 - C_1)$$

$\Rightarrow f'(x) = 2ax + b$

On integrating,

we get $f(x) = ax^2 + bx + c$

Where c is an arbitrary constant

Since, f has maximum at $x = 5/2$

$\Rightarrow f'(5/2) = 0 \Rightarrow 5a + b = 0$ ... (i)

Also, $f(0) = 2 \Rightarrow c = 2$

and $f(1) = 1 \Rightarrow a + b + c = 1$ ... (ii)

On solving equation (i) and (ii) for a, b, we get $a = \frac{1}{4}$, $b = -\frac{5}{4}$

Thus, $f(x) = \frac{1}{4}x^2 - \frac{5}{4}x + 2$

Sol 11:

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ +\sin\left(\theta-\frac{2\pi}{3}\right) & +\cos\left(\theta-\frac{2\pi}{3}\right) & +\sin\left(2\theta-\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}$$

Now, $\sin\left(\theta+\frac{2\pi}{3}\right) + \sin\left(\theta-\frac{2\pi}{3}\right)$

$$= 2\sin\left(\frac{\theta+\frac{2\pi}{3}+\theta-\frac{2\pi}{3}}{2}\right)\cos\left(\frac{\theta+\frac{2\pi}{3}-\theta+\frac{2\pi}{3}}{2}\right)$$

$= 2\sin\theta\cos\frac{2\pi}{3} = 2\sin\theta\cos\left(\pi-\frac{\pi}{3}\right)$

$= -2\sin\theta\cos\frac{\pi}{3} = -\sin\theta$

and $\cos\left(\theta+\frac{2\pi}{3}\right) + \cos\left(\theta-\frac{2\pi}{3}\right)$

$$= 2\cos\left(\frac{\theta+\frac{2\pi}{3}+\theta-\frac{2\pi}{3}}{2}\right)\cos\left(\frac{\theta+\frac{2\pi}{3}-\theta+\frac{2\pi}{3}}{2}\right)$$

$= 2\cos\theta\cos\left(\frac{2\pi}{3}\right) = 2\cos\theta\left(-\frac{1}{2}\right) = -\cos\theta$

and $\sin\left(2\theta+\frac{4\pi}{3}\right) + \sin\left(2\theta-\frac{4\pi}{3}\right)$

$$= 2\sin\left(\frac{2\theta+\frac{4\pi}{3}+2\theta-\frac{4\pi}{3}}{2}\right)\cos\left(\frac{2\theta+\frac{4\pi}{3}-2\theta+\frac{4\pi}{3}}{2}\right)$$

$= 2\sin2q\cos\frac{4\pi}{3} = 2\sin2q\cos\left(\pi+\frac{\pi}{3}\right)$

$= -2\sin2q\cos\frac{\pi}{3} = -\sin 2\theta$

$$\therefore \Delta = \begin{vmatrix} \sin\theta & \cos\theta & \sin2\theta \\ -\sin\theta & -\cos\theta & -\sin2\theta \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}$$

= 0 (since, R_1 and R_2 are proportional)

Sol 12: (B)

$BBT = \left(A^{-1}A^T\right)\left(A^{-1}A^{-T}\right)^T = \left(A^{-1}A^T\right)\left(A.\left(A^{-1}\right)^T\right)$

$= A^{-1}.\left(A^TA\right)A^T.\left(A^{-1}\right)^T = A^{-1}\left(AA^T\right)\left(A^{-1}\right)^T$

$= \left(A^{-1}A\right)A^T.\left(A^{-1}\right)^T = A.\left(A^{-1}\right) = \left(A^{-1}A\right)^T = I$

Sol 13: (D) $AA^T = 9T$

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9I$$

$$\Rightarrow \begin{vmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{vmatrix} \Rightarrow \begin{vmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{vmatrix}$$

Equation $a+4+2b = 0 \Rightarrow a+2b = -4$ ... (i)

$2a+2-2b = 0 \Rightarrow 2a-2b = -2$ ... (ii)

& $a^2+4+b^2 = 0 \Rightarrow a^2+b^2 = 5$ ... (iii)

Solving $a = -2, b = -1$

Sol 14: (B) $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} A\,\text{adj}\,A = AA^T$

$$\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$$\begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix} = \begin{bmatrix} 25a^2+b^2 & 15-2b \\ 15-2b & 13 \end{bmatrix}$$

Equate, $10a+3b = 25a^2+b^2$

and $10a+3b = 13$

and $15a-2b = 0$

$\frac{a}{2}=\frac{b}{15}=k$ (let)

Solving $a=\frac{2}{5}, b=3$

So, $5a+b=5\times\frac{2}{5}+3=5$

JEE Advanced/Boards

Exercise 1

Sol 1: (a) $|A| = a$, $(B) = (\text{adj}A)$, $|B| = b$

$$\frac{1}{2}S=\frac{a}{b}+\frac{a^2}{b^3}+\frac{a^3}{b^5}+....$$

$$=\frac{a}{b}\left[1+\frac{a}{b^2}+\left(\frac{a}{b^2}\right)^2+....\right]$$

$$=\frac{a}{b}\left[\frac{1}{1-\frac{a}{b^2}}\right]=\frac{a}{b}\left[\frac{b^2}{b^2-a}\right]=\frac{ab}{b^2-a}$$

$$S=\frac{2ab}{b^2-a}$$

$b > a$, $a = 3$

$|B| = |A|^{n-1} = |A|^{3-1} = (3)^2 = 9 = b$

$\Rightarrow (ab^2 + a^2b + 1)\ \frac{2(3)9}{9^2-3} = (3.\ (9)^2 + 3^2.\ 9 + 1)\ \frac{2(3)9}{9^2-3}$

$= (1 + 81 + 243)\ \frac{6.9}{78} = \frac{6.9}{78} = 225$

(b) $|A| = -2$, $|B| = 1$

$(A^{-1})\ (\text{adj } B^{-1})\ \text{adj}(2A^{+1})$

$\Rightarrow |A^{-1}|\ |\text{adj } B^{-1}|\ 2^{3-1}\ |\text{adj } A|$

$= \frac{1}{-2}\times 1\times 2^2\times(-2)^2 = \frac{4\times4}{-2} = \frac{16}{-2} = -8$

Sol 2: $A = [a_{ij}]$, $a_{ij}\in\{0, 1, 2, 3, 4\}$

$a_{11} + a_{12} + a_{21} + a_{22} = 4$

(i) $T_r(A) = a_{11} + a_{22} = 4$

and $a_{11}, a_{22}\in\{0, 1, 2, 3, 4\}$

total possibilities = 0 + 4 = 4 + 0 = 1 + 3 = 3 + 1 = 2 + 2

$\Rightarrow 5$

(ii) A is invertible $\Rightarrow$ so $|A|\neq 0$

$a_{11}\ a_{22} - a_{21}a_{21}\neq 0$

$a_{11}a_{22}\neq a_{21}\ a_{12}$

total way $\rightarrow \frac{2\times(41+3)}{{}^3C_2}=\frac{2}{3}.\ 27 = 18$

(iii) $|A|_{max} - |A|_{min}$

$\Rightarrow |A|_{max} = a_{11}a_{22} - a_{21}a_{12}$

$(a_{11}\ a_{22})_{max} = 4\ (\because\ a_{11} + a_{22} + a_{12} + a_{12} = 4)$

and + $(a_{21}.\ a_{12})_{max} = 3$

$a_{21}\ a_{12} = 0$ with $a_{12}\ a_{22} = 4 = (2)\ (2)$

So $|A|_{max} = 2(2) - 0 = 4$

$|A|_{min} = 0 - 2(2) = -4$

$|A|_{max} - |A|_{min} = 4 - (-4) = 8$

(iv) A is symmetric or skew symmetric or both |det A| is divisible by 2

So |det A| can be 0, 2, 4

$A\Rightarrow\begin{bmatrix}4&0\\0&0\end{bmatrix},\begin{bmatrix}0&0\\0&4\end{bmatrix},\begin{bmatrix}2&0\\0&2\end{bmatrix},\begin{bmatrix}2&1\\1&0\end{bmatrix},\begin{bmatrix}0&2\\2&0\end{bmatrix}$

Total no. $\rightarrow 5$

Sol 3: $A=\begin{bmatrix}4&-4&5\\-2&3&-3\\3&-3&4\end{bmatrix}$

$A_{11} = 12 - 4 = 3$ A_{13} =..................... .

$A_{12} = -9 + 8 \Rightarrow -A_{21} = -15 + 16 = 1$

$A_{22} = 16 - 15 = 1$, $-A_{23} = -12 + 12 = 0$

$A_{31} = 12 - 15 = -3$, $-A_{32} = -10 + 12 = 2$

$A_{33} = 12 - 8 = 4$

$\text{adj}A=\begin{bmatrix}3&+1&-3\\-1&1&2\\-3&0&4\end{bmatrix}$

$|A| = 4[A_{11}] - 4(A) + 5[A_{13}]$

$= 4(3) - 4(-1) + 5(-3) = 12 + 4 - 15 = 1$

$A^{-1}=\frac{\text{adj}A}{|A|}=\begin{bmatrix}3&+1&-3\\-1&1&2\\-3&0&4\end{bmatrix}$

$A^{-2}=\begin{bmatrix}3&+1&-3\\-1&1&2\\-3&0&4\end{bmatrix}\begin{bmatrix}3&+1&-3\\-1&1&2\\-3&0&4\end{bmatrix}=\begin{bmatrix}17&4&-19\\-10&0&13\\-21&-3&25\end{bmatrix}$

Sol 4: (a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}_{3\times3}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}_{2\times2}$

$$BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}_{2\times3}$$

Assume $P = \begin{bmatrix} P_1 & P_2 & P_3 \\ P_4 & P_5 & P_6 \end{bmatrix}_{2\times3}$

$$BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B^{-1}BPA = PA = B^{-1}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$|B| = 8 - 9 = -1$

$$\text{adj}B = \frac{1}{-1}\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$$

$$PA = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix}$$

$|A| = 1[4 -3]- 1 [2 - 2] + 1 [6 - 8] = 1 - 2 = - 1$

$$\text{adj}A = \begin{bmatrix} 1 & 3-1 & 1-4 \\ 0 & -1 & 2-1 \\ 6-8 & 2-3 & +2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 1 \\ -2 & -1 & +2 \end{bmatrix},$$

$$A^{-1} = \begin{bmatrix} -1 & -2 & 3 \\ 0 & -1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$$

$$PAA^{-1} = \begin{bmatrix} -4 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix}A^{-1} = P$$

$$P = \begin{bmatrix} -4 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix}\begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$$

$$P = \begin{bmatrix} -4 & 7 & -7 \\ 3 & -5 & 5 \end{bmatrix}$$

(b) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\cdot A\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix}$

assume $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$

$|B| = 4 - 3 = 1$ $|C| = -9 - 10 = -19$

$\text{adj}B = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\text{adj}C$

$= \begin{bmatrix} -3 & -2 \\ -5 & 3 \end{bmatrix}$, $C^{-1} = \frac{1}{19}\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$

$$\Rightarrow BAC = \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix}$$

$$B^{-1}BAC = AC = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 4-3 & 8+1 \\ -6+6 & -12-2 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 9 \\ 0 & -14 \end{bmatrix} \Rightarrow ACC^{-1}$$

$$= A = \begin{bmatrix} 1 & 9 \\ 0 & -14 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} - \frac{1}{19}$$

$$A = \frac{1}{19}\begin{bmatrix} 3+45 & 2-27 \\ -70 & 42 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 48 & -25 \\ -70 & 42 \end{bmatrix}$$

Sol 5: $A = \begin{bmatrix} \ell^2-3 & P & 0 \\ 0 & m^2-8 & q \\ r & 0 & n^2-15 \end{bmatrix}$

$A^2 = A$[$\because$ A is idempotent matrix]

$A^2 =$

$$\begin{bmatrix} (\ell^2-3)^2+0 & p(\ell^2-3)+p[m^2-8] & pq \\ qr & (m^2-8)^2 & q(m^2-8)+q(n^2-15) \\ r(\ell^2-3)+r(n^2+5) & rp & (n^2-15)^2 \end{bmatrix}$$

$$= \begin{bmatrix} \ell^2-3 & p & 0 \\ 0 & m^2-8 & q \\ r & 0 & n^2-15 \end{bmatrix}$$

compare elements

$\Rightarrow (1^2 - 3)^2 = \ell^2 - 3 \Rightarrow 1^2 - 3 = 0 \text{ or } 1$

$\ell = \pm\sqrt{3} \text{ or } \pm\sqrt{4} = \pm 2$

$p[1^2 - 3 + m^2 - 8] = p \Rightarrow p = 0 \text{ or } 1^2 + m^2 - 11 = 1$

$rp = 0 \Rightarrow r = 0 \text{ or } p = 0$

$(n^2 - 15)^2 = n^2 - 15 \Rightarrow n^2 - 15 = 1 \text{ or } 0$

$q[(m^2 - 8) + n^2 - 15] = q \Rightarrow q = 0 \text{ or } m^2 + n^2 - 23 = 0 + 1$

$(m^2 - 8)^2 = m^2 - 8 \Rightarrow m^2 - 8 = \text{ or } 1$

$m = \pm\sqrt{8} \text{ or } \pm\sqrt{9} = \pm 3$

if, $1_1 m_1 n, q, q, r \in z$

$S = \{0, \pm2, \pm3, \pm4\}$

$\Rightarrow$ Sum of products of elements = $2^2+3^2+4^2=29$

Sol 6: $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$F(x).\ F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos x\cos y - \sin x\sin y & -\cos x\sin y - \sin x\cos y & 0 \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos y\cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$[F(x)]^{-1} = F(-x)$

L. H. S. $\Rightarrow |F(x)| = \cos^2(x) + \sin^2 x - 1$

$\text{adj}[F(x)] = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$[F(x)]^{-1} = \dfrac{\text{adj}F(x)}{1} = \begin{bmatrix} \cos(-x) & -\sin(-x) & 0 \\ \sin(-x) & \cos(-x) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(-x)$

L. H. S. = R. H. S.

Hence proved

Sol 7: $A_n = [a_{ij}]$, $B_n = [b_{ij}]$

$a_{ij} = \dfrac{2i+j}{32n}$, $b_{ij} = \dfrac{3i-i}{2^{2n}}$

$l = \lim\limits_{n\to\infty} \text{Tr}[3A_1 + 3^2A_2 + 3^3A_3 + \ldots + 3^nAn + \ldots.]$

For A_n Tr(A) = $a_{11} + a_{22} + a^{33}$

$= \dfrac{(2(1)+1)+2(2)+2+2(3)+3}{3^{2n}} = \dfrac{9+6+3}{3^{2n}} = \dfrac{18}{3^{2n}}$

$\text{Tr}(3^{nA}\ n) = \dfrac{3^n 18}{3^{2n}} = \dfrac{18}{3^n}$

$\text{Tr}\left(\sum_{n=1}^{\infty} 3^n An\right) = \sum_{i=1}^{\infty} \text{Tr}(3^n A_3) = 18\left[\dfrac{1}{3} + \dfrac{1}{3^2} + \ldots\ldots \dfrac{1}{3^\infty}\right]$

$= 18\left[\dfrac{1}{3}\left[\dfrac{1}{1-\dfrac{1}{3}}\right]\right] = \dfrac{18}{3} \times \dfrac{8}{3-1} = \dfrac{18}{2}$

$\text{Tr}(B_n) = \dfrac{3(1)-1+3(2)-2+3(3)-3}{2^{2n}} = \dfrac{2+4+6}{2^{2n}} = \dfrac{12}{2^{2n}}$

$\text{Tr}(2^n B_n) = \dfrac{12}{2^n}$

$m = \sum_{n=1}^{a} [\text{Tr}(2^n B_n)] = 12\left[\dfrac{1}{2} + \dfrac{1}{22} + \ldots\right]$

$= 12\left[\dfrac{1}{2}\left[\dfrac{1}{1-\dfrac{1}{2}}\right]\right] = \dfrac{12}{2}\cdot 2 = 12$

l + m = 12 + 9 = 21

Sol 8: A is 3 × 3 matrix

$A_{11} = a_{33} = 2$, all other $a_{ij} = 1$

$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $\begin{matrix} R_1 \Rightarrow R_1 - R_2 \\ R_3 \Rightarrow R_3 - R_1 \end{matrix}$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} R_2 \Rightarrow R_2 - R_1 - R_2$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$A^{-1} = I = xA^2 + yA + zI$

$I = (x + y + z)I$

$(x + y + z) = 1$

Sol 9: $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & 3 \end{bmatrix}$

$C = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 10 \\ 13 \\ 9 \end{bmatrix}$, Cb = D

$|C| = 2[2-1] + 1[1-2] + 1[2-2] = 2 - 1 = 1$

$$\text{adj}C = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix} = |C|\, C^{-1} = C^{-1}$$

$C^{-1}Cb = C^{-1}D$

$$b = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}_{3\times3} \begin{bmatrix} 10 \\ 13 \\ 9 \end{bmatrix}_{3\times1}$$

$$b = \begin{bmatrix} 10-9 \\ 10+13 \\ -13+18 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$|A| = 1[6 + 3] + 2[3 - 6] + 2[-2 -2] = 9 - 6 - 8 = - 5$

$$\text{adj}A = \begin{bmatrix} 6+3 & -8 & 2 \\ -3 & 3-2 & 1 \\ -4 & 3 & 2-4 \end{bmatrix} = \begin{bmatrix} 9 & -8 & 2 \\ -3 & 1 & 1 \\ -4 & 3 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}A = -\frac{1}{5}\text{adj}A$$

$AX = b$

$$X = A^{-1}b = -\frac{1}{5}\begin{bmatrix} 9 & -8 & 2 \\ -3 & 1 & 1 \\ -4 & 3 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$X = -\frac{1}{5}\begin{bmatrix} 9-24+10 \\ -3+3+5 \\ -4+9-10 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -5 \\ +5 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Sol 10: $A = \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$

$$AB = \begin{bmatrix} -2x+7x & 28x-28x & 14x-14x \\ 0 & 1 & 0 \\ -x+x & 14x-2-4x & 7x-2x \end{bmatrix}$$

$$= \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix}$$

$|AB| = 5x\,[5x] = 25x^2$

$$(AB)^{-1} = \frac{1}{25x^2}\begin{bmatrix} 5x & 0 & 0 \\ 0 & 25x^2 & 0 \\ 0 & -50x^2+10x & 5x \end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix} \frac{1}{5x} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{-10x+2}{5x} & \frac{1}{5x} \end{bmatrix} = AB = \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix}$$

$\Rightarrow x = 1/5$

$$AB = \begin{bmatrix} \frac{-5}{5} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & \frac{-5}{5} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & -1 \end{bmatrix}$$

$(AB)^2 = (AB)\,(AB) = (AB)\,(AB)^{-1} = I$

$\text{Tr}[AB+(AB)^2+(AB)^3+\ldots+(AB)^{100}]$

$= \text{Tr}[AB+I+AB+I+\ldots+I]$

$= \text{Tr}[50AB+50I]=50\,\text{Tr}(AB)+50\text{Tr}(I)$

$= 50[-1+1-1]+50\,[1+1+1] =-50+3(50)=100$

Sol 11: $M_n = [m_{ij}]$order $= n$

$1 \le i \le n,\ m_{ij} = 10;$

$1 \le i \le n-1,\ m_{i+1},\ I = m_i,\ i + 1 = 3$

All other entries in Mn are zero

$$M_3 = \begin{bmatrix} 10 & 3 & 0 \\ 3 & 10 & 3 \\ 0 & 3 & 10 \end{bmatrix}, |M_3| = 10[100-9] + 3\,[-30]$$

$= 1000 - 90 - 90 = 820$

$$M_2 = \begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix} \Rightarrow |M_2| = 100 - 9 = 91$$

$D_3 - 9D_2 = 820 - 9(91) = 820 - 819 = 1$

Sol 12: $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$$\begin{aligned} x+y+2z &= 1 \\ 3x+2y+z &= 7 \\ 2x+y+3z &= 2 \end{aligned} \quad AB = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4I_3$$

$$_1B^{-1} = \frac{A}{4} = \begin{bmatrix} \frac{-5}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{7}{4} & \frac{1}{4} & \frac{-5}{4} \\ \frac{1}{4} & \frac{-1}{4} & \frac{1}{4} \end{bmatrix}$$

$|B| = 1[6 - 1] + 1[2 - 9] + 2[3 - 4]$

$= 5 - 7 - 2 = -4$

$x = \frac{|X|}{|B|}, |X| \begin{vmatrix} 1 & 1 & 2 \\ 7 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = [6 - 1] + 7[2 - 3]$

$+ 2[1 - 4] = + 5 - 7 - 6 = -8$

$x = \frac{-8}{-4} = 2, Y = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 7 & 1 \\ 2 & 2 & 3 \end{vmatrix}$

$= 1[21 - 2] + 1[2 - 9] + 2[6 - 14] = 19 - 7 - 16 = -4$

$y = \frac{-4}{-4} = 1, Z = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 7 \\ 2 & 1 & 2 \end{vmatrix}$

$= 1[4 - 7] + 1[14 - 6] + 1[3 - 4] = -3 + 8 - 1 = 4$

or $\Rightarrow Bx = C$

$x = B^{-1}C$

$z = \frac{-4}{-4} = -1$

$(x, y, z) = (2, 1, -1)$

$$x = \begin{bmatrix} \frac{-5}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{7}{4} & \frac{1}{4} & \frac{-5}{4} \\ \frac{1}{4} & \frac{-1}{4} & \frac{1}{4} \end{bmatrix}_{3\times3} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}_{3\times1} = \begin{bmatrix} 1 \\ +1 \\ -1 \end{bmatrix}$$

Sol 13: $\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -1 \end{bmatrix}$

$|D| = \begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix} = 3[-8a - 9] - 2[18 - 5a] + 1[5 + 16] \backslash$

$= -24a - 27 - 36 + 10a + 21 = -14a - 42$

(i) System has a unique solution $101 \neq 0$

$-140 - 42 \neq 0$

$a \neq -\frac{42}{14} = -3$

$a \neq -3$ and $b \in R$

(ii) At $a = -3$ has no solution $\Rightarrow a = -3$

$$\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$$

$R_2 \Rightarrow R_2 + 2R_3$

$$\begin{bmatrix} 3 & -2 & 1 \\ 5+4 & -8+2 & 9-6 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3-2 \\ -1 \end{bmatrix} = \begin{bmatrix} b \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & 1 \\ 9 & -6 & 3 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 1 \\ -1 \end{bmatrix}$$

Compare row 2nd and 1st

$3x - 2y + z = b$ ... (i)

$9x - 6y + 3z = 1$

$3x - 2y + z = \frac{1}{3}$ ... (ii)'

From equation (i) and (ii)

$b = \frac{1}{3}$

For no. solution $a = -3$ and $b \neq \frac{1}{3}$

(iii) Has infinitely solution

so $a = -3$ and $b = \frac{1}{3}$

so $|D| = 0$ and $|D|_x = 0$

Sol 14: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$X = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix}$

(a) $AX = B - I$

$X \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$

$|A| = 4 - 6 = -2$

$\text{adj}A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$\Rightarrow A^{-1} = -\frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \frac{1}{|A|} \text{adj}A$

so $A^{-1}AX = X = A^{-1}(B - I)$

$$X = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 8-2 & 4+2 \\ 6+1 & -3-1 \end{bmatrix}$$

$$X = -\frac{1}{2}\begin{bmatrix} 6 & 6 \\ -5 & -4 \end{bmatrix} = \begin{bmatrix} -3 & -3 \\ \frac{5}{2} & 2 \end{bmatrix}$$

(b) $(B - I)X = IC = C$

$$B - I = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$$

$|B - I|$ Z$[-1] - 1 = -3$

$\text{adj}(B - I) = \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$, $(B - I)^{-1}$

$$= \frac{\text{adj}(B-I)}{|B-I|} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$$

$$X = (B - I)^{-1}C = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$X = \frac{1}{3}\begin{bmatrix} 1+2 & 2+4 \\ 1-4 & 2-8 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 & 6 \\ -3 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

(c) $CX = A$

$$|C| = \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 4 - 4 = 0$$

So C^{-1} does not exit $\Rightarrow$ Y has no solution

Sol 15: A is orthogonal matrix

$\Rightarrow AA' = A'A = I_n$

and $B = AP$, P is non-singular

if A is orthogonal, so A^{-1} is also orthogonal

$B = AP$

$BB^{-1} = APB^{-1}$

$I = APB^{-1}$

$A^{-1} = A^{-1}APB^{-1}$

$A^{-1} = PB^{-1}$

A^{-1} is orthogonal, so PB^{-1} is also orthogonal

Sol 16: $M\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$; $M^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Assume $M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

so $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$a_{11} - a_{12} = -1; + a_{21} - a_{22} = 2$ ... (i)

$$M^2 = \begin{bmatrix} a_{11}^2 + a_{12}a_{21} & a_{11}a_{12} + a_{12}a_{22} \\ a_{11}a_{21} + a_{21}a_{22} & a_{21}a_{12} + a_{22}^2 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_{11}^2 + a_{12}a_{21} & a_{11}a_{12} + a_{12}a_{22} \\ a_{11}a_{21} + a_{21}a_{22} & a_{21}a_{12} + a_{22}^2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$a_{11}^2 + a_{12}a_{21} + a_{12}a_{22} = 1$ ... (ii)

$\Rightarrow a_{11}a_{21} + a_{21}a_{22} - a_{21}a_{12} + a_{22}^2 = 0$

$\Rightarrow a_{11}[a_{11} - a_{12}] + a_{12}[a_{21} + a_{22}] = 1$

$\Rightarrow a_{11}(-1) + a_{12}(2) = 1$

$\Rightarrow 2a_{12} - a_{11} = 1$

$\Rightarrow a_{12} + 1 = 1 \Rightarrow a_{12} = 0$

$\Rightarrow a_{11} = -1$

$\Rightarrow a_{21}[a_{11} - a_{12}] + a_{22}[a_{21} - a_{22}] = 0$

$\Rightarrow a_{21}[-1] + a_{22}[2] = 0$

$\Rightarrow 2a_{22} - a_{21} = 0$

$-[a_{21} - a_{22} - a_{22}] = 0$

$2 - a_{22} = 0$

$\Rightarrow a_{22} = 2$

$\Rightarrow a_{21} = 4$

$$M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix}$$

$|M - XI| = 0$

$$\begin{vmatrix} -1-x & 0 \\ 4 & 2-X \end{vmatrix} = 0$$

$(1 + x)(x - 2) = 0 \Rightarrow x = -1$ or $x = 2$

$5x_1 + 2x_2 = 5(2) + 2(-1)\ 2 > -1 = 10 - 2 = 8$

Sol 17: $A_1 = 1$, $A_2 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$,

$$A_3 = \begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix} \text{...............}$$

No. of element in $A_n = n^2$

For $A_n = 10^2 = 100$, (10 in each row)

S_n = sum of all element of A_n

So $S_1 = 1$, $S_2 = 2 + 3 + 4 + 5$

$S_9 = 1 + 2 + 3 + \ldots\ldots\ldots m$

Where $m = 1 + 2^2 + 3^2 + 4^2 + \ldots\ldots + 9^2$

$= \dfrac{(2n+1)n(n+1)}{6} = \dfrac{9(18+1)(9+1)}{6}$

$= \dfrac{3}{2} \times 10 \times 19 = 285$

So $i_n\ a_{10} \Rightarrow a_{11} = 285 + 1 = 286$

$a_{22} = 286 + 11$

$a_{nn} = 286 + (n-1)11$

$tr(A) = \sum_{i=1}^{10} a_{ij} = 286 \times 10 + [11 + 11(2)$

$+ 3(11) + \ldots\ldots + 9(11)]$

$= 286^o + 11\,[1 + 2 + \ldots + 9]$

$= 286^o + 11 \times \dfrac{9 \times 10^5}{2} = 286^o + 11 \times 45 = 3355$

Sol 18: $I_{n',m} = \int_0^1 \dfrac{x^n}{x^m - 1}\,dx\ \forall\ n, m$

$I_{n',m} = \int_0^1 \dfrac{x^n}{x^m + 1}\,dx\ \forall\ x > m, n, m \in N$

(a)$A = [a_{ij}]_{3\times 3}$

$a_{ij} = \begin{cases} I_{6+i,3} \quad -I_{i+3,3} & , i = j \\ 0 & , i \neq j \end{cases}$

$a_{11} = I_{6+1,3} - I_{1+3,3} = I_{7,3} - I_{4,3}$

$= \int_0^1 \dfrac{x_7 dx}{x^3 - 1} - \int_0^1 \dfrac{x^4 dx}{x^3 - 1} = \int_0^1 \dfrac{x^7 - x^4}{x^3 - 1}\,dx$

$= \int_0^1 x^4 \left(\dfrac{x^3 - 1}{x^3 - 1}\right) dx = \left[\dfrac{x^5}{5}\right]_0^1 = \dfrac{1}{5}$

$a_{22} = I_{8,3} - I_{5,3} = \int_0^1 x^5 dx = \left[\dfrac{x^6}{6}\right]_0^1 = \dfrac{1}{6}$

$a_{33} = I_{8+1,3} - I_{6,3} = \int_0^1 x^6 \left(\dfrac{x^3 - 1}{x^3 - 1}\right) dx = \left[\dfrac{x^7}{7}\right]_0^1 = \dfrac{1}{7}$

$A = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{7} \end{bmatrix}$, $|A| = \dfrac{1}{5}.\dfrac{1}{6}.\dfrac{1}{7} = \dfrac{1}{210}$

$\text{Adj } A = \begin{bmatrix} \frac{1}{42} & 0 & 0 \\ 0 & \frac{1}{35} & 0 \\ 0 & 0 & \frac{1}{30} \end{bmatrix}$, $A^{-1} = \dfrac{1}{|A|}$

$\text{adj}A = \dfrac{1}{\frac{1}{210}} \begin{bmatrix} \frac{1}{42} & & \\ & \frac{1}{35} & \\ & & \frac{1}{30} \end{bmatrix}$

$Tr(A^{-1}) = 210\left[\dfrac{1}{42} + \dfrac{1}{35} + \dfrac{1}{30}\right] = 5 + 6 + 7 = 18$

(b) $A = \begin{bmatrix} J_{6,5} & 72 & J_{11,5} \\ J_{7,5} & 63 & J_{12,5} \\ J_{8,5} & 56 & J_{13,5} \end{bmatrix}$

$B = \begin{bmatrix} J_{6,5} & 72 & J_{11,5} \\ J_{7,5} & 63 & J_{12,5} \\ J_{8,5} & 56 & J_{13,5} \end{bmatrix}$

$\det(A) = -72\,[J_{7,5}\,J_{13,5} - J_{12,5}\,J_{8,5}] + \ldots\ldots\ldots\ldots\ldots$

$J_{n,\alpha}\,J_{m,\alpha} - J_{N,\alpha}\,J_{M,\alpha}$

If $n + m = N + N$, the $\int_0^1 \left(\dfrac{x^{n+m}}{x^{x+1}} - \dfrac{x^{n+m}}{x^{\alpha+1}}\right) dx = 0$

So det (A) = 0

$|B| = 72\,[I_{12'}\,5I_{8,5} - I_{7,5}\,I_{13',5}] + \ldots\ldots\ldots$

Sum as above 12 + 8 = 7 + 13

So, |B| = 0

det(A) – det (B) = 0

Sol 19: $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$

P is orthogonal matrix $\Rightarrow Q = PAPT$,

$R = PTQ^K P$, $S = PBPT$, $T = PTS^K P$

Sol 20: $A \to p,q,t; B \to s; C \to p,r; D \to r$

$A_{2\times2} = [a_{ij}]$

Elements are 0, 1, 2, 4

(A) $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$|A| = a_{11}\,a_{22} - a_{21}a_{12}$

If $|A| > 0 \Rightarrow a_{11}a_{12} > a_{21}\,a_{12}$

$\begin{bmatrix} 1 & 0 \\ 4 & 2 \end{bmatrix} \Rightarrow |A| = 2$

$\begin{vmatrix} 2 & 1 \\ 0 & 4 \end{vmatrix} \Rightarrow 8, \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} = 4$

(B) If |A| = m, then is also a matrix $\Rightarrow |A| = -m$

So for all matrix, have one –ve det (A) matrix so Σ det(A) = 0

(C) Least value of det(A) = 2 or – 2

$|adj(adj(adjA))| = ((\pm2)^{2-1})^{2-1})^{2-1} = \pm2$, 2 or –2

(D) det(A) is algebraically least = – 8

$4A^{-1} = \dfrac{4adjA}{|A|} = \dfrac{4}{-8}\ adjA = \left(\dfrac{1}{-2}\right)(adj\ A)$

$|4A^{-1}| = |-Z^{-1}adjA| = (-Z)^2\ |A|^{2-1}$

$= \dfrac{1}{4} \times -8 = -2$

Exercise 2

Single Correct Choice Type

Sol 1: (B) Let [A, B] = AB – BA

[[A, B], C] + [[B, C], A] + [[C, A], B]

⇒ [[A, B], C] = [AB – BA, C] = (AB – BA)C – C(AB – BA)

= ABC – BAC – CAB + CBA ...(i)

[[B, C], A] = [BC, – CB, A] = (BC – CB)A – A(BC – CB)

= BCA – CBA – ABC + ACB ...(ii)

[[C, A], B] = [CA – AC, B] = (CA – AC) B – B(CA – AC)

= CAB – A, B – BCA + BAC ...(iii)

sum of equation (i), (ii) & (iii)

[[A, B]C] + [[B, C], A] + [[C, A], B] = ABC – BAC + BAC – ABB +....... = 0

Sol 2: (A) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 2 & 3 & \alpha \end{bmatrix}$

$f(x) = x^3 - 8x^2 + bx + \gamma$

a satisfies f(x) = 0

Sol 3: (A) two rowed unit matrix $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$F_2^2 = I_2$

So square root of $I_2 = I_2 = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ (given)

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$\alpha = 1 = \delta, \gamma = \beta = 0$

Sol 4: (A) $A = \begin{bmatrix} 4 & 2i \\ i & 1 \end{bmatrix}$ (A – 2I) (A – 3I) = ?

$A - 2I = \begin{bmatrix} 4 & 2i \\ i & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4-2 & 2i \\ i & 1-2 \end{bmatrix} = \begin{bmatrix} 2 & 2i \\ i & -1 \end{bmatrix}$

$A - 3J = \begin{bmatrix} 4 & 2i \\ i & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4-3 & 2i \\ i & 1-3 \end{bmatrix} = \begin{bmatrix} 1 & 2i \\ i & -2 \end{bmatrix}$

$(A - 2J)\ (A - 3J) = \begin{bmatrix} 2 & 2i \\ i & -1 \end{bmatrix} \begin{bmatrix} 1 & 2i \\ i & -2 \end{bmatrix}$

$= \begin{bmatrix} 2(1)+2i(i) & 4i-4i \\ i-i & 2i(i)-1(-2) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = null matrix

Sol 5: (D) $A = \begin{bmatrix} 1 & \cos(\alpha-\beta) & \cos(\alpha-\gamma) \\ \cos(\beta-\alpha) & 1 & \cos(\beta-\gamma) \\ \cos(\gamma-\alpha) & \cos(\gamma-\beta) & 1 \end{bmatrix}$

$|A = 1[1 - \cos(\beta-\gamma)\cos(\gamma-\beta)] + \cos(\alpha-\beta)[\cos(\beta - r)\cos(\gamma-\alpha) - \cos(\beta-\alpha)]$

$+\cos(\alpha-\gamma)[\cos(\beta-\alpha)\cos(\gamma-\beta) - \cos(\gamma-\alpha)]$

$(\because \cos(A) = \cos(-A))$

$= 1 - \cos^2(\beta-\gamma) + 2\cos(\alpha-\beta)\cos(\beta-\gamma)\cos(\gamma-\alpha) - \cos^2(\beta-\alpha) = \cos^2(\alpha-\gamma)$

$= 1 - \left[\cos\dfrac{(\alpha+\beta-\gamma-\alpha+\gamma-\beta)}{2}\right]^2$

$= 1 - \cos^2 0 = 1 - 1 = 0$

Sol 6: (C) $A = \begin{bmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{bmatrix}$

matrix A is non singular

$|A| \neq 0$

$$\begin{bmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{bmatrix} \neq 0$$

$\Rightarrow (x + a) [(x + b) (x + c) - bc] + b[ac - a(x + c)] + c[ab - a(x + b)] \neq 0$

$\Rightarrow (x + a) [x^2 + x(b + c)] + b [ac - ax - ac] + (c) (-ax) \neq 0$

$\Rightarrow x^3 + ax^2 + x^2(b + c) + ax(b + c) - abx - acx \neq 0$

$\Rightarrow x^3 + x^2(a + b + 1) \neq 0$

$\Rightarrow x^2[x + (a + b + c)] \neq 0$

$\Rightarrow x \neq 0$ and $x \neq -(a + b + c)$

Sx $x = R - \{0, -(a + b + c)\}$

Sol 7: (B) A is skew symmetric matrix

$A^2 = A$ and $B^TB = B$

$B^TB = B$

Multiply with $B^{-1} \Rightarrow (B^TB)B^{-1} = BB^{-1} = I$

$B^TI = B^T = I$

$B^T = I$. So $B = I$

$X = (A + B) (A - B)$

$X = A^2 - AB + BA - B^2 (\because B = I)$

$X = A - A + A - I = A - I$

$X^7 = (A - I)^T = A^T - I$

$X^7X = (A^T - 1) (A - I)$

$= AA^T - A^T - A + I$

$A^T = -A (\because$ A is skew symmetric)

$X^7X = -AA - A + A + I$

$= -A^2 + I = -A + I = I - A$

Sol 8: (C) Z, and Z_2 are uni modular complex

$$\begin{bmatrix} \bar{z}_1 - z_2 \\ \bar{z}_2 \quad z_1 \end{bmatrix}^{-1} \begin{bmatrix} z_1 & z_2 \\ \bar{z}_2 & \bar{z}_1 \end{bmatrix}^{-1} = A \text{ (assume)}$$

$$= \begin{bmatrix} \bar{z}_1 z_1 + z_2\bar{z}_2 & \bar{z}_1 z_2 - z_2\bar{z}_1 \\ \bar{z}_2 z_1 - z_1\bar{z}_2 & \bar{z}_2 z_2 + z_1\bar{z}_1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} = \frac{1}{\det(A)} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

Sol 9: (C) $\begin{bmatrix} \frac{1}{25} & x \\ 0 & \frac{1}{25} \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix}^{-2}$

$$= \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix}^{-1} \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix}^{-1} = \frac{1}{25}\begin{bmatrix} 5 & a \\ 0 & 5 \end{bmatrix}^{\frac{1}{25}} \begin{bmatrix} 5 & a \\ 0 & 5 \end{bmatrix}$$

$$= \frac{1}{625} - \begin{bmatrix} 5 & a \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 5 & a \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1/25 & x \\ 0 & 1/25 \end{bmatrix}$$

$$= \frac{1}{625}\begin{bmatrix} 25 & 5a + a5 \\ 0 & 25 \end{bmatrix} = \frac{1}{625}\begin{bmatrix} 25 & 10a \\ 0 & 25 \end{bmatrix}$$

$$x = \frac{10a}{625} = \frac{2a}{125}$$

Sol 10: (D) $A^2 = I$

$|A| = 1$, $B = (\text{adj } A)^{-1}$

$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \text{adj}(A)$

$(A^{-1})^{-1} = (\text{adj}A)^{-1}$

$A = (\text{adj } A)^{-1} = B$ given

$A = B$

$A^2 = I$

$AA = AB = I$

$AB = AA = BA = I$

$\Rightarrow B \neq I$ we can't say that $B = I$

Sol 11: (B) adj A = Border of both = 3×3

Adj (3AB)= 3^{3-1} adj (AB)

$= 9(\text{adj } B) (\text{adj}A) = 9(\text{adj } B)B = 9|B| = I_3$

$\therefore \text{adj}(AB) = (\text{adj}B) (\text{adj } A)$

Sol 12: (C) $A^T + B = 0$

$A = \text{adj}B$, $\text{tr}(A) = 1$, $A^2 = A$

$\text{tr}\{\text{adj } (A^TB)\}$

$\Rightarrow A^T + B = 0$

$\Rightarrow A^T = -B$

$\Rightarrow tr[(adj\ B)\ (adj\ A^T)]$

$\Rightarrow tr[A\ adj(-B)]$

$\Rightarrow tr(A(-1)^{n-1}A]$

$\Rightarrow (-1)^{n-1}\ tr(A^2) = (-1)^{n-1}\ tr(A)$

$\Rightarrow (-1)^{n-1}\ (-1) = (-1)^n$

Sol 13: (C) C = A + B

$|C|^2 = |A|^2\ |I - (A^{-1}B)^2|$

AB = BAC = A + B

$\Rightarrow |C| = |A + B| = |A|\ [I + A^{-1}B]$

$|C|^2 = |A|^2\ |I - A^{-1}B|\ |I + A^{-1}B|$ ... (ii)

Equation $\frac{(2)}{(1)} \Rightarrow \frac{|C|^2}{|C|}$

$= \frac{|A|^2 |I - A^{-1}B||I + A^{-1}B|}{|A||I + A^{-1}B|}$

$|C| = A^{-1}\ |I - A^{-1}B| = |A - B|$

$|C| = |A - B|$

Previous Years' Questions

Sol 1: (A) | A | ≠ 0, as non-singular

$\therefore \begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$

$\Rightarrow 1(1 - c\omega) - a\ (\omega - cw^2) + b\ (w^2 - w^2) \neq 0$

$\Rightarrow 1 - c\omega - a\omega + acw^2) \neq 0$

$\Rightarrow (1 - c\omega)\ (1 - a\omega) \neq 0$

$\Rightarrow a \neq \frac{1}{\omega}, c \neq \frac{1}{\omega}$

$\Rightarrow a = \omega, c = \omega$ and $b \Rightarrow \{\omega, w^2\}$

$\Rightarrow$ 2 solutions

Sol 2: (C) Given, $M^T = -M$, $N^T = -N$

and MN = NM ... (i)

$\therefore M^2N^2(M^TN)^{-1}\ (MN^{-1})^T$

$\Rightarrow M^2N^2N^{-1}(M^T)^{-1}(N^{-1})^TM^T$

$\Rightarrow M^2N(NN^{-1})(-M)^{-1}(N^T)^{-1}(-M)$

$\Rightarrow M^2\ NI(-M^{-1})(-N^{-1})(-M)$

$\Rightarrow -M^2\ NM^{-1}N^{-1}M$

$\Rightarrow -M \cdot (MN)M^{-1}N^{-1}\ M$

$\Rightarrow -M(NM)M^{-1}N^{-1}\ M$

$\Rightarrow -MN(NN^{-1})N^{-1}\ M$

$\Rightarrow -M(NN^{-1})M$

$\Rightarrow -M^2$

Note: Here, non-singular word should not be used, since there is no non-singular 3 x 3 skew-symmetric matrix.

Sol 3: Let $\Delta = \begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

Applying $R_2 \to R_2 - (R_1 + R_3)$, we get

$\Delta = \begin{vmatrix} x^2+x & x+1 & x-2 \\ -4 & 0 & 0 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

Applying $R_1 \to R_1 + \frac{x^2}{4}\ R_2$

and $R_3 \to R_3 + \frac{x^2}{4}\ R_2$, w

$\Delta = \begin{vmatrix} x & x+1 & x-2 \\ -4 & 0 & 0 \\ 2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

Applying $R_3 \to R_3 - 2R_1$

$= \begin{vmatrix} x+0 & x+1 & x-2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$

$= \begin{vmatrix} x & x & x \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$

$= x\begin{vmatrix} 1 & 1 & 1 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$

$\Rightarrow \Delta = Ax + B$

Where $A = \begin{vmatrix} 1 & 1 & 1 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$ and $B = \begin{vmatrix} 0 & 1 & -2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$

Sol 4: The given system of equation

$3x - y + 4z = 3$

$x + 2y - 3z = -2$

$6x + 5y + \lambda z = -3$

Has at least one solution, if $\Delta \neq 0$

$$\therefore \Delta = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{vmatrix} \neq 0$$

$\Rightarrow 3(2\lambda + 15) + 1(\lambda + 18) + 4(5 - 12) \neq 0$

$\Rightarrow 7(\lambda + 5) \neq 0 \Rightarrow \lambda \neq -5$

For $\lambda = -5$

$\Rightarrow \Delta = 0$

$$\text{Then, } \Delta_1 = \begin{vmatrix} 3 & -1 & 4 \\ -2 & 2 & -3 \\ -3 & 5 & -5 \end{vmatrix} = 0$$

$$\Delta_2 = \begin{vmatrix} 3 & 3 & 4 \\ 1 & -2 & -3 \\ 6 & -3 & -5 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 3 \\ 1 & 2 & -2 \\ 6 & 5 & -3 \end{vmatrix} = 0$$

$\Delta_1 = \Delta_2 = \Delta_3 = 0$

Sol 5: The system of equations has non-trivial solution, if $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

Expanding along C_1, we get

$\Rightarrow \sin 3\theta \cdot (28 - 21) - \cos 2\theta (-7 - 7) + 2(-3 - 4) = 0$

$\Rightarrow 7\sin 3\theta + 14\cos 2\theta - 14 = 0$

$\Rightarrow \sin 3\theta + 2\cos 2\theta - 2 = 0$

$\Rightarrow 3\sin\theta - 4\sin^3\theta + 2(1 - 2\sin^2\theta) - 2 = 0$

$\Rightarrow \sin\theta\,(4\sin^2\theta + 4\sin\theta - 3) = 0$

$\Rightarrow \sin\theta\,(2\sin\theta - 1)(2\sin\theta + 3) = 0$

$\Rightarrow \sin\theta = 0, \sin\theta = 1/2$

(neglecting $\sin\theta = -3/2$)

$\Rightarrow \theta = n\pi, n\pi + (-1)^n \pi/6, n \in Z$

Sol 6: Given, $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2 - 3n \end{vmatrix}$

$$\therefore = \begin{vmatrix} \sum_{a=1}^{n}(a-1) & n & 6 \\ \sum_{a=1}^{n}(a-1)^2 & 2n^2 & 4n-2 \\ \sum_{a=1}^{n}(a-1)^3 & 3n^3 & 3n^2 - 3n \end{vmatrix}$$

= Applying $C_3 \to C_3 - 6C_1$

$$= \frac{n^3(n-1)}{12}\begin{vmatrix} 1 & 1 & 0 \\ 2n-1 & 6n & 0 \\ n-1 & 6n & 0 \end{vmatrix} = 0$$

$\Rightarrow \sum_{a=1}^{n} \Delta_a = c$, (c = 0 ie, constant)

Sol 7: Let $\Delta = \begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix}$

Applying $R_1 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ a-p & 0 & r-c \end{vmatrix} = c\begin{vmatrix} a-p & q-b \\ a-p & 0 \end{vmatrix} + (r-c)\begin{vmatrix} p & b \\ a-p & q-b \end{vmatrix}$$

$= -c(a-p)(q-b) + (r-c)[p(q-b) - b(a-p)]$

$= -c(a-p)(q-b) + p(r-c)(q-b) - b(r-c)(a-p)$

Since, $\Delta = 0$

$\Rightarrow -c(a-p)(q-b) + p(r-c)(q-b) - b(r-c)(a-p) = 0$

[On dividing both side by Radding 204 th side and –x $+(\sin\alpha)y - (\cos\alpha)z = 0$ has non-

$(a-p)(q-b)(r-c)]\ \dfrac{b}{b-a} + \dfrac{c}{r-c} + \dfrac{b}{q-b} + 2 = 2$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + 0 + \frac{r}{r-c} + 0 = 2$$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

Sol 8: Given, $D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$

Taking n!, (n + 1)! and (n + 2)! Common from R_1, R_2 and R_3 respectively.

$$\therefore D = n!(n+1)!(n+2)!\begin{vmatrix} 1 & (n+1) & (n+1)(n+2) \\ 1 & (n+2) & (n+2)(n+3) \\ 1 & (n+3) & (n+3)(n+4) \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$, we get

$$D = n!(n+1)!(n+2)!\begin{vmatrix} 1 & (n+1) & (n+1)(n+2) \\ 0 & 1 & 2n+4 \\ 0 & 1 & 2n+6 \end{vmatrix}$$

Expanding along C_1, we get

$D = (n!)(n + 1)!(n + 2)![(2n + 6) - (2n + 4)]$

$D = (n!)(n + 1)!(n + 2)![2]$

On dividing both side by $(n!)^3$

$$\Rightarrow \frac{D}{(n!)^3} = \frac{(n!)(n!)(n+1)(n!)(n+1)(n+2)2}{(n!)^3}$$

$$\Rightarrow \frac{D}{(n!)^3} = 2(n + 1)(n + 1)(n + 2)$$

$$\Rightarrow \frac{D}{(n!)^3} = 2(n^3 + 4n^2 + 5n + 2) = 2n(n^2 + 4n + 5) + 4$$

$$\Rightarrow \frac{D}{(n!)^3} - 4 = 2n(n^2 + 4n + 5)$$

Which shows that $\left[\frac{D}{(n!)^3} - 4\right]$ is divisible by n.

Sol 9: Given, $\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

and $-x + (\sin\alpha)y - (\cos\alpha)z = 0$ has non-trivial solution.

$\therefore \Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$\Rightarrow \lambda(-\cos^2\alpha - \sin^2\alpha) - \sin\alpha(-\cos\alpha + \sin\alpha) + \cos\alpha(\sin\alpha + \cos\alpha) = 0$

$\Rightarrow -\lambda + \sin\alpha\cos\alpha + \sin\alpha\cos\alpha - \sin^2\alpha + \cos^2\alpha = 0$

$\Rightarrow \lambda = \cos 2\alpha + \sin 2a$

$\left(\because -\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}\right)$

$\therefore -\sqrt{2} \le \lambda \le \sqrt{2}$ ... (i)

Again, when $\lambda = 1$, $\cos 2\alpha + \sin 2\alpha = 1$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos 2\alpha + \frac{1}{\sqrt{2}}\sin 2\alpha = \frac{1}{\sqrt{2}}$$

$\Rightarrow \cos(2\alpha - \pi/4) = \cos\pi/4$

$\therefore 2\alpha - \pi/4 = 2n\pi \pm \pi/4$

$\Rightarrow 2\alpha = 2n\pi - \pi/4 + \pi/4$ or $2\alpha = 2n\pi + \pi/4 + \pi/4$

$\therefore \alpha = n\pi$ or $n\pi + \pi/4$

Sol 10: Given,

$$\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} a^2x-aby-ac & bx+ay & cx+a \\ abx+a^2y & -ax+by-c & cy+b \\ acx+a^2 & cy+b & -ax-by+c \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + bC_2 + cC_3$

$$\Rightarrow \frac{1}{a}\begin{vmatrix} (a^2+b^2+c^2)x & ay+bx & cx+a \\ (a^2+b^2+c^2)y & by-c-ax & b+cy \\ a^2+b^2+c^2 & b+cy & c-ax-by \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{a}\begin{vmatrix} x & ay+bx & cx+a \\ y & by-c-ax & b+cy \\ 1 & b+cy & c-ax-by \end{vmatrix} = 0$$

$(\because a^2 + b^2 + c^2 = 1)$

Applying $C_2 \to C_2 - bC_1$

and $C_3 \to C_3 - cC_1$

$$\Rightarrow \frac{1}{a}\begin{vmatrix} x & ay & a \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{ax}\begin{vmatrix} x^2 & axy & ax \\ y & -c-ax & b \\ 1 & cy & ax-by \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + yR_2 + R_3$

$$\Rightarrow \frac{1}{ax}\begin{vmatrix} x^2+y^2+1 & 0 & 0 \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{ax}[(x^2 + y^2 + 1)\{(-c - ax)(-ax - by) - b(cy)\}] = 0$$

$$\Rightarrow \frac{1}{ax}[(x^2 + y^2 + 1)(acx + bcy + a^2x^2 + abxy - bcy)] = 0$$

$$\Rightarrow \frac{1}{ax}[(x^2 + y^2 + 1)(acx + a^2x^2 + abxy)] = 0$$

$$\Rightarrow \frac{1}{ax}[ax(x^2 + y^2 + 1)(c + ax + by)] = 0$$

$\Rightarrow (x^2 + y^2 + 1)(ax + by + c) = 0$

$\Rightarrow ax + by + c = 0$

Which represents a straight line.

Sol 11: (A) $\Delta = 1\left(1 - c\omega - a\left(\omega - \omega^2 c\right) + b(0)\right)$

$\Delta = 1c\omega - a\omega + \omega^2 ac$

$\Delta = 1 - \omega(c + a) + \omega^2 ac$

$c = \omega \;\; a = \omega^2$ singular

$c = \omega^2 \;\; a = \omega$ singular

$c = \omega \;\; a = \omega$ non singular

$c = \omega^2 \;\; a = \omega^2$ singular

for every pair (a, c) there are two possible values of b hence 2 matrices.

Sol 12: Let $M = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow a_2 = -1, b_2 = 2, c_2 = 3$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} . \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow a_1 - a_2 = 1$$

$\Rightarrow a_1 = 0,\ b_1 = 3,\ c_1 = 3$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} . \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} \Rightarrow c_1 + c_2 + c_3 = 12$$

$\Rightarrow c_3 = 12 - 5 = 7$

$\therefore$ Sum of diagonal elements $= a_1 + b_2 + c_3 = 0 + 2 + 7 = 9$

Sol 13: (D) There seems to be an ambiguity in the question since 3 × 3 skew-symmetric matrices can't be non-singular.

[**Property:** Determinant of an odd order skew-symmetric matrix is always zero]

P is a 3×3 matrix

Let $P = \begin{bmatrix} a & b & c \\ \alpha & \beta & \gamma \\ \iota & m & n \end{bmatrix}$

$P^T = \begin{bmatrix} a & \alpha & \iota \\ b & \beta & m \\ c & \gamma & n \end{bmatrix}$

$P^T = 2P + I$

$$\begin{bmatrix} a & \alpha & \iota \\ b & \beta & m \\ c & \gamma & n \end{bmatrix} = \begin{bmatrix} 2a & 2b & 2c \\ 2\alpha & 2\beta & 2\gamma \\ 2\iota & 2m & 2n \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & \alpha & \iota \\ b & \beta & m \\ c & \gamma & n \end{bmatrix} = \begin{bmatrix} 2a+1 & 2b & 2c \\ 2\alpha & 2\beta+1 & 2\gamma \\ 2\iota & 2m & 2n+1 \end{bmatrix}$$

$= 2b = \alpha, b = 2\alpha$ .It is possible when $b = \alpha, = 0$

Similarly,, $c = \iota = 0$

$m = \gamma = 0$

The matrix P is $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

So , $PX = -X$

Sol 14: (D) $P = \left[a_{ij}\right]$

$Q = \left[b_{ij}\right]$

$b_{ij} = 2^{i+j}.a_{ij}$

$b_{11} = 2^2 a_{11} \;\; b_{21} = 2^3.a_{21} \;\; b_{31} = 2^4 a_{31}$

$b_{12} = 2^3 a_{12} \;\; b_{22} = 2^4.a_{22} \;\; b_{32} = 2^5 a_{32}$

$b_{13} = 2^4 a_{13} \;\; b_{23} = 2^5.a_{22} \;\; b_{33} = 2^6 a_{33}$

Given $P = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 2$

$$Q = \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix}$$

$$Q = 2^2.2^3.2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$Q = 2^2.2^3.2^4 \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix}$$

$$Q = 2^2.2^3.2^4.2^2.2^1\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$Q = 2^2.2^3.2^4.2^2.2^1.2^1 = 2^{13}$$

Sol 15: (B, C, D) $P^2 = 0$ only when n is multiple of 3.

$$\text{E.g:} \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix} . \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore P^2 \neq 0$ when $n = 55, 56, 58$

Sol 16: (A, B) $MN = NM$

$N^2M = N(NM) = (NM)N = (MN)N = MN^2$

$\left(M - N^2\left(M + N^2\right)\right) = M^2 + MN^2 - N^2M - N^4 = M^2 - N^4$

As $M - N^2 \neq 0 \Rightarrow \left|M + N^2\right| = 0$

$\left|M^2 + MN^2 = \left|M\left(M + \_N^2\right)\right|\right| = \left|M\right|M + N^2\right||$

$= 0 \Rightarrow \left|M + N^2\right| = 0$

Sol 17: (D) When roots are purely imaginary.

Then the form of equation is $x^2 + K = 0$

where K is positive no.

Let $p(x) = x^2 + K$

$p(p(x)) = (p(x))^2 + K$

$p(p(x)) = \left(x^2 + K\right)^2 + K$

$p(p(x)) = x^4 + 2Kx^2 + K \Rightarrow p(p(x)) = 0$

$x^4 + 2Kx^2 + K = 0$

All coefficients are positive and no odd degree of x are present.

Sol 18: (A) $z = \dfrac{-1 + i\sqrt{3}}{2} = \omega$

$$p = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$$p^2 = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}\begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$$= \begin{bmatrix} (-\omega)^{2r} + \left(\omega^{2s}\right)^2 & \omega^{2s}(-\omega)^r + \omega^r\omega^{2s} \\ \omega^{2s}(-\omega)^r + \omega^r\omega^{2s} & \omega^{2r}\left(\omega^r + \omega\right) \end{bmatrix} = -I \text{ (Given)}$$

r	s	r	s
1	1	1	1
2	2	3	3

Total no. pairs = 1

Sol 19: (B, C) $\left(\dfrac{P}{K}\right).Q = I \qquad \therefore Q = \left(\dfrac{P}{K}\right)^{-1}$

Comparing P_{23} we get,

$\dfrac{-K}{8} = \dfrac{-K(3\alpha + 4)}{12\alpha + 20} \Rightarrow \alpha = -1$

Also $|P||Q| = K^3$

$\therefore (12\alpha + 20)\dfrac{K^2}{2} = K^3$

K = 6α + 10 = 4

Sol 20: (B)

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 96 & 12 & 1 \end{bmatrix}$$

$$\therefore P^n = \begin{bmatrix} 1 & 0 & 0 \\ 4n & 1 & 0 \\ 8\left(n^2 + n\right) & 4n & 1 \end{bmatrix}$$

$$\therefore P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 8 \times \left(n^2 + n\right) & 4n & 1 \end{bmatrix}$$

$P^{50} - Q = I$

Equation we get $200 - q_{21} = 0 \Rightarrow q^{21} = 200$

$400 \times 51 - q_{31} = 0$

$q_{31} = 400 \times 51$

$200 - q_{32} = 0 \Rightarrow q_{32} = 200$

$\dfrac{q_{31} + q_{32}}{q_{21}} = \dfrac{400 \times 51 + 200}{200} = 103$