

Solved Examples

JEE Main/Boards

Example 1: An electric bulb of volume 250cm^3 was sealed off during manufacture at the pressure of 10^{-3}mm of Hg at 27°C . Find the number of air molecules in the bulb.

Sol: $PV = \frac{N}{A}RT = N\frac{R}{A}T = NkT$

Let N be the number of air molecules in the bulb.

$$V_1 = 250\text{cm}^3, P_1 = 10^{-3}\text{mm of Hg},$$

$$T_1 = 273 + 27 = 300^\circ\text{K}$$

As $P_1V_1 = NkT$ where k is constant, then

$$10^{-3} \times 250 = N \cdot k \cdot 300 \quad \dots (i)$$

At N.T.P., one mole of air occupies a volume of 22.4 litre,

$$V_0 = 22400\text{cm}^3, P_0 = 760\text{ mm of Hg},$$

$$T = 273^\circ\text{K and } N_0 = 6 \times 10^{23}\text{ molecules}$$

$$\therefore 760 \times 22400 = 6 \times 10^{23} \times k \times 273 \quad \dots (ii)$$

Dividing equation (i) by equation (ii), we get

$$\frac{10^{-3} \times 250}{760 \times 22400} = \frac{N \times 300}{6 \times 10^{23} \times 273}$$

$$N = \frac{10^{-3} \times 250 \times 6 \times 10^{23} \times 273}{760 \times 22400 \times 300} = 8.02 \times 10^{15}\text{ mole}$$

Example 2: One gram-mole of oxygen at 27°C and one atmospheric pressure is enclosed in a vessel.

(a) Assuming the molecules to be moving with V_{rms} , find the number of collisions per second which molecules make with one square meter area of the vessel wall.

(b) The vessel is next thermally insulated and moved with a constant speed v_0 . It is then suddenly stopped.

The process results in a rise of the temperature of the gas by 1°C . Calculate the speed v_0 .

Sol: Formula based: $n = \frac{P}{kT}$ & $V_{\text{rms}} = \sqrt{\frac{3N_A kT}{M_n}}$. Recall

the assumption of KTG. Kinetic energy changed to internal energy.

$$(a) n = P/kT$$

where k = Boltzmann constant =

$$1.38 \times 10^{-23} \text{ J/mol/K}$$

$$P = 1 \text{ atmosphere} = 1.01 \times 10^5 \text{ n/m}^2,$$

$$T = 27^\circ\text{C} = 300^\circ\text{K}$$

$$n = \frac{1.01 \times 10^5}{1.38 \times 10^{-23} \times 300} = 2.44 \times 10^{25} \text{ m}^{-3}$$

The root mean square velocity V_{rms} is given by

$$V_{\text{rms}} = \sqrt{\left(\frac{3RT}{M_m}\right)} = \sqrt{\left(\frac{3N_A kT}{M_m}\right)} \quad (\because R = N_A k)$$

$$N_A = \text{Avogadro number} = 6.02 \times 10^{23} \text{ molecule/mole}$$

$$M_m = 32 \text{ gm} = 32 \times 10^{-3} \text{ kg.}$$

$$V_{\text{rms}} = \sqrt{\left(\frac{36.02 \times 10^{23} \times 1.38 \times 10^{-23} \times 300}{32 \times 10^{-3}}\right)} = 483.4 \text{ m/s.}$$

Since each molecule may be moving at a given instant, along any of the six directions (i.e. $\pm X$, $\pm Y$ and $\pm Z$) only (1/6)th of the total molecules contained in the volume along positive and negative x , y , z directions, move towards the wall. The number of collisions per second with one square meter area of the vessel wall will be

$$= \frac{1}{6} n v_{\text{rms}} = \frac{1}{6} \times 2.44 \times 10^{25} \times 483.4 = 1.97 \times 10^{27}.$$

$$(b) \text{ K.E.} = \frac{1}{2} m v_0^2$$

$$\text{Heat energy gained} = C_v \Delta T = C_v \times 1 = C_v$$

$$C_p - C_v = R \text{ or } C_v = \frac{R}{\gamma - 1}$$

$$\therefore \gamma = 1.41; \frac{1}{2} m v_0^2 = C_v = \frac{R}{\gamma - 1}$$

$$v_0 = \sqrt{\left(\frac{2R}{m(\gamma - 1)}\right)} = \sqrt{\left(\frac{2 \times 8.31 \times 1}{32 \times 10^{-3} \times 1.41 - 1}\right)} = 35.6 \text{ m/sec.}$$

Example 3: An ideal gas having initial pressure P , volume V and temperature T is allowed to expand adiabatically until its volume becomes $5.66V$ while its temperature fall to $T/2$.

(a) How many degrees of freedom do the gas molecules have?

(b) Obtain the work done by the gas during the expansion as a function of the initial pressure P and volume V .

$$\text{Sol: (a) } y = \frac{\frac{f}{2} + 1}{\frac{f}{2}} \text{ or } y = 1 + \frac{2}{f}$$

$$(b) W = \frac{nR}{1-\gamma} (T_f - T_i)$$

(a) For adiabatic process

$$TV^{\gamma-1} = T_1 V_1^{\gamma-1}; V_1 = 5.66V \text{ and } T_1 = T/2$$

$$\therefore TV^{\gamma-1} = \frac{T}{2} \times (5.66V)^{\gamma-1}; (5.66)^{\gamma-1} = 2$$

$$(\gamma - 1) \log 5.66 = \log 2$$

$$\gamma - 1 = \frac{\log 2}{\log 5.66}; \gamma - 1 = \frac{0.3010}{0.7528} = 0.4$$

$$\therefore \gamma = 1.4 \text{ (diatomic gas)}$$

The degrees of freedom of gas molecules = 5

(b) Work done during adiabatic change, W , is given by

$$W = \frac{1}{1-\gamma} [P_1 V_1 - PV]$$

For an ideal gas equation,

$$\frac{PV}{T} = \frac{P_1 V_1}{T_1} \text{ or } P_1 = \frac{T_1 V}{T V_1} P; P_1 = \frac{1}{2} \times \frac{1}{5.66} P = \frac{P}{11.32}$$

$$W = \frac{1}{1-1.4} \left[\frac{P}{11.32} \times 5.66V - PV \right]$$

$$= \frac{PV}{0.4} \left[1 - \frac{5.66}{11.32} \right] = \frac{PV}{0.8} = 1.25 PV.$$

Example 4: Two moles of helium gas with γ equal to $\frac{5}{3}$

are initially at temperature 27°C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value.

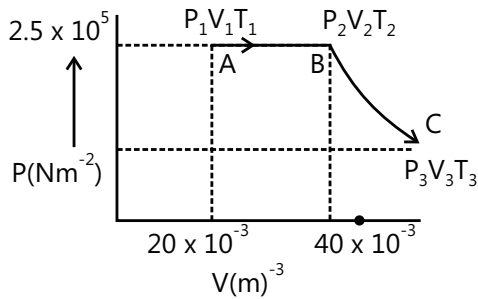
(a) Sketch the process on P-V diagram.

(b) Find the final value of volume and pressure of the gas.

(c) Find the work done by the gas under isobaric and adiabatic processes and total work done.

Sol: Relate P_1, V_1 and T_1 with P_2, V_2, T_2 and P_3, V_3, T_3 . Calculate the work alone of process AB, BC separately.

(a) The sketch of P-V diagram is shown in the figure.



(b) At A, $V_1 = 20 \text{ l} = 20 \times 10^{-3} \text{ m}^3$, $T_1 = 27^\circ\text{C} = 300\text{K}$; $P_1 = ?$

For an ideal gas, $PV = nRT$.

$$P_1 = \frac{nRT_1}{V_1} = \frac{2 \times 8.31 \times 300}{20 \times 10^{-3}} = 2.5 \times 10^5 \text{ Nm}^{-2}$$

At B, $P_2 = P_1 = 2.5 \times 10^5 \text{ Nm}^{-2}$, $V_2 = 40 \times 10^{-3} \text{ m}^3$

For isobaric process,

$$\frac{T_2}{T_1} = \frac{V_2}{V_1} = \frac{40 \times 10^{-3}}{20 \times 10^{-3}} = 2 \quad T_2 = 2T_1 = 2 \times 300 = 600\text{K}$$

The gas now undergoes adiabatic expansion from B to C.

$$T_3 V_3^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\frac{V_3}{V_2} = \left(\frac{T_2}{T_3} \right)^{\frac{1}{\gamma-1}} = \left(\frac{600}{300} \right)^{1/(5/3-1)} = 2^{3/2} = 2\sqrt{2} = 2.83$$

Final Volume =

$$V_3 = V_2 \times 2.83 = 40 \times 10^{-3} \times 2.83 = 113 \times 10^{-3} \text{ m}^3$$

Final pressure P_3 is given by $P_3 V_3^\gamma = P_2 V_2^\gamma$

$$P_3 = P_2 \times \left(\frac{V_2}{V_3} \right)^\gamma = 2.5 \times 10^5 \times \left(\frac{40 \times 10^{-3}}{113 \times 10^{-3}} \right)^{5/3}$$

$$= 2.5 \times 10^5 \times (0.353)^{5/3} = 2.5 \times 10^5 \times 0.176$$

$$= 4.41 \times 10^5 \text{ Nm}^{-2}$$

(c) Work done during isobaric process along AB = $P(V_2 - V_1)$

$$= 2.5 \times 10^5 \times 20 \times 10^{-3} = 5000 \text{ J}$$

Work done during adiabatic process along

$$BC = \frac{nR(T_2 - T_3)}{\gamma - 1} = \frac{2 \times 8.31 \times (600 - 300)}{\frac{5}{3} - 1}$$

$$\text{Total work done} = 5000 + 7479 = 11479 \text{ J.}$$

Example 5: Two moles of helium gas undergo a cyclic process as shown in the figure.

Assuming the gas to be ideal, calculate the following quantities in this process.

(a) The net change in the heat energy

(b) The net work done

(c) The net change in internal energy

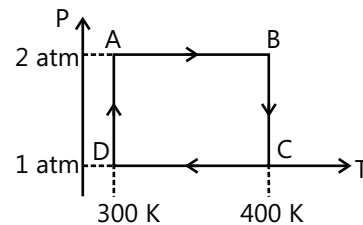
Sol: Calculate for isobaric and isothermal process only.

(a) AB is isobaric process. The work done during this process from A to B:

$$W_{AB} = P(V_2 - V_1) = nR(T_2 - T_1)$$

$$\text{or } W_{AB} = 2 \times 8.3 \times (400 - 300) = 1160 \text{ joule}$$

Work done during isothermal process from B to C:



$$W_{BC} = nRT_C \log_e(V_2/V_1) = nRT_C \log_e(P_2/P_1)$$

$$= 2 \times 8.3 \times 400 \times 2.303 \log_{10} 2$$

$$= 2 \times 8.3 \times 400 \times 2.303 \times 0.3010 = 4602.9 \text{ joule}$$

Work done during isobaric process from C to D:

$$W_{CD} = nR(T_D - T_C) = 2 \times 8.3 \times (300 - 400) = -1660 \text{ joule}$$

Work done during isothermal process from D to A:

$$W_{DA} = nRT_D \log_e(P_D/P_A) = nRT_D \log_e 2$$

$$= 2 \times 8.03 \times 300 \times 2.0303 \times 0.3010 = -3452.2 \text{ Joule}$$

$$\text{Net workdone} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= 1660 + 4602.9 - 1660 - 3452.2$$

$$= 1150.7 \text{ joule}$$

(b) First law of thermodynamics gives

$$\Delta Q = \Delta U + \Delta W$$

As $\Delta U = 0$, in cyclic process

$$\therefore \Delta Q = \Delta W = 1150.7 \text{ joule}$$

The heat given to the system = 1150.7 joule

(c) As the gas returns to its original state, there is no change in internal energy.

Example 6: An ideal gas is taken a cyclic thermodynamic process through four steps. The amount of heat involved in these steps are $Q_1 = 5960\text{ J}$, $Q_2 = -5585\text{ J}$, $Q_3 = -2980\text{ J}$ and $Q_4 = 3645\text{ J}$ respectively. The corresponding works involved are $W_1 = 2200\text{ J}$, $W_2 = -825\text{ J}$, respectively.

(a) Find the value of W_4

(b) What is the efficiency of the cycle?

Sol: $Q_{\text{Total}} = W_{\text{Total}}$ as $\Delta U = 0$ in cyclic process,

$$\eta = \frac{W_T}{(Q \text{ absorbed})} \text{ only}$$

As the process is cyclic, $\Delta U = 0$

Net heat absorbed by the system

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

$$= 5960 - 5585 - 2980 + 3645 = 1040\text{ J}$$

Net work performed

$$W = W_1 + W_2 + W_3 + W_4$$

$$= 2200 - 825 - 1100 + W_4 = 275 + W_4$$

According to the first law of thermodynamics

$$Q = \Delta U + W ; 1040 = 0 + 275 + W_4$$

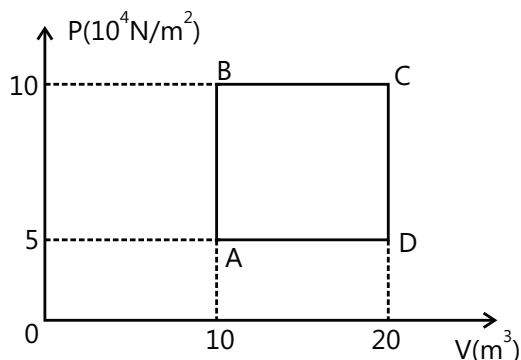
$$\therefore W_4 = 1040 - 275 = 765\text{ joule.}$$

$$\text{Efficiency } \eta = \frac{\text{Work done}(W)}{\text{Heat absorbed}(Q_1 + Q_4)}$$

$$= \frac{275 + 765}{5960 + 3645} = \frac{1040}{9605} = 0.1082$$

Percentage efficiency = 10.82%

Example 7: A sample of 2 kg of monoatomic Helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC as shown in the figure.



Given molecular mass of Helium = 4

(a) What is the temperature of Helium in each of the states A, B, C and D?

(b) Is there any way of telling afterwards which sample of Helium went through the process ABC and which went through the process ADC? Write Yes or No.

(c) How much is the heat involved in each of the processes ABC and ADC?

Sol: Work = Area under P-V curve hence, work done in ABC is more than in ADC so is the heat (Q). At C, system's thermodynamic states are same, it can't be determined how they are achieved.

$$(a) n = \frac{2 \times 10^3}{4} = 500$$

At A, $P_A V_A = nRT_A$ or $T_A = (P_A V_A / nR)$

$$\therefore T_A = \frac{(5 \times 10^4) \times 10}{500 \times 831} = 120.33\text{ K}$$

$$\text{Similarly, } T_B = \frac{(10 \times 10^4) \times 10}{500 \times 831} = 240.66\text{ K}$$

$$T_C = \frac{(10 \times 10^4) \times 20}{500 \times 831} = 481.32\text{ K}$$

$$T_D = \frac{(5 \times 10^4) \times 20}{500 \times 831} = 240.66\text{ K}$$

(b) No

(c) For process ABC:

Change in internal energy $\Delta U = nC_V \Delta T$

$$(\Delta U)_{ABC} = n \left(\frac{3}{2} R \right) \Delta T$$

$$= 500 \times \left(\frac{3}{2} \times 8.3 \right) \times [481.32 - 120.33] = 2.25 \times 10^6\text{ J}$$

Work done $(\Delta W)_{ABC} = 10 \times (10 \times 10^4)$

$$= 10^6\text{ J } (\Delta Q)_{ABC} = (\Delta U)_{ABC} + (\Delta W)_{ABC}$$

$$= 2.25 \times 10^6 + 10^6 = 3.25 \times 10^6\text{ J}$$

For process ADC:

$$(\Delta W)_{ADC} = 5 \times 10^4 [20 - 10] = 0.5 \times 10^6\text{ J}$$

$$(\Delta U)_{ADC} = 2.25 \times 10^6$$

$$\therefore (\Delta Q)_{ADC} = (2.25 \times 10^6) + (0.5 \times 10^6)$$

$$= 2.75 \times 10^6\text{ J}$$

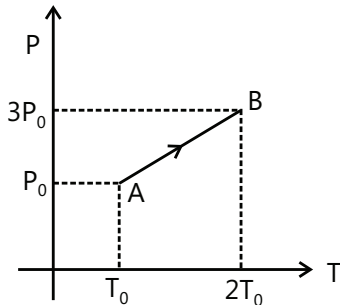
Example 8: Pressure versus temperature graph of an ideal gas is as shown in figure. Density of the gas at point A is ρ_0 . Density at B would be

Sol: Use relation between density and temperature, given below.

$$\rho = \frac{PM}{RT} \propto \frac{P}{T}$$

Pressure

$$\left(\frac{P}{T}\right)_A = 2\left(\frac{P_0}{T_0}\right)$$



$$\text{and } \left(\frac{P}{T}\right)_B = \frac{3}{2}\left(\frac{P_0}{T_0}\right) = \frac{3}{4}\left(\frac{P}{T}\right)_A$$

$$\therefore \rho = \frac{3}{4}\rho_A = \frac{3}{4}\rho_0$$

Example 9: The root mean square (rms) speed of hydrogen molecules at a certain temperature is 300 m/s. If the temperature is doubled and hydrogen gas dissociates into atomic hydrogen the rms speed will become

Sol: Formula based $V_{\text{rms}} \propto \sqrt{T}$.

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

T is doubled and M is halved. Therefore, rms speed will become two times or 600 m/s.

Example 10: The changes in temperature of an ideal gas, when its volume changes from V to 2V in the process $P = aV$, is (Here a is a positive constant)

Sol: Use relation between P and V.

$$P \propto V \quad (P = aV)$$

Therefore, pressure and volume both are doubled or temperature becomes four times ($T \propto PV$)

JEE Advanced/Boards

Example 1: A gaseous mixture enclosed in a vessel of volume V consists of one gram mole of a gas A with $\gamma(C_p/C_v) = 5/3$ and another has B with $\gamma = 7/5$ at a certain temperature T. The gram molecular weights of

the gases A and B are 4 and 32 respectively. The gases A and B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $PV^{19/13} = \text{constant}$, in adiabatic processes.

(a) Find the number of gram moles of the gas B in the gaseous mixture.

(b) Compute the speed of sound in the gaseous mixture at $T = 300\text{K}$.

(c) If T is raised by 1 K from 300 K, find the percentage change in the speed of sound in the gaseous mixture.

(d) The mixture is combined adiabatically to $1/5^{\text{th}}$ of its initial volume V. Find the change in its adiabatic compressibility in terms of the given quantities.

Sol: γ of the mixture is known from equation of process. Therefore C_v can be known. Compare C_v (mixture)

$$= \frac{n_A C_{vA} + n_B (C_v)_B}{(n_A + n_B)}$$

(a) As the gaseous mixture follows the equation $PV^{19/13} = \text{constant}$, then for the mixture of the gas $\gamma = 19/13$.

$$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = 1 + \frac{R}{C_v} = \frac{19}{13}$$

$$\frac{R}{C_v} = \frac{6}{13}, \therefore C_v = \frac{13R}{6}$$

$$C_p = C_v + R = \frac{19}{6}R$$

For gas A, $\gamma_A = 5/3$; For gas B, $\gamma_B = 7/5$.

$$(C_v)_A = \frac{3}{2}R \text{ and } (C_v)_B = \frac{5}{2}R$$

$$(C_p)_A = \frac{5}{2}R \text{ and } (C_p)_B = \frac{7}{2}R$$

Let n_A and n_B be the number of kg moles in gas A and gas B respectively.

$$n_A = 1 \text{ gm mole} = 10^{-3} \text{ kg.mole}$$

As the gases have fixed volume, no work is done by the gas and vessel system. In the adiabatic process, no heat is exchanged with the surroundings, the internal energy of the system will remain constant.

$$\therefore (n_A + n_B)C_v dT = n_A(C_v)_A dT + n_B(C_v)_B dT$$

$$(n_A + n_B)C_v = n_A(C_v)_A + n_B(C_v)_B$$

$$(1 \times 10^{-3} + n_B) \left(\frac{13}{6}R \right) = (1 \times 10^{-3}) \frac{3}{2}R + n_B \left(\frac{5}{2}R \right)$$

$$13 \times 10^{-3} + 13n_B = 9 \times 10^{-3} + n_B(15)$$

$$2n_B = 4 \times 10^{-3}$$

$$\therefore n_B = \frac{4 \times 10^{-3}}{2} = 2 \times 10^{-3} \text{ kgmole} = 2 \text{ gm mole}$$

(b) The speed of sound in gaseous mixture is given by

$$v = \sqrt{\left(\frac{\gamma RT}{M}\right)}$$

where R is gas constant and M is equivalent gram molecular weight of gaseous mixture. Let M_A and M_B be the gram molecular weights of gases A and B respectively, then

$$n_A M_A + n_B M_B = (n_A + n_B) M$$

$$\text{or } M = \frac{n_A M_A + n_B M_B}{n_A + n_B} = \frac{(1 \times 4) + (2 \times 32)}{1 + 2}$$

$$= \frac{68}{3} \text{ gm} = \frac{68}{3} \times 10^{-3} \text{ kg}$$

$$\therefore v = \sqrt{\left[\left(\frac{19}{13}\right) \times \frac{8.3 \times 300 \times 3}{68 \times 10^{-3}}\right]} = 400.7 \text{ m/s}$$

(c) At temperature T, $v = \sqrt{\left(\frac{\gamma RT}{M}\right)}$

$$T' = (300 + 1) \text{ K}, v' = \sqrt{\left(\frac{\gamma RT'}{M}\right)}$$

$$\therefore \frac{v'}{v} = \sqrt{\left(\frac{T'}{T}\right)} = \sqrt{\left(\frac{301}{300}\right)} \text{ or } \frac{v'}{v} - 1 = \left[\sqrt{\left(\frac{301}{300}\right)} - 1\right]$$

$$\text{The percentage change in speed of sound} = \frac{v' - v}{v} \times 100$$

$$= \left[\sqrt{\frac{301}{300}} - 1\right] \times 100 = 0.17\%$$

(d) Adiabatic compressibility =

$$\beta_{\text{mt}} = -\frac{1}{V} \left(\frac{dv}{dp}\right)_{\text{mt}} = -\frac{1}{\gamma P}$$

$$\Delta\beta = \frac{1}{\gamma P} \left[1 - \frac{1}{(5)^\gamma}\right] = \frac{13}{3 \times 19 RT} \left[1 - \frac{1}{(5)^\gamma}\right] = 2.487 \times 10^{-3}$$

Example 2: At 27°C, two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabatically to a volume 2V. Calculate

(a) The final temperature of the gas

(b) Change in its internal energy

(c) The workdone by the gas during this process

Sol: Formula $TV^\gamma = \text{constant}$, $\Delta U = n C_V \Delta T$ $W = -\Delta U$ used.

(a) For adiabatic expansion

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}; 300 \times (V)^{\gamma-1} = T_2 (2V)^{\gamma-1}$$

$$T_2 = 300 \times \frac{1}{(2)^{\gamma-1}} = \frac{300}{(2)^{2/3}} \quad (\because \gamma = 5/3)$$

$$\therefore T_2 = 188.99 \text{ K}$$

(b) Change in thermal energy is given by

$$\Delta U = n C_V (T_2 - T_1) = 2 \times (3R/2) (188.99 - 300)$$

$$= 2 \times \left(\frac{3 \times 8.3}{2}\right) \times (-111.09) = -2767.5 \text{ J}$$

The negative sign indicates that there is a decrease in the internal energy.

(c) For adiabatic process,

$$\Delta W + \Delta U = 0 \text{ or } \Delta W = -\Delta U$$

The workdone by the gas is given by

$$\Delta W = -\frac{nR(T_2 - T_1)}{\gamma - 1} = -\frac{2 \times 8.3 \times (188.99 - 300)}{(5/3 - 1)}$$

$$= 2764.2 \text{ J}$$

Example 3: Two moles of an ideal monoatomic gas, initially at pressure p_1 and volume V_1 undergo an adiabatic compression until its volume is V_2 . Then the gas is given heat Q at constant volume V_2 .

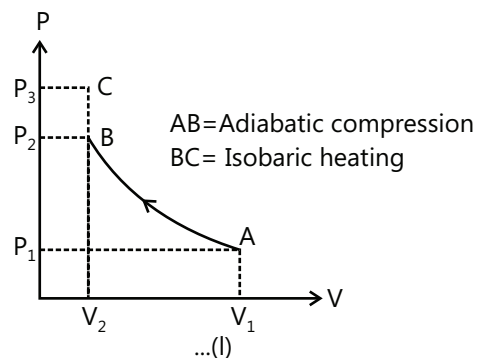
(a) Sketch the complete process on a P-V diagram.

(b) Find the total workdone by the gas, the total change in its internal energy and the final temperature of the gas.

[Give your answers in terms of p_1, V_1, V_2, Q and R].

Sol: Use formula $TV^{\gamma-1} = \text{constant}$ and $W = \frac{nR\Delta T}{1-\gamma}$

(a) P-V diagram is shown in the figure where AB is adiabatic compression and BC is isobaric heating



(b) Let T_1 and T_2 be the initial temperature and the temperature after adiabatic compression respectively.

For adiabatic compression

$$V_1^{\gamma-1}T_1 = V_2^{\gamma-1}T_2$$

$$T_1 = \frac{p_1 V_1}{2R}, \gamma = \frac{5}{3} \text{ for monoatomic gas}$$

$$\therefore V_1^{2/3} \left(\frac{p_1 V_1}{2R} \right) = V_2^{2/3} T_2 \therefore T_2 \left(\frac{p_1 V_1^{5/3}}{2R V_2^{2/3}} \right) \quad \dots(i)$$

For isochoric process at temperature T_3 , heat supplied is Q .

$$\therefore Q = nC_V dT ; Q = 2.3 / 2R(T_3 - T_2)$$

$$\frac{Q}{3R} = T_3 - T_2 ; \therefore T_3 = \frac{Q}{3R} + T_2 = \frac{Q}{3R} + \frac{p_1 V_1^{5/3}}{2R V_2^{2/3}} \quad \dots (ii)$$

The total work done by the gas is equal to the work done in adiabatic process plus the work done in isochoric process when $W_{BC} = 0$

$$\therefore W_{\text{Total}} = W_{AB} + W_{BC} = W_{AB}$$

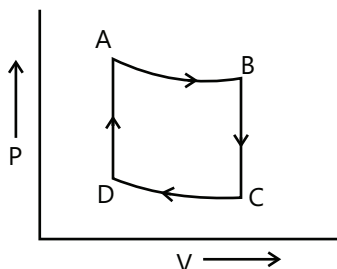
$$W_{\text{Total}} = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{2R}{2/3} \left[\frac{p_1 V_1}{2R} - \frac{p_1 V_1^{5/3}}{2R V_2^{2/3}} \right] = \frac{3}{2} p_1 V_1 \left[1 - \left(\frac{V_1}{V_2} \right)^{2/3} \right]$$

Change in internal energy $\Delta U = nC_V(T_3 - T_1)$

$$\therefore \Delta U = 2x \left(\frac{3}{2} R \right) \left[\frac{Q}{3R} + \frac{p_1 V_1^{5/3}}{2R V_2^{2/3}} - \frac{p_1 V_1}{2R} \right]$$

$$= Q + \frac{3}{2} \frac{p_1 V_1^{5/3}}{V_2^{2/3}} - \frac{3}{2} p_1 V_1 = Q + \frac{3}{2} p_1 V_1 \left[\left(\frac{V_1}{V_2} \right)^{2/3} - 1 \right]$$

Example 4: One mole of monoatomic ideal gas is taken through the cycle shown in figure.



- A \rightarrow B Adiabatic expansion
- B \rightarrow C Cooling at constant volume
- C \rightarrow D Adiabatic compression
- D \rightarrow A Heating at constant volume

The pressure and temperature at A, B etc., are denoted by $P_A, T_A; P_B, T_B$ etc. respectively.

- Given $T_A = 1000\text{K}, P_B = (2/3)P_A$ & $P_C = (1/3)P_A$. Calculate
- The work done by the gas in the process A \rightarrow B
 - The heat lost by the gas in the process B \rightarrow C and
 - Temperature T_D given $(2/3) = 0.85$ and $R = 8.31 \text{ J/mol K}$.

Sol: Use the relation for the respective processes. Such as T/P relation in adiabatic process.

(a) As for adiabatic change

$$PV^\gamma = \text{constant}$$

$$\text{i.e. } P \left(\frac{nRT}{P} \right)^\gamma = \text{constant} \quad [\text{as } PV = nRT]$$

$$\text{i.e. } \frac{T^\gamma}{P^{\gamma-1}} = \text{constant}$$

$$\text{where } \gamma = \frac{5}{3}$$

$$\text{i.e. } T_B = T_A \left(\frac{2}{3} \right)^{1-\frac{1}{\gamma}} = 1000 \left(\frac{2}{3} \right)^{2/5} = 850\text{K}$$

$$\text{so } W_{AB} = \frac{nR[T_i - T_f]}{\gamma - 1} = \frac{1 \times 8.31 [1000 - 850]}{[(5/3) - 1]}$$

$$\text{i.e. } W_{AB} = (3/2) \times 8.31 \times 150 = 1869.75\text{J}$$

(b) For B \rightarrow C, $V = \text{constant}$ so $\Delta W = 0$

so from first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W = \mu C_V \Delta T + 0$$

$$\text{or } \Delta Q = 1x \left(\frac{3}{2} R \right) (T_C - 850) \quad \text{as } C_V = \frac{3}{2} R$$

Now along path BC, $V = \text{constant}; P \propto T$

$$\text{i.e. } \frac{P_C}{P_B} = \frac{T_C}{T_B}, T_C = \frac{(1/3)P_A}{(2/3)P_A} \times T_B = \frac{T_B}{2} = \frac{850}{2} = 425\text{K} \quad \dots(ii)$$

$$\text{So, } \Delta Q = 1x \frac{3}{2} \times 8.31 (425 - 850) = -5297.625\text{J}$$

[Negative heat means heat is lost by the sys.]

(c) D \rightarrow A process is isochoric

$$\frac{P_D}{P_A} = \frac{T_D}{T_A}, \text{ i.e. } P_D = P_A \frac{T_D}{T_A}$$

But C and D are on the same adiabatic

$$\left(\frac{T_D}{T_C}\right)^\gamma = \left(\frac{P_D}{P_C}\right)^{\gamma-1} = \left(\frac{P_A T_D}{P_C T_A}\right)^{\gamma-1}$$

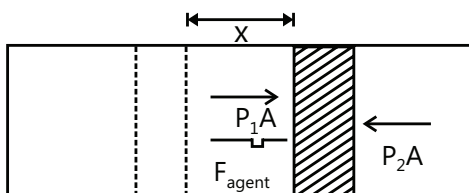
$$\text{or } (T_D)^{1/\gamma} = T_C \left[\frac{P_A}{P_C T_A}\right]^{1-\frac{1}{\gamma}}$$

$$\text{i.e. } T_C^{3/5} = \left(\frac{T_B}{2}\right) \left[\frac{P_A}{(1/3)P_A 1000}\right]^{2/5}$$

$$\text{i.e. } T_D^{3/5} = \left[\frac{1}{2} \left(\frac{2}{3}\right)^{2/3} \times 1000\right] \left[\frac{3}{1000}\right]^{2/5}$$

$$\text{i.e. } T_D = 500\text{K}$$

Example 5: A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal part of volume V_0 , in which an ideal gas is contained under the same pressure P_0 and at the same temperature.



What work has to be performed in order to increase isothermally the volume of one of gas η times compared to that of the other by slowly moving piston?

Sol: Apply isothermal condition on both compartments. Then, proceed to find V_f (left)/ V_f (right).

Let the agent move as shown.

In equilibrium position, $P_1 A + F_{\text{agent}} = P_2 A$

$$F_{\text{agent}} = (P_2 - P_1)A$$

Elementary work done by the agent

$$F_{\text{agent}} dx = (P_2 - P_1) A dx = (P_2 - P_1) dV \quad \dots (i)$$

Applying $PV = \text{constant}$ for two parts, we have

$$P_1(V_0 + Ax) = P_0 V_0 \text{ and } P_2(V_0 - Ax) = P_0 V_0$$

$$P_1 = \frac{P_0 V_0}{(V_0 + Ax)} \text{ and } P_2 = \frac{P_0 V_0}{(V_0 - Ax)}$$

$$\therefore P_2 - P_1 = \frac{P_0 V_0 (2Ax)}{V_0^2 - A^2 x^2} = \frac{2P_0 V_0 V}{V_0^2 - V^2}$$

When the volume of the left end is η times the volume of right end, we have $(V_0 + V) = \eta(V_0 - V)$

$$V = \left(\frac{\eta - 1}{\eta + 1}\right) V_0 \quad \dots (ii)$$

The work done by the agent is given by

$$\begin{aligned} W &= \int_0^V (P_2 - P_1) dV = \int_0^V \frac{2P_0 V_0 V}{V_0^2 - V^2} dV \\ &= -P_0 V_0 [\ln(V_0^2 - V^2)]_0^V = -P_0 V_0 [\ln(V_0^2 - V^2) - \ln V_0^2] \\ &= -P_0 V_0 \left[\ln \left\{ V_0^2 - \left(\frac{\eta - 1}{\eta + 1}\right)^2 V_0^2 \right\} - \ln V_0^2 \right] \\ &= -P_0 V_0 [\ln \{4\eta / (\eta + 1)^2\}] = P_0 V_0 \ln \left[\frac{(\eta + 1)^2}{4\eta} \right]. \end{aligned}$$

Example 6: An ideal gas has a density of 1.78 kg / m^3 is contained in a volume of $44.8 \times 10^{-3} \text{ m}^3$. The temperature of the gas is 273 K . The pressure of the gas is $0.01 \times 10^5 \text{ Pa}$. The gas constant $R = 8.31 \text{ J-K}^{-1} \text{ mole}^{-1}$.

(a) What is the root mean square velocity of the air molecules?

(b) How many moles of gas are present?

(c) What is the gas?

(d) What is the internal energy of the gas?

Sol: Use relation V_{rms} and P given. $m = \rho V = nM$ also.

$$(a) P = \frac{1}{3} \rho V^2 \Rightarrow$$

$$V_{\text{rms}} = \left(\frac{3P}{\rho}\right)^{1/2} = \left(\frac{(3)(0.01 \times 10^5 \text{ N/m}^2)}{1.78 \text{ kg/m}^3}\right)^{1/2}$$

$$= 4.13 \times 10^2 \text{ m/s}$$

$$(b) PV = n_m RT \Rightarrow$$

$$n_m = \frac{PV}{RT} = \frac{(0.01 \times 10^5 \text{ Nm}^{-2})(44.8 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ JK}^{-1} \text{ mole}^{-1})(273 \text{ K})}$$

$$= 2.0 \text{ moles}$$

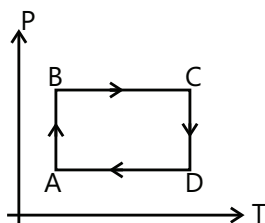
$$M_{\text{molar}} = \frac{\rho V}{n_m} = \frac{(1.78 \text{ kg/m}^3)(44.8 \times 10^{-3} \text{ m}^3)}{(2.0 \text{ moles})}$$

$$= 340.0 \times 10^{-3} \text{ kg / mole}$$

(c) This ideal gas is Argon.

(d) Internal energy of monoatomic gas $= 3/2 nRT$.

Example 7: Plot P-V, V-T and ρ -T graph corresponding to the P-T graph for an ideal gas shown in the figure.



Sol: Look for parameter which is constant in the each process.

Process AB is an isothermal process with $T = \text{constant}$ and $P_B > P_A$.

P-V graph: $P \propto \frac{1}{V}$ i.e., P-V graph is a hyperbola with $P_B > P_A$ and $V_B > V_A$.

V-T graph: $T = \text{constant}$. Therefore, V-T graph is a straight line parallel to V-axis with $V_B > V_A$.

ρ -T graph: $\rho = \frac{PM}{RT}$ or $\rho \propto P$.

As T is constant. Therefore, ρ -T graph is a straight line parallel to ρ -axis with $\rho_B > \rho_A$ as $P_B > P_A$.

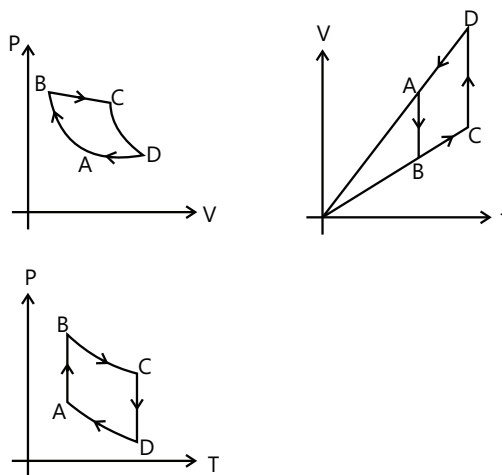
Process BC is isobaric process with $P = \text{constant}$ and $T_C > T_B$.

P-V graph: As P is constant. Therefore, P-V graph is a straight line parallel to V-axis with $V_C > V_B$ (because $V \propto T$ in an isobaric process)

V-T graph: In an isobaric process $V \propto T$, i.e., V-T graph is a straight line passing through the origin, with $T_C > T_B$ and $V_C > V_B$.

ρ -T graph: $\rho \propto \frac{1}{T}$ (when $P = \text{constant}$), i.e., ρ -T graph is a hyperbola with $T_C > T_B$ and $\rho_C > \rho_B$.

There is no need to discuss C-D and D-A processes as they are opposite to AB and BC respectively. The corresponding three graphs are shown above.



JEE Main/Boards

Exercise 1

Q.1 Although the r.m.s. speed of gas molecules is of the order of the speed of sound in that gas yet on opening a bottle of ammonia in one corner of a room, its smell takes time in reaching the other corner. Explain Why?

Q.2 The pressure of a gas at -173°C temperature is 1 atmosphere, keeping the volume constant, to what temperature should the gas be heated so that its pressure becomes 2 atmosphere.

Q.3 Explain (i) Boyle's law (ii) Charle's law. Why they are not applicable to real gases at all states?

Q.4 State and explain (i) Guy Isac's law and (ii) Gas equation. Distinguish clearly between R and r for a gas.

Q.5 State the postulates of Kinetic Theory of gases. Explain the pressure exerted by an ideal gas.

Q.6 Find an expression for the pressure exerted by a gas and establish its relation with kinetic energy of the gas.

Q.7 From Kinetic Theory of gases, explain kinetic interpretation of temperature and absolute zero.

Q.8 Explain the concept of mean free path.

Q.9 Explain what is meant by Brownian Motion?

Q.10 The density of water is 1000kg/m^3 . The density of water vapour at 100°C and 2 atmospheric pressure is 0.6kg m^3 . The volume of a molecule multiplied by the

total number gives what is called, molecular volume. Estimate the ratio (or fraction) of the molecular volume to the total volume occupied by the water vapour under the above conditions of temperature and pressure.

Q.11 A 3000cm^3 tank contains oxygen at 20°C and a gauge pressure of $2.5 \times 10^6\text{Pa}$. Find the mass of the oxygen in the tank. Take $1\text{ atm} = 10^5\text{ Pa}$.

Q.12 Calculate the r.m.s. velocity of air molecules at N.T.P. Given that 22400 c.c. of gas at N.T.P. weight 64 gm .

Q.13 How many collisions per second does each molecule of a gas make, when average speed of the molecule is 500ms^{-1} and mean free path is $2.66 \times 10^{-7}\text{ m}$?

Q.14 Calculate the mean free path of gas molecules, if number of molecules per cm^3 is 3×10^{19} and diameter of each molecule is 2\AA .

Q.15 The diameter of a gas molecules is $2.4 \times 10^{-10}\text{m}$. Calculate the mean free path at N.T.P. Given Boltzmann constant $k = 1.38 \times 10^{-23}\text{ J molecule}^{-1}\text{ K}^{-1}$.

Q.16 Which molecules, ice at 0°C or water 0°C have greater potential energy and why?

Q.17 An ideal gas is compressed at a constant temperature, will its internal energy increases or decrease?

Q.18 Which type of motion of the molecules is responsible for internal energy of a monoatomic gas?

Q.19 The volume of an ideal gas is V at a pressure P . On increasing the pressure by ΔP , the change in volume of the gas is (ΔV_1) under isothermal conditions and (ΔV_2) under adiabatic conditions, Is $\Delta V_1 > \Delta V_2$ or vice-versa and why?

Q.20 200 joule of work is done on a gas to reduce its volume by coming it. If this change is done under adiabatic conditions, find out the change in internal energy of the gas and also the amount of heat absorbed by the gas?

Q.21 Give briefly the concept of internal energy.

Q.22 Define the four thermodynamic processes. What is meant by indicator diagram?

Q.23 State the sign conventions used in all thermodynamic processes.

Q.24 What do you learn by applying first law of thermodynamics to isothermal and adiabatic processes?

Q.25 Explain what is meant by isothermal operations. Give some examples.

Q.26 What are adiabatic operations? Enumerate some examples. State equations representing these operations.

Q.27 Obtain an expression for work done by a gas in isothermal expansion.

Q.28 Derive an expression for work done in an adiabatic process.

Q.29 What are cyclic and non cyclic processes? Calculate work done in such processes.

Q.30 What are reversible and irreversible processes? Give some examples of each.

Q.31 What is a heat engine? Obtain an expression for its efficiency.

Q.32 A tyre pumped to a pressure of 3 atmosphere suddenly bursts. Calculate the fall in temperature due to adiabatic expansion. The temperature of air before expansion is 27°C and value of $\gamma = 1.4$.

Q.33 A quantity of air at 27°C and atmospheric pressure is suddenly compressed to half its original volume. Find the final (i) pressure and (ii) temperature. Given γ for air = 1.42 .

Q.34 A Cylinder containing one gram mole of gas was put on boiling water bath and compressed adiabatically till its temperature rose by 70°C . Calculate the work done and increase in energy of the gas, $\gamma = 1.5$, $R = 2\text{ cal. mole}^{-1}\text{ K}^{-1}$.

Q.35 One gram mole of an ideal gas at S.T.P. is subjected to reversible adiabatic expansion to double its volume. Find the change in internal energy in the process. Take $\gamma = 1.4$.

Q.36 If 1 gram oxygen at 760mm pressure and 0°C has its volume doubled in an adiabatic change, calculate the change in internal energy. Take $R=2 \text{ cal. mole}^{-1} \text{ K}^{-1}$, $J=4.2 \text{ J cal}^{-1}$ and $\gamma =1.4$.

Exercise 2

Single Correct Choice Type

Q.1 Find the approx. number of molecules contained in a vessel of volume 7 litres at 0°C at $1.3 \times 10^5 \text{ Pascal}$

- (A) 2.4×10^{23} (B) 3×10^{23}
(C) 6×10^{23} (D) 4.8×10^{23}

Q.2 An ideal gas mixture filled inside a balloon expands according to the relation $PV^{2/3} = \text{constant}$. The temperature inside the balloon is

- (A) Increasing (B) Decreasing
(C) Constant (D) Can't be said

Q.3 A rigid tank contains 35 kg of nitrogen at 6 atm. Sufficient quantity of oxygen is supplied to increase the pressure to 9 atm, while the temperature remains constant. Amount of oxygen supplied to the tank is:

- (A) 5 kg (B) 10 kg (C) 20 kg (D) 40 kg

Q.4 At temperature $T \text{ K}$, the pressure of 4.0g argon in bulb is p . The bulb is put in a bath having temperature higher by 50K than the first one. 0.8g of argon gas had to be removed to maintain original pressure. The temperature T is equal to

- (A) 510 K (B) 200 K (C) 100 K (D) 73 K

Q.5 When 2 gms of a gas are introduced into an evacuated flask kept at 25°C the pressure is found to be one atmosphere. If 3 gms of another gas added to the same flask the pressure becomes 1.5 atmosphere. The ratio of the molecular weights of these gases will be

- (A) 1: 3 (B) 3: 1 (C) 2: 3 (D) 3: 2

Q.6 During an experiment, an ideal gas obeys an addition equation of state $P^2V = \text{constant}$. The initial temperature and pressure of gas are T and V respectively. When it expands to volume $2V$, then its temperature will be?

- (A) T (B) $\sqrt{2} T$ (C) $2 T$ (D) $2\sqrt{2} T$

Q.7 A barometer tube, containing mercury, is lowered in a vessel containing mercury until only 50 cm of the tube is above the level of mercury in the vessel. If the atmospheric pressure is 75 cm of mercury, what is the pressure at the top of the tube?

- (A) 33.3 kPa (B) 66.7 kPa
(C) 3.33 MPa (D) 6.67 MPa

Q.8 A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T . The pressure of the gas is P . An identical vessel containing one mole of He gas (molar mass 4) at a temperature $3T$ has a pressure of

- (A) $P/8$ (B) P (C) $2P$ (D) $8P$

Q.9 The ratio of average translational kinetic energy to rotational kinetic energy of a diatomic molecule temperature T is

- (A) 3 (B) $7/5$ (C) $5/3$ (D) $3/2$

Q.10 One mole of an ideal gas at STP is heated in an insulated closed container until the average speed of its molecules is doubled. Its pressure would therefore increase by factor.

- (A) 1.5 (B) $\sqrt{2}$ (C) 2 (D) 4

Q.11 One mole of an ideal gas is contained within a cylinder by a frictionless piston and is initially at temperature T . The pressure of the gas is kept constant while it is heated and its volume doubles. If R is molar gas constant, the work done by the gas in increasing its volume is?

- (A) $RT \ln 2$ (B) $1/2RT$ (C) RT (D) $3/2 RT$

Q.12 A polyatomic gas with six degrees of freedom does 25J work when it is expanded at constant pressure. The heat given to the gas is?

- (A) 100J (B) 150J (C) 200J (D) 250J

Q.13 In thermodynamic process pressure of a fixed mass of gas is changed in such a manner that the gas release 30 joule of heat and 18 joule of work was done on the gas. If the initial internal energy of the gas was 60 joule, then, the final internal energy will be?

- (A) 32 joule (B) 48 joule
(C) 72 joule (D) 96 joule

Q.14 An ideal gas undergoes an adiabatic process obeying the relation $PV^{4/3} = \text{constant}$. If its initial temperature is 300 K and then its pressure is increased upto four times its initial value, then the final temperature is (in Kelvin)?

- (A) $300\sqrt{2}$ (B) $300\sqrt[3]{2}$
(C) 600 (D) 1200

Q.15 1 kg of a gas does 20 kJ of work and receives 16 kJ of heat when it is expanded between two states. A second kind of expansion can be found between the initial and final state which requires a heat input of 9 kJ. The work done by the gas in the second expansion is:

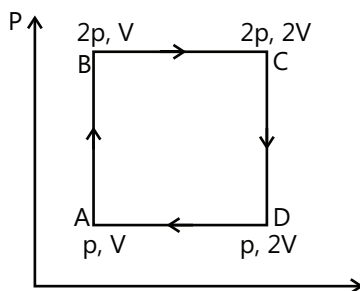
- (A) 32 kJ (B) 5 kJ
(C) -4 kJ (D) 13 kJ

Q.16 A mixture of ideal gases 7 kg of nitrogen and 11 kg of CO_2 . Then (Take γ for nitrogen and CO_2 as 1.4 and 1.3 respectively)

- (A) Equivalent molecular weight of the mixture is 36.
(B) Equivalent molecular weight of the mixture is 18.
(C) γ for the mixture is 5/2
(D) γ for the mixture is 4/3

Previous Years' Questions

Q.1 An ideal mono-atomic gas is taken round the cycle ABCD as shown in the P-V diagram (see figure). The work done during the cycle is **(1983)**



- (A) PV (B) $2PV$
(C) $\frac{1}{2}pV$ (D) Zero

Q.2 At room temperature, the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. The gas is **(1984)**

- (A) H_2 (B) F_2 (C) O_2 (D) Cl_2

Q.3 70 cal of heat are required to raise the temperature of 2 moles of an ideal diatomic gas at constant pressure from 30°C to 35°C . The amount of heat required (in calorie) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is **(1985)**

- (A) 30 (B) 50 (C) 70 (D) 90

Q.4 If one mole of a monatomic gas ($\gamma = 5/3$) is mixed with one mole of a diatomic gas ($\gamma = 7/5$), the value of γ for the mixture is **(1988)**

- (A) 1.40 (B) 1.50 (C) 1.53 (D) 3.07

Q.5 The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root mean square velocity of the gas molecules is v , at 480 K it becomes **(1996)**

- (A) $4v$ (B) $2v$ (C) $v/2$ (D) $v/4$

Q.6 The average translational energy and the rms speeds of molecules in a sample of oxygen gas at 300 K are $6.21 \times 10^{-21} \text{ J}$ and 484 m/s respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour) **(1997)**

- (A) $12.42 \times 10^{-21} \text{ J}$, 968 m/s
(B) $8.78 \times 10^{-21} \text{ J}$, 684 m/s
(C) $6.21 \times 10^{-21} \text{ J}$, 968 m/s
(D) $12.42 \times 10^{-21} \text{ J}$, 684 m/s

Q.7 A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T . The pressure of the gas is p . An identical vessel containing one mole of the gas (molar mass 4) at a temperature $2T$ has a pressure of **(1997)**

- (A) $p/8$ (B) p (C) $2p$ (D) $8p$

Q.8 Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V . The mass of the gas in A is m_A and that in B is m_B . The gas in each cylinder is now allowed to expand isothermally to the same final volume $2V$. The changes in the pressure in A and B are found to be ΔP and $1.5 \Delta p$ respectively. Then **(1998)**

- (A) $4m_A = 9m_B$ (B) $2m_A = 3m_B$
(C) $3m_A = 2m_B$ (D) $9m_A = 4m_B$

Q.9 A mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T . Neglecting all vibrational modes, the total internal energy of the system is
(1999)

- (A) $4 RT$ (B) $15 RT$ (C) $9 RT$ (D) $11RT$

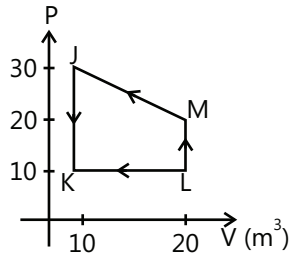
Q.10 A monoatomic ideal gas, initially at temperature T_1 , is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by
(2000)

- (A) $(L_1/L_2)^{2/3}$ (B) (L_1/L_2)
(C) L_2/L_1 (D) $(L_2/L_1)^{2/3}$

Q.11 An ideal gas is expanding such that $pT^2 = \text{constant}$. The coefficient of volume expansion of the gas is
(2008)

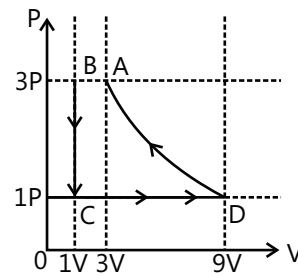
- (A) $\frac{1}{T}$ (B) $\frac{2}{T}$ (C) $\frac{3}{T}$ (D) $\frac{4}{T}$

Q.12 Match the following for the given process (2006)



Column I	Column II
(A) Process $J \rightarrow K$	(p) $Q > 0$
(B) Process $K \rightarrow L$	(q) $W < 0$
(C) Process $L \rightarrow K$	(r) $W > 0$
(D) Process $M \rightarrow J$	(s) $Q < 0$

Q.13 One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P-V diagram. column II gives the characteristics involved in the cycle. Match them with each of the processes given in column I.
(2011)

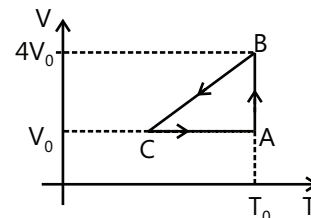


Column I	Column II
(A) Process $A \rightarrow B$	(p) Internal energy decreases
(B) Process $B \rightarrow C$	(q) Internal energy increases
(C) Process $C \rightarrow D$	(r) Heat is lost
(D) Process $D \rightarrow A$	(s) Heat is gained
	(t) Work is done on the gas

Q.14 For an ideal gas

- (A) The change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_V(T_2 - T_1)$, where C_V is the molar heat capacity at constant volume and n the number of moles of the gas
(B) The change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process
(C) The internal energy does not change in an isothermal process
(D) No heat is added or removed in an adiabatic process.
(1989)

Q.15 One mole an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in figure. Its pressure at A is P_0 . Choose the correct option(s) from the following
(2010)



- (A) Internal energies at A and B are the same
(B) Work done by the gas in process AB is $P_0 V_0 \ln 4$
(C) Pressure at C is $\frac{P_0}{4}$
(D) Temperature at C is $\frac{T_0}{4}$

Q.16 The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1} . The speed of sound in helium (He) at the same temperature will be (assumed both gases to be ideal) **(2008)**

- (A) 460 ms^{-1} (B) 500 ms^{-1}
(C) 650 ms^{-1} (D) 1420 ms^{-1}

Q.17 An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1 . The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2 . If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be **(2008)**

- (A) $\frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$ (B) $\frac{P_1 V_1 T_1 + P_2 V_2 T_2}{P_1 V_1 + P_2 V_2}$
(C) $\frac{P_1 V_1 T_2 + P_2 V_2 T_1}{P_1 V_1 + P_2 V_2}$ (D) $\frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_1 + P_2 V_2 T_2}$

Q.18 One kg of a diatomic gas is at a pressure of $8 \times 10^4 \text{ N/m}^2$. The density of the gas is 4 kg m^{-3} . What is the energy of the gas due to its thermal motion? **(2009)**

- (A) $3 \times 10^4 \text{ J}$ (B) $5 \times 10^4 \text{ J}$
(C) $6 \times 10^4 \text{ J}$ (D) $7 \times 10^4 \text{ J}$

Q.19 Assuming the gas to be ideal the work done on the gas in taking it from A to B is **(2009)**

- (A) 200 R (B) 300 R (C) 400 R (D) 500 R

Q.20 The work done on the gas in taking it from D to A is **(2009)**

- (A) -414 R (B) $+414 \text{ R}$
(C) -690 R (D) $+690 \text{ R}$

Q.21 The net work done on the gas in the cycle ABCDA is **(2009)**

- (A) Zero (B) 276 R
(C) 1076 R (D) 1904 R

Q.22 A diatomic ideal gas is used in a Car engine as the working substance. If during the adiabatic expansion part of the cycle, volume of the gas increases from V to $32V$ the efficiency of the engine is **(2010)**

- (A) 0.5 (B) 0.75 (C) 0.99 (D) 0.25

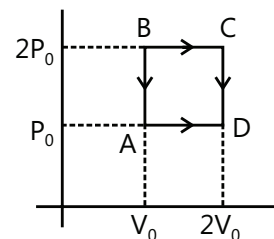
Q.23 The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is **(2010)**

- (A) $\frac{b^2}{2a}$ (B) $\frac{b^2}{12a}$ (C) $\frac{b^2}{4a}$ (D) $\frac{b^2}{6a}$

Q.24 Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively: **(2011)**

- (A) 372 K and 330 K (B) 330 K and 268 K
(C) 310 K and 248 K (D) 372 K and 310 K

Q.25 Helium gas goes through a cycle ABCDA (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly: (Assume the gas to be close to ideal gas) **(2012)**



- (A) 15.4% (B) 9.1% (C) 10.5% (D) 12.5%

Q.26 A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be **(2012)**

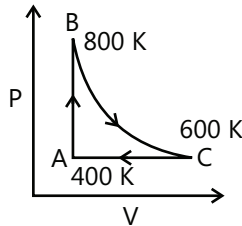
- (A) Efficiency of Carnot engine cannot be made larger than 50%
(B) 1200 K
(C) 750 K
(D) 600 K

Q.27 An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M . The piston and the cylinder have equal cross sectional area A . When the piston is in equilibrium, the volume of

the gas is V_0 and its pressure is P_0 . The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency: **(2013)**

- (A) $\frac{1}{2\pi} \frac{A\gamma P_0}{V_0 M}$ (B) $\frac{1}{2\pi} \frac{V_0 M P_0}{A^2 \gamma}$
 (C) $\frac{1}{2\pi} \sqrt{\frac{A^2 \gamma P_0}{M V_0}}$ (D) $\frac{1}{2\pi} \sqrt{\frac{M V_0}{A \gamma P_0}}$

Q.28 One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement **(2014)**



- (A) The change in internal energy in whole cyclic process is 250R
 (B) The change in internal energy in the process CA is 700R
 (C) The change in internal energy in the process AB is -350R
 (D) The change in internal energy in the process BC is -500R

Q.29 Consider a spherical shell of radius R at temperature T . The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $P = \frac{1}{3} \left(\frac{U}{V} \right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is **(2015)**

- (A) $T \propto e^{-R}$ (B) $T \propto e^{-3R}$
 (C) $T \propto \frac{1}{R}$ (D) $T \propto \frac{1}{R^3}$

Q.30 A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:

- (i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
 (ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy changes of the body in the two cases respectively is **(2015)**

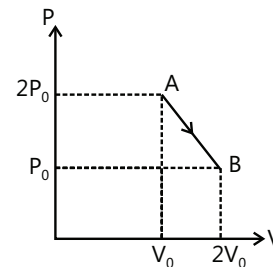
- (A) $\ln 2, 4 \ln 2$ (B) $\ln 2, \ln 2$
 (C) $\ln 2, 2 \ln 2$ (D) $2 \ln 2, 8 \ln 2$

Q.31 Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^n , where V is the volume of the

gas. The value of n is: $\left(\gamma = \frac{C_p}{C_v} \right)$ **(2015)**

- (A) $\frac{3\gamma+5}{6}$ (B) $\frac{3\gamma-5}{6}$
 (C) $\frac{\gamma+1}{2}$ (D) $\frac{\gamma-1}{2}$

Q.32 'n' moles of an ideal gas undergoes a process A → B as shown in the figure. The maximum temperature of the gas during the process will be: **(2016)**



- (A) $\frac{3P_0 V_0}{2nR}$ (B) $\frac{9P_0 V_0}{2nR}$ (C) $\frac{9P_0 V_0}{nR}$ (D) $\frac{9P_0 V_0}{4nR}$

Q.33 An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by $PV^n = \text{constant}$, then n is given by (Here C_p and C_v are molar specific heat at constant pressure and constant volume, respectively): **(2016)**

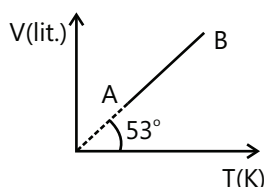
- (A) $n = \frac{C - C_p}{C - C_v}$ (B) $n = \frac{C_p - C}{C - C_v}$
 (C) $n = \frac{C - C_v}{C - C_p}$ (D) $n = \frac{C_p}{C_v}$

JEE Advanced/Boards

Exercise 1

Q.1 A closed vessel of volume V_0 contains oxygen at a pressure P_0 and temperature T_0 . Another closed vessel of the same volume V_0 contains helium at a pressure P_0 and temperature $T_0/2$. Find ratio of the masses of oxygen to the helium.

Q.2 V-T curve for 2 moles of a gas is straight line as shown in the graph here. Find the pressure of gas at A.



Q.3 A gas is undergoing an adiabatic process. At a certain stage A, the values of volume and temperature = (V_0, T_0) and the magnitude of the slope of V-T curve is m . Find the value of C_p and C_v .

Q.4 Find the molecular mass of a gas if the specific heats of the gas are $C_p = 0.2 \text{ cal/gm}^\circ\text{C}$ and $C_v = 0.15 \text{ cal/gm}^\circ\text{C}$. [Take $R = 2 \text{ cal/mole}^\circ\text{C}$]

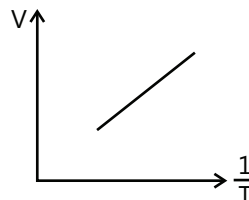
Q.5 The average degrees of freedom per molecules for a gas is 6. The gas performs 25 J of work when it expands at constant pressure. Find the heat absorbed by the gas.

Q.6 A mixture of 4gm helium and 28 gm of nitrogen in enclosed in a vessel of constant volume 300 K. Find the quantity of heat absorbed by the mixture to double the room mean velocity of its molecules. ($R = \text{Universal gas constant}$)

Q.7 One mole of an ideal gas is compressed from 0.5 lit to 0.25 lit. During the compression, $23.04 \times 10^2 \text{ J}$ of work is done on the gas and heat is removed to keep the temperature of the gas constant at all times. Find the temperature of the gas. (Take universal gas constant $R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$)

Q.8 Ideal diatomic gas is taken through a process $\Delta Q = 2\Delta U$. Find the molar heat capacity for the process (where ΔQ is the heat supplied and ΔU is change in internal energy)

Q.9 An ideal gas has a molar heat capacity C_v at constant volume. Find the molar heat capacity of this gas as a function of volume, if the gas undergoes the process: $T = T_0 e^{aV}$.

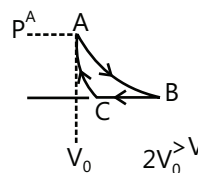


Q.10 One mole of an ideal monoatomic gas undergoes a process as shown in the figure. Find the molar specific heat of the gas in the process.

Q.11 The pressure of an ideal gas changes with volumes as $P = aV$ where 'a' is a constant. One mole of this gas is expanded to 3 times its original volume V_0 . Find

- (i) The heat transferred in the process.
- (ii) The heat capacity of the gas

Q.12 In a cycle ABCA consisting of isothermal expansion AB, isobaric compression BC and adiabatic compression CA, find the efficiency of cycle.

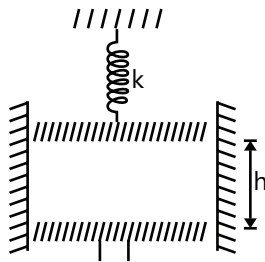


(Given: $T_A = T_B = 400\text{K}$, $\gamma = 1.5$)

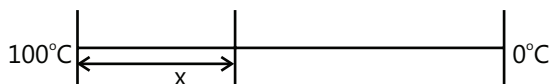
Q.13 A highly conduction solid cylinder of radius a and length ℓ is surrounded by co-axial layer of a material having thermal conductivity K and negligible heat capacity. Temperature of surrounding space (out side the layer) is T_0 , which is higher than temperature of the cylinder. If heat capacity per unit volume of cylinder material is s and outer radius of the layer is b , calculate time required to increase temperature of the cylinder from T_1 to T_2 . Assume end faces to be thermally insulated.

Q.14 A vertical brick duct (tube) is filled with cast iron. The lower end of the duct is maintained at a temperature T_1 which is greater than the melting point T_m of cast iron

and the upper end at a temperature T_2 which is less than the temperature of the melting point of cast iron. It is given that the conductivity of liquid cast iron is equal to k times the conductivity solid cast iron. Determine the fraction of the duct filled with molten metal.



Q.15 A lagged stick of cross section area 1cm^2 and length 1m is initially at a temperature of 10°C . It is then kept between 2 reservoirs of temperature 100°C and 0°C . Specific heat capacity is $10\text{ J/kg}^\circ\text{C}$ and linear mass density is 2 kg/m . Find



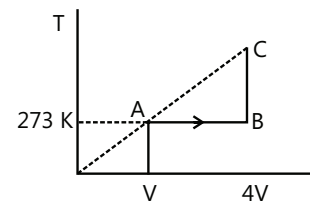
- (a) Temperature gradient along the rod in steady state.
 (b) Total heat absorbed by the rod to reach steady state.

Q.16 A cylindrical block of length 0.4 m and area of cross-section 0.04m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400K and the initial temperature of the disc is 300K . If the thermal conductivity of the material of the cylinder is 10 watt/m-K and the specific heat of the material of the disc in 600 J/kg-K , how long will it take for the temperature of the disc to increase to 350K ? Assume, for purposes of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

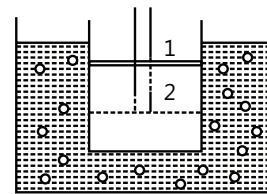
Q.17 A liquid takes 5 minutes to cool from 80°C to 50°C . How much time will it take to cool from 60°C to 30°C ? The temperature of surrounding is 20°C . Use exact method.

Q.18 An ideal gas at NTP is enclosed in an adiabatic vertical cylinder having area of cross section $A = 27\text{ cm}^2$, between two light movable pistons as shown in the figure. Spring with force constant $k = 3700\text{ N/m}$ is in a relaxed state initially. Now the lower piston is moved upwards a height $h/2$, h being the initial length of gas column. It is observed that the upper piston moves up by a distance $h/16$. Find h taking γ for the gas to be 1.5 . Also find the final temperature of the gas.

Q.19 At a temperature of $T_0 = 273^\circ\text{K}$, two moles of an ideal gas undergoes a process as shown. The total amount of heat imparted to the gas equals $Q = 27.7\text{ kJ}$. Determine the ratio of molar specific heat capacities.



Q.20 A cylinder containing a gas is closed by a movable piston. The cylinder is submerged in an ice-water mixture. The piston is quickly pushed down from position 1 to position 2. The piston is held at position 2 until the gas is again at 0°C and then slowly raised back to position 1. Represent the whole process on P-V diagram. If $m = 100\text{ gm}$ of ice are melted during the cycle, how much work is done on the gas, Latent heat of ice = 80 cal/gm .

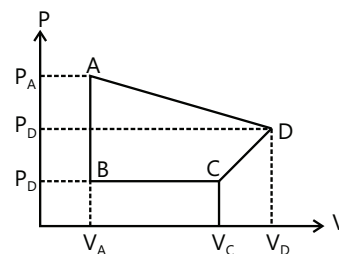


Q.21 A parallel beam of particles of mass m moving with velocities v impinges on a wall at an angle θ to its normal. The number of particles per unit volume in the beam is n . If the collision of particles with the wall is elastic, then find the pressure exerted by this beam on the wall.

Q.22 For the thermodynamic process shown in the figure.

$$P_A = 1 \times 10^5 \text{ Pa}; P_B = 0.3 \times 10^5 \text{ Pa}$$

$$P_D = 0.6 \times 10^5 \text{ Pa}; V_A = 0.20 \text{ litre } V_D = 1.30 \text{ litre}$$



- (a) Find the work performed by the system along path AD.
 (b) If the total work done by the system along the path ADC is 85 J find the volume at point C.
 (c) How much work is performed by the system along the path CDA?

Exercise 2

Single Correct Choice Type

Q.1 A perfect gas of a given mass is heated first in small vessel and then in a large vessel, such that their volumes remain unchanged. The P-T curves are

- (A) Parabolic with same curvature
 (B) Parabolic with different curvature
 (C) Linear with same slopes
 (D) Linear with different slopes

Q.2 An open and wide glass tube is immersed vertically in mercury in such a way that length 0.05 m extends above mercury level. The open end of the tube is closed and the tube is raised further by 0.43 m. The length of air column above mercury level in the tube will be? Take $P_{\text{atm}} = 76\text{cm}$ of mercury.

- (A) 0.215 m (B) 0.2 m (C) 0.1 m (D) 0.4 m

Q.3 A container X has volume double that of container Y and both are connected by a thin tube. Both contains same ideal gas. The temperature of X is 200K and that of Y is 400 K. If mass of gas in X is m then Y it will be:

- (A) $m/8$ (B) $m/6$ (C) $m/4$ (D) $m/2$

Q.4 An ideal gas of Molar mass M is contained in a vertical tube of height H, closed at both ends. The tube is accelerating vertically upwards with acceleration g. Then, the ratio of pressure at the bottom and the mid point of the tube will be

- (A) $\exp[2MgH/RT]$ (B) $\exp[-2MgH/RT]$
 (C) $\exp[MgH/RT]$ (D) MgH/RT

Q.5 Two monoatomic ideal gas at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. The temperature of the mixture will be?

- (A) $\frac{T_1 + T_2}{n_1 + n_2}$ (B) $s \frac{T_1}{n_1} + \frac{T_2}{n_2}$
 (C) $\frac{n_2 T_1 + n_1 T_2}{n_1 + n_2}$ (D) $\frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$

Q.6 At temperature T, N molecules of gas A each having mass m and at the same temperature 2N molecules of gas B each having mass 2m are filled in a container. The

mean square velocity of molecules of gas B is v^2 and mean square of x component of velocity of molecules of gas A is w^2 . The ratio of w^2 / v^2 is?

- (A) 1 (B) 2 (C) 1/3 (D) 2/3

Q.7 A reversible adiabatic path on a P-V diagram for an ideal gas passes through state A where $P=0.7 \times 10^5 \text{ N/m}^2$ and $v = 0.0049\text{m}^3$. The ratio of specific heat of the gas is 1.4. The slope of path at A is?

- (A) $2.0 \times 10^7 \text{ Nm}^{-5}$ (B) $1.0 \times 10^7 \text{ Nm}^{-5}$
 (C) $-2.0 \times 10^7 \text{ Nm}^{-5}$ (D) $-1.0 \times 10^7 \text{ Nm}^{-5}$

Q.8 A cylinder made of perfectly non conducting material closed at both ends is divided into two equal parts by a heat proof piston. Both parts of the cylinder contain the same masses of a gas at a temperature $t_0 = 27^\circ\text{C}$ and pressure $P_0 = 1 \text{ atm}$. Now if the gas in one of the parts is slowly heated to $t = 57^\circ\text{C}$ while the temperature of first part is maintained at t_0 the distance moved by the piston from the middle of the cylinder will be (length of the cylinder = 84cm)

- (A) 3cm (B) 5cm (C) 2cm (D) 1cm

Q.9 A vessel contains an ideal monoatomic gas which expands at constant pressure, when heat Q is given to it. Then the work done in expansion is:

- (A) Q (B) $\frac{3}{5}Q$ (C) $\frac{2}{5}Q$ (D) $\frac{2}{3}Q$

Multiple Correct Choice Type

Q.10 Two bodies A and B have thermal emissivities of 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies radiate energy at the same rate. The wavelength λ_B , corresponding to the maximum spectral radiancy in the radiation from B, is shifted from the wavelength corresponding to the maximum spectral radiancy in the radiation from A by $1.00 \mu\text{m}$. If the temperature of A is 5802 K,

- (A) The temperature of B is 1934K
 (B) $\lambda_B = 1.5 \mu\text{m}$
 (C) The temperature of B is 11604 K
 (D) The temperature of B is 2901 K

Q.11 During an experiment, an ideal gas is found to obey a condition $VP^2 = \text{constant}$. The gas is initially at a temperature T, pressure P and volume V. The gas expands to volume 4V.

- (A) The pressure of gas changes to $\frac{P}{2}$
- (B) The temperature of gas changes to $4T$
- (C) The graph of above process on the P-T diagram is parabola
- (D) The graph of above process on the P-T diagram is hyperbola.

Q.12 The total kinetic energy of translatory motion of all the molecules of 5 litres of nitrogen exerting a pressure P is 3000 J.

- (A) The total K.E. of 10 litres of N_2 at a pressure of $2P$ is 3000 J
- (B) The total K.E. of 10 litres of He at a pressure of $2P$ is 3000 J
- (C) The total K.E. of 10 litres of O_2 at a pressure of $2P$ is 20000 J
- (D) The total K.E. of 10 litres of Ne at a pressure of $2P$ is 12000 J

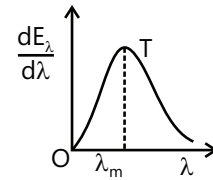
Q.13 A container holds 10^{26} molecules/ m^3 , each of mass 3×10^{-27} kg. Assume that $1/6$ of the molecules move with velocity 2000 m/s directly towards one wall of the container while the remaining $5/6$ of the molecules move either away from the wall or in perpendicular direction, and all collisions of the molecules with the wall are elastic

- (A) Number of molecules hitting $1 m^2$ of the wall every second is 3.33×10^{28} .
- (B) Number of molecules hitting $1 m^2$ of the wall every second is 2×10^{29} .
- (C) Pressure exerted on the wall by molecules is 24×10^5 Pa.
- (D) Pressure exerted on the wall by molecules is 4×10^5 Pa.

Q.14 Two gases have the same initial pressure, volume and temperature. They expand to the same final volume, one adiabatically and the other isothermally

- (A) The final temperature is greater for the isothermal process
- (B) The final pressure is greater for the isothermal process
- (C) The work done by the gas is greater for the isothermal process.
- (D) All the above options are incorrect

Q.15 The figure shows a radiant energy spectrum graph for a black body at a temperature T. Choose the correct statement(s)



- (A) The radiant energy is not equally distributed among all the possible wavelengths
- (B) For a particular wavelength the spectral intensity is maximum
- (C) The area under the curve is equal to the total rate at which heat is radiated by the body at that temperature
- (D) None of these

Q.16 Two metallic sphere A and B are made of same material and have got identical surface finish. The mass of sphere A is four times that of B. Both the spheres are heated to the same temperature and placed in a room having lower temperature but thermally insulated from each other.

- (A) The ratio of heat loss of A to that of B is $2^{4/3}$.
- (B) The ratio of heat loss of A to that of B is $2^{2/3}$.
- (C) The ratio of the initial rate of cooling of A to that of B is $2^{-2/3}$.
- (D) The ratio of the initial rate of cooling of A to that of B is $2^{-4/3}$.

Q.17 50 gm ice at $10^\circ C$ is mixed with 20 gm steam at $100^\circ C$. When the mixture finally reaches its steady state inside a calorimeter of water equivalent 1.5 gm then: [Assume calorimeter was initially at $0^\circ C$, take latent heat of vaporization of water = $1 \text{ cal/gm}^\circ C$, specific heat capacity of ice = $0.5 \text{ cal/gm}^\circ C$]

- (A) Mass of water remaining is: 67.4 gm
- (B) Mass of water remaining is: 67.87 gm
- (C) Mass of steam remaining is: 2.6 gm
- (D) Mass of steam remaining is: 2.13 gm

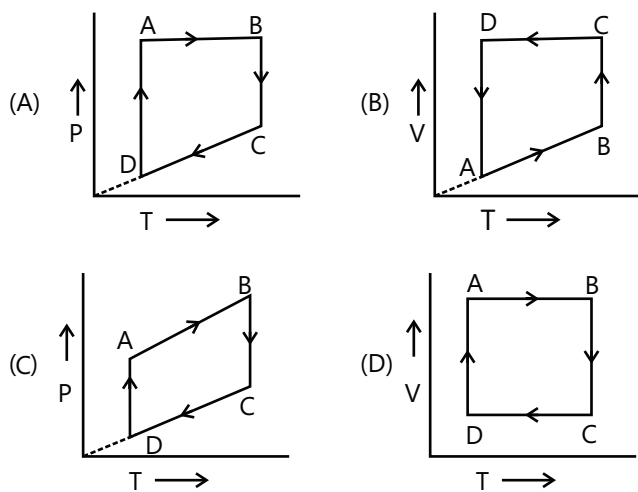
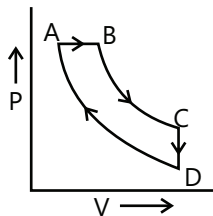
Q.18 A gas expands such that its initial and final temperature are equal. Also, the process followed by the gas traces a straight line on the P-V diagram:

- (A) The temperature of the gas remains constant throughout.
- (B) The temperature of the gas first increases and then decreases.

(C) The temperature of the gas first decreases and then increases.

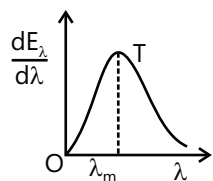
(D) The straight line has a negative slopes

Q.19 A cyclic process ABCD is shown in the P-V diagram. Which of the following curves represents the same process if BC & DA are isothermal processes.



Comprehension Type

Paragraph 1:



Q.20 The figure shows a radiant energy spectrum graph for a black body at a temperature T .

Choose the correct statement (s)

(A) The radiant energy is not equally distributed among all the possible wave lengths

(B) For a particular wavelength the spectral intensity is maximum

(C) The area under the curve is equal to the total rate at which heat is radiated by the body at that temperature.

(D) None of these

Q.21 If the temperature of the body is raised to higher temperature T' , then choose the correct statement(s)

(A) The intensity of radiation for every wavelength increases

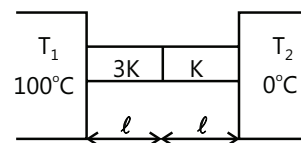
(B) The maximum intensity occurs at a shorter wavelength

(C) The area under the graph increases

(D) The area under the graph is proportional to the fourth power of temperature

Paragraph 2:

Two rods A and B of same cross-sectional area A and length l connected in series between a source ($T_1 = 100^\circ\text{C}$) and a sink ($T_2 = 0^\circ\text{C}$) as shown in figure. The rod is laterally insulated



Q.22 The ratio of the thermal resistance of the rod is

(A) $\frac{R_A}{R_B} = \frac{1}{3}$ (B) $\frac{R_A}{R_B} = 3$

(C) $\frac{R_A}{R_B} = \frac{3}{4}$ (D) $\frac{4}{3}$

Q.23 If T_A and T_B are the temperature drops across the rod A and B, then

(A) $\frac{T_A}{T_B} = \frac{3}{1}$ (B) $\frac{T_A}{T_B} = \frac{1}{3}$

(C) $\frac{T_A}{T_B} = \frac{3}{4}$ (D) $\frac{T_A}{T_B} = \frac{4}{3}$

Q.24 If G_A and G_B are the temperature gradients across the rod A and B, then

(A) $\frac{G_A}{G_B} = \frac{3}{1}$ (B) $\frac{G_A}{G_B} = \frac{1}{3}$

(C) $\frac{G_A}{G_B} = \frac{3}{4}$ (D) $\frac{G_A}{G_B} = \frac{4}{3}$

Paragraph 3:

In fluids heat transfer takes place and molecules of the medium takes very active part. The molecules take energy from high temperature zone and move towards low temperature zone. This method is known

as convection, when we require heat transfer with fast phase, we use some mechanism to make the flow of fluid on the body fast. The rate of loss of heat is proportional to velocity of fluid (v), and temperature difference (ΔT) between the body any fluid, of course more the surface area of body more the rate of loss of heat. We can write

$$\text{the rate of loss of heat as } \frac{dQ}{dt} = KAv\Delta T$$

Where K is positive constant.

Now answer the following questions

Q.25 A body is being cooled with fluid. When we increase the velocity of fluid 4 times and decrease the temperature difference $1/2$ time, the rate of loss of heat increases.

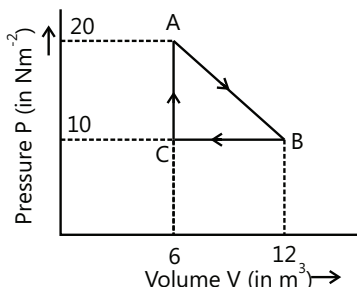
- (A) Four times (B) Two times
(C) Six times (D) No change

Q.26 In the above question if mass of the body increased two times, without change in any of the other parameters, the rate of cooling

- (A) Decreases
(B) Increases
(C) No effect of change of mass
(D) None of these

Paragraph 4:

When a thermo-dynamic process is shown on P-V diagram, area under curve represents work done during process. During cyclic process work done is area enclosed. The P-V graph for a thermodynamically system is shown in figure.



Q.27 The work done by the system in the process A to B is

- (A) 90 J (B) 60 J (C) 0 J (D) 30J

Q.28 The work done in the process B to C is

- (A) -90 J (B) -60 J (C) 0 J (D) -30J

Q.29 The work done in the complete cycle ABCA is

- (A) 90 J (B) 60 J (C) 0 J (D) 30 J

Paragraph 5:

Five moles of helium are mixed with two moles of hydrogen to form a mixture. Take molar mass of helium $M_1 = 4g$ and that of hydrogen $M_2 = 2g$

Q.30 The equivalent molar mass of the mixture is

- (A) 6g (B) $\frac{13g}{7}$
(C) $\frac{18g}{7}$ (D) None

Q.31 The equivalent degree of freedom f of the mixture is

- (A) 3.57 (B) 1.14 (C) 4.4 (D) None

Q.32 The equivalent value of γ is

- (A) 1.59 (B) 1.53 (C) 1.56 (D) None

Q.33 If the internal energy of He sample of 100J and that of the hydrogen sample is 200J, then the internal energy of the mixture is

- (A) 900J (B) 128.5J
(C) 171.4J (D) 300J

Match the Column

Q.34 An ideal gas at pressure P and volume V is expanded to volume $2V$. Column I represents the thermodynamic processes used during expansion. column II represents the work during these processes in the random order.

Column I	Column II
(p) isobaric	(x) $\frac{PV(1-2^{1-\gamma})}{\gamma-1}$
(q) isothermal	(y) PV
(r) adiabatic	(z) $PV/n2$

The correct matching of column I and column II is given by:

- (A) p-y, q-z, r-x (B) p-y, q-x, r-z
(C) p-x, q-y, r-z (D) p-z, q-y, r-x

Previous Years' Questions

Q.1 When the ideal diatomic gas is heated at constant pressure the fraction of the heat energy supplied which increases the internal energy of the gas is **(1990)**

- (A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{3}{7}$ (D) $\frac{5}{7}$

Q.2 A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to per N_2 molecule is **(1998)**

- (A) 1: 1
 (B) 1: 2
 (C) 2: 1
 (D) Depends on the moment of inertia of the two molecules

Q.3 Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is **(1998)**

- (A) 30 K (B) 18 K (C) 50 K (D) 42 K

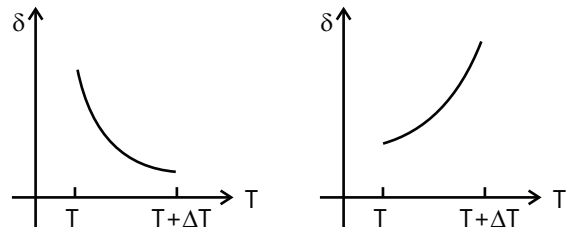
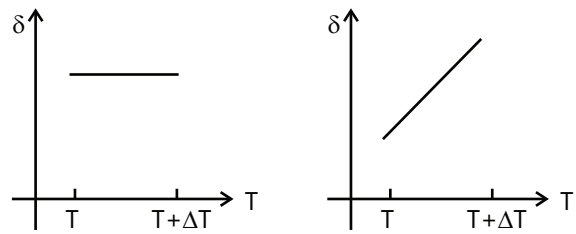
Q.4 The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is **(1999)**

- (A) $\sqrt{2/7}$ (B) $\sqrt{1/7}$
 (C) $(\sqrt{3})/5$ (D) $(\sqrt{6})/5$

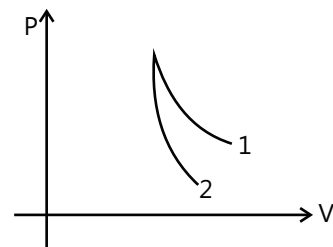
Q.5 Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic, then **(2000)**

- (A) $W_2 > W_1 > W_3$; (B) $W_2 > W_3 > W_1$
 (C) $W_1 > W_2 > W_3$; (D) $W_1 > W_3 > W_2$

Q.6 An ideal gas is initially at temperature T and volume V . Its volume is increased by ΔV due to an increase in temperature ΔT , pressure remaining constant. The quantity $\delta = V / V\Delta T$ varies with temperature as **(2000)**

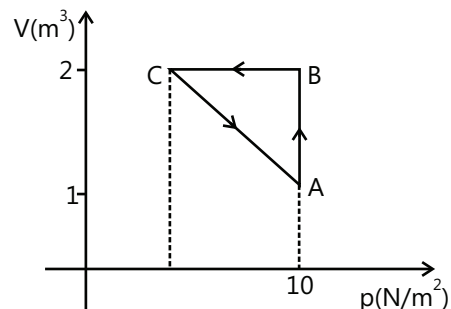


Q.7 P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to **(2001)**



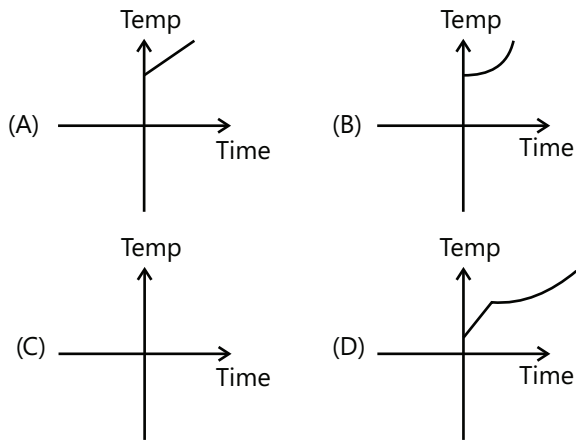
- (A) He and O_2 (B) O_2 and He
 (C) He and Ar (D) O_2 and N_2

Q.8 An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5J, the work done by the gas in the process $C \rightarrow A$ is **(2002)**



- (A) -5J (B) -10 J (C) -15 J (D) -20 J

Q.9 Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represent the variation of temperature with time? **(2004)**



Q.10 An ideal gas expands isothermally from a volume V_1 to V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is P_1 and final pressure is P_3 . The total work done is W . Then, (2004)

- (A) $P_3 > P_1, W > 0$ (B) $P_3 = P_1, W > 0$
 (C) $P_3 > P_1, W < 0$ (D) $P_3 = P_1, W = 0$

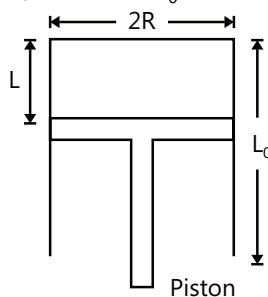
Q.11 Statement-I The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume

Statement-II The molecules of a gas collide with each other and the velocities of the molecules change due to the collision. (2007)

- (A) If statement-I is true, statement-II is true; statement-II is the correct explanation for statement-I
 (B) If statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I
 (C) If Statement-I is true; statement true is false
 (D) If Statement-I is false; statement-II is true

Paragraph 1:

A fixed thermally conducting cylinder has a radius R and height L_0 . The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface as shown in the figure. The atmospheric pressure is P_0 .



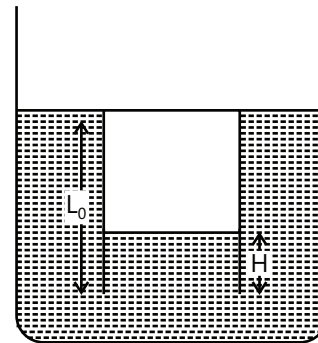
Q.12 The piston is now pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and the piston will then be (2007)

- (A) P_0 (B) $\frac{P_0}{2}$
 (C) $\frac{P_0}{2} + \frac{Mg}{\pi R^2}$ (D) $\frac{P_0}{2} - \frac{Mg}{\pi R^2}$

Q.13 While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is (2007)

- (A) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg} \right) (2L)$ (B) $\left(\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0} \right) (2L)$
 (C) $\left(\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0} \right) (2L)$ (D) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg} \right) (2L)$

Q.14 The piston is take completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ . In equilibrium, the height H of the water column in the cylinder satisfies (2007)



- (A) $\rho g(L_0 - H)^2 + p_0(L_0 - H) + L_0 p_0 = 0$
 (B) $\rho g(L_0 - H)^2 - p_0(L_0 - H) - L_0 p_0 = 0$
 (C) $\rho g(L_0 - H)^2 + p_0(L_0 - H) - L_0 p_0 = 0$
 (D) $\rho g(L_0 - H)^2 - p_0(L_0 - H) + L_0 p_0 = 0$

Q.15 C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then, (2009)

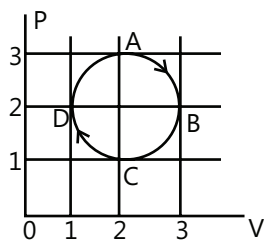
- (A) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(C) $\frac{C_p}{C_v}$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(D) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

Q.16 The figure shows the P-V plot an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then, **(2009)**



- (A) The process during the path $A \rightarrow B$ is isothermal
 (B) Heat flows out of the gas during the path $B \rightarrow C \rightarrow D$
 (C) Work done during the path $A \rightarrow B \rightarrow C$ is zero
 (D) Positive work is done by the gas in the cycle ABCDA

Match the Columns

Q.17 Column I contains a list of processes involving expansion of an ideal gas. Match this with column II describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 x 4 matrix given in the ORS. **(2008)**

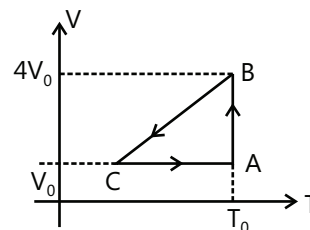
Column I	Column II
(A) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened. <div style="text-align: center;"> </div>	(p) The temperature of the gas decreases

Column I	Column II
(B) An ideal monoatomic gas expands to twice its original volume such that its pressure $p \propto \frac{1}{V^2}$, where V is the volume of the gas.	(p) The temperature of the gas increases or remains constant
(C) An ideal monoatomic gas expands to twice its original volume such that its pressure $p \propto \frac{1}{V^{4/3}}$, where V is its volume.	(q) The gas loses heat
(D) An ideal monoatomic gas expands such that its pressure p and volume V follows the behaviour shown in the graph. <div style="text-align: center;"> </div>	(r) The gas gains heat

Q.18 A real gas behaves like an ideal gas if its **(2010)**

- (A) Pressure and temperature are both high
 (B) Pressure and temperature are both low
 (C) Pressure is high and temperature is low
 (D) Pressure is low and temperature is high

Q.19 One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0 . Choose the correct option(s) from the following: **(2010)**



- (A) Internal energies at A and B are the same
 (B) Work done by the gas in process AB is $P_0 V_0 \ln 4$
 (C) Pressure at C is $\frac{P_0}{4}$
 (D) Temperature at C is $\frac{T_0}{4}$

Q.20 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1 , the work done in the process is **(2011)**

- (A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$ (C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

Q.21 A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of

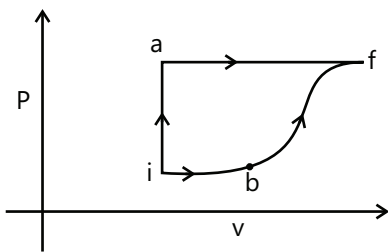
the rms speeds $\left(\frac{V_{\text{rms}}(\text{helium})}{V_{\text{rms}}(\text{argon})}\right)$ is **(2012)**

- (A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

Q.22 Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is **(2013)**

- (A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D) 8 : 9

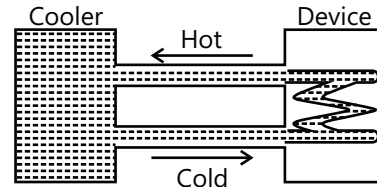
Q.23 A thermodynamic system is taken from an initial state i with internal energy $U_i = 100$ J to the final state f along two different paths iaf and ibf , as schematically shown in the figure. The work done by the system along the paths af , ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf , ib and bf are Q_{iaf} , Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J, the ratio Q_{bf} / Q_{ib} is **(2014)**



Q.24 A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T . Assuming the gases are ideal, the correct statement(s) is(are) **(2015)**

- (A) The average energy per mole of the gas mixture is $2RT$.
 (B) The ratio of speed of sound in the gas mixture to that in helium gas is.
 (C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/2$.
 (D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is.

Q.25 A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C . The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is $4.2 \text{ kJ kg}^{-1} \text{ K}^{-1}$ and the density of water is 1000 kg m^{-3}) **(2016)**

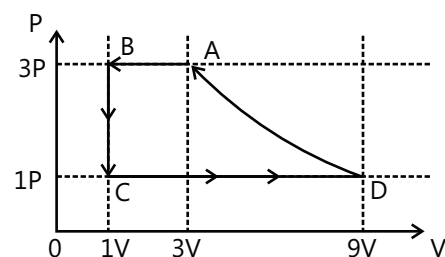
- (A) 1600 (B) 2067 (C) 2533 (D) 3933

Q.26 A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it upto height H . Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.

[Take atmospheric pressure = $1.0 \times 10^5 \text{ N/m}^2$, density of water = 1000 kg/m^3 and $g = 10 \text{ m/s}^2$. Neglect any effect of surface tension.] **(2009)**

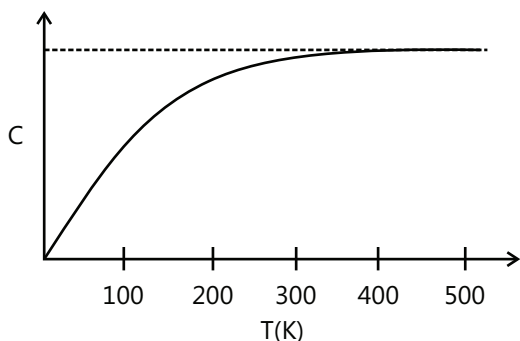
Q.27 A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i , the value of a is **(2010)**

Q.28 One mole of a monatomic gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II give the characteristics involved in the cycle. Match them with each of the processes given in column I. **(2011)**



Column I	Column II
(A) Process A \rightarrow B	(p) Internal energy decreases
(B) Process B \rightarrow C	(q) Internal energy increases.
(C) Process C \rightarrow D	(r) Heat is lost
(D) Process D \rightarrow A	(s) Heat is gained
	(t) Work is done on the gas

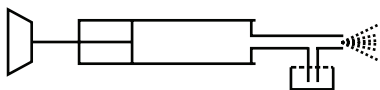
Q.29 The figure shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation. **(2013)**



- (A) The rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T .
- (B) Heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400 – 500 K.
- (C) There is no change in the rate of heat absorption in range 400 – 500 K.
- (D) The rate of heat absorption increases in the range 200 – 300 K.

Paragraph 1:

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere. **(2014)**



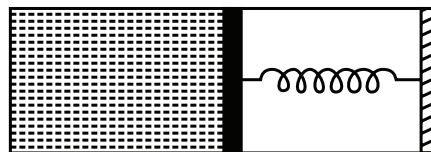
Q.30 If the piston is pushed at a speed of 5 mms^{-1} , the air comes out of the nozzle with a speed of

- (A) 0.1 ms^{-1} (B) 1 ms^{-1} (C) 2 ms^{-1} (D) 8 ms^{-1}

Q.31 If the density of air is ρ_a and that of the liquid ρ_ℓ , then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

- (A) $\sqrt{\frac{\rho_a}{\rho_\ell}}$ (B) $\sqrt{\rho_a \rho_\ell}$ (C) $\sqrt{\frac{\rho_\ell}{\rho_a}}$ (D) ρ_ℓ

Q.32 An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1 , pressure P_1 and volume V_1 and the spring is in its relaxed state.



The gas is then heated very slowly to temperature T_2 , pressure P_2 and volume V_2 . During this process the piston moves out by a distance x . Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are) **(2015)**

- (A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4} P_1 V_1$
- (B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1 V_1$
- (C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3} P_1 V_1$
- (D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6} P_1 V_1$

Q.33 A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5 \text{ Pa}$ and volume $V_i = 10^{-3} \text{ m}^3$ changes to a final state at $P_f = (1/32) \times 10^5 \text{ Pa}$ and $V_f = 8 \times 10^{-3} \text{ m}^3$ in an adiabatic quasi-static process, such that $P^3 V^5 = \text{constant}$. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volume V_f . The amount of heat supplied to the system in the two-step process is approximately **(2016)**

- (A) 112 J (B) 294 J (C) 588 J (D) 813 J

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q. 11 Q.20 Q.32
 Q.33 Q.34 Q.35
 Q.36

Exercise 2

Q.4 Q.5 Q.15
 Q.16 Q.17 Q.20
 Q.28 Q.29 Q.30

JEE Advanced/Boards

Exercise 1

Q.12 Q.15 Q.18
 Q.19 Q.22

Exercise 2

Q.8 Q.10 Q.11

Answer Key

JEE Main/Boards

Exercise 1

Q.2 $P_1 / T_1 = P_2 / T_2$ or $T_2 = P_2 T_1 / P_1 = \frac{2 \times (273 - 173)}{1} = 200\text{K} = -73^\circ\text{C}$

Q.10 6×10^{-4}

Q.11 0.103 kg

Q.13 $1.88 \times 10^9 \text{ s}^{-1}$

Q.14 $1.87 \times 10^{-7} \text{ m}$

Q.15 $1.47 \times 10^{-7} \text{ m}$

Q.17 No, because internal energy of an ideal gas depends only on temperature of the gas

Q.18 Translational motion of molecules.

Q.19 $\frac{\Delta V_1}{\Delta V_2} = \gamma$. As $\gamma > 1$, $\therefore (\Delta V_1) > (\Delta V_2)$

Q.20 Internal energy increasing by 200 J. Heat absorbed is zero.

Q.32 80.8°C

Q.33 (i) 2.675 atm (ii) 128.3°C

Q.34 -1176 joule, -280 cal

Q.35 - 1374 J

Q.36 - 43.38 J

Exercise 2

Single Correct Choice Type

Q.1 A	Q.2 A	Q.3 C	Q.4 B	Q.5 A	Q.6 B
Q.7 A	Q.8 C	Q.9 D	Q.10 D	Q.11 C	Q.12 A
Q.13 B	Q.14 A	Q.15 D	Q.16 A		

Previous Years' Questions

Q.1 A	Q.2 A	Q.3 B	Q.4 B	Q.5 B	Q.6 D
Q.7 C	Q.8 C	Q.9 D	Q.10 D	Q.11 C	
Q.12 A → s; B → r, p; C → p; D → s	Q.13 A → p; r, t; B → p, r; C → q, s; D → r, t	Q.14 A, B, C, D			
Q.15 A, B	Q.16 D	Q.17 A	Q.18 B	Q.19 C	Q.20 A
Q.21 B	Q.22 B	Q.23 C	Q.24 D	Q.25 A	Q.26 C
Q.27 C	Q.28 D	Q.29 C	Q.30 B	Q.31 C	Q.32 D
Q.33 A					

JEE Advanced/Boards

Exercise 1

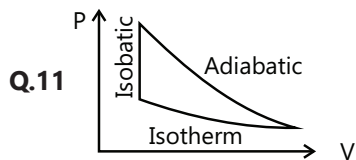
Q.1 4:1

$$Q.3 \frac{mRT_0}{V_0} \left(1 + \frac{T_0 m}{V_0} \right) R$$

Q.5 100J

Q.7 400 K

$$Q.9 C_V + \frac{R}{\alpha V}$$



$$(i) \left(\frac{\gamma+1}{\gamma-1} \right) 4aV_0^2, (ii) \left(\frac{\gamma+1}{\gamma-1} \right) \frac{R}{2}$$

Q.2 1.25×10^4 Pa

Q.4 The molar mass of the gas is 40 gm,

Q.6 3600 R

Q.8 5R

$$Q.10 \frac{R}{2}$$

$$Q.12 1 - \frac{3 \left(1 - \frac{1}{2^{1/3}} \right)}{\ln 2}$$

$$Q.13 \frac{a^2 s}{2K} \log_e \left(\frac{b}{a} \right) \log_e \left(\frac{T_0 - T_1}{T_0 - T_2} \right)$$

$$Q.14 \frac{l_1}{l} = \frac{k(T_1 - T_m)}{k(T_1 - T_m) + (T_m - T_2)}$$

Q.15 (a) -100°C/m , (b) 1000 J

Q.16 166.3 sec

Q.17 10 minutes

Q.18 1.6 m, 364 K

Q.19 1.63

Q.20 8000 cal.

Q.21 $2mv^2 \cos^2 \theta$

Q.22 (a) $W_{AD} = 88\text{J}$, (b) $V_C = 1.23\text{litre}$,
(c) $W_{CDA} = -85\text{J}$

Exercise 2

Single Correct Choice Type

Q.1 D Q.2 C Q.3 C Q.4 C Q.5 D Q.6 D
 Q.7 C Q.8 C Q.9 C

Multiple Correct Choice Type

Q.10 A, B Q.11 A, D Q.12 C, D Q.13 A, D Q.14 A, B, C Q.15 A, B
 Q.16 A, C Q.17 A, C Q.18 B, D Q.19 A, B

Comprehension Type

Paragraph 1: Q.20 A, B Q.21 A, B, C, D Paragraph 2: Q.22 A Q.23 B Q.24 B
 Paragraph 3: Q.25 B Q.26 A Paragraph 4: Q.27 A Q.28 B Q.29 D
 Paragraph 5: Q.30 D Q.31 A Q.32 C Q.33 D

Match the Column

Q.34 A

Previous Years' Questions

Q.1 D Q.2 A Q.3 D Q.4 C Q.5 A Q.6 C
 Q.7 B Q.8 A Q.9 C Q.10 C Q.11 B Q.12 A
 Q.13 D Q.14 C Q.15 B, D Q.16 B, D
 Q.17 A → q; B → p, r; C → p, s; D → q, s Q.18 D Q.19 A, B Q.20 A
 Q.21 D Q.22 D Q.23 2 Q.24 A, B, D Q.25 B Q.26 6
 Q.27 4 Q.28 A → p, r, t; B → p, r; C → q, s; D → r, t Q.29 A, B, C, D Q.30 C
 Q.31 A Q.32 B or A, B, C Q.33 C

Solutions

JEE Main/Boards

Exercise 1

Sol 1: The speed of molecules follows directly from measuring the pressure and density—you don't need to know the size of molecules. In standard kinetic theory, collisions with other molecules have always been ignored, because the molecules were tiny. Though, they

aren't. So, the molecules are not going in straight line. Thus, it takes time for the smell to spread in the room.

Sol 2: Initial

$$P_i = 1 \text{ atm}$$

$$T_0 = -173^\circ\text{C} = 100 \text{ K}$$

$$V_i = V$$

Finally

$$P_f = 2 \text{ atm}$$

$$T_f = ?$$

$$V_f = V = \text{const.}$$

By ideal gas law

$$PV = nRT$$

$$\Rightarrow V = \frac{T}{P} = \text{constant}$$

$$\therefore \frac{T_i}{P_i} = \frac{T_f}{P_f}$$

$$\Rightarrow T_f = \frac{T_i \times P_f}{P_i} = \frac{100 \times 2}{1}$$

$$= 200 \text{ K} = -73^\circ\text{C}$$

Sol 3: Boyle's law states that absolute pressure exerted by a given mass of an ideal gas is inversely proportional to the volume it occupies if the temperature and amount of gas remains unchanged within a closed system.

$$\text{i.e. } P \propto \frac{1}{V}$$

[Keeping temp. and amt. of gas fixed]

Charles's law

Law of volumes: - $V \propto T$

[Given pressure and amount of gas remains constant]

These laws are not applicable to real gases since in real gases size of particles is not negligible, collisions of particles are not elastic and there are attractive forces between particles.

Sol 4: (i) Guy Isaac's law: - The pressure of a gas of fixed mass and fixed volume is directly proportional to the gas absolute temperature.

$$\text{i.e. } P \propto T.$$

[Keeping volume and amount of gas fixed]

$$\Rightarrow \frac{P}{T} = \text{constant}$$

(ii) Ideal gas equation: - $PV = nRT$

$R \rightarrow$ Gas constant

$r \rightarrow$ Radius of gas molecule

Sol 5: Postulates of Kinetic Theory Gases (KTG):

1. Gases consist of particles in random motion.
2. Volume of molecule is negligible.

3. Attraction or repulsion forces between any two particles are negligible

4. The collisions between gas molecules or molecule and wall of container are completely elastic meaning no energy is gained or lost from collisions.

5. The time it takes to collide is negligible

6. All gases at a given temperature have same Kinetic Energy.

7. Motion of particles is random

8. Effect of gravity on gas molecule is neglected

9. Average momentum of gas molecule is zero.

Expression for the pressure and v_{rms} of a gas-

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

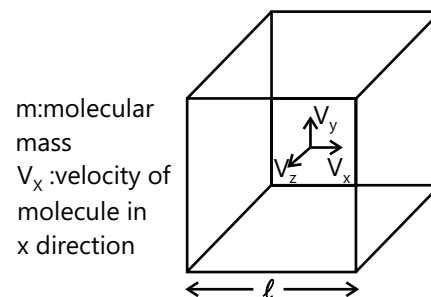
By postulates of KTG we know

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$$

{ $\langle \rangle$ means average}

$$\therefore \langle v^2 \rangle = 3 \langle v_x^2 \rangle$$

Since collision is elastic



Change in momentum in one collision

$$(\Delta P) = 2m v_x$$

$$\text{Time interval between collision } (\Delta t) = \frac{2l}{v_x}$$

$$\therefore \text{Force by one molecule} = \frac{\Delta P}{\Delta t} = \frac{mv_x^2}{l}$$

$$\therefore \text{Total force} = \frac{m}{l} [v_{x1}^2 + v_{x2}^2 + \dots + v_{xn}^2]$$

[Assuming n molecules in the vessel]

$$= \frac{mn}{l} \frac{[v_{x1}^2 + v_{x2}^2 + \dots + v_{xn}^2]}{n} = \frac{M}{l} \langle v_x^2 \rangle$$

$M = mn$: - Total mass

$$\Rightarrow \text{Total force} = \frac{M}{3l} \langle v^2 \rangle$$

$$\text{Pressure (P)} = \frac{\text{Force}}{\text{Area}}$$

Assume vessel to be a cube.

$$\therefore P = \frac{\frac{1}{3} M \langle v^2 \rangle}{\ell \times \ell} = \frac{1}{3} \frac{M}{\ell^3} \langle v^2 \rangle$$

$$\Rightarrow P = \frac{1}{3} \rho v_{\text{rms}}^2$$

$$[\text{Since volume} = \ell^3 \therefore (M/\ell^3) = \rho \text{ and } v_{\text{rms}} = \sqrt{\langle v^2 \rangle}]$$

Sol 6: Expression for pressure has been found in previous question.

$$\text{i.e. } P = \frac{1}{3} \rho v_{\text{rms}}^2$$

$$P = \frac{1}{3} \left(\frac{M}{V} \right) v_{\text{rms}}^2$$

$$\Rightarrow PV = \frac{1}{3} n M_m v_{\text{rms}}^2$$

[M_m : molecular weight]

Using gas equation we get

$$PV = \frac{1}{3} n M_m v_{\text{rms}}^2$$

$$\Rightarrow \frac{1}{2} M_m v_{\text{rms}}^2 = \frac{3}{2} PV_m ; V_m \text{ is molar volume}$$

L.H.S. is the expression for kinetic energy per mole of gas.

Sol 7: We know that molar kinetic energy

$$= \frac{3}{2} PV_m = \frac{3}{2} RT \text{ [From previous question]}$$

$$\therefore \text{Kinetic energy per molecule} = \frac{1}{N_A} \left[\frac{3}{2} RT \right]$$

$$= \frac{1}{N_A} \times \frac{3}{2} K N_A T = \frac{3}{2} KT$$

{ N_A : Avogadro's number

K: Boltzmann constant}

$$\Rightarrow \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} kT \text{ [m: molecular mass]}$$

$$\therefore T = \frac{m v_{\text{rms}}^2}{3k} = \frac{2}{3k} \times \text{kinetic energy per molecule.}$$

\therefore The average energy of the molecule is proportional to absolute temperature. Absolute zero is the temperature at which the kinetic energy of the molecules becomes zero. i.e. they stop.

Sol 8: Mean free path is the average distance travelled by the molecules between successive collisions.

Sol 9: Brownian motion is the random motion of particles in a fluid. Resulting from their collision with the other atoms or molecules in the fluid.

$$\text{Sol 10: Volume of } m \text{ kg of molecules} = \frac{m}{1000}$$

$$\text{Volume occupied by water vapour} = \frac{m}{0.6}$$

$$\therefore \text{Required ratio} = \frac{\frac{m}{1000}}{\frac{m}{0.6}} = 6 \times 10^{-4}$$

Note: Here water is assumed to be completely compact. Therefore, by using density of water we calculate molecular volume.

$$\text{Sol 11: } V = 3000 \text{ cm}^3 = 3000 \times 10^{-6} \text{ m}^3$$

$$T = 20^\circ \text{C} = 293 \text{ K}$$

$$P = 2.5 \times 10^6 \text{ Pa} + 10^5 \text{ Pa} = 2.6 \times 10^6 \text{ Pa}$$

$$n = \frac{PV}{RT} = \frac{2.6 \times 10^6 \times 3000 \times 10^{-6}}{8.31 \times 293} \approx 3.20 \text{ c/moles}$$

\therefore Mass of O_2 in tank

$$= 3.204 \times 32 \approx 102.5 \text{ gm} = \approx 0.103 \text{ kg}$$

$$\text{Sol 12: } v_{\text{rms}} = \sqrt{\frac{3RT}{M_m}} = \sqrt{\frac{3 \times 8.314 \times 273}{64 \times 10^{-3}}}$$

$$\approx 326.18 \text{ m/s}$$

Sol 13: Average time per collision

$$= \frac{2.66 \times 10^{-7} \text{ m}}{500 \text{ m sec}^{-1}} = 5.32 \times 10^{-10} \text{ sec}$$

\therefore No. of collision per second

$$= \frac{1}{5.32 \times 10^{-10}} \approx 1.88 \times 10^9 \text{ sec}$$

$$\text{Sol 14: Mean free path estimate} = \frac{1}{\sqrt{2} \pi d^2 n_v}$$

d: diameter of molecule, n_v molecules per unit volume

$$n_v = 3 \times 10^{19} \text{ cm}^{-3} = 3 \times 10^{25} \text{ m}^{-3}$$

$$d = 2 \times 10^{-10} \text{ m}$$

∴ Mean free path

$$= \frac{1}{1.414 \times 3.14 \times (2 \times 10^{-10})^2 \times 3 \times 10^{25}}$$

$$= 0.0187 \times 10^{-5} \text{ m}$$

$$\approx 1.87 \times 10^{-7} \text{ m}$$

Sol 15: Mean free path (\bar{x}) = $\frac{1}{\sqrt{2}\pi d^2 n_v}$

$$n_v = \frac{nN_A}{V}$$

n: number of moles in volume V

$$N_A = \text{Avogadro's number} = 6.02 \times 10^{23}$$

By gas equation: - $\frac{n}{V} = \frac{P}{RT}$

$$\therefore n_v = \frac{PN_A}{RT}$$

$$\therefore \bar{x} = \frac{RT}{\sqrt{2}\pi d^2 PN_A}$$

$$= \frac{8.314 \times 273}{1.414 \times 3.14 \times (2.4 \times 10^{-10})^2 \times 10^5 \times 6.02 \times 10^{23}}$$

$$\approx 1.47 \times 10^{-7} \text{ m}$$

Sol 16: Water has higher potential energy. As we already know that kinetic energy depends on temperature. So, as we cool water to ice its K.E. remains constant at 0°C. Therefore, the heat energy removed from water, is accounted as decrease in potential energy of ice.

Sol 17: We know that internal energy is a state function and depends on temperature.

$$\therefore \Delta U = 0 \rightarrow \text{No change}$$

Sol 18: Translational motion of molecules.

Sol 19: For isothermal

$$PV = \text{constant}$$

$$\Rightarrow P\Delta V_1 + V\Delta P = 0$$

$$\Rightarrow \frac{\Delta V_1}{\Delta P} = -\frac{V}{P} \quad \dots\dots(i)$$

For adiabatic process

$$PV^\gamma = \text{constant}$$

$$\Rightarrow V^\gamma \Delta P + P^\gamma V^{\gamma-1} \Delta V_2 = 0$$

$$\Rightarrow \frac{\Delta V_2}{\Delta P} = +\frac{1}{\gamma} \left(\frac{-V}{P} \right) \quad \dots\dots(ii)$$

By (i) and (ii) we get

$$\frac{\Delta V_2}{\Delta P} = \frac{1}{\gamma} \left(\frac{\Delta V_1}{\Delta P} \right)$$

$$\Rightarrow \frac{\Delta V_1}{\Delta V_2} = \gamma$$

As $\gamma > 1$

$$\therefore \Delta V_1 > \Delta V_2$$

Sol 20: $\Delta W = -200 \text{ J}$

$\Delta Q = 0$ [adiabatic conditions]

By 1st law of thermodynamics:-

$$\Delta Q = \Delta U + \Delta W$$

$$\Rightarrow \Delta U = +200 \text{ J}$$

Sol 21: The internal energy is the total energy contained by a thermodynamics system. It is the energy needed to create the system but excludes the energy due to external force fields. E.g. Kinetic, energy if the sample is moving or potential energy if the sample is at a height from the ground. Internal energy has two major components i.e. kinetic and internal potential energy.

Sol 22: Four thermodynamics process are:-

(i) Isothermal process: Temperature remains constant. i.e. $PV = \text{constant}$ [$\therefore \Delta T = 0$]

$$\therefore \Delta U = nC_v \Delta T = 0$$

(ii) Adiabatic process:- No flow of heat either into or out from the system.

$$\text{i.e. } \Delta Q = 0 = \Delta U + W$$

$$\therefore \Delta U = -W$$

(iii) Isochoric process:- Volume of the system remains constant.

$$\text{i.e. } V = \text{constant}$$

$$\Rightarrow \Delta V = 0$$

$$\therefore W = \int PdV = 0$$

$$\text{Thus, } \Delta\theta = \Delta U$$

(iv) Isobaric process:- Pressure remains constant

$$\text{i.e. } P = \text{constant}$$

$$\Delta P = 0$$

$$W = P(V_f - V_i)$$

Indicator diagram: are graphical representation of cyclic variations of pressure and volume within a heat engine.

Sol 23: Sign Convention

$\Delta Q \rightarrow +ve$ for heat supplied to gas

$\rightarrow -ve$ for heat rejected by gas

$\Delta W \rightarrow +ve$ for work done by gas

$\rightarrow -ve$ for work done on gas

Sol 24: Isothermal process

$$\Delta T = 0$$

$$\therefore \Delta U = nC_v \Delta T = 0$$

By 1st law of thermodynamics:-

$$\therefore \Delta Q = \Delta U + \Delta W \Rightarrow \Delta Q = \Delta W$$

No change in internal energy

Work done by system = heat supplied to the system.

Adiabatic Process:-

$$\Delta Q = 0$$

$$\text{So, } \Delta Q = \Delta U + \Delta W$$

$$\Rightarrow \Delta U = -\Delta W$$

No heat is supplied to the system

Change in internal energy = Work done on gas

Sol 25: Isothermal Operation: Operations/ process in which temperature stays constant.

Example:- System immersed in a large constant temperature ice-water bath.

Sol 26: Adiabatic operations are those operations in which neither heat enters or leaves a system.

Example:- A system having a perfectly non-conducting boundary.

$$Q = \Delta U + W$$

$$\therefore Q = 0$$

$$\therefore \Delta U + W = 0 \Rightarrow \Delta U = -W$$

Sol 27: $\Delta T = 0$

$$\Delta U = nC_v \Delta T = 0$$

$$\therefore \Delta W = \Delta Q - \Delta U \Rightarrow \Delta W = \Delta Q$$

$$\Delta W = \int_{V_i}^{V_f} P dV$$

$$\text{We know, } PV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$\therefore \Delta W = \int_{V_i}^{V_f} \frac{nRT dV}{V} = nRT \ln V \Big|_{V_i}^{V_f} = nRT \ln \frac{V_f}{V_i}$$

$$\Rightarrow \Delta W = nRT \ln \frac{V_f}{V_i} = nRT \ln \frac{P_i}{P_f}$$

Sol 28: For adiabatic process

$$\Delta Q = 0$$

$$\therefore \Delta U = -\Delta W$$

$$\therefore nC_v dT = -PdV$$

...(i)

By equation of gas:- $PV = nRT$

$$\Rightarrow PdV + VdP = nRdT$$

$$\therefore ndT = \frac{PdV + VdP}{R}$$

\therefore Substituting in (i) we get :-

$$C_v \left(\frac{PdV + VdP}{R} \right) = -PdV$$

$$\Rightarrow (C_v + R) PdV = -C_v VdP$$

$$\Rightarrow \frac{C_p dV}{C_v V} = \frac{-dP}{P} \quad [\because C_v + R = C_p]$$

$$\Rightarrow \gamma \frac{dV}{V} = \frac{-dP}{P}$$

On integrating both sides, we get

$$\gamma \int_{V_i}^{V_f} \frac{dV}{V} = - \int_{P_i}^{P_f} \frac{dP}{P} \Rightarrow \gamma \ln \frac{V_f}{V_i} = \ln \frac{P_i}{P_f}$$

$$\Rightarrow \frac{P_i}{P_f} = \left(\frac{V_f}{V_i} \right)^\gamma \Rightarrow P_i V_i^\gamma = P_f V_f^\gamma = k \text{ (say)}$$

$$\therefore PV^\gamma = \text{const.} = k \Rightarrow P = \frac{k}{V^\gamma}$$

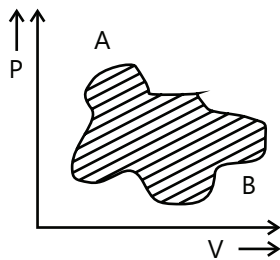
$$\therefore W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} \frac{k}{V^\gamma} dV$$

$$\Rightarrow W = k \frac{V^{-\gamma+1}}{-\gamma+1} \Big|_{V_i}^{V_f} = \frac{kV_f^{1-\gamma} - kV_i^{1-\gamma}}{1-\gamma}$$

$$= \frac{(P_f V_f^\gamma) V_f^{1-\gamma} - (P_i V_i^\gamma) V_i^{1-\gamma}}{1-\gamma}$$

$$\Rightarrow W = \frac{P_f V_f - P_i V_i}{1-\gamma} = \frac{nR}{1-\gamma} (T_f - T_i)$$

Sol 29: Cyclic processes:- In cyclic process, thermodynamic process start from the same state and end at the initial state.



Since, initial and final states are same,

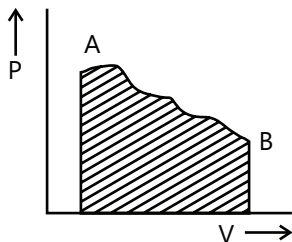
Thus $\Delta U = 0$,

$\therefore \Delta Q = \Delta W$

$\therefore |\Delta Q| = |\Delta W| = \text{Area enclosed by the curve.}$

Sign of ΔW is '+' if process is clockwise and '-' if process is in anti-clockwise.

Non-cyclic process: If initial and final states are not same.



$$\Delta W = \int_A^B P dV = \text{Area enclosed by P-V curve}$$

Sol 30: Reversible process: - The process in which the system and surroundings can be restored to the initial state from the final state without producing any change in the thermodynamic properties of the universe.

Ex: - An infinitesimal compression of a gas in a cylinder.

Irreversible process:- In irreversible process system is not in equilibrium throughout the process. Initial state can't be obtained from final state without producing changes in the universe.

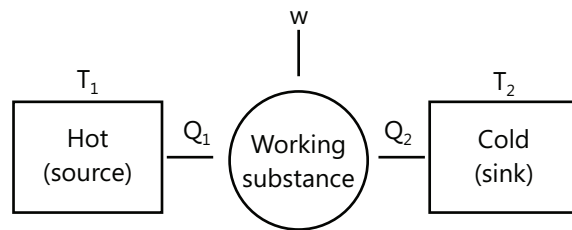
Example:- (i) Processes having friction.

(ii) Heat transfer through a finite temperature difference.

Sol 31: Heat engine is a device which convert heat energy into mechanical energy by using a cyclic process.

$$W = Q_1 - Q_2$$

$$\text{Efficiency: } \eta = \frac{W}{Q_{\text{given}}}$$



Here, Q_1 is the heat given to the substance whereas Q_2 is the heat released.

$$\therefore Q_{\text{given}} = Q_1$$

$$\therefore \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\therefore \text{Efficiency } (\eta) = 1 - \frac{\text{Heat released}}{\text{Heat absorbed}}$$

Two types of heat engines:-

- (i) Internal combustion engines
- (ii) External combustion engines

Sol 32: For adiabatic process

$$PV^\gamma = \text{constant}$$

$$\text{We know } V = \frac{nRT}{P}$$

$$\therefore P \left(\frac{nRT}{P} \right)^\gamma = \text{const.}$$

$$\Rightarrow P^{1-\gamma} T^\gamma = \text{const.}$$

$$P_i = 3 \text{ atm; } P_f = 1 \text{ atm}$$

$$T_i = 300 \text{ K; } T_f = ?$$

$$\therefore T_f^\gamma = \frac{P_i^{1-\gamma} T_i^\gamma}{P_f^{1-\gamma}}$$

$$\Rightarrow T_f^\gamma = \left(\frac{3}{1} \right)^{1-1.4} (300)^{1.4}$$

$$\Rightarrow T_f = 3^{\frac{-0.4}{1.4}} \times 300 \approx 219.2 \text{ K}$$

$$\therefore \text{Fall in temperature} = T_i - T_f = 80.8^\circ \text{C}$$

Sol 33: $T_i = 27^\circ\text{C} = 300 \text{ K; } T_f = ?$

$$P_i = 1 \text{ atm; } P_f = ?$$

$$V_i = V; V_f = V/2$$

(i) $PV^\gamma = \text{constant}$

$$\therefore P_f = \left(\frac{V_i}{V_f} \right)^\gamma P_i = (2)^{1.42} \times 1 \approx 2.675 \text{ atm}$$

$$(ii) PV^\gamma = \text{constant}$$

$$\Rightarrow \frac{nRT}{V} V^\gamma = \text{constant}$$

$$\Rightarrow TV^{\gamma-1} = \text{constant}$$

$$\therefore T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} T_i$$

$$= (2)^{0.42} \times 300$$

$$\Rightarrow T \approx 128.38^\circ\text{C}$$

Sol 34: $n = 1$

$$T_i = T_i; T_f = (T + 70) \text{ K}$$

$$W = \frac{nR}{1-\gamma} [T_f - T_i] = \frac{1 \times 2}{1-1.5} [70] = -280 \text{ Cal}$$

$$\therefore \text{Work done} = -1176 \text{ joules}$$

$$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = 280 \text{ Cal}$$

Work done on gas can be seen by increase in temperature of the gas, which accounts for 280 calories of energy.

Sol 35: $n = 1$

$$V_i = V_i; \Delta Q = 0$$

$$V_f = 2V_i; \Delta U = -\Delta W = \frac{-nR}{1-\gamma} [T_f - T_i]$$

$$T_i = 273 \text{ K}$$

$$TV^{\gamma-1} = \text{constant}$$

$$\therefore T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} \times T_i$$

$$\Rightarrow T_f = (0.5)^{0.4} \times 273 = 206.89 \text{ K}$$

$$\therefore \Delta U = -\frac{1 \times 8.314}{1-1.4} [206.89-273]$$

$$\Rightarrow \Delta U \approx -1374 \text{ Joule}$$

Sol 36: $V_i = V_i; V_f = 2V$

$$P_i = 760 \text{ mm of Hg} = 1 \text{ atm} \approx 10^5 \text{ Pascal}$$

$$T_i = 273 \text{ K}, n = \frac{1}{32}$$

$$\therefore \Delta Q = 0$$

For adiabatic process:-

$$TV^{\gamma-1} = \text{constant}$$

$$\therefore T_f = \left(\frac{V_i}{V_f}\right)^{\gamma-1} T_i$$

$$= \left(\frac{1}{2}\right)^{1.4-1} \times 273 = 206.89 \text{ K}$$

$$\therefore \Delta U = \frac{nR}{1-\gamma} (T_f - T_i)$$

$$= \frac{-\left(\frac{1}{32}\right) \times (2 \times 4.2)}{1-1.4} (206.89 - 273)$$

$$\approx -43.38 \text{ Joule}$$

Exercise 2

Sol 1: (A) $V = 7L = 7 \times 10^{-3} \text{ m}^3$

$$T = 273 \text{ K}$$

$$P = 1.3 \times 10^5 \text{ Pa}$$

$$\therefore m = \frac{PV}{RT} = \frac{1.3 \times 10^5 \times 7 \times 10^{-3}}{8.314 \times 273} \approx 0.4 \text{ moles}$$

$$\therefore \text{No. of molecules} = 0.4 \times 6.02 \times 10^{23}$$

$$= 2.4 \times 10^{23} \text{ molecules}$$

Sol 2: (A) $PV^{2/3} = \text{const.}$

$$\Rightarrow \frac{nRT}{V} V^{2/3} = \text{const.}$$

$$\Rightarrow TV^{-1/3} = \text{const.}$$

$$\Rightarrow T = \text{const.} \times V^{1/3}$$

\therefore On increasing volume, temperature increase

Sol 3: (C) $T = \text{constant}, V = \text{constant},$

$$n_i = \frac{35}{28} = 1.25 \text{ Kmoles}$$

$$\therefore \frac{P}{n} = \frac{RT}{V} = \text{const.}$$

$$\therefore \frac{P_f}{n_f} = \frac{P_i}{n_i}$$

$$\Rightarrow n_f = \frac{P_f}{P_i} \times n_i = \frac{9}{6} \times \frac{35}{28} \approx 1.875 \text{ kmoles}$$

\therefore Moles of O_2 supplied

$$= 1.875 - 1.25 = 0.625 \text{ kmoles}$$

\therefore Amount of oxygen = 20 kg

$$\text{Sol 4: (B)} \quad PV = \frac{4}{M} RT \quad \dots(i)$$

$$PV = \frac{(4-0.8)}{M} R(T+50) = \frac{3.2R}{M} (T+50) \quad \dots(ii)$$

By (i) and (ii) we get,

$$\frac{4}{M} RT = \frac{3.2R}{M} (T+50) \Rightarrow 4T = 3.2T + 3.2 \times 50$$

$$\Rightarrow T = 200 \text{ K}$$

Sol 5: (A) For $P = 1 \text{ atm}$, $m_1 = 2 \text{ gms}$, $T = 298 \text{ K}$

$$\therefore 1 \times V = \frac{2R}{M_1} \times 298 \quad \dots(i)$$

For $P = 1.5 \text{ atm}$, $m_1 = 2 \text{ gm}$, $m_2 = 3 \text{ gm}$, $T = 298 \text{ K}$.

$$\therefore 1.5 \times V = \left[\frac{2}{M_1} + \frac{3}{M_2} \right] R \times 298 \quad \dots(ii)$$

Using (ii)/(i) we get,

$$1.5 = \frac{\left[\frac{2}{M_1} + \frac{3}{M_2} \right]}{\frac{2}{M_1}} \Rightarrow 3 = \left[2 + \frac{3M_1}{M_2} \right] \Rightarrow \frac{M_1}{M_2} = \frac{1}{3}$$

$$\text{Sol 6: (B)} \quad P = \frac{nRT}{V}$$

$$\therefore \left(\frac{nRT}{V} \right)^2 V = \text{constant} \Rightarrow \frac{T^2}{V} = \text{constant}$$

$$\therefore T_f = \left(\frac{V_f}{V_i} \right)^{\frac{1}{2}} \times T_i = \sqrt{2} T$$

$$\text{Sol 7: (A)} \quad P_{\text{at top of tube}} + P_{\text{due to 50 cm of Hg}} = P_{\text{atmospheric}}$$

$$\Rightarrow P_{\text{at top of tube}} = (75 - 50) \text{ cm of Hg}$$

$$= 25 \text{ cm of Hg}$$

$$= 25 \times \frac{10^5}{75} \text{ Pa} = 33.3 \text{ k Pa}$$

Sol 8: (C)

For O_2

$$n = 1$$

$$T$$

$$P$$

$$V$$

For H

$$n = 1$$

$$2T$$

$$P'$$

$$V$$

$$\therefore PV = 1 \times RT \quad \therefore P'V = 1 \times R \times (2T)$$

$$\Rightarrow PV = RT \quad \Rightarrow P' = 2P$$

Sol 9: (D) For diatomic molecule

Translational degree of freedoms = 3

Rotational degree of freedoms = 2

$$\text{Ratio} = \frac{3KT}{2KT} = \frac{3}{2}$$

Sol 10: (D) We know

$$v_{\text{rms}} = 1.085 v_{\text{avg}}$$

$$\Rightarrow \sqrt{\frac{3P}{\rho}} = 1.085 v_{\text{avg}}; \rho \text{ is density}$$

In this case ρ is constant

$$\therefore P \propto (v_{\text{avg}})^2$$

$$\therefore \frac{P_f}{P_i} = \left[\frac{(v_{\text{avg}})_f}{(v_{\text{avg}})_i} \right]^2 = 4$$

Sol 11: (C) Frictionless piston means, no generation of heat due to the motion of piston.

$$n = 1$$

$$T_i = T$$

$$P = \text{const.}$$

$$V_i = V$$

$$V_f = 2V$$

Initially:-

$$PV = RT$$

$$\text{Work done} = P\Delta V = P(2V - V) = PV = RT$$

Sol 12: (A) $f = 6$

$$\Delta W = 25 \Rightarrow P\Delta V = 25 \Rightarrow nR\Delta T = 25 \text{ J}$$

[By using gas equation]

$$\therefore P = \text{constant}$$

$$\Delta Q = \Delta U + \Delta W \text{ [By 1st law of thermodynamics]}$$

$$\text{and } \Delta U = \frac{nf}{2} R\Delta T = \frac{6}{2} \times 25 = 75 \text{ J}$$

$$\therefore \Delta Q = 100 \text{ J}$$

Alternative Method

$$\text{At constant pressure, } \Delta Q = nC_p \Delta T = C_p \frac{P\Delta V}{R}$$

$$[\Rightarrow PV = nRT]$$

$$P\Delta V = nR\Delta T \text{ at const. pressure}]$$

$$\Delta W = P\Delta V = 25 \text{ J}$$

$$f = 6; \quad C_p = \left(1 + \frac{f}{2}\right) R = 4R$$

$$\therefore \Delta Q = (4R) \left[\frac{25}{R}\right] = 100 \text{ J}$$

Sol 13: (B) $\Delta Q = -30 \text{ J}$

$$\Delta W = -18 \text{ J}$$

By 1st law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\Rightarrow \Delta U = -30 - (-18) = -12 \text{ J}$$

$$\Rightarrow U_B - U_i = -12$$

$$\therefore U_B = -12 + 60 = 48 \text{ Joules}$$

Sol 14: (A) $T_i = 300 \text{ K}$ $T_f = ?$
 $P_i = P$ $P_f = 4P$

By gas equation we know: $-V = \frac{nRT}{P}$

$$\therefore PV^{4/3} = \text{constant}$$

$$\Rightarrow P \left(\frac{nRT}{P}\right)^{4/3} = \text{constant} \Rightarrow \frac{T^{4/3}}{P^{1/3}} = \text{constant}$$

$$\therefore T_f = \left(\frac{P_f}{P_i}\right)^{3/4} \times T_i = \left(\frac{4P}{P}\right)^{3/4} \times 300 \text{ K}$$

$$\Rightarrow T_f = 300 \sqrt{2} \text{ K}$$

Sol 15: (D) For first kind of expansion:-

$$\Delta W = 20 \text{ KJ}$$

$$\Delta Q = 16 \text{ KJ}$$

$$\therefore \Delta U = \Delta Q - \Delta W = -4 \text{ KJ}$$

Since, U is a state function. Therefore, value of ΔU in both expansions remain same.

Thus, for second expansion:-

$$\Delta U = -4 \text{ KJ}, \Delta Q = 9 \text{ KJ}$$

\therefore By first law of the thermodynamics:-

$$\Delta W = \Delta Q - \Delta U = 13 \text{ KJ}$$

Sol 16: (A) $n_{N_2} = \frac{7 \text{ kg}}{28} = 0.25 \text{ k moles}$

$$f_{N_2} = \frac{2}{\gamma - 1} = \frac{2}{1.4 - 1} = 5$$

$$n_{CO_2} = \frac{11 \text{ kg}}{44} = 0.25 \text{ k moles}$$

$$f_{CO_2} = \frac{2}{1.3 - 1} = \frac{2}{0.3} = \frac{20}{3}$$

\therefore Equivalent molecular weight

$$= \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \frac{m_1 + m_2}{n_1 + n_2}$$

$$= \frac{7 + 11}{0.25 + 0.25} = 36 \text{ gm}$$

$$\gamma_{\text{mix}} \neq \frac{n_1 \gamma_1 + n_2 \gamma_2 + \dots}{n_1 + n_2 + \dots}$$

$$f_{\text{mix}} = \frac{n_1 f_1 + n_2 f_2 + \dots}{n_1 + n_2}$$

$$= \frac{\left(0.25 \times 5 + 0.25 \times \frac{20}{3}\right)}{0.25 + 0.25} \approx \frac{35}{6}$$

$$\therefore \gamma_{\text{mix}} = 1 + \frac{2}{f_{\text{mix}}} = 1 + \frac{2}{35/6} = \frac{47}{35} = 1.34$$

Previous Years' Questions

Sol 1: (A) Work done in a cyclic process = area between the cycle

$$= AB \times BC = (2P - P) \times (2V - V) = PV$$

Note if cycle is clockwise (p on y-axis and V on x-axis) work done is positive and if it is anticlockwise work done is negative.

Sol 2: (A) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

Room temperature $T = 300 \text{ K}$

$$\therefore 1930 = \sqrt{\frac{3 \times 8.31 \times 10^3 \times 300}{M}}$$

$$\therefore M = 2.0 \text{ g/mol or the gas is } H_2.$$

Sol 3: (B) $Q_1 = nC_p \Delta T$, $Q_2 = nC_v \Delta T$, $\frac{Q_2}{Q_1} = \frac{C_v}{C_p} = \frac{1}{\gamma}$

$$\text{or } Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4} = 50 \text{ cal}$$

Sol 4: (B) $\gamma_1 = \frac{5}{3}$ means gas is monatomic or $C_{V_1} = \frac{3}{2}R$

$\gamma_2 = \frac{7}{5}$ means gas is diatomic or $C_{V_2} = \frac{5}{2}R$

C_V (of the mixture)

$$= \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}R\right)}{1+1} = 2R$$

C_p (of the mixture) = $C_V + R = 3R$

$$\therefore \gamma_{\text{mixture}} = \frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$$

Sol 5: (B) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

i.e., $v_{\text{rms}} \propto \sqrt{T}$

When temperature is increased from 120 K to 480 K (i.e., four times), the root mean square speed will become $\sqrt{4}$ or 2 times i.e., $2v$.

Sol 6: (D) The average translational KE = $\frac{3}{2} kT$ which is directly proportional to T , while rms speed of molecules is given by

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ i.e., } v_{\text{rms}} \propto \sqrt{T}$$

When temperature of gas is increased from 300 K to 600 K (i.e., 2 times), the average translational KE will increase to 2 times and rms speed to $\sqrt{2}$ or 1.414 times.

$$\therefore \text{Average translational KE} = 2 \times 6.21 \times 10^{-21} \text{ J} \\ = 12.42 \times 10^{-21} \text{ J}$$

$$\text{And } v_{\text{rms}} = (1.414)(484) \text{ m/s} \approx 684 \text{ m/s}$$

Sol 7: (C) $PV = nRT$ or $P = \frac{nRT}{V}$ or $P \propto T$

If V and n are same. Therefore, if T is doubled pressure also becomes two times i.e., $2p$.

Sol 8: (C) Process is isothermal. Therefore

$$T = \text{constant. } \left(p \propto \frac{1}{V} \right)$$

Volume is increasing, therefore, pressure will decrease.

In chamber A \rightarrow

$$-\Delta P = (p_A)_i - (p_A)_f = \frac{n_A RT}{V} - \frac{n_A RT}{2V} \\ = \frac{n_A RT}{2V} \quad \dots (i)$$

In chamber B \rightarrow

$$-1.5\Delta P = (p_B)_i - (p_B)_f = \frac{n_B RT}{V} - \frac{n_B RT}{2V} \\ = \frac{n_B RT}{2V} \quad \dots (ii)$$

From Equation (i) and (ii)

$$\frac{n_A}{n_B} = \frac{1}{1.5} = \frac{2}{3} \text{ or } \frac{m_A/M}{m_B/M} = \frac{2}{3} \text{ or } \frac{m_A}{m_B} = \frac{2}{3} \text{ or } 3m_A \\ = 2m_B$$

Sol 9: (D) Internal energy of n moles of an ideal gas at temperature T is given by

$$U = n \left(\frac{f}{2} RT \right)$$

where, f = degrees of freedom

$$= 5 \text{ for } O_2 \text{ and } 3 \text{ for Ar}$$

Hence, $U = U_{O_2} + U_{Ar}$

$$= 2 \left(\frac{5}{2} RT \right) + 4 \left(\frac{3}{2} RT \right) = 11 RT$$

Sol 10: (D) During adiabatic expansion, we know

$$TV^{\gamma-1} = \text{constant}$$

$$\text{or } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

For a monoatomic gas, $\gamma = \frac{5}{3}$

$$\therefore \frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\gamma-1} = \left(\frac{AL_2}{AL_1} \right)^{\frac{5}{3}-1}$$

(A = Area of cross-section of piston)

$$= \left(\frac{L_2}{L_1} \right)^{2/3}$$

Sol 11: (C) $pT^2 = \text{constant}$

$$\therefore \left(\frac{nRT}{V} \right) T^2 = \text{constant or } T^3 V^{-1} = \text{constant}$$

Differentiating the equation, we get

$$\frac{3T^2}{V} dT - \frac{T^3}{V^2} dV = 0$$

$$\text{or } 3dT = \frac{T}{V} \cdot dV \quad \dots\dots (i)$$

From the equation, $dV = V\gamma dT$

$$\gamma = \text{coefficient of volume expansion of gas} = \frac{dV}{V \cdot dT}$$

$$\text{From Equation (i) } \gamma = \frac{dV}{V \cdot dT} = \frac{3}{T}$$

Sol 12: A \rightarrow s; B \rightarrow r, p; C \rightarrow p; D \rightarrow s

In process J \rightarrow K V is constant whereas p is decreasing. Therefore, T should also decrease.

$$\therefore W = 0, \Delta U = -ve \text{ and } Q < 0$$

In process K \rightarrow L p is constant while V is increasing. Therefore, temperature should also increase.

$$W > 0, \Delta U > 0 \text{ and } Q > 0$$

In process L \rightarrow M This is inverse of process J \rightarrow K.

$$\therefore W = 0, \Delta U > 0 \text{ and } Q > 0$$

In process M \rightarrow J

V is decreasing. Therefore, $W < 0$

$$(PV)_J < (PV)_M$$

$$\therefore T_f < T_M \text{ or } \Delta U < 0$$

Therefore, $Q < 0$

Sol 13: A \rightarrow p, r, t; B \rightarrow p, r; C \rightarrow q, s; D \rightarrow r, t

Internal energy $\propto T \propto PV$

$$\text{This is because } U = \frac{nf}{2} RT = \frac{f}{2} (PV)$$

Here, n = number of moles, f = degree of freedom.

\therefore If the product PV increases then internal energy will increase and if product decreases the internal energy will decrease.

Further, work is done on the gas, if volume of gas decreases. For heat exchange,

$$Q = W + \Delta U$$

Work done is area under p-V graph. If volume increases work done by gas is positive and if volume decrease work done by gas is negative. Further ΔU is positive if product of PV is increasing and ΔU is negative if product of PV is decreasing.

If heat is taken by the gas, Q is positive and if heat is lost by the gas, Q is negative.

Keeping the above points in mind the answer to this question is as under.

(A) \rightarrow (p, r, t); (B) \rightarrow (p, r);

(C) \rightarrow (q, s); (D) \rightarrow (r, t)

Sol 14: (A) $\Delta U = nC_V \Delta T = nC_V (T_2 - T_1)$ in all processes.

(B) In adiabatic process $\Delta Q = 0$

$$\therefore \Delta U = -\Delta W \text{ or } |\Delta U| = |\Delta W|$$

(C) In isothermal process $\Delta T = 0$

$$\therefore \Delta U = 0 \text{ (as } \Delta U = nC_V \Delta T)$$

(D) In adiabatic process $\Delta Q = 0$

Sol 15: (A, B) $T_A = T_B \quad \therefore U_A = U_B$

$$W_{AB} = (1)(R)T_0 \ln \left(\frac{V_f}{V_i} \right) = RT_0 \ln \left(\frac{4V_0}{V_0} \right) = p_0 V_0 \ln(4)$$

Information regarding p and T at C cannot be obtained from the given graph. Unless it is mentioned that line BC passes through origin or not.

Sol 16: (D) No option is correct

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{v_1}{v_2} = \frac{\sqrt{\gamma_1 M_2}}{\sqrt{\gamma_2 M_1}} = \sqrt{\frac{\frac{7}{5} \times 4}{\frac{5}{3} \times 32}}$$

$$\frac{460}{v_2} = \sqrt{\frac{21}{25 \times 8}} \Rightarrow v_2 = \frac{460 \times 5 \times 2\sqrt{2}}{\sqrt{21}} = 1420 \text{ m/s}$$

Sol 17: (A) $U = U_1 + U_2$

$$T = \frac{(P_1 V_1 + P_2 V_2) T_1 T_2}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$$

Sol 18: (B) Thermal energy corresponds to internal energy

Mass = 1 kg

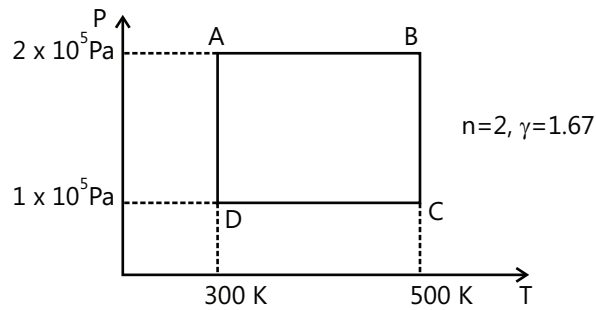
Density = 8 kg/m³

$$\Rightarrow \text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{1}{8} \text{ m}^3$$

Pressure = 8×10^4 N/m²

$$\therefore \text{Internal Energy} = \frac{5}{2} P \times V = 5 \times 10^4 \text{ J}$$

Sol 19: (C) $W_{AB} = \Delta Q - \Delta U = \int_n C_p dT - \int_n C_v dT$ (at constant pressure)



$$= n(C_p - C_v)dt$$

$$= nRdT = 2 \times R \times (500 - 300) = 400 R$$

Sol 20: (A) At constant temperature (isothermal process)

$$\begin{aligned} W_{DA} &= nRT \ln\left(\frac{P_1}{P_2}\right) = 2.303 \times 2R \times 300 \log\left(\frac{10^5}{2 \times 10^5}\right) \\ &= 2.303 \times 600 \log\left(\frac{1}{2}\right) \\ &= 0.693 \times 600R = -414R \end{aligned}$$

Sol 21: (B) Net work done in a cycle = $W_{AB} + W_{BC} + W_{CB} + W_{BA}$

$$\begin{aligned} &= 400 R + 2 \times 2.303 \times 500 R \ln 2 - 400R - 414 R \\ &= 1000R \times \ln 2 - 600R \times \ln 2 = 400R \times \ln 2 = 276R \end{aligned}$$

Sol 22: (B) The efficiency of cycle is

$$\eta = 1 - \frac{T_2}{T_1}$$

for adiabatic process

$$TV^{\gamma-1} = \text{constant}$$

For diatomic gas $\gamma = \frac{7}{5}$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_1 = T_2 \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_1 = T_2 (32)^{\frac{7}{5}-1} = T_2 (2^5)^{2/5} = T_2 \times 4$$

$$T_1 = 4T_2$$

$$\eta = \left(1 - \frac{1}{4}\right) = \frac{3}{4} = 0.75$$

$$\text{Sol 23: (C)} \quad U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

$$U(x = \infty) = 0$$

$$\text{as, } F = -\frac{dU}{dx} = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$$

At equilibrium, $F = 0$

$$\therefore x^6 = \frac{2a}{b}$$

$$\therefore U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4a}$$

$$\therefore D = [U(x = \infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

$$\text{Sol 24: (D)} \quad \eta_1 = \frac{T_1 - T_2}{T_1} = \frac{1}{6}$$

$$\text{and } \eta_2 = \frac{T_1 - (T_2 - 62)}{T_1} = \frac{1}{3} \Rightarrow \frac{T_1 - T_2}{T_1} + \frac{62}{T_1} = \frac{1}{3}$$

$$\frac{1}{6} + \frac{62}{T_1} = \frac{1}{3} \Rightarrow \frac{62}{T_1} = \frac{1}{6}$$

$$\therefore T_1 = 62 \times 6 = 372K$$

$$\frac{T_1 - T_2}{T_1} = \frac{1}{6} \Rightarrow 1 - \frac{T_2}{T_1} = \frac{1}{6}$$

$$\Rightarrow \frac{T_2}{372} = \frac{5}{6} \Rightarrow T_2 = 310K$$

Sol 25: (A) Work done in complete cycle = Area under P-V graph = $P_0 V_0$

From A to B, heat given to the gas

$$= n C_v \Delta T = n \frac{3}{2} R \Delta T = \frac{3}{2} V_0 \Delta P = \frac{3}{2} P_0 V_0$$

From B to C, heat given to the system

$$= n C_v \Delta T = n \left(\frac{5}{2} R\right) \Delta T = \frac{5}{2} (2P_0) \Delta V = 5P_0 V_0$$

From C to D and D to A, heat is rejected.

$$\text{Efficiency, } \eta = \frac{\text{Work done by gas}}{\text{Heat given to the gas}} \times 100$$

$$\eta = \frac{P_0 V_0}{\frac{3}{2} P_0 V_0 + 5P_0 V_0} = 15.4\%$$

$$\text{Sol 26: (C)} \quad \frac{40}{100} = \frac{500 - T_s}{500}, T_s = 300\text{K}$$

$$\frac{600}{100} = \frac{T - 300}{T} \Rightarrow T = 750\text{K}$$

Sol 27: (C)

$$\frac{Mg}{A} = P_0 \quad P_0 V_0^\gamma = PV^\gamma$$

$$Mg = P_0 A \dots \dots \dots (i) \quad P_0 A X_0^\gamma = PA(x_0 - x)^\gamma$$

Let piston is displaced by x $P = \frac{P_0 A x_0^\gamma}{(x_0 - x)^\gamma}$

$$Mg = \left(\frac{P_0 x_0^\gamma}{(x_0 - x)^\gamma} \right) A = F_{\text{restoring}}$$

$$P_0 A \left(1 - \frac{x_0^\gamma}{(x_0 - x)^\gamma} \right) = F_{\text{restoring}} \quad [x_0 - x \approx x_0]$$

$$F = \frac{\gamma P_0 A x}{x_0}$$

$$\therefore f = \frac{1}{2\pi} \sqrt{\frac{\gamma P_0 A}{x_0 M}} = \frac{1}{2\pi} \sqrt{\frac{\gamma P_0 A^2}{M V_0}}$$

$$\text{Sol 28: (D)} \quad \Delta U = n C_V \Delta T = 1 \times \frac{5R}{2} \Delta T$$

For BC, $\Delta T = -200\text{K}$

$$\Rightarrow \Delta U = -500R$$

$$\text{Sol 29: (C)} \quad p = \frac{1}{3} \frac{U}{V}$$

$$\frac{nRT}{V} \propto \frac{1}{3} T^4$$

$$VT^3 = \text{const}$$

$$\frac{4}{3} \pi R^3 T^3 = \text{const}$$

$$TR = \text{const} \Rightarrow T \propto \frac{1}{R}$$

Sol 30: (B) Since entropy is a state function, therefore change in entropy in both the processes should be same.

$$\text{Sol 31: (C)} \quad \text{Since } \tau = \frac{1}{n\pi\sqrt{2}v_{\text{rms}}d^2}$$

$$n \propto \frac{1}{V} \text{ and } v_{\text{rms}} \propto \sqrt{T}$$

$$\Rightarrow \tau \propto \frac{V}{\sqrt{T}}$$

$$n = C_1 V^{-1} < v > = C_2 T^{1/2}$$

$$\text{Since } TV^{\gamma-1} = \text{constant} \Rightarrow \tau \propto V^{\frac{\gamma+1}{2}}$$

$$\text{Sol 32: (D)} \quad P - P_0 = -\frac{P_0}{V_0}(V - 2V_0)$$

$$P = 3P_0 - \frac{P_0}{V_0}V \quad \dots(i)$$

$$\frac{nRT}{V} = 3P_0 - \frac{P_0}{V_0}V$$

$$nRT = 3P_0V - \frac{P_0}{V_0}V^2$$

Differentiate w.r.t. Volume

$$3P_0 - \frac{2P_0}{V_0}V = 0 \Rightarrow V = \frac{3V_0}{2}$$

Putting in equ (i)

$$P = 3P_0 - \frac{P_0}{V_0} \left(\frac{3V_0}{2} \right) = \frac{3P_0}{2}$$

Now, $PV = xRT$

$$\frac{9P_0 V_0}{4} = nRT \Rightarrow T = \frac{9 P_0 V_0}{4 xR}$$

$$\text{Sol 33: (A)} \quad C = C_V + \frac{R}{1-n}$$

$$C - C_V = \frac{C_P - C_V}{1-n}; \quad 1-n = \frac{C_P - C_V}{C - C_V}$$

$$n = 1 - \frac{C_P - C_V}{C - C_V} = \frac{C - C_P}{C - C_V}$$

JEE Advanced/Boards

Exercise 1

Sol 1: For O_2 For He

$$P = P_0 \qquad P = P_0$$

$$V = V_0 \qquad V = V_0$$

$$T = T_0 \qquad T = T_0/2$$

$$M = 32 \text{ gm} \qquad M = 4 \text{ gm}$$

$$PV = nRT$$

$$\Rightarrow \frac{m}{M} T = \frac{PV}{R} = \text{constant}$$

$$\therefore \frac{m_{O_2}}{M_{O_2}} T_{O_2} = \frac{m_{He}}{M_{He}} T_{He}$$

$$\Rightarrow \frac{m_{O_2}}{32} \times T_0 = \frac{m_{He}}{4} \times T_0/2 \Rightarrow \frac{m_{O_2}}{m_{He}} = 4$$

Sol 2: We have, $V = \left(\frac{nR}{P}\right) T$ and we know that $V - T$ curve is a straight line.

\therefore Pressure is constant and its slope = $\frac{nR}{P} = \tan 53^\circ$

$$\Rightarrow \frac{2 \times 8.314 \times 10^3 \text{ Pa L / moleK}}{P} = \frac{4}{3}$$

$$\Rightarrow P \approx 1.25 \times 10^4 \text{ Pa}$$

Sol 3: $PV^\gamma = \text{constant}$

$$\Rightarrow TV^{\gamma-1} = \text{constant}$$

$$\Rightarrow V^{\gamma-1} dT + (\gamma-1) V^{\gamma-2} T dV = 0$$

$$\Rightarrow \frac{dT}{T} = \frac{-V}{(\gamma-1)T} \Rightarrow \left| \frac{dT}{T} \right| = \frac{V}{(\gamma-1)T}$$

$$\Rightarrow m = \frac{+V_0}{(\gamma-1)T_0} \Rightarrow \frac{1}{\gamma-1} = + \frac{mT_0}{V_0}$$

$$\therefore C_V = \frac{R}{\gamma-1} = + \frac{mT_0 R}{V_0}$$

And $C_p = C_V + R$

$$= \frac{(V_0 + mT_0)R}{V_0} = \frac{mRT_0}{V_0} \left(1 + \frac{T_0 m}{V_0} \right) R$$

Sol 4: C_p' and C_V' be molar specific heats.

$$\therefore C_p' = C_p \times M = 0.2 M \text{ cal/mole } ^\circ\text{C}$$

$$C_V' = C_V \times M = 0.15 M \text{ cal/mole } ^\circ\text{C}$$

$$\text{and } C_p' - C_V' = R$$

$$\Rightarrow 0.2 M - 0.15 M = 2$$

$$\Rightarrow M = \frac{2}{0.05} = 40 \text{ gms}$$

Sol 5: $f = 6$

At constant pressure

$$\Delta Q = nC_p \Delta T$$

$$= C_p \times \frac{P\Delta V}{R}$$

[$PV = nRT \Rightarrow P \Delta V = nR \Delta T$ at constant pressure]

$$= \frac{C_p}{R} \times \Delta W$$

$$C_p = \left(1 + \frac{f}{2} \right) R = 4R$$

And $\Delta W = 25 \text{ J}$

$$\Delta Q = \frac{4R}{R} \times 25 = 100 \text{ J}$$

Sol 6: He N_2

$$m = 4 \text{ gm} \qquad m_{N_2} = 28 \text{ gm}$$

$$\therefore n_{He} = \frac{4}{4} = 1 \text{ mole} \qquad n_{N_2} = \frac{28}{28} = 1 \text{ mole}$$

$$f = 3 \qquad f = 5$$

$$\therefore C_V = \frac{3}{2} \qquad \therefore C_V = 5/2 R$$

$$\therefore C_{V_{\text{mix}}} = \frac{n_{He} C_{V_{He}} + n_{N_2} C_{V_{N_2}}}{n_{He} + n_{N_2}}$$

$$\therefore C_{V_{\text{mix}}} = \frac{3R + 5R}{1+1} = 2R$$

$$\text{We know } v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

\therefore To double the v_{rms} temperature must be made 4 times the original temperature.

$$\therefore T_f = 4 T_i = 1200 \text{ K}$$

$$\therefore \Delta T = 900 \text{ K}$$

$$\begin{aligned}\therefore \Delta Q &= n C_V \Delta T \text{ [At constant volume]} \\ &= (n_{\text{He}} + n_{\text{N}_2}) C_{V_{\text{mix}}} \Delta T \\ &= 2 \times 2R \times 900 = 3600 R\end{aligned}$$

Sol 7: T = constant

$$\therefore \Delta U = 0$$

$$\text{Thus } \Delta W = -23.04 \times 10^2 \text{ J}$$

$$\Rightarrow nRT \ln \frac{V_f}{V_i} = -23.04 \times 10^2$$

$$\Rightarrow 8.31 \times T \ln \frac{0.25}{0.50} = -23.04 \times 10^2$$

$$\therefore T = \frac{+23.04 \times 10^2}{8.31 \times \ln 2} \approx 400 \text{ K}$$

Sol 8: Let C be the molar heat capacity of this gas.

$$\therefore \Delta Q = nC\Delta T$$

$$\text{And we know } \Delta U = nC_V \Delta T$$

$$\therefore \Delta Q = 2\Delta U$$

$$\therefore nC \Delta T = 2nC_V \Delta T$$

$$\Rightarrow C = 2C_V = 2 \left(\frac{5}{2}R \right) = 5R$$

Sol 9: C → Molar heat capacity of the process

By 1st law of thermodynamics:-

$$dQ = dU + dW$$

$$\Rightarrow nC dT = nC_V dT + dW$$

$$dW = PdV$$

$$= \frac{nRT}{V} dV = nR \frac{T_0 e^{\alpha V}}{V} dV$$

$$T = T_0 e^{\alpha V}$$

$$\therefore dT = \alpha T_0 e^{\alpha V} dV$$

$$\Rightarrow dV = \frac{dT}{\alpha T_0 e^{\alpha V}}$$

$$\Rightarrow dW = \frac{nR}{\alpha V} dT$$

Substituting value of dW in (i) we get

$$nC dT = nC_V dT + \frac{nR}{\alpha V} dT$$

$$\therefore C = C_V + \frac{R}{\alpha V}$$

Sol 10: From graph, it is clear that V and 1/T have a linear relationship

$$\therefore V = \frac{m}{T}$$

m is the slope of straight line

$$\Rightarrow VT = \text{constant}$$

$$\Rightarrow PV^2 = \text{constant} = k \text{ (say)}$$

$$\Delta W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} kV^{-2} dV$$

$$= -kV^{-1} \Big|_{V_i}^{V_f} = -[kV_f^{-1} - kV_i^{-1}]$$

$$= -[(P_f V_f^2) V_f^{-1} - (P_i V_i^2) V_i^{-1}] = -[P_f V_f - P_i V_i]$$

$$= -nR\Delta T.$$

$$\text{Thus } \Delta Q = \Delta U + \Delta W$$

$$\Rightarrow nC \Delta T = nC_V \Delta T - nR \Delta T$$

$$\Rightarrow C = C_V - R$$

$$\Rightarrow C = \frac{3}{2}R - R = \frac{R}{2}$$

$$\text{Sol 11: (i) } \Delta W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} aV dV$$

$$= \frac{aV^2}{2} \Big|_{V_i}^{V_f} = \frac{aV_f^2 - aV_i^2}{2} = \frac{\left(\frac{P_f}{V_f}\right)V_f^2 - \left(\frac{P_i}{V_i}\right)V_i^2}{2}$$

$$\dots(i) \quad \left(a = \frac{P_f}{V_f} = \frac{P_i}{V_i}\right)$$

$$\Rightarrow \Delta W = \frac{P_f V_f - P_i V_i}{2} = \frac{nR\Delta T}{2}$$

$$\Delta U = nC_V \Delta T = \frac{nR}{\gamma - 1} \Delta T$$

$$\Delta Q = \Delta W + \Delta U$$

$$= \frac{nR\Delta T}{2} + \frac{nR}{\gamma - 1} \Delta T = nR\Delta T \left[\frac{1}{2} + \frac{1}{\gamma - 1} \right]$$

$$= \frac{nR\Delta T}{2} \left[\frac{\gamma + 1}{\gamma - 1} \right]$$

....(i)

We know P = aV

$$\Rightarrow \frac{nRT}{V} = aV \Rightarrow T = \frac{aV^2}{nR}$$

$$\therefore T_i = \frac{aV_0^2}{R} \text{ and } T_f = \frac{9aV_0^2}{R}$$

$$\therefore \Delta T = \frac{8aV_0^2}{R}$$

Substituting (ii) in (i)

$$\text{Thus } \Delta Q = \frac{R}{2} \times \frac{8aV_0^2}{R} \left(\frac{\gamma+1}{\gamma-1} \right) = \left(\frac{\gamma+1}{\gamma-1} \right) 4aV_0^2$$

(ii) C → Heat capacity of gas

$$\text{So } \Delta Q = nC\Delta T$$

Using (i) we get

$$nC\Delta T = \frac{nR\Delta T}{2} \left(\frac{\gamma+1}{\gamma-1} \right)$$

$$\Rightarrow C = \left(\frac{\gamma+1}{\gamma-1} \right) \frac{R}{2}$$

Sol 12: At A

$$V_A = V_0, T_A = 400 \text{ K}$$

$$\therefore P_A = \frac{400nR}{V_0}$$

At B

$$V_B = 2V_0, T_B = 400 \text{ K}, P_B = \frac{400nR}{2V_0}$$

At C

$$V_C = ?, T_C = ?, P_C = \frac{400nR}{2V_0}$$

∴ AC is adiabatic process

∴ $PV^\gamma = \text{constant}$

$$\Rightarrow V_C = \left(\frac{P_A}{P_C} \right)^{\frac{1}{\gamma}} V_A = (2)^{\frac{1}{1.5}} V_0 \Rightarrow V_C = 2^{2/3} V_0$$

$$\Rightarrow T_C = \frac{P_C V_C}{nR} = 400 \times (2)^{-1/3}$$

For process AB

$$\Delta U = 0$$

$$\Delta Q = \Delta W = nR \times (400) \times \ln \left(\frac{2V_0}{V_0} \right)$$

$$= 400 nR \ln 2$$

For process BC

$$\Delta U = nC_V \Delta T = \frac{nR}{\gamma-1} \times [400 (2)^{-1/3} - 400]$$

$$= -2 \times 400 nR [1-2^{-1/3}]$$

$$\Delta Q = nC_p \Delta T = -\frac{n\gamma R}{\gamma-1} \times 400 [1-2^{-1/3}]$$

$$= -3 \times 400 nR [1-2^{-1/3}]$$

$$\Delta W = -400 nR [1-2^{-1/3}]$$

.....(ii)

For process AC

$$\Delta Q = 0$$

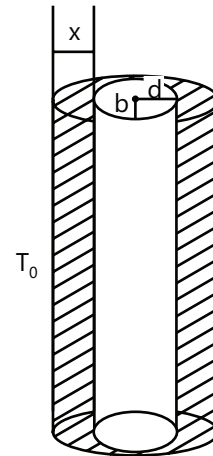
$$\Delta U = nC_V \Delta T = 2 \times 400 nR [1-2^{-1/3}]$$

$$\Delta W = -\Delta U = -2 \times 400 nR [1-2^{-1/3}]$$

$$\text{Efficiency (e)} = 1 - \frac{Q_{\text{released}}}{Q_{\text{absorbed}}}$$

$$= 1 - \frac{3 \times 400 nR [1-2^{-1/3}]}{400 nR \ln 2} = 1 - \frac{3(1-2^{-1/3})}{\ln 2}$$

$$\text{Sol 13: } \frac{dQ}{dt} = \frac{-kAdT}{dx} = \frac{-k[2\pi(b-x) \times \ell]dT}{dx}$$



On integrating with respect to dx we get

$$\Rightarrow \int_0^{b-a} \frac{dQ}{dt} \times \frac{1}{2\pi(b-x)} dx = \int_a^{b-a} -k\ell \frac{dT}{dx} \times dx$$

$$\Rightarrow \frac{dQ}{dt} \times \frac{-\ln(b-x)}{2\pi} \Big|_0^{b-a} = -k\ell \Delta T$$

$$\frac{dQ}{dt} \cdot \ln \frac{b}{a} = -2\pi k\ell (T - T_0)$$

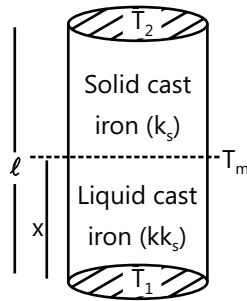
$$\left[(\pi a^2 \ell) 3 \frac{dT}{dx} \right] \ln \left(\frac{b}{a} \right) = -2\pi k\ell (T - T_0)$$

⇒ On integrating w.r.t. t we get

$$\Rightarrow \frac{-\pi a^2 \ell \ln \left(\frac{b}{a} \right)}{2\pi k\ell} \int_{T_1}^{T_2} \frac{dT}{T - T_0} = \int_0^t dt$$

$$\Rightarrow \frac{a^2 \ell \ln \left(\frac{b}{a} \right)}{2K} \ln \left(\frac{T_0 - T_1}{T_0 - T_2} \right) = t$$

Sol 14: $k_s \rightarrow$ Conductivity of solid cast iron. At the point of junction of solid cast iron and liquid cast iron, temperature would be equal to melting point of iron.



$$\left(\frac{dQ}{dt}\right)_{\text{solid cast iron}} = \left(\frac{dQ}{dt}\right)_{\text{liquid cast iron}}$$

$$\Rightarrow \frac{-k_s A [T_2 - T_m]}{l - x} = \frac{-k_l [T_m - T_1]}{x}$$

$$\Rightarrow \frac{x}{l - x} = \frac{k [T_m - T_1]}{T_2 - T_m}$$

$$\Rightarrow \frac{x}{l} = \frac{k [T_m - T_1]}{(T_2 - T_m) + k (T_m - T_1)} = \frac{k (T_1 - T_m)}{k (T_1 - T_m) + (T_m - T_2)}$$

Sol 15: (a) Temperature gradient = $\frac{dT}{dx}$

$$= \frac{(0 - 100)^\circ\text{C}}{(1 - 0)\text{m}} = -100^\circ\text{C/m}$$

$$(b) \frac{dT}{dx} = \frac{T - 100}{x - 0} = -100^\circ\text{C/m}$$

$$\Rightarrow T = 100 (1 - x)^\circ\text{C}$$

Total heat absorbed

$$= \int \text{mass} \times \text{specific heat} \times \text{change of temp}$$

$$= \int_0^1 (2dx)(10)(T - 0) = 20 \int_0^1 100(1 - x)dx$$

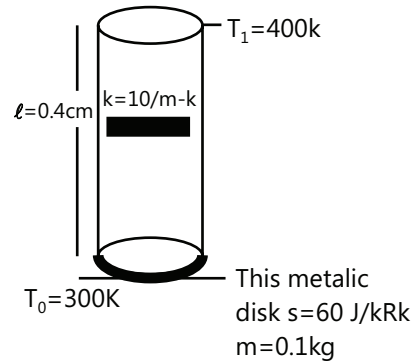
$$= 2000 \left[x - \frac{x^2}{2} \right]_0^1 = 1000 \text{ J}$$

$$\text{Sol 16: } \frac{dQ}{dt} = \frac{-kA(T - 400)}{l}$$

$$\Rightarrow ms \frac{dT}{dt} = \frac{-kA(T - 400)}{l}$$

$$\Rightarrow \frac{msl}{kA} \frac{dT}{T - 400} = -dt$$

Area of cross-section(A)=0.04m²



On integrating we get

$$\Rightarrow - \frac{msl}{kA} \int_{300}^{350} \frac{dT}{T - 400} = \int_0^t dt$$

$$\Rightarrow t = \frac{-0.4 \times 600 \times 0.4}{10 \times 0.04} \lambda n \left| \frac{350 - 400}{300 - 400} \right|$$

$$\approx 166.3 \text{ sec}$$

Sol 17: $(T - T_0) = (T_1 - T_0) e^{-kt}$

$$\Rightarrow t = \frac{1}{k} \lambda n \left(\frac{T_1 - T_0}{T - T_0} \right)$$

T_0 : Surrounding temp

T_1 : Initial temp of object

T : Final temp of object

$$\therefore 5 = \frac{1}{k} \lambda n \left(\frac{80 - 20}{50 - 20} \right) = \frac{1}{R} \lambda n (2)$$

$$\text{and } t = \frac{1}{k} \lambda n \left(\frac{60 - 20}{30 - 20} \right) = \frac{1}{k} \lambda n (2)^2 = 2 \times \frac{1}{k} \lambda n 2$$

$$\therefore t = 10 \text{ min}$$

Sol 18: $A = 27 \text{ cm}^2 = 27 \times 10^{-4} \text{ m}^2$

$k = 3700 \text{ N/m}$

$P_i = 1 \text{ atm} = 10^5 \text{ Pa}$

$T_i = 273 \text{ K}$

$V_i = Ah.$

Initial contraction in spring

$$= \frac{P_i \times A}{k} = \frac{10^5 \times 27 \times 10^{-4}}{3700} = \frac{27}{370} \text{ m}$$

$P_f = ?$

$$T_f = ?$$

$$V_f = \frac{9hA}{16}$$

Since, spring contract by $h/16$ length

$$\therefore \text{Force exerted by spring} = k \left(\frac{h}{16} + \frac{27}{370} \right)$$

Avg. pressure due to spring (P_0)

$$= \frac{k}{A} \left(\frac{h}{16} + \frac{27}{370} \right)$$

Since process is adiabatic

$$\therefore P_f = \left(\frac{V_i}{V_f} \right)^\gamma P_i$$

$$\left(\frac{16}{09} \right)^{3/2} 10^5 = \frac{4 \times 16}{27} \times 10^5$$

P_f must be equal to P_0 for equilibrium

$$\text{Thus, } \frac{k}{A} \left[\frac{h}{16} + \frac{27}{370} \right] = \frac{4 \times 16}{27} \times 10^5$$

$$\Rightarrow \frac{h}{16} + \frac{27}{370} = \frac{4 \times 16 \times 10^5 \times 27 \times 10^{-4}}{27 \times 3700}$$

$$\Rightarrow h = \frac{4 \times (16)^2}{370} - \frac{27 \times 16}{370}$$

$$\Rightarrow h = 1.6 \text{ m}$$

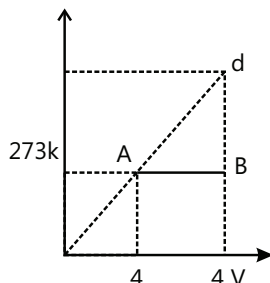
In adiabatic process $TV^{\gamma-1} = \text{constant}$

$$\Rightarrow T_f = \left(\frac{V_i}{V_f} \right)^{\gamma-1} T_i$$

$$= \left(\frac{16}{9} \right)^{1/2} \times 273$$

$$\Rightarrow T_f = 364 \text{ K}$$

Sol 19: Process AC is isobaric



$$\therefore T_C = \frac{V_C}{V_A} T_A = \frac{4V}{V} \times T_A$$

$$\Rightarrow T_C = 4T_A$$

$$\Delta Q_{AB} = \Delta U + \Delta W$$

$$= nRT_A \lambda n 4$$

$$\Delta Q_{BC} = \Delta U + \Delta W$$

$$= nC_V [T_C - T_A] = 3n C_V T_A$$

$$\therefore \Delta Q_{AB} + \Delta Q_{BC} = 27.7 \times 10^3 \text{ J}$$

$$\Rightarrow nRT_A \lambda n 4 + 3 n C_V T_A = 27700$$

$$\Rightarrow R \lambda n 4 + 3C_V = \frac{27700}{2 \times 273} \approx 50.73$$

$$\therefore C_V \approx 13.068 \text{ J/mol-K}$$

$$\frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V} \approx 1.63$$

Sol 20: Initially

$$T_i = 0^\circ\text{C} = 273\text{K} = T_1$$

$$P_i = P_1$$

$$V_i = V_1$$

From (1) to (2)

Process is adiabatic

$$\Delta Q = 0$$

$$\Delta U = -\Delta W$$

$$P = P_2$$

$$T = T_2$$

$$V = V_2 < V_1$$

At (2)

Process is isochoric

$$\therefore V = \text{constant}$$

$$\therefore V = V_2$$

$$\therefore P = P_3 < P_1$$

$$T = 273 \text{ K} = T_1 < T_2$$

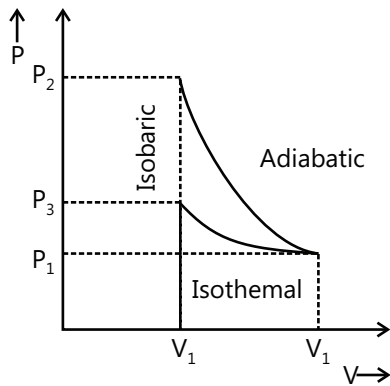
From (2) to (1)

Process is isothermal

$$T = 273 \text{ K} = T_1$$

$$V = V_1$$

$$\therefore P = P_1$$



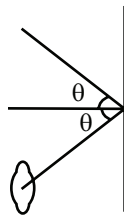
$$\Delta Q_{\text{process}} = 100 \times 80 = 8000 \text{ cal}$$

$$\Delta U_{\text{process}} = 0 \text{ cal}$$

$$\therefore \Delta W + \Delta U = \Delta Q$$

$$\therefore \Delta W = 8000 \text{ cal}$$

Sol 21: Number of molecules hitting 1m^2 of wall per second (N) = $nv \cos \theta \frac{\text{molecules}}{\text{m}^2 \text{ sec}}$



Change in momentum of 1 molecule

$$= m (2v \cos \theta) = 2mv \cos \theta$$

\therefore Pressure exerted = N \times change of momentum of 1 molecule

$$= nv \cos \theta \times 2mv \cos \theta = 2mv^2 \cos^2 \theta$$

Sol 22: (a) Work along path A-D

= Area under curve AD

$$= \frac{1}{2} [P_A + P_D] \times [V_D - V_A]$$

$$= \frac{1}{2} \times 1.6 \times 10^5 \times 1.1 \times 10^{-3} = 88 \text{ J}$$

$$(b) \Delta W_{\text{ADC}} = \Delta W_{\text{AD}} + \Delta W_{\text{DC}} \Rightarrow \Delta W_{\text{DC}} = -3 \text{ J}$$

$$\therefore - \left[\frac{1}{2} [P_D + P_B] \times [V_D - V_C] \right] = -3$$

$$\Rightarrow 0.9 \times 10^5 (1.3 - V_C) \times 10^{-3} = 6$$

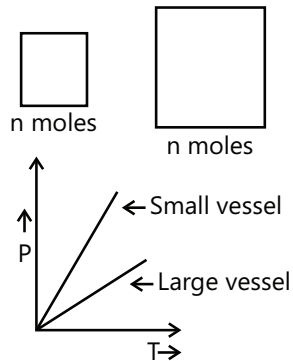
$$\Rightarrow V_C \approx 1.23 \text{ L}$$

$$(c) W_{\text{CDA}} = -W_{\text{ADC}} = -85 \text{ J}$$

Exercise 2

Single Correct Choice Type

Sol 1: (D)



$V = \text{constant}$

and $PV = nRT$

$$\Rightarrow P = \left(\frac{nR}{V} \right) T$$

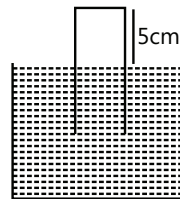
$\therefore P = mT$ where m is a constant

\therefore P-T curve for both vessels will be linear but with different slopes, since the constant volumes have different values in both cases.

Sol 2: (C) Initially

$$P_i = 76 \text{ cm of Hg}$$

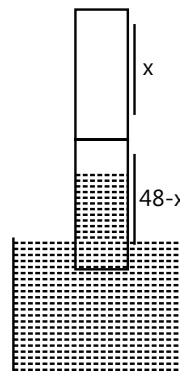
$$V_i = 5A$$



Finally

$$P_f = (76) - (48 - x) = (28 + x) \text{ cm of Hg}$$

$$V_f = xA$$



Since, temperature remains constant

$$\therefore P_i V_i = P_f V_f$$

$$\Rightarrow 76 \times 5A = (28 + x) \times A$$

$$\Rightarrow x^2 + 28x - 380 = 0$$

$$\Rightarrow (x + 38)(x - 10) = 0$$

$$\Rightarrow x = 10 \text{ cm or } x = -38 \text{ cm}$$

$x = -38$ rejected since x can't be -ve

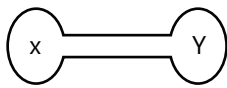
Sol 3: (C)

Container X

Container Y

2V

V



Process (P) in both vessels will be same.

Thus $V \propto nT$

$$\therefore n \propto \frac{V}{T}$$

$$\therefore n_x = k \frac{2V}{200} \text{ [k is some constant]}$$

$$n_y = \frac{kV}{400}$$

$$\therefore \frac{n_x}{n_y} = \frac{4}{1}$$

Since gas in both container is same

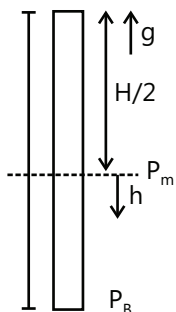
$$\therefore \frac{m_x}{m_y} = \frac{n_x}{n_y} = 4$$

$$\therefore m_y = \frac{m_x}{4} = \frac{m}{4}$$

Sol 4: (C) Pressure gradient will develop due to the upward acceleration so

$$\frac{dP}{dh} = 2\rho g$$

$$\frac{dP}{dh} = \frac{PM}{RT} 2g$$



$$\int_{P_m}^{P_B} \frac{dP}{P} = \int_0^{H/2} \frac{2Mg}{RT} dh$$

$$\lambda n \left(\frac{P_B}{P_m} \right) = \frac{MgH}{RT}$$

$$\frac{P_B}{P_m} = \exp(MgH/RT)$$

Sol 5: (D) Since there is no loss of energy

\therefore Sum of change of internal energies must be zero.

$$\text{i.e. } \Delta U_1 + \Delta U_2 = 0$$

$$\Rightarrow n_1 C_v [T - T_1] + n_2 C_v [T - T_2] = 0$$

$$\Rightarrow (n_1 + n_2) T = n_1 T_1 + n_2 T_2$$

$$\Rightarrow T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

Sol 6: (D) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{\text{rms}}^2 \propto \frac{1}{M}$

$$\therefore \frac{(v_{\text{rms}}^2)_B}{(v_{\text{rms}}^2)_A} = \frac{M_A}{M_B} = \frac{m \times N_A}{2m \times N_A} = \frac{1}{2} \quad \dots(i)$$

Since by the postulate of KTG that, molecules move in random motions.

$$\therefore \langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle \text{ and } v^2 = v_x^2 + v_y^2 + v_z^2$$

$$\therefore \langle v^2 \rangle = 3 \langle v_x^2 \rangle$$

$$\text{Thus } (v_{\text{rms}}^2)_A = 3w^2 \text{ \& } (v_{\text{rms}}^2)_B = w^2$$

$$\text{Thus, by (i) we get: } -\frac{v^2}{3w^2} = \frac{1}{2} \Rightarrow \frac{w^2}{v^2} = \frac{2}{3}$$

Sol 7: (C) $P = 0.7 \times 10^5 \text{ N/m}^2$

$$V = 0.0049 \text{ m}^3$$

$$\gamma = \frac{C_p}{C_v} = 1.4 = \text{ratio of specific heats}$$

$$\frac{C_v}{C_p} \neq 1.4 \text{ since, we know } C_p > C_v$$

In reversible adiabatic process

$$PV^\gamma = \text{constant}$$

$$\therefore V^\gamma \Delta P + P^\gamma V^{\gamma-1} \Delta V = 0$$

$$\Rightarrow \frac{\Delta P}{\Delta V} = \frac{-\gamma P}{V} = -\frac{1.4 \times 0.7 \times 10^5}{0.0049}$$

$$\Rightarrow \frac{dP}{dV} = -2 \times 10^7 \text{ Nm}^{-5}$$

Sol 8: (C) Equal mass of same gas \Rightarrow Equal moles

Initially

(X) n moles $T = 300 \text{ K}$ $P = 1 \text{ atm}$ $V = 42 \text{ A}$	n moles (Y) $T = 300 \text{ K}$ $P = 1 \text{ atm}$ $V = 42 \text{ A}$
42cm	42cm
Area of cross-section = A	

Using gas equation we get

$$PV = nRT$$

$$\Rightarrow 42 A = nR 300$$

$$\Rightarrow \frac{nR}{A} = \frac{42}{300}$$

Finally

(X) n moles $T = 300 \text{ K}$ $P_f = P$ $V = (42+x) A$	n moles $T = 300 \text{ K}$ $P = 1 \text{ atm}$ $V = 42 A$
(42+x)cm	42-x

Using gas equation we get

$$P(42+x)A = nR 330$$

$$P(42-x)A = nR 300$$

(ii)/(iii) gives

$$\frac{42+x}{42-x} = \frac{330}{300}$$

$$\Rightarrow \frac{2x}{42-x} = \frac{30}{300}$$

$$\Rightarrow 20x = 42 - x$$

$$\Rightarrow x = 2 \text{ cm}$$

Sol 9: (C) At constant pressure

$$W = P\Delta V$$

$$\text{For monoatomic: } -C_p = \frac{5}{2}R$$

$$\text{and } Q = nC_p \Delta T$$

$$\Rightarrow Q = n \times \frac{5R}{2} \times \frac{P\Delta V}{nR} = \frac{5}{2}W$$

$$\therefore W = \frac{2}{5}Q$$

Multiple Correct Choice Type

Sol 10: (A, B) $e_A = 0.01$ and $e_B = 0.81$

$$A_A = A_B$$

$$E_A = E_B$$

$$\Rightarrow e_A \sigma A_A T_A^4 = e_B \sigma A_B T_B^4$$

$$\Rightarrow 0.01 T_A^4 = 0.81 T_B^4$$

$$\Rightarrow T_B = \frac{1}{3} \times T_A$$

$$\Rightarrow T_B = \frac{1}{3} \times 5802 = 1934 \text{ K}$$

By Wien's displacement law

$$\lambda_m T = \text{constant} = 2.93 \times 10^{-3} \text{ mK}$$

$$\therefore \lambda_{m_A} = 0.5 \mu\text{m}$$

Since, it is given in the question that

$$\lambda_{m_B} = 1 \mu\text{m} + \lambda_{m_A}$$

$$\therefore \lambda_{m_B} = 1.5 \mu\text{m}$$

Sol 11: (A, D) $P^2 V = \text{const.}$

$$\therefore P_f = \left(\frac{V_i}{V_f}\right)^{1/2} \times P_i = \left(\frac{V}{4V}\right)^{1/2} P = P/2$$

$$P^2 \left(\frac{nRT}{P}\right) = \text{constant}$$

$$\Rightarrow PT = \text{constant}$$

$\therefore P - T$ curve is hyperbola

$$\text{and } T_f = \left(\frac{P_i}{P_f}\right) T_i = 2T$$

Sol 12: (C, D) $P = 3000 \text{ J}$

$$V = 5 \ell$$

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M_m}}$$

$$\Rightarrow \frac{1}{2} (nM_m) v_{\text{rms}}^2 = \frac{3}{2} nRT = \frac{3}{2} PV$$

$$\text{Translational K.E. of all molecules} = \frac{3}{2} PV$$

$$\therefore \frac{3}{2} P \times 5 \ell = 3000 \text{ J}$$

$$\therefore P = 400 \text{ J/l}$$

Total K.E. of 10 ℓ of gas at Pressure 2P for a monoatomic

$$\text{gas} = \frac{3}{2} \times 800 \times 10 = 12000 \text{ J}$$

Total K.E. of 10 ℓ, 2P pressure for a diatomic gas = $\frac{5}{2} PV$

$$= \frac{5}{2} \times 800 \times 10 = 2000 \text{ Joules}$$

Note: In options its asked total K.E. not translational K.E.

Sol 13: (A, D) No. of molecules moving towards wall

$$\text{per unit volume} = \frac{1}{6} \times 10^{26} \frac{\text{molecules}}{\text{m}^3}$$

∴ No. of molecules hitting 1m² of the wall every sec (n)

$$= \left(\frac{1}{6} \times 10^{26} \right) \times (2000)$$

$$= 3.33 \times 10^{28} \frac{\text{molecules}}{\text{m}^2 \text{ sec}}$$

Pressure exerted = (change in momentum) × no. of molecules putting wall per unit area per sec

$$= (3 \times 10^{-27} \text{ kg/molecules}) \times (2 \times 2000 \text{ m/s}) \times 3.33 \times 10^{28} \text{ molecules/m}^2 \text{ sec}$$

$$\approx 4 \times 10^5 \text{ Pa}$$

Sol 14: (A, B, C) Isothermal process Adiabatic process

$$PV = \text{constant} \quad PV^\gamma = \text{constant}$$

$$\Rightarrow \frac{T}{V} = \text{constant} \quad \Rightarrow TV^{\gamma-1} = \text{constant}$$

$$\therefore T_f = \frac{V_i}{V_f} \times T_i \quad \Rightarrow T_f = \left(\frac{V_i}{V_f} \right)^{\gamma-1} T_i$$

(Since there is expansion)

$$\therefore \frac{V_f}{V_i} > 1 \quad \Rightarrow \frac{V_i}{V_f} < 1$$

$$\text{Thus} \left(\frac{V_i}{V_f} \right)^{\gamma-1} < 1$$

Since $\gamma-1$ is +ve

(∴ Isothermal temp. > adiabatic temp.)

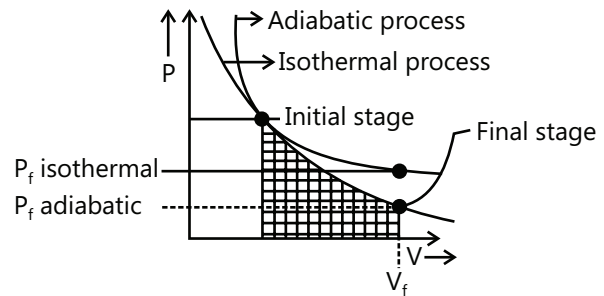
$$P_f = \frac{V_i}{V_f} \times P_i \quad P_f = \left(\frac{V_i}{V_f} \right)^\gamma \times P_i$$

(Since $\gamma > 1$ and $\frac{V_i}{V_f} < 1$)

$$\therefore \frac{V_i}{V_f} > \left(\frac{V_i}{V_f} \right)^\gamma$$

∴ Isothermal pressure > Adiabatic pressure

Alternative method:



From graph, it is very clear, that

$$P_{f \text{ isothermal}} > P_{f \text{ adiabatic}}$$

and for same volume and moles of gas

$$T \propto P \quad [\because PV = nRT]$$

$$\text{Thus, } T_{f \text{ isothermal}} > T_{f \text{ adiabatic}}$$

$$\text{Work done} = \int_{V_i}^{V_f} P dV = \text{area under P-V diag}$$

$$\text{Thus, (work done)}_{\text{isothermal}} > (\text{work done})_{\text{adiabatic}}$$

Sol 15: (A, B) Area under the curve gives the rate at which heat per unit surface is radiated by the body.

i.e. total rate of heat radiation = (Area under the curve) × (Surface area of the body)

$$\text{Sol 16: (A, C)} \quad \rho = \frac{m}{V}$$

$$\Rightarrow \rho \times \frac{4}{3} \pi r^3 = m$$

$$\Rightarrow r \propto (m)^{1/3}$$

And area of sphere (A) $\propto r^2$

$$\therefore A \propto (m)^{2/3}$$

$$\therefore \frac{A_A}{A_B} = (4)^{2/3} = (2)^{4/3}$$

$$\therefore \text{Ratio of heat loss} = \frac{eA_A \sigma (T - T_0)^4}{eA_B \sigma (T - T_0)^4}$$

$$= \frac{A_A}{A_B} = (2)^{4/3}$$

By Newton's law of cooling:

$$\frac{dQ}{dt} = ms(\text{COOH})_2 = -k(T - T_0)$$

$$\Rightarrow \frac{dT}{dt} = \frac{-k}{ms} (T - T_0)$$

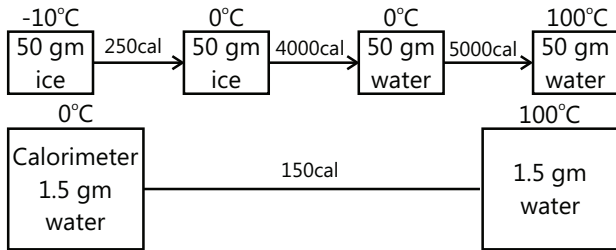
Where $k = 4e A \sigma T_0^3$

$$\therefore \frac{dT}{dt} \propto \frac{A}{m}$$

$$\therefore \left(\frac{dT}{dt}\right)_A = \frac{A_A}{m_A} = \frac{(2)^{4/3}}{4} = 2^{-2/3}$$

$$\left(\frac{dT}{dt}\right)_B = \frac{A_B}{m_B}$$

Sol 17: (A, C)



\therefore Heat absorbed by ice and calorimeter to reach 100°C water

$$= 250 + 4000 + 5000 + 150 = 9400 \text{ cal}$$

\therefore Amount of steam converted into water

$$= \frac{9400}{540} = 17.4 \text{ gm}$$

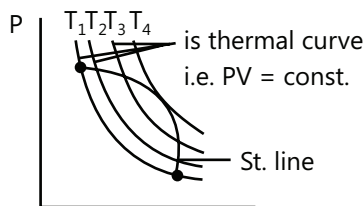
\therefore Amount of water remaining = 50 + 17.40 = 67.4 gm

\therefore Amount of steam remaining = 20 - 17.4 = 2.6 gm

Sol 18: (B, D) $\Delta T = 0$

The slope of straight line can't be +ve. Since, $T \propto PV$ and if slope is +ve, then both P and V are increasing. Therefore, temperature will always increase.

$$T_1 < T_2 < T_3 < T_4$$



Thus, from graph it can be seen, that temperature first increases and then decrease.

Sol 19: (A, B) Process AB is isochoric: - i.e. P = constant

$$PV = nRT$$

$$\Rightarrow V = \left(\frac{nR}{P}\right)T$$

\therefore V is a linear function of T with slope $\frac{nR}{P}$ and passing through origin in V-T curve.

Secondly for process CD: P-T will be a linear curve passing through origin.

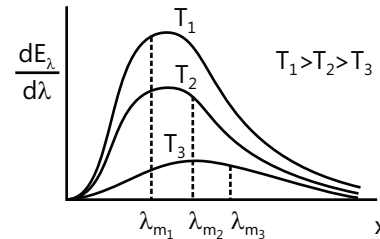
Comprehension Type

Paragraph 1:

Sol 20: (A, B) Area under the curve gives the rate at which heat per unit surface is radiated by the body.

i.e. total rate of heat radiation = (Area under the curve) \times (Surface area of the body)

Sol 21: (A, B, C, D) $\lambda_m T = \text{constant}$ [By Wien's Displacement Law]



$$\text{Area under graph} = E_\lambda = e \sigma T^4 \propto T^4$$

Paragraph 2:

Sol 22: (A)

$A_A = A$	$A_B = A$
$l_A = l$	$l_B = l$
$k_A = 3k$	$k_B = k$

$$\therefore R_A = \frac{l_A}{k_A A_A} = \frac{l}{3kA} \quad R_B = \frac{l_B}{k_B A_B} = \frac{l}{kA}$$

$$\therefore \frac{R_A}{R_B} = \frac{1}{3}$$

Sol 23: (B) Rate at which heat flows from A

= Rate at which heat flows from B

$$\Rightarrow \left(\frac{dQ}{dt}\right)_A = \left(\frac{dQ}{dt}\right)_B$$

$$\Rightarrow \frac{T_A}{R_A} = \frac{T_B}{R_B} \Rightarrow \frac{T_A}{T_B} = \frac{R_A}{R_B} = \frac{1}{3}$$

$$\text{Sol 24: (B)} \quad G_A = \frac{T_A}{\ell_A} = \frac{T_A}{\ell} \quad \text{and} \quad G_B = \frac{T_B}{\ell_B} = \frac{T_B}{\ell}$$

$$\therefore \frac{G}{G_B} = \frac{T_A}{T_B} = \frac{1}{3}$$

Paragraph 3:

$$\text{Sol 25: (B)} \quad \left(\frac{dQ}{dt}\right)_{\text{initially}} = kAV \Delta T$$

$$\left(\frac{dQ}{dt}\right)_{\text{finally}} = kA(4V) \left(\frac{\Delta T}{2}\right) = 2 \left(\frac{dQ}{dt}\right)_{\text{initially}}$$

Sol 26: (A) If all the parameters are kept constant then

$$\frac{dQ}{dt} = ms \frac{dT}{dt} = kA v \Delta T$$

$$\therefore \frac{dT}{dt} = \frac{kAv\Delta T}{ms}$$

Paragraph 4:

Sol 27: (A) w_{ab} = Area below curve AB

$$= \frac{1}{2} [10 + 20] \times (12 - 6)$$

$$= \frac{1}{2} \times 30 \times 6 = 90 \text{ J}$$

Sol 28: (B) w_{BC} = -Area below curve BC

$$= -10 [12 - 6] = -60 \text{ J}$$

Note:- w_{BC} is -ve, since volume is decreasing

Sol 29: (D) w_{CA} = 0 since volume is constant

$$\therefore w_{ABCA} = w_{AB} + w_{BC} + w_{CA} = (90) + (-60) + 0 = 30 \text{ J}$$

Paragraph 5:

$$n_{\text{He}} = 5 \text{ moles} \quad n_{\text{H}_2} = 2 \text{ moles}$$

$$M_{\text{He}} = 4\text{g} \quad M_{\text{H}_2} = 2\text{g}$$

$$\text{Sol 30: (D)} \quad \text{Equivalent molar mass} = \frac{n_{\text{He}} M_{\text{He}} + n_{\text{H}_2} M_{\text{H}_2}}{n_{\text{He}} + n_{\text{H}_2}}$$

$$= \frac{5 \times 4 + 2 \times 2}{5 + 2} = \frac{24}{7} \text{ gm}$$

Sol 31: (A) $f_{\text{H}_2} = 5$ and $f_{\text{He}} = 3$

$$\therefore f_{\text{mix}} = \frac{n_{\text{H}_2} f_{\text{H}_2} + n_{\text{He}} f_{\text{He}}}{n_{\text{H}_2} + n_{\text{He}}}$$

$$= \frac{2 \times 5 + 5 \times 3}{2 + 5} = \frac{25}{7} = 3.57$$

Sol 32: (C) $\therefore r_{\text{mix}} = 1 + \frac{2}{f_{\text{mix}}} = 1 + \frac{2}{25/7} = 1.56$

Sol 33: (D) Since internal energy is an extensive property

$$\therefore U_{\text{mix}} = U_{\text{He}} + U_{\text{H}_2} = 200 + 100 = 300 \text{ J}$$

Match the Columns

Sol 34: (A) $V_i = V$ $V_f = 2V$

$$P_i = P$$

(P) \rightarrow (y) isobaric process

$$W = P\Delta V = PV$$

(q) \rightarrow (z) isothermal

$$W = nRT \ln \frac{V_f}{V_i} = PV \ln 2$$

(r) \rightarrow (x) Adiabatic

$$P_f = \left(\frac{V_i}{V_f}\right)^\gamma \quad P_i = 2^\gamma P$$

$$W = \frac{P_f V_f - P_i V_i}{1 - \gamma} = \frac{(2^{-\gamma} P)(2V) - PV}{1 - \gamma}$$

$$\Rightarrow W = \frac{PV(1 - 2^{1-\gamma})}{\gamma - 1}$$

Previous Years' Questions

Sol 1: (D) The desired fraction is

$$f = \frac{\Delta U}{\Delta Q} = \frac{nC_V \Delta T}{nC_p \Delta T} = \frac{C_V}{C_p} = \frac{1}{\gamma}$$

$$\text{or } f = \frac{5}{7} \left(\text{as } \gamma = \frac{7}{5} \right)$$

Sol 2: (A) Average kinetic energy per molecule per degree of freedom = $\frac{1}{2}kT$. Since, both the gases are diatomic and at same temperature (300 K), both will have the same number of rotational degree of freedom i.e., two. Therefore, both the gases will have the same average rotational kinetic energy per molecule

$$= 2 \times \frac{1}{2}kT \text{ or } kT$$

Thus, ratio will be 1: 1.

Sol 3: (D) A is free to move, therefore, heat will be supplied at constant pressure

$$\therefore dQ_A = nC_p dT_A \quad \dots (i)$$

B is held fixed, therefore, heat will be supplied at constant volume.

$$\therefore dQ_B = nC_v dT_B \quad \dots (ii)$$

But $dQ_A = dQ_B$ (given)

$$nC_p \Delta T_A = nC_v \Delta T_B \therefore dT_B = \left(\frac{C_p}{C_v} \right) dT_A$$

$$= \gamma(dT_A) \quad [\gamma = 1.4 \text{ (diatomic)}]$$

$$(dT_A = 30 \text{ K})$$

$$= (1.4)(30 \text{ K})$$

$$\therefore dT_B = 42 \text{ K}$$

Sol 4: (C) Speed of sound in an ideal gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\therefore v \propto \sqrt{\frac{\gamma}{M}} \quad (T \text{ is same for both the gases})$$

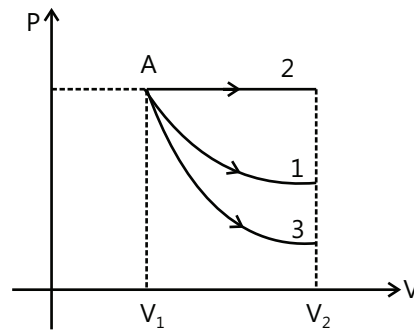
$$\therefore \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{\gamma_{N_2} \cdot M_{He}}{\gamma_{He} \cdot M_{N_2}}} = \sqrt{\frac{7/5 \left(\frac{4}{28} \right)}{5/3}} = \sqrt{3}/5$$

$$\gamma_{N_2} = \frac{7}{5} \text{ (Diatomic)}$$

$$\gamma_{He} = \frac{5}{3} \text{ (Monoatomic)}$$

Sol 5: (A) The corresponding p–V graphs (also called indicator diagram) in three different processes will be as shown:

Area under the graph gives the work done by the gas



$$(\text{Area})_2 > (\text{Area})_1 > (\text{Area})_3$$

$$\therefore W_2 > W_1 > W_3$$

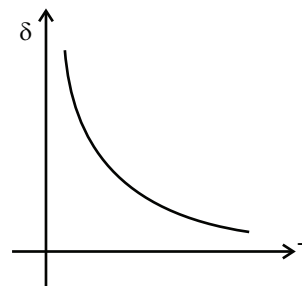
Sol 6: (C) For an ideal gas : $PV = nRT$

For $P = \text{constant}$

$$P\Delta V = nR\Delta T$$

$$\therefore \frac{\Delta V}{\Delta T} = \frac{nR}{p} = \frac{nR}{\frac{nRT}{V}} = \frac{V}{T}$$

$$\therefore \frac{\Delta V}{V\Delta T} = \frac{1}{T} \text{ or } \delta = \frac{1}{T}$$



Therefore, δ is inversely proportional to temperature T . i.e., when T increases, δ decreases and vice-versa.

Hence, δ - T graph will be a rectangular hyperbola as shown in the above figure.

Sol 7: (B) In adiabatic process

$$\text{Slope of P-V graph, } \frac{dP}{dV} = -\gamma \frac{P}{V}$$

Slope $\propto \gamma$ (with negative sign)

From the given graph,

$$(\text{slope})_2 > (\text{slope})_1$$

$$\therefore \gamma_2 > \gamma_1$$

Therefore, 1 should correspond to O_2 ($\gamma = 1.4$) and 2 should correspond to He ($\gamma = 1.67$).

Sol 8: (A) $\Delta W_{AB} = P\Delta V = (10)(2 - 1) = 10 \text{ J}$

$\Delta W_{BC} = 0$ (as $V = \text{constant}$)

From first law of thermodynamics

$$\Delta Q = \Delta W + \Delta U$$

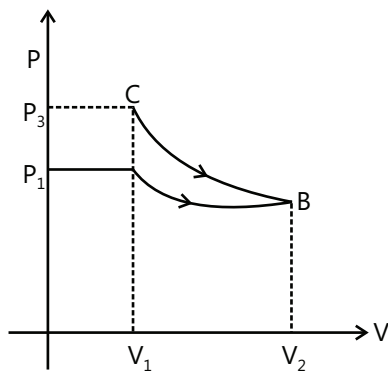
$\Delta U = 0$ (process ABCA is cyclic)

$$\therefore \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$$

$$\begin{aligned} \therefore \Delta W_{CA} &= \Delta Q - \Delta W_{AB} - \Delta W_{BC} \\ &= 5 - 10 - 0 = -5 \text{ J} \end{aligned}$$

Sol 9: (C) Temperature of liquid oxygen will first increase in the same phase. Then, phase change (liquid to gas) will take place. During which temperature will remain constant. After that temperature of oxygen in gaseous state will further increase.

Sol 10: (C) Slope of adiabatic process at a given state (P, V, T) is more than the slope of isothermal process. The corresponding P - V graph for the two processes is as shown in figure.



In the graph, AB is isothermal and BC is adiabatic.

$W_{AB} = \text{positive}$ (as volume is increasing)

and $W_{BC} = \text{negative}$ (as volume is decreasing) plus,

$|W_{BC}| > |W_{AB}|$, as area under p - V graph gives the work done.

Hence, $W_{AB} + W_{BC} = W < 0$

From the graph itself, it is clear that $P_3 > P_1$

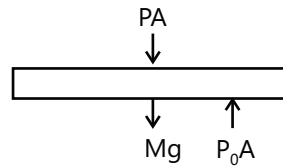
Note: At point B, slope of adiabatic (process BC) is greater than the slope of isothermal (process AB)

Sol 11: (B) Total translational kinetic energy

$$= \frac{3}{2} nRT = \frac{3}{2} PV = 1.5 PV$$

Sol 12: (A) Since it is open from top. Pressure will be P_0 .

Sol 13: (D) Let p be the pressure in equilibrium.



Then, $P_A = P_0A - Mg$

$$P = P_0 - \frac{Mg}{A} = p_0 - \frac{Mg}{\pi R^2}$$

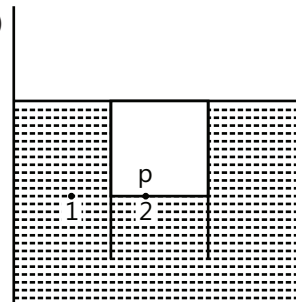
Applying, $P_1V_1 = P_2V_2$

$$\therefore P_0(2AL) = (P)(AL')$$

$$\therefore L' = \frac{2P_0L}{P} = \left(\frac{P_0}{P_0 - \frac{Mg}{\pi R^2}} \right) (2L)$$

$$= \left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg} \right) (2L)$$

Sol 14: (C)



$$p_1 = p_2$$

$$P_0 + \rho g(L_0 - H) = P \quad \dots\dots(i)$$

Now, applying $P_1V_1 = P_2V_2$ for the air inside the cylinder, we have

$$P_0(L_0) = P(L_0 - H)$$

$$\therefore p = \frac{P_0L_0}{L_0 - H}$$

Substituting in Equation (i), we have

$$P_0 + \rho g(L_0 - H) = \frac{P_0L_0}{L_0 - H}$$

$$\text{or } \rho g(L_0 - H)^2 + P_0(L_0 - H) - P_0L_0 = 0$$

Sol 15: (B, D) In case of free expansion under adiabatic conditions, change in internal energy $\Delta U = 0$

\therefore Internal energy and temperature will remain constant.

$$(B) P \propto \frac{1}{V^2}$$

$$\therefore PV^2 = \text{constant} \quad \dots\dots\dots (i)$$

$$\text{or} \left(\frac{nRT}{V} \right) V^2 = \text{constant}$$

$$\therefore T \propto \frac{1}{V} \quad \dots\dots\dots (ii)$$

If volume is doubled, temperature will decrease as per equation (ii).

Further, molar heat capacity in process $PV^x = \text{constant}$ is

$$C = C_v + \frac{R}{1-x}$$

From equation (i), $x = 2$

$$\therefore C = \frac{3}{2}R + \frac{R}{1-2} = -\frac{R}{2}$$

Since, molar heat capacity is positive, according to $Q = nC\Delta T$, Q will be negative if ΔT is negative. Or gas loses heat if temperature is decreasing.

$$(C) P \propto \frac{1}{V^{4/3}}$$

$$PV^{4/3} = \text{constant}$$

$$\therefore \left(\frac{nRT}{V} \right) V^{4/3} = \text{constant}$$

$$\therefore T \propto \frac{1}{V^{1/3}}$$

Further, with increase in volume temperature will decrease.

$$\text{Here, } x = \frac{4}{3}$$

$$\therefore C = \frac{3}{2}R + \frac{R}{1-\frac{4}{3}} = -1.5R$$

As molar heat capacity is negative, Q will be positive if ΔT is negative. Or gas gains heat with decrease in temperature.

$$(D) T \propto PV$$

In expansion from V_1 to $2V_1$, product of PV is increasing. Therefore, temperature will increase. Or $\Delta U = +ve$. Further, in expansion work done is also positive.

Hence, $Q = W + \Delta U = +ve$ or, gas gains heat.

Sol 16: (B, D) For monoatomic gas

$$C_v = \frac{3}{2}R, C_p = \frac{5}{2}R$$

$$\text{For diatomic gas, } C_v = \frac{5}{2}R, C_p = \frac{7}{2}R$$

Sol 17: A \rightarrow q; **B** \rightarrow p, r; **C** \rightarrow p, s; **D** \rightarrow q, s

(A) P-V graph is not rectangular hyperbola. Therefore, process A – B is not isothermal.

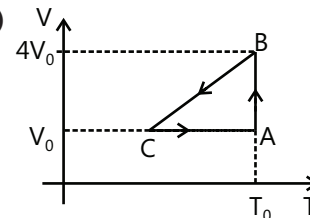
(B) In process BCD, product of PV (therefore temperature and internal energy) is decreasing. Further, volume is decreasing. Hence, work done is also negative. Hence, Q will be negative or heat will flow out of the gas.

(C) $W_{ABC} = \text{positive}$

(D) For clockwise cycle on p- v diagram with P on y-axis, net work done is positive.

Sol 18: (D) At low pressure and high temperature inter molecular forces become ineffective. So a real gas behaves like an ideal gas.

Sol 19: (A, B)



$$\therefore U = \frac{f}{2}nRT$$

Where f , n , R are constants. Also temperature T is same at A and B.

$$\therefore U_A = U_B$$

Also,

$$\Delta W_{AB} = nRT_0 \ln \left(\frac{V_f}{V_i} \right) = nRT_0 \ln \frac{4V_0}{V_0} = nRT_0 \ln 4 = P_0 V_0 \ln 4$$

Sol 20: (A) $TV^{\lambda-1} = C$

$$T_1(5.6)^{2/3} = T_2(0.7)^{2/3} \Rightarrow T_2 = T_1(8)^{2/3} = 4T_1$$

$$\therefore \Delta w(\text{work done on the system}) = \frac{nR\Delta T}{\gamma-1} = \frac{9}{8}RT_1$$

$$\text{Sol 21: (D)} \quad V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\text{Required ratio} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$$

$$\text{Sol 22: (D)} \quad PV = nRT = \frac{m}{M} RT$$

$$\Rightarrow PM = \rho RT$$

$$\frac{\rho_1}{\rho_2} = \frac{P_1 M_1}{P_2 M_2} = \left(\frac{P_1}{P_2}\right) \times \left(\frac{M_1}{M_2}\right) = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$

Here ρ_1 and ρ_2 are the densities of gases in the vessel containing the mixture.

$$\text{Sol 23: (2)} \quad U_b = 200 \text{ J}, U_i = 100 \text{ J}$$

Process iaf

Process	W (in Joule)	ΔU (in Joule)	Q (in Joule)
ia		0	
af		200	
Net	300	200	500

$$\Rightarrow U_f = 400 \text{ Joule}$$

Process ibf

Process	W (in Joule)	ΔU (in Joule)	Q (in Joule)
ia	100	50	150
af	200	100	300
Net	300	150	450

$$\Rightarrow \frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$$

$$\text{Sol 24: (A, B, D)} \quad U = nC_{V1}T + nC_{V2}T$$

$$= 1 \times \frac{5}{2}RT + 1 \times \frac{3}{2}RT = 4RT$$

$$\Rightarrow 2C_{V_{\text{mix}}}T = 4RT$$

$$\text{Average energy per mole} = 2RT \Rightarrow C_{V_{\text{mix}}} = 2R$$

$$\frac{C_{\text{mix}}}{C_{\text{He}}} = \sqrt{\left(\frac{\lambda_{\text{mix}}}{\lambda_{\text{He}}}\right) \left(\frac{M_{\text{He}}}{M_{\text{mix}}}\right)} = \sqrt{\frac{3}{2} \times \frac{3}{5} \times \frac{4}{3}} = \sqrt{\frac{6}{5}}$$

$$\frac{V_{\text{rmsHe}}}{V_{\text{rmsH}_2}} = \sqrt{\frac{M_{\text{H}_2}}{M_{\text{He}}}} = \frac{1}{\sqrt{2}}$$

$$\text{Sol 25: (B)} \quad \text{Rate of heat generated } \frac{dQ}{dt} = 3kW$$

Let at any time 't', temperature of cooler = T

Rate of cooling:

$$ms \frac{dT}{dt} = 3kW - P$$

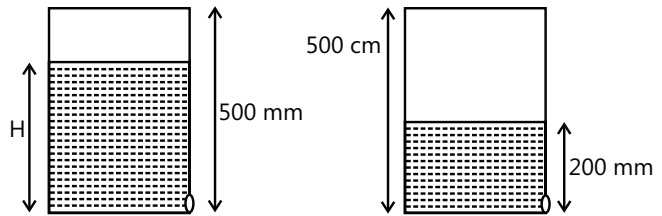
$$\int_{10}^{30} dT = \frac{(3kW - P)}{ms} \int_0^3 dt$$

$$30 - 10 = \frac{(3kW - P) \times 3 \times 3600}{120 \times 4.2 \times 10^3}$$

$$3kW - P = \frac{20 \times 120 \times 42}{3 \times 36} = \frac{2800}{3}$$

$$P = 3000 - 933 = 2067 \text{ W}$$

$$\text{Sol 26: (6)} \quad P = P_0 - \rho gh = 98 \times 10^3 \text{ N/m}^2$$



$$P_0 V_0 = PV$$

$$10^5 [A(500 - H)] = 98 \times 10^3 [A(500 - 200)]$$

$$H = 206 \text{ mm}$$

$$\text{Level fall} = 206 - 200 = 6 \text{ mm}$$

$$\text{Sol 27: (4)} \quad TV^{\gamma-1} = \text{constant}$$

$$TV^{7/5-1} = aT \left(\frac{V}{32}\right)^{7/5-1}$$

$$\therefore a = 4$$

$$\text{Sol 28: A} \rightarrow p, r, t; \text{ B} \rightarrow p, r; \text{ C} \rightarrow q, s; \text{ D} \rightarrow r, t$$

Process A \rightarrow B \rightarrow Isobaric compression

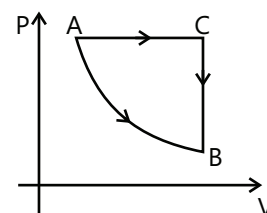
Process B \rightarrow C \rightarrow Isochoric process

Process C \rightarrow D \rightarrow Isobaric expansion

Process D \rightarrow A \rightarrow Polytropic with $T_A = T_D$

Sol 29: (A, B, C, D) Option (A) is correct because the graph between (0 - 100 K) appears to be a straight line upto a reasonable approximation.

Option (B) is correct because area under the curve in the temperature range (0 - 100 K) is less than in range (400 - 500 K.)



Option (C) is correct because the graph of C versus T is constant in the temperature range (400 - 500 K)

Option (D) is correct because in the temperature range (200 - 300 K) specific heat capacity increases with temperature.

Sol 30: (C) By $A_1V_1 = A_2V_2$

$$\Rightarrow \pi(20)^2 \times 5 = \pi(1)^2 V_2 \Rightarrow V_2 = 200 \text{ m}^3$$

Sol 31: (A) $\frac{1}{2}\rho_a V_a^2 = \frac{1}{2}\rho_\ell V_\ell^2$

For given V_a

$$V_\ell \propto \sqrt{\frac{\rho_a}{\rho_\ell}}$$

Sol 32: (B or A, B, C) P (pressure of gas) = $P_1 + \frac{kx}{A}$

$$W = \int P dV = P_1(V_2 - V_1) + \frac{kx^2}{2} = P_1(V_2 - V_1) + \frac{(P_2 - P_1)(V_2 - V_1)}{2}$$

$$\Delta U = nC_V \Delta T = \frac{3}{2}(P_2 V_2 - P_1 V_1)$$

$$Q = W + \Delta U$$

$$\text{Case I: } \Delta U = 3P_1 V_1, W = \frac{5P_1 V_1}{4}, Q = \frac{17P_1 V_1}{4}, U_{\text{spring}} = \frac{P_1 V_1}{4}$$

$$\text{Case II: } \Delta U = \frac{9P_1 V_1}{2}, W = \frac{7P_1 V_1}{3}, Q = \frac{41P_1 V_1}{6}, U_{\text{spring}} = \frac{P_1 V_1}{3}$$

Note: A and C will be true after assuming pressure to the right of piston has constant value P_1 .

Sol 33: (C) For adiabatic process

$$P^3 V^5 = \text{constant}$$

$$P V^{\frac{5}{3}} = \text{constant}$$

$$\gamma = \frac{5}{3} \text{ gas is monoatomic}$$

For process AC

$$\Delta Q_1 = n C_p \Delta T = n \left(\frac{5}{2} R \right) \Delta T = \frac{5}{2} P \Delta V$$

$$\Delta Q_1 = \frac{5}{2} \times 10^5 \times (8 - 1) \times 10^{-3}$$

$$\Delta Q_1 = 17.5 \times 10^2 \text{ J} = 1750 \text{ J}$$

For process CD

$$\Delta Q_2 = n C_V \Delta T = n \left(\frac{3}{2} R \right) \Delta T = \frac{3}{2} V (\Delta P)$$

$$\Delta Q_2 = \frac{3}{2} \times 8 \times 10^{-3} \times \left(-1 + \frac{1}{32} \right) \times 10^5$$

$$\Delta Q_2 = \frac{-93}{8} \times 10^2 = -11.625 \times 10^2$$

$$\Delta Q_{\text{net}} = 1750 - 1162 = 588 \text{ J}$$