PROBLEM SOLVING TACTICS

(a) Suppose two liquids of densities r_1 and r_2 having masses m_1 and m_2 are mixed together.

Then the density of the mixture will be = $\dfrac{(m_1 + m_2)}{\left(\dfrac{m_1}{\rho_1} + \dfrac{m_2}{\rho_2}\right)}$

If two liquids of densities r_1 and r_2 having volume V_1 and V_2 are mixed, then the density of the mixture will be

$\dfrac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$.

(b) When solving questions on Bernoulli's always assume a reference level and calculate the heights from the reference level.

FORMULAE SHEET

Fluid Statics:

1. Density = $\frac{\text{mass}}{\text{volume}}$, S.I. units: kg/m^3

2. Specific gravity / Relative density / Specific density = $\frac{\text{Ratio of its density}}{\text{Ratio of density of water at } 4°C}$,

S.I. units: No units

3. If two liquids of volume V_1 and V_2 and densities d_1 and d_2 respectively are mixed then the density d of the mixture is $d = \frac{V_1 d_1 + V_2 d_2}{V_1 + V_2}$; If $V_1 = V_2$ then $d = \frac{d_1 + d_2}{2}$

4. If two liquids of densities d_1 and d_2 and masses m_1 and m_2 respectively are mixed together,

then the density d of the mixture is $d = \frac{m_1 + m_2}{\frac{m_1}{d_1} + \frac{m_2}{d_2}}$; if $m_1 = m_2$ then $d = \frac{2d_1 d_2}{d_1 + d_2}$

5. Pressure = $\frac{\text{Normal component of force}}{\text{Area on which force acts}} = \frac{f}{A}$, S.I. units: N/m^2, Pa

6. Pressure P acting at the bottom of an open fluid column of height h and density d is

= 1.013 × 10^5 Pa = 1.013 × 10^5 Pa = 1.013 × 10^6 dynes/cm^2= 76 cm of Hg = 760 torr = 1.013 bars.

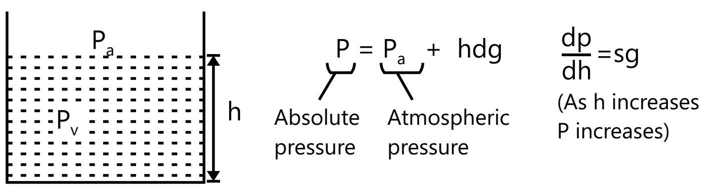


Figure 9.56

$P - P_a = hdg$

gauge pressure = absolute – atmospheric pressure.

7.

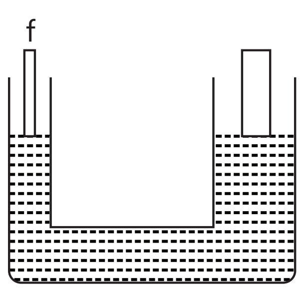


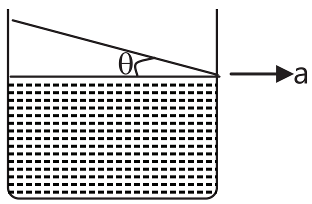
Figure 9.57

Area of smaller piston, a; area of larger piston, A, f is applied on the smaller piston

Force F developed on the larger piston $\frac{F}{A} = \frac{f}{a}$

$\therefore \quad F = \frac{fA}{a}$

8. Beaker is accelerated in horizontal direction



$\tan\theta = \frac{a}{g}$

a is the acceleration of the beaker in horizontal direction.

Figure 9.58

9. Beaker is accelerated and it has components of acceleration a_x, and a_y in x and y directions respectively.

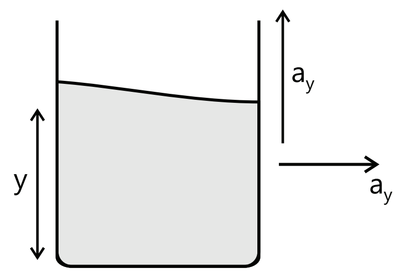


Figure 9.59

P increases with depth $\rightarrow \frac{dP}{dy} = p(g + ay)$

P is the density of the fluid.

ρ is the density of the fluid. $\frac{dP}{dx} = -pax$

10. Buoyant force $F = V_1\rho_1(\vec{g} - \vec{a})$

V_1 = immersed volume of liquid

ρ_1 = density of liquid

g = acceleration due to gravity

a = acceleration of body dipped inside liquid.

11. Body floats when Buoyant force balances the weight of the body.

$\underset{\text{(Buoyant force)}}{V_i\rho_2 g} = \underset{\text{(Weight of body)}}{V_b\rho_b g}$

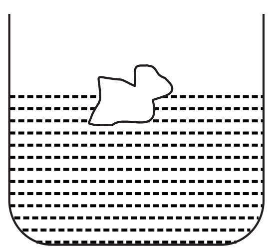


Figure 9.60

V_b, ρ_b = volume and density of body.

V_i = Volume of the immersed part of body.

ρ_2 = density of liquid.

Fraction of volume immersed $\frac{V_i}{V_b} = \frac{\rho_b}{\rho_2}$

% of volume immersed $\frac{V_i}{V_b} \times 100 = \frac{\rho_b}{\rho_2} \times 100.$

12. Apparent weight of a body inside a fluid is $W_{app} = W_{act}$ – Upthrust

$$W_{app} = V_b g (\rho_b - \rho_2)$$

$V_{b'}$, δ_b = volume and density of body.

V_i = Volume of the immersed part of body.

ρ_2 = density of liquid.

13. General equation of continuity

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ Generally $\rho_1 = \rho_2$ i.e., density is uniform.

A_1 & A_2 are area of cross-section at point P and Q.

V_1 & V_2 are velocities of the fluid at point P and Q.

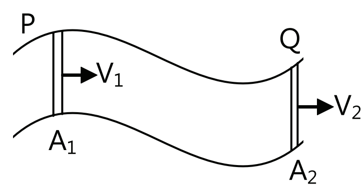


Figure 9.61

14. Bernoulli's Equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$$

i.e.,

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

Pressure Height Velocity at the point

at that point from the reference level

$$\frac{P}{\rho g} + \frac{V^2}{2g} + h = \text{constant}$$

Pressure head Velocity head gravitational head

15. Volumetric flow $Q = A\nu = \frac{dV}{dt}$ A – Area of cross section; ν – Velocity; V– Volume

S.I. unit = $\frac{m^3}{S}$

16. Torricelli Theorem:

$V = \sqrt{2gh}$ → height

velocity of efflux

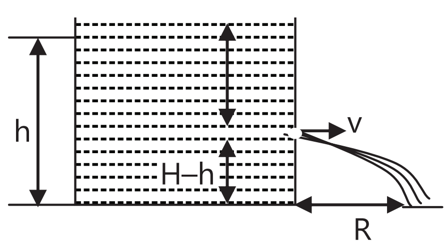


Figure 9.62

Range $R = 2\sqrt{h(H-h)}$

Range is maximum at $h = \frac{H}{2}$ and $R_{max} = H$

A_b – Area of orifice

A – Area of cross-section of the container.

Time taken to fall from H_1 to H_2 = $t \times \frac{A}{A_0}\sqrt{\frac{2}{g}}$

17. Viscous Force $F = \eta A \frac{dv}{dy}$

↓

coefficient of viscosity

L – Length of pipe

P_1 and P_2 are pressure at two ends of pipe.

R – Radius of pipe.

When liquid is flowing through a tube, velocity of flow of a liquid at distance from the axis.

$V = \frac{P}{4\eta L}\left(r^2 - x^2\right)$. Velocity distribution curve is a parabola.

18. Stoke's Law: Formula for the viscous force on a sphere

$F = 6\pi\eta rv$ (η – coefficient of viscosity)

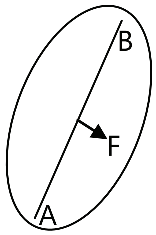
(r – radius of sphere)

(v – velocity of sphere)

$V_T = \frac{2}{9}r^2\frac{(\rho-\sigma)g}{\eta}$ (ρ – density of sphere)

($\propto$ – density of fluid)

19. Surface Tension



$T = \frac{F}{L}$

F is the total force acting on either side of AB.

L is length of AB.

Figure 9.63

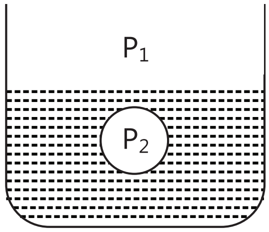
20. Surface Energy: dW = TdA

Surface Tension $T = \frac{dV}{dA} = \frac{\text{Surface energy}}{\text{Area}}$

21. Pressure inside the soap bubble is P, then

$P - P_0 = \frac{4T}{R}$

22. Air Bubble Inside a Liquid



$$P_2 - P_1 = \frac{2T}{R}$$

Figure 9.64

R – radius of bubble

T – surface tension force

23. Capillary Rise

$h = \dfrac{2T\cos\theta}{r\rho g}$ r = is the radius of capillary tube

θ = angle of contact