7. NUCLEAR CHEMISTRY AND ENVIRONMENTAL CHEMISTRY

NUCLEAR CHEMISTRY

1. INTRODUCTION

Nuclear chemistry is deals with nuclear reaction, radioactivity, nuclear processes and nuclear properties. It includes the study of radioactivity and use of radioactive sources for a range of processes.

1.1 Stability of Nucleus

Some nuclei are stable while others are unstable. The stability of an atom has been explained in terms of Columbic forces of attraction and forces of motion. The stability of the nucleus cannot be explained in terms of these forces because of similar charged particles present in nucleus. No single rule allows us to predict whether a particular nucleus is radioactive and how it might decay.

2. SOME IMPORTANT TERMINOLOGIES

2.1 Mass Defect

The mass difference between a nucleus and its constituent nucleons responsible for binding energy, is called the mass defect.

2.2 Binding Energy

The total energy given out during binding up of nucleons in the nucleus is known as **binding energy.** Greater the binding energy, lesser is the energy level of nucleus and thus, more is its stability.

Therefore, $B.E. = \Delta m \times c^2$ erg (m in g) (c in cm/sec)

$= 1.6605 \times 10^{-24} \times \Delta m' \times c^2$ erg ($\Delta m'$ in amu)

$= 1.6605 \times 10^{-24} \times \Delta m' \times (2.9979 \times 10^{10})^2$ erg

$= 14.923 \times 10^{-4} \times \Delta m'$ erg $= 14.923 \times 10^{-11} \times \Delta m'$ J (10^7 erg = 1 J)

$= \frac{14.923 \times 10^{-11} \times \Delta m'}{1.602 \times 10^{-19}}$ eV (1.602×10^{-19} J = 1 eV)

$$= \frac{14.923 \times 10^{-11} \times \Delta m'}{1.602 \times 10^{-19} \times 10^{6}} \text{MeV} \qquad (10^6 \text{ eV} = 1 \text{ MeV})$$

$$\simeq 931.478 \times \times \Delta m' \text{ MeV} \qquad \text{(Mega or Million electron volt)}$$

If m′ = 1 amu then, B.E. = 931.478 MeV, i.e., decay of 1 amu produces 931.478 MeV energy.

Also, 1 amu = 931.478 MeV

B.E. per nucleon, $\bar{B} = \frac{\text{Total B.E.}}{\text{No. of nucleons}}$

B.E. / nucleon has been found to increase with increase in atomic number (Fig. 1) and becomes maximum for $_{26}$Fe at 8.78 MeV. After Fe, it continuously decreases and becomes almost constant at 7.6 MeV for $_{82}$Pb and onwards.

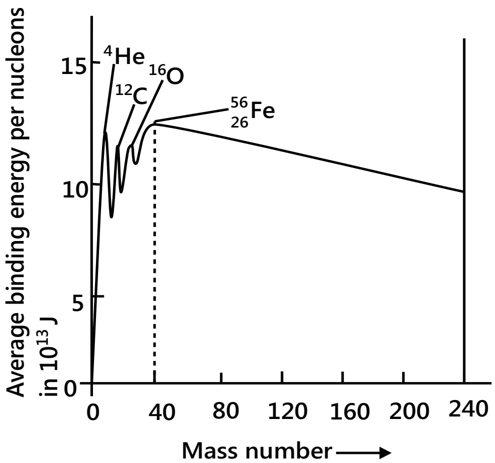

Figure 7.1: Plot of nuclear binding energy per nucleon against the mass number for naturally occurring nucleoids

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The greater the binding energy per nucleon, the more stable is the nucleus and thus $^{56}_{26}$Fe is the most stable nucleus.

Vaibhav Krishan (JEE 2009, AIR 22)

Illustration 1: The atomic masses of Li, He and proton are 6.041amu, 4.002602 amu and 1.00715 amu respectively.

Calculate the energy evolved in the reaction: $^7_3\text{Li} + ^1_1\text{H} \longrightarrow 2^4_2\text{He} + \Delta E$ **(JEE MAIN)**

Sol: First determine the total mass change after the reaction, it is calculated as follows,

Δm = (total mass of reactant)-(total mass of product)

And then determine the energy using the following expression

B.E =931.478 × × Δm′ MeV

Mass of elements undergoing decay = Mass of Li + Mass of proton = 7.01823 + 1.00715 = 8.02538 amu

Mass of products after decay = 2 × Mass of helium = 2 × 4.00387 = 8.00774 amu

∴ Mass decayed = 8.02538 – 8.00774 = 0.01764 amu

∴ Energy evolved during reaction = 0.01764 × 931.478 = 16.43 MeV

Illustration 2: Calculate the loss in mass during the change: $^7_3\text{Li} + ^1_1\text{H} \longrightarrow 2^4_2\text{He} + 17.25 \text{ MeV}$ **(JEE MAIN)**

Sol: Here change in energy is given by using the following relation loss in mass can be determined.

∵ ΔE = Δm × 931.478; ∴ $\Delta m = \frac{\Delta E}{931.478} = \frac{17.25}{931.478} = 0.0185$ amu; Δm = 3.07 × 10^{-26} g

Illustration 3: Calculate the mass defect and binding energy per nucleon for an alpha particle whose mass is 4.0028 amu, m_p = 1.0073 and m_n = 1.0087. **(JEE ADVANCED)**

Sol: Mass defect can be calculated by calculating the mass difference between a nucleus and its constituent nucleons. From the calculated mass defect binding energy can be calculated as B.E =931.478 × × Δm′ MeV

Mass of an α-particle = Mass of 2p + Mass of 2n = 2 × 1.0073 + 2 × 1.0087 = 4.032 amu

$\because$ Actual mass of α-particle = 4.0028 amu

$\therefore$ Mass decay = 4.032 – 4.0028 = 0.0292 amu, Also, B.E. = 0.0292 × 931.478 = 27.20 MeV

$\therefore$ B.E./nucleons = (27.19/4) = 6.80 MeV

Illustration 4: An isotopic species of lithium hydride $^6Li^2H$ is used as a potential nuclear fuel following the nuclear reaction: $^6_3Li^2_1H \longrightarrow 2^4_2He$

Calculate the expected power production of megawatt (Mw) associated with 1.00 g of $^6Li^2H$ per day assuming 100% efficiency. Given, ^6_3Li = 6.01512 amu; ^2_1H = 2.01410 amu and ^4_2He = 4.00260 amu. **(JEE ADVANCED)**

Sol: Mass decay, Δm per molecule of LiH = m($^6_3Li^2_1H$) – 2 × m ^4_2He

= (6.01512 + 2.01410) – 2 × 4.0026 = 0.02402 amu

Thus, energy produced during the mass decay

= Δm × 931.478 MeV = 0.02402 × 931.478 = 22.37 MeV = 22.37 × 10^6eV

= 22.37 × 10^6 × 1.602 × 10^{-19} J= 3.58 × 10^{-12} J

Now energy produced for 1 mole of LiH = 3.58 × 10^{-12} × 6.023 × 10^{23} = 21.56 × 10^{11} J mol^{-J}

Energy produced for 1 g of $^6Li^2H$ = $\frac{21.55\times10^{11}}{8}Jg^{-1}$ per day

Energy produced for 1 g of $^6Li^2H$ per sec = $\frac{21.55\times10^{11}}{8\times24\times3600}Jg^{-1}s^{-1}$ = 3.12 × 10^6 wg^{-1} = 3.12 Mwg^{-1} (Js^{-1} = 1w–1–)

2.3 The Neutron To Proton N/Z

Neutrons apparently help to hold protons together within the nucleus. The number of neutrons necessary to create a stable nucleus increases rapidly as the number of protons increases; the number of neutron to proton ratio of stable nuclei increases with increasing atomic number. The area within which all stable nuclei are found is known as the **belt of stability**. The majority of radioactive nuclei occur outside this belt.

The type of radioactive decay that a particular radio isotope will undergo depends to a large extent on its neutron to proton ratio.

(a) A nucleus with high n/p ratio which is placed above the belt of stability emits a b-particle in order to lower n/p ratio and move towards the belt of stability.

$$^1_0n \longrightarrow {^1_1p} + {^0_{-1}e} + \bar{\nu} \quad (\bar{\nu} \text{ is Anti neutrino})$$

e.g., $$^{82}_{34}Se \longrightarrow {^{82}_{36}Kr} + 2\,{^0_{-1}e}$$

(b) A nucleus with low n/p ratio (less than 1) which is placed below the belt of stability either emits protons or undergoes electron capture.

$$^1_1p \longrightarrow {^1_0n} + {^0_{+1}e} \;;\; \text{e.g. } {^1_1p} \longrightarrow {^1_0n} + {^0_{+1}e}$$

(Position emission)

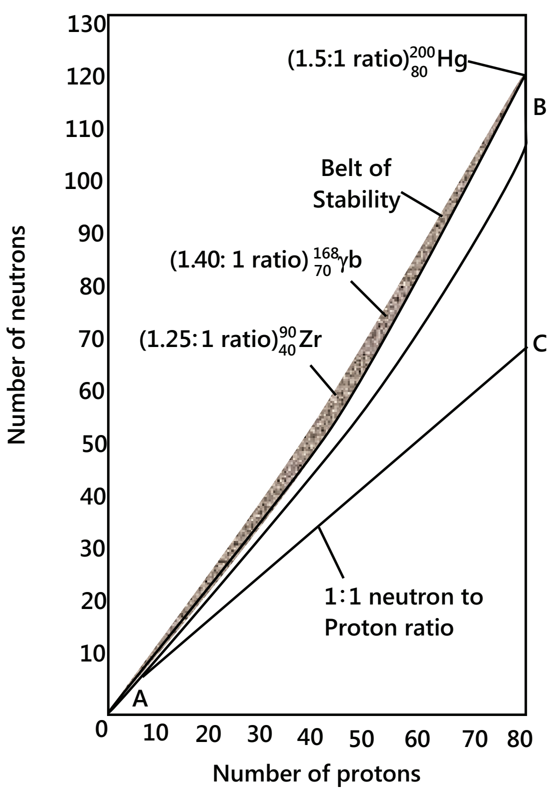

Figure 7.2: Plot of number of neutron vs number of proton

$^{1}_{+1}P + ^{0}_{-1}e \longrightarrow ^{1}_{0}n + X-rays$ (Electron capture); e.g. $^{133}_{56}Ba + \underset{(K-electron)}{^{0}_{-1}e} \longrightarrow ^{133}_{55}Cs + X-rays$

(c) The nuclei lying beyond the upper right edge (i.e., nuclei with atomic number > 83) outside the belt of stability undergo α-emission. Emission of an α-particle decreases both the number of protons and neutrons and thereby increases n/p ratio.

$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$; Thus, $\frac{n}{z} \geq 1.0$, unstable nuclei; $\frac{n}{z} \geq 1.0$, stable nuclei

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$\frac{n}{z} \geq 1.0$, unstable nuclei; $\frac{n}{z} \leq 1.0$, stable nuclei

- The number of stable nuclides is the maximum when both Z and n are even numbers. About 60% of stable nuclides have both Z and n even.
- The number of stable nuclides in which either the Z or n is odd is about one third of those, where both are even.

Nikhil Khandelwal (JEE 2009, AIR 94)

Table 7.1: Number of Stable nuclides

Z	n	Number of stable nuclides
even	even	166
even	odd	57
odd	even	53
odd	odd	8

2.3.1 The Magic Numbers

The nuclear shell model of nuclear structure is analogous to the electron shell model of atomic structure. Just as certain numbers of electrons (2, 8, 18, 36, 54 and 86) correspond to stable closed shell electron configuration, certain number of nucleons leads to closed shell in nuclei. The protons and neutrons can achieve a closed shell. Nuclei with 2, 8, 20, 28, 50 or 82 protons or 2, 8, 20, 28, 50, 82 or 126 neutrons correspond to closed nuclear shell. Closed shell nuclei are more stable than those that do not have a closed shell. These numbers of nucleons that correspond to closed nuclear shells are called magic numbers.

Illustration 5: There is an analogy between the stability of the nucleus ^{208}Pb and the lack of reactivity of argon gas. Comment. **(JEE MAIN)**

Sol: $^{40}_{18}Ar$ has a closed shell of electrons (2, 8, 8) which limits its reactivity. ^{208}Pb has a closed shell of protons and neutrons which leads to nuclear stability.

Illustration 6: Of the isotopes $^{114}_{48}Cd$, $^{118}_{50}Sn$ and $^{114}_{50}In$, which is likely to be radioactive and why? **(JEE MAIN)**

Sol: $^{114}_{49}In$ It has odd number of protons and odd number of neutrons.

2.3.2 Radioactivity

Henry Becquerel (1891) observed the spontaneous emission of invisible, penetrating rays from potassium uranyl sulphate $K_2UO_2(SO_4)_2$, which influenced photographic plate in dark and were able to produce luminosity in substances like ZnS. Later on, Marie Curie and her husband Pierre Curie named this phenomenon of spontaneous emission of penetrating rays as radioactivity. They also pointed out that radioactivity is a characteristic property of an unstable or excited nucleus, i.e., a nuclear property and is independent of all the external conditions, the nature of other atoms associated with the unstable atom but depends upon the amount of unstable atom.

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Radioactivity is a nucleus phenomenon and does not depend upon environmental conditions.

Rutherford identified two types of these penetrating rays and named them alpha (a) and beta (b) particles. Later on P. Villard identified and named, the third category as gamma (g) rays.

Vaibhav Krishan (JEE 2009, AIR 22)

Table 7.2: Properties of α, β-particles and γ-rays, Becquerel radiations

S.N.	Properties	Alpha	Beta	Gamma
1.	Nature	Fast moving He nuclei	Fast moving electron	High energy radiations
2.	Notation	$_2He^4$ or α	$_{-1}e^0$ or β	γ or $^0_0\gamma$
3.	Charge	2 unit (+ve)	1 unit (–ve)	No charge
4.	Typical source	Ra-226	C-14	Tc-99 m
5.	Velocity	1/10 of light	33% to 90% of light	Same as light waves
6.	Nature of path	Straight line	Crooked	Waves
7.	Relative penetrating power	1 or (0.01 mm of Al foil)	100 or (0.1 cm of Al foil)	10,000 or (8 cm lead or 25 cm steel)
8.	Travel distance in air	2-4 cm	200-300 cm	500 cm
9.	Tissue depth	0.05 mm	4-5 mm	50 cm or more
10.	Shielding	Paper, clothing	Heavy clothing, labcoats, gloves	Lead, thick concrete
11.	Mass g/particle	6.65×10^{-24}	9.11×10^{-28}	0
12.	Relative ionizing power	10,000	100	1
13.	Electrical or magnetic field's influence	Deflected towards –ve pole	Deflected towards +ve pole	Not deflected

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- β-particles are deflected more towards magnetic or electric field than α-particles because of their lower mass,
- γ-rays are identical to X-rays but their wavelength is much shorter than X-rays.

Saurabh Gupta (JEE 2010, AIR 443)

Illustration 7: Why does α-particles cause more damage to tissues than β-particles? **(JEE MAIN)**

Sol: Explain this by considering penetrating power of α-particles and β-particles.

Penetrating power of α being less and thus they provide all their energy at one spot damaging tissues.

Illustration 8: Why is an α-emitter more hazardous to an organism internally then externally, whereas γ-emitter is equally hazardous internally and externally? **(JEE ADVANCED)**

Sol: α-particles move relatively slowly and cannot penetrate too much externally. However inside the body α-particles give up their energy to surrounding tissues. Gamma rays move at the speed of light and have too much penetrating power. Thus, they are equally hazardous internally and externally.

3. THEORY OF NUCLEAR INTEGRATION: RUTHERFORD AND SODDY

The ejection of α, β particles and γ-rays from a radioactive material has been satisfactorily explained by Rutherford and Soddy.

Step I: An excited nucleus (a nucleus of low B.E. or higher energy level) tries to attain lower energy level in order to attain stability. Therefore, α-particles are emitted from the nucleus as an energy carrier.

Step II: During α-decay no doubt that energy level comes down but n/p ratio increases and therefore to decrease n/p ratio and attain stability, nucleus undergoes neutron decay or neutron transformation to show emission of β-particles. $_{0}^{1}n \longrightarrow {}_{+1}^{1}P + {}_{-1}^{0}e + v$

The energy of β-particles does not account for the difference in energy between the parent and daughter nuclei during neutron decay and therefore missing energy gave the existence of another particle called anti-neutrino.

Step III: The resultant nucleus after α-, β-emission still possesses higher energy level than required for its stability and this difference of energy comes out in form of γ-rays. Thus, gammaradiations are given off by the nuclei in an excited state.

Therefore, α, β-emissions are primary whereas γ-emissions are secondary emissions.

Illustration 9: Though a nucleus does not contain any particle of-ve charge, still the nucleus of a radioactive element emits β-rays. Explain. **(JEE MAIN)**

Sol: Neutron decay on account of higher n/p brings in β-emission: $_{0}^{1}n \longrightarrow {}_{1}^{1}p + {}_{?1}^{0}e + \bar{v}$

Illustration 10: Explain with reason the nature of emitted particle by: **(JEE ADVANCED)**

(A) $_{20}^{38}Ca$ (B) $_{18}^{35}Ar$ (C) $_{32}^{80}Ge$ (D) $_{79}^{173}Au$ (E) $_{20}^{40}Ca$ (F) $_{11}^{22}Na$ (G) $_{92}^{238}U$ (H) $_{16}^{35}S$ (I) $_{9}^{17}F$ (J) $_{4}^{10}Be$

Sol: By taking into account neutron to proton ratio suggest the nature of particle emitted by different isotopes.

(A) $_{20}^{38}Ca$ It has n/p = $\frac{18}{20}$ =0.9, which lies below the belt of stability and thus, positron emitter.

$_{20}^{38}Ca \longrightarrow {}_{19}^{38}K + {}_{+1}^{0}e$

(B) $_{18}^{35}Ar$: It has = $\frac{n}{p} = \frac{17}{18}$ =0.994, which lies below the belt of stability and thus, positron emitter

$_{18}^{35}Ar \longrightarrow {}_{17}^{35}Cl + {}_{+1}^{0}e$

(C) $_{32}^{80}Ge$: It has n/p = $\frac{48}{32}$ = 1.5, which lies above the belt of stability and thus, β-emitter.

$_{32}^{80}G \longrightarrow {}_{33}^{80}As + {}_{-1}^{0}e$

(D) $_{79}^{173}Au$: It has $\frac{n}{p} = \frac{94}{79}$ = 1.19, which lies above the belt of stability and thus, β-emitter,

$_{79}^{173}Au \longrightarrow {}_{80}^{173}Hg + {}_{-1}^{0}e$

(E) $_{11}^{22}Na$: It has both magic numbers p = 20, n =20 and thus, stable.

(F) $_{11}^{22}Na$: n/p = 1 and shows, positron emission. $_{11}^{22}Na \longrightarrow {}_{11}^{22}Na + {}_{+1}^{0}e$

(G) $^{238}_{92}U$ = n/p = 1.58 and shows, α-emission. $^{238}_{92}U \longrightarrow ^{234}_{90}Th + ^{4}_{2}He$

(H) $^{10}_{4}Be$: n/p = 1.18 and shows, (β-emission. $^{35}_{16}S \longrightarrow ^{354}_{17}Cl + ^{0}_{-1}e$

(I) $^{10}_{4}Be$: n/p = 0.88 and shows, positron emission. $^{17}_{9}F \longrightarrow ^{17}_{8}O + ^{0}_{+1}e$

(J) $^{10}_{4}Be$: n/p = 1.5, and shows β-emission. $^{10}_{4}Be \longrightarrow ^{10}_{5}B + ^{0}_{-1}e$

3.1 SODDY-FAJAN'S Rule or Group Displacement Law

(a) A radioactive atom on losing an α-particle shows a loss in mass no. by 4 units and a loss in atomic no. by 2 units. Thus, the newly formed element occupies two positions left to the parent element in the periodic table.

$$\underset{\text{II gp}}{^{226}_{88}Ra} \xrightarrow{-\alpha} \underset{\text{zero gp}}{^{222}_{86}Rn} \xrightarrow{-\alpha} \underset{\text{VI gp}}{^{218}_{84}Po}$$

(b) A radioactive atom on decay of a β-particle shows a gain in its atomic no. by 1 unit whereas, mass no. remains the same. Thus, newly formed element occupies one position right to the parent element in the periodic table.

$$\underset{\text{IV gp}}{^{214}_{82}Pb} \xrightarrow{-\beta} \underset{\text{V gp}}{^{214}_{83}Bi} \xrightarrow{-\beta} \underset{\text{VI gp}}{^{214}_{84}Po}$$

Illustration 11: $^{14}_{6}C$ Nuclide undergoes β-decay. What stable nuclide is formed? Give equation. **(JEE MAIN)**

Sol: $^{14}_{6}C \longrightarrow ^{14}_{7}N + ^{0}_{-1}e$

Illustration 12: Which one is more chemically reactive and which is more radioactive among $^{1}_{1}H$, $^{2}_{1}H$, $^{3}_{1}H$

(JEE MAIN)

Sol: Reactivity order: $^{1}_{1}H > ^{2}_{1}H > ^{3}_{1}H$, Radioactive: only $^{3}_{1}H$ is radioactive and β-emitter.

Illustration 13: Which of the following processes given below gives rise to (i) an increase in atomic no. (ii) an increase in n/p ratio (iii) a decrease in atomic no. (iv) a decrease in n/p ratio (v) emission of X-rays definitely.

(a) α-emission (b) β-emission (c) positron emission (d) K-electron capture **(JEE ADVANCED)**

Sol: (i) Increase in at. No: β-emission

(ii) Increase in n/p ratio: α-emission, K-electron capture

(iii) Decrease in at. no: positron emission, α-emission

(iv) Decrease in n/p ratio: β-emission

(v) X-rays emission: K-electron capture

Illustration 14: Calculate no. of α- and β-particles emitted when $^{238}_{92}U$ changes into radioactive $^{206}_{82}Pb$

(JEE ADVANCED)

Sol: Let a, b are the number of α-particles and β-particles given out during the change of $^{238}_{92}U$ to $^{206}_{82}Pb$, then

$$^{238}_{92}U \longrightarrow ^{206}_{82}Pb + a\,^{4}_{2}He + b\,^{0}_{-1}e$$

Equating mass no. on both sides of nuclear reaction 238=206 + 4a +b × 0 $\therefore$ a = 8

Now equating atomic no. on both sides of nuclear reaction

92 = 82+2a + b (–1) = 82 + 2 × 8 + b (–1) $\therefore$ b = 6

Thus, No. of α-particles emitted out = 8; No. of β-particles emitted out = 6

4. RATE OF RADIOACTIVE DECAY

Radioactive disintegration is an example of first order reaction, i.e. the rate of decay is directly proportional to the no. of atoms (amount) of the element present at that particular time.

A $\rightarrow$ Decay product (N_0 = No. of atoms at t = 0)

(N = No. of atoms left after t = t)

$-\frac{dN}{dt} \propto [N]^1 = \lambda N$. The negative sign indicates the decreasing trend of N with increasing time.

Where λ is the proportionality constant and is known as decay or disintegration or radioactive constant. Integration of this equation gives,

$-\ln N = \lambda + A$ (A is integration constant)

At t = 0, $N = N_0$ $\therefore$ $A = -\ln N_0$ $\therefore -\ln N = \lambda - \ln N_0$

OR $\ln \frac{N_0}{N} = \lambda$ or $\frac{N_0}{N} = e^{\lambda}$ ... (i)

OR $N = N_0 e^{-\lambda}$ ... (ii)

OR $\lambda = \frac{1}{t} \ln \frac{N_0}{N}$ ... (iii)

OR $\lambda = \frac{2.303}{t} \log_{10} \frac{N_0}{N}$ ... (iv)

OR $N = N_0 10^{-\lambda/2.303}$ ... (v)

Characteristics of rate of decay

(a) Rate of decay continuously decreases with time and obeys Ist order kinetics.

(b) Rate of decay as well as λ are independent of P and T.

(c) (a) Unit of rate of decay: disintegration per time (b) Unit of decay constant: time^{-1}.

(d) Time required to complete a definite fraction is independent of initial number of atoms (amount) of radioactive species, i.e. time required to complete nth fraction, $t_{1/n}$ $t_{1/n} \propto [N_0]^0$

(e) Fraction of atoms decayed or degree of decay (α) in time, t

$$\alpha = \frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0} = 1 - e^{-\lambda t} \quad \text{... (vi)}$$

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Time required to complete a definite fraction is independent of initial number of atoms (amount) of radioactive species,

Neeraj Toshniwal (JEE 2009, AIR 21)

Half-life period: $t = \frac{2.303}{\lambda}\log_{10}\left(\frac{N_0}{N}\right)$ If $t = t_{1/2}$; $N = \frac{N_0}{2}$, then $t_{1/2} = \left(\frac{2.303}{\lambda}\right) \log_{10}\left[\frac{N_0}{(N_0/2)}\right]$

or $t_{1/2} = \frac{2.303 \log_{10} 2}{\lambda} = \frac{2.303 \times 0.3010}{\lambda}$ $t_{1/2} = \frac{0.693}{\lambda}$ ... (vi)

It is clear from this expression that $t_{1/2}$ is independent of the initial no. of atoms.

Note: $t_{1/2}$ is independent of the initial no of atoms.

Average life: $T_{av} = \dfrac{\text{Sum of lives of all atoms}}{\text{Total number of atoms}}$

Average life: $T_{av} = \dfrac{\text{Total life of all atoms}}{N_0} = \int_0^{\infty} \dfrac{t\,dN}{N_0} = \int_0^{\infty} \dfrac{-t\lambda N dt}{N_0}$ $\left(\because -\dfrac{dN}{dt} = \lambda.N\right)$

$$= \int_0^{\infty} \frac{-t\lambda N_0 e^{-\lambda t} dt}{N_0} \qquad (\because N = N_0 e^{-\lambda})$$

$T_{av} = \dfrac{1}{\lambda} = \int_0^{\infty} -t\lambda e^{-\lambda t}\,dt = \dfrac{1}{\lambda}$; $N = N_0 e^{-\lambda}$, if $t = \dfrac{1}{\lambda} = N_0 e^{-1} = \dfrac{N_0}{e} = \dfrac{N_0}{2.718} = 0.37\ N_0$

Decay constant can also be defined as the reciprocal of time in which radioactive atoms of a species reduces to 37% of its initial value.

Amount left after I half = $\dfrac{N_0}{2}$; Amount left after II half = $\dfrac{N_0}{2^2}$

$\therefore$ Amount left after n halves = $\dfrac{N_0}{2^n}$ $\therefore$ Amount decayed after n halves = $N_0 - \dfrac{N_0}{2^n} = \dfrac{N_0[2^n - 1]}{2^n}$

Total time (T) = no. of halves × half life $T = n \times t_{1/2}$

Note: Amount left after n halves = $\dfrac{N_0}{2^n}$

Illustration 15: Half-life for a radioactive substance is 5 hours. Calculate the % left in 2.5 and 10 hours? When will whole of the matter disappear? Will the decay law always apply? **(JEE MAIN)**

Sol: $T = n \times t_{1/2}$, $n = \dfrac{2.5}{5} = \dfrac{1}{2}$

$\therefore$ Amount left in 2.5 hours= $\dfrac{100}{(2)^{1/2}}$ = 70.71 % Also, $T = n \times t_{1/2}$; $n = \dfrac{10}{5} = 2$

$\therefore$ Amount left = $\dfrac{100}{2^2}$ =25%

The whole matter will never disappear. The decay law does not apply when the matter contains minimum amount or number of atoms say one atom.

Illustration 16: The number of radioactive atoms of a radio isotope falls to 12.5% in 90 days. Compute the half-life and decay constant of isotope. **(JEE MAIN)**

Sol: Using the equation of radioactive decay first calculate the disintegration constant and then by using the relation of disintegration constant and half-life term calculate the half-life of the radioactive atom.

Given, N = 12.5, N_0 = 100, t = 90 days

$\therefore \quad \lambda = \dfrac{2.303}{t}\log\dfrac{N_0}{N} = \dfrac{2.303}{90}\log\dfrac{100}{12.5} = 2.31 \times 10^{-2}\ day^{-1}$ and $t_{1/2} = \dfrac{0.693}{\lambda} = \dfrac{0.693}{2.31\times10^{-2}}$ = 30 days

Illustration 17: Prove that time required for 99.9% decay of a radioactive species is almost 10 times to its half-life period. **(JEE ADVANCED)**

Sol: N_0 = 100, for 99.9% decay N = 100 – 99.9 = 0.1

$$\therefore\ t_{99.9\%} = \frac{2.303}{\lambda}\log\frac{100}{0.1} = \frac{2.303}{\lambda}\log 1000 = \frac{2.303}{\lambda}\times 3 \qquad \text{... (i)}$$

Also for 50% decay: N = 100 – 50 = 50

$t_{50\%} = \frac{2.303}{\lambda}\log\frac{100}{50} = \frac{2.303}{\lambda}\log 2 = \frac{2.303}{\lambda}\times 0.3010$ ... (ii)

By equation (i) and (ii) $t_{99.9\%} \approx t_{50\%} \times 10$

Illustration 18: The rate of decay of a radioactive species is r_1 at time t_1 and r_2 at time t_2. If $t_2 > t_1$, what is the mean life of the sample? Also calculate the number of atoms disintegrated in the time $(t_2 - t_1)$ if half-life is $t_{1/2}$.

(JEE ADVANCED)

Sol: $r_1 = \lambda.\ N_1$ ... (i)

$r_2 = \lambda.\ N_2$ ... (ii)

$\frac{r_1}{r_2} = \frac{N_1}{N_2}$ Let initial rate be $r_0 t_1 = \frac{2.303}{\lambda}\log\frac{r_0}{r_1}$ $t_2 = \frac{2.303}{\lambda}\log\frac{r_0}{r_2}$

$t_2 - t_1 = \frac{2.303}{\lambda}[\log r_0 - \log r_2 - \log r_0 + \log r_1]$; $t_2 - t_1 = \frac{2.303}{\lambda}\log\frac{r_1}{r_2}$

Also Average life is $\frac{1}{\lambda} = \frac{t_2 - t_1}{2.303\log\frac{r_1}{r_2}}$

From (i) and (ii) $\frac{r_1 - r_2}{\lambda} = N_1 - N_2$ (atoms decayed) in time $(t_2 - t_1)$ $\therefore N_1 - N_2 = \frac{r_1 - r_2}{0.693} \times t_{1/2}$

5. ACTIVITY OF A RADIOACTIVE SUBSTANCE: ITS DETECTION AND UNITS

Activity = Rate of disintegration of radioactive substance $-\frac{dN}{dt} \propto N$ or $-\frac{dN}{dt} = IN$

or Activity = λN Activity = $\frac{0.693 \times N}{t_{1/2}}$ i.e. Activity = $\frac{0.6931 \times \text{Number of atoms present}}{\text{Half-life}}$

This shows that the activity of a radioactive substance is inversely proportional to its half-life. The greater the half-life of the substance, lesser is its activity and vice-versa.

MASTERJEE CONCEPTS

The greater the half-life of the substance, lesser is its activity and vice-versa.

Curie: If a radioactive substance disintegrates at the rate of 3.7×10^{10} disintegrations per second, its activity is said to be one Curie, i.e., one Curie = 1 Ci = 3.7×10^{10} disintegrations per sec (dps). Sometimes smaller units are also used viz.

1 millicurie = 1m Ci = 10^{-3} Ci ; 1 microcurie = 1 μ Ci = 10^{-6} Ci

The S.I. unit of radioactivity is proposed as Becquerel or Bq which refers to one dps.

Rohit Kumar (JEE 2012, AIR 79)

Illustration 19: Calculate the time in which the activity of an element reduces to 90% of its original value. The half-life period of element is 1.4×10^{10} year. **(JEE MAIN)**

Sol: Here activity is given, as radioactive decay is directly proportional to no of atoms substitute the value for $\frac{N_0}{N_1}$ in the equation of radioactive decay and calculate the time.

Given $r_1 = (90/100)r_0$ at time t

$\therefore \quad \frac{r_0}{r_1} = \frac{100}{90}$ at time t ; $\quad \because$ Rate of decay $\propto$ No. of atoms

$\frac{r_0}{r_1} = \frac{N_0}{N_1} = \frac{100}{90}$ Now, $t = \frac{2.303}{\lambda}\log\frac{N_0}{N_1}$; $t = \frac{2.303 \times 1.4 \times 10^{10}}{0.693}\log\frac{100}{90}$ $t = 2.128 \times 10^9$ year

Illustration 20: What mass of ^{14}C with $t_{1/2}$ = 5730 years has activity equal to one curie? **(JEE MAIN)**

Sol: 1 curie = 3.7×10^{10} disintegration per second; Rate = 3.7×10^{10} dps

Since, Rate = λ × No. of atoms; $3.7 \times 10^{10} = \frac{0.693}{5730 \times 365 \times 24021560 \times 60}$ × Number of atoms

No of atoms = 9.65×10^{21} Q 6.023×10^{23} atoms of ^{14}C = 14 g

9.65×10^{21} atoms of $^{14}C = \frac{14 \times 9.65 \times 10^{21}}{6.023 \times 10^{23}}$ = 0.2243 g

Illustration 21: The decay constant for an α-decay of ^{232}Th is 1.58×10^{-10} sec^{-1}. Find the number of α-decays that occur for a 1 g sample in 365 days. **(JEE ADVANCED)**

Sol: By using the expression for radioactive decay calculate the no of atom undergoing radioactive decay from this Find the number of α-decay that occur from 1 g sample in 365 day.

$t = \frac{2.303}{\lambda}\log\frac{N_0}{N}$ ($\because$ N_0/N is ratio and thus it may be taken as ratio of atoms, weights or g-atoms of elements undergoing decay)

$\therefore 365 \times 24 \times 60 \times 60 = \frac{2.303}{1.58 \times 10^{-10}}\log\frac{1}{N}$ (N_0 = 1 g) $\quad \therefore$ N = 0.995 g

$\therefore$ Amount of Th undergoing decay = $N_0 - N$ = 1 – 0.995 = 0.005 g $\because$ ^{232}Th is an α-emitter

$\because$ 232 g Th on decay produces 6.023×10^{23} α-atom

$\therefore$ 0.005 g Th on decay produces = $\frac{6.023 \times 10^{23} \times 0.005}{232}$ = 1.298×10^{19} α-atoms

Illustration 22: Calculate the decay constant and the average life of ^{55}Co radio nuclide if its activity is known to decrease 4% per hour. Assume that decay product of ^{55}Co is non-radioactive. **(JEE ADVANCED)**

Sol: $A_0 = \lambda N_0$ $\quad A = \lambda.N \quad \frac{A_0}{A} = \frac{N_0}{N} = e^{\lambda t}$ $\quad$ or $\quad \frac{A}{A_0} = e^{-\lambda t}$

The decrease in A_0 when t = 1 hr is 4%, i.e. a decrease of 0.04 A_0, i.e. 0.04

Activity (A) left after 1 hr = $A_0 - 0.04\,A_0 = 0.96\,A_0$

$\therefore \frac{A_0}{0.96A_0} = e^{\lambda.1}$ or $\quad 0.96 = e^{-\lambda}$ $\quad \lambda = 4.08 \times 10^{-2}\ hr^{-1} = 1.14 \times 10^{-5}\ sec^{-1}$

6. RADIOACTIVE SERIES

A series of nuclear reactions that begins with an unstable nucleus and terminates with a stable one is known as radioactive series or a nuclear disintegration series.

$^{232}_{90}Th \longrightarrow {}^{208}_{82}Pb + 6\,{}^{4}_{2}He + 4\,{}^{0}_{-1}e$ (4n series)

$^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + 8\,{}^{4}_{2}He + 6\,{}^{0}_{-1}e$ (4n + 2 series)

$^{235}_{92}U \longrightarrow {}^{207}_{82}Pb + 7\,{}^{4}_{2}He + 4\,{}^{0}_{-1}e$ (4n + 3 series)

The parent element, intermediates and final stable element of ^{232}Th series have masses which are integral multiples of 4. That is why this series is known as 4n series. Similarly, ^{238}U and ^{235}U series are known as (4n + 2) and (4n + 3) series.

Illustration 23: In which radioactive series does the following appear during disintegrations **(JEE MAIN)**

(A) $^{228}_{89}Ac$ (B) $^{227}_{89}Ac$ (D) $^{214}_{84}Po$

Sol: (A) 4n, (B) 4n + 3, (C) 4n + 2

7. RADIOACTIVE EQUILIBRIUM

Radioactive change is an irreversible process but it shows equilibrium when a daughter element disintegrates at the same rate at which it is formed from parent element.

$$\underset{\text{Parent}}{A} \longrightarrow \underset{\substack{\text{Intermediate}\\ \text{daughter}}}{B} \longrightarrow C$$

Maximum yield of daughter element: A radioactive element A decays to give a daughter element B which further decays to another daughter element C and so on till a stable element is formed (A→B→C). Also if the number of daughter atoms at t = 0 is zero and parent atom is much more lived than daughter (i.e., $\lambda_A < \lambda_B$), where λ_A and λ_B are decay constant of A and B respectively, then number of atoms of daughter element B after time t is, $N_B = \frac{N_0\lambda_A}{\lambda_B - \lambda_A}[e^{-\lambda_A t} - e^{-\lambda_B . t}]$

Maximum activity of daughter element can be expressed as t_{max}: $t_{max.} = \frac{2.303}{\lambda_B - \lambda_A}\log_{10}\left[\frac{\lambda_B}{\lambda_A}\right]$

Illustration 24: The half-life of ^{212}Pb is 10.6 hours. It undergoes decay to its daughter (unstable) element ^{212}Bi of half-life 60.5 minutes. Calculate the time at which the daughter element will have maximum activity? **(JEE ADVANCED)**

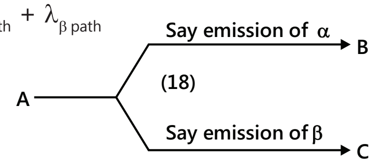
Sol: First find out the disintegration constant for parent and daughter element. By substituting the value in the following equation will give you t_{max} (time at which daughter element will have maximum activity)

$t_{max.} = \frac{2.303}{\lambda_{Bi} - \lambda_{Pb}}\log_{10}\frac{\lambda_{Bi}}{\lambda_{Pb}}$; $\lambda_{Pb} = \frac{0.693}{10.6 \times 60} = 1.0896 \times 10^{-3}$; $\lambda_{Bi} = \frac{0.693}{60.5} = 11.45 \times 10^{-3}$

$t_{max.} = \frac{2.303}{\lambda_{Bi} - \lambda_{Pb}}\log_{10}\frac{\lambda_{Bi}}{\lambda_{Pb}} = \frac{2.303}{10.3604 \times 10^{-3}}\log_{10}\frac{11.45 \times 10^{-3}}{1.0896 \times 10^{-3}}$ $t_{max.} = 227.1$ minute

Parallel path decay: A radioactive element A decays to B and C in two parallel paths as:

The average decay constant for the element A can be expressed as: $\lambda_{average} = \lambda_{\alpha\ path} + \lambda_{\beta\ path}$



$\lambda_{\alpha\ path}$ = [Fractional yield of B] × $\lambda_{average}$

$\lambda_{\beta\ path}$ = [Fractional yield of C] × $\lambda_{average}$

Illustration 25: The mean lives of a radioactive substance are 1620 years and 405 years for α-emission and β-emission respectively. Find out the time during which three-fourth of a sample will decay if it is decaying both by α-emission and β-emission simultaneously. **(JEE ADVANCED)**

Sol: For successive α, β-emissions.

λ average = $\lambda_\alpha + \lambda_b$ $= \frac{1}{1620} + \frac{1}{405} = \frac{5}{1620}$ year^{-1}

Let $\frac{3}{4}$ sample decays in time t hr, then $N = (N_0/4)$; $\because t = \frac{2.303}{\lambda}\log\frac{N_0}{N}$; $\therefore t = \frac{2.303 \times 1620}{5}\log 4 = 449.24$ year

8. SOME IMPORTANT TERMS

Isotopes: 1.Atoms of same the element having the same atomic no. but different mass no. are known as isotopes, e.g., ${}^{16}_{8}O$, ${}^{17}_{8}O$, ${}^{18}_{8}O$

Isobars: 1. Atoms of different elements having same mass no. are known isobars, e.g. ${}^{40}_{18}Ar$, ${}^{40}_{19}K$, ${}^{40}_{20}Ca$

Isotones: 1. Atoms having same no. of neutrons are called isotones, e.g. ${}^{2}_{1}H$ and ${}^{3}_{2}He$

2. Mass no. – atomic no. = constant (i.e. no. of neutrons)

Isoelectronics: 1. Atom and ions having same no. of electrons are called as isoelectronics.

2. E.g. N^{3-}, O^{2-}, F^{-}, Ne, Na^{+} Mg^{2+}, Al^{3+}

Isodiaphers: 1. Atoms having the same difference of neutron and proton or same isotopic number.

2. Nuclide and its decay product formed after β-emission are called isodiaphers.

Isosters: 1. Molecules having the same no. of atoms and same no. of electrons.

2. E.g. CO_2 and N_2O.

Nuclear Isomers: Nuclides having identical atomic no. and mass no. but differing in radioactive decay are known as nuclear isomers.

E.g. ${}^{60}Co$ and ${}^{60m}Co$, ${}^{69}Zn$ and ${}^{69m}Zn$, ${}^{80}Br$ and ${}^{80m}Br$ or like U_A and Uz.

The symbol m with mass no. represents the metastable state of parent element.

$${}^{60m}Co \xrightarrow[\text{transition}]{\text{Isomeric}} {}^{60}Co + \gamma\text{-rays}$$

Illustration26: Match the followings: **(JEE MAIN)**

A. Isotopes	A. ${}^{16}_{8}O$ and ${}^{17}_{8}O$
B. Isobars	B. Na^{+}, Mg^{2+}, F^{-}
C. Nuclear isomers	C. ${}^{2}_{1}H$ and ${}^{3}_{2}He$
D. Isosters	D. U_A and U_Z
E. Isotones	E. CO_2 and N_2O
F. Isoelectronics	F. ${}^{Z}_{A}X$, ${}^{Z-4}_{A-2}Y$
G. Isodiaphers	G. ${}^{40}_{20}Ca$ and ${}^{40}_{19}K$

Sol: A → p, B → v, C → s, D → t, E → r, F → q, G → u

9. NUCLEAR REACTIONS

Types of nuclear reactions: Some of the nuclear reactions are cited below:

(a) **Induced radioactivity:** The phenomenon of converting stable nuclei into unstable nuclei by the interaction of nucleons or a nuclear reaction yielding a product of nuclei of radioactive nature, is known as induced or artificial radioactivity (Irene Curie and F. Joliot).

$${}^{10}_{5}B + {}^{4}_{2}He \longrightarrow {}^{13}_{6}C + {}^{1}_{1}H$$

$${}^{10}_{5}B + {}^{4}_{2}He \longrightarrow {}^{13}_{7}N + {}^{1}_{0}n \xrightarrow[\text{Artificial radioactivity}]{\text{Nuclear reaction, Vis-a-Vis}} {}^{13}_{6}C + {}^{0}_{+1}e$$

(b) **Nuclear Fission:**

(i) The phenomenon of splitting up of a heavy nucleus, on bombarΔment with slow speed neutrons, into two fragments of comparable mass, with the release of two or more fast moving neutrons and a large amount of energy, is known as nuclear fission.

(ii) $^{235}_{92}U + ^{1}_{0}n + \longrightarrow ^{235}_{92}U \longrightarrow$
- $^{140}_{56}Ba + ^{93}_{36}Kr + 3\,^{1}_{0}n$
- $^{144}_{54}Xe + ^{90}_{38}Sr + 2\,^{1}_{0}n$
- $^{144}_{55}Cs + ^{90}_{37}Rb + 2\,^{1}_{0}n$

(iii) A loss in mass occurs releasing a vast quantity of energy $\simeq 2.041 \times 10^{10}$ kJ per mol of ^{235}U

(iv) Fission of 1 g ^{235}U releases energy = $\dfrac{20.41 \times 10^9}{235}$ kJ = 8.68×10^7 kJ

Fission of 1 mole ^{235}U releases energy = 20.41×10^9 kJ

Fission of one atom of ^{235}U releases energy = 211.5 MeV

(v) The secondary neutrons (formed during fission) further cause fission and thus, set up chain reaction, giving out huge amounts of energy. Nuclear fission reactions are self-multiplying or self-sustaining reactions. Once, a nuclear fission starts, it continues till the end and does not need bombardment of neutron from outside as neutrons formed by decay of ^{235}U are used for further decay with a small lump of mass of ^{235}U. Most of the neutrons released during fission escape but if the mass of ^{235}U exceeds a definite value (called critical mass) neutrons emitted during fission (on an average 2.5 neutrons per ^{235}U nucleus) are absorbed by ^{235}U to develop a chain reaction. This minimum amount of fissionable nuclei which develops a self-sustaining chain reaction is called critical mass. If mass of fissionable nuclei is lesser (i.e. subcritical mass) than the critical mass, the neutrons released during fission escape and the fission stops, but if the mass of fissionable nuclei is more (i.e. super critical mass) than the critical mass, the fission develops violently producing explosion. For a chain reaction, multiplication factor (K) should be greater than 1 and is given by:

$$K = \frac{\text{Number of neutrons produced in one step of fission}}{\text{Number of neutrons produced in one step to this fission}}$$

(vi) Nuclear fission is an uncontrolled reaction in an atom bomb whereas, in nuclear reactors, it is controlled by using a control rod of boron, steel or Cd which capture some of the neutrons so that chain reaction does not become violent, slowing down the speed of neutrons using moderators, e.g., D_2O, graphite, so that neutrons can be captured more readily by the fuel. A circulating coolant (water, molten Na) is employed to extract the heat from the reactor which is used for power production. The coolant liquid can also serve as the neutron moderator.

(vii) ^{238}U does not show fission by slow speed neutrons and that is why refining of uranium is necessary before it is used as nuclear fuel in nuclear reactors. Natural uranium consists of 99.3% ^{238}U + 0.7% ^{235}U.

(viii) A breeder reactor is one that produces more fissionable nuclei than it consumes, e.g. when $^{238}_{92}U$ is bombarded with fast neutron, it produces ^{239}Pu, a fissionable nuclei.

$$^{238}_{92}U + ^{1}_{0}n \longrightarrow ^{239}_{92}U \longrightarrow ^{239}_{93}Np + ^{0}_{-1}e\ ;\ ^{239}_{93}Np \longrightarrow ^{239}_{94}Pu + ^{0}_{-1}e\ ;\ ^{239}_{94}Pu + ^{1}_{0}n \rightarrow \text{Fission products} + 2 \text{ or } 3\left(^{1}_{0}n\right)$$

(c) Nuclear Fusion:

(i) The phenomenon of joining up of two light nuclei into a heavier nucleus is called fusion,

$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n$; ($\Delta H = -17.6$ MeV)

(ii) Huge amount of energy is required to overpower the Coulombic forces of repulsion between two nuclei which is obtained by triggering on nuclear fission.

(iii) About 0.231% of total mass decay occurs to liberate fantastically high energy during fusion.

(iv) The temperature corresponding to nuclear fusion is about 1.2×10^7 K. This requisite condition for fusion reaction exists in the stars and in the sun. Although the sun's surface temperature is only about 6000 K, its internal temperature is as high as 1.5 × 10^7 K. Under these conditions, H nuclei undergo fusion to form helium nuclei and in the process a continuous emission of solar energy occurs. Therefore, fusion is also referred as thermonuclear reactions.

(v) It is an uncontrolled reaction and the principle is used in the formation of H-bombs.

Illustration 27: Complete the following: **(JEE MAIN)**

(i) ${}_4^9Be + {}_2^4He \longrightarrow {}_6^{12}C + ?$; (ii) ${}_1^3T + {}_1^2D \longrightarrow {}_2^4He + ?$;

Sol: (i) ${}_4^9Be + {}_2^4He \longrightarrow {}_6^{12}C + {}_0^1n$; (ii) ${}_1^3T + {}_1^2D \longrightarrow {}_2^4He + {}_0^1n$;

Illustration 28: When ^{24}Mg is bombarded with neutrons, protons are ejected. Complete the equation and report the new element formed. **(JEE MAIN)**

Sol: ${}_{12}^{24}Mg + {}_0^1n \longrightarrow {}_{11}^{24}Na + {}_1^1H$

Illustration 29: Write a balanced equation for the following nuclear reaction

${}_{92}^{235}U$ bombarded with neutron to given ${}_{38}^{90}Sr$ and ${}_{54}^{143}Xe$ **(JEE ADVANCED)**

Sol: ${}_{92}^{235}U + {}_0^1n \longrightarrow {}_{38}^{90}Sr + {}_{54}^{143}Xe + 3{}_0^1n$

Illustration 30: What are X and Y in each of the reaction? **(JEE MAIN)**

(a) ${}_{42}^{99}Mo \longrightarrow {}_{43}^{99}Tc + X$; ${}_{43}^{99}Tc \longrightarrow {}_{44}^{99}Ru + Y$

(b) ${}_7^{14}N + {}_2^4He \longrightarrow {}_8^{17}O + X$ (c) ${}_1^1H + {}_2^7Li \longrightarrow 2X$ (d) $2\,{}_1^2D \longrightarrow {}_2^3He + X$

Sol: (a) ${}_{42}^{99}Mo \longrightarrow {}_{-1}^{0}e + {}_{43}^{99}TC$ ${}_{43}^{99}Tc \longrightarrow {}_{-1}^{0}e + {}_{44}^{99}Ru$

(b) ${}_7^{14}N + {}_2^4He \longrightarrow {}_8^{17}O + {}_1^1H$ (c) ${}_1^1H + {}_3^7Li \longrightarrow 2{}_2^4He$ (d) $2\,{}_1^2D \longrightarrow {}_2^3He + {}_0^1n$

10. APPLICATIONS OF RADIOACTIVITY AND RADIOISOTOPE

(a) Artificial transmutation: Many isotopes of elements which were either found in traces or not found in nature were made by nuclear reactions.

${}_7^{14}N + {}_0^1n \longrightarrow {}_6^{14}C + {}_1^1p$	(n, p) type;	${}_1^1H + {}_0^1n \longrightarrow {}_1^2H + \gamma$	(n, γ) type
${}_{13}^{27}Al + {}_0^1n \longrightarrow {}_{13}^{26}Al + 2\,{}_0^1n$	(n, 2n) type;	${}_{19}^{19}F + {}_2^4He \longrightarrow {}_{10}^{22}Ne + {}_1^1p$	(α, p) type
${}_{26}^{59}Fe + {}_2^4He \longrightarrow {}_{29}^{63}Cu + {}_{-1}^{0}e$	(α, β) type;	${}_8^{16}O + {}_1^1p \longrightarrow {}_7^{13}N + {}_2^4He$	(p, α) type
${}_5^{11}B + {}_1^1p \longrightarrow {}_6^{11}C + {}_0^1n$	(p, n) type;	${}_6^{12}C + {}_1^1p \longrightarrow {}_7^{13}N + \gamma$	(p, γ) type
${}_4^9Be + {}_1^1p \longrightarrow {}_4^8Be + {}_1^2H$	(p, D) type;	${}_{15}^{31}P + {}_1^2H \longrightarrow {}_{15}^{13}P + {}_0^1n$	(D, p) type
${}_{13}^{27}Al + \gamma \longrightarrow {}_{11}^{24}Na + 2{}_1^1p + {}_0^1n$			

(b) Dating: The determination of the age of minerals and rocks, an important part of geological studies involves determination of either a species formed during a radioactive decay or the residual activity of an isotope which is undergoing decay. For example ${}_{92}^{238}U$ undergoes a decay ($t_{1/2}$ 4.5 × 10^9 yrs) series forming a stable isotope ${}_{82}^{206}Pb$ and

He. Helium obtained as a result of decay of $^{238}_{92}U$ has almost certainly been formed from α-particles. Thus, if ^{238}U and He contents are known in a rock we can determine the age of rock sample (1 g of ^{238}U in equilibrium with its decay products produces about 10^{-7} g He in a year). Also, by assuming that initially the rock does not contain ^{206}Pb and it is present in rock due to decay of ^{238}U, we can calculate the age of rocks and mineral by measuring the ratio of ^{238}U and ^{206}Pb. The amount of ^{206}Pb is supposed to be obtained by decay of ^{238}U Thus, $^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + 8^{4}_{2}He + 6^{0}_{-1}e$

Mole of ^{238}U left = N at time t i.e. N_t ; Mole of ^{206}Pb formed = N′ at time t

∵ Initial mole of ^{238}U = N + N′ (at time 0) i.e., (N_0)

Thus, time t can be evaluated by $t = \frac{2.303}{\lambda}\log\frac{N_0}{N_t}$

Illustration 31: On analysis, a sample of uranium are was found to contain 0.277 g of $^{206}_{82}Pb$ and 1.667 g of $^{238}_{92}U$. The half-life period of ^{238}U is 4.51 × 10^9 years. If all the lead were assumed to have come from decay of $^{238}_{92}U$, what is the age of earth? **(JEE ADVANCED)**

Sol: Given, at time t; $^{238}_{92}U$ = 1.667 g = (1.667/238) mole ; $^{206}_{82}Pb$ =0.277 g = (0.277/206) mole

Since, all lead has been formed from ^{238}U, therefore moles of U decayed = Moles of Pb formed = (0.277 / 206)

∴ Total moles of U before decay (N_0) = Moles of U at time t (N) + Moles of U decayed

$= \frac{1.667}{238} \times \frac{0.277}{206}$ $\because t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303\times 4.51\times 10^9}{0.693}\log\frac{(1.667/238)+(0.277/206)}{(1.667/238)}$ ∴ t = 1.147 × 10^9 years

Illustration 32: The activity of the hair of an Egyptian mummy is 7 disintegrations $minute^{-1}$ of ^{14}C. Find the age of the mummy, given $t_{0.5}$ of ^{14}C is 5770 years and disintegration ratio of fresh sample of ^{14}C is 14 disintegration $minute^{-1}$ **(JEE MAIN)**

Sol: r_0 = 14 dpm and r_1 = 7 dpm ∴ $\frac{r_0}{r} = \frac{14}{7} = 2 = \frac{N_0}{N}$ ($\because r_0 \propto N_0$)

$t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303\times 5770}{0.693}\log 2$ = 5771 year

Illustration 33: The half period of ^{14}C is 5760 years. A piece of wood when buried in the earth had 1% ^{14}C. Now as charcoal it has only 0.25% ^{14}C. How long has the piece of wood been buried? **(JEE MAIN)**

Sol: Given, $N_{0_{^{14}C}}$ = 1% ; $N_{^{14}C}$ = 0.25% and $t_{1/2}$ = 5760 year

$t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303\times 5760}{0.693}\log\frac{1}{0.25}$ = 11524 year