Basic Mathematics

PROBLEM-SOLVING TACTICS

(a) The main thing to remember about surds and working them out is that it is about manipulation. Changing and manipulating the equation so that you get the desired result. Rationalizing the denominator is all about manipulating the algebra expression.

(b) Strategy for Solving Equations containing Logarithmic and Non-Logarithmic Expressions:

(i) Collect all logarithmic expressions on one side of the equation and all constants on the other side.

(ii) Use the Rules of Logarithms to rewrite the logarithmic expressions as the logarithm of a single quantity with coefficient of 1.

(iii) Rewrite the logarithmic equation as an equivalent exponential equation.

(iv) Solve for the variable.

(v) Check each solution in the original equation, rejecting apparent solutions that produce any logarithm of a negative number or the logarithm of 0. Usually, a visual check suffices!

Note: The logarithm of 0 is undefined

(c) Logarithmic series

(i) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\infty$
(ii) $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\infty$
(iii) $\ln(x+1) - \ln(1-x) = \ln\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots\right)$
(iv) $\ln(1+x) + \ln(1-x) = \ln(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \ldots\ldots\right)$

FORMULAE SHEET

(a) Laws of indices

(i) $a^0 = 1$, $(a \neq 0)$	**(ii)** $a^{-m} = \frac{1}{a^m}$, $(a \neq 0)$
(iii) $a^{m+n} = a^m.a^n$, where m and n are real numbers	**(iv)** $a^{m-n} = \frac{a^m}{a^n}$

(v) $\left(a^m\right)^n = a^{mn}$	**(vi)** $a^{p/q} = \sqrt[q]{a^p}$
(vii) $(ab)^n = a^n b^n$	**(viii)** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(b) Some Important Identities

(i) $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$
(ii) $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$
(iii) $a^2 - b^2 = (a + b)(a - b)$
(iv) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
(v) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
(vi) $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 + b^2 - ab)$
(vii) $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + b^2 + ab)$
(viii) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = a^2 + b^2 + c^2 + 2abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$
(ix) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$
(x) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
(xi) $a^4 - b^4 = (a + b)(a - b)(a^2 + b^2)$
(xii) $a^4 + a^2 + 1 = (a^2 + 1)^2 - a^2 = (1 + a + a^2)(1 - a + a^2)$

(c) Laws of Surds

(i) $\sqrt[n]{a} = a^{\frac{1}{n}}$	**(ii)** $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$
(iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	**(iv)** $\left(\sqrt[n]{a}\right)^n = a$
(v) $\left(\sqrt[m]{\sqrt[n]{a}}\right) = \sqrt[mn]{a}$	**(vi)** $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

(d) Logarithm formulas

(i) $\log_a(M.N) = \log_a M + \log_a N$	**(ii)** $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$
(iii) $y = \log_a x \Leftrightarrow a^y = x(a, x > 0, a \neq 1)$	**(iv)** $\log_a M^x = x \log_a .M$

(v) $\log_{a^x} M = \frac{1}{x}\log_a M \quad (x \neq 0)$	**(vi)** $\log_b a = \frac{\log_c a}{\log_c b} = \frac{\log a}{\log b} \quad (c > 0, c \neq 1)$
(vii) $\log_a b = \frac{1}{\log_b a}$	**(viii)** $\log_b a \,.\, \log_c b \,.\, \log_d c = \log_d a$
(ix) $a^{\log_e c} = c^{\log_e a}$	**(x)** $\log_b a \,.\, \log_a b = 1$
(xi) $e^{x \ln a} = a^x$	**(xii)** $\log_a 1 = 0$ and $\log_a a = 1$

(e) Exponential series

(i) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ...\infty$	**(ii)** $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + ... + \frac{(-1)^n}{n!}x^n + ...\infty$
(iii) $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ...\infty$	**(iv)** $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ...\infty$
(v) $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + ...\infty$	**(vi)** $\frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + ...\infty$
(vii) $a^x = 1 + x(\ln a) + \frac{x^2}{2!}(\ln a)^2 + .. \ (a > 0)$	