5. WORK, ENERGY AND POWER

1. INTRODUCTION

This chapter explains the concepts of work and energy and how these quantities are related to each other. The law of conservation of energy is an important tool in physics, for the analysis of motion of a system of particles or bodies, and in understanding various phenomena in nature. When the nature of forces involved in a process are not exactly known, or when we want to avoid complicated calculations, then the law of conservation of energy proves to be an indispensable tool in solving many problems. The importance of energy cannot be explained in words. The progress of science and civilization is based on finding new ways to efficiently use the energy available in nature in various forms. Energy is required by a person to perform his/her daily activities, as well as to run our automobiles and machines. Depletion of natural energy resources is a major concern these days. The efficiency of energy utilization processes and quantity of energy sources harnessed by a country determines the pace of its economic development.

2. WORK

2.1 Work

In physics, a force is said to do work only when it acts on a body, and if there is a consequential displacement of the point of application in the direction of the force.

For example, say if a constant force F displaces a body through displacement s then the work done, W, is given by

$W = Fs\cos\theta = \vec{F}.\vec{s}$

where s is magnitude of displacement and θ is angle between force and displacement. The SI unit of work is Joule or Newton-metre.

Sign Convention of Work

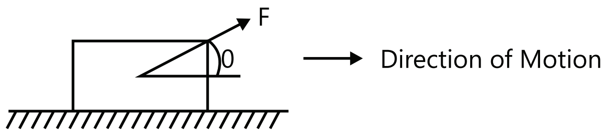


Figure 5.1: Motion of block in directon of applied force

We now define the sign convention of work as follows:

When $0 < \theta < 90^0$,

then $W = Fs\ \cos\theta$ is positive

i.e., when the force constantly supports the motion of a body,work done by that force is said to be positive.

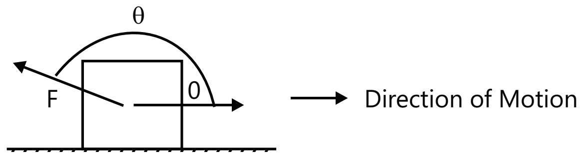


Figure 5.2: Motion of block

then $W = Fs\cos\theta = -ve$

i.e., in this case force is not truly supporting the motion of the body and hence the work done by that force is said to be negative.

2.2 Nature of Work

Work done is signified by the equation: $\vec{F}.\vec{S}$

Based on this equation, three possible situations are possible regarding the nature or sign of the work done as listed here under:

(a) To begin with, the work done is said to be positive if the angle between the force and the displacement vectors is an acute angle.

E.g., when a horse pulls a cart on a level road, the work done by the horse is positive.

(b) Second, the work done is zero if the force and the displacement vectors are perpendicular to each other.

E.g., when a body is moved along a circular path by a string, then the work done due to the string is zero.

(c) The last possible situation is that the work done is said to be negative if the angle between the force and the displacement vectors is an obtuse angle.

E.g., when a body slides over a rough surface, the resultant work done due to the frictional force is negative. (It is pertinent here to remember the fact that the angle between the force and the displacement is 180 degrees.)

MASTERJEE CONCEPTS

Students should be able to deduce that by positive work, force is actually doing what it is meant for, i.e. force wants to move a body in certain direction and if it moves in that direction then it's positive work.

Anurag Saraf (JEE 2011, AIR 226)

Illustration 1: Assume that a body is displaced from $\overrightarrow{r_A} = (2m, 4m, -6m)$ to $\overrightarrow{r_B} = (6i - 4j + 2k)m$ under a constant force $\vec{F} = (2i + 3j - k)N$. Now, calculate the total work done. **(JEE MAIN)**

Sol: The work done by the constant force $\vec{F}$ during displacement $\vec{S}$ of a particle is scalar product of force and displacement and is given by $W = \vec{F} \bullet \vec{S}$

$$\overrightarrow{r_A} = \left(2\hat{i} + 4\hat{j} - 6\hat{k}\right)m \quad \vec{S} = \overrightarrow{r_B} - \overrightarrow{r_A} = \left(6\hat{i} - 4\hat{j} + 2\hat{k}\right) - \left(2\hat{i} + 4\hat{j} - 6\hat{k}\right) = 4\hat{i} - 8\hat{j} + 8\hat{k}$$

$$W = \vec{F}.\vec{S} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right).\left(4\hat{i} - 8\hat{j} + 8\hat{k}\right) = 8 - 24 - 8 = (-24\hat{j})$$

Illustration 2: A block of total mass 5 kg is being raised vertically upwards with the help of a string attached to it and it rises with an acceleration of 2 m/s^2. Find the work done due to the tension in the string if the block rises by 2.5 m. Also, calculate the work done due to the gravity and the net work done. **(JEE ADVANCED)**

Sol: The tension in the string is acting vertically upwards and the block is also moving vertically upwards, so the work done by the tension will be positive. The force of gravity is acting vertically downwards so the work done by gravity will be negative.

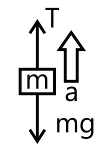


Figure 5.3

Let us first calculate the tension T.

From the force diagram T-mg = 5a; T = 5(9.8 + 2) = 59 N.

As it is clear that both T and displacement S are in the same direction (upwards), then work done by the tension T is W based on which we calculate that $W = Ts = 59(2.5) = 147.5\,J$.

Now, work done due to gravity = –mgs = –5(9.8) (2.5) = –122.5 J

Therefore, net work done on the block = work done by T + work done by mg = 147.5+ (–122.5) = 25 J.

MASTERJEE CONCEPTS

Point of application of force also plays a major role.

Zero work is done by a force in following cases: -If the point of application of force is not changed in space but the body moves. If body doesn't move but the point of application of force moves.

Nivvedan (JEE 2009, AIR 113)

3. WORK DONE BY A VARIABLE FORCE

We need to be aware of the fact that when the force is an arbitrary function of position, then we need the principles of calculus to evaluate the work done by it. The Fig. 5.4 given here under shows F (x) as some function x. We now begin our evaluation in this regard by replacing the actual variation of the force by a series of small steps. In the Fig. 5.4 provided, the area under each segment of the curve is approximately equal to the area of a rectangle. Based on the height of the rectangle, the amount of work done is given by the relation, $\Delta W_n = F_n \Delta X_n$. Therefore, the total work done is approximately given by the summation of the areas of both the recangles: $W \approx \sum F_n \Delta X_n$. As the number of the steps is reduced, the tops portions of the rectangle more closely resemble the actual curve shown in the Fig.5.4. In limit $\Delta x \to o$, which is equivalent to letting the number of steps to be infinite, the discrete sum is replaced by a continuous integral. $W = \int_{x_1}^{x_2} F(x)\,dx = \text{area under the } F - x \text{ curve and the } x - \text{axis}$

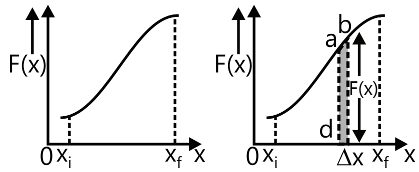


Figure 5.4: Work done on particle by variable force

Illustration 3: A force $F = (10 + 0.50X)$ is observed to act on a particle in the x direction, where F is in newton and x in meter. Find the actual work done by this force during a displacement from x=0 to x=2.0 m. **(JEE MAIN)**

Sol: If a particle is being displaced under action of variable force, the work done by this force is calculated as $W = \int_{s_1}^{s_2} \vec{F} \bullet d\vec{s}$.

As we know that the force is a variable quantity, we shall find the work done in a small displacement from x to x +

dx and then integrate the resultant value to calcuate the total work done. The work done in this small displacement is calculated as

$dW = \vec{F}.\vec{dx} = (10 + 0.50x)dx$. Thus, $W = \int_0^{2.0} (10 + 0.50x)dx \quad = \left[10x + 0.50\frac{x^2}{2}\right]_0^{2.0} = 21$ J.

4. CONSERVATIVE AND NON-CONSERVATIVE FORCES

A force is said to be of the conservative category if the work done by it in moving a particle from one point to another does not depend upon the path taken but depends only upon the initial and final positions. The work done by a conservative force around a closed path calculated to be zero. Gravitational force, electric force, spring force, etc. are some of the examples of this category. Basically, all central forces are conservative forces. In contrast, if the work done by a force in moving a body from one point to another depends upon the path followed, then the force is said to be of the nonconservative category. The work done by such a force around a closed path cannot be zero. For example, both the frictional and viscous forces work in an irreversible manner and hence a definite part of energy is lost in overcoming these frictional forces. (Mechanical energy is converted to other energy forms such as heat, sound, etc.). Therefore, these forces are of the nonconservative category.

5. WORK DONE AGAINST FRICTION

We know that the frictional force always acts opposite to the direction of motion (and hence direction of the displacement); therefore, the work done by the frictional force is always on the negative side. Further, the work done by the frictional force is invariably lost in the form of heat and sound energy and thus it is a nonconservative force.

MASTERJEE CONCEPTS

The work done by the frictional force is either negative or zero, but never positive. The frictional force always resists the attempted work done along a horizontal surface. Work done along a horizontal surface is given by: $-\mu mgl$, where

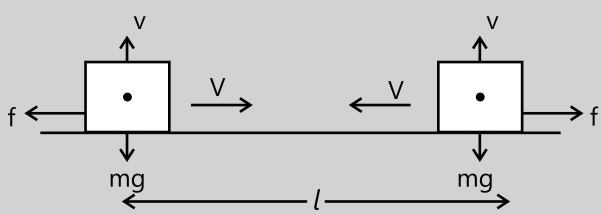


Figure 5.5

m is the mass of the object ;

μ is the coefficient of friction

g is the acceleration due to gravity ($9.8 m/s^2$)

l is the distance traveled by the block along the rough surface

Similarly, work done along an inclined surface with an angle θ from horizontal is given by $-\mu mgl\cos\theta$

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 4: It is observed that a block of mass 4 kg slides down a plane inclined at 37° with the horizontal. The length of the plane is calculated to be of 3 m. The value of the coefficient of sliding friction between the block and the plane is 0.2. Based on the above, find the work done due to the gravity, the frictional force, and the normal reaction between the block and the plane. **(JEE MAIN)**

Sol: Normal reaction is always perpendicular to the inclined plane hence it is perpendicular to the displacement and thus the work done by it is zero. Whereas the frictional force is in opposite direction to the displacement and hence the work done by the firctional force is negative. The work done by the component of gravitational force along the inclined plane will be positive.

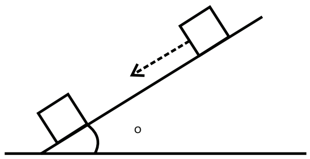


Figure 5.6

Total force acting on the block moving on inclined plane constitutes frictional force, normal reaction due to ground and gravitational force acting on wire. The work done on block is given as $W = Fs\cos\theta$

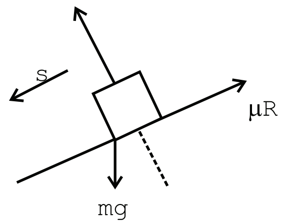


Figure 5.7

As the normal reaction is perpendicular to the point of displacement, work done by the normal reaction R = R s cos 90° = 0. The magnitude of displacement s = 3 m and the angle between force of gravity (mg) and displacement is equal to (90°–37°).

Therefore, work done by gravity = mgs cos (90°–37°)

$= mgs\sin 37^0 = 4 \times 9.8 \times 3 \times 3/5 = 70.56\,J$

Work done by friction = $-(\mu R)s = -(\mu\, mg\cos 37°)s = -0.2 \times 4 \times 9.8 \times 4/5 \times 3 = -18.816$ J.

6. POWER

Power is defined as the rate at which the actual work is done. If an amount of work ΔW is done in time Δt, then average power, $P_n = \dfrac{\Delta W}{\Delta t}$ and instantaneous power, $P = \lim\limits_{\Delta t \to 0}\left(\dfrac{\Delta W}{\Delta t}\right) = \dfrac{dW}{dt}$.

It is a well-known fact that work done by a force F on an object that has infinitesimally small displacement ds is dw=F.ds. Then, instantaneous power, $P = \dfrac{dW}{dt} = \dfrac{\vec{F}d\vec{s}}{dt} = \vec{F}.\vec{v}$.

The S I unit of power is Watt (W) or Joule/second (J/s) and it is a scalar quantity. Dimensions of power is $M^1L^2T^{-3}$.

Illustration 5: A block of mass m is allowed to slide down a fixed smooth inclined plane of angle θ and length ℓ. Calculate the magnitude of power developed by the gravitational force when the block reaches the bottom.

(JEE ADVANCED)

Sol: The power dlivered by the force $\vec{F}$ is the scalar product of the force and velocity i.e. $P = \vec{F}.\vec{v}$

When body reaches bottom of the inclined plane the velocity of of body is $v = \sqrt{2gh} = \sqrt{2g \bullet \ell\sin\theta}$ and the angle between velocity and vertical will be $(90-\theta)^o$. $P = \vec{F}.\vec{V} = mg\sin\theta\sqrt{2g\ell\sin\theta} = \sqrt{2m^2g^3\ell\sin^3\theta}$.

Illustration 6: A particle of mass m is moving in a circular path of constant radius r such that its centripetal accelecration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the force acting on it is

(JEE MAIN)

(A) $2\pi mk^2r^2$ (B) mk^2r^2t (C) $\dfrac{(mk^4r^2t^5)}{3}$ (D) Zero

Sol: (B) As the centripetal force is perpendicular to the direction of the velocity, the work done and power delivered by the centripetal force will be zero, whereas the tangential force is in the direction of the velocity so the power delivered to the particle of mass m is $P = F_t \bullet v$

Here $a_c = k^2rt^2$ or $\dfrac{v^2}{r} = k^2rt^2$ or $v = krt$

Therefore, tangential acceleration, $a_t = \dfrac{dv}{dt} = kr$ or tangential force, $F_t = m\,a_t = m\,kr$

However, only tangential force does work. Power = F_tv = (mkr)(krt) or Power = mk^2r^2t

7. ENERGY

Generally, the energy of a body is signified by the body's capacity to do work. It is a scalar quantity and shares the same unit as that of work (Joule in SI unit). In mechanics, both kinetic and potential energies are involved with dynamics of the body.

7.1 Potential Energy

7.1.1. Potential Energy

Potential energy of a body is the energy possessed by virtue of its position or due to its state. It is independent of the way in which the body is transformed to this state. Although it is a relative parameter, it depends upon its value at reference level. We can define the change in potential energy as the negative of work done by the conservative force in operation in carrying a body from a reference position to the position under consideration.

7.1.2 Definition

$\Delta U = -W_{AB}$ where A is the initial state, B is the final state, and W_{AB} is the total work done by conservative forces. We know that potential energy depends upon the work done by conservative force only. Hence, it cannot be defined for the nonconservative force (s). This is because of the proven fact that in this type work done depends upon the path followed alone.

7.1.3 Gravitational Potential Energy (GPE)

Suppose if we lift a block through some height (h) from A to B, then the work is done defying the gravity. The work done in such a case is stored normally in the form of gravitational potential energy of the block-energy system. Therefore, we can write that work done in raising the block = (mg)h. This is exactly equal to the increase in gravitational potential energy (GPE) of the block.

If the center of a body of mass m is raised by a height h, then increase in GPE = mgh

If the center of a body of mass m is lowered by a distance h, decrease in GPE = mgh

7.1.4 Elastic Potential Energy

Suppose when a spring is elongated (or compressed), then work is done against the restoring force of the spring. This resultant work done is stored in the spring in the form of elastic potential energy.

7.1.5 Nature of Restoring Force

Suppose if a spring is extended or compressed by a distance x, the spring then exerts a restoring force so as to oppose this change.

MASTERJEE CONCEPTS

GPE is always thought of as only of block. But to be more specific it is the energy of block-earth system. Potential energy never comes in context of a single particle. It is always for a configuration. In the case of GPE, writers however generally skip writing "Earth" each time.

Chinmay S Purandare (JEE 2012, AIR 698)

7.1.6 Spring

In case of a spring, natural length of the spring is assumed to be the reference point and correspondingly is always assigned zero potential energy (This is a universal assumption.). However, in gravity, we can choose any point as our reference and hence assign it any value of potential energy.

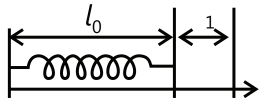


Figure 5.8: Energy stored in stretched spring

For Stretching

$$U_f - U_i = -\int_i^f \vec{F}.d\vec{S}\ ; U_f - 0 = -\int_0^{x_i} kx(-i)(dx)i\ ; U = \frac{1}{2}kx_1^2$$

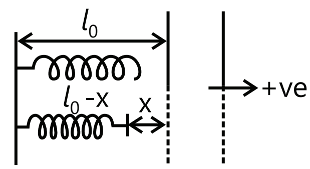


Figure 5.9: Energy stored in compressed spring

For Compression

$$U_f - U_i = -\int_i^f \vec{F}.d\vec{S} = -\int_0^{x_i} kxi(dx)(-i) = U = \frac{1}{2}kx^2$$

Thus, if the spring is either stretched or compressed from natural length by x the corresponding potential energy is $1/2kx^2$

7.1.7 Relationship between Force and Potential Energy

Now, let us discuss the relationship between force and potential energy.

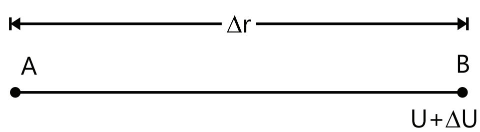


Figure 5.10

Let us assume that a body is taken from A to B in such away that there is no net change in its kinetic engery. Then

$\Rightarrow$ Work done $= -$change in P.E. ; $F\Delta r = U-(U+\Delta U) = -\Delta U$

$$\Rightarrow \quad F_{avg} = -\left(\frac{\Delta U}{\Delta r}\right) \text{ if } \Delta r \to 0; \quad F = -\lim_{\Delta r \to 0}\frac{\Delta u}{\Delta r} = -\frac{\partial U}{\partial r}$$

7.2 Kinetic Energy

Kinetic energy (KE) is the energy of a body possessed by virtue of its motion alone. Therefore, a body of mass m and moving with a velocity v has a kinetic energy $E_k = \frac{1}{2}mv^2$.

We already know that velocity is a relative parameter; therefore, KE is also a relative parameter.

We provide a detailed account on kinetic energy after presenting the concept of conservation of mechanical energy.

8. EQUILIBRIUM

We have already studied in the chapter on "Laws of Motion" that a body is said to be in translatory equilibrium only if net force acting on the body is zero, i.e., $\vec{F}_{net} = 0$

However, if the forces are conservative, then $F = -\frac{dU}{dr}$; for equilibrium, then

F = 0; Thus, $-\frac{dU}{dr}=0$, or $\frac{dU}{dr}=0$

i.e., exactly at the equilibrium position the slope of U-r graph is zero or the potential energy is optimum (maximum or minimum or constant). Equilibria are of three types, i.e., stable equilibrium, unstable equilibrium, and neutral equilibrium. Further, the situations where F = 0 and dU/dr = 0 can be obtained only under three conditions as specified hereunder.

(a) If $\frac{d^2U}{dr^2} > 0$, then it is stable equilibrium;

(b) If $\frac{d^2U}{dr^2} < 0$, then it is unstable equilibrium; and

(c) If $\frac{d^2U}{dr^2} = 0$, then it is neutral equilibrium.

MASTERJEE CONCEPTS

A system always wants to minimize its energy. The above equilibriums are categorized only on this basis. Stable indicates that if system is disturbed slightly, from these configuration, it would try to come back to its original state (position of energy minima). For unstable equilibrium, a slight disturbance would cause the system to find some other suitable configuration (position of energy maxima). A neutral equilibrium is generally found when U becomes constant and each position is a state of equilibrium. A slight disturbance has no after reactions and the new state is also an equilibrium position.

Anurag Saraf (JEE 2011, AIR 226)

Illustration 7: The potential energy of a particle of mass 5 kg, moving in xy plane, is given by U = (–7x + 24y)J where x and y being in meters. Initially (at t=0), the particle is at the origin and has velocity $\vec{v}=\left(14.4\hat{i}+4.2\hat{j}\right)m/s$.

Then Calculate (a) the acceleration of the particle and (b) the direction of acceleration of the particle. (c) The speed of the particle at t = 4 s. **(JEE MAIN)**

Sol: If particle has potential energy U then corresponding conservative force, is $F=-\frac{dU}{dr}$ and according to the Newton's second law of motion $\vec{F}=m\vec{a}$. The direction of acceleration is calculated as $\tan\theta=\frac{a_y}{a_x}$.

(a) Acceleration,

$$F_x=\frac{\delta U}{\delta x}, F_y=-\frac{\delta U}{\delta y} \Rightarrow F_x=7N, \quad F_y=-24N; \quad \Rightarrow a_x=7/5,\ a_y=-24/5$$

(b) Direction of acceleration $\theta=\tan^{-1}\left(\frac{a_y}{a_x}\right)$;

(c) $\vec{v}=\vec{u}+\vec{a}\,t$; $v_x=14.4+\frac{7}{5}\times 4=20$; $v_y=4.2-\frac{24}{5}\times 4=(-15)$

Illustration 8: The potential energy of a particle in a certain field has the form U= a/r² – b/r, where a and b are positive constants and r is the distance from the center of the field. Find the value of r_0 corresponding to equilibrium position of the particles and hence examine whether this position is stable. **(JEE ADVANCED)**

Sol: Conservative force acting on the particle is $F=-\frac{dU}{dr}$. Under stable equilibrium particle has minimum potential energy while potential energy is maximum in case of unstable equilibrium.

$U(r) = a/r^2 - b/r$

$\text{Force} = F = -\frac{dU}{dr} = -\left(\frac{-2a}{r^3} + \frac{b}{r^2}\right)$; $\quad F = -\frac{(br - 2a)}{r^3}$

At equilibrium, then $F = \frac{dU}{dr} = 0$

Hence, br – 2a = 0 at equilibrium.

Further, $r = r_0 = 2a/b$ corresponds to equilibrium.

At stable equilibrium, the potential energy of a particle is at its minimum, whereas at unstable equilibrium, it is the maximum. From the principles of calculus, we know that for minimum value around a point $r = r_0$, the first derivative should be zero and the second derivative should be invariably positive.

For minimum potential energy, the applicable conditions are

$$\frac{dU}{dr} = 0 \quad \text{and} \quad \frac{d^2U}{dr^2} > 0 \quad \text{at} \quad r = r_0$$

However, we have already used dU/dr = 0 to obtain $r = r_0 = 2a/b$.

Now, in a similar way let us investigate the second derivative.

$$\frac{d^2U}{dr^2} = \frac{d}{dr}\left(\frac{dU}{dr}\right) = \frac{d}{dr}\left(-\frac{2a}{r^3} + \frac{b}{r^2}\right) = \frac{6a}{r^4} - \frac{2b}{r^3}$$

$$\text{At } r = r_0 = 2a/b, \ \frac{d^2U}{dr^2} = \frac{6a - 2br_0}{r_0^4} = \frac{2a}{r_0^4} > 0.$$

Based on our calculations, the potential energy function U(r) has a minimum value only when $r_0 = 2a/b$. Therefore, we conclude that the system has stable equilibrium only at the minimum potential energy state.

9. WORK ENERGY THEOREM

Suppose that a particle is acted upon by various forces and consequently undergoes a displacement. Then there is a change in its kinetic energy by an amount equal to the total (net) work (W_{net}) done on the particle by all the forces.

i.e., $W_{net} = K_f - K_i = \Delta K$ ... (i)

We call the above expression as the work-energy theorem.

Expression (i) is valid irrespective of the fact that whether the forces are constant or varying and whether the path followed by the particle is straight or curved.

We further elaborate expression (i) as follows:

$W_c + W_{NC} + W_{Oth} = \Delta K$ ... (ii)

where W_c is the work done by conservative forces

W_{sc} is the work done by nonconservative forces

W_{oth} is the work done by all other forces which are not included in the category of conservative, nonconservative, and pseudo forces.

'Since $W_c = \Delta U$' (based on definition of potential energy), therefore, expression (ii) can be accordingly modified as

$W_{NC} + W_{oth} = \Delta K + \Delta U = \Delta(K + U) = \Delta E$ ... (iii)

In expression (iii), the term K + U = E is known as the mechanical energy of the system.

Illustration 9: Find how much will mass "m" rise if 4 m falls away. Block are at rest and in equilibrium **(JEE MAIN)**

Sol: Initially the block is at rest. When the block rises to the maximum height, it again comes to rest momentarily. So, by work energy theorem the total work done on the block by force of gravity and spring force is zero.

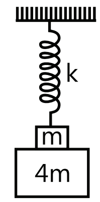


Figure 5.11

Applying work energy theorem (WET) on a block of mass m

$W_g + W_{sp} = K.E._f - K.E._i$

Let the final displacement of the block from the initial equilibrium is x. Then

$$-mg\left(\frac{5mg}{k}+X\right)+\frac{1}{2}k\left(\frac{25m^2g^2}{k^2}\right)-\frac{1}{2}kx^2=0;\frac{1}{2}kx^2+mgx-\frac{15m^2g^2}{2k}=0\ ;\qquad x=\frac{3mg}{k}$$

MASTERJEE CONCEPTS

Whenever there is frictional force, energy is dissipated which is equal to work done by frictional force and the dissipated energy converts into heat. Practically, machine handlers do a lot of things to minimize friction and reduce energy losses by applying lubricants and rollers in their parts.

Yashwanth Sandupatla (JEE 2012, AIR 821)

Illustration 10: A body of mass m was slowly hauled up the hill as shown in the Fig. 5.12 provided by a force F which at each point was directed along a tangent to the trajectory. Find the work done due to this force if the height of the hill is h, the length of its base is l, and the coefficient of friction is μ. **(JEE ADVANCED)**

Sol: As block hauls slowly, the kinetic energy will not change throughout the motion. And the sum of the work done by applied force, gravitational force, normal reaction and frictional force will be zero as per work energy theorem.

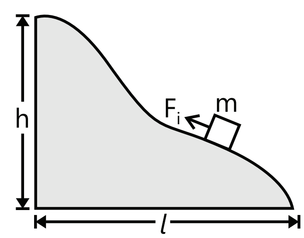


Figure 5.12

The four forces that are acting on the body are listed hereunder.

(a) Weight (mg),

(b) Normal reaction (N),

(c) Friction (f), and

(d) The applied force (F)

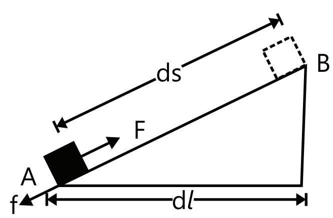


Figure 5.13

According to the principle of work-energy theorem

$W_{net} = \Delta$ KE or $W_{mg}+W_N+W_f+W_F = 0$ ... (i)

Here, $\Delta KE=0$, because $K_i=0=K_f$ $\therefore W_{mg} = -mgh; W_N = 0$

(This is because the normal reaction is perpendicular to displacement at all the points.)

W_f can be calculated as $f=\mu mg\cos\theta$

$$\therefore\quad (dW_{AB})_f = -f\,ds = -(\mu\,mg\cos\theta)ds = -\mu\,mg(dl)\ \text{(as } ds\cos\theta = dl)$$

$$\therefore\quad f = -\mu\,mg\sum dl = -\mu\,mgl$$

Substituting these values in Eq. (i), we obtain the expression $W_F = mgh+\mu mgl$.

Note: Here again, if we desire to solve this problem without using the concept of work-energy theorem, then we will first evaluate magnitude of applied force $\vec{F}$ at different locations following which we will then integrate $\left(=\vec{F}.\vec{d}r\right)$ with proper limits.

10. KINETIC ENERGY

Now, let us attempt to develop a relationship between the work done and the change in speed of a particle. Based on the Fig. 5.14 provided, we observe that the particle moves from point P_1 to P_2 under the action of a net force $\vec{F}$

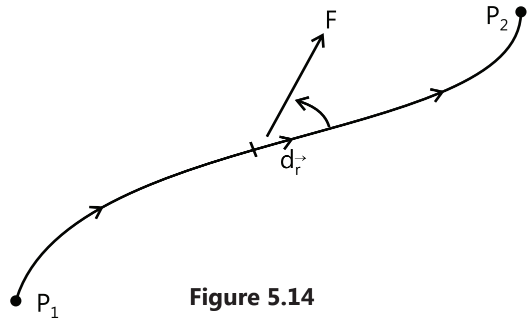


Figure 5.14

$$W = \int_{P_1}^{P_2} \vec{F}.d\vec{r} \; ; \; \vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} \; ; \; d\vec{r} = dx\,\hat{i} + d_y \hat{j} + dz\,\hat{k}$$

$$W = \int_{P_2}^{P_2} \left(F_x dx + F_Y d_y + F_z d_z\right)$$

It is very clear for us now that a particle moves along a curved path from point P_1 to P_2, only when acted upon by a force F that varies in both magnitude and direction. $F_x = ma_x = \frac{m d\nu_x}{dt}$; $\int_{P_1}^{P_2} F_x dx = \int_{P_1}^{P_2} m\frac{dV_x}{dt}dx$

Treating now ν_x as a function of position, we obtain:

$$\frac{d\nu_x}{dt} = \frac{d\nu_x}{dx}\left(\frac{dx}{dt}\right) = \frac{d\nu_x}{dx}.\nu_x = \nu_x\frac{d\nu_x}{dx}; \; \therefore \int_{P_1}^{P_2} F_x d_x = \int_{P_1}^{P_2} m\frac{d\nu_x}{dt}dx = \int_{P_1}^{P_2} m\nu_x\frac{d\nu_x}{dx}dx = \int_{P_1}^{P_2} m\nu_x d\nu_x = \frac{1}{2}m\nu_x^2\int_{\nu_{x1}}^{\nu_{x2}} = \frac{1}{2}m\left(\nu^2_{x2} - \nu^2_{x1}\right)$$

ν_{x1} = velocity in x-direction at P_1; ν_{x2} = velocity in x-direction at P_2.

We now apply the same principle for terms in y and z.

$$W = \frac{1}{2}M\left[\nu_{x2}^{\;2} + \nu_{y2}^{\;2} + \nu_{z2}^{\;2} - \left(\nu_{x1}^{\;2} + \nu_{y1}^{\;2} + \nu_{z1}^{\;2}\right)\right] \quad = \frac{1}{2}M\left(\nu_2^2 - \nu_1^2\right); \quad W = \frac{1}{2}m\nu_2^2 - \frac{1}{2}m\nu_1^2$$

Define: $K = \frac{1}{2}m\nu^2 \equiv$ Kinetic energy of particle

KE: Potential of a particle to do work by virtue of its velocity.

We know that the work done on the particle by the net force equals the change in KE of the particle.

$W = K_2 - K_1$ or $\Rightarrow W = \Delta K$ Work–Energy Theorem.

For a particle $\vec{P} = M\vec{v}$ (linear momentum); $\therefore K = \frac{1}{2m}P^2$

Regarding KE, the following two points are very significant.

(a) Since, both m and v^2 are always positive, KE is always positive and hence does not depend on the directional parameter of motion of the body.

(b) KE depends on the frame of reference. For example, the KE of a person of mass m in a train moving with speed v is zero in the frame of train, whereas in the frame of earth the KE is $\frac{1}{2}mv^2$ for the same person.

MASTERJEE CONCEPTS

Energy can never be negative.

No! Only kinetic energy can't be negative. If anyone generally speaks about energy, it means the sum of potential and kinetic energies. However, we can always choose such a reference in which this sum is negative. Hence, total energy can be negative.

Anurag Saraf (JEE 2011, AIR 226)

Illustration 11: A uniform chain of length ℓ and mass m overhangs a smooth table with its two-third parts lying on the table. Find the kinetic energy of the chain as it completely slips off the table. **(JEE MAIN)**

Sol: The initial kinetic energy of the chain is zero. When chain start slipping off table the loss in its potential energy is equal to the gain in its kinetic energy.

Let us take the potential energy at the table as zero. Now, consider a part dx of the chain at a depth x below the surface of the table. The mass of this part is $dm = \frac{m}{\ell}dx$ and hence its potential energy is $-(m/\ell\ dx)gx$.

The potential energy of the one-third of the chain that overhangs is given by $U_1 = \int_0^{\ell/3} -\frac{m}{\ell}gx\,dx$

$$= -\left[\frac{m}{\ell}g\left(\frac{x^2}{2}\right)\right]_0^{\ell/3} = -\frac{1}{18}mg\ell$$

However, this is also the potential energy of the full chain in the initial position; this is because the part lying on the table has zero potential energy. Now, we can calculate the potential energy of the chain when it completely slips off the table as

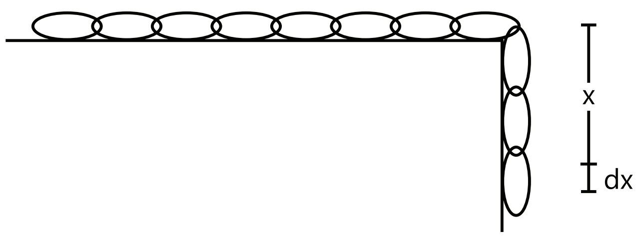


Figure 5.15

$U_2 = \int_0^{\ell} -\frac{m}{\ell}gx\,dx = -\frac{1}{2}mg\ell$ The loss in potential energy is $= \left(-\frac{1}{18}mg\ell\right) - \left(-\frac{1}{2}mg\ell\right) = \frac{4}{9}mg\ell$.

Basically, this should be equal to the gain in the KE in this case. However, the initial KE is zero. Hence, the KE of the chain as it completely slips off the table is $\frac{4}{9}mg\ell$.

Illustration 12: A block of mass m is pushed against a spring of spring constant k fixed at one end to a wall. The block can slide on a frictionless table as shown in the Fig. 5.16. The natural length of the spring is taken as L_0 and it is compressed to half its natural length when the block is released. Now, based on the above find the velocity of the block as a function of its distance x from the wall. **(JEE ADVANCED)**

Sol: The block will move under action of restoring force of spring when spring is released. The block will have constant kinetic energy when it looses contact with the spring. In this process the energy of system will be conserved as there are no external forces acting on the system. (Spring + block system)

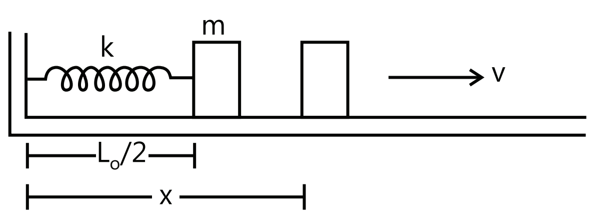


Figure 5.16

When the block is released, naturally the spring pushes it toward right. The velocity of the block keep on inreasing till the block loses contact with the spring and thereafter moves with constant velocity.

Initially, the compression of the spring is $L_0/2$. But when the distance of the block from the wall becomes x, where $x < L_0$, the compression is $(L_0 - x)$. Applying the principle of conservation of energy

$$\frac{1}{2}k\left(\frac{L_0}{2}\right)^2 = \frac{1}{2}k(L_0 - x)^2 + \frac{1}{2}mv^2 \text{. Solving this, } v = \sqrt{\frac{k}{m}}\left[\frac{L_0^2}{4} - (L_0 - x)^2\right]^{1/2}$$

Thus, when the spring acquires its natural length, then $x = L_0$ and $v = \sqrt{\frac{k}{m}}\frac{L_0}{2}$. Thereafter, the velocity of the block remains constant.

11. MOTION IN A VERTICAL CIRCLE

Let us consider a particle of mass m attached to one end of a string and rotated in a vertical circle of radius r with centre O. The speed of the particle will decrease as the particle travels from the lowest point to the highest point but increases in the reverse direction due to acceleration due to gravity.

Thus, if the particle is moving with velocity v at any instant at A, (where the string is subtending an angle θ with the vertical), then the forces acting on the particle are tension T in the string directed toward AO and weight mg acting downward.

Further, the net force T–mg cos θ is directed toward the cenetr and hence provides the centripetal force

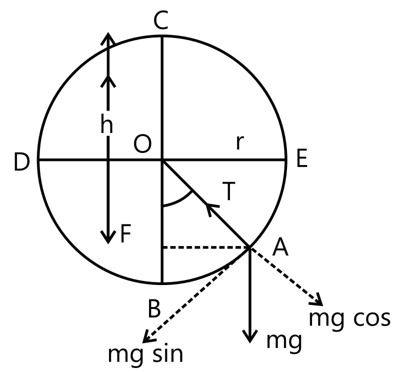


Figure 5.17: Motion in vertical circle

$$T - mg\cos\theta = \frac{mv^2}{r}; T = m\left(g\cos\theta + \frac{v^2}{r}\right)$$

If v_0 is the speed of the particle at the highest point, then the velocity increases as the particle falls through any height h. However, if it falls from C to A, then the vertical distance h is given by

$$h = CF = CO + OF = CO + OA\cos\theta = r + r\cos\theta; h = r(1 + \cos\theta)$$

$v^2 = v_0^2 + 2gh = v_0^2 + 2gr(1 + \cos\theta)$ (Because there is no actual work done due to the influence of tension)

(i) At the highest point C, $\theta = 180^0$

$$\text{Tension at } C = T_c = m\left[\frac{v_0^2}{r} + g\cos(180)\right] = m\left[\frac{v_0^2}{r} - g\right] \quad \text{... (i)}$$

The particle will now fall because the string will slacken if T_c is negative. Therefore, the minimum velocity at the highest point is corresponding to the situation where T_c is just zero, i.e., when $m\left[\frac{v_0^2}{r} - g\right] = 0$, or $v_0 = \sqrt{rg}$

(ii) At the lowest point B, $\theta = 0$, tension T_B is given by $T_B = m\left[\frac{v_B^2}{r} + g\right]$

where v_B is velocity at B. $v_B^2 = v_0^2 + 4rg = rg + 4rg = 5rg$; $(\text{using } v^2 = u^2 + 2gh)$; $v_B = \sqrt{5rg}$ ... (ii)

Minimum tension at B when the particle completes the circle is given by $T_B = m\left[\frac{5rg}{r} + g\right] = 6mg$

At the point E, when $\theta = 90^0$, $T_E = \frac{mv_E^2}{r}$

Where velocity at E is given by $V_E = V_c^2 + 2rg = rg + 2rg = 3rg$; $V_E = \sqrt{3rg}$

Tension at E corresponding to speed V_E is $T_E = m\left(\frac{3rg}{r}\right) = 3mg$

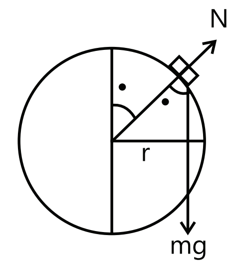


Figure 5.18

(iii) In another case the particle of mass m is not tied to the string but is moving along a circular track of radius r and has normal reaction N. However, it is moving with a velocity v and its radius vector is subtending an angle θ with the vertical, then $mg\cos\theta - N = \frac{mv^2}{r}$.

At the highest point, $mg - N = \frac{mv^2}{r}$; when ... (iii)

$N = 0, V = \sqrt{rg}$ Therefore, $V = \sqrt{rg}$ is the minimum speed with which the particle can move at the highest point without losing contact.

Condition of Looping the Loop $(u \geq \sqrt{5gR})$

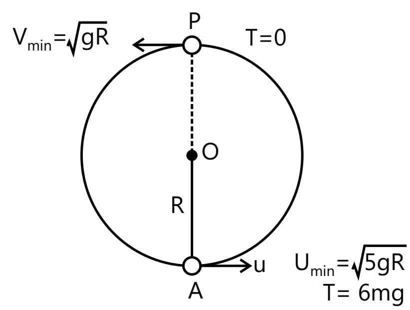


Figure 5.19

The particle will complete the circle only if the string does not slack even at the highest point (θ=π). Thus, tension in the string should be obviously greater than or equal to zero (T ≥ 0) at θ=π. In the critical case, however, by substituting T = 0 and θ=π in Eq. (iii), we obtain

$mg = \frac{mv_{min}^2}{R}$ or $v_{min}^2 = gR$ or $v_{min} = \sqrt{gR}$ (at the highest point)

Further, by substituting $\theta = \pi$ in Eq. (i), h = 2R

Therefore, from Eq. (ii) $u_{min}^2 = v_{min}^2 + 2gh$ or $u_{min}^2 = gR + 2g(2R)$ or $u_{min} = \sqrt{5gR}$

Thus, if $u \geq \sqrt{5gR}$, then the particle will complete the circle.

At u = $\sqrt{5gR}$, the velocity at the highest point is $v = \sqrt{gR}$ and the tension in the string is zero.

By substituting θ = 0° and $v = \sqrt{5gR}$ in Eq. (iii), we get T = 6mg or in the critical condition tension in the string at the lowest position is 6mg as shown in the Fig. 5.19. If $u < \sqrt{5gR}$, then the following two cases are possible.

Condition of Leaving the Circle $(\sqrt{2gR} < u < \sqrt{5gR})$

If $u < \sqrt{5gR}$, then the tension in the string will be zero before reaching the highest point. From Eq. (iii), tension in the string is zero (T=0) where, $\cos\theta = \frac{-v^2}{Rg}$ or $\cos\theta = \frac{2gh - u^2}{Rg}$

Now, by substituting, this value of $\cos\theta$ in Eq. (i), we obtain $\frac{2gh - u^2}{Rg} = 1 - \frac{h}{R}$ or $h = \frac{u^2 + Rg}{3g} = h_1$ (say) ... (iv)

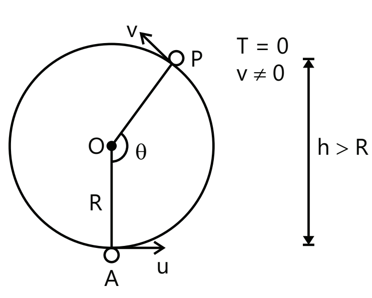


Figure 5.20

Or, in other words, we can say that at height h_1 tension in the string becomes zero. Further, if $u < \sqrt{5gR}$, then the velocity of the particle becomes zero when $0 = u^2 - 2gh$ or $h = \dfrac{u^2}{2g} = h_2$ (say) ... (v)

i.e., at height h_2 velocity of the particle becomes zero. Now, the particle will move out from the circle if tension alone in the string becomes zero but not the velocity or T=0 but $v \neq 0$. This is possible only when $h_1 < h_2$ or $\dfrac{u^2 + Rg}{3g} < \dfrac{u^2}{2g}$ or $2u^2 + 2Rg < 3u^2$ or $u^2 > 2Rg$ or $u > \sqrt{2Rg}$.

Therefore, if $\sqrt{2gR} < u < \sqrt{5gR}$, the particle moves out from the circle.

From Eq.(iv), we observe that h >R if $u^2 > 2Rg$. Thus, the particle, will move out of the circle when h>R or $90° < \theta < 180°$. This situation is shown in the Fig. 4.75.

$\sqrt{2gR} < u < \sqrt{5gR}$ or $90° < \theta < 180°$

Note, however, that after leaving the circle, the particle will follow a parabolic path.

Condition of Oscillation $(0 < u < \sqrt{2gR})$

The particle will oscillate, however, only if velocity of the particle becomes zero but not tension in the string. Or, in other words, v = 0, but $T \neq 0$. This is possible only when $h_2 < h_1$.

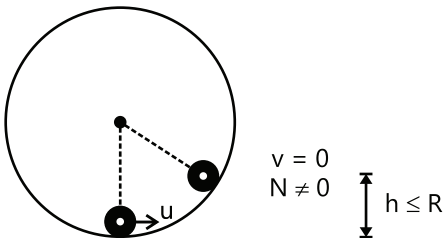


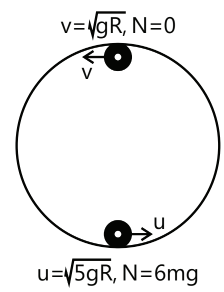
Figure 5.21

Or $\dfrac{u^2}{2g} < \dfrac{u^2 + Rg}{3g}$ or $3u^2 < 2u^2 + 2Rg$ or $u^2 < 2Rg$ or $u < \sqrt{2Rg}$

Moreover, if $h_1 = h_2$, $u = \sqrt{2Rg}$ then both tension and velocity becomes zero simultaneously.

Further, from Eq (iv), we observe that $h \leq R$ if $u \leq \sqrt{2Rg}$. Thus, for $0 < u \leq \sqrt{2gR}$, the particle oscillates in the lower half of the circle $(0° < \theta \leq 90°)$. This situation is shown in the Fig. 5.21. $(0 < u < \sqrt{2gR})$ or $(0° < \theta \leq 90°)$

Note: The above three conditions have been derived for a particle that is moving only in a vertical circle and attached to a string. The same conditions apply, however, if a particle moves inside a smooth spherical shell also of radius R. The only difference here is that the tension is replaced by the normal reaction N.

Condition of Looping the Loop is $(u \geq \sqrt{5gR})$	 **Figure 5.22**

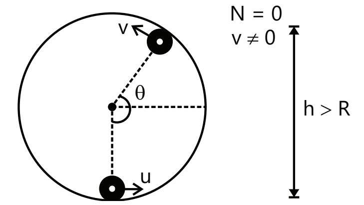
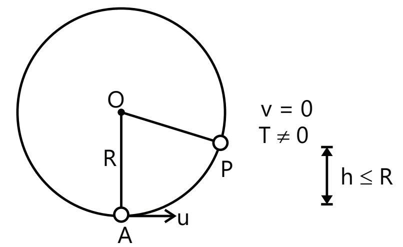
Condition of Leaving the Circle $(\sqrt{2gR} < u < \sqrt{5gR})$	 **Figure 5.23**
Condition of Oscillation $(0 < u < \sqrt{2gR})$	 **Figure 5.24**

Illustration 31: A heavy particle hanging from a fixed point by a light inextensible string of length l is projected horizontally with speed $\sqrt{gl}$. Now, find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle. **(JEE ADVANCED)**

Sol: Loss in the kinetic energy of the particle is equal to the gain in the potential energy. Apply Newton's second law along the direction of the string.

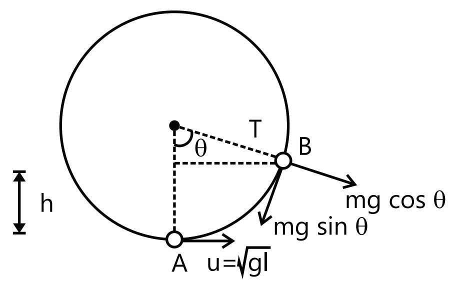


Figure 5.25

Let T = mg at angle θ as shown in the Fig. 5.25.

$H = l\,(1-\cos\theta)$ ... (i)

Applying the principle of conservation of mechanical energy between points A and B, we obtain $\frac{1}{2}m(u^2 - v^2) = mgh$

Here, $u^2 = gl$ ... (ii)

and v = speed of particle in position B $\therefore \quad v^2 = u^2 - 2gh$ ... (iii)

Further, $T - mg\cos\theta = \frac{mv^2}{l}$ or $mg - mg\cos\theta = \frac{mv^2}{l}$ (T = mg)

Or $\quad v^2 = gl(1-\cos\theta)$ ... (iv)

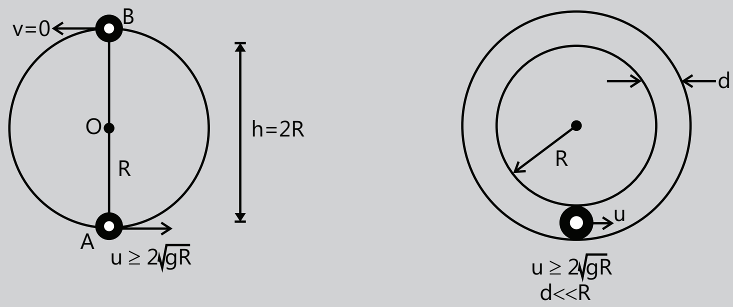
Now, by substituting the values of v^2, u^2 and h from Eqs. (iv), (ii) and (i) in Eq. (iii), we obtain

$gl(1-\cos\theta) = gl - 2gl(1-\cos\theta)$ or $\cos\theta = \frac{2}{3}$ or $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

Further, by substituting $\cos\theta = \frac{2}{3}$ in Eq. (iv), we obtain $v = \sqrt{\frac{gl}{3}}$

MASTERJEE CONCEPTS

If a particle of mass m is connected to a light rod and whirled in a vertical circle of radius R, then to complete the circle, the minimum velocity of the particle at the bottommost point is not $\sqrt{5gR}$. Because, in this case, velocity of the particle at the topmost point can be zero also. Using conservation of mechanical energy between points A and B as shown in Fig. 5.26(a) we get



(a) (b)

Figure 5.26

$\frac{1}{2}m(u^2 - v^2) = mgh$ or $\frac{1}{2}mu^2 = mg(2R)$ (as v = 0) $\therefore$ $u = 2\sqrt{gR}$

Therefore, the minimum value of u in the case is $2\sqrt{gR}$.

Same is the case when a particle is compelled to move inside a smooth vertical tube as shown in Fig 5.26(b).

Anurag Saraf (JEE 2011, AIR 226)

12. A BODY MOVING INSIDE A HOLLOW TUBE

Our discussion above holds good in this case too, but instead of tension in the string we have the normal reaction of the surface. If we take N is the normal reaction at the lowest point, then $N - mg = \frac{mv_1^2}{r}$; $N = m\left(\frac{v_1^2}{r} + g\right)$ However, at the highest point of the circle, $N + mg = \frac{mv_2^2}{r}$

$N = m\left(\frac{v_2^2}{r} - g\right)$; $N \geq 0$ $\Rightarrow$ Implies the condition $V_1 \geq \sqrt{5rg}$

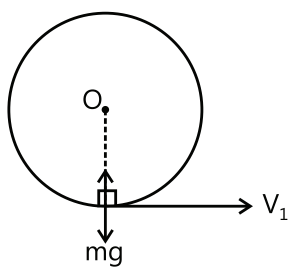


Figure 5.27: Block moving inside hollow sphere

In the same way as shown above, all the other equations similarly can be obtained by just replacing tension T by reaction N.

13. BODY MOVING ON A SPHERICAL SURFACE

Consider that the small body of mass m is placed on top of a smooth sphere whose radius is r.

Now, if the body slides down the surface, at what point does it fly off the surface?

Consider the point C where the mass is, at a certain instant. Now, the acting forces are the normal reaction R and the weight mg. Further, the radial component of the weight is mg $\cos\theta$ acting toward the center. The centripetal force in this case is taken as $mg\cos\theta - R = \frac{mv^2}{r}$

where v is the velocity of the body at O.

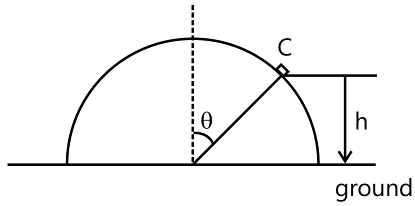


Figure 5.28: Motion of body on spherical surface

$$R = m\left(g\cos\theta - \frac{v^2}{r}\right) \quad \text{... (i)}$$

Now, it is clear that the body flies off the surface at the point where R becomes zero.

$$\text{i.e.., } g\cos\phi - R = \frac{mv^2}{r} \quad \text{... (ii)}$$

To find v, we apply the principle of conservation of energy

$$\text{i.e., } \frac{1}{2}mv^2 = mg(BN) = mg\,(OB - ON) = mgr\,(1-\cos\phi)$$

$$v^2 = 2rg\,(1-\cos\phi); \qquad 2(1-\cos\phi) = \frac{v^2}{rg} \quad \text{... (iii)}$$

From equations (ii) and (iii), we obtain

$$\cos\phi = 2 - 2\cos\phi; \qquad 3\cos\phi = 2$$

$$\cos\phi = \frac{2}{3}; \qquad \phi = \cos^{-1}\left(\frac{2}{3}\right) \quad \text{... (iv)}$$

This exactly denotes the angle at which the body goes off the surface. The height from the ground of that point is

$$= AN = r(1+\cos\phi) = r\left(1+\frac{2}{3}\right) = \frac{5}{3}\,r$$

Illustration 32: A point mass m starts from rest and slides down the surface of a frinctionless solid sphere of radius R as shown in the Fig. 5.29 provided. At what angle will this body break off the surface of the sphere? Also, find the velocity with which it will break off. **(JEE MAIN)**

Sol: As the block slides down, the loss in potential energy is equal to gain in kinetic energy and at time of break off, the normal reaction from the sphere on block is zero.

Applying princliple of conservation of energy (COE), at the points A and B

$$mgR\,(1-\cos\theta) = \frac{1}{2}mv^2 \quad \text{... (i)}$$

Force equation in this equation is $mg\cos\theta - N = mv^2/R$ ... (ii)

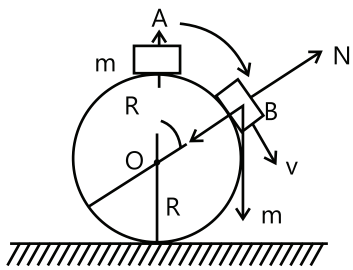


Figure 5.29

N = 0 for break off.

$$\therefore v = \sqrt{gR\cos\theta} \quad \text{... (iii)}$$

Replacing this value in (i)

We get $\cos\theta = 2/3$ Putting this in (iii) we get $v = \sqrt{\frac{2}{3}gR}$.

Illustration 33: A heavy particle is suspended by a string of length ℓ. The horizontal velocity of the particle is v_0. However, the string becomes slack at some angle and the particle proceeds on a parabolic path. Find the value of v_0 if the particle passes through the point of suspension. **(JEE ADVANCED)**

Sol: While particle moves in vertical circle, the tension in the string provides the necessary centripetal force. The loss in kinetic energy is equal to the gain in potential energy. At point the string become slack the tension in the string is zero.

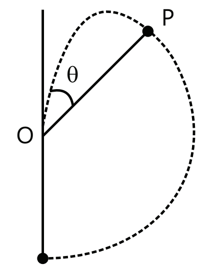


Figure 5.30

Let us suppose the string becomes slack when the particle reaches the point P. We now assume that the string OP makes an angle θ with the upward vertical. Further, the only force acting on the particle at the point P is its weight mg. Further, the radial component of the force is $mg\cos\theta$. Now, as the particle moves along the circle upto P,

$$mg\cos\theta = m\left(\frac{v^2}{\ell}\right) \Rightarrow v^2 = g\ell\cos\theta \qquad \text{... (i)}$$

where v is its speed at the point P. Now, applying the principle of conservation of energy

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mg\ell(1+\cos\theta) \text{ or } v^2 = v_0^2 - 2g\ell(1+\cos\theta) \qquad \text{... (ii)}$$

From (i) and (ii), $v_0^2 = 2g\ell(1+\cos\theta) = g\ell\cos\theta$ or $v_0^2 = g\ell(2+3\cos\theta)$ ... (iii)

From hereon, the particle follows a parabolic path due to acceleration due to gravity. Then as it passes through the point of suspension O, the equations for horizontal and vertical motion give

$$\ell\sin\theta = (v\cos\theta)t \quad \text{and} \quad -\ell\cos\theta = (v\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad -\cos\theta = (v\sin\theta)\left(\frac{\ell\sin\theta}{v\cos\theta}\right) - \frac{1}{2}g\left(\frac{\ell\sin\theta}{v\cos\theta}\right)^2$$

or, $$-\cos^2\theta = \sin^2\theta - \frac{1}{2}g\frac{\ell\sin^2\theta}{v^2\cos\theta}$$

or, $$-\cos^2\theta = 1 - \cos^2\theta - \frac{1}{2}\frac{g\ell\sin^2\theta}{g\ell\cos^2\theta} \quad [\text{From(i)}]$$

or, $1 = \frac{1}{2}\tan^2\theta$ or, $\tan\theta = \sqrt{2}$

From (iii), $v_0 = \left[g\ell(2+\sqrt{3})\right]^{1/2}$

14. VARIOUS FORMS OF ENERGY: THE LAW OF CONSERVATION OF ENERGY

Conservation of Energy

We observe that in many processes the sum of both the kinetic and potential energies does not remain a constant. This may be due to the influence of dissipative forces such as friction.

(a) The more general form of law of conservation of energy was established by taking into account other forms of energy such as thermal, electrical, chemical, nuclear, etc.

(b) The charges in all forms of energy is given by: $\Delta KE + \Delta U + \Delta(\text{all other forms of energy}) \equiv 0$

This is what we mean by the law of conservation of energy and it is one of the most important principles of physics.

"The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one body to another, but the total amount remains constant."

PROBLEM-SOLVING TACTICS

(a) One should always isolate the known and unknown quantities and write equations and solve them.

(b) The next step would be to find out a way from unknown to known quantities and write equations and solve them.

(c) One should always be very careful in doing so to avoid silly mistakes such as unit change of parameter.

(d) Energy is scalar in nature. However, get a clear idea of what is being gained or lost by which entity.

(e) Physical visualization of any problem will always help in increaseaing confidence in solving equations pertaining to the same.

(f) Further, problems involving integration would be easy to understand if you go event by event and then solve.

(g) Special cases and boundary conditions of circular motion are definitely recommended to be mastered because many problems break down to these special cases just after few manipulations.

FORMULAE SHEET

S. NO.	DESCRIPTION	FORMULA
1	Kinetic energy of the particle	$K(v) = \frac{1}{2}mv^2 = \frac{1}{2}m\vec{v}.\vec{v}$
2	Work done by force F	$W = \vec{F}.\vec{r}$ (here $\vec{r}$ is total displacement)
3	Work done by variable force	$w = \int \vec{F}.d\vec{r}$
4	Power generated by force F acting on body	$P = \frac{dW}{dt} = \vec{F}.\frac{d\vec{r}}{dt} = \vec{F}.\vec{v}$
5	Increase in Kinetic Energy = Decrease in Potential Energy	$KE = -\Delta U$
6	Energy conservation principle	$\Delta K + \Delta U = 0;\ \frac{1}{2}mv^2 = mgh$ or, $v = \sqrt{2gh}$
7	For a Spring work done W	$W = \int_{x_1}^{x_2} -kx\,dx = \frac{1}{2}k\left(x_1^2 - x_2^2\right)$
8	Work-Energy principle	$W_{net} = \Delta KE = K_f - K_i$
9	Work done by variable forces in short range	For $\vec{F} = \vec{F}_1 + \vec{F}_2 + ...$ $W = \int \vec{F}.d\vec{r} = \int\left(\vec{F}_1 + \vec{F}_2 + ...\right).\,d\vec{r}$
10	For conservative forces, change in potential energy	$U_f - U_i = -\int_{r_i}^{r_f} \vec{F}.d\vec{r}$
11	Elastic Potential Energy	$U = \frac{1}{2}kx^2$