PROBLEM-SOLVING TACTICS

To avoid confusion while using signs in equations of motion, the following points need to be considered:

(a) Assuming any one direction to be positive, the other automatically becomes negative.

Generally, vertically up is considered as positive and right side is taken as positive.

(b) Write down the values of velocity, displacement and acceleration according to the sign convention.

(c) On completion of sign convention, then simply use the equations of motions.

FORMULAE SHEET

Position vector of point A with respect to O:

$\vec{r}_A = \overrightarrow{OA} = x_A\hat{i} + y_A\hat{j} + z_A\hat{k}$

$\overrightarrow{AB} = \vec{r}_s - \vec{r}_A$

$\overrightarrow{AB} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$

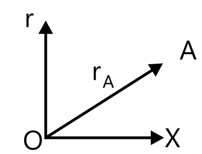

Figure 2.69

For un-accelerated Motion: Distance = Speed x Time

Displacement = Velocity x Time

v_{av} = Average speed $= \frac{\Delta s}{\Delta t}$; $\vec{v}_{av}$ = Average Velocity $= \frac{\Delta \vec{r}}{\Delta t}$

$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$ and $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{d\vec{r}}{dt}$

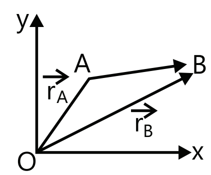

Figure 2.70

(a) Uniform motion is due to constant Velocity

Average acceleration has the same direction as of change in velocity

$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$

(b) One-dimensional Uniformly Accelerated Motion

- $v = u + at,\ s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$ and $v_{avg} = (v + u)/2$

(i) Maximum height attained by a particle, thrown upwards from ground with initial velocity u is $h = \frac{u^2}{2g}$

(c) Displacement of particle in t^th second of its motion , $s_t = u + \frac{a}{2}(2t - 1)$

- Time taken to reach maximum height = u/g

(i) Velocity of a particle (u=0) when it touches the ground when dropped from a height h is,

$v = \sqrt{2gh}$

(ii) In (b) time of collision with ground $t = \sqrt{2h/g}$

- If acceleration of particle is not constant, basic equations of velocity and acceleration are used, i.e.

(i) $\vec{v}=\frac{d\vec{s}}{dt}$ or $\vec{v}=\frac{d\vec{r}}{dt}$ (ii) $\vec{a}=\frac{d\vec{v}}{dt}$ (iii) $d\vec{s}=\vec{v}\,dt$ (iv) $d\vec{v}=\vec{v}\,dt$

- For one dimensional motion,

(i) $v=\frac{ds}{dt}$ (ii) $a=\frac{dv}{dt}=v\frac{dv}{ds}$

(iii) $ds=v\,dt$ and (iv) $dv=a\,dt$ or $v\,dv=a\,ds$

- If $z=\frac{dy}{dx}$ or $\frac{y}{x}$, the value of z at any point on x-y graph can be obtained by the slope of the graph at that point.

(i) slope of displacement–time graph gives velocity $\left(\text{as } v=\frac{ds}{dt}\right)$

(ii) slope of velocity–time graph gives acceleration $\left(\text{as } a=\frac{dv}{dt}\right)$

If z = yx, y (dx), or x (dy), the value of z between x_1 and x_2 or y_1 and y_2 can be obtained by the area of graph between x_1 and x_2 or y_1 and y_2

(iii) velocity–time graph gives displacement (as ds=v dt)

(iv) acceleration–time graph gives change in velocity (as dv= adt).

relative velocity of A with respect to B (written as $\vec{v}_{AB}$) is $\vec{v}_{AB}=\vec{v}_A-\vec{v}_B$

Similarly, relative acceleration of A with respect to B is $\vec{a}_{AB}=\vec{a}_A-\vec{a}_B$

$\vec{v}_{AB}=-\vec{v}_{BA}$ or $\vec{a}_{BA}=-\vec{a}_{AB}$

- In case of a one dimensional motion when we can treat the vectors as scalars by assigning positive to one direction and negative to another, the above equations can be written as $v_{AB}=v_A-v_B$ and $a_{AB}=a_A-a_B$