Efficiency of the cycle, $\eta = \left(1 - \frac{T_2}{T_1}\right) x100$ or $\eta = \left(1 - \frac{300}{1100}\right) x100 = 72.72\%$

PROBLEM-SOLVING TACTICS

(a) Most of the problems of T_{max}, P_{max} and V_{max} are solved by differentiating. Sometimes the graph will be given and sometimes, the equation will be given. For problems of finding P_{max} , you will require either the P-V or P-T equation.

(b) From the sections of thermodynamic, graphical questions are usually asked so one must have a thorough understanding of PV diagrams at least. It is generally advised to convert other graphs namely TV and PT graph to PV graph before solving to visualize it easily. Also in many cases we can directly get asked quantities like work done because it involves integration of area under the PV graph.

(c) Silly mistakes must be avoided because there is not much variety in problems and not so much to think. Remember the following points:-

- **(i)** Be very careful in signs of Q and W while solving questions of thermodynamics.
- **(ii)** Be careful in noting the type of gas used in question whether it is monatomic or diatomic.
- **(iii)** Be careful with units of given quantities and asked quantities.
- **(iv)** You can list more from your personal experience.

(d) It would be helpful to note keywords in a question (like noting down slow and fast changes because slow changes imply reversible change whereas fast change imply irreversible change).

(e) It is always helpful to make a separate list of known quantities and asked quantities and then think of a way, a link between them.

(f) Questions are mostly based on various reversible or irreversible processes which have their own set of formulae. And no more processes can be found other than these. Hence, these concepts must be strong and you should know the algorithm of each one thoroughly.

FORMULAE SHEET

Kinetic Theory of Gases

$$PV = \frac{1}{3} n M_m {V_{rms}}^2 = nRT$$

Kinetic energy per mole of gas = $E = \frac{1}{2} M_m {V_{rms}}^2 = \frac{3}{2} RT$

$$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

$$V_{av} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8P}{\pi\rho}}$$

M is the molar mass in kilogram per mole.

$$V_{mp} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2P}{\rho}}$$

$$V_{rms} : V_{av} : V_{mps} :: \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}\,;$$

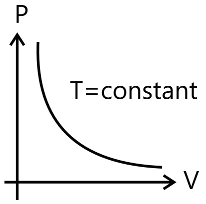
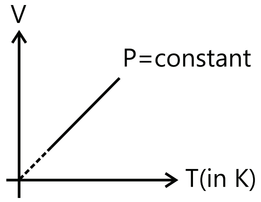
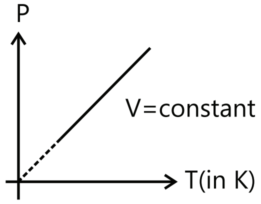
$$V_{mps} : V_{rms} : V_{av} :: 1 : 1.224 : 1.28$$

The mean free path of a gas molecule is the average distance between two successive collisions. It is represented by λ.

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 \rho}$$

Here, σ = Diameter of the molecule
k = Boltzmann's constant

Thermodynamics

(a)	**Boyle's law:** According to this law, for a given mas of a gas the volume of a gas at constant temperature (called isothermal process) is inversely proportional to its pressure, i.e., $V \propto \frac{1}{P}$ (T= constant) or PV = constant	 Figure 13.18
(b)	**Charle's law:** According to this law, for a given mass of a gas, the volume of a gas at constant pressure (called isobaric process) is directly proportional to its absolute temperature, i.e., $V \propto T$ (P= constant) or $\frac{V}{T} = \text{constant}$	 Figure 13.19
(c)	**Gay Lussac's law or Pressure law:** According to this law, for a given mass of a gas the pressure of a gas at constant volume (called isochoric process) is directly proportional to its absolute temperature, i.e., $P \propto T$ (V= constant) or $\frac{P}{T} = \text{constant}$	 Figure 13.20
(d)	**Avogadro's law:** According to this law, at same temperature and pressure equal volumes of all gases contain equal number of molecules.	

Ideal gas equation	$PV = nRT$ OR $\rho = \frac{PM}{RT}$
Degree of freedom of monoatomic gas	$f = 3$ (all translational)

Degree of freedom of a diatomic and linear polyatomic gas	f = 5 (3 translational + 2 rotational) at room temperatures and f = 7 (3 translational + 2 rotational + 2 vibrational) at high temperatures.
Degree of freedom of nonlinear polyatomic gas	f = 6 (3 translational + 3 rotational) at room temperatures and f = 8 (3 translational + 3 rotational + 2 vibrational) at high temperatures.
Degree of freedom of solid	An atom in a solid has no degrees of freedom for translational and rotational motion. At high temperatures due to vibration along 3 axes it has 3 x 2 = 6 degrees of freedom. f = 6 (all vibrational) at high temperatures.

$$C = \frac{\Delta Q}{n\Delta T} \quad \text{or} \quad \Delta Q = nC\Delta T$$

For a gas the value of C depends on the process through which its temperature is raised.

Types of gas	**Degrees of freedom**	**Total energy**	**C_P**	**C_V**	γ
Monoatomic	3	$\frac{3}{2}RT$	$\frac{5}{2}R$	$\frac{3}{2}R$	1.67
Diatomic	5	$\frac{5}{2}RT$	$\frac{7}{2}R$	$\frac{5}{2}R$	1.4
Polyatomic(≥ 3)	6	3RT	4R	3R	1.33

$$\text{Ratio of specific heat :- } \gamma = \frac{C_P}{C_V} = \frac{f+2}{f} \begin{cases} \xrightarrow{\text{monoatomic}} 5/3 = 1.67 \\ \xrightarrow{\text{diatomic}} 7/5 = 1.4 \\ \xrightarrow{\text{polyatomic}} 4/3 = 1.33 \end{cases}$$

Note: C of a gas depends on the process of that gas, which can be infinite in types.

and $f = \frac{2}{\gamma - 1}$; $C_v = \frac{R}{\gamma - 1}$; $C_P = \frac{\gamma R}{\gamma - 1}$

Often the first law must be used in its differential form, which is dU = dQ – dW

This can also be written as dQ = dU + dW

Δ Q is +ve for heat supplied

Δ Q is -ve for heat rejected

1. Δ W = +ve for work done by the gas (in expansion of gas)

   Δ W = -ve for work done on the gas (in contraction of gas)
2. For change in internal energy of the gas, apply $\Delta U = nC_v\Delta T$ or in differential form, $dU = nC_v dt = \frac{f}{2}nRdT$
3. For heat transfer, apply $Q = nC\Delta T$

   or in differential form $dQ = nCdT$

   $dW = PdV$

$$W = \int dW = \int_{V_i}^{V_f} PdV \quad = \text{Area under P-V curve}$$

Isothermal	**Q = W**	**0**	$nRT\ln\left(\frac{V_f}{V_i}\right) = nRT\ln\left(\frac{P_i}{P_f}\right)$
Adiabatic	0	$nC_V\Delta T$	$\frac{P_iV_i - P_fV_f}{\gamma - 1} = -\Delta = \frac{nR\Delta T}{\gamma - 1}$
Isobaric	$nC_P\Delta T$	$nC_V\Delta T$	$P(V_f - V_i)$
Isochoric	$Q = \Delta U$ $= nC_V\Delta T$	$nC_V\Delta T$	0

$(\text{Slope})_{\text{isothermal}} = -\frac{P}{V}$ $\quad\quad$ $(\text{Slope})_{\text{adiabatic}} = -\gamma\left(\frac{P}{V}\right)$

$$\eta = \left(\frac{\begin{array}{c}\text{Work done by the working substance}\\ \text{(an ideal gas in our case) during a cycle}\end{array}}{\text{Heat supplied to the gas during the cycle}}\right) \times 100 = \frac{W_{\text{Total}}}{Q_{+ve}} \times 100 = \left\{1 - \left|\frac{Q_{-ve}}{Q_{+ve}}\right|\right\} \times 100$$

For Carnot Engine: $\eta = 1 - \frac{T_2}{T_1}$

Solved Examples

JEE Main/Boards

Example 1: An electric bulb of volume 250cm^3 was sealed off during manufacture at the pressure of 10^{-3}mm of Hg at 27°C. Find the number of air molecules in the bulb.

Sol: $PV = \frac{N}{A}RT = N\frac{R}{A}T = NkT$

Let N be the number of air molecules in the bulb.

$V_1 = 250\text{cm}^3$, $P_1 = 10^{-3}$mm of Hg,

$T_1 = 273 + 27 = 300°\text{K}$

As $P_1V_1 = NkT$ where k is constant, then

$10^{-3} \times 250 = N.k.300$ ... (i)

At N.T.P., one mole of air occupies a volume of 22.4 litre,

$V_0 = 22400\,\text{cm}^3$, $P_0 = 760$ mm of Hg,

T = 273° K and $N_0 = 6 \times 10^{23}$ molecules

$\therefore 760 \times 22400 = 6\text{x}10^{23} \times k \times 273$ ... (ii)

Dividing equation (i) by equation (ii), we get

$$\frac{10^{-3} \times 250}{760 \times 22400} = \frac{N \times 300}{6 \times 10^{23} \times 273}$$

$$N = \frac{10^{-3} \times 250 \times 6 \times 10^{23} \times 273}{760 \times 22400 \times 300} = 8.02\text{x}10^{15} \text{ mole}$$

Example 2: One gram-mole of oxygen at 27°C and one atmospheric pressure is enclosed in a vessel.

(a) Assuming the molecules to be moving with V_{rms}, find the number of collisions per second which molecules make with one square meter area of the vessel wall.

(b) The vessel is next thermally insulated and moved with a constant speed ν_0. It is then suddenly stopped.

The process results in a rise of the temperature of the gas by 1°C. Calculate the speed ν_0.

Sol: Formula based: $n = \frac{P}{kT}$ & $V_{rms} = \sqrt{\frac{3N_AkT}{M_n}}$. Recall the assumption of KTG. Kinetic energy changed to internal energy.