PROBLEM SOLVING TACTICS

Step 1: First, construct a big schematic diagram of the physical situation. Then, while reading and rereading the problem statement construct your diagram accordingly including every available information from the statement on the diagram. Thereafter, if applicable, attach appropriate symbols to each important parameter in the problem irrespective of the fact that whether the value of the parameter is known or not. Eventually, make straight lines straight, parallel lines parallel, perpendicular lines perpendicular, etc., to the best of your ability in order to avoid confusion later on.

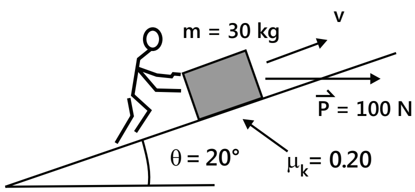


Figure: 4.69

Example at right. (Note how the following items recorded from the statement of the problem into specific items on the drawing: "... 30 kg suitcase ... moves upward ... 20 degree incline ... applied horizontal force of 100 N ... kinetic coefficient of friction ... is 0.20. ")

Step 2: Select a «system» to which you intend to apply Newton›s second law. In some problems, however, there may be more than one candidate for the "system." You may not choose the best one always the first time. That should not be a case for worry; just choose another one and do it again.

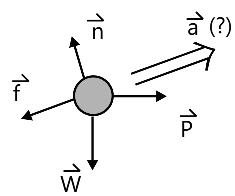
Example: We will select the "suitcase" as our system because it is the thing to which many obvious forces are being applied and it is the thing whose acceleration we want to find.

Step 3: Identify all the forces acting on «the system.» You can do this by drawing a dotted line around the system chosen in step 2 and identifying all physical objects that come in contact with the system. Each of these will exert a definite force on the system. Then, look for «field» forces—those forces that act without touching through the intermediary of a field of some sort. We know that in introductory mechanics the only "field" force is the force of gravity. It is a force exerted by the earth (or some other very massive body) on the system through the intermediary of the gravitational field. **Important!** It should be understood that every force on a system is exerted by some physical object exterior to the system. If you cannot identify that object and the method of interaction (contact or field), then the force DOES NOT EXIST! Listed here are the some commonly encountered forces and some tips on dealing with them:

(a) Ropes or strings: These exert «tension» forces on the system in question. They are always directed away from the system and along the direction of extension of the rope or string used.

(b) Contacts with surfaces: We generally split up the force due to contact with a surface into two components called the "normal"—meaning "perpendicular"—force and the "frictional" force. The normal force is generally a "push" type of force directed toward the system, unless the surface is sticky enabling it to exert a «pull» type of force. In contrast, the frictional force is parallel to the surface, opposes motion or potential motion (i.e., a system on the verge of «slipping») and is often assumed to be related to the normal force through a «coefficient of friction.» Readers may kindly refer to the discussion on the topic, "coefficient of friction" in this regard.

(c) Hinges or Pins: These exert forces of arbitrary magnitude and direction as required so as to ensure that the point of attachment remains stationary.



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(d) General pushes or pulls: If a working problem specifies that some object is being pushed or pulled in some direction, then you may have to assume that the force specified is being exerted by some physical object. Therefore, it is very important that you do not forget to include the same.

(e) Air resistance: Air may is not visible, but it is very likely that it does establish a physical contact with your system. Quite often we neglect air resistance because its effects are deemed negligible. However, if a problem specifies a certain amount of air resistance is involved or tells you that the air resistance depends in some way on velocity or other parameters, then do not forget to include it.

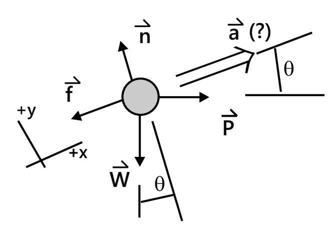


Figure: 4.71

(f) Gravitational force: We are aware of the fact that the gravitational force—commonly called the "weight" of the system—is the only force that acts without being in physical contact with the system (at least until you learn about electric and magnetic forces later on.) Generally, it acts in the downward direction (by definition!) and is equal to the mass of the system times the local gravitational field strength g—commonly, but misleadingly called "the acceleration due to gravity."

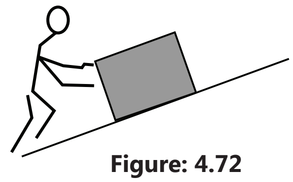


Figure: 4.72

The example at right. Here, we find two objects in contact with the system—one being the "surface" and other one "pusher." Thus, we find a total of four forces—the normal force, the frictional force (from the surface), the push (from the pusher), and the weight (due to the only force—so far—that acts without needing to touch—gravity.)

Step 4: Draw an "FBD." Now, the system may be represented by a simple circle or square; however, we want to focus our attention on the forces on and the resulting acceleration of the system. Now, draw each force with its tail at the surface of the system extending in the proper direction. Further, include the acceleration vector as well, but distinguish it from the force vectors by drawing it in a different way.

Example at the right. In this example, note that the normal force is directed perpendicular to the surface (not shown in the FBD), the frictional force is directed opposite to the direction that the system slips with respect to the surface, the push is in its given direction, and the weight is directed "down." We also show the acceleration as a different-looking vector that is directed upward along the plane, but we do not know this for certain; it may be directed downward along the incline. To keep us remind ourselves of this fact, we put a "(?)" next to the acceleration vector.

Step 5: Now, pick a coordinate system and hence determine the angles that the forces and accelerations make with the coordinate axes. It is usually "clever" and preferable to pick a coordinate system that minimizes the number of unknown vectors that will have to be broken down into components. The answers you obtain thereafter must and will be independent of your choice of coordinate system, but clever choices will help us to arrive at equations that are more easy to solve. However, you may need to do some geometrical work on another sheet of paper to figure out how the angles are related to those given in the problem statement.

Example at right. In this example, we have chosen a coordinate system that requires us to determine the components of only the weight and the push—the two forces about which we know a lot. These two forces lie at the angle theta (given in the problem statement as 20 degrees) from one of the axis directions.

Step 6: Now, write Newton's second law. This law is the basic physical principle you are applying; i.e., the "starting point" for your calculations. Just proceed to do it! Example: $\sum \vec{F} = m\vec{a}$

Step 7: Thence, apply the basic equation to this problem. Now, simply write what the "sum of forces" is in this case. If the acceleration is zero, then use that fact to simplify the equation too. Example: $\vec{n} + \vec{f} + \vec{P} + \vec{W} = m\vec{a}$

Step 8: Now, continue by writing the component equations. This is simply a matter of recognizing that every vector equation is shorthand for two (or, more generally, three) scalar equations. Then, simply rewrite the vector equation for each component direction with each vector quantity rewritten as the corresponding component. Examples: $x: n_x + f_x + P_x + W_x = ma_x$ and $n_y + f_y + P_y + W_y = ma_y$

Step 9: Now, determine what each component is in terms of the vector magnitude and trigonometric functions of the associated angles. In this step, it is imperative that we explicitly indicate the signs of the vector components. This is also a good time to explicitly substitute "mg" for "W" if you really happen to know the mass of the system.

Example: Notice that the normal force is purely in the +y direction, the frictional force is purely in the –x direction, the push has a positive x-component and a negative y-component, the weight has negative x- and y-components, and the assumed acceleration is purely in the +x direction. Thus, we have: $x:\ 0 + (-f) + (+P\cos\theta) + (-mg\sin\theta) = m(+a)$

$y:\ (+n) + 0 + (-P\sin\theta) + (-mg\cos\theta) = m(0)$

Step 10: To conclude this procedure, as a final step, simplify the resulting equations and figure out where to go from here. This is the end of "the method." Example: $P\cos\theta - f - mg\sin\theta = ma$ $\ n - P\sin\theta - mg\cos\theta = 0$

FORMULAE SHEET

(a) $F_1 = F\cos\theta =$ component of $\vec{F}$ along AC $\quad F_2 = F\sin\theta =$ component of $\vec{F}$ perpendicular to AC

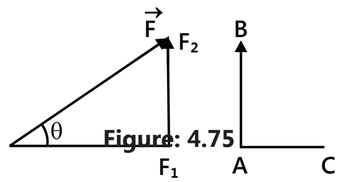


Figure: 4.73

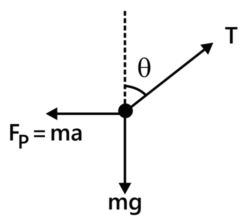
(b) $F = \frac{dp}{dt} = \frac{d(mv)}{dt}$

(c) $\sum \vec{F} = \vec{F}_{net} = m\vec{a}$ or $\vec{a} = \frac{\vec{F}_{net}}{m}$

$\sum F_x = 0 \Rightarrow T\sin\theta = ma$

$\sum F_y = 0 \Rightarrow T\cos\theta = mg$

$\theta = \tan^{-1}\left(\frac{a}{g}\right)$



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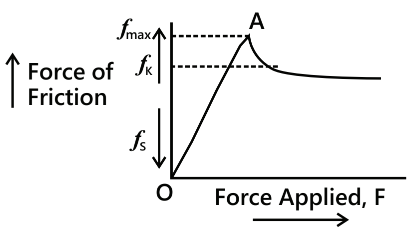
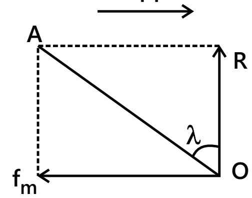
(d) **Impulse = change in momentum** $\quad \vec{F}\Delta t = m\vec{v_f} - m\vec{V_o}$

(e) $\mu = \frac{f_{max}}{R}$



(f) $f_{max} = f_{limiting} = \mu_s R$



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(g) **Angle of Friction:** $\quad \tan\lambda = \frac{f_{max}}{R} = \mu \quad$ or $\lambda = \tan^{-1}(\mu)$

(h) **Pseudo force:** F = –ma; where m = mass of the object, a = acceleration of the reference frame

(i) A particle in circular motion may have two types of velocities as listed hereunder.

(i) Linear velocity v and

(ii) Angular velocity ω.These two are related by the equation $\quad v = R\omega$ (R = radius of circular path)

(j) Acceleration of a particle in a circular motion may have two components as listed hereunder.

(i) Tangential component (a_t) and

(ii) Normal or radial component (a_n).

As the name suggests, the tangential component is tangential to the circular path, given by a_t = rate of change of speed

(k) $\frac{dv}{dt} = \frac{d|\vec{v}|}{dt} = R\alpha$ where α = angular acceleration = rate of change of angular velocity = $\frac{d\omega}{dt}$

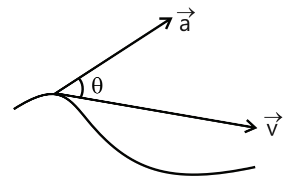
The normal or radial component, also known as centripetal acceleration is toward the center and is given by

$a_n = R\omega^2 = \frac{v^2}{R}$

(l) Net acceleration of a particle is the resultant of two perpendicular components, a_n and a_t. Hence, $a = \sqrt{a_n^2 + a_t^2}$

(m) Tangential component a_t is responsible for change of speed of a particle. This can be positive, negative or zero, depending upon the situation whether the speed of the particle is increasing, decreasing or remains constant.

(n) In general, in any curvilinear motion, direction of instantaneous velocity is tangential to the path, while acceleration may assume any direction. If we resolve the acceleration in two normal directions, one parallel to velocity and another perpendicular to velocity, then the first component is a_t while the other is a_n.



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Thus, a_t = component of $\vec{a}$ along $\vec{v}$ = $a\cos\theta = \frac{\vec{a}.\vec{v}}{v} = \frac{dv}{dt} = \frac{d|\vec{v}|}{dt}$ = rate of change of speed.

Further, a_n = component of $\therefore a_{1x} = a_{2x} = a_{3x}$ perpendicular to $\vec{v} = \sqrt{a^2 - a_t^2} = \frac{v^2}{R}$

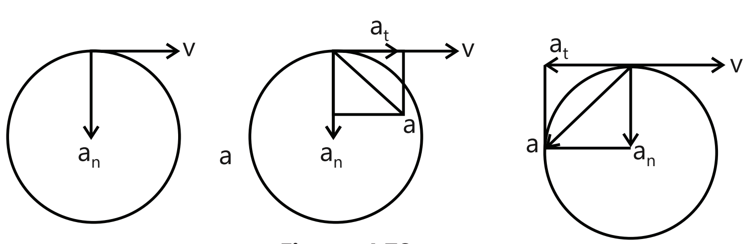
Here, v is the speed of the particle at that instant and R is called the radius of curvature to the curvilinear path at that point.

(o) In $a_t = a\cos\theta$, if θ is acute, a_t will be positive and speed increases. However, if θ is obtuse a_t will be negative and speed will decrease. If θ is $90°$, a_t is zero and speed will remain constant.

(p) Now, depending upon the value of a_t, circular motion may be of three types as listed hereunder.

(i) Uniform circular motion in which speed remains constant or $a_t = 0$.

(ii) Circular motion of increasing speed, in which a_t is positive.



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(iii) Circular motion of decreasing speed, in which a_t is negative.