14. HEAT TRANSFER

1. INTRODUCTION

Heat can be transformed from one place to another place by the three processes - conduction, convection and radiation. In conduction, the heat flows from a place of higher temperature to a place of lower temperature through a stationary medium. The molecules of the medium oscillate about their equilibrium positions more violently at a place of higher temperature and collide with the molecules of adjacent position, thus transferring a part of their energy to these molecules which now vibrate more violently. Thus heat can be transmitted by collision of molecules. In metals, the conduction of heat takes place by the movement of free electrons. In the cases of liquids and gases, the heat is transferred not only by collision but also by motion of heated molecules which carry the heat in such media. This process is called convection. When a liquid in a vessel is heated, the lighter molecules present in the lower layer of the liquid get heated which rise to the surface of the liquid and cold molecules at the surface go towards the bottom of vessel. These are convection currents and are the major means of heat transport in fluids. Radiation is mode of transfer of heat in which the heat travels directly from one place to another without the role of any intervening medium. The heat from the sun propagates mostly through vacuum to reach the earth by the process of radiation.

2. CONDUCTION

The figure shows a rod whose ends are in thermal contact with a hot reservoir at temperature T_1 and a cold reservoir at temperature T_2. The sides of the rod are insulated, hence heat transfer is only along the rod and not through its sides. The molecules at the hot reservoir have greater vibrational energy. This energy is transferred by collisions to the atoms at the end face of the rod. These atoms in turn transfer the energy to their neighbors further along the rod. Such transfer of heat through a substance in which heat is transported without direct mass transport is called conduction.

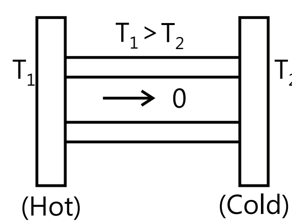


Figure 14.1

The quantity of heat conducted Q in time t across a slab of length L, area of cross-section A and steady state temperature θ_1 and θ_2 at respective hot and cold ends is given by $Q = \frac{kA(\theta_1 - \theta_1)t}{L}$, where k is the coefficient of thermal conductivity which is equal to the quantity of heat flowing per unit time through unit area of cross-section of a material per unit length along the direction of flow of heat.

Units of k are kilocalorie/meter second degree centigrade or $J.m^{-1}sec^{-1}K^{-1}$. In C.G.S. units,

k is expressed in $cal cm^{-1}(°C)^{-1}sec^{-1}$

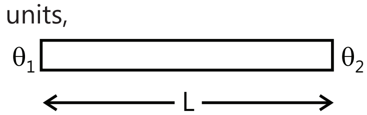


Figure 14.2

The temperature Gradient / (unit distance) $= -\frac{d\theta}{dx}$

$$\therefore \quad Q = -kA\left(\frac{d\theta}{dx}\right)t; \qquad \frac{\Delta Q}{\Delta t} = -kA\,\frac{dT}{dx}$$

The quantity dT/dx is called the temperature gradient. The minus sign indicates that dT/dx is negative along the direction of the heat flow, i.e., heat flows from a higher temperature to a lower one.

$$\frac{dT}{dx} = H = \frac{\Delta t}{L/kA} = \frac{\Delta T}{R}$$

Here ΔT = temperature difference (TD) and $R = \frac{L}{kA}$ = Thermal resistance of the rod.

MASTERJEE CONCEPTS

This relation is mathematically equivalent to Ohm's Law and can be used very effectively in solving problems effectively by considering temperature analogous to potential and heat transferred per unit time as current.

Nivvedan (JEE 2009, AIR 113)

Heat flow through a conducting rod	Current flow through a resistance
Heat current $H = \frac{dQ}{dt}$ =Rate of heat flow $H = \frac{\Delta T}{R} = \frac{T(\text{temp diff})}{R}$ where $R = \frac{L}{kA}$ and k = Thermal conductivity	Electric current $i = \frac{dq}{dt}$ = Rate of charge flow $i = \frac{\Delta V}{R} = \frac{PD(\text{potential diff})}{R}$; $\quad R = \frac{i}{\sigma A}$ σ = Electrical conductivity.

3. GROWTH OF ICE ON PONDS

When temperature of the atmosphere falls below 0°C, the water in the pond starts freezing. Let at time t thickness of ice in the pond is y and atmospheric temperature is -T°C. The temperature of water in contact with the lower surface of ice will be 0°C.

Using $\frac{dQ}{dt} = L\left(\frac{dm}{dt}\right)$; $\qquad \frac{TD}{R} = L\frac{d}{dt}\{A\rho y\}$ $\qquad$ (A = Area of pond)

$$\therefore \frac{\left[0-(-T)\right]}{(y/kA)} = LA\rho.\frac{dy}{dt} \quad \therefore -\frac{dy}{dt} = \frac{kT}{\rho}.\frac{1}{Ly}$$ where L -> Latent heat of fusion

And hence time taken by ice to grow a thickness y $\; t = \frac{\rho L}{kT}\int_0^y y\,dy$ or $t = \frac{1}{2}\frac{\rho L}{kT}y^2$

Time does not depend on the area of pond.

MASTERJEE CONCEPTS

Time taken by ice to grow on ponds is independent of area of the pond and it is only dependent only the thickness of ice sheet.

Vaibhav Krishnan (JEE 2009, AIR 22)

4. SERIES AND PARALLEL CONNECTION OF RODS

4.1 Series Connection

Consider two rods of thermal resistances R_1 and R_2 joined one after the other as shown in figure. The free ends are kept at temperatures T_1 and T_2 with $T_1>T_2$. In steady state, any heat that goes through the first rod also goes through the second rod. Thus, the same heat current passes through the two rods. Such a connection of rods is called a series connection.

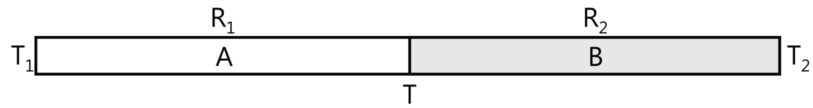


Figure 14.3

Suppose, the temperature of the junction is T, the heat current through the first rod is,

$$i=\frac{\Delta Q}{\Delta t}=\frac{T_1-T}{R_1} \text{ or } T_1-T=R_1 i \qquad \text{... (i)}$$

and that through the second rod is $i=\frac{\Delta Q}{\Delta t}=\frac{T-T_2}{R_2}$ or $T-T_2=R_2 i$ ... (ii)

Adding (i) and (ii) $\quad T_1-T_2=\left(R_1+R_2\right)i$ or $i=\frac{T_1-T_2}{R_1+R_2}$

Thus, the two rods together is equivalent to a single rod of thermal resistance R_1+R_2.

If more than two rods are joined in series, the equivalent thermal resistance is given by, $R=R_1+R_2+R_3+\ldots$

4.2 Parallel Connection

Now, suppose the two rods are joined at their ends as shown in figure. The left end of both the rods are kept at temperature T_1 and the right ends are kept at temperature T_2.

So the same temperature difference is maintained between the ends of each rod. Such a connection of rods is called a parallel connection. The heat current going through the first rod is $i_1=\frac{\Delta Q_1}{\Delta t}=\frac{T_1-T_2}{R_1}$

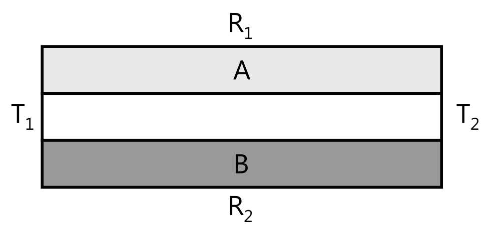


Figure 14.4

and that through the second rod is $i_2=\frac{\Delta Q_2}{\Delta t}=\frac{T_1-T_2}{R_2}$

The total heat current going through the left end is $i=i_1+i_2=(T_1-T_2)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

or $i=\frac{T_1-T_2}{R}$

Where $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$ ... (i)

5. RADIAL FLOW OF HEAT THROUGH A CYLINDRICAL TUBE

Consider a cylindrical tube of length l and respective inner and outer radii as r_1 and r_2. If the heat flows radially i.e., perpendicular to the axis of the tube from the steady state temperatures θ_1 at the inner surface to the temperature θ_2 at the outer surface, then the rate of heat flowing through an element of shell lying between radius r and r+ dr is given by $\Delta Q=-k\left(2\pi r\ell\right)\frac{d\theta}{dr}$ where $d\theta$ is temperature difference across the shell.

It can be integrated for total heat flow per second.

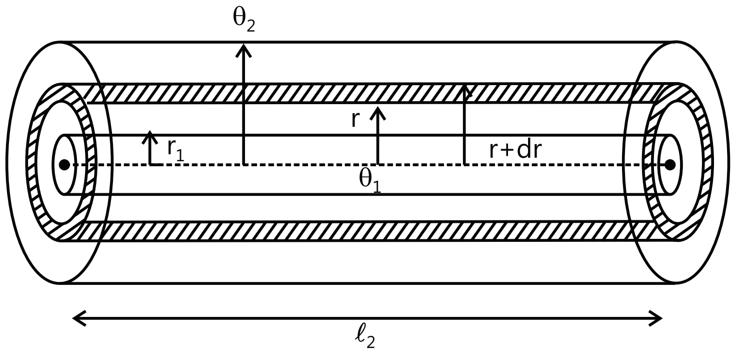


Figure 14.5

∴ Total heat flowing per second,

$$Q = \frac{2\pi k\ell(\theta_1 - \theta_2)}{\int_{r_1}^{r_2} \frac{dr}{r}}; \quad Q = \frac{2\pi k\ell(\theta_1 - \theta_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

MASTERJEE CONCEPTS

No mass movement of matter occurs in conduction. Solids are better conductors than liquids, liquids are better conductors than gases.

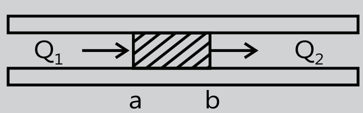


Figure 14.6

Consider a section ab of a rod as shown in figure. Suppose Q_1 heat enters into the section at 'a' and Q_2 leaves at 'b', then $Q_2 < Q_1$.

Part of the energy Q_2 - Q_1 is utilized in raising the temperature of section ab and the remaining is lost to the atmosphere through ab. If heat is continuously supplied from the left end of the rod, a stage comes when temperature of the section becomes constant. In that case $Q_1 = Q_2$ if rod is insulated from the surroundings (or loss through ab is zero). This is called the steady state condition. Thus, in steady state temperature of different sections of the rod becomes constant (but not same).

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 1: One face of a copper cube of edge 10 cm is maintained at 100°C and the opposite face is maintained at 0°C. All other surfaces are covered with an insulating material. Find the amount of heat flowing per second through the cube. Thermal conductivity of copper is 385 $Wm^{-1}\,°C^{-1}$. **(JEE MAIN)**

Sol: Always consider the A which perpendicular to the flow of heat.

The heat flows from the hotter face towards the colder face. The area of cross section perpendicular to the heat flow is $A = (10\text{cm})^2$

The amount of heat flowing per second is $\frac{\Delta Q}{\Delta t} = KA\frac{T_1 - T_2}{X} = \left(385\text{Wm}^{-1}°\text{C}^{-1}\right) \times (0.1\text{m})^2 \times \frac{(100°\text{C} - 0°\text{C})}{0.1\text{m}} = 3850\text{W}.$

Illustration 2: A cylindrical block of length 0.4 m and area of cross-section 0.04m^2 is placed coaxially on a thin metallic disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at the constant temperature of 400 K and initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 100 watt/m-K and the specific heat of the material of the disc is 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350 K? Assume, for the purpose of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

(JEE ADVANCED)

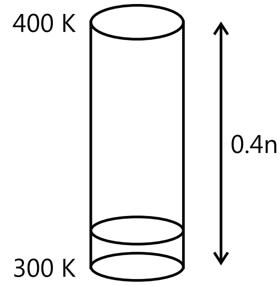


Figure 14.7

Sol: Write the equation rate of heat transfer at any temperature 'T' for the disc. Rate of heat transfer proportional to rate of change in temperature.

As heat is conducted from the cylinder to the disc, the temperature of the disc increases. If the temperature of the disc at some instant is T, then rate of flow of heat through the cylinder at that instant is $\frac{dQ}{dt}=\frac{KA\left(400-T\right)}{L}$ ... (i)

If dT is the further increase in the temperature of the disc in the infinitesimal time interval dt,

then $\frac{dQ}{dt}=ms\frac{dT}{dt}$ ... (ii)

Where m is the mass of the disc and c is its specific heat.

From equations (i) and (ii)

$$\frac{KA\left(400-T\right)}{L}=ms\frac{dT}{dt};\ dt=\frac{msL}{KA}\left[\frac{dT}{400-T}\right]$$

Integrating we get, $$t=\frac{msL}{KA}\int_{300}^{350}\frac{dT}{400-T}=\frac{msL}{KA}\times 2.303\log_{10}\left[\frac{400-300}{400-350}\right]$$

$$=\frac{0.4\times 600\times 0.4}{10\times 0.04}\times 2.303\times 0.3010=166s.$$

6. CONVECTION

In this process, actual motion of heated material results in transfer of heat from one place to another. For example, in a hot air blower, air is heated by a heating element and is blown by a fan. The air carries the heat wherever it goes. When water is kept in a vessel and heated on a stove, the water at the bottom gets heated due to conduction through the vessel's bottom. Its density decreases and consequently it rises. Thus, the heat is carried from bottom to the top by the actual movement of the parts of the water. If the heated material is forced to move, say by a blower or by a pump, the process of heat transfer is called forced convection. If the material moves due to difference in density, it is called natural or free convection.

MASTERJEE CONCEPTS

The convection currents created in a room by a radiator means that the warm air is circulated around and the warming is more uniform than just being the air around the radiator. When heating water on a stove, the convection currents created by the rising hot water means that all the water gets heated instead of just the water at the very bottom of the pan. Some rainfall is also caused by moist air being heated and rising, then cooling quickly and allowing the water vapor to condense into rain.

Anand K (JEE 2011, AIR 47)

7. RADIATION

The third means of energy transfer is radiation which does not require a medium. The best known example of this process is the radiation from Sun. All objects radiate energy continuously in the form of electromagnetic waves. The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature. This is known as the **Stefan's law** and is expressed in equation form as $P = \sigma AeT^4$

Here P is the power in watts(J/s) radiated by object, A is the surface area in m^2,e lies between 0 and 1 and is called **emissivity** of the object and σ is universal constant called Stefan's constant, which has the value, $\sigma = 5.67\times10^{-8}\,W/m^2-K^4$.

8. PERFECTLY BLACK BODY

A body that absorbs all the radiation incident upon it and has as emissivity equal to 1 is called a perfectly black body. A black body is also an ideal radiator. It implies that if a black body and an identical another body is kept at the same temperature, then the black body will radiate maximum power as is obvious from equation $P = \sigma AeT^4$

This is also because e=1 for a perfectly black body while for any other body, e<1.

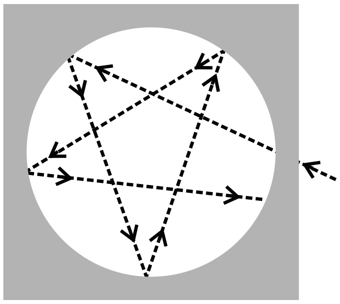


Figure 14.8

MASTERJEE CONCEPTS

Always remember that black body is a perfect absorber and emitter of light. At temperatures higher than the surrounding, it is the most shining thing and at lower temperatures it is the darkest thing.

There is no perfect black body. Materials like black velvet or lamp black come close to being ideal black bodies, but the best practical realization of an ideal black body is a small hole leading into a cavity, as this absorbs 98% of the radiation incident on them.

GV Abhinav (JEE 2012, AIR 329)

Illustration 3: A solid copper sphere of density ρ, specific heat c and radius r is at temperature T_1. It is suspended inside a chamber whose walls are at temperature 0K. What is the time required for the temperature of sphere to drop to T_2? Take the emissivity of the sphere to be equal to e. **(JEE MAIN)**

Sol: Heat lost by radiation cause temperature to fall.

The rate of loss of energy due to radiation, $P = \sigma AeT^4$. This rate must be equal to $mc\frac{dT}{dt}$ Hence, $-mc\frac{dT}{dt} = \sigma AeT^4$

Negative sign is used as temperature decreases with time. In this equation,

$$m = \left(\frac{4}{3}\pi r^3\right)\rho \text{ and } A = 4\pi r^2 \quad \therefore -\frac{dT}{dt} = \frac{3e\sigma}{\rho cr}T^4 \text{ or } -\int_0^1 dt = \frac{\rho cr}{3e\sigma}\int_{T_1}^{T_2}\frac{dT}{T^4}; \quad t = \frac{\rho cr}{9e\sigma}\left(\frac{1}{T_2^3} - \frac{1}{T_1^3}\right)$$

9. ABSORPTIVE POWER 'a'

"It is defined as the ratio of the radiant energy absorbed by a body in a given time to the total radiant energy incident on it in the same interval of time."

$$a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

As a perfectly black body absorbs all radiations incident on it, the absorptive power of perfectly black body is maximum and unity.

10. SPECTRAL ABSORPTIVE 'a_λ'

This absorptive power 'a' refers to radiations of all wavelengths (or the total energy) while the spectral absorptive power is the ratio of radiant energy absorbed by a surface to the radiant energy incident on it for a particular wavelength λ. It may have different values for different wavelengths for a given surface. Let us take an example, suppose a = 0.6, a_λ = 0.4 for 1000 Å and a_λ = 0.7 for 2000 Å for a given surface. Then it means that this surface will absorb only 60% of the total radiant energy incident on it. Similarly it absorbs 40% of the energy incident on it corresponding to 1000 Å and 70% corresponding to 2000 Å. The spectral absorptive power a_λ is related to absorptive power a through the relation $a = \int_0^\infty a_\lambda d\lambda$

11. EMISSIVE POWER 'e'

(Don't confuse it with the emissivity e which is different from it, although both have the same symbols e).

"For a given surface it is defined as the radiant energy emitted per second per unit area of the surface." It has the units of W/m^2 or J/s-m^2, for a black body $e = \sigma T^4$

Note: Absorptive power is dimensionless quantity where emissive power is not.

12. SPECTRAL EMISSIVE POWER

Similar to the definition of the spectral absorptive power, it is emissive power for a particular wavelength λ.

Thus, $e = \int_0^\infty e_\lambda d\lambda$

13. KIRCHHOFF'S LAW

The ratio of emissive power to absorptive power is the same for all bodies at a given temperature and is equal to the emissive power E of a blackbody at that temperature. Thus,

$$\frac{E(\text{body})}{a(\text{body})} = E(\text{blackbody})$$

Kirchhoff's law tells that if a body has high emissive power, it should also have high absorptive power to have the ratio e/a same. Similarly, a body having low emissive power should have low absorptive power. Kirchhoff's law may be easily proved by a simple argument as described below.

Consider two bodies A and B of similar geometrical shapes placed in an enclosure. Suppose A is any random body and B is a blackbody. In thermal equilibrium, both the bodies will have the same temperature as the temperature of the enclosure. Suppose an amount ΔU of radiation falls on the body A in a given time Δt. As A and B have the same geometrical shapes, the radiation falling on the blackbody B is also ΔU. The blackbody absorbs all of this ΔU. As the temperature of the blackbody remains constant, it also emits an amount ΔU of radiation in that time. If the emissive power of the blackbody is e_0, we have $\Delta U \propto E_0$ or $\Delta U = kE_0$ ... (i)

where k is constant.

Let the absorptive power of A be a. Thus, it absorbs an energy of a ΔU of the radiation falling on it in time Δt. As its temperature remains constant, it must also emit the same energy a ΔU in that time. If the emissive power of the body A is e, we have $a\,\Delta U = ke$ ... (ii)

The same proportionality constant k is used in (i) and (ii) because the two bodies have identical geometrical shapes and radiation emitted in the same time Δt is considered.

From (i) and (ii),

$$a = \frac{E}{E_0} \text{ or } \frac{E}{a} = E_0 \text{ or } \frac{E(\text{body})}{a(\text{body})} = E(\text{blackbody})$$

MASTERJEE CONCEPTS

It can be thought like, good absorber is a good emitter because at some point of time, it might have stored energy because it is a good absorber. Now as soon as the temperature of the surrounding becomes low than that of the body, this energy starts decreasing until the steady state is reached. Hence, it must be a good emitter too.

Good absorbers for a particular wavelength are also good emitters of the same wavelength.

Anurag Saraf (JEE 2011, AIR 226)

14. STEFANS-BOLTZMANN LAW

The energy of thermal radiation emitted per unit time by a blackbody of surface area A is given by $u = \sigma A T^4$ ... (i)

Where is a universal constant known as Stefan Boltzmann constant and T is its temperature on absolute scale. The measured value of σ is 5.67×1^{-8} Wm^{-2} K^{-4}. Equation (i) itself is called the Stefan-Boltzmann law. Stefan had suggested this law based on his experimental data on radiation and Boltzmann derived it from thermo dynamical analysis. The law is also quoted as Stefan's law and the constant σ as Stefan constant.

A body which is not a blackbody, emits less radiation than given by equation (i). It is, however, proportional to T^4. The energy emitted by such a body per unit time is written as $u = e\sigma A T^4$ ... (ii)

Where e is a constant for the given surface having a value between 0 and 1. This constant is called the emissivity of the surface. It is zero for completely reflecting surface and is unity for a blackbody.

Using Kirchhoff's law $\frac{E(\text{body})}{E(\text{blackbody})} = a$ ... (i)

Where a is the absorptive power of the body. The emissive power E is proportional to the energy radiated per unit time, that is, proportional to u. Using above equations, $\frac{e\sigma A T^4}{\sigma A T^4} = a$ or e=a.

Thus, emissivity and absorptive power have the same value.

Consider a body of emissivity e kept in thermal equilibrium in a room at temperature T_0.

The energy of radiation absorbed by it per unit time should be equal to the energy emitted by it per unit time. This is because the temperature remains constant. Thus, the energy of the radiation absorbed per unit time is $u = e\sigma A T_0^4$.

Now suppose the temperature of the body is changed to T but room temperature remains T_0. The energy of the thermal radiation emitted by the body per unit time is $u = e\sigma A T^4$.

The energy absorbed per unit time by the body is $u_0 = e\sigma A T_0^4$.

Thus, the net loss of thermal energy per unit time is $\Delta u = u - u_0 = e\sigma A(T^4 - T_0^4)$ ... (iii)

Illustration 4: A blackbody of surface area 10cm² is heated to 127°C and is suspended in a room at temperature 27°C. Calculate the initial rate of loss of heat from the body to the room. **(JEE MAIN)**

Sol: Heat lost by radiation and gained by absorption.

For a blackbody at temperature T, the rate of emission is $u = \sigma A T^4$. When it is kept in a room at temperature T_0, the rate of absorption is $u_0 = \sigma A T_0^4$.

The net rate of loss of heat is $u - u_0 = \sigma A(T^4 - T_0^4)$

Here $A = 10 \times 10^{-4}\,m^2$ $T = 400K$ $T_0 = 300K$

Thus, $u - u_0 = (5.67 \times 10^{-8}\,Wm^{-2}K^{-4})(10 \times 10^{-4}\,m^2)(400^4 - 300^4)K^4 = 0.99W$

Illustration 5: Energy falling on 1.0 area placed at right angles to a sun beam just outside the earth's atmosphere is 1.35 K joule in one second. Find sun's surface temperature. Mean distance of earth from sun is $1.50 \times 10^8 km$, mean distance of sun= $1.39 \times 10^6 km$ and Stefan's constant= 5.67×10^{-8} watt $m^{-2}K^{-4}$. **(JEE MAIN)**

Sol: $\sigma A_{sun}\, T^4 = S \times A_{earth}$

The temperature of the sun is given by

$$T^4 = \frac{S}{\sigma}\left(\frac{R}{r}\right)^2$$

$$\frac{S}{\sigma} = \frac{1.35 kJ/m^2 - sec}{5.67 \times 10^{-8}\ watt/m^2 - K^2} = \frac{135 \times 10^3\ watt/m^2}{5.67 \times 10^{-8}\ watt/m^2 - K^4} = 2.38 \times 10^{10} K^4$$

$$\frac{R}{r} = \frac{1.50 \times 10^8 km}{0.695 \times 10^6 km} = 215.8$$

$$\therefore T^4 = (2.38 \times 10^{10} K^4)(215.8)^2 = 1108 \times 10^{12} K^4$$

$$T = 5.770 \times 10^3 K \text{ or } T = 5770K$$

15. NEWTON'S LAW OF COOLING

The rate of cooling of a body is directly proportional to the difference of temperature of the body over its surroundings.

If a body at temperature θ_1 is placed in surroundings at lower temperature θ_2, the rate of cooling is given by $\frac{dQ}{dt} \propto (\theta_1 - \theta_2)$ where dQ is the quantity of heat lost in time dt.

Newton's law of cooling gives $\frac{dQ}{dt} = -k\left(\theta_1 - \theta_2\right)$ where k is constant.

If a body of mass m and specific heat s loses a temperature $d\theta$ in time dt, then $\frac{dQ}{dt} = ms\frac{d\theta}{dt} = -k(\theta_1 - \theta_2)$

Illustration 6: A liquid cools from 70°C to 60°C in 5 minutes. Calculate the time taken by the liquid to cool from 60°C to 50°C, if the temperature of the surrounding is constant at 30°C. **(JEE MAIN)**

Sol: Use newton's law cooling and taking temperature of the body is average of initial and final value.

The average temperature of the liquid in the first case is $\theta_1 = \frac{70°C + 60°C}{2} = 65°C$

The average temperature difference from the surrounding is $\theta_1 - \theta_0 = 65°C - 30°C = 35°C$.

The rate of fall of temperature is $-\frac{d\theta_1}{dt} = \frac{70°C - 60°C}{5\text{mins}} = 2°C\text{min}^{-1}$.

From Newton's law of cooling, $2°C\text{min}^{-1} = bA(35°C)$ Or $bA = \frac{2}{35\text{min}}$ ... (i)

In the second case, the average temperature of the liquid is $\theta_2 = \frac{60°C + 50°C}{2} = 55°C$

So that, $\theta_2 - \theta_0 = 55°C - 30°C = 25°C$

If it takes a time t to cool down from 60°C to 50°C, the rate of fall in temperature is $-\frac{d\theta_2}{dt} = \frac{60°C - 50°C}{t} = \frac{10°C}{t}$.

From Newton's law of cooling and (i), $\frac{10°C}{t} = \frac{2}{35\text{min}} \times 25°C$ Or $t = 7\text{min}$.

Illustration 7: At midnight, with the temperature inside your house at 70°F and the temperature outside at 20°F, your furnace breaks down. Two hours later, the temperature in your house has fallen to 50°F. Assume that the outside temperature remains constant at 20°F. At what time will the inside temperature of your house reach 40°F? **(JEE ADVANCED)**

Sol: Newton's law of cooling, follow logarithm curve in cooling.

The boundary value problem that models this situation is

$$\frac{dT}{dt} = k(20 - T) \qquad \begin{array}{l} T(0) = 70 \\ T(2) = 50 \end{array}$$

Where time 0 is midnight. The solution of this boundary value problem is $T = 20 + 50\left(\frac{3}{5}\right)^{t/2}$

This is obtained by solving above differential equation.

Note (for the purpose of a reasonableness check) that this formula given us

$T(0) = 20 + 50\left(\frac{3}{5}\right)^{0/2} = 70.$ and $T(2) = 20 + 50\left(\frac{3}{5}\right)^{2/2} = 50.$

To find when the temperature in the house will reach 40°F, we must solve equation $20 + 50\left(\frac{3}{5}\right)^{t/2} = 40$

The solution of this equation is $t = 2\left(\frac{\ln(2/5)}{\ln(3/5)}\right) \approx 3.6$

Thus, the temperature in the house will reach 40°F a little after 3.30 a.m.

MASTERJEE CONCEPTS

Newton's law of cooling can also be thought in the context of Stefan-Boltzmann law by considering the temperature difference between the body and the surroundings very close to zero, i.e. it can be considered as a special case of the latter.

Vijay Senapathi (JEE 2011, AIR 71)

16. WIEN'S DISPLACEMENT LAW

At ordinary temperatures (below about 600°C), the thermal radiation emitted by bodies is invisible, most of them lie in wavelengths longer than visible light. The figure shows how the energy of a black body radiation varies with temperature and wavelength. As the temperature of the black body increases, two different behaviors are observed. The first effect is that the peak of the distribution shifts to shorter wavelengths. This shift is found to satisfy the following relationship called Wien's displacement law.

$\lambda_{max}T=b$. Here b is a constant called Wien's constant. The value of this constant in SI unit is 2.898×10^{-3} m-K. Thus, $\lambda_{max}\alpha 1/T$

Here λ_{max} is the wavelength corresponding to the maximum spectral emissive power e_λ.

The second effect is that the total amount of energy the black body emits per unit area per unit time ($=\sigma T^4$) increases with fourth power of absolute temperature T.

This is also known as emissive power. We know

$e=\int_0^\infty e_\lambda d\lambda=$ Area under graph, e_λ Vs $\lambda=\sigma T^4$

Area $\propto T^4$ $\;A_2=(2)^4=16A_1$

Thus, if the temperature of the black body is made two fold, λ_{max} remains half while the area becomes 16 times.

MASTERJEE CONCEPTS

Have you ever wondered how do scientists calculate the temperature of sun and other stars? It is through this law.

Ankit Rathore (JEE Advanced 2013, AIR 158)

Illustration 8: The light from the sun is found to have a maximum intensity near the wavelength of 470 nm. Assuming that the surface of the sun emits as a blackbody, calculate the temperature of the surface of the sun.

(JEE MAIN)

Sol: Formula of Wien's displacement law.

For a blackbody, λ_m T=0.288 cmK. Thus, $T=\dfrac{0.288\,\text{cmK}}{470\,\text{nm}}=6130\text{K}$

Illustration 9: What is the wavelength of the brightest part of the light from our next closest star, Proxima Centauri? Proxima Centauri is a red dwarf star about 4.2 light years away from us with an average surface temperature of 3,042 Kelvin?

(JEE MAIN)

Sol: $\lambda_{max}\,T=b$

We don't really need the distance to solve this. All we need is the surface temperature to plug into our Wien's law equation

Wavelength λ_{max} in meters = $\dfrac{0.0029\text{meters}-K}{3.042K}$ which is 0.000000953 meters.

We can convert this to nanometers and we get a peak wavelength of 953 nm.

Illustration 10: Two bodies A and B have thermal emissivity of 0.1 and 0.81 respectively. The outer surface areas of the two bodies are identical. These two bodies emit total radiative power at the same rate. The wavelength λ_B corresponding to the maximum spectral radiancy in the radiation from B is 1.0 μm larger than the wavelength λ_A corresponding to the maximum spectral radiancy in the radiation from A. If the temperature of body A is 5802 K, find (a) temperature of (B) and (b) λ_B. **(JEE MAIN)**

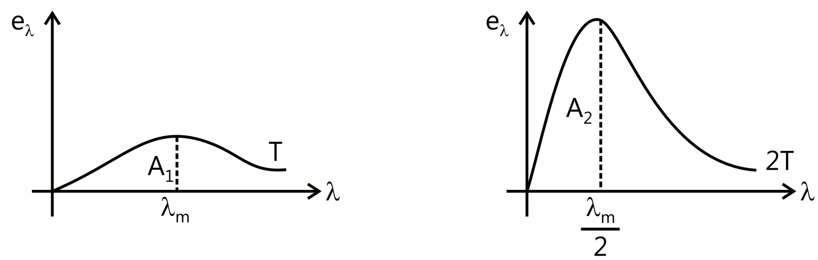


Figure 14.9

Sol: By equating their emissive power, ratio of temperatures (a) could be calculated.

(a) Power radiated from A $= P_A = E_A A = e_A \sigma T_A^4 A$

Power radiated from B $= P_B = E_A A = e_B \sigma T_B^4 A$

Where A is surface area of both the bodies as $P_1 = P_2$, $e_A T_A^4 = e_B T_B^4$

$$\therefore 0.01T_A^4 = 0.81T_B^4 \;\therefore \left[\frac{T_B}{T_A}\right]^4 = \left[\frac{0.01}{0.81}\right] = \left[\frac{1}{81}\right]; \frac{T_B}{T_A} = \left[\frac{1}{3}\right] \qquad \text{or } T_B = \frac{1}{3}\times T_A = \frac{1}{3}\times 5802 = 1934K$$

(b) $\lambda_m T$ = constant as per Wien's law

$$\therefore \lambda_A T_A = \lambda_B T_B \text{ or } \frac{\lambda_B}{\lambda_A} = \frac{T_A}{T_B} = 3 \;;\; \lambda_A = \frac{\lambda_B}{3}; \qquad \lambda_B - \lambda_A = 1\mu m, \lambda_B - \frac{\lambda_B}{3} = \frac{2\lambda_B}{3} = 1\mu m$$

$$\therefore \lambda_B = \frac{1\times 3}{2} = 1.5\mu m$$

17. SOLAR CONSTANT AND TEMPERATURE OF SUN

Solar constant is defined as the amount of radiation received from the sun at the earth per minute per cm^2 of a surface placed at right angle to the solar radiation at a mean distance of the earth from the sun. Assuming that the absorption of solar radiation by the atmosphere near the earth is negligible, the value of solar constant, S, is equal to 1.94 $cal.cm^{-2}\,min^{-1}$.

The temperature of the sun, T, is given as follows $T^4 = \dfrac{S}{\sigma}\left(\dfrac{R}{r}\right)^2$

Where S is solar constant, σ is Stefan's constant, R is mean distance of earth from sun and r is radius of sun.

PROBLEM-SOLVING TACTICS

1. Problems of conduction can be easily solved by making analogy with current electricity (Problems like calculation of net conductance of series and parallel connection. Actually, the way in which steady state is achieved in heat transfer and current electricity is very similar. At steady state considering a cylindrical rod, potential at each point becomes constant in current electricity and so does temperature in heat transfer. The amount of charge transferred per unit time is related in same way to potential as that of heat energy transferred relates to temperature difference and the constant of proportionality have similar properties.)

2. Most of the problems involve concepts of integration, so be careful with infinitesimal elements. Basically, try to be physically involved in the problem and understand it event by event so that you learn more. Toughness in most of the questions is involved only in its mathematical analysis.

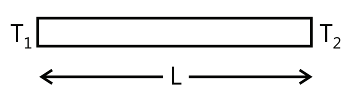
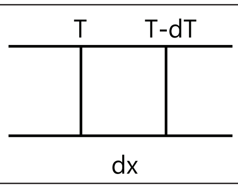
3. Problems from radiation and law of cooling also generally involve integration which becomes necessary to do at times. However an approximate approach is also available in case of law of cooling useful in solving problems without involving integration.

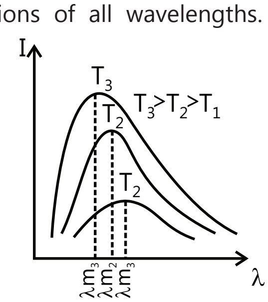
4. Laws must be carefully known because many questions directly focus on understanding of laws rather than involving calculations (Example - If temperature of a body is doubled, find the ratio of maximum wavelength for final and initial state.)

5. Noting down the known and asked quantities and thinking of a link between them will always prove to be a good way.

6. Questions from this topic usually come in a hybrid involving concepts of other topics like thermodynamics, gaseous state and calorimetry. So one must be strong in their concepts too!!

FORMULAE SHEET

S. No.	Term	Descriptions
1.	Conduction	Due to vibration and collision of medium particles.
2.	Steady state	In this state heat absorption stops and temperature gradient throughout the rod becomes constant i.e. $\frac{dT}{dx} = \text{constant}$.
3.	Before steady state	Temp of rod at any point changes. **Note:** If specific heat of any substance is zero, it can be considered always to be in steady state.
4.	Ohm's law for thermal Conduction in Steady state	Let the two ends of rod of length L is maintained At temp T_1 and $T_2 \left(T_1 > T_2\right)$ Thermal Current $\frac{dQ}{dT} = \frac{T_1 - T_2}{R_{Th}}$. Where $R_{Th} = \frac{L}{KA}$ (L is length of material, K is coefficient of thermal conductivity, A is area of cross- section) 
5.	Differential form of Ohm's law	$\frac{dQ}{dT} = KA\frac{dT}{dx}$ $\frac{dT}{dx}$ = Temperature gradient 
6.	Convection	Heat transfer due to movement of medium particles.

7.	Radiation	Every body radiates electromagnetic radiation of all possible wavelength at all temp>0 K
8.	Stefan's Law	Rate of heat emitted by a body at temp T K from per unit area $E = \sigma T^4 J/sec/m^2$ Radiation power $\frac{dQ}{dT} = P = \sigma A T^4 watt$ If body is placed in a surrounding of temperature T_s $\frac{dQ}{dT} = \sigma A(T^4 - T_s^{\ 4})$ valid only for black body Emissivity or emmisive power $e = \frac{\text{heat from general body}}{\text{heat from black body}}$ If temp of body falls by dT in time dt $\frac{dT}{dt} = \frac{eA\sigma}{ms}\left(T^4 - T_s^{\ 4}\right)$ (dT/dt=Rate of cooling)
9.	Newton's law of cooling	If temp difference of body with surrounding is small i.e. $T = T_s$ Then, $\frac{dT}{dt} = \frac{4eA\sigma}{ms} T_s^{\ 3}\left(T - T_s\right)$ So $\frac{dT}{dt} \propto \left(T - T_s\right)$
10.	Average form of Newton's law of cooling	If a body cools from T_1 to T_2 in time δt $\frac{T_1 - T_2}{\delta t} = \frac{K}{mS}\left(\frac{T_1 + T_2}{2} - T_S\right)$ (Used generally in objective questions) $\frac{dT}{dt} = \frac{K}{mS}(T - T_S)$ (For better results use this generally in subjective)
11.	Wien's black body radiation	At every temperature (>0K) a body radiates energy radiations of all wavelengths. According to Wien's displacement law if the wavelength corresponding to maximum energy is λ_m then λ_m T=b where b= is a constant(Wien's Constant) T=Temperature of body 

Solved Examples

JEE Main/Boards

Example 1: A copper rod 2 m long has a circular cross section of radius 1 cm. One end is kept at 100°C and other at 0°C, and the surface is insulated so that negligible heat is lost through the surface. Find

(a) The thermal resistance of bar

(b) The thermal current H

(c) The temperature gradient $\frac{dT}{dx}$

(d) The temperature 25 cm from hot end. Thermal conductivity of copper is 401 W/m-K

Sol: Recall the formula of heat transfer.

(a) Thermal resistance

$$R = \frac{1}{kA} = \frac{1}{k(\pi r^2)} \text{ or } R = \frac{2}{(401)(\pi)(10^{-2})^2} = 15.9 K/W$$

(b) Thermal current, $H = \frac{\Delta T}{R} = \frac{\Delta\theta}{R} = \frac{100}{15.9}$ or

$H = 6.3W$

(c) Temperature gradient

$$= \frac{0 - 100}{2} = -50 K/m = -50°C/m$$