Solved Examples

JEE Main/Boards

Example 1: Evaluate $\sqrt[3]{72.3}$, if $\log_{10} 72.3 = \bar{1}.8591$

Sol: Here consider x = $\sqrt[3]{72.3}$. Now by applying log on both sides and solving using logarithm formula we will get value of $\sqrt[3]{72.3}$.

Let x = $\sqrt[3]{72.3}$, Then, log x = log $(72.3)^{1/3}$

$\Rightarrow \log_{10} x = \frac{1}{3} \log_{10} 72.3$ ... (i)

$\log 72.3 = \log\left(0.723 \times 10^2\right)$

$\Rightarrow \log 0.723 + \log 10^2 = \bar{1}.8591 + 2 = 1.8591$ ... (ii)

$\Rightarrow \log_{10} x = \frac{1}{3} \times 1.8591$

$\Rightarrow \log_{10} x = 0.6197$; $\Rightarrow$ x=antilog (0.6197)

$\Rightarrow$ x = 4.166 (using antilog table)

Example 2: Using logarithm, find the value of 6.45 × 981.4

Sol: Consider x = 6.45 × 981.4 and then apply log on both sides and solve by using $\log_a(M.N) = \log_a M + \log_a N$ and log table.

Then, $\log_{10} x = \log_{10} (6.45 \times 981.4)$

$= \log_{10} 6.45 + \log_{10} 981.4$

= 0.8096 + 2.9919 (using log table)

$\therefore$ x = antilog (3.8015) = 6331 (using antilog table)

Example 3: Find minimum value of x satisfying

$|x-3| + 2|x+1| = 4$

Sol: Similar to illustration 25.

Case-I: When $x < -1$

$-1(x-3) - 2(x+1) = 4$

$\Rightarrow -x + 3 - 2x - 2 = 4$; $\Rightarrow -3x + 1 = 4$

$\Rightarrow 3x = -3$; $\Rightarrow x = -1$

$\because$ x < – 1 so, x = –1 is not possible

Case-II: When –1 ≤ x < 3

$\Rightarrow$ – (x – 3) + 2 (x + 1) = 4 $\Rightarrow$ – x + 3 + 2x + 2 = 4

$\Rightarrow x+5=4 \quad \Rightarrow x=-1$; So, x = - 1 is a solution.

Case-III: When x ≥ 3 is taken, (x – 3) +2 (x + 1) = 4

$\Rightarrow$ 3x –1 = 4 $\Rightarrow$ x = 5/3 $\Rightarrow$ Therefore, no solution

Result x = –1 is the only solution.

Example 4: Let

$\log_3 N=\alpha_1+\beta_1, \log_5 N=\alpha_2+\beta_2, \log_7 N=\alpha_3+\beta_3$

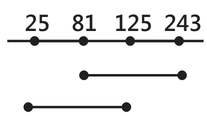
where $\alpha_1, \alpha_2, \alpha_3 \in I$ and $\beta_1, \beta_2, \beta_3 \in [0,1)$ then

(i) Find number of integral values of N if α_1 = 4 and α_1 = 2

(ii) Find the largest integral value of N if α_1 = 5, α_2 = 3, α_3 = 2

(iii) Difference of largest and smallest integral values

Sol: Here by using $y=\log_a x \Leftrightarrow a^y=x$ we can obtain values of N. After that by drawing a number line we will get the required answer.



(i) $N=3^{4+\beta_1}$ and $N=5^{2+\beta_2}$

$N=[3^4,3^5)$ and $N=[5^2,5^3]$

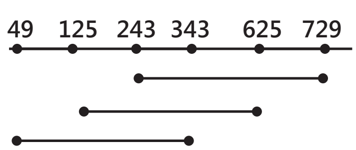
N = [81, 243) and N = [25, 125)

So [81, 125] is the common part hence the no. of integral values of N are 125 – 81 = 44

(i) $N=3^{5+\beta_1}, N=\left[3^5,3^6\right), N=\left[243,729\right)$

$N=5^{3+\beta_2}, N=\left[5^3,5^4\right), N=\left[125,625\right)$

$N=7^{2+\beta_3}, N=[7^2,7^3], N=\left[49,343\right)$



Common part is [243, 343]. So largest integral value = 342

(b) Difference of largest and smallest values

= 342 – 243 = 99

Example 5: Find the number of zeros in, x = $(0.35)^{12}$, Given $\log_{10}(7)$ = 0.8451, $\log_{10}(2)$ = 0.3010

Sol: By applying $\log_{10}$ on both sides and using logarithm formulae we will get the result.

$\log_{10} x=12\log_{10}\left(\frac{35}{100}\right)$

$\log_{10} x=12\log 35-\log_{10} 100=12$

$\left[\log_{10} 7+\log_{10} 5-2\right]=12$

$\left[\log_{10} 7+\log_{10} 10-\log_{10} 2-2\right]$

= 12[.8451 + 1 – .3010 –2] = 12 [.5441 –1]

$\log_{10}x$ = – 12 + 6.5292

$\log_{10} x=-12+6+0.5292=-6+0.5292=\bar{6}.5292$

So $x=10^{-6}.10^{-5212}$

Hence the number of zeros after the decimal = 5

Example 6: Find the number of zeros in, 2^{-40}

Sol: Consider 2^{-40} = x and solve as in illustration 5.

$x=\frac{1}{2^{40}}=2^{-40}$

$\log_{10} x=-40\log_{10} 2=-40\left[.3010\right]=-12.0400$

$\log_{10} x=\left(-12-0.04\right)+1-1 \Rightarrow \log_{10} x=-13+0.96$

$\Rightarrow x=10^{-13}.10^{0.96}$

Number of zeros = 12

Example 7: Find the number of digits for $x=3^{12}\times 2^8$

Sol: By applying $\log_{10}$ on both sides and then using a log table we can solve the problem above.

$\log_{10} x=12\log_{10} 3+8\log_{10} 2$

$\left(0.4771\right)+8\left(0.3010\right)=5.7252+2.4080$

$\log_{10} x=8.1332 \Rightarrow x=\left(10^8\right)10^{0.1332}$

No. of digits = 8 + 1 = 9

Example 8: Solve $x^{\log_{\sqrt{x}}(x-2)}=9$

Sol: Here, by using $\log_{a^x} M=\frac{1}{x}\log_a M$ we can solve the problem above.

$x^{\log_{\sqrt{x}}(x-2)}=9 \Rightarrow x^2\log x\ (x\text{-}2) = 9$

$\Rightarrow x^{\log_x(x-2)^2}=9$ where x > 0, $x\neq 1$

$\Rightarrow \left(x-2\right)^2=9; \Rightarrow x-2=\pm 3$

$\Rightarrow$ x = – 1, x = 5

But x = –1 is rejected as x should be greater than 0.

Example 9: $\log_3\left(\log_{1/2}^2 x-3\log_{1/2} x+5\right)=2$

Sol: $\log_3\left(\log_{1/2}^2 x-3\log_{1/2} x+5\right)=2$

$\Rightarrow \log_{1/2}^2 x - 3\log_{1/2} x + 5 = 9$;

Let $\log_{1/2}(x) = t \quad \Rightarrow t^2 - 3t - 4 = 0$

(t – 4) (t + 1) = 0 $\Rightarrow$ t = 4, t = –1

$\Rightarrow \log_{1/2} x = 4, \log_{1/2} x = -1$

x= 1/16, x = 2

Example 10: Solve $\dfrac{1-2\left(\log_{10} x^2\right)^2}{\log_{10} x - 2\left(\log_{10} x\right)^2} = 1$

Sol: Simply by putting $\log_{10}$ x = t we can solve the problem above.

$\dfrac{1-2\left(\log_{10} x^2\right)^2}{\log_{10} x - 2\left(\log_{10} x\right)^2} = 1$, Let $\log_{10}$ x = t

$\Rightarrow \dfrac{1-2(2t)^2}{t-2t^2} = 1 \Rightarrow 1 - 8t^2 = t - 2t^2$

$\Rightarrow 6t^2 + 3t - 2t - 1 = 0 \Rightarrow 3t(2t+1) - 1(2t+1) = 0$

t = 1/3, t = –1/2 $\Rightarrow$ log x = 1/3, log x = $-\dfrac{1}{2}$

$x = 10^{1/3}, x = 10^{-1/2}$

Example 11 $\left(\dfrac{1}{5}\right)^{\log_{10}^2 x - \log_{10} x} = \dfrac{1}{125}.5^{\log_{10} x - 1}$

Sol: By using $a^{m-n} = \dfrac{a^m}{a^n}$ we can evaluate the problem above.

$\left(\dfrac{1}{5}\right)^{\log_{10}^2 x - \log_{10} x} = 5^{\left(\log_{10} x - 1\right) - 3} \quad \therefore 5^{\log x - \log^2 x} = 5^{\log x - 4}$

$\Rightarrow \log x - \log^2 x = \log x - 4 \quad \Rightarrow \log^2 x = 4$

$x = 10^2, x = 10^{-2}$

JEE Advanced/Boards

Example 1: Solve,

$\log_{3x+7}\left(9 + 12x + 4x^2\right) + \log_{2x+3}\left(6x^2 + 23x + 21\right) = 4$

Sol: Here $6x^2 + 23x + 21$

$= (2x+3)(3x+7)$ and $\left(9 + 12x + 4x^2\right) = (2x+3)^2$.

Hence substitute it in the above equation and solve using the logarithm formula.

Given that

$\log_{3x+7}\left(9 + 12x + 3x^2\right) + \log_{2x+3}\left(6x^2 + 23x + 21\right) = 4$

$\log_{3x+7}(2x+3)^2 + \log_{2x+3}\left[(2x+3)(3x+7)\right] = 4$ Let

$\log_{3x+7}(2x+3) = A$; $2A + 1 + \dfrac{1}{A} = 4$

$\Rightarrow 2A^2 - 3A + 1 = 0$; $2A^2 - 2A - A + 1 = 0$

$\Rightarrow$ 2A (A –1) –1 (A –1) = 0; A = 1/2, A = 1

$\Rightarrow \log_{3x+7}(2x+3) = 1/2$

For $A = \dfrac{1}{2}, 2x + 3 = \sqrt{3x+7}$

$\Rightarrow 4x^2 + 9 + 12x = 3x + 7$; $4x^2 + 9x + 2 = 0$

$\Rightarrow 4x^2 + 8x + x + 2 = 0 \quad \Rightarrow 4x(x+2) + 1(x+2) = 0$

$\Rightarrow x = \dfrac{-1}{4}, x = -2$; For $A = 1, \log_{3x+7} 2x + 3 = 1$

$\Rightarrow$ 2x + 3 = 3x + 7

$\Rightarrow$ x = –4 also 2x + 3 > 0, 3x + 7 > 0

$\Rightarrow$ x > –3/2, x > – 7/3

$\Rightarrow x = \dfrac{-1}{4}$ (–4 and –2 will be rejected)

Example 2: Solve, $(x)^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \left(\frac{5}{4}\right)} = \sqrt{2}$

Sol: By taking $\log_x$ on both sides and solving we will get the result.

Taking log on both sides to the base x

$\log_x\left[(x)\right]^{\frac{3}{4}(\log_2 x)^2 + (\log_2 x) - \frac{5}{4}} = \log_x\left(\sqrt{2}\right)$

$\dfrac{3}{4}(\log_2 x)^2 + (\log_2 x) - \dfrac{5}{4} = \dfrac{1}{2}\log_x 2$

Let $\log_2 x = t$; $\dfrac{3}{4}t^2 + t - \dfrac{5}{4} = \dfrac{1}{2t}$

$3t^3 + 4t^2 - 5t = 2 \Rightarrow 3t^3 + 4t^2 - 5t - 2 = 0$

$\Rightarrow 3t^3 + 3t^2 - 6t + t^2 + t - 2 = 0 \Rightarrow (3t+1)\left(t^2 + t - 2\right) = 0$

$\Rightarrow (3t+1)(t+2)(t-1) = 0$; $\Rightarrow t = 1, -2, -\dfrac{1}{3}$

Putting t = $\log_2 x$

$\log_2 x = 1 \Rightarrow x = 2$; $\log_2 x = -2 \Rightarrow x = \frac{1}{4}$

$\log_2 x = -1/3 \Rightarrow x = 1/(2)^{1/3}$

Example 3: Solve $|x-1|^{\log_3 x^2 - 2\log_x 9 - 6} = (x-1)$

Sol: As a^x is defined for a > 0 so (x –1) > 0. Therefore by taking log on both side we can solve it.
Now taking log on both sides

$$\left(\log_3 x^2 - 2\log_x 9\right)\log(x-1) = \log(x-1)$$

$$\left(2\log_2 x - \frac{2}{\log_{3^2} x} - 1\right)\log(x-1) = 0$$

Either log (x – 1) = 0 ⇒ x = 2

Let $\log_3 x = t$

(2t – 4/t – 7) = 0 ⇒ $2t^2 - 4 - 7t = 0$

⇒ $2t^2 - 8t + t - 4 = 0$ ⇒ 2t(t – 4) + 1 (t – 4) = 0

t = 4, t = –1/2

$\log_3 x = 4$ or $\log_3 x = -1/2$

$x = (3)^4$ or $x = (3)^{-1/2}$

x = 81, x = $1/\sqrt{3}$

For x = $\frac{1}{\sqrt{3}}$ log (x – 1) is not defined, so x = 2 or x = 81.

Example 4: Solve,

$$\log_4\left(x^2 - 1\right) - \log_4(x-1)^2 = \log_4\sqrt{(4-x)^2}$$

Sol: By using formula $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$ and using modulus inequalities we can solve the problem above.

$$\log_4\frac{\left(x^2-1\right)}{(x-1)^2} = \log_4|4-x|\left[\because \sqrt{x^2} = |x|\right]$$

$$\Rightarrow \log_4\frac{(x-1)(x+1)}{(x-1)^2} = \log_4|4-x|$$

So we have $\frac{(x+1)}{(x-1)} = |4-x|$

or (x + 1) = (x – 1) |4 – x|

Case-I: 4 – x > 0 or x < 4 then (x + 1) = (x – 1) (4 – x)

$\Rightarrow x + 1 = 4x - x^2 - 4 + x \Rightarrow x^2 - 4x + 5 = 0$

$\Rightarrow x = \frac{4 \pm \sqrt{16-20}}{2}, x \notin R$

which is not possible

Case-II: (4 – x) < 0 or x > 4 then (x + 1) = (x – 1) (x – 4)

$\Rightarrow x + 1 = x^2 - 5x + 4$ $\Rightarrow x^2 - 6x + 3 = 0$

$\Rightarrow x^2 - 6x + 3 = 0$ $\Rightarrow x = \frac{6 \pm \sqrt{24}}{2}$

$\Rightarrow x = \frac{6 \pm 2\sqrt{6}}{2}$ $\Rightarrow x = 3 \pm \sqrt{6}$

$\because x > 4$ is taken, hence x = 3 + $\sqrt{6}$

Example 5: If the sum of all solutions of the equation $\left[(x)^{\log_{10} 3}\right]^2 - \left(3^{\log_{12} x}\right) - 2 = 0$ is $(a)^{\log b}$ where b and c arerelatively prime and a, b c ∈ N then (a + b + c) = ?

Sol: Here by putting $3^{\log_{10} x}$ = t and solving we will get the result.

$$\left((3)^{\log_{10} x}\right)^2 - \left(m^{\log_{10} x}\right) - 2 = 0$$

Let e $3^{\log_{10} x}$ = t then

$\Rightarrow t^2 - t - 2 = 0$; $\Rightarrow t^2 - 2t + t - 2 = 0$

$\Rightarrow t(t-2) + 1(t-2) = 0$; $\Rightarrow (t+1)(t-2) = 0$

Case-I: $t = -1$; $\Rightarrow t = -1$ & $t = 2$ $\Rightarrow 3^{\log_{10} x} = -1$

Exponential value cannot be negative

Case-II: t = 2; $3^{\log_{10} x} = 2$

Taking $\log_3$ both side

$\log_3(3)^{\log_{10} x} = \log_3 2 \Rightarrow \log_{10} x = \log_3 2 \Rightarrow x = 10^{\log_3 2}$

Comparing by $(a)^{\log_b c}$ we get

a = 10 , b = 3, c = 2

∴ a + b + c = 10 + 3 + 2 = 15

Example 6: Find the number of zeros after decimal before a significant digit in $(9/8)^{-100}$.

Sol: By putting x = $\left(\frac{9}{8}\right)^{-100}$ and applying $\log_{10}$ on both side we will get the result.

Let x = $\left(\frac{9}{8}\right)^{-100}$

$\Rightarrow \log_{10} x = -100\left[\log_{10} 9 - \log_{10} 8\right]$

$\Rightarrow \log_{10} x = -100\left[2\log_{30} 3 - 3\log_{10} 2\right]$

$\Rightarrow \log_{10} x = -100\left(2\times 0.4771 - 3\times 0.3010\right)$
$= -100\left[0.9542 - 0.9030\right] = -100\left[0.0512\right] = -5.12$

$\log_{30} x = \left(-5 - 0.12\right) + 1 - 1$

$\log_{10} x = \overline{6}.88 \Rightarrow x = 10^{-6} \times 10^{0.88}$

$\therefore$ Number of zeros before any significant digits = 5

Example 7: Solve $\log_4\left(2\log_3\left(1+\log_2\left(1+3\log_2 x\right)\right)\right) = 1/2$

Sol: Here by using $y = \log_a x \Leftrightarrow a^y = x$ we can solve it.

$\log_4\left(2\log_3\left(1+\log_3\left(3\log_3 x\right)\right)\right) = 1/2$

$\Rightarrow 2\log_3\left(1+\log_2\left(1+3\log_3 x\right)\right) = 2$

$\Rightarrow \log_3\left(1+\log_2\left(1+3\log_3 x\right)\right) = 1$

$\Rightarrow 1+\log_2\left(1+3\log_3 x\right) = 3 \Rightarrow \log_2\left(1+3\log_3 x\right) = 2$

$\Rightarrow 1+3\log_3 x = 4 \Rightarrow 3\log_3 x = 3 \Rightarrow \log_3 x = 1 \Rightarrow x = 3$

Example 8: Solve $\log_{0.5x} x - 7\log_{16x} x^3 + 40\log_{4x} \sqrt[4]{x} = 0$

Sol: By using $\log_b a = \dfrac{\log a}{\log b}$ we can reduce the given equation to $\dfrac{\log_2 x}{\log_2 0.5x} - \dfrac{7\log_2 x^3}{\log_2 16x} + \dfrac{40\log_2 \sqrt[4]{x}}{\log_2 4x} = 0$ and then by putting $\log_2 x = t$ we can solve it.

Let $\log_2 x = t$

$\Rightarrow \dfrac{t}{-1+t} - \dfrac{7(3t)}{4+t} + \dfrac{10t}{2+t} = 0 \Rightarrow \dfrac{t}{t-1} - \dfrac{21t}{t+4} + \dfrac{10t}{t+2} = 0$

$$\Rightarrow t\left\{\frac{(t+4)(t+2) - 21(t-1)(t+2) + 10(t-1)(t+4)}{(t-1)(t+4)(t+2)}\right\} = 0$$

$$\Rightarrow t\left\{\frac{t^2+6t+8-21t^2-21t+42+10t^2+30t-40}{(t-1)(t+4)(t+2)}\right\} = 0$$

$$\Rightarrow t\left\{\frac{-10t^2+15t+10}{(t-1)(t+4)(t+2)}\right\} = 0$$

$\Rightarrow t = 0, -\dfrac{1}{2}, 2$ $\quad\therefore \log_2 x = 0 \Rightarrow x = 1$

$\log_2 x = -\dfrac{1}{2} \Rightarrow x = \dfrac{1}{\sqrt{2}}$ and $\log_2 x = 2 \Rightarrow x = 4$

Example 9: Solve, $\log_2\left(x/4\right) = \dfrac{15}{\log_2 \frac{x}{8} - 1}$

Sol: Simply by putting $\log_2$ (x) = t and using basic logarithmic formula we can solve the problem above.

$\log_2\left(x/4\right) = \dfrac{15}{\log_2 \frac{x}{8} - 1} \Rightarrow \left(\log_2 x - 2\right) = \dfrac{15}{\left(\log_2 x - 3\right) - 1}$

Let $\log_2$ (x) = t

$\Rightarrow t - 2 = \dfrac{15}{t-4} \quad \Rightarrow t2 - 6t + 8 = 15$

$\Rightarrow t^2 - 6t - 7 = 0 \Rightarrow (t-7)(t+1) = 0$

$\Rightarrow t = 7, t = -1 \quad \Rightarrow \log_2 x = 7$ and $\log_2 x = -1$

$\Rightarrow x = 2^7$ and $x = 2^{-1}$

Example 10: Solve, $\sqrt{\log_2\left(2x^2\right)\log_4\left(16x\right)} = \log_4 x^3$

Sol: By using $\log_a(M.N) = \log_a M + \log_a N$ we can reduce the given equation to $\sqrt{\left(1+2\log_2 x\right)\left(2+\frac{1}{2}\log_2 x\right)} = \frac{3}{2}\log_2 x$.

After that putting $\log_2 x = t$ we will get the result.

$\sqrt{\log_2\left(2x^2\right)\log_4\left(16x\right)} = \log_4 x^3$

$\Rightarrow \sqrt{\left(1+2\log_2 x\right)\left(2+\dfrac{1}{2}\log_2 x\right)} = \dfrac{3}{2}\log_2 x$

Let $\log_2 x = t$

$\Rightarrow \sqrt{\left(1+2t\right)\left(2+\dfrac{t}{2}\right)} = \dfrac{3}{2}t$

$\Rightarrow \left(1+2t\right)\left(\dfrac{4+t}{2}\right) = \dfrac{9t^2}{4} \Rightarrow \left(2t+1\right)\left(t+4\right) = \dfrac{9t^2}{2}$

$\Rightarrow 2.\left(2t^2+9t+4\right) = 9t^2$

$\Rightarrow 5t^2 - 18t - 8 = 0 \Rightarrow 5t^2 - 20t + 2t - 8 = 0$

$\Rightarrow 5t\left(t-4\right) + 2\left(t-4\right) = 0; t = -2/5, t = 4$

But $t \neq 4 \Rightarrow x = 6$ and logx = –2/5 is Not Possible

$\therefore\ t = -\dfrac{2}{5} \quad \Rightarrow \log_2 x = -\dfrac{2}{5} \quad \therefore x = 2^{-2/5}$

JEE Main/Boards

Exercise 1

Q.1 Solve

(i) $\log_{16} 32$

(ii) $\log_8 16$

(iii) $\log_{1/3}(1/9)$

(iv) $\log_{2\sqrt{3}}(1728)$

(v) $\log_2 \cos 45°$

(vi) $\log_2(\log_2 4)$

(vii) $\log_3(\tan 30°)$

Q.2 Prove the following

(i) $\log_5 \sqrt{5\sqrt{5\sqrt{5 - \infty}}} = 1$

(ii) $\log_{0.125}(8) = -1$

(iii) $\log_{1.5}(0.\bar{6}) = -1$

(iv) $\log_{2.25}(0.\bar{4}) = -1$

(v) $\log_{10}(0.\bar{9}) = 0$

Q.3 Find the no. of digits in

(i) 2^{100} (ii) 3^{10}

Q.4 Solve

(i) $\log_{x-1} 3 = 2$

(ii) $\log_3(3^x - 8) = 2 - x$

(iii) $\log_{5-x}(x^2 - 2x + 65) = 2$

(iv) $\log_3(x+1) + \log_3(x+3) = 1$

(v) $x^{2\log_{10} x} = 10.x^2$

(vi) $x^{\frac{\log_{10} x + 5}{3}} = 10^{5 + \log_{10} x}$

(vii) $x^{\log_3 x} = 9$

Q.5 $1 - \log 5 = \frac{1}{3}\left(\log\frac{1}{2} + \log x + \frac{1}{3}\log 5\right)$

Q.6 $\log x - \frac{1}{2}\log\left(x - \frac{1}{2}\right) = \log\left(x + \frac{1}{2}\right) - \frac{1}{2}\log\left(x + \frac{1}{8}\right)$

Q.7 $x^{\frac{\log_{10} x + 7}{4}} = 10^{\log_{10} x + 1}$

Q.8 $\left(\frac{\log_{10} x}{2}\right)^{\log^2_{10} x + \log_{10} x^2 - 2} = \log_{10}\sqrt{x}$

Q.9 $\sqrt{\log_2 x} - \log_2 8x + 1 = 0$

Q.10 $\log_{1/3} x - 3\sqrt{\log_{1/3} x} + 2 = 0$

Q.11 $\left(a^{\log_b x}\right)^2 - 5a^{\log_b x} + 6 = 0$

Q.12 $\log_4(x^2 - 1) - \log_4(x-1)^2 = \log_4\left(\sqrt{(4-x)^2}\right)$

Q.13 $2\log_3\frac{x-3}{x-7} + 1 = \log_3\frac{x-3}{x-1}$

Q.14 $\log_x(9x^2)\log_3^2 x = 4$

Q.15 $\log_{0.5x} x^2 + 14\log_{16x} x^2 + 40\log_{4x}\sqrt{x} = 0$

Q.16 $\log_3\left(\log^2_{1/2} x - 3\log_{1/2} x + 5\right) = 2$

Q.17 $\log_2(x/4) = \frac{15}{\log_2\frac{x}{8} - 1}$

Q.18 $\frac{1}{2}\log_{10}(5x-4) + \log_{10}\sqrt{x+1} = 2 + \log_{10} 0.18$

Q.19 $\log_{10} x^2 = \log_{10}(5x-4)$

Q.20 $\frac{1}{6}\log_2(x-2) - \frac{1}{3} = \log_{1/8}\sqrt{3x-5}$

Q.21 $\frac{\log_{10}\left(\sqrt{x+1} + 1\right)}{\log_{10}\left(\sqrt[3]{x-40}\right)} = 3$

Q.22 $1-\frac{1}{2}\log_{10}(2x-1)=\frac{1}{2}\log_{10}(x-9)$

Q.23 $\log_{10}(3x^2+7)-\log_{10}(3x-2)=1$

Q.24 $\left(1+\frac{1}{2x}\right)\log_{10}3+\log_{10}2=\log_{10}\left(27-3^{1/x}\right)$

Q.25 $\frac{1}{2}\log_{10}x+3\log_{10}\sqrt{2+x}=\log_{10}\sqrt{x(x+2)}+1$

Q.26 $\log_2(4^x+1)=x+\log_2(2^{x+3}-6)$

Q.27 $\log_{\sqrt{5}}(4^x-6)-\log_{\sqrt{5}}(2^x-2)=2$

Q.28 $\log_{10}(3^x-2^{4-x})=2+\frac{1}{4}\log_{10}16-\frac{x\log_{10}4}{2}$

Q.29 $\log_{10}(\log_{10}x)+\log_{10}(\log_{10}x^4-3)=0$

Q.30 $\log_3(9^x+9)=\log_3 3^x(28-2.3^x)$

Exercise 2

Single Correct Choice Type

Q.1 $\frac{1}{\log_{\sqrt{bc}}abc}+\frac{1}{\log_{\sqrt{ac}}abc}+\frac{1}{\log_{\sqrt{ab}}abc}$

has the value equal to

(A) 1/2 (B) 1 (C) 2 (D) 4

Q.2 The equation, $\log_2(2x^2)+\log_2 x.x^{\log_x(\log_2 x+1)}$

$+\frac{1}{2}\log_4 2x^4+2^{-3\log_{1/2}(\log_2 x)}=1$ has

(A) Exactly one real solution (B) Two real solutions

(C) 3 Real solutions (D) No solution

Q.3 Number of zeros after decimal before a significant figure in $(75)^{-10}$ is:

(Use $\log_{10}2 = 0.301$ and $\log_{10}3= 0.477$)

(A) 20 (B) 19 (C) 18 (D) None

Q.4 If $5x^{\log_2 3}+3^{\log_2 x}=162$ then logarithm of x to the base 4 has the value equal to

(A) 2 (B) 1 (C) –1 (D) 3/2

Q.5 $(x)^{\log_{10}^2 x+\log_{10}x^3+3}=\frac{2}{\frac{1}{\sqrt{x+1}-1}-\frac{1}{\sqrt{x+1}+1}}$

where $x_1 > x_2 > x_3$, then

(A) $x_1+x_3=2x_2$ (B) $x_1.x_3=x_2^2$

(C) $x_2=\frac{2x_1x_2}{x_1+x_2}$ (D) $x_1^{-1}+x_1^{-1}=x_3^{-1}$

Q.6 Let $x=2^{\log 3}$ and $y=3^{\log 2}$ where base of the logarithm is 10, then which one of the following holds good?

(A) 2x < y (B) 2y < x (C) 3x = 2y (D) y = x

Q.7 Number of real solution(s) of the equation

$|x-3|^{3x^2-10x+3}=1$ is-

(A) Exactly four (B) Exactly three

(C) Exactly two (D) Exactly one

Q.8 If x_1 and x_2 are the roots of the equation $\sqrt{2010}x^{\log_{2010}x}=x^2$, then find the cyphers at the end of the product (x_1x_2)

(A) 1 (B) 3 (C) 2 (D) 4

Q.9 Let x = 2 or x = 3 satisfy the equation, $\log_4(x^2+bx+c)=1$. Then find the value of |bc|.

(A) 50 (B) 60 (C) 40 (D) 55

JEE Advanced/Boards

Exercise 1

Q.1 Let A denotes the value of

$$\log_{10}\left(\frac{ab+\sqrt{(ab)^2-4(a+b)}}{2}\right)+\log_{10}\left(\frac{ab-\sqrt{(ab)^2-4(a+b)}}{2}\right)$$

when a = 43 and b = 57 and B denotes the value of the expression $\left(2^{\log_6 18}\right).\left(3^{\log_6 3}\right)$. Find the value of (A.B).

Q.2 Simplify:

(a) $\log_{1/3}\sqrt[4]{729\sqrt[3]{9^{-1}.27^{-4/3}}}$ (b) $\frac{\log_b(\log_b N)}{\log_b a}$

Q.3 (a) Which is smaller? 2 or $\left(\log_\pi 2+\log_2 \pi\right)$

(b) Prove that $\log_3 5$ and $\log_2 7$ are both irrational.

Q.4 Find the square of the sum of the roots of the equation $\log_3x\cdot\log_4x\cdot\log_5x=\log_3x\cdot\log_4x+\log_4x\cdot\log_5x+\log_5x\cdot\log_3x$.

Q.5 Find the value of the expression

$$\frac{2}{\log_4(2000)^6}+\frac{3}{\log_5(2000)^6}$$

Q.6 Simplify: $\frac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt6}3}}}{409}\left(\left(\sqrt7\right)^{\frac{2}{\log_{25}7}}-(125)^{\log_5 6}\right)$

Q.7 Simplify: $5^{\log_{1/5}(1/2)}+\log_{\sqrt2}\frac{4}{\sqrt7+\sqrt3}+\log_{1/2}\frac{1}{10+2\sqrt{21}}$

Q.8 Given that $\log_2 a = s$, $\log_4 b = s^2$ and $\log_{c^2} 8 = \frac{2}{s^3+1}$

Write $\log_2\frac{a^2b^5}{C^4}$ as function of 's' (a, b, c > 0) (c ≠ 1).

Q.9 Prove that $\frac{\log_2 24}{\log_{96}2}-\frac{\log_2 192}{\log_{12}2}=3$

Q.10 Prove that $a^x - b^y = 0$ wher $x = \sqrt{\log_a b}$ and $y=\sqrt{\log_b a}$, a>0, b>0 & b≠1.

Q.11 (a) Solve for x, $\frac{\log_{10}(x-3)}{\log_{10}(x^2-21)}=\frac{1}{2}$

(b) log (log x) + log ($\log x^3$ –2) = 0; where base of log is 10 everywhere

(c) $\log_x 2.\log_{2x}2 = \log_{4x}2$

(d) $5^{\log x} + 5x^{\log 5} = 3$ (a > 0); where base of log is a

Q.12 Solve the system of equations:

$\log_a x \log_a (xyz) = 48$

$\log_a y \log_a (xyz) = 12$

$\log_a z \log_a (xyz) = 84$

Q.13 Let 'L' denotes the antilog of 0.4 to the base 1024. and 'M' denotes the nuber of digits in 6^{10} (Given $\log_{10}2$ = 0.3010, $\log_{10}3$ = 0.4771) and 'N' denotes the number of positive integers which have the characteristic 2, when base of the logarithm is 6. Find the value of LMN.

Q.14 Prove the identity.

$\log_aN . \log_bN + \log_bN . \log_cN + \log_cN . \log_a$

$N = \frac{\log_a N\log_b N\log_c N}{\log_{abc}N}$

Q.15 If x, y > 0, $\log_y x + \log_x y = \frac{10}{3}$ and xy = 144, then $\frac{x+y}{2}=\sqrt{N}$ where N is a natural number, find the vaue of N.

Q.16 If $\log_{10}2$ = 0.0310, $\log_{10}3$ = 0.4771. Find the number of integers in:

(a) 5^{200}

(b) 6^{15}

(c) The number of zeros after the decimal in 3^{-100}.

Q.17 $\log_5 120 + (x-3) - 2\log_5(1-5^{x-3}) = -\log_5(2-5^{x-4})$

Q.18 $\log_{x+1}(x^2+x-6)^2 = 4$

Q.19 $x + \log_{10}(1+2^x) = x\log_{10}5 + \log_{10}6$

Q.20 If 'x' and 'y' are real numbers such that, $\log_{10}(2x-3x)=\log_{10}x+\log_{10}y$, find $\frac{x}{y}$

Q.21 If a = $\log_{12}18$ and b = $\log_{24}54$ then find the value of ab + 5 (a – b)

Q.22 Find the value of $\log_3 x$ if following is true

$$\sqrt{\log_9(9x^4)\log_3(3x)}=\log_3 x^3$$

Q.23 Positive numbers x, y and z satisfy xyz = 10^{81} and $(\log_{10}x)(\log_{10}yz)+(\log_{10}y)(\log_{10}z)=468$. Find the value of $(\log_{10}x)^2+(\log_{10}y)^2+(\log_{10}z)^2$.

Q.24 Find the sum of all solutions of the equation

$$3^{(\log_9 x)^2}-\frac{9}{2}\log_9 x+5=3\sqrt{3}$$

Q.25 Let a, b, c, d are positive integers such that $\log_a b = 3/2$ and $\log_c d = 5/4$. If (a – c) = 9, find the value of (b – d).

Q.26 Find the product of the positive roots of the equation $\sqrt{(2008)}(x)^{\log_{2008}x}=x^2$

Q.27 Find x satisfying the equation

$$\log_{10}^2\left(1+\frac{4}{x}\right)+\log_{10}^2\left(1-\frac{4}{x+4}\right)=2\log_{10}^2\left(\frac{2}{x-1}-1\right)$$

Q.28 Solve: $\log_3\left(\sqrt{x}+\left|\sqrt{x}-1\right|\right)$

$=\log_9\left(4\sqrt{x}-3+4\left|\sqrt{x}-1\right|\right)$

Q.29 Prove that

$$2^{\left(\sqrt{\log_a\sqrt[4]{ab}+\log_b\sqrt[4]{ab}}-\sqrt{\log_a\sqrt[4]{b/a}+\log\sqrt[4]{a/b}}\right)\sqrt{\log_a b}}=\begin{cases}2 & \text{if } b\ge a>1\\ 2^{\log_a b} & \text{if} 1<b<a\end{cases}$$

Q.30 Find the value of x satisfying the equation

$$\sqrt{\left[\log_3(3x)^{1/3}+\log_x(3x)^{1/3}\right]\log_3 x^3}+\sqrt{\left[\log_3\left(\sqrt{\frac{x}{3}}\right)^{1/3}+\log_x\left(\frac{3}{x}\right)^{1/3}\right]\log_3 x^3}=2$$

Q.31 Let $a=(\log_7 81)(\log_{6561}625)(\log_{125}216)(\log_{1296}2401)$

b denotes the sum of the roots of the equation $x^{\log_2 x}=(2x)^{\log_2\sqrt{x}}$ and c denotes the sum of all natural solution of the equation |x + 1| + |x – 4| = 7.

Find the value of (a + b) ÷ c.

Exercise 2

Single Correct Choice Type

Q.1 Number of ordered pair(s) satisfying simultaneously, the system of equations, $2^{\sqrt{x}+\sqrt{y}}=256$ and $\log_{10}\sqrt{xy}-\log_{10}1.5=1$, is:

(A) Zero (B) Exactly one

(C) Exactly two (D) More than two

Q.2 Let ABC be a triangle right angled at C. The value of $\frac{\log_{b+c}a+\log_{c-b}a}{\log_{b+c}a.\log_{c-b}a}$ (b + c ≠, c – b ≠ 1) equals

(A) 1 (B) 2 (C) 3 (D) ½

Q.3 Let B, C, P and L be positive real number such that log (B · L) + log (B · P) = 2; log (P · L) + log (P · C) = 3; log (C · B) + log (C · L) = 4. The value of the product (BCPL) equals (base of the log is 10)

(A) 10^2 (B) 10^3 (C) 10^4 (D) 10^9

Q.4 If the equation $\frac{\log_{12}(\log_8(\log_4 x))}{\log_5(\log_4(\log_y(\log_2 x)))}=0$ has a solution for 'x' when c < y < b, y ≠ a, where 'b' is as large as possible and 'c' is an small as possible, then the value of (a + b + c) is equal to

(A) 18 (B) 19 (C) 20 (D) 21

Q.5 For N > 1, the product

$\frac{1}{\log_2 N}\cdot\frac{1}{\log_N 8}\cdot\frac{1}{\log_{32} N}\cdot\frac{1}{\log_N 128}$ simplifies to

(A) $\frac{3}{7}$ (B) $\frac{3}{7\ell n2}$ (C) $\frac{3}{5\ell n2}$ (D) $\frac{5}{21}$

Q.6 Let $N = 10^{3\log 2-2\log(\log 10^3)+\log((\log 10^6)^2)}$ where base of the logarithm is 10. The characteristics of the logarithm of N to the base 3, is equal to

(A) 2 (B) 3 (C) 4 (D) 5

Q.7 If $x=\frac{\sqrt{10}+\sqrt{2}}{2}$ and $y=\frac{\sqrt{10}-\sqrt{2}}{2}$, then the value of $\log_2(x^2+xy+y^2)$, is equal to

(A) 0 (B) 2 (C) 3 (D) 4

Q.8 The sum $\sqrt{\frac{5}{4}+\sqrt{\frac{3}{2}}}+\sqrt{\frac{5}{4}-\sqrt{\frac{3}{2}}}$ is equal to

(A) $\tan\frac{\pi}{3}$ (B) $\cot\frac{\pi}{3}$ (C) $\sec\frac{\pi}{3}$ (D) $\sin\frac{\pi}{3}$

Q.9 Suppose that x < 0. Which of the following is equal to $\left|2x-\sqrt{(x-2)^2}\right|$

(A) x – 2 (B) 3x – 2 (C) 3x + 2 (D) – 3x + 2

Q.10 Solution set of the inequality $3^x(0.333.....)^{x-3}\le(1/27)^x$ is:

(A) [3/2, 5] (B) $(-\infty,3/2]$

(C) $(2,\infty)$ (D) None of these

Q.11 Solution set of the inequality $\left(\frac{1}{5}\right)^{\frac{2x+1}{1-x}}>\left(\frac{1}{5}\right)^{-3}$ is-

(A) $(-\infty,-2)\cup(1,\infty)$ (B) (1, 4)

(C) $(-\infty,1)\cup(2,\infty)$ (D) None of these

Q.12 The set of all x satisfying the equation $x^{\log_3 x^2+(\log_3 x)^2-10}=\frac{1}{x^2}$ is-

(A) {1, 9} (B) $\left\{9,\frac{1}{81}\right\}$ (C) $\left\{1,4,\frac{1}{81}\right\}$ (D) $\left\{1,9,\frac{1}{81}\right\}$

Q.13 If $\frac{(\ln x)^2-3\ln x+3}{\ln x-1}<1$, then x belongs to:

(A) (0, e) (B) (1, e) (C) (1, 2e) (D) (0, 3e)

Multiple Correct Choice Type

Q.14 The number $N=\frac{1+2\log_3 2}{(1+\log_3 2)^2}+\log_6^2 2$

when simplified reduces to-

(A) A prime number

(B) An irrational number

(C) A real number is less than $\log_3\pi$

(D) A real which is greater than $\log_7 6$

Q.15 The value of x satisfying the equation, $2^{2x}-8.2^x=-12$ is

(A) $1+\frac{\log 3}{\log 2}$ (B) $\frac{1}{2}\log 6$ (C) $1+\log\frac{3}{2}$ (D) 1

Q.16 If $\left(\sqrt{5\sqrt{2}-7}\right)^x+6\left(\sqrt{5\sqrt{2}+7}\right)^x=7$,

then the value of x can be equal to-

(A) 0 (B) $\log_{(5\sqrt{2}-7)}36$

(C) $\frac{-2}{\log_6(5\sqrt{2}+7)}$ (D) $\log_{\sqrt{5\sqrt{2}-7}}6$

Assertion Reasoning Type

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Q.17 Statement-I: $\sqrt{\log_x\cos(2\pi x)}$ is a meaningful quantity only if x ∈ (0, ¼) ∪ (3/4, 1).

Because

Statement-I: If the number N > 0 and the base of the logarithm b (greater than zero not equal to 1) both lie on the same side of unity then $\log_b N > 0$ and if they lie on different side of unit then $\log_b N < 0$.

Q.18 Statement-I: $\log_2\left(2\sqrt{17-2x}\right) = 1 + \log_2(x-1)$ has a solution.

because

Statement-II: Change of base in logarithms is possible.

Q.19 Consider the following statements

Statement-I: The equation $5^{\log_5\left(x^3+1\right)} - x^2 = 1$ has two distinct real solutions.

Because.

Statement-II: $a^{\log_a N} = N$ when a > 0, a ≠ 1 and N > 0.

Comprehension Type

Paragraph 1: Equations of the form (i) f ($\log_a x$) = 0, a > 0, a ≠ 1 and (ii) g ($\log_x A$) = 0, A > 0, then Eq. (i) is equivalent to f(t) = 0, where t = $\log_a x$. If $t_1, t_2, t_3, \ldots\ldots\ldots t_k$ are the roots of f(t) = 0, then $\log_a x = t$, $\log_a x = t_2, \ldots\ldots$, $x = t_k$ and eq. (ii) is equivalent to f(y) = 0, where y $\log_x A$. If $y_f, y_2, y_3, \ldots., y_k$ are the root of f(y) = 0, then $\log_x A = y_1$, $\log_{x'} A = y_2, \ldots\ldots, \log_x A = y_k$.

On the basis of above information, answer the following questions.

Q.20 The number of solution of the equation $\log_x^3 10 - 6\log_x^2 10 + 11\log_x 10 - 6 = 0$ is:

(A) 0 (B) 1 (C) 2 (D) 3

Match the Columns

Q.21

Column-I	Column-II
(A) The value of x for which the radical product $\sqrt{3\sqrt{x}-\sqrt{7x+\sqrt{4x-1}}}\sqrt{2x+\sqrt{4x-1}}\sqrt{3\sqrt{x}+\sqrt{7x+\sqrt{4x-1}}}$ is equal to 13, is not greater than	(p) 4
(B) Let P(x) = $x^7 - 3x^5 + x^3 - 7x^2 + 5$ and Q (x) = x – 2. The remainder of $\frac{P(x)}{Q(x)}$ is not smaller than	(q) 7
(C) Given a right triangle with side of length a, b and c and area equal to $a^2 + b^2 - c^2$. The ratio of the larger to the smaller leg of the triangle is	(r) 10
(D) If a, b and c ∈ N such $\left(\sqrt[3]{4}+\sqrt{2}-2\right)\left(a\sqrt[3]{4}+b\sqrt[3]{2}+c\right) = 20$ Then the value of (a + b – c), is not equal to	(s) 17

Q.22

Column I	Column II
(A) The expression $x = \log_2 \log_9 \sqrt{6+\sqrt{6+\sqrt{6+\ldots\ldots\ldots\infty}}}$ simplifies to	(p) An integer
(B) The number $N = 2^{(\log_2 3.\log_3 4.\log_4 5.....\log_{99} 100)}$ simplifies to	(q) A prime
(C) The expression $\frac{1}{\log_5 3}+\frac{1}{\log_6 3}-\frac{1}{\log_{10} 3}$ simplifies to	(r) A natural
(D) The number N = $\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$ simplifies to	(s) A composite

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.2 Q.3 Q.15
Q.25 Q.26

Exercise 2

Q.3 Q.5 Q.9
Q.10

JEE Advanced/Boards

Exercise 1

Q.6 Q.12 Q.14
Q.16 Q.23

Exercise 2

Q.4 Q.6 Q.11
Q.15 Q.17

Answer Key

JEE Main/Boards

Exercise 1

Q.1 (i) $\frac{5}{4}$ (ii) $\frac{4}{3}$ (iii) 2 (iv) 6 (v) $-\frac{1}{2}$ (vi) 1 (vii) $-\frac{1}{2}$

Q.3 (i) 31 (ii) 5

Q.4 (i) $1+\sqrt{3}$ (ii) 2 (iii) –5 (iv) 0 (v) $10^{\frac{\sqrt{3}+1}{2}}, 10^{\frac{1-\sqrt{3}}{2}}$ (vi) $\frac{1}{10^5}, 1000$

(vii) $3^{\sqrt{2}}, 3^{-\sqrt{2}}$

Q.5 $\frac{2^4}{5^{1/3}}$ **Q.6** 1 **Q.7** 10^{-4}, 10 **Q.8** 10^{-3}, 10, 10^2

Q.9 2, 16 **Q.10** 1/3, $(1/3)^4$ **Q.11** $2^{\log_a b, 3\log_a b}$ **Q.12** $3+\sqrt{6}$

Q.13 – 5 **Q.14.** 3, 1/9 **Q.15** $2^{\left(-1+\sqrt{\frac{17}{5}}\right)}, 2^{\left(-1-\sqrt{\frac{17}{5}}\right)}$ **Q.16** 1/16, 2

Q.17 $2^7, 2^{-1}$ **Q.18** 8, $-\frac{41}{5}$ **Q.19** 4, 1 **Q.20** 3

Q.21 48 **Q.22** 13 **Q.23** 1, 9 **Q.24** $\frac{1}{4}, \frac{1}{2}$

Q.25 98 **Q.26** 0 **Q.27** 2 **Q.28** 3

Q.29 $(10)^{-1/4}$, (10) **Q.30** (–1), 2

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** D **Q.3** C **Q.4** D **Q.5** B **Q.6** D

Q.7 B **Q.8** C **Q.9** A

JEE Advanced/Boards

Exercise 1

Q.1 12 **Q.2** (a) 1 (b) $\log_b N$ **Q.3** (a) 2 **Q.4** $(61)^2$

Q.5 1/6 **Q.6** 1 **Q.7** 6 **Q.8** $2s + 10s^2 - 3(s^3 + 1)$

Q.11 (a) 5 (b) 10 (c) $2^{\pm\sqrt{2}}$ (d) $2^{-\log_5 a}$ **Q.12** (a^4, a, a^7) or (a^{-4}, a^{-1}, a^{-7}) **Q.13** 23040

Q.15 507 **Q.16** (a) 140 (b) 12 (c) 47 **Q.17** -0.410 **Q.18** 1

Q.19 1 **Q.20** 4/9 **Q.21** 1 **Q.22** $\frac{5+3\sqrt{5}}{10}$

Q.23 5625 **Q.24** 2196 **Q.25** 93 **Q.26** $(2008)^2$

Q.27 $\sqrt{2}, \sqrt{6}$ **Q.28** $[0, 1] \cup \{4\}$ **Q.30** $[1/3, 3] - \{1\}$ **Q.31** 1

Exercise 2

Single Correct Choice Type

Q.1 C **Q.2** B **Q.3** B **Q.4** B **Q.5** D **Q.6** B

Q.7 C **Q.8** A **Q.9** D **Q.10** D **Q.11** B **Q.12** D

Q.13 A

Multiple Correct Choice Type

Q.14 C, D **Q.15** A, D **Q.16** A , B, C, D

Assertion Reasoning Type

Q.17 D **Q.18** B **Q.19** B

Comprehenstion Type

Q.20 D

Match the Columns

Q.21 $A \rightarrow q, r, s; B \rightarrow p, q, r, s; C \rightarrow p; D \rightarrow r$ **Q.22** $A \rightarrow p; B \rightarrow p, r, s; C \rightarrow p, r; D \rightarrow p, q, r$

Solutions

JEE Main/Boards

Exercise 1

Sol 1: (i) $\log_{16}32 = \log_{2^4} 2^5$

we know $\log_{x^n} y^m = \frac{m}{n}\log_x y$

$\Rightarrow \log_{2^4} 2^5 = \frac{5}{4}\log_2 2 = \frac{5}{4}$

(ii) $\log_8 16 = \log_{2^3} 2^4 = \frac{4}{3}\log_2 2 = \frac{4}{3}(1) = \frac{4}{3}$

(iii) $\log_{1/3}(1/9) = \log_{1/3}(1/3)^2 = 2\log_{1/3}(1/3) = 2.(1) = 2$

(iv) $\log_{2\sqrt{3}}(1728) = \log_{2\sqrt{3}}(2\sqrt{3})^6$

$= 6\log_{2\sqrt{3}} 2\sqrt{3} = 6(1) = 6$

(v) $\log_2 \cos 45° = \log_2 \frac{1}{\sqrt{2}} = \log_2 (2)^{-\frac{1}{2}} = -\frac{1}{2}\log_2 2 = -\frac{1}{2}$

(vi) $\log_2 (\log_2 4)$

$\Rightarrow \log_2\left(\log_2 2^2\right)$

$\Rightarrow \log_2\left(2\log_2 2\right)$

$\Rightarrow \log_2 2 \quad \because \log_a a = 1$

$\Rightarrow 1$

(vii) $\log_3 (\tan 30°)$

$\because \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \log_3\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \log_3\left(3^{\frac{-1}{2}}\right)$

$\Rightarrow \frac{-1}{2}\log_3 3$

$\Rightarrow \frac{-1}{2}$

Sol 2: (i) $\log_5\sqrt{5\sqrt{5\sqrt{5..........\infty}}}$

Let $5\sqrt{5\sqrt{5..........\infty}} = x$

$\Rightarrow 5\sqrt{x} = x \Rightarrow 5 = x^{1/2} \Rightarrow x = 25$

$\log_5\sqrt{x} = \log_5\sqrt{25} = \log_5 5 = 1$

(ii) $\log_{0.125} 8 = \log_{0.5}^{2} \Rightarrow \frac{3}{3}\log_{0.5}^{2}$

$\Rightarrow \log_{1/2}^{2} \Rightarrow \log_{(2)^{-1}}^{2} \Rightarrow -1.\log_2^2 = -1$

(iii) $\log(0.\bar{6})$

$\because 0.\bar{6} = 0.6666............ = \frac{2}{3}$

$\Rightarrow \log_{3/2}\left(\frac{3}{2}\right) = \log_{3/2}\left(\frac{3}{2}\right)^{-1} = -1$

(iv) $\log_{2.25}(0.\bar{4})$

$\Rightarrow 0.\bar{4} = 0.444....... = \frac{4}{9} \Rightarrow 2.25 = \frac{225}{100} = \frac{9}{4}$

$\Rightarrow \log_{9/4}\left(\frac{4}{9}\right) = \log_{9/4}\left(\frac{9}{4}\right)^{-1} = 1$

(v) $\log_{10}(0.\bar{9})$

$0.\bar{9} = 0.99999 = 1 \Rightarrow \log_{10} 1 = 0$

Sol 3: We have to find out no. of digits in

(i) $2^{100} = x$ (Assume)

$\Rightarrow \log_{10}x = \log_{10}2^{100} = 100\log_{10}2 = 100(0.3010) = 30.10$

$\Rightarrow x = 10^{30.103} = 10^{30}(10)^{0.10}$

Total no. of digit = 30 + 1 = 31

(ii) $x = 3^{10}$

$\Rightarrow \log_{10}x = \log_{10}3^{10} = 10\log_{10}3 = 10(0.47712) = 4.7712$

$\Rightarrow x = 10^{4.7712} = 10^4 \times 10^{0.7712}$

Total no. of digits = 4 + 1 = 5

Sol 4: (i) $\log_{x-1} 3 = 2$ ($x \neq 1, 2$)

$\frac{1}{2}\log_{x-1}3 = 1 \Rightarrow \log_{x-1}3^{1/2} = 1$

$3^{1/2} = x - 1 \Rightarrow x = 1 + \sqrt{3}$

(ii) $\log_3(3^x - 8) = 2 - x$

$\Rightarrow (3^x - 8) = (3)^{2-x} = 3^2 . 3^{-x} = 9.3^{-x} \Rightarrow 3^x - 9.3^{-x} = 8$

Assume $3^x = y$

$\Rightarrow y - \frac{9}{y} = 8 \Rightarrow y^2 - 9 = 8y \Rightarrow y^2 - 8y - 9 = 0$

$\Rightarrow y = \frac{8 \pm \sqrt{8^2 + 4(1)(9)}}{2(1)} \Rightarrow y = \frac{8 \pm \sqrt{64+36}}{2} = \frac{8 \pm \sqrt{100}}{2}$

$\Rightarrow y = 4 \pm 5 = 9, -1$

Terefore, $3^x = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2$

$3^x = -1 \Rightarrow$ no solution

Hence $x = 2$

(iii) $\log_{5-x}(x^2 - 2x + 65) = 2$

$\Rightarrow x^2 - 2x + 65 = (5 - x)^2 = x^2 + 5^2 - 2(5)x$

$\Rightarrow -2x + 65 = 25 - 10x \Rightarrow 10x - 2x = 25 - 65 = -40$

$\Rightarrow 8x = -40 \Rightarrow x = -\frac{40}{8} = -5$

(iv) $\log_3(x + 1) + \log_3(x + 3) = 1$

$\Rightarrow \log_3[(x + 1) . (x + 3)] = 1$

$\Rightarrow (x+1)(x+3) = 3 \Rightarrow x^2 + x + 3x + 3(1) = 3$

$\Rightarrow x^2 + 4x = 0 \Rightarrow x(x + 4) = 0 \Rightarrow x = 0, -4$

But at $x = -4$, equation is

$\log_3(-4 + 1) + \log_3(-4 + 3) = 1$

It can't be –ve so $x \neq -4 \Rightarrow x = 0$

(v) $x^{2 \log x} = 10\, x^2$

Take logarithms is both sides

$\log_{10}(x^{2 \log x}) = \log_{10} 10x^2$

$2 \log_{10}x\, (\log_{10}x) = \log_{10}10 + \log_{10}x^2$

$2\log_{10}x(\log_{10} x) = 1 + 2\log_{10} x$

Assume $\log_{10} x = y$ ... (i)

$\Rightarrow 2y(y) = 1 + 2y \Rightarrow 2y^2 = 1 + 2y \Rightarrow 2y^2 - 2y - 1 = 0$

$\Rightarrow y = \frac{2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$

$y = \frac{2 \pm \sqrt{4+8}}{4} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$

So from equation (i)

$\log_{10}x = \frac{1 \pm \sqrt{3}}{2} \Rightarrow x = 10^{\frac{(1+\sqrt{3})}{2}}$ and $10^{\frac{(1-\sqrt{3})}{2}}$

(vi) $x^{\frac{\log_{10} x+5}{3}} = 10^{5+\log_{10} x}$

take logarithm (base 10) on both side

$\log_{10}\left[x^{\frac{\log_{10} x+5}{3}}\right] = \log_{10} 10^{5+\log x}$

$\Rightarrow \left(\frac{\log_{10} x + 5}{3}\right)\log_{10} x = \left(5 + \log_{10} x\right)\log_{10} 10$

$\Rightarrow \left(\frac{5 + \log_{10} x}{3}\right)\log_{10} x = \left(5 + \log_{10} x\right)$

$\Rightarrow \log_{10}x = 1(3) = 3 \Rightarrow x = 10^3 = 1000$

2nd solution $\Rightarrow 5 + \log x = 0$

$\Rightarrow \log_{10}x = -5 \Rightarrow x = 10^{-5}$

(vii) $x^{\log_3 x} = 9$

Take logarithm (base 3) in both sides

$\log_3[x^{\log_3 x}] = \log_3 9 = \log_3 3^2 = 2\log_3 3$

$\Rightarrow (\log_3 x)^2 = 2 \Rightarrow |\log_3 x| = 2^{1/2} \Rightarrow \log_3 x = \pm \sqrt{2}$

$\Rightarrow x = 3^{\sqrt{2}}, 3^{-\sqrt{2}}$

Sol 5: $1 - \log_{10} 5 = \frac{1}{3}\left(\log_{10}\frac{1}{2} + \log_{10} x + \frac{1}{3}\log_{10} 5\right)$

$3\left(1 - \log_{10} 5\right) = \log_{10}\frac{1}{2} + \log_{10} x + \frac{1}{3}\log_{10} 5$

$3 - \log_{10} 5 = \log_{10}\frac{1}{2} + \log_{10} 5^{1/3} + \log_{10} x$

$\Rightarrow 3 = \log_{10} 5^3 + \log_{10}\frac{1}{2} + \log_{10} 5^{1/3} + \log_{10} x$

$\Rightarrow 3 = \log_{10}\left[5^3 \times \frac{1}{2} \times 5^{1/3}\right] + \log_{10} x$

$\Rightarrow \log_{10} x = 3 - \log_{10}\left[5^{3+\frac{1}{3}} \times \left(\frac{1}{2}\right)\right]$

$\log_{10} x = \log_{10} 10^3 - \log_{10}\left(5^{10/3} \times 2^{-1}\right)$

$$= \log_{10}\left(\frac{10^3}{5^{10/3}2^{-1}}\right) = \log_{10}\frac{5^3\times 2^3}{5^{10/3}\times 2^{-1}}$$

$$= \log_{10}[5^{\frac{9-10}{3}}\,2^{3+1}] = \log_{10}[5^{-1/3}\,2^4]$$

$$\log_{10} x = \log_{10}\frac{2^4}{5^{1/3}} \Rightarrow x = \frac{2^4}{5^{1/3}}$$

Sol 6: $\log_{10} x - \frac{1}{2}\log_{10}\left(x-\frac{1}{2}\right) = \log_{10}\left(x+\frac{1}{2}\right)$

$$-\frac{1}{2}\log_{10}\left(x+\frac{1}{8}\right)$$

$$2\log_{10} x - \log_{10}\left(x-\frac{1}{2}\right) = 2\log_{10}\left(x+\frac{1}{2}\right) - \log_{10}\left(x+\frac{1}{8}\right)$$

$$\log_{10} x^2 - \log_{10}\left(x-\frac{1}{2}\right) = \log_{10}\left(x+\frac{1}{2}\right)^2 - \log_{10}\left(x+\frac{1}{8}\right)$$

$$\Rightarrow \log_{10}\left(\frac{x^2}{x-\frac{1}{2}}\right) = \log\left[\frac{\left(x+\frac{1}{2}\right)^2}{\left(x+\frac{1}{8}\right)}\right]$$

$$\Rightarrow \log_{10}\left(\frac{x^2}{x-\frac{1}{2}}\right) - \log_{10}\left[\frac{\left(x+\frac{1}{2}\right)^2}{\left(x+\frac{1}{8}\right)}\right] = 0$$

$$\Rightarrow \log_{10}\left[\frac{x^2}{\left(x-\frac{1}{2}\right)}\times\frac{x+\frac{1}{8}}{\left(x+\frac{1}{2}\right)^2}\right] = 0$$

$$\Rightarrow \left(\frac{x^2}{x-\frac{1}{2}}\right)\left(\frac{x+\frac{1}{8}}{\left(x+\frac{1}{2}\right)^2}\right) = 1 \Rightarrow \frac{x^2\left(x+\frac{1}{8}\right)}{\left(x^2-\frac{1}{4}\right)\left(x+\frac{1}{2}\right)} = 1$$

$$\Rightarrow x^2\left(x+\frac{1}{8}\right) = \left(x^2-\frac{1}{4}\right)\left(x+\frac{1}{2}\right)$$

$$\Rightarrow x^3 + \frac{x}{8} = x^3 + \frac{x^2}{2} - \frac{x}{4} - \frac{1}{4}\left(\frac{1}{2}\right)$$

$$\Rightarrow x^3 + \frac{x^2}{8} = x^3 + \frac{x^2}{2} - \frac{x}{4} - \frac{1}{8}$$

$$\Rightarrow x^2 = 4x^2 - 2x - 1 \Rightarrow 3x^2 - 2x - 1 = 0$$

$$\Rightarrow x = \frac{2\pm\sqrt{4+12}}{6} \Rightarrow x = \frac{2\pm 4}{6}$$

$$x = 1, -\frac{1}{3} \quad \text{at} \quad x = 1, -\frac{1}{3}$$

$$2\log(-2) = \log(4)$$

Which is not possible $\Rightarrow x = 1$

Sol 7: $x^{\frac{\log_{10} x+7}{4}} = 10^{\log_{10} x+1}$

Take logarithm on both sides

$$\log_{10}\left(x^{\frac{\log_{10} x+7}{4}}\right) = \log_{10}\left(10^{\log_{10} x+1}\right)$$

$$\Rightarrow \left(\frac{\log_{10} x+7}{4}\right)(\log_{10} x) = (\log_{10} x+1)\log_{10} 10$$

$\Rightarrow$ Assume $\log_{10} x = y \Rightarrow \left(\frac{y+7}{4}\right)(y) = y + 1$

$\Rightarrow y^2 + 7y = 4(y + 1) = 4y + 4$

$\Rightarrow y^2 + 7y - 4y - 4 = 0 \Rightarrow y^2 + 3y - 4 = 0$

$\Rightarrow (y + 4)(y - 1) = 0 \Rightarrow y = -4$ and $+1$

$\Rightarrow \log_{10} x = -4$ or 1

Hence $x = 10^{-4}$ or 10

Sol 8: $\left(\frac{\log_{10} x}{2}\right)^{\log_{10}^2 x+\log_{10} x^2-2} = \log_{10}\sqrt{x}$

$$\Rightarrow \left(\log_{10} x^{1/2}\right)^{\log_{10}^2 x+\log_{10} x^2-2} = \log_{10} x^{1/2}$$

$\Rightarrow \log_{10}^2 x + \log_{10} x^2 - 2 = 1$ or $\log_{10} x^{1/2} = 1$

$\Rightarrow \log_{10}^2 x + 2\log_{10} x - 2 = 1;\quad \log_{10} x = 2 \Rightarrow x = 10^2$

$\Rightarrow \log^2 x + 2\log x - 2 = 0$

Assume that $\log x = y$

$\Rightarrow y^2 + 2y - 2 = 1 \Rightarrow y^2 + 2y - 2 - 1 = 0$

$\Rightarrow (y + 3)(y - 1) = 0$

$y = -3$ or $y = 1$

$\log_{10} x = -3$ or $\log_{10} x = 1$

$\Rightarrow x = 10^{-3}$ or $10^1 \Rightarrow x = 10^{-3}, 10, 10^2$

Sol 9: $3\sqrt{\log_2 x} - \log_2 8x + 1 = 0$

$\Rightarrow 3\sqrt{\log_2 x} = \log_2 2^3 x - 1 \Rightarrow 3\sqrt{\log_2 x} = 2 + \log_2 x$

Assume that $\log_2 x = y$

$\Rightarrow 3\sqrt{y} = 2 + y$

Square on both sides

$\Rightarrow (3\sqrt{y})^2 = (2 + y)^2 \Rightarrow 9y = 2^2 + y^2 + 2(2)(y)$

$\Rightarrow 9y = 4 + y^2 + 4y \Rightarrow y^2 - 5y + 4 = 0$

$\Rightarrow (y - 4)(y - 1) = 0 \Rightarrow y = 4$ or $y = 1$

$\Rightarrow \log_2 x = 4$ or $\log_2 x = 1$

$\Rightarrow x = 2^4$ or $x = 2^1 \Rightarrow x = 16$ or 2

Sol 10: $\log_{1/3} x - 3\sqrt{\log_{1/3} x} + 2 = 0$

$\log_{1/3} x + 2 = 3\sqrt{\log_{1/3} x}$

Assume that $\log_{1/3} x = y$ .... (i)

$\Rightarrow y + 2 = 3\sqrt{y} \Rightarrow y = 4$ or $y = 1$ [Refer above solution]

$\log_{1/3} x = 4$ or $\log_{1/3} x = 1$

$\Rightarrow x = \left(\frac{1}{3}\right)^4$ or $x = \left(\frac{1}{3}\right)^1 \Rightarrow x = \frac{1}{81}$ or $\frac{1}{3}$

Sol 11: $\left(a^{\log_b x}\right)^2 - 5x^{\log_b x} + 6 = 0$

Assume that $x = b^y$

$\Rightarrow (a^y)^2 - 5(a^{\log_b x}) + 6 = 0 \Rightarrow a^{2y} - 5a^y + 6 = 0$

$\Rightarrow (a^y - 3)(a^y - 2) = 0 \Rightarrow a^y = 2, 3$

$y = \log_a 2, \log_a 3 \quad \therefore x = 2^{\log_a b}, 3^{\log_a b}$

Sol 12: $\log_4(x^2 - 1) - \log_4(x - 1)^2 = \log_4(\sqrt{(4-x)^2})$

$\log_4\left(\frac{x^2-1}{(x-1)^2}\right) = \log_4(\sqrt{(4-x)^2})$

$\frac{x^2-1}{(x-1)^2} = \sqrt{(4-x)^2}$

$\Rightarrow \frac{(x-1)(x+1)}{(x-1)^2} = \sqrt{(4-x)^2} \quad ; \quad x \neq 1,$

$\Rightarrow \frac{x+1}{(x-1)} = \sqrt{(4-x)^2} \quad \Rightarrow \frac{x+1}{(x-1)} = |4 - x|$

Case-I: When $4 - x \geq 0$

$\Rightarrow \frac{x+1}{x-1} = 4 - x \Rightarrow (x + 1) = (4 - x)(x - 1)$

$\Rightarrow x + 1 = 4x - 4 - x^2 + x \Rightarrow x^2 - 4x - x + x + 1 + 4 = 0$

$\Rightarrow x^2 - 4x + 5 = 0 \Rightarrow x = \frac{4 \pm \sqrt{4^2 - 4(5)(1)}}{2(1)}$

$\Rightarrow x = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm \sqrt{-4}}{2}$ (no solution)

Case-II: When $4 - x < 0$

$\Rightarrow \frac{x+1}{x-1} = x - 4 \Rightarrow x + 1 = (x - 1)(x - 4) = x^2 + 4 - x - 4x$

$\Rightarrow x^2 - 4x - x - x + 4 - 1 = 0 \Rightarrow x^2 - 6x + 3 = 0$

$\Rightarrow x = \frac{6 \pm \sqrt{6^2 - 4(3)(1)}}{2(1)} \Rightarrow x = \frac{6 \pm \sqrt{36-12}}{2}$

$= \frac{6 \pm \sqrt{24}}{2} = 3 \pm \sqrt{6}$

But $x > 4$

So, $x = 3 + \sqrt{6}$

Sol 13: $2\log_3 \frac{x-3}{x-7} + 1 = \log_3 \frac{x-3}{x-1}$

$\log_3\left(\frac{x-3}{x-7}\right)^2 + \log_3 3 = \log_3 \frac{x-3}{x-1}$

$\log_3\left[\frac{(x-3)^2}{(x-7)^2} \times 3\right] = \log_3 \frac{x-3}{x-1}$

$\Rightarrow \frac{3(x-3)^2}{(x-7)^2} = \frac{(x-3)}{(x-1)} \Rightarrow 3(x - 3)(x - 1) = (x - 7)^2$

$\Rightarrow 3x^2 + 9 - 3x - 9x = x^2 - 14x + 49$

$\Rightarrow 2x^2 + 2x - 40 = 0 \Rightarrow x^2 + x - 20 = 0$

$\Rightarrow (x + 5)(x - 4) = 0$

$x = -5$ or $x = 4$

At $x = 4$, equation is $2\log_3\left(\frac{4-3}{4-7}\right) + 1 = \log \frac{4-3}{4-7}$

$\frac{4-3}{4-7} = \frac{+1}{-3} \Rightarrow$ –ve which is not possible

Hence $x \neq 4$, $x = -5$

Sol 14: $\log_x (9x^2)\log_3^2 x = 4 \Rightarrow (\log_x 3^2 x^2)(\log_3 x)^2 = 4$

$$\Rightarrow 2[\log_x 3x]\left[\frac{\log_e x}{\log_e 3}\right]^2 = 4$$

We know that $\log_m n = \dfrac{\log_e n}{\log_e m}$

$$\Rightarrow 2[\log_x 3 + \log_x x]\left[\frac{\log_e x}{\log_e 3}\right]^2 = 4$$

$$\Rightarrow \left[\frac{\log_e 3}{\log_e x}+1\right]\left[\frac{\log_e x}{\log_e 3}\right]^2 = 2$$

$$\Rightarrow \frac{\log_e 3}{\log_e x} \times \frac{(\log_e x)^2}{(\log_e 3)^2} + (\log_3 x)^2 = 2$$

$\Rightarrow \log_3 x + (\log_3 x)^2 = 2$

Assume that $\log_3 x = y$

$\Rightarrow y^2 + y = 2 \Rightarrow y^2 + y - 2 = 0 \Rightarrow (y + 2)(y - 1) = 0$

$\Rightarrow y = -2$ or $y = 1$

Now, we have $\log_2 x = -2$ or $\log_3 x = 1$

$\Rightarrow x = 3^{-2}$ or $x = 3^{+1}$

Hence, $x = \dfrac{1}{9}$ or $x = 3$

Sol 15: $\log_{0.5x} x^2 + 14 \log_{16x} x^2 + 40 \log_{4x} \sqrt{x} = 0$

$$\frac{\log_2 x^2}{\log_2(0.5x)} + \frac{14\log_2 x^2}{\log_2(16x)} + \frac{40\log_2 \sqrt{x}}{\log_2(4x)} = 0$$

Assume that $\log_2 x = y$

$$\Rightarrow \frac{2y}{\log_2 .2^{-1} + y} + \frac{28y}{\log_2 2^4 + y} + \frac{20y}{\log_2 2^2 + y} = 0$$

$$\Rightarrow \frac{y}{y-1} + \frac{14y}{y+4} + \frac{10y}{y+2} = 0$$

$$y = 0 \text{ or } \left(\frac{1}{y-1} + \frac{14}{y+4} + \frac{10}{y+2}\right) = 0$$

$\Rightarrow \log_2 x = y \Rightarrow x = 2^y = 2^\circ = 1$

or $(y + 4)(y + 2) + 14(y - 1)(y + 2) + 10(y - 1)(y + 4) = 0$

$\Rightarrow y^2 + 8 + 6y + 14y^2 - 28 + 14y + 10y^2 - 40 + 30y = 0$

$\Rightarrow 25y^2 + 50y - 60 = 0$

$\Rightarrow y^2 + 2y - \dfrac{60}{25} = 0 \Rightarrow y^2 + 2y - \dfrac{12}{5} = 0$

$$\Rightarrow y = -\frac{2-\sqrt{(2)^2 - 41\left(-\frac{12}{5}\right)}}{2(1)} \quad \Rightarrow y = \frac{2-\sqrt{2\left(1+\frac{12}{5}\right)}}{2}$$

$$\Rightarrow y = \frac{2-2\sqrt{\frac{5+12}{5}}}{2} \quad \Rightarrow y = -1 \pm \sqrt{\frac{17}{5}}$$

Now, we have $\log_2 x = y$

$$\Rightarrow x = 2^{\left(-1+\sqrt{17/5}\right)} \text{ or } 2^{\left(-1-\sqrt{17/5}\right)}$$

Sol 16: $\log_3[\log_{1/2}^2 x - 3\log_{1/2} x + 5] = 2$

Assume that $\log_{1/2} x = y$

$\Rightarrow \log_3[y^2 - 3y + 5] = 2 \Rightarrow y^2 - 3y + 5 = 9$

$\Rightarrow y^2 - 3y - 4 = 0 \Rightarrow (y - 4)(y + 1) = 0$

$\Rightarrow y = 4$ or $y = -1 \Rightarrow \log_{1/2} x = 4$ or $\log_{1/2} x = -1$

$$\Rightarrow x = \left(\frac{1}{2}\right)^4 \text{ or } x = \left(\frac{1}{2}\right)^{-1}$$

$\Rightarrow x = \dfrac{1}{16}$ or $x = 2.$

Sol 17: $\log_2(x/4) = \dfrac{15}{\log_2 \frac{x}{8} - 1}$

$$\Rightarrow \log_2 x - \log_2 4 = \frac{15}{\log_2 x - \log_2 8 - 1}$$

$\Rightarrow$ Assume that $\log_2 x = y$

$$y - 2 = \frac{15}{y-3-1} = \frac{15}{y-4}$$

$\Rightarrow (y - 2)(y - 4) = 15 \Rightarrow y^2 - 6y + 8 = 15$

$\Rightarrow y^2 - 6y - 7 = 0 \Rightarrow (y - 7)(y + 1) = 0$

$y = 7$ or $y = -1$

Now, we have $\log_2 x = 7$ or $\log_2 x = -1$

Hence $x = 2^7$ or $x = 2^{-1}$

Sol 18: $\dfrac{1}{2}\log_{10}(5x - 4) + \log_{10}\sqrt{x+1} = 2 + \log_{10} 0.18$

$\Rightarrow \log_{10}(5x - 4) + 2\log_{10}\sqrt{x+1} = 2[2 + \log_{10} 0.18]$

$\Rightarrow \log_{10}(5x - 4) + \log_{10}(x + 1) = 4 + 2\log_{10} 0.18$

$\Rightarrow \log_{10}[(5x - 4)(x + 1)] = 4 + \log_{10}(0.18)^2$

$\Rightarrow \log_{10}[(5x-4)(x+1)] = \log_{10}[10^4 \times (0.18)^2]$

$\Rightarrow (5x-4)(x+1) = 10^4(0.0324) = 324$

$\Rightarrow 5x^2 + x - 4 = 324 \Rightarrow 5x^2 + x - 328 = 0$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(5)(-328)}}{(10)}$$

$$x = -\frac{-1 \pm \sqrt{1+20(328)}}{10} = \frac{-1 \pm \sqrt{6561}}{10}$$

$$x = \frac{-1 \pm 81}{10} = 8, -\frac{41}{5},$$

Also it is clear that $x > 4/5$

$\therefore x = \frac{-41}{5}$ is rejected

Sol 19: $\log_{10}x^2 = \log_{10}(5x-4)$

$\Rightarrow x^2 = 5x - 4 \Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow (x-4)(x-1) = 0 \Rightarrow x - 4 = 0$or $x - 1 = 0$

Hence, $x = 4, 1$

Sol 20: $\frac{1}{6}\log_2(x-2) - \frac{1}{3} = \log_{1/8}\sqrt{3x-5}$

$$\Rightarrow \frac{1}{6}\log_2(x-2) - \frac{1}{3} = \log_{2^{-3}}\sqrt{3x-5}$$

$$\Rightarrow \frac{1}{6}\log_2(x-2) - \frac{1}{3} = -\frac{1}{3}\log_2\sqrt{3x-5}$$

$$\Rightarrow \frac{1}{2}\log_2(x-2) - 1 = -\log_2\sqrt{3x-5}$$

$\Rightarrow \log_2(x-2) + 2\log_2\sqrt{3x-5} = 2$

$\Rightarrow \log_2(x-2) + \log_2(3x-5) = \log_2 2^2$

$\Rightarrow (x-2)(3x-5) = 4 \Rightarrow 3x^2 + 10 - 6x - 5x = 4$

$\Rightarrow 3x^2 - 11x + 6 = 0$

$$x = \frac{11 \pm \sqrt{121 - 4(3)(6)}}{2(3)} = \frac{11 \pm \sqrt{121-72}}{6}$$

$$\Rightarrow x = \frac{11 \pm \sqrt{49}}{6} = 3, \frac{2}{3}$$

At $x = \frac{2}{3}$, eq. $\Rightarrow \frac{1}{6}\log_2\left(\frac{1}{3} - 2\right) - \frac{1}{3}$

$= \log_{1/8}\sqrt{3\frac{(2)}{3} - 5} = \sqrt{-3}$ (Not a possible solution)

So $x = 3$

Sol 21: $\frac{\log_{10}(\sqrt{x+1}+1)}{\log_{10}(x-40)^{1/3}} = 3 \Rightarrow \frac{\log_{10}(\sqrt{x+1}+1)}{\frac{1}{3}\log_{10}(x-40)} = 3$

$\log_{10}(\sqrt{x+1} + 1) = \log_{10}(x-40)$

$\sqrt{x+1} + 1 = x - 40 \Rightarrow \sqrt{x+1} = x - 41$

On squaring both sides

$x + 1 = (x-41)^2 = x^2 + 41^2 - 2(41)x$

$\Rightarrow x^2 - 82x - x + 41^2 - 1 = 0 \Rightarrow x^2 - 83x + 1680 = 0$

$$x = 83 \pm \frac{\sqrt{(83)^2 - 4(1680)(1)}}{2(1)} = \frac{83 \pm \sqrt{169}}{2} = \frac{83 \pm 13}{2} = 48, 35$$

Now, for $x = 35$

The given equations yields $\frac{\log_{10}\sqrt{35+1}+1}{\log_{10}3\sqrt{35-40}} = 3\sqrt{-5}$

Wich is not a possible solutions

Hence $x \neq 35$ and $x = 48$

Sol 22: $1 - \frac{1}{2}\log_{10}(2x-1) = \frac{1}{2}\log_{10}(x-9)$

$\Rightarrow 2 - \log_{10}(2x-1) = \log_{10}(x-9)$

$\Rightarrow \log_{10}(x-9) + \log_{10}(2x-1) = 2$

$\Rightarrow \log_{10}(x-9)(2x-1) = \log_{10}10^2$

$\Rightarrow (x-9)(2x-1) = 100$

$\Rightarrow 2x^2 - 18x - x + 9 = 100 \Rightarrow 2x^2 - 19x - 91 = 0$

$$x = \frac{19 \pm \sqrt{19^2 - 4(2)(-91)}}{2(2)} = \frac{19 \pm \sqrt{1089}}{4} = 13, -\frac{7}{2}$$

But $x = -\frac{7}{2}$ is not in the domain hence $x = 13$

Sol 23: $\log_{10}(3x^2 + 7) - \log_{10}(3x-2) = 1$

$$\log_{10}\left(\frac{3x^2+7}{3x-2}\right) = 1 = \log_{10}10$$

$\frac{3x^2+7}{3x-2} = 10$ and $3x^2 + 7 = 10(3x-2)$

$\Rightarrow 3x^2 + 7 = 30x - 20 \Rightarrow 3x^2 - 30x + 27 = 0$

$\Rightarrow x^2 - 10x + 9 = 0 \Rightarrow (x-1)(x-9) = 0$

Hence $x = 9, 1$

Sol 24: $\left(1+\frac{1}{2x}\right)\log_{10}3 + \log_{10}2 = \log_{10}(27 - 3^{1/x})$

$\Rightarrow \log_{10} 3^{(1+1/2x)} + \log_{10}2 = \log_{10}(27 - 3^{1/x})$

$\Rightarrow \log_{10}2\times(3)^{1+1/2x} = \log_{10}(27-3^{1/x})$

$\Rightarrow 2 \times 3^{1+1/2x} = 27 - 3^{1/x}$

Assume that $3^{1/x} = y$

$\Rightarrow 2\times3\times\sqrt{y} = 27 - y$

On squaring both sides, we get

$\Rightarrow 2^2 \times 3^2 \times y = (27 - y)^2 \Rightarrow 36y = 27^2 + y^2 - 2(27)y$

$\Rightarrow y^2 - 54y - 36y + 27^2 = 0 \Rightarrow y^2 - 90y + 27^2 = 0$

$(y - 81)(y - 9) = 0 \Rightarrow y = 81, 9$

$\therefore\ x = \frac{1}{\log_3 y} \Rightarrow x = \frac{1}{\log_3 81}$ or $\frac{1}{\log_3 9} = \frac{1}{4}, \frac{1}{2}$

Clearly $3^{1/x} < 27\ \therefore x > \frac{1}{3}$

So x = 1/4 is not valid

Sol 25: $\frac{1}{2}\log_{10} x + 3\log_{10}\sqrt{2+x} = \log_{10}\sqrt{x(x+2)} + 1$

$\log_{10} x + 6\log_{10}\sqrt{2+x} = 2\log_{10}\sqrt{x(x+2)} + 2$

$\Rightarrow \log_{10} x + \log_{10}(2 + x)^3 - \log_{10}[x(x + 2)] = 2$

$\Rightarrow \log_{10}\left[\frac{x(2+x)^3}{x(x+2)}\right] = \log_{10}{}^{2}$

$\Rightarrow (2 + x)^2 = 100 \Rightarrow 2 + x = \pm 100$

$x\left[\begin{array}{l} 100-2=98 \\ -100-2=-102\end{array}\right.$

Here, x = – 102 does not satisfy the equation

Hence x = 98

Sol 26: $\log_2(4^x + 1) = x + \log_2(2^{x+3} - 6)$

$\log_2(4^x + 1) = \log_2 2^x + \log_2(2^{x+3} - 6)$

$\Rightarrow \log_2(4^x + 1) = \log_2[2^x[2^{x+3} - 6]]$

$\Rightarrow 4^x + 1 = 2^x[2^x 2^3 - 6]$

Assume that $2^x = y$

$\Rightarrow y^2 + 1 = y(8y - 6) \Rightarrow y^2 + 1 = 8y^2 - 6y$

$\Rightarrow 7y^2 - 6y - 1 = 0 \Rightarrow (y - 1)(7y + 1) = 0$

$y = 1$or $y = -\frac{1}{7}$

$2^x = 1$ or $2^x = -\frac{1}{7}$ (not valid)

$\Rightarrow x = 0$ and so, x = 0 is only solution.

Sol 27: $\log_{\sqrt{5}}(4^x - 6) - \log_{\sqrt{5}}(2^x - 2) = 2$

$\log_{\sqrt{5}}\left(\frac{4^x-6}{2^x-2}\right) = 2 \Rightarrow \frac{4^x-6}{2^x-2} = 5$

Assume that $2^x = y$

$\Rightarrow \frac{y^2-6}{y-2} = 5 \Rightarrow y^2 - 6 = 5(y - 2) = 5y - 10$

$\Rightarrow y^2 - 5y - 6 + 10 = y^2 - 5y + 4 = 0$

$\Rightarrow (y - 4)(y - 1) = 0$

$y = 4$ or $y = 1$

$\Rightarrow 2^x = 4$ or $2^x = 1$

$\Rightarrow x = 2$ or $x = 0$

x = 0 does not satisfy the equation, hence x = 2

Sol 28: $\log_{10}(3^x - 2^{4-x}) = 2 + \frac{1}{4}\log_{10}16 - \frac{x\log_{10}4}{2}$

$\Rightarrow \log_{10}(3^x-2^{4-x})=\log_{10}10^2+\frac{1}{4}\log_{10}2^4-\frac{x\log 2^2}{2}$

$\Rightarrow \log_{10}(3^x-2^{4-x})=\log_{10}100+\frac{4}{4}\log_{10}2-\frac{x\times2\log_{10}2}{2}$

$\Rightarrow \log_{10}(3^x - 2^{4-x}) = \log_{10}[100 \times 2] - \log 2^x$

$\Rightarrow \log_{10}(3^x - 2^{4-x}) = \log_{10}\frac{(200)}{2^x}$

$\Rightarrow 3^x - \frac{2^4}{2^x} = \frac{200}{2^x} \Rightarrow 3^x.2^x - 2^4 = 200$

$\Rightarrow 6^x = 200 + 2^4 \Rightarrow 216 = 6^3 \Rightarrow x = 3$

Sol 29: $\log_{10}(\log_{10} x) + \log_{10}(\log_{10} x^4 - 3) = 0$

$\log_{10}[(\log_{10} x)(\log_{10} x^4 - 3)] = 0$

$\Rightarrow (\log_{10} x)(\log_{10}x^4 - 3) = 1$

$(\log_{10} x)(4\log_{10} x - 3) = 1$

Assume that $\log_{10} x = y$

$\Rightarrow y(4y - 3) = 1;\ 4y^2 - 3y = 1$

$\Rightarrow 4y^2 - 3y - 1 = 0 \Rightarrow (y - 1)(4y + 1) = 0$

$\Rightarrow y = 1$ or $y = -\frac{1}{4}$

$\log_{10} x = 1$ or $\log_{10}x = -\frac{1}{4}$

$\Rightarrow x = 10$ or $x = 10^{-\frac{1}{4}}$

for $x \pm 10^{-\frac{1}{4}}$ given log function is not defined.

Hence, x = 10

Sol 30: $\log_3(9^x + 9) = \log_3 3^x(28 - 2.3^x)$

$\Rightarrow 9^x + 9 = 3^x(28 - 2 . 3^x)$

Assume that $3^x = y$

So $9^x = (3^2)^x = (3^x)^2 = y^2$

$\Rightarrow y^2 + 9 = y(28 - 2y) \Rightarrow y^2 + 9 = 28y - 2y^2$

$\Rightarrow 3y^2 - 28y + 9 = 0 \Rightarrow (3y - 1)(y - 9) = 0$

This gives $y = 9, \frac{1}{3}$

Hence, x = 2, –1

Exercise 2

Single Correct Choice Type

Sol 1: (B) $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ac}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$

$= \frac{\log_{10}\sqrt{bc}}{\log_{10} abc} + \frac{\log_{10}\sqrt{ac}}{\log_{10} abc} + \frac{\log_{10}\sqrt{ab}}{\log_{10} abc}$

$= \frac{\log_{10}\sqrt{bc} + \log_{10}\sqrt{ac} + \log_{10}\sqrt{ab}}{\log_{10} abc}$

$= \frac{\log_{10}\sqrt{bc}\sqrt{ac}\sqrt{ab}}{\log_{10} abc} = \frac{\log_{10} abc}{\log_{10} abc} = 1$

Sol 2: (D) $\log_2(2x^2) + \log_2 x.\ x^{\log_x(\log_2 x+1)}$

$+ \frac{1}{2}\log_4 2x^4 + 2^{-3\log_{1/2}(\log_2 x)} = 1$

$\Rightarrow \log_2(2x^2) + (\log_2 x)\ (x)^{\log_x(\log_2 x+1)}$

$+ \frac{1}{2}\log_4 4^{1/2}x^4 + 2^{-3\log_{1/2}(\log_2 x)} = 1$

$\Rightarrow 1 + 2\log_2 x + (\log_2 x)\,(x)^{\log_x(\log_2 x+1)}$

$+ \frac{1}{4}\log_4 4 + \frac{4}{2}\log_4 x + 2^{3\log_2(\log_2 x)} = 1$

$\Rightarrow 1 + 2\log_2 x + (\log_2 x + 1)(\log_2 x) + \frac{1}{4}$

$+ \log_2 x + (2)^{\log_2(\log_2 x)^3} = 1$

$\Rightarrow 1 + 2\log_2 x + (\log_2 x)(\log_2 x + 1) + \frac{1}{4}$

$+ \log_2 x + (\log_2 x)^3 = 1$

Assume $\log_2 x = y$

$\Rightarrow 2y + y(y + 1) + \frac{1}{4} + y + y^3 = 0$

$\Rightarrow y^3 + 4y + y^2 + \frac{1}{4} = 0$

Differential of equation is $\frac{d}{dy}[y^3 + 4y + y^2 + \frac{1}{4}] = 0$

$\Rightarrow 3y^2 + 4 + 2y = 0 \Rightarrow y = -\frac{-2 \pm \sqrt{2^2 - 4(4)(3)}}{2(3)}$

$y = \frac{-2 \pm \sqrt{-48 + 4}}{6}$

No solution so there is no minima and maximum

At $y = 0 \Rightarrow f(y) = 0 + 0 + 0 + \frac{1}{4} > 0$

$y = -1$, $f(y) = (-1)^3 + 4(-1) + (-1)^2 + \frac{1}{4}$

$\Rightarrow -1 - 4 + 1 + \frac{1}{4} = -4 + \frac{1}{4} = -\frac{15}{4} < 0$

It mean f(y) is zero some where $-1 < y < 0$

So $\log_2 x < 0$

But in equation (original) $\log_2 x$ should be positive so there is no solution

Sol 3: (C) $x = (75)^{-10}$

$\log_{10}x = \log_{10}(75)^{-10} = -10\log_{10}75 = -10\log_{10}\left(100 \times \frac{3}{4}\right)$

$= -10[\log_{10}10^2 + \log_{10}3 - \log_{10}2^2]$

$= -10[2 + 0.477 - 2(0.301)] = -18\ 75$

$\Rightarrow x = 10^{-18.75} = 10^{-19} \times 10^{-0.25}$

Number of zeros = 18

Sol 4: (D) $5x^{\log_2 3} + 3^{\log_2 x} = 162$

Assume $x = 2^y \Rightarrow 5.2^{y\log_2 3} + 3^{\log_2 2^y} = 162$

$\Rightarrow 5.2^{\log_2 3^y} + 3^{y\log_2 2} = 162 \Rightarrow 5.3^y + 3^y = 6.3^y = 162$

$3^y = \frac{162}{6} = 27 = 3^3$

$y = 3;\ x = 2^y = 2^3 = 8$

$\log_4 x = \log_4 8 = \log_4 (4)^{3/2} = \frac{3}{2}$

Sol 5: (B) $(x)^{\log_{10}^2 x + \log_{10} x^3 + 3}$

$= \dfrac{2}{\dfrac{1}{\sqrt{x+1}-1} - \dfrac{1}{\sqrt{x+1}+1}} = B$ (Assume)

$B = \dfrac{2}{\dfrac{1}{\sqrt{x+1}-1} - \dfrac{1}{\sqrt{x+1}+1}} = \dfrac{2}{\dfrac{\sqrt{x+1}+1-\sqrt{x+1}+1}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}}$

$B = ((\sqrt{x+1})^2 - (1)^2 = x + 1 - 1 = x$

So $(x)^{\log_{10}^2 x + 3\log_{10} x + 3} = x \Rightarrow x = 1$

Or $\Rightarrow$ Assume $\log_{10} x = y$

$\Rightarrow y^2 + 3y + 3 = 1 \Rightarrow y^2 + 3y + 2 = 0$

$\Rightarrow (y + 2)(y + 1) = 0$

$y = -2$ or $y = -1$

$\log_{10} x = -2$ or $\log_{10} x = -1$

$x = 10^{-2}, 10^{-1}$

$x_1, x_2, x_3 = 1, 10^{-1}, 10^{-2}$

$x_1 . x_3 = 1.10^{-2} = (10^{-1})^2 = (x_2)^2$

Sol 6: (D) $x = 2^{\log 3}$, $y = 3^{\log 2}$

$x = 2^{\log 3} = 3^{\log 2} = y$

As $a^{\log_n m} = m^{\log_n a}$

Sol 7: (B) $|x-3|^{3x^2-10x+3} = 1$; $x \neq 3$

Or if $|x - 3| = 1$

$\Rightarrow x = 2$ or 4 is solution

If $x - 3 \neq 0$ then $3x^2 - 10x + 3 = 0$ is another sol^n

$3x^2 - 10x + 3 = 0 \Rightarrow (3x\ \ 1)(x\ \ 3) = 0$

$x = +3$ or $= \frac{+1}{3}$

But $x \neq 3$; so, $x = \frac{1}{3}$

total solution $\Rightarrow x = \frac{1}{3}, 2, 4$

Sol 8: (C) x_1 and x_2 are roots of the equation

$\sqrt{2010}\ x^{\log_{2010} x} = x^2$

Assume that $x = (2010)^y$

$\Rightarrow (2010)^{1/2}\ (2010)^{y \log_{2010} (2010)^y} = (2010)^{2y}$

$\Rightarrow (2010)^{1/2}\ (2010)^{y^2} = (2010)^{2y}$

$\Rightarrow y^2 + \frac{1}{2} = 2y \Rightarrow y^2 - 2y + \frac{1}{2} = 0$

$\Rightarrow y = \dfrac{2 \pm \sqrt{2^2 - 4(1)(1/2)}}{2} = \dfrac{2 \pm \sqrt{2}}{2} = 1 \pm \dfrac{1}{\sqrt{2}}$

$x_1 x_2 = (2010)^{1-\frac{1}{\sqrt{2}}}\ (2010)^{1+\frac{1}{\sqrt{2}}} = (2010)^2 = (201 \times 10)^2$

No. of zeros in $x_1 x_2 = 2$

Sol 9: (A) Given that $x = 2$ or $x = 3$ satisfy the equation

$\log_4 (x^2 + bx + c) = 1 = \log_4 4$

$\Rightarrow x^2 + bx + c - 4 = 0$

$\Rightarrow b = 2 + 3 = 5$ and $c - 4 = 2 . 3 \Rightarrow c = 10$

$bc = 10(-5) = -50$

$|bc| = 50$

JEE Advanced/Boards

Exercise 1

Sol 1: $B = (2^{\log_6 18}).(3^{\log_6 3})$

$\Rightarrow B = 2^{\log_6 (6\times3)}.\ 3^{\log_6 3} \Rightarrow B = 2^{\log_6 6 + \log_6 3}.\ 3^{\log_6 3}$

$\Rightarrow B = 2^{1+\log_6 3}\ 3^{\log_6 3} = 2 \times 2^{\log_6 3}.\ 3^{\log_6 3}$

$\Rightarrow B = 2\{6\}^{\log_6 3} = 2.3 = 6$

$A = \log_{10} \dfrac{ab + \sqrt{(ab)^2 - 4(a+b)}}{2} + \log_{10} \dfrac{ab - \sqrt{(ab)^2 - 4(a+b)}}{2}$

$A = \log_{10} \left[\dfrac{ab + \sqrt{(ab)^2 - 4(a+b)}}{2} \times \dfrac{ab - \sqrt{(ab)^2 - 4(a+b)}}{2} \right]$

$= \log_{10} \left[\dfrac{(ab)^2 - ((ab)^2 - 4(a+b))^{2/2}}{4} \right]$

$= \log_{10} \left[\dfrac{(ab)^2 - (ab)^2 + 4(a+b)}{4} \right] = \log_{10} \dfrac{4(a+b)}{4}$

$= \log_{10}(a + b) = \log_{10}(43 + 57) = \log_{10}100 = 2$

$\Rightarrow A = 2$ and $B = 6$

Hence, AB = 12

Sol 2: (a) $\log_{1/3} \sqrt[4]{729\sqrt[3]{9^{-1}.27^{-4/3}}}$

$= \log_{1/3} \sqrt[4]{729\sqrt[3]{3^{-2}.3^{-4}}}$

$= \log_{1/3} \sqrt[4]{729.3^{-2}} = \log_{1/3} \sqrt[4]{81} = \log_{1/3}3 = 1$

(b) $a^{\frac{\log_b(\log_b N)}{\log_b a}} = a^x$ say

$x = \dfrac{\log_b(\log_b N)}{\log_b a} = \log_a(\log_b N)$

So $a^x = a^{\log_a(\log_b N)} = \log_b N$

Sol 3: (a) $\log_\pi 2 + \log_2 p$

$\Rightarrow \dfrac{\log 2}{\log \pi} + \dfrac{\log \pi}{\log 2}$ Assume that $\dfrac{\log 2}{\log \pi} = x$ (+ve always)

$(2 < \pi < 10) \Rightarrow x + \dfrac{1}{x} = c$ (Assume)

$x^2 - cx + 1 = 0 \Rightarrow x = \dfrac{c \pm \sqrt{c^2 - 4}}{2}$

For x to be real $c^2 - 4 \geq 0$

$c^2 \geq 4 \Rightarrow c \geq 2 \Rightarrow c = 2 \Rightarrow x = 1$

For all other value c > 2 (Not Possible)

Here, $\log_\pi 2 + \log_2\pi$ is greater than 2

(b) For $\log_3 5$ and $\log_2 7$

Assume that $\log_3 5$ is rational $\therefore \log_3 5 = a \Rightarrow 5 = 3^a$

This is not possible when a is rational $\therefore$ a is irrational

Similarly, $\log_2 7 = b$ assuming b is rational gives $7 = 2^b$

Which is not possible, so b is irrational.

Sol 4: $\log_3 x . \log_4 x . \log_5 x = \log_3 x.\log_4 x + \log_4 x. \log_5 x + \log_5 x \log_3 x$

Assume that $\log_{10} x = y$

$\Rightarrow \dfrac{\log_{10} x.\log_{10} x.\log_{10} x}{\log_{10} 3 \log_{10} 4 \log_{10} 5}$

$= \dfrac{\log_{10} x\log_{10} x}{\log_{10} 3\log_{10} 4} + \dfrac{\log_{10} x\log_{10} x}{\log_{10} 4.\log_{10} 5} + \dfrac{\log_{10} x.\log_{10} x}{\log_{10} 5.\log_{10} 3} \Rightarrow y^3$

$= (\log_{10}5)y^2 + (\log_{10}3)y^2 + (\log_{10}4)y^2$

$y^3 = y^2[\log_{10}5 + \log_{10}3 + \log_{10}4]$

$\Rightarrow y^3 = y^2[\log_{10}(3.4.5)] = y^2 \log_{10} 60$

$\Rightarrow y = 0$ or $y = \log_{10}60$

$\Rightarrow \log_{10} x = 0$ or $y = \log_{10}x = \log_{10}60$

$\Rightarrow x = 1$ or $x = 60$

Sum of roots = 1 + 60 = 61

Square of sum of roots = $(61)^2 = 3721$

Sol 5: $\dfrac{2}{\log_4(2000)^6} + \dfrac{3}{\log_5(2000)^6}$

$\dfrac{2}{6\log_4(2000)} + \dfrac{3}{6\log_5(2000)}$

$= \dfrac{1}{6}\left[\dfrac{2}{\log_4(4^2 \times 5^3)} + \dfrac{3}{\log_5(5^3 \times 4^2)}\right]$

$= \dfrac{1}{6}\left[\dfrac{2}{\log_4 4^2 + \log_4 5^3} + \dfrac{3}{\log_5 5^3 + \log_5 4^2}\right]$

$= \dfrac{1}{6}\left[\dfrac{2}{2 + 3\log_4 5} + \dfrac{3}{3 + 2\log_5 4}\right]$

$= \dfrac{1}{6}\left[\dfrac{2}{2 + \dfrac{3\log_{10} 5}{\log_{10} 4}} + \dfrac{3}{3 + \dfrac{2\log_{10} 4}{\log_{10} 5}}\right]$

$= \dfrac{1}{6}\left[\dfrac{2\log_{10} 4}{2\log_{10} 4 + 3\log_{10} 5} + \dfrac{3\log_{10} 5}{3\log_{10} 5 + 2\log_{10} 4}\right]$

$= \dfrac{1}{6}\left[\dfrac{2\log_{10} 4 + 3\log_{10} 5}{2\log_{10} 4 + 3\log_{10} 5}\right] = \dfrac{1}{6}$

Sol.6 $\dfrac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right)$

$= \dfrac{9^{2\log_9 5} + 3^{3\log_3 \sqrt{6}}}{409}\left((\sqrt{7})^{2\log_7 25} - (25)^{\frac{3}{2}\log_{25} 6}\right)$

$= \dfrac{9^{\log_9 5^2} + 3^{\log_3(\sqrt{6})^3}}{409}[7^{\log_7 25} - 25^{\log_{25} 6^{3/2}}]$

$= \dfrac{5^2 + (\sqrt{6})^3}{409}[25 - 6^{3/2}] = \dfrac{(5^2)^2 - (6^{3/2})^2}{409}$

$= \dfrac{(25)^2 - 6^3}{409} = \dfrac{409}{409} = 1$

Sol 7: $(5)^{\log_{1/5}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}} \dfrac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2} \dfrac{1}{10+2\sqrt{21}}$

$= 5^{\log_5 2} + \log_{2^{\frac{1}{2}}} \left(\dfrac{4}{\sqrt{7}+\sqrt{3}}\right) + \log_{2^{-1}} \dfrac{1}{10+2\sqrt{21}}$

$= 2 + \log_2 \left(\dfrac{4}{\sqrt{7}+\sqrt{3}}\right)^2 + \log_2 10+2\sqrt{21}$

$= \left(\dfrac{4}{\sqrt{7}+\sqrt{3}}\right)^2 = \dfrac{16}{7+3+2\sqrt{7}\sqrt{3}} = \dfrac{16}{10+2\sqrt{21}}$

$= 2 + \log_2 \dfrac{16}{10+2\sqrt{21}} \left(10+2\sqrt{21}\right)$

$= 2 + \log_2 2^4 = 2 + 4 = 6$

Sol 8: $\log_2 a = s \Rightarrow a = 2^s$

$\log_4 b = s^2 \Rightarrow b = 4^{s^2} = (2)^{2s^2}$

and $\log_{c^2} 8 = \dfrac{2}{s^3+1} \Rightarrow 8^{\frac{1}{2}} = c^{\frac{2}{s^3+1}}$

$\Rightarrow c = (2^{3/2})^{\frac{s^3+1}{2}}$; $c = 2^{\frac{3(s^3+1)}{4}}$

Then $\dfrac{a^2b^5}{c^4} = \dfrac{(2^s)^2(2^{2s^2})^5}{\left(2^{3\frac{(s^3+1)}{4}}\right)^4} = \dfrac{2^{2s}\ 2^{10s^2}}{2^{3(s^3+1)}} = (2)^{(2s+10s^2-3(s^3+1))}$

$\Rightarrow \log_2 \dfrac{a^2b^5}{c^4} = (2s+10s^2-3(s^3+1))$

Sol 9: $\dfrac{\log_2 24}{\log_{96} 2} - \dfrac{\log_2 192}{\log_{12} 2}$

$\Rightarrow$ We know that $\log_m n = \dfrac{1}{\log_n m}$

$\Rightarrow (\log_2 96)(\log_2 24) - (\log_2 192)(\log_2 12)$

Where, $\log_2 24 = \log_2 12 \times 2 = \log_2 12 + \log_2 2$

$\Rightarrow \log_2 96(\log_2 12 + \log_2 2) - \log_2(96\times 2) \log_2 12$

$\Rightarrow \cancel{\log_2 96 . \log_2 12} + \log_2 96 - \cancel{\log_2 96 . \log_2 12} - \log_2 12$

$\Rightarrow \log_2 \left(2^3 \times 12\right) - \log_2 12 \Rightarrow 3 + \log_2 12 - \log_2 12 = 13$

Sol 10: We have to prove that

$a^x - b^y = 0$, where $x = \sqrt{\log_a b}$

and $y = \sqrt{\log_b a} \Rightarrow x^2 = \log_a b$

$y^2 = \log_b a \Rightarrow y^2 = \dfrac{1}{x^2} \Rightarrow x^2y^2 = 1$

$xy = 1$ $(x, y > 0)$ now $a^x - b^y = (b^{y^2})^x - (a^{x^2})^y$

$\Rightarrow (b^{xy})^y - (a^{xy})^x \Rightarrow b^y - a^x \Rightarrow a^x - b^y = b^y - a^x = -(a^x - b^y)$

$\Rightarrow a^x - b^y + a^x - b^y = 0 \Rightarrow 2(a^x - b^y) = 0 \Rightarrow a^x - b^y = 0$

Sol 11: (a) $\dfrac{\log_{10}(x-3)}{\log_{10}(x^2-21)} = \dfrac{1}{2}$

$\Rightarrow 2\log_{10}(x-3) = \log_{10}(x^2-21)$

$\Rightarrow \log_{10}(x-3)^2\ \ \log_{10}(x^2-21) = 0 \Rightarrow \log_{10} \dfrac{(x-3)^2}{(x^2-21)} = 0$

$\Rightarrow \dfrac{(x-3)^2}{x^2-21} = 1 \Rightarrow x^2 + 3^2 - 2(3)x = x^2 - 21$

$\Rightarrow 9 - 6x = -21 \Rightarrow 6x = 9 + 21 \Rightarrow x = \dfrac{30}{6} = 5$

(b) $\log(\log x) + \log(\log x^3 - 2) = 0$

$\Rightarrow \log[\log x(\log x^3-2)] = 0 \Rightarrow (\log x)(\log x^3 - 2) = 1$

$\Rightarrow (\log x)(3 \log x - 2) = 1$ Assume that $\log x = y$

$\Rightarrow y(3y-2) = 1 \Rightarrow 3y^2 - 2y - 1 = 0$

$\Rightarrow 3y(y-1) + 1(y-1) = 0 \Rightarrow y = -\dfrac{1}{3}$ or $y = 1$

$\Rightarrow \log_{10} x = -\dfrac{1}{3}$ or $\log_{10} x = 1 \Rightarrow x = (10)^{-\frac{1}{3}}$ or $x = 10^1$

At $x = 10^{-1/3}$ equation does not satisfy

Hence, $x = 10$

(c) $\log_x 2 . \log_{2x} 2 = \log_{4x} 2$

$\Rightarrow \dfrac{1}{\log_2 x} . \dfrac{1}{\log_2 2x} = \dfrac{1}{\log_2 4x}$

$\Rightarrow \log_2 2^2 + \log x = (\log_2 x)(\log_2 2 + \log_2 x)$

Assume $\log_2 x = y$

$\Rightarrow 2 + y = y(1 + y) \Rightarrow 2 + y = y^2 + y$

$\Rightarrow y^2 = 2 \Rightarrow y = \pm\sqrt{2}$

$\log x = \pm \sqrt{2}$

$\log_2 x = +\sqrt{2}$ or $\log_2 x = -\sqrt{2}$

$x = (2)^{\sqrt{2}}$ or $x = 2^{-\sqrt{2}}$

(d) $5^{\log_a x} + 5x^{\log_a 5} = 3$, $(a > 0)$

Assume $x = a^y$

$\Rightarrow 5^{\log_a a^y} + 5a^{y\log_a 5} = 3 \Rightarrow 5^y + 5a^{\log_a 5^y}$

$= 5^y + 5.5^y = 6.5^y = 3 \Rightarrow 5^y = \frac{3}{6} = \frac{1}{2} = 2^{-1}$

Take logarithm (base 5) both side

$\Rightarrow \log_5 5^y = \log_5 2^{-1} \Rightarrow y = \log_5 2^{-1}$

So $x = a^y = a^{\log_5 2^{-1}}$

$\Rightarrow x = 2^{-\log_5 a}$

Sol 12: $\log_a x \log_a(xyz) = 48$ ... (i)

$\log_a y \log_a(xyz) = 12$ ... (ii)

$\log_a z \log_a(xyz) = 84$ ... (iii)

When sum of all equation is taken

$\log_a(xyz)[\log_a x + \log_a y + \log_a z]$

$= 48 + 12 + 84 = 144 = 12^2$

$(\log_a(xyz))(\log_a(xyz)) = 12^2$

$(\log_a xyz)^2 = 12^2 \Rightarrow \log_a xyz = 12\ (\pm 1)$

In equation

(i) $\log_a x\ (\pm 12) = 48$

$\log_a x = \pm 4 \Rightarrow x = a^4, a^{-4}$

(ii) $\log_a y(\pm 12) = 12$

$\log_a y = \pm 1 \Rightarrow y = a, a^{-1}$

(iii) $\log_a z(\pm 12) = 84$

$\log_a z = \pm 7 \Rightarrow z = a^7, a^{-7}$

$(x, y, z) = (a^4, a, a^7)$ or (a^{-4}, a^{-1}, a^{-7})

Sol 13: Given

L = antilog of 0.4 to the base 1024

$\Rightarrow L = (1024)^{0.4} = (2^{10})^{0.4} = 2^4 = 16$

L = 16

And M is the number of digits in 6^{10}

$\Rightarrow \log_{10} 6^{10} = 10\log_{10} 6 \Rightarrow 10[0.7761] = 7.761$

$\Rightarrow 6^{10} = 10^{7.761} = 10^7.10^{0.761}$

No. of digits = 7 + 1 = 8 $\therefore$ M = 8

$\Rightarrow \log_6 6^2 = 2$ (characteristic 2)

$\Rightarrow \log_6 6^3 = 3$ (characteristic 3)

Total no. of positive integers which have the characteristic 2(between 6^2 and 6^3) = $6^3 - 6^2$

= 216 – 36 = 180

LMN = 16 × 8 × 180 = 23040

Sol 14: $\log_a N.\log_b N + \log_b N.\log_c N + \log_c N.\log_a N$ ... (i)

$$= \frac{\log_a N.\log_b N.\log_c N}{\log_{abc} N}$$

We know that $\log_x y = \frac{\log y}{\log x}$

So, in equation (i) at R.H.S, we have

$$= \frac{\frac{\log N}{\log a}\cdot\frac{\log N}{\log b}\cdot\frac{\log N}{\log c}}{\frac{\log N}{\log abc}} = \frac{(\log N)^2 \log abc}{(\log a)\log b(\log c)}$$

$$= \frac{\log N^2(\log a + \log b + \log c)}{\log a \log b \log c}$$

$$= \frac{(\log N)(\log N)}{\log b \log c} + \frac{\log N \log N}{\log a \log c} + \frac{\log N \log N}{\log a \log b}$$

$= \log_a N \log_b N + \log_a N \log_c N + \log_b N \log_c N$

R.H.S. = L.H.S.

Sol 15: x, y> 0 and $\log_y x + \log_x y = \frac{10}{3}$

$$\Rightarrow \frac{\log_{12} x}{\log_{12} y} + \frac{\log_{12} y}{\log_{12} x} = \frac{10}{3}$$

Assume that $\frac{\log_{12} x}{\log_{12} y} = a$

$\Rightarrow a + \frac{1}{a} = \frac{10}{3} \Rightarrow 3a^2 - 10a + 3 = 0$

$\Rightarrow (3a\ \ 1)(a\ \ 3) = 0 \Rightarrow a = 3, \left(\frac{1}{3}\right)$

So $\frac{\log_{12} x}{\log_{12} y} = 3 \Rightarrow$ add + 1 both side

$$\frac{\log_{12} x}{\log_{12} y} + 1 = 3 + 1 = 4 \Rightarrow \frac{\log x + \log y}{\log_{12} y} = 4$$

$\Rightarrow \dfrac{\log_{12}(xy)}{\log_{12} y} = \dfrac{\log_{12} 12^2}{\log y} = 4 \Rightarrow \dfrac{2}{\log_{12} y} = 4$

$\log_{12} y = \dfrac{2}{4} = \dfrac{1}{2} \Rightarrow y = 12^{1/2}$

So $x = \dfrac{144}{y} = 144 \times 12^{-\frac{1}{2}} = 12^{2-\frac{1}{2}} = 12^{\frac{3}{2}}$

$\dfrac{x+y}{2} = \sqrt{N}$

$\Rightarrow \dfrac{(x+y)^2}{2^2} = N \Rightarrow x^2 + y^2 + 2xy = 4N$

$\Rightarrow (12^{3/2})^2 + (12^{1/2}) + 2(144) = 4\,N$

$\Rightarrow 12^3 + 12 + 2 \times 144 = 4\,N$

$4N = 2028 \Rightarrow N = \dfrac{2028}{4} \Rightarrow N = 507$

Sol 16: (a) $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$

$\Rightarrow 5^{200} = x$ (Assume)

$\log_{10} x = \log_{10} 5^{200} = 200\log_{10}5$

$= 200\log_{10}\dfrac{10}{2} = 200(\log_{10}10 - \log_{10}2)$

$= 200(1 - 0.3010) = 200(0.699) = 139.8$

$\Rightarrow x = 10^{139} \times 10^{0.8}$

no. of digits in x = 139 + 1 = 140

(b) $x = 6^{15} \Rightarrow \log_{10}x = \log_{10} 6^{15} = 15 \log_{10} 6$

$= 15(\log 2 + \log 3) = 15 \times (0.778) = 11.67$

$\therefore x = 10^{11.67} = 10^{11}\, 10^{0.67}$

No. of digits in x = 11 + 1 = 12

(c) Number of zeros after the decimal in $3^{-100} = (x)$ (Assume)

$\log x = \log 3^{-100} = -100 \log_{10}3 = -100(0.4771) = -47.71$

So $x = 10^{-47.71} = 10^{-47} \times 10^{-0.71}$

$\therefore$ No. of zeros = 47

Sol 17: $\log_5 120 + (x-3) - 2\log_5(1-5^{x-3}) = -\log_5(2-5^{x-4})$

$\Rightarrow \log_5 120 + (x-3) - \log_5(1-5^{x-3})^2 + \log_5(2-5^{x-4}) = 0$

$\Rightarrow \log_5 \dfrac{120 \times 5^{x-3} \times (2-5^{x-4})}{(1-5^{x-3})^2} = 0$

$\Rightarrow \dfrac{120 \times 5^{x-3} \times (2-5^{x-4})}{(1-5^{x-3})^2} = 1$

$\Rightarrow \dfrac{120}{5^3}\, 5^x \left[2 - \dfrac{5^x}{5^4}\right] = 1^2 + 5^{2(x-3)} - 2(5^{x-3})$

Assume that $5^x = y$

$\Rightarrow \dfrac{120}{5 \times 5 \times 5}\, y \left[2 - \dfrac{y}{25 \times 25}\right] = 1 + y^2\, 5^{-6} - \dfrac{2 \times y}{5^3}$

Multiply by 5^6

$\Rightarrow 5^3 \times 120y[2 - y\, 5^{-4}] = 5^6 + y^2 - 2x5^3y$

$\Rightarrow 5^3 \times 240y - \dfrac{120y^2}{5} = 5^6 + y^2 - 2 \times 5^3y$

$\Rightarrow 5^3 \times 240y - 24y^2 = 5^6 + y^2 - 2 \times 5^3y$

$5^4 \times 48y - 25y^2 = 5^6 - 10 \times 5^2y$

Divide by 5^2 on the both side

$5^2 \times 48y - y^2 = 5^4 - 10y$

$\Rightarrow y^2 - y(10 + 5^2 \times 48) + 5^4 = 0$

$\Rightarrow y^2 - 1210y + 625 = 0$

$\Rightarrow y = \dfrac{1210 \pm \sqrt{(1210)^2 - 4(1)(625)}}{2}$

$\Rightarrow y = \dfrac{1210 \pm 1208.96}{2}$

y = 0.51675 or y = 1209. 48 (Rejected)

$5^x = y = 0.51675 \Rightarrow x = \log_5 y$

Hence, x = – 0.410

Sol 18: Given that $\log_{x+1} (x^2 + x - 6)^2 = 4$

$\Rightarrow (x^2 + x - 6)^2 = (x + 1)^4 \Rightarrow (x^2 + x - 6) = \pm (x + 1)^2$

When +ve case is taken $\rightarrow x^2 + x - 6 = (x + 1)^2$

(and $x^2 + x - 6 \ge 0$)

$x^2 + x - 6 = x^2 + 1 + 2x$

$x = -6 - 1 = -7$

In the given equation, base is x + 1 = – 7 + 1 = – 6 which is negative

So $x \neq -7$

When -ve case is taken $\rightarrow x^2 + x - 6 < 0$

$\Rightarrow x^2 + x - 6 = -(x + 1)^2 \Rightarrow x^2 + x - 6 = -x^2 - 1 - 2x$

$\Rightarrow 2x^2 + 3x - 5 = 0 \Rightarrow (2x + 5)(x - 1) = 0$

$x = -\frac{5}{2}$ or $x = 1$

$x = -\frac{5}{2}$ also does not satisfy equation

So x = 1

Sol 19: Given that $x + \log_{10}(1 + 2^x) = x\log_{10}5 + \log_{10}6$

$\Rightarrow \log_{10}10^x + \log_{10}(1+2^x) = \log_{10}5^x + \log_{10}6$

$\Rightarrow \log_{10}[10^x(1 + 2^x)] = \log_{10}[5^x\, 6]$

$\Rightarrow 10^x\,(1 + 2^x) = 6\, 5^x \Rightarrow 10^x + 20^x = 5^x\, 6$

Divide by 5^x on the both the sides

$\Rightarrow \frac{10^x}{5^x} + \frac{20^x}{5^x} = \frac{6\times 5^x}{5^x} = 6$

$\Rightarrow 2^x + 4^x = 6 \Rightarrow 2^x + 2^{2x} = 6$

Assume that $2^x = y$

$\Rightarrow y + y^2 = 6 \Rightarrow y^2 + y - 6 = 0$

$\Rightarrow (y - 2)(y + 3) = 0 \Rightarrow y = -3$ or $y = 2$

$\Rightarrow 2^x = -3$ or $2^x = 2 \Rightarrow 2^x = -3$ is not possible so, $2^x = 2$

Therefore, the real solution $\Rightarrow x = 1$

Sol 20: $2\log_{10}(2y - 3x) = \log_{10}x + \log_{10}y$

We have to find $\left(\frac{x}{y}\right)$

$\Rightarrow \log_{10}(2y - 3x)^2 = \log_{10}(xy) \Rightarrow 4y^2 - 12xy + 9x^2 = xy$

Let x = ky

$\Rightarrow 4y^2 - 12ky^2 + 9k^2 y^2 = ky^2 \Rightarrow 9k^2 - 13k + 4 = 0$

$\Rightarrow (9k - 4)(k\;\; 1) = 0 \Rightarrow k = 1, \frac{4}{9}$

If $k = 1 \Rightarrow x = y \Rightarrow 2y - 3x$ is –ve

$\therefore \frac{x}{y} = \frac{4}{9}$

Sol 21: We have $a = \log_{12}18$ and $b = \log_{24}54$

$\Rightarrow a = \frac{\log_2 18}{\log_2 12} = \frac{2\log_2 3 + 1}{2 + \log_2 3}$

$\Rightarrow (a - 2)\log_2 3 = 1 - 2a$ ... (i)

Similarly $b = \frac{\log_2 54}{\log_2 24} = \frac{3\log_2 3 + 1}{3 + \log_2 3}$

$\Rightarrow (b - 3)\log_2 3 = 1 - 3b$ ... (ii)

Dividing E.q. (i) and (ii), we get

$(a - 2)(1 - 3b) = (1 - 2a)(b - 3)$

$\Rightarrow 2a(b - 3) + (a - 2)(1 - 3b) = b - 3$

$\Rightarrow 2ab - 6a + a - 3ab - 2 + 6b = b - 3$

$\Rightarrow - ab - 5a + 5b + 1 = 0 \Rightarrow 5(b - a) - ab + 1 = 0$

$\Rightarrow 5(a - b) + ab = 1$

Sol 22: $\sqrt{\log_9(9x^4)\log_3(3x)} = \log_3 x^3$

$\Rightarrow \sqrt{(1 + 4\log_3 x)[1 + \log_3 x]} = 3\log x$

Assume that $\log_3 x = y$

$\Rightarrow (1 + 4y)(1 + y) = (3y)^2 = 9y^2$

$\Rightarrow 1 + 4y^2 + 4y + y = 9y^2 \Rightarrow 5y^2 - 5y - 1 = 0$

$\Rightarrow y = \frac{5 \pm \sqrt{5^2 - 4(-1)(5)}}{2(5)} = \frac{5 \pm \sqrt{25 + 20}}{10}$

$y = \frac{5 \pm \sqrt{45}}{10} = \frac{5 \pm \sqrt{3^2 \times 5}}{10} = \frac{5 \pm 3\sqrt{5}}{10}$

In equation (i) $\log_3 x > 0$

Hence, $y = \frac{5 + 3\sqrt{5}}{10}$

Sol 23: Given that $xyz = 10^{81}$

$(\log_{10}x)(\log_{10}yz) + (\log_{10}y)(\log_{10}z) = 468$

We know that $(a + b + c)^2$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= a^2 + b^2 + c^2 + 2a(b + c) + 2bc$ ... (i)

$\Rightarrow \log_{10}x(\log_{10} y + \log z) + (\log_{10} y)\;\; (\log_{10} z) = 468$

Assume that log10x = a, $\log_{10} y = b$ and $\log_{10} z = c$

$\Rightarrow a(b + c) + bc = 468$

From equation (i)

$2a(b+c) + 2bc = (a+b+c)^2 - (a^2+b^2+c^2)$

$\Rightarrow 2a(b + c) + 2bc = 2 \times 468 = 936$

$\Rightarrow (a + b + c)^2 - (a^2 + b^2 + c^2) = 936$

$\Rightarrow a + b + c = \log_{10}x + \log_{10}y + \log_{10}z$

$= \log_{10}xyz = \log_{10}10^{81} = 81$

$\Rightarrow 81^2 - (a^2 + b^2 + c^2) = 936$

$a^2 + b^2 + c^2 = 81^2 - 936 = 5625$

$\Rightarrow (\log_{10} x)^2 + (\log_{10} y)^2 + (\log_{10} z)^2 = 5625$

Sol 24: Sum of all solution of equation

$\Rightarrow [3]^{(\log_9 x)^2 - \frac{9}{2}\log_9 x + 5} = 3\sqrt{3}$

$\Rightarrow (3)^{(\log_9 x)^2 - \frac{9}{2}\log_9 x + 5} = (3)^{3/2}$

$\Rightarrow (\log_9 x)^2 - \frac{9}{2}\log_9 x + 5 = \frac{3}{2}$

Assume that $\log_9 x = y$

$\Rightarrow y^2 - \frac{9}{2}y + 5 = \frac{3}{2} \Rightarrow y^2 - \frac{9}{2}y + 5 - \frac{3}{2} = y^2 - \frac{9}{2}y + \frac{7}{2} = 0$

$\Rightarrow 2y^2 - 9y + 7 = 0$

$\Rightarrow (2y - 7)(y - 1) = 0 \;\Rightarrow y = \frac{7}{2};\; y = 1$

$\log_9 x = \frac{7}{2} \log_9 x = 1$

$\Rightarrow x = (9)^{7/2} = 3^7; x = 9$

Sum of solution $= 3^7 + 9 = 2196$

Sol 25: a, b, c, d > 0

$\because \log_a b = \frac{3}{2}$ and $\log_c d = \frac{5}{4}$, $a - c = 9$

$\frac{\log_{10} b}{\log_{10} a} = \frac{3}{2};\; \frac{\log_{10} d}{\log_{10} c} = \frac{5}{4}$

$2\log_{10} b = 3\log_{10} a$

$4\log_{10} d = 5\log_{10} c$

$b = a^{\frac{3}{2}}, d = c^{\frac{5}{4}}$

$\therefore$ a should be perfect square and c should be perfect power of 4

Let a = 25, c = 16

$\therefore b = (5)^3 = 125 \Rightarrow \quad d = (16)^{5/4} = 32 \quad \therefore b - d = 93$

Sol 26: Refer Sol 11 of Ex 2 JEE Main

Sol 27:

$\log_{10}^2\left[1 + \frac{4}{x}\right] + \log_{10}^2\left[1 - \frac{4}{x+4}\right] = 2\log_{10}^2\left[\frac{2}{x-1} - 1\right]$

$\log_{10}^2\left[\frac{x+4}{x}\right] + \log_{10}^2\left[\frac{x+4-4}{x+4}\right] = 2\log_{10}^2\left[\frac{2-(x-1)}{x-1}\right]$

$\log_{10}^2\left(\frac{x+4}{x}\right) + \log_{10}^2\left(\frac{x}{x+4}\right) = 2\log_{10}^2\left(\frac{2-x+1}{x-1}\right)$

We know $\log_{10}\frac{1}{x} = -\log_{10}x$. So $\left(\log_{10}\frac{1}{x}\right)^2 = (\log_{10} x)^2$

$\Rightarrow \log_{10}^2\left(\frac{x+4}{x}\right) + \log_{10}^2\left(\frac{x+4}{x}\right) = 2\log_{10}^2\left(\frac{3-x}{x-1}\right)$

$\log_{10}^2\left(\frac{x+4}{x}\right) = \log_{10}^2\left(\frac{3-x}{x-1}\right)$

So $\frac{x+4}{x} = \frac{3-x}{x-1}$ or $\frac{x}{x+4} = \left(\frac{3-x}{x-1}\right)$

$x^2 + 4x - x - 4 = 3x - x^2$ or $x^2 - x = 3x + 12 - x^2 - 4x$

$\Rightarrow 2x^2 - 4 = 0$ or $2x^2 = 12 \Rightarrow x^2 = 2$ or $x^2 = 6$

$x = \pm\sqrt{2}$ or $x = \pm\sqrt{6}$

$x = \sqrt{2}$ and $-\sqrt{6}$ do not satisfy equation

So $x = \sqrt{2}, \sqrt{6}$

Sol 28: $\log_3(\sqrt{x} + |\sqrt{x} - 1|) = \log_9(4\sqrt{x} - 3 + 4|\sqrt{x} - 1|)$

$\log_3\left(\sqrt{x} + |\sqrt{x} - 1|\right) = \frac{1}{2}\log_3\left(4\sqrt{x} + -3 + 4|\sqrt{x} - 1|\right)$

$\Rightarrow 2\log_3(\sqrt{x} + |\sqrt{x} - 1|) = \log_3(4\sqrt{x} - 3 + 4|\sqrt{x} - 1|)$

$\Rightarrow \log_3(\sqrt{x} + |\sqrt{x} - 1|)^2 = \log_3(4\sqrt{x} - 3 + 4|(\sqrt{x} - 1|)$

$\Rightarrow (\sqrt{x} + |\sqrt{x} - 1|)^2 = (4\sqrt{x} - 3 + 4|\sqrt{x} - 1|)$

$x + (\sqrt{x} - 1)^2 + 2\sqrt{x}|\sqrt{x} - 1| = 4\sqrt{x} - 3 + 4|\sqrt{x} - 1|$

(i) Assume $(\sqrt{x} - 1) < 0$

$\Rightarrow |\sqrt{x} - 1| = 1 - \sqrt{x}$

$\Rightarrow x + x + 1 - 2\sqrt{x} + 2\sqrt{x}(1 - \sqrt{x}) = 4\sqrt{x} - 3 + 4(1 - \sqrt{x})$

$\Rightarrow 1 + 2x - 2\sqrt{x} + 2\sqrt{x} - 2x = 4\sqrt{x} - 3 + 4 - 4\sqrt{x}$

1 = 1 always correct

So $\sqrt{x} - 1 < 0$ and $x > 0$

$\sqrt{x} < 1$

$\Rightarrow x \in (0, 1)$ and if $\sqrt{x} - 1 \ge 0$, $\sqrt{x} > 0$

$x + x + 1 - 2\sqrt{x} + 2\sqrt{x}(\sqrt{x} - 1) = 4\sqrt{x} - 3 + 4(\sqrt{x} - 1)$

$\Rightarrow 2x + 1 - 2\sqrt{x} + 2x - 2\sqrt{x} = 4\sqrt{x} - 3 - 4 + 4\sqrt{x}$

$\Rightarrow 4x + 1 + 7 - 4\sqrt{x} = 8\sqrt{x} \Rightarrow 4x - 12\sqrt{x} + 8 = 0$

$\Rightarrow x - 3\sqrt{x} + 2 = 0 \Rightarrow (\sqrt{x} - 2)(\sqrt{x} - 1) = 0$

$\Rightarrow \sqrt{x} - 2 = 0$ or $\sqrt{x} - 1 = 0 \Rightarrow x = 4$ or $x = 1$

Put condition was $\Rightarrow \sqrt{x} - 1 \ge 0$

So $x = [0, 1] \cup \{4\}$

Sol 29:

$$2^{\left(\sqrt{\log_a \sqrt[4]{ab} + \log_b \sqrt[4]{ab}} - \sqrt{\log_a \sqrt[4]{b/a} + \log_b \sqrt[4]{a/b}}\right)\cdot\sqrt{\log_a b}} = 2^x$$

$$\Rightarrow x = \left(\sqrt{\frac{1}{4}(\log_a (a\times b) + \log_b (a\times b))} - \sqrt{(\log_a ba^{-1} + \log_b ab^{-1})\frac{1}{4}}\right)\sqrt{\log_a b}$$

$$x = \frac{1}{2}\left[\sqrt{1 + \log_a b + 1 + \log_b a} - \sqrt{-1 + \log_a b - 1 + \log_b a}\right]\sqrt{\log_a b}$$

$$x = \frac{1}{2}\left[\sqrt{2\log_a b + 1 + (\log_a b)^2} - \sqrt{-2\log_a b + (\log_a b)^2 + 1}\right]$$

We know $\log_a b = \frac{1}{\log_b a}$

$$x = \frac{1}{2}\left(\sqrt{(1 + \log_a b)^2} - \sqrt{(\log_a b - 1)^2}\right)$$

$$x = \frac{1}{2}(|1 + \log_a b| - |\log_a b - 1|)$$

When $\log_a b \ge 1 \Rightarrow b \ge a > 1$

$$x = \frac{1}{2}(1 + \log_a b - \log_a b + 1) = \frac{1}{2}\times 2 = 1$$

so $2^x = 2^1 = 2$ (when $b \ge a > 1$)

When $\log_a b < 1$

$\Rightarrow b < a,\ a, b > 1$

$$\Rightarrow x = \frac{1}{2}[1 + \log_a b - (1 - \log_a b)]$$

$$x = \frac{1}{2}[1 + \log_a b + \log_a b] = \frac{1}{2}2\log_a b$$

$x = \log_a b$

$2^x = 2^{\log_a b}$ (if $1 < b < a$)

Sol 30: $\sqrt{[\log_3 (3x)^{1/3} + \log_x (3x)^{1/3}]\log_3 x^3} +$

$$\sqrt{\left[\log_3\left(\frac{x}{3}\right)^{\frac{1}{3}} + \log_x\left(\frac{3}{x}\right)^{\frac{1}{3}}\right]\log_3 x^3}$$

Assume that

$$A = \sqrt{\left[\frac{1}{3}\log_3 (3x) + \frac{1}{3}\log_x (3x)\right]\log_3 x^3}$$

$$\Rightarrow \sqrt{\frac{3}{3}\left[(\log_3 x + 1) + (\log_x 3 + 1)\right]\log_3 x}$$

$$A = \sqrt{(2\log_3 x + (\log_3 x)^2 + 1)}$$

$A = |\log_3 x + 1|$

$$\text{And } B = \sqrt{\left(\left(\log_3 \frac{x}{3}\right)\frac{1}{3} + \frac{1}{3}\left(\log_x \frac{3}{x}\right)\right)\log_3 x^3}$$

$$\Rightarrow \sqrt{\frac{3}{3}\left[\log_3 x - 1 + \log_x 3 - 1\right]\log_3 x}$$

$$B = \sqrt{((\log_3 x)^2 - 2\log_3 x + 1)}$$

$$B = \sqrt{(\log_3 x - 1)^2} = |\log_3 x - 1|$$

$A + B = 2 \Rightarrow |\log_3 x + 1| + |\log_3 x - 1| = 2$

$\log_3 x \ge 1 \Rightarrow x \ge 3$

$A + B \Rightarrow \log_3 x + 1 + \log_3 x - 1 = 2\log_3 x = 2$

$\log_3 x = 1 \Rightarrow x = 3$

$x \ge 3$ and $x = 3 \Rightarrow x = 3$

If $\log_3 x < 1$ and $\log_3 x + 1 > 0 \Rightarrow x < 3$ and $x > \frac{1}{3}$

$A + B \Rightarrow \log_3 x + 1 - (\log_3 x - 1)$

$= \log_3 x + 1 - \log_3 x + 1 = 2 = 2$(always)

So $x \in \left(\frac{1}{3}, 3\right)$

$\log_3 x \le -1 \Rightarrow x \le \frac{1}{3}$

$A + B = -(\log_3 x + 1) - (\log_3 x - 1)$

$= -\log_3 x - 1 - \log_3 x + 1 = -2\log_3 x = 2$

$\Rightarrow \log_3 x = -1 \Rightarrow x = 3^{-1} = \frac{1}{3}$

$x \ge \frac{1}{3}$ and $x = \frac{1}{3} \Rightarrow x = \frac{1}{3}$

So $x = \left[\frac{1}{3}, 3\right] - \{1\}$

$x \neq 1$ because base can't be 1

Sol 31: $a = (\log_7 81)(\log_{6561} 625)(\log_{125} 216)(\log_{1296} 2401)$

$\Rightarrow a = (\log_7 3^4)\ (\log_{3^8} 5^4)\ (\log_{5^3} 6^3)\ (\log_{6^4} 7^4)$

$\Rightarrow a = 4(\log_7 3)\frac{4}{8}(\log_3 5)(\log_5 6)\left(\frac{3}{3}\right)\left(\frac{4}{4}\right)\log_6 7$

$\Rightarrow a = \frac{2\log_{10} 3}{\log_{10} 7}\frac{\log_{10} 5}{\log_{10} 3}\frac{\log_{10} 6}{\log_{10} 5}\frac{\log_{10} 7}{\log_{10} 6} = 2$

$\Rightarrow$ and b = sum of roots of the equation

$x^{\log_2 x} = (2x)^{\log_2 \sqrt{x}}$

$x^{\log_2 x} = (2x)^{\log_2 x^{1/2}}$

Take logarithm (base x) both sides

$\log_x x^{\log_2 x} = \log_x (2x)^{\log_2 x^{1/2}}$

$(\log_2 x)(1) = \log_2 x^{1/2}[\log_x (2x)]$

$\log_2 x = \frac{1}{2}\log_2 x(\log_x 2 + 1)$

$\log_2 x = 0 \Rightarrow x = 1$ or $2 = \log_x 2 + 1$

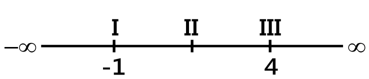
$\log_x 2 = 1 \Rightarrow x = 2$

$x_1 + x_2 = 1 + 2 = 3$

$b = 3$

and c = sum of all natural solution of equation

$|x + 1| + |x - 4| = 7$



If $x < -1 \rightarrow |x + 1| = -1 - x$

$|x - 4| = 4 - x$

$\Rightarrow$ eq.$\rightarrow -1 - x + 4 - x = 3 - 2x = 7$

$\Rightarrow 2x = 3 - 7 = -4 \Rightarrow x = -\frac{4}{2} = -2$

If $x > 4 \rightarrow |x + 1| = x + 1$

$|x - 4| = x - 4$

Eq. $\rightarrow x + 1 + x - 4 = 2x - 3 = 7$

$\Rightarrow 2x = \frac{7+3}{1} = 10?2x = 10 \Rightarrow x = \frac{10}{2} = 5$

If $-1 < x < 4$

$\Rightarrow |x + 1| \rightarrow 1 + x$

$|x - 4| \rightarrow 4 - x$

$\Rightarrow 1 + x + 4 - x = 5 \neq 7$

So no solution for this region $\rightarrow x = 5$ and -2

But -2 is not natural no.

So $c = 5$

$a + b = 2 + 3 = 5$

$(a + b) \div c = \frac{5}{5} = 1$

Exercise 2

Single Correct Choice Type

Sol 1: (C) $2^{\sqrt{x}+\sqrt{y}} = 256$ and $\log_{10}\sqrt{xy} - \log_{10} 1.5 = 1$

$\Rightarrow 2^{\sqrt{x}+\sqrt{y}} = 256 = 2^8$

$\Rightarrow \sqrt{x} + \sqrt{y} = 8$ ... (i)

and $\log_{10}\sqrt{xy} = 1 + \log_{10} 1.5 = \log_{10} 10 + \log_{10} 1.5$

$\log_{10}\sqrt{xy} = \log_{10}(10 \times 1.5) = \log_{10} 15$

$\Rightarrow \sqrt{xy} = 15 \Rightarrow xy = 15^2 = 225$

$|\sqrt{x} - \sqrt{y}| = \sqrt{(\sqrt{x}+\sqrt{y})^2 - 4\sqrt{xy}}$

$= \sqrt{8^2 - 4\times 15} = \sqrt{64 - 60}$

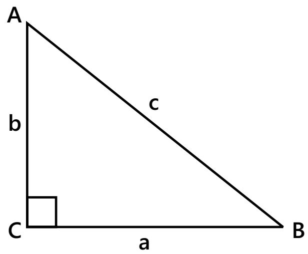
$|\sqrt{x} - \sqrt{y}| = \sqrt{4} = 2$

$\sqrt{x} + \sqrt{y} = 8$

$\Rightarrow$ If $\sqrt{x} > \sqrt{y} \Rightarrow (x, y) = (25, 9)$

$\Rightarrow$ If $\sqrt{x} < \sqrt{y} \Rightarrow (x, y) = (9, 25)$

Sol 2: (B)



$\Rightarrow c^2 = a^2 + b^2 \Rightarrow c^2 - b^2 = a^2$

$$\frac{\log_{b+c} a + \log_{c-b} a}{\log_{b+c} a \cdot \log_{c-b} a} = \frac{\frac{\log_a a}{\log_a (b+c)} + \frac{\log_a a}{\log_a (c-b)}}{\frac{\log_a a}{\log_a (b+c)} \frac{\log_a a}{\log_a (c-b)}}$$

$= \left(\log_a (c-b) + \log_a (b+c)\right) = \log_a(c^2 - b^2) = 2$

Sol 3: (B) B, C, P, and L are positive number

$\therefore$ log(B.L) + log(B.P) = 2; log(P.L) + log(P.C) = 3

and log(C.B) + log(C.L) = 4

Adding all the above equations, we have

log[B.L.B.P.P.L.P.C.C.B.C.L] = 2 + 3 + 4 = 9

$\log(BCPL)^3 = 9 \Rightarrow 3\log BCPL = 9$

$\Rightarrow \log BCPL = \frac{9}{3} = 3$

$\therefore BCPL = 10^3$

Sol 4: (B) $\frac{\log_{12}(\log_8(\log_4 x))}{\log_5(\log_4(\log_y(\log_2 x)))} = 0$

$c < y < b, y \neq a$

where 'b' is as large as possible and 'c' is as small as possible.

$\Rightarrow \log_{12}(\log_8(\log_4 x)) = 0 \Rightarrow \log_8(\log_4 x) = 1 = \log_8 8$

$\log_4 x = 8 \Rightarrow x = 4^8 = 2^{2\times 8} = 2^{16}$

and $\log_5(\log_4(\log_y(\log_2 x))) \neq 0$

$\Rightarrow \log_5(\log_4(\log_y(\log_2 2^{16}))) \neq 0$

$\Rightarrow \log_5(\log_4(\log_y 16)) \neq 0, y \neq 1$

$\Rightarrow \log_4(\log_y 16) \neq 1 \Rightarrow \log_y 16 \neq 4$

$\Rightarrow \log_{2^4} y \neq \frac{1}{4} \Rightarrow \frac{1}{4}\log_2 y \neq \frac{1}{4} \Rightarrow \log_2 y \neq 1 \Rightarrow y \neq 2$

$\log_4(\log_y 16) \neq 0 \Rightarrow \log_y 16 \neq 1$

$\log_{16} y \neq 1 \Rightarrow y \neq 16$

$\log_4(\log_y 16) > 0$

$\log_y 16 > 1 \Rightarrow y < 16$

$\log_y 16 > 0$

$\Rightarrow$ a = 2, b = 16, c = 1

a + b + c = 2 + 16 + 1 = 19

Sol 5: (D) $\frac{1}{\log_2 N} \cdot \frac{1}{\log_N 8} \cdot \frac{1}{\log_{32} N} \cdot \frac{1}{\log_N 128}$

$$= \frac{\log_2}{\log N} \cdot \frac{\log N}{3\log_2} \cdot \frac{5\log_2}{\log N} \cdot \frac{\log N}{7\log_2} = \frac{5}{21}$$

Sol 6: (B) $N = 10^p$; $p = \log_{10} 8 - \log_{10} 9 + 2\log_{10} 6$

$p = \log\left(\frac{8.36}{9}\right) = \log_{10} 32$

$\therefore N = 10^{\log_{10} 32} = 32$

Hence characteristics of $\log_3 32$ is 3

Sol 7: (C) $\log 2\left(\left(x+y\right)^2 - xy\right)$

But x + y $= \sqrt{2}$; $\quad xy = \frac{10-2}{4} = 2$

$\log_2(10-2) = \log_2 8 = 3$

Sol 8: (A) Let $x = \sqrt{\frac{5}{4} + \sqrt{\frac{3}{2}}} + \sqrt{\frac{5}{4} - \sqrt{\frac{3}{2}}}$

$\Rightarrow x^2 = \frac{5}{2} + 2\sqrt{\frac{25}{16} - \frac{3}{2}} = \frac{5}{2} + 2.\frac{1}{4} = 3$

$\Rightarrow x = \sqrt{3} = \tan\frac{\pi}{3}$

Sol 9: (D) $y = |2x - |x-2|| = |2x - (2-x)| = |3x - 2|$ as $x < 0$
Hence $y = 2 - 3x$

Sol 10: (D) $3^x(0.333\ldots)^{(x-3)} \leq \left(\frac{1}{27}\right)^x$

$\Rightarrow 3^x\left(\frac{1}{3}\right)^{x-3} \leq \left(\frac{1}{3^3}\right)^x = \left(\frac{1}{3}\right)^{3x}$

$\Rightarrow 3^x\, 3^{-(x-3)} = 3^x \,.\, 3^{3-x} \leq \left(\frac{1}{3}\right)^{3x}$

$3^3 = 27 \le \left(\frac{1}{3}\right)^{3x} = 3^{-3x}$

$3 \le -3x \Rightarrow -x \ge 1 \Rightarrow x \le -1$

$x \in [-\infty, -1]$

Sol 11: (B) $\left(\frac{1}{5}\right)^{\frac{2x+1}{1-x}} > \left(\frac{1}{5}\right)^{-3}$

$\frac{2x+1}{1-x} < -3$

$2x + 1 < -3(1-x) = -3 + 3x$ (if $(1-x) > 0$)

$\Rightarrow 2x + 1 < -3 + 3x \Rightarrow 3x - 2x > 1 + 3 = 4$

$\Rightarrow x > 4 \Rightarrow x > 4$ and $x < 1$ which implies no solution

If $x > 1 \Rightarrow 1 - x < 0 \Rightarrow \frac{2x+1}{1-x} < -3$

$\Rightarrow \frac{2x+1}{1} > -3(1-x) = 3x - 3$

$\Rightarrow 3x - 2x < 1 + 3 = 4 \Rightarrow x < 4$ and $x > 1 \Rightarrow x \in (1, 4)$

Sol 12: (D) $x^{\log_3 x^2 + (\log_3 x)^2 - 10} = \frac{1}{x^2} = x^{-2}$

$\Rightarrow \log_3 x^2 + (\log_3 x)^2 - 10 = -2$

Assume $\log_3 x = y \rightarrow 2y + y^2 - 10 = -2$

$\Rightarrow y^2 + 2y - 10 + 2 = y^2 + 2y - 8 = 0$

$\Rightarrow (y + 4)(y - 2) = 0 \Rightarrow y = -4$ or $y = 2$

$x = 3^{-4} = \frac{1}{81}$; $x = 9$

$x = \left\{1, 9, \frac{1}{81}\right\}$

Sol 13: (A) $\frac{(ln)^2 - 3\,l\text{n}x + 3}{l\text{n}x - 1} < 1$

If $ln\,x - 1 > 0 \Rightarrow ln\,x > 1 \Rightarrow x > e$

$\Rightarrow (ln\,x)^2 - 3ln\,x + 3 < 1[(ln(x)) - 1]$

Assume $ln\,x = y$

$\Rightarrow y^2 - 3y + 3 < y - 1 \Rightarrow y^2 - 3y - y + 3 + 1 < 0$

$\Rightarrow y^2 - 4y + 4 < 0 \Rightarrow (y-2)^2 < 0$ always false

So if $\ln x < 1 \Rightarrow x < e$ and $x > 0$

$y^2 - 3y + 3 > (y - 1) \Rightarrow y^2 - 3y - y + 3 + 1 > 0$

$y^2 - 4y + 4 > 0 \Rightarrow (y-2)^2 > 0$ always true

So, $x \in (0, e)$

Multiple Correct Choice Type

Sol 14: (C, D) $N = \frac{1 + 2\log_3 2}{(1 + \log_3 2)^2} + \log_6^2 2$

$N = \frac{1 + 2\log_3 2}{(1 + \log_3 2)^2} + \left(\frac{\log_3 2}{\log_3 6}\right)^2$

Assume that $\log_3 2 = y$

$\Rightarrow N = \frac{1+2y}{(1+y)^2} + \frac{y^2}{(\log_3 2 + \log_3 3)^2}$

$\Rightarrow N = \frac{1+2y}{(1+y)^2} + \frac{y^2}{(1+y)^2} = \frac{y^2 + 2y + 1}{(1+y)^2}$

$\Rightarrow N = \frac{(1+y)^2}{(1+y)^2} = 1$

And $\pi = 3.147 > 3$ and $7 > 6$

So, $\log_3 \pi > 1$ and $\log_7 6 < 1$

Sol 15: (A, D) $2^{2x} - 8.2^x = -12$

Assume that $2^x = y$

$\Rightarrow y^2 - 8y = -12 \Rightarrow (y - 6)(y - 2) = 0 \Rightarrow y = 6$ or $y = 2$

$2^x = 6$; $2^x = 2^1$

$x \log_{10} 2 = \log_{10} 6 = \log_{10}(2 \times 3)$

$x = \frac{\log_{10} 2 + \log_{10} 3}{\log_{10} 2} = 1 + \frac{\log_{10} 3}{\log_{10} 2}; x = 1$

Sol 16: (A, B, C, D) $\left(\sqrt{5\sqrt{2} - 7}\right)^x + 6\left(\sqrt{5\sqrt{2} + 7}\right)^x = 7$

Assume $x = \log_{\sqrt{5\sqrt{2}-7}} y$

$\Rightarrow \left(\sqrt{5\sqrt{2} - 7}\right)^{\log_{\sqrt{5\sqrt{2}-7}} y} + 6\left(\sqrt{5\sqrt{2} + 7}\right)^{\log_{\sqrt{5\sqrt{2}-7}} y} = 7$

$\sqrt{5\sqrt{2} - 7} = \sqrt{5\sqrt{2} - 7} \times \frac{\sqrt{5\sqrt{2} + 7}}{\sqrt{5\sqrt{2} + 7}}$

$= \frac{\sqrt{50 - 49}}{\sqrt{5\sqrt{2} + 7}} = \left(\sqrt{5\sqrt{2} + 7}\right)^{-1}$

$\Rightarrow y + 6\left(\sqrt{5\sqrt{2} + 7}\right)^{-\log_{\sqrt{5\sqrt{2}+7}} y} = 7 \Rightarrow y + 6y^{-1} = 7$

$\Rightarrow y^2 + 6 = 7y \Rightarrow y^2 - 7y + 6 = 0$

$\Rightarrow (y - 6)(y - 1) = 0$ which gives $y = 6$ or $y = 1$

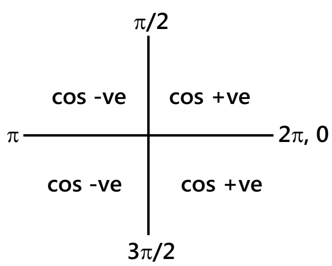
$x = \log_{\sqrt{5\sqrt{2}-7}} 6$ or $x = \log_{\sqrt{5\sqrt{2}-7}} 1 = 0$

$\Rightarrow x = \log_{(5\sqrt{2}-7)^{1/2}} 6 = 2\log_{(5\sqrt{2}-7)} 6 = \log_{(5\sqrt{2}-7)} 36$

$$x = \frac{2}{\log_6(5\sqrt{2}-7)} = \frac{-2}{\log_6(5\sqrt{2}+7)}$$

Assertion Reasoning Type

Sol 17: (D) Statement-I



$\sqrt{\log_x \cos(2\pi x)}$ is a meaningful quantity only if

$x \in \left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right)$

$\cos 2\pi x > 0 \Rightarrow \frac{\pi}{2} > 2\pi x > 0$

$\frac{1}{4} > x > 0$ and $x \neq 1$, $x > 0$

$\frac{3\pi}{2} < 2\pi x < 2\pi \Rightarrow \frac{3}{4} < x < 1$

So $x \in \left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right)$

But also $\log_x \cos(2\pi x) > 0 = \log_x 1$

$\cos 2\pi x > 1$ which is never possible

So statement-I is false

Statement-II If the number N > O and the base of the logarithm b(greater than zero not equal to)

Both lie on the same side of unity than $\log_b N > 0$ and if they lie on the different side of unity then $\log_b N < 0$ statement-II is true

Sol 18: (B) Statement-I

$\log_2(2\sqrt{17-2x}) = 1 + \log_2(x - 1)$ has a solution

$\Rightarrow 1 + \log_2(\sqrt{17-2x}) = 1 + \log_2(x - 1)$

$\Rightarrow \sqrt{17-2x} = (x - 1)$

Squaring both sides

$\Rightarrow 17 - 2x = (x - 1)^2 = x^2 - 2x + 1$

$\Rightarrow 17 = x^2 + 1 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

$\Rightarrow x \neq -4$ does not satisfy equation in statement-I

So x = 4. x has a solution

Statement-II

"Change of base in logarithm is possible" which is true but not the correct explanation for statement-I.

Sol 19: (B) Statement-I: $5^{\log_5(x^3+1)} - x^2 = 1$ have two distinct real solutions.

Statement-II: $a^{\log_a N} = N$ when $a > 0$ $a \neq 1$, $N > 0$

$\Rightarrow 5^{\log_5 x^3+1} - x^2 = 1$

$[5^{\log_5(x^3+1)} = x^3 + 1]$ from statement-II

$\Rightarrow x^3 + 1 - x^2 = 1 \Rightarrow x^3 - x^2 = 0$

$\Rightarrow x^3 = x^2 \Rightarrow x = 0$ or 1

Statement-I is true and II is true and II is not the correct explanation for statement -I.

Comprehension Type

Paragraph 1:

Sol 20: (D) $\log_x^3 10 - 6\log_x^2 10 + 11 \log_x 10 - 6 = 0$

Assume that $\log_x 10 = y$

$\Rightarrow y^3 - 6y^2 + 11y - 6 = 0$

$f(y) = y^3 - 6y^2 + 11y - 6$

$\frac{df(y)}{dy} = 3y^2 - 12y + 11 \to 0$

$$\Rightarrow y = \frac{12 \pm \sqrt{12^2 - 4 \times 3 \times 11}}{2(3)} = \frac{12 \pm \sqrt{12}}{6}$$

There is maxima and minima at

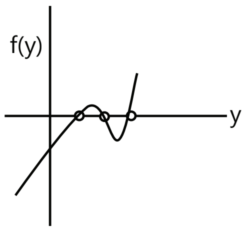
$$y = \frac{12 \pm \sqrt{12}}{6} = 2 \pm \frac{\sqrt{6}\sqrt{2}}{6} = 2 \pm \frac{\sqrt{2}}{6} = 2 \pm \frac{1}{\sqrt{3}}$$

At $y = 2 + \frac{1}{\sqrt{3}}$

$y^3 - 6y^2 + 11y - 6$ is negative and at $y = 2 - \frac{1}{\sqrt{3}}$,

Equation $y^3 - 6y^2 + 11y - 6$ is positive

So there is total 3 solutions for this equation



Match the Columns

Sol 21: A → q, r, s; B → p, q, r, s; C → p; D → r

(A)

$$\sqrt{3\sqrt{x}-\sqrt{7x+\sqrt{4x-1}}}\ \sqrt{2x+\sqrt{4x-1}}\ \sqrt{3\sqrt{x}+\sqrt{7x+\sqrt{4x-1}}} = 13$$

$$\sqrt{(3\sqrt{x}-\sqrt{7x+\sqrt{4x-1}})(3\sqrt{x}+\sqrt{7x+\sqrt{4x-1}})(\sqrt{2x+\sqrt{4x-1}})}$$

$$= \sqrt{\left(3\sqrt{x}\right)^2-\left(\sqrt{7x+\sqrt{4x-1}}\right)^2\left(2x+\sqrt{4x-1}\right)}$$

$$= \sqrt{\left(9x-7x-\sqrt{4x-1}\right)\left(2x+\sqrt{4x-1}\right)}$$

$$= \sqrt{\left(2x-\sqrt{4x-1}\right)\left(2x+\sqrt{4x-1}\right)}$$

$$= \sqrt{\left(2x\right)^2-\left(4x-1\right)} = 13 \Rightarrow \sqrt{\left(4x\right)^2-4x-1} = 13$$

$$\Rightarrow \left(2x-1\right) = 13 \Rightarrow x = \frac{14}{2} = 7$$

(B) $P(x) = x^7 - 3x^5 + x^3 - 7x^2 + 5$

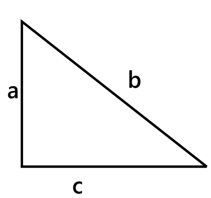
$Q(x) = x - 2$

Remainder $\frac{P(x)}{Q(x)}$

$Q(x) = 0$ at $x = 2$

So $P(2) = 2^7 - 3(2)^5 + 2^3 - 7(2)^2 + 5 = 17$

(C)



Area of triangle is

Area = $a^2 + b^2 - c^2$

Also, we have $b^2 = a^2 + c^2$

So area = $a^2 + (a^2 + c^2) - c^2 = \frac{1}{2} \times a \times c = \frac{ac}{2}$

$\Rightarrow 2a^2 = \frac{ac}{2} \Rightarrow 4 = \frac{ac}{a^2} = \frac{a}{c}$

$\Rightarrow$ ratio $= \frac{c}{a} = 4$

(D) a, b, c ∈ N

$\therefore ((4)^{1/3} + (2)^{1/3} - 2)(a(4)^{1/3}+b(2)^{1/3}+c)=20$

$= (2^{2/3}+2^{1/3}-2)(a2^{2/3} + b2^{1/3}+c)=20$

$\Rightarrow a(2^{4/3} + 2 - 2.2^{2/3}) + b[2^{3/3} + 2^{2/3} - 2.2^{1/3}] + c(2^{2/3}+2^{1/3}-2^{3/3})=20$

$\Rightarrow 2^{1/3}(2a - 2b + c) + 2^{3/3}(a + b - c) + 2^{2/3}(-2a + b + c)= 20$

$\Rightarrow a + b - c = \frac{20}{2} = 10$

Sol 22: A → p; B → p, r, s; C → p, r; D → p, q, r

(A) $x = \log_2 \log_9 \sqrt{6+\sqrt{6+\sqrt{6+......\infty}}}$

Assume that $x = \log_2 \log_9 y$

$\Rightarrow y = \sqrt{6+\sqrt{6+\sqrt{6+......\infty}}} = \sqrt{6+y}$

$\Rightarrow y^2 = 6+ y \Rightarrow y^2 - 6 - y = 0$

$\Rightarrow (y - 3)(y + 2) = 0 \Rightarrow y = 3$ or $y = -2$, $y \neq -2$

$\therefore$ y = 3

$x = \log_2 \log_9 3 = \log_2 \log_9 (9)^{1/2}$

$\Rightarrow x = \log_2\left(\frac{1}{2}\right) = \log_2 2^{-1} = -1$

$\Rightarrow x = -1$ is an integer

(B) $N = 2^{(\log_2 3.\log_3 4.\log_4 5......\log_{99} 100)}$

$N = 2^x$ (Assume)

$\Rightarrow x = \frac{\log 3}{\log 2}.\frac{\log 4}{\log 3}...\frac{\log 100}{\log 99} = \frac{\log 100}{\log 2} = \log_2 100$

$N = 2^{\log_2 100} = 100$

N = 100 which is a composite, integer, natural number

(C) $\frac{1}{\log_5 3} + \frac{1}{\log_6 3} \quad \frac{1}{\log_{10} 3}$

$$\Rightarrow \frac{\log 5}{\log 3} + \frac{\log 6}{\log 3} - \frac{\log 10}{\log 3} = \left(\frac{\log 5 + \log 6 - \log 10}{\log 3}\right)$$

$$\Rightarrow \frac{\log(5 \times 6 \div 10)}{\log 3} = \frac{\log 3}{\log 3} = 1$$

$\Rightarrow$ 1 is natural and integer number

(D) $N = \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$

$$N = \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{(3-\sqrt{5})^2}}}$$

$$N = \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+(-\sqrt{5}+3)}} = \sqrt{2+\sqrt{5}-\sqrt{9-4\sqrt{5}}}$$

$$N = \sqrt{2+\sqrt{5}-\sqrt{(\sqrt{5})^2+(2)^2-2(2)\sqrt{5}}}$$

$$N = \sqrt{2+\sqrt{5}-\sqrt{(\sqrt{5}-2)^2}} = \sqrt{2+\sqrt{5}-\sqrt{5}+2} = \sqrt{4} = 2$$

2 is natural prime and an integer.