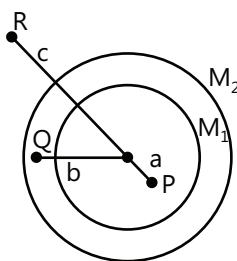


Solved Examples

JEE Main/Boards

Example 1: Two concentric shells of mass M_1 and M_2 are as shown. Calculate the gravitational force on m due to M_1 at points P, Q and R.

Sol: For a particle of mass m , lying at a distance r from the center of the spherical shell of mass M , the gravitational force of attraction is $\left(\frac{GMm}{r^2}\right)$. If the particle is lying inside the spherical shell then the force of gravitation on it is zero.



At P, $F = 0$ At Q, $F = \frac{GM_1m}{b^2}$

At R, $F = \frac{G(M_1 + M_2)m}{c^2}$

Example 2: Find the potential energy of gravitational interaction of a point mass m and a thin uniform rod

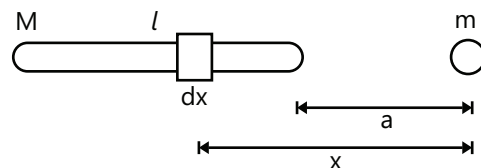
of mass M and length l , if they are located along a straight line at a distance a from each other.

Sol: The gravitational potential energy is given by $U = \frac{Gm_1m_2}{r}$ where m_1 and m_2 are point masses.

Consider the gravitational potential energy of interaction between the point mass m and an infinitesimal element of the rod of mass dm . The total potential energy will be the summation of energy of interaction of all the small elements.

Consider small element dx of the rod whose mass

$$dm = \frac{M}{l}dx$$



$$\Rightarrow dU = -\frac{Gm\left(\frac{M}{l}dx\right)}{x}$$

$$\Rightarrow U = \int dU = -\frac{GmM}{l} \int_a^{a+l} \frac{dx}{x} = -\frac{GmM}{l} [\ln x]_a^{a+l}$$

$$\Rightarrow U = -\frac{GmM}{l} \log_e \left(\frac{a+l}{a}\right)$$

Example 3: If the radius of the earth contracts to half of its present value without change in its mass, what will be the new duration of the day?

Sol: The angular momentum of the earth is given by

$$L = I\omega = \frac{2}{5}MR^2\omega \text{ since earth is considered to be sphere}$$

of uniform mass density. As there is no external force is acting on the earth, the angular momentum of the earth must remain constant after the radius of earth reduces to half of its original size. The time period of

$$\text{revolution is } T = \frac{2\pi}{\omega}.$$

$$\text{Present angular momentum of earth } L_1 = I\omega = \frac{2}{5}MR^2\omega$$

New angular momentum because of change in radius

$$L_2 = \frac{2}{5}M\left(\frac{R}{2}\right)^2\omega'$$

If external torque is zero then angular momentum must be conserved

$$L_1 = L_2$$

$$\frac{2}{5}MR^2\omega = \frac{1}{4} \times \frac{2}{5}MR^2\omega' \text{ i.e., } \omega' = 4\omega$$

$$T' = \frac{1}{4}T = \frac{1}{4} \times 24 = 6h$$

Example 4: Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle.

Sol: As the particles go around the circle they always remain diametrically opposite to each other. To sustain their respective circular motion the necessary centripetal acceleration is provided by the gravitation force of attraction between them.

The particles will always remain diametrically opposite so that the force on each particle will be directed along the radius. Consider the motion of one of the particles.

The force on the particle is $F = \frac{Gm^2}{4R^2}$. If the speed is v , its acceleration is v^2/R .

Thus, by Newton's law,

$$\frac{Gm^2}{4R^2} = \frac{mv^2}{R} \quad \text{Or, } v = \sqrt{\frac{Gm}{4R}}$$

Example 5: Three particles A, B and C, each of mass m , are placed in a line with $AB=BC=d$. Find the gravitational force on a fourth particle P of same mass, placed at a distance d from the particle B on the perpendicular bisector of the line AC.

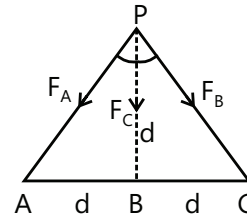
Sol: The gravitational force acting on the particle P due

to each of other particles is given by $F = \frac{Gm^2}{(r)^2}$ where

r is the separation between P and the other particle. As the force is vector quantity the resultant force on particle P has to be found by vector addition.

The force at P due to A is

$$F_A = \frac{Gm^2}{(AP)^2} = \frac{Gm^2}{2d^2} \text{ along PA. The force at P due to C is}$$



$$F_C = \frac{Gm^2}{(CP)^2} = \frac{Gm^2}{2d^2} \text{ along PC. The force at P due to B is}$$

$$F_B = \frac{Gm^2}{d^2} \text{ along PB}$$

The resultant of F_A , F_B and F_C will be along PB. Clearly $\angle APB = \angle BPC = 45^\circ$

$$\text{Components of } F_A \text{ along PB} = F_A \cos 45^\circ = \frac{Gm^2}{2\sqrt{2}d^2}$$

$$\text{Component of } F_C \text{ along PB} = F_C \cos 45^\circ = \frac{Gm^2}{2\sqrt{2}d^2}$$

$$\text{Component of } F_B \text{ along PB} = \frac{Gm^2}{d^2}$$

Hence, the resultant of the three forces is

$$\frac{Gm^2}{d^2} \left(\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + 1 \right) = \frac{Gm^2}{d^2} \left(1 + \frac{1}{\sqrt{2}} \right) \text{ along PB.}$$

Example 6: What is the fractional decrease in the value of free-fall acceleration g for a particle when it is lifted from the surface to an elevation h ? ($h < R$)

Sol: The gravitational acceleration g at height h is given by $g = \frac{GM}{(R+h)^2}$. As here $R \gg h$ then $g \approx \frac{GM}{R^2}$. The

fractional decrease in g at height h above the surface of the earth is given by $\frac{\Delta g}{g}$.

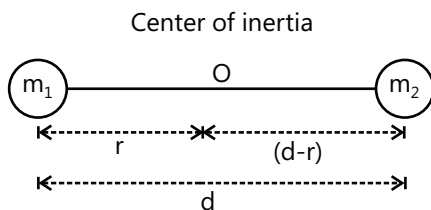
The acceleration due to gravity is $g = \frac{GM}{R^2}$

$$\therefore \frac{\Delta g}{\Delta R} = \frac{-2GM}{R^3} \quad (\text{Differentiating})$$

$$\Rightarrow \frac{dg}{h} = \frac{-2GM}{R^2} \frac{1}{R} \Rightarrow \frac{dg}{g} = -2 \left(\frac{h}{R} \right)$$

Example 7: A double star is a system of two stars moving around the center of inertia of the system due to gravitation. Find the distance between the components of the double star, if its total mass equals M and period of revolution is T .

Sol: Each star is moving in circular orbit whose center is at the combined center of inertia. Find the radius of orbit of one of the stars in terms of the separation between them and find the orbital velocity of the star in terms of d .



The situation is shown in the above figure

$$\text{Here } m_1 r = m_2 (d - r)$$

$$\therefore (m_1 + m_2) r = m_2 d$$

$$r = \frac{m_2 d}{(m_1 + m_2)}$$

$$\text{Also } M = (m_1 + m_2)$$

As gravitational force provides the necessary centripetal force for rotation, we have

$$G \frac{m_1 m_2}{d^2} = \frac{m_1 v_1^2}{r} = \frac{m_1 v_1^2 (m_1 + m_2)}{m_2 d}$$

$$\therefore v_1 = \left[\frac{G m_2^2}{(m_1 + m_2) d} \right]^{1/2} = m_2 \left[\frac{G}{M d} \right]^{1/2}$$

$$\text{Now } T = \frac{2\pi}{\omega} = \frac{2\pi r}{v_1} = \frac{2\pi r}{m_2 \sqrt{G/Md}} = \frac{2\pi d^{3/2}}{\sqrt{GM}} \quad \left(\text{as } r = \frac{m_2 d}{M} \right)$$

$$\therefore \frac{T}{2\pi} = \frac{d^{3/2}}{\sqrt{GM}} \quad \text{or } d = \sqrt[3]{\left(\frac{T}{2\pi} \right)^2 GM}$$

Example 8: Find the distance of a point from the earth's center where the resultant gravitational field due to the earth and the moon is zero. The mass of the earth is 6.0×10^{24} kg and that of the moon is 7.4×10^{22} kg. The distance between the earth and the moon is 4.0×10^5 km.

Sol: If a body is placed between moon and the earth then it is under action of gravitational force due to earth and moon simultaneously. When the gravitational field due to earth is equal in magnitude but opposite in direction to gravitational field due to moon then the net field is zero.

The point must be on the line joining the centers of the earth and the moon and in between them. If the distance of the point from the earth is x , the distance from the moon is $(4.0 \times 10^5 \text{ km} - x)$. The magnitude of the gravitational field due to the earth is

$$E_1 = \frac{GM_e}{x^2} = \frac{G \times 6 \times 10^{24} \text{ kg}}{x^2}$$

and the magnitude of the gravitational field due to the moon is

$$E_2 = \frac{GM_m}{(4.0 \times 10^5 \text{ km} - x)^2} = \frac{G \times 7.4 \times 10^{22} \text{ kg}}{(4.0 \times 10^5 \text{ km} - x)^2}$$

These fields are in opposite directions. For the resultant field to be zero $E_1 = E_2$,

$$\text{Or, } \frac{6 \times 10^{24} \text{ kg}}{x^2} = \frac{7.4 \times 10^{22} \text{ kg}}{(4.0 \times 10^5 \text{ km} - x)^2}$$

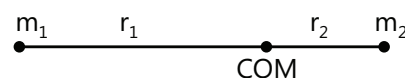
$$\text{Or, } \frac{x}{4.0 \times 10^5 \text{ km} - x} = \sqrt{\frac{6 \times 10^{24}}{7.4 \times 10^{22}}} = 9$$

$$\text{Or, } x = 3.6 \times 10^5 \text{ km}$$

Example 9: A planet of mass m_1 revolves around the sun of mass m_2 . The distance between the sun and the planet is r . Taking into consideration the motion of the sun, find the total energy of the system assuming the orbits to be circular.

Sol: The gravitational pull between sun and planet provides the necessary centripetal acceleration to make them revolve in circular orbits with same angular velocities. The center of each circular orbit will be at the combined center of mass but their radii will be different.

Both the planet and the sun revolve around their center of mass with same angular velocity (say ω)



$$r = r_1 + r_2 \quad \dots (i)$$

$$m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = \frac{G m_1 m_2}{r^2} \quad \dots (ii)$$

Solving Eqs. (i) and (ii), we get

$$r_1 = r \left(\frac{m_2}{m_1 + m_2} \right)$$

$$r_2 = r \left(\frac{m_1}{m_1 + m_2} \right)$$

$$\omega^2 = \frac{G(m_1 + m_2)}{r^3}$$

And now, total energy of the system is $E = \text{P.E.} + \text{K.E.}$

$$\text{or } E = -\frac{G m_1 m_2}{r} + \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2$$

Substituting the values of r_1 , r_2 and ω^2 , we get

$$E = -\frac{G m_1 m_2}{2r}$$

Example 10: Two particles A and B of masses 1 kg and 2 kg respectively are kept 1 m apart and are released to move under mutual attraction. Find the speed of A when that of B is 3.6 cm/hour. What is the separation between the particles at this instant?

Sol: As the particles A and B are initially at rest, the system has potential energy only, but as they move towards each other the loss in potential energy is equal to gain in kinetic energy. As particle is moving under their mutual interaction, the linear momentum system must be conserved.

The linear momentum of the pair A+B is initially zero. As only mutual attraction is taken into account – which is internal when A+B is taken as the system – the linear momentum will remain zero. The particles move in opposite directions. If the speed of A is v when the speed of B is 3.6 cm/hour = 10^{-5} m/s,

$$(1\text{kg})v = (2\text{kg})(10^{-5}\text{ms}^{-1})$$

$$\text{or, } v = 2 \times 10^{-5} \text{ms}^{-1}$$

The potential energy of the pair is $-\frac{G m_A m_B}{R}$ with usual symbols. Initial potential energy

$$= -\frac{6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2 \times 2\text{kg} \times 1\text{kg}}{1\text{m}}$$

$$= -13.34 \times 10^{-11} \text{J}$$

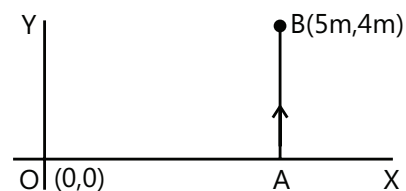
If the separation at the given instant is d , using conservation of energy,

$$-13.34 \times 10^{-11} \text{J} + 0$$

$$= -\frac{13.34 \times 10^{-11} \text{J} - m}{d} + \frac{1}{2} (2 \text{ kg}) (10^{-5} \text{ m/s})^2 + \frac{1}{2} (1 \text{ kg}) (2 \times 10^{-5} \text{ m/s})^2$$

Solving this, $d = 0.31 \text{m}$.

Example 11: The gravitational field in a region is given by $\vec{E} = (10 \text{Nkg}^{-1})(\vec{i} + \vec{j})$. Find the work done by an external agent to slowly shift a particle of mass of 2 kg from the point (0,0) to a point (5m, 4m).



Sol: As the particle is moving slowly, the kinetic energy of the particle remains zero during its motion. The work done by the external agent to move the particle is given

$$\text{by } W = -\Delta U = \int_i^f \vec{F} \cdot d\vec{r}$$

As the particle is slowly shifted, its kinetic energy remains zero. The total work done on the particle is thus zero. The work done by the external agent should be negative of the work done by the gravitational field.

$$\text{The work done by the field is } dW = -dU = \int_i^f \vec{F} \cdot d\vec{r}$$

Consider the figure. Suppose the particle is taken from O to A and then from A to B. The force on the particle is

$$\vec{F} = m\vec{E} = (2\text{kg})(10\text{Nkg}^{-1})(\vec{i} + \vec{j}) = (20\text{N})(\vec{i} + \vec{j})$$

The work done by the field during the displacement OA is

$$W_1 = \int_0^{5\text{m}} F_x dx = \int_0^{5\text{m}} (20\text{N}) dx = 20\text{N} \times 5\text{m} = 100\text{J}$$

Similarly, the work done in displacement AB is

$$W_2 = \int_0^{4\text{m}} F_y dy = \int_0^{4\text{m}} (20\text{N}) dy = (20\text{N})(4\text{m}) = 80\text{J}$$

Thus, the total work done by the field, as the particle is shifted from O to B, is 180 J.

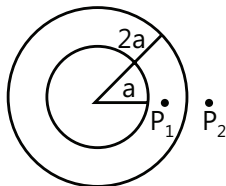
The work done by the external agent is -180 J.

Note that the work is independent of the path so that we can choose any path convenient to us from O to B.

Example 12: A uniform solid sphere of mass M and radius ' a ' is surrounded symmetrically by a uniform thin and spherical shell of equal mass and radius $2a$. Find the gravitational field at a distance

- (a) $\frac{3}{2}a$ from the center, (b) $\frac{5}{2}a$ from the center.

Sol: If the particle is inside the spherical shell then the gravitation field due to the shell is zero. The gravitational field at distance r from the center of the sphere is given by $E = \frac{GM}{r^2}$.



Given figure shows the situation. The point p_1 is at a distance $\frac{3}{2}a$ from the center and p_2 is at a distance $\frac{5}{2}a$ from the center. As p_1 is inside the cavity of the thin spherical shell, the field here due to the shell is zero. The field due to the solid sphere is

$$E = \frac{GM}{\left(\frac{3}{2}a\right)^2} = \frac{4GM}{9a^2}$$

This is also the resultant field. The direction is towards the center. The point p_2 is outside the sphere as well as the shell. Both may be replaced by single particles of the same mass at the center. The field due to each of them is

$$E' = \frac{GM}{\left(\frac{5}{2}a\right)^2} = \frac{4GM}{25a^2}$$

The resultant field is $E = 2E' = \frac{8GM}{25a^2}$ towards the center.

Example 13: A planet of mass m revolves in an elliptical orbit around the sun so that its maximum and minimum distance from the sun are equal to r_a and r_p respectively. Find the angular momentum of this planet relative to the sun.

Sol: At the apogee and perigee the radius vector is perpendicular to the velocity vector of the plane. Use the law of conservation of angular momentum and energy at these two points.

Using conservation of angular momentum $mv_p r_p = mv_a r_a$

As velocities are perpendicular to the radius, vectors at apogee and perigee, $v_p r_p = v_a r_a$

Using conservation of energy,

$$-\frac{GMm}{r_p} + \frac{1}{2}mv_p^2 = -\frac{GMm}{r_a} + \frac{1}{2}mv_a^2$$

By solving, the above equations,

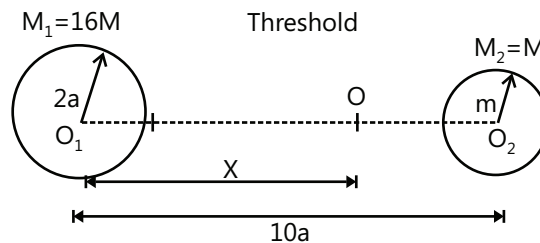
$$v_p = \sqrt{\frac{2GM r_a}{r_p(r_p + r_a)}}; L = mv_p r_p = m \sqrt{\frac{2GM r_p r_a}{(r_p + r_a)}}$$

JEE Advanced/Boards

Example 1: The distance between the centers of two stars is $10a$. The masses of these stars are M and $16M$ and their radii, ' a ' and ' $2a$ ' respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G , M and a .

Sol: At a certain distance from the centers of the stars, the gravitational fields due to the stars are equal in magnitude but opposite in direction. As the body of mass m is projected from the surface of larger star towards the surface of smaller star, the kinetic energy lost by the body is equal to gain of its potential energy when it reaches at the point of zero field.

Let O be the point along O_1O_2 where gravitational intensities due to both the stars balance each other.



Let $O_1O = x$

$$\therefore \frac{GM_1}{x^2} = \frac{GM_2}{(10a - x)^2}$$

$$\text{Or } 16(10 - x)^2 = x^2 \text{ or } x = 8a$$

Potential energy of the body on surface of larger star,

$$U_1 = -\frac{Gm(16M)}{2a} - \frac{GmM}{8a} = -\frac{65GMm}{8a}$$

Potential energy at

$$O = -\frac{GMm}{2a} - \frac{G(16M)m}{8a} = -\frac{5GMm}{2a} = U_0$$

$$\text{As } U_1 + \frac{1}{2}mv_{\min}^2 = U_0$$

$$\therefore -\frac{65GMm}{8a} + \frac{1}{2}mv_{\min}^2 = -\frac{5GMm}{2a}$$

$$\text{or } \frac{1}{2}mv_{\min}^2 = \frac{65GMm}{8a} - \frac{5GMm}{2a} = \frac{45}{8} \frac{GMm}{a}$$

$$v_{\min}^2 \frac{45}{4} \frac{GM}{a} = \frac{9 \times 5}{4} \times \frac{GM}{a}$$

$$v_{\min} = \frac{3}{2} \times \sqrt{\frac{5GM}{a}}$$

Example 2: Two masses m_1 and m_2 , at an infinite distance from each other are initially at rest, start interacting gravitationally. Find their velocity of approach when they are at a distance r apart.

Sol: As the masses move towards each other gain in kinetic energy is equal to loss in gravitational potential energy. This problem is best solved in center of mass frame where the total kinetic energy of masses depends on the square of velocity of approach.

Let v_r be their velocity of approach. From conservation of energy:

Increase in kinetic energy = decrease in gravitational potential energy

$$\text{Or } \frac{1}{2}\mu v_r^2 = \frac{Gm_1 m_2}{r} \quad \dots (i)$$

$$\text{Here, } \mu = \text{reduced mass} = \frac{m_1 m_2}{m_1 + m_2}$$

Substituting in Eq. (i), we get

$$v_r = \sqrt{\frac{2G(m_1 + m_2)}{r}}$$

Example 3: A satellite is revolving round the earth in a circular orbit of radius e and velocity v_o . A particle is projected from the satellite in forward direction with relative velocity $v = (\sqrt{5/4} - 1)v_o$. Calculate its

minimum and maximum distance from earth's center during subsequent motion of the particle.

Sol: As the particle is projected from the satellite while the satellite is still in circular motion, the net velocity of the particle is sum of velocity relative to satellite and the velocity of the satellite. As the particle is still bound to the gravitational attraction of the earth, the orbit of the particle will be ellipse. The point of projection is perigee. Conserve the angular momentum at the apogee and perigee.

The orbital speed of satellite is

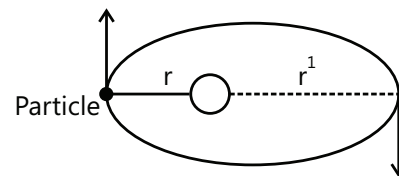
$$v_o = \sqrt{\frac{GM}{r}} \quad \dots (i)$$

Where M = mass of earth

Absolute velocity of particle would be:

$$v_p = v + v_o = \sqrt{\frac{5}{4}} v_o = \sqrt{1.25} v_o \quad \dots (ii)$$

Since, v_p lies between orbital velocity and escape velocity, path of the particle would be an ellipse with r being the minimum distance.



Let r' be the maximum distance and v_p' its velocity at

the moment. $v_p = \sqrt{\frac{5}{4}} v_o$

Then, from the conservation of angular momentum and conservation of mechanical energy, we get

$$mv_p r = mv_p' r' \quad \dots (iii)$$

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r} = \frac{1}{2}mv_p'^2 - \frac{GMm}{r'} \quad \dots (iv)$$

Solving the above Eqs. (i), (ii), (iii) and (iv), we get $r' = \frac{5r}{3}$ and r .

Hence, the maximum and minimum distance are $\frac{5r}{3}$ and r respectively.

Example 4: An earth satellite is revolving in a circular orbit of radius 'a' with velocity v_o . A gun is in the satellite and is aimed towards the earth. A bullet is fired from the gun with muzzle velocity $\frac{v_o}{2}$. Neglecting resistance offered by cosmic dust and recoil of gun, calculate maximum and minimum distance of bullet from the center of earth during its subsequent motion.

Sol: Conserve the angular momentum and energy of the particle between the points, the point of projection and at perigee. At perigee velocity is perpendicular to radius.

The orbital speed of the satellite is

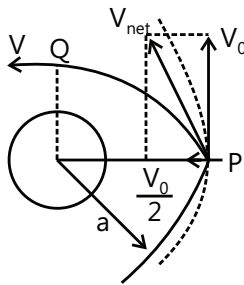
$$v_o = \sqrt{\frac{GM}{a}} \quad \dots (i)$$

From conservation of angular momentum at P and Q we have

$$mav_o = mvr$$

$$\text{Or } v = \frac{av_o}{r} \quad \dots (ii)$$

From conservation of mechanical energy at P and Q we have



$$\frac{1}{2}m \left(v_o^2 + \frac{v_o^2}{4} \right) - \frac{GMm}{a} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$\text{or } \frac{5}{8}v_o^2 - \frac{GM}{a} = \frac{v^2}{2} - \frac{GM}{r}$$

Substituting values of v and v_o from Eqs. (i) and (ii), we get

$$\frac{5}{8} \frac{GM}{a} - \frac{GM}{a} = \frac{a^2}{r^2} \left(\frac{GM}{2a} \right) - \frac{GM}{r}$$

$$\text{or } -\frac{3}{8a} = \frac{a}{2r^2} - \frac{1}{r} \quad \text{or } -3r^2 = 4a^2 - 8ar$$

$$\text{or } 3r^2 - 8ar + 4a^2 = 0$$

$$\text{or } r = \frac{8a \pm \sqrt{64a^2 - 48a^2}}{6}$$

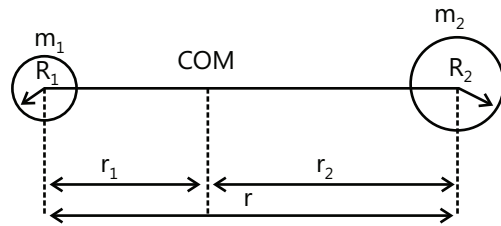
$$\text{or } r = \frac{8a \pm 4a}{6} \quad \text{or } r=2a \text{ and } \frac{2a}{3}$$

Hence, the maximum and minimum distance are $2a$ and $\frac{2a}{3}$ respectively.

Example 5: Binary stars of comparable masses m_1 and m_2 rotate under the influence of each other's gravity with a time period T . If they are stopped suddenly in their motion, find their relative velocity when they collide with each other. The radii of the stars are R_1 and R_2 respectively. G is the universal constant of gravitation.

Sol: They rotate about center of mass, such that the necessary centripetal acceleration for the rotational motion is provided by the gravitational force of attraction. As the stars start approaching each other and collide, the loss in the gravitational energy of system is equal to the gain in the kinetic energy of the system. Find the initial separation in terms of the time period.

Both the stars rotate about their center of mass (COM).



For the position of COM

$$\frac{r_1}{m_2} = \frac{r_2}{m_1} = \frac{r_1 + r_2}{m_1 + m_2} = \frac{r}{m_1 + m_2} \quad (r = r_1 + r_2)$$

$$\text{Also, } m_1 r_1 \omega^2 = \frac{Gm_1 m_2}{r^2} \quad \text{or } \omega^2 = \frac{Gm_2}{r_1^2} \quad \left(\omega = \frac{2\pi}{T} \right)$$

$$\text{But, } r_1 = \frac{m_2 r}{m_1 + m_2}$$

$$\therefore \omega^2 = \frac{G(m_1 + m_2)}{r^3}$$

$$\text{Or } r = \left\{ \frac{G(m_1 + m_2)}{\omega^2} \right\}^{1/3} \quad \dots (i)$$

Applying conservation of mechanical energy we have

$$-\frac{Gm_1 m_2}{r} = -\frac{Gm_1 m_2}{(R_1 + R_2)} + \frac{1}{2} \mu v_r^2 \quad \dots (ii)$$

Here, $\mu = \text{reduced mass} = \frac{m_1 m_2}{m_1 + m_2}$ and

$v_r = \text{relative velocity between the two stars.}$

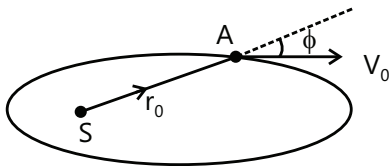
From Eq. (ii), we find that

$$\begin{aligned} v_r^2 &= \frac{2Gm_1 m_2}{\mu} \left(\frac{1}{R_1 + R_2} - \frac{1}{r} \right) \\ &= \frac{2Gm_1 m_2}{m_1 m_2} \left(\frac{1}{R_1 + R_2} - \frac{1}{r} \right) \\ &= 2G(m_1 + m_2) \left(\frac{1}{R_1 + R_2} - \frac{1}{r} \right) \end{aligned}$$

Substituting the value of r from Eq. (i), we get

$$v_r = \sqrt{2G(m_1 + m_2) \left[\frac{1}{R_1 + R_2} - \left\{ \frac{4\pi^2}{G(m_1 + m_2) T^2} \right\}^{1/3} \right]}$$

Example 6: Find the maximum and minimum distance of the planet A from the sun S, if at a certain moment of time it was a distance r_0 and travelling with the velocity v_0 with the angle between the radius vector and velocity vector being equal to ϕ .



Sol: As the planet revolves around the sun, the mechanical energy of the system is conserved. Conserve the angular momentum between the given point and apogee.

At minimum and maximum distance velocity vector (\vec{v}) makes an angle of 90° with radius vector. Hence, from conservation of angular momentum,

$$mv_0 r_0 \sin \phi = mrv \quad \dots(i)$$

Here, m is the mass of the planet.

From energy conservation law it follows that.

$$\frac{mv_0^2}{2} - \frac{GMm}{r_0} = \frac{mv^2}{2} - \frac{GMm}{r} \quad \dots(ii)$$

Here, M is the mass of the sun.

Solving Eqs. (i) and (ii) for r , we get we values of r , one is r_{\max} and another is r_{\min} . So,

$$r_{\max} = \frac{r_0}{2-K} \left(1 + \sqrt{1-K(2-K)\sin^2 \phi} \right)$$

$$\text{and } r_{\min} = \frac{r_0}{2-K} \left(1 - \sqrt{1-K(2-K)\sin^2 \phi} \right)$$

$$\text{Here, } K = \frac{r_0^2 v_0^2}{GM}$$

Example 7: The density inside a solid sphere of radius 'a' is given by $\rho = \rho_0 a/r$, where ρ_0 is the density at the surface and r denotes the distance from the center. Find the gravitational field due to this sphere at a distance of '2a' from its center.

Sol: The given mass distribution is having spherical symmetry. Any spherically symmetrical body can be replaced by a point particle of the same mass situated at the center of the spherical body. The gravitational field due to the sphere at the point $2a$ from the center of the sphere is given by $E = \frac{GM}{(2a)^2}$

The field is required at a point outside the sphere. Dividing the sphere in concentric shells, each shell can be replaced by a point particle at its center having mass equal to the mass of the shell. Thus, the whole sphere can be replaced by a point particle at its center having mass equal to the mass of the given sphere. If the mass of the sphere is M , the gravitational field at the given point is

$$E = \frac{GM}{(2a)^2} = \frac{GM}{4a^2} \quad \dots(i)$$

The mass M may be calculated as follows: Consider a concentric shell of radius r and thickness dr . Its volume is

$$dV = (4\pi^2) dr \text{ and its mass is}$$

$$dM = \rho dV = \left(\rho_0 \frac{a}{r} \right) (4\pi^2 dr) = 4\rho_0 a dr.$$

The mass of the whole sphere is

$$M = \int_0^a 4\rho_0 a dr = 2\pi\rho_0 a^3$$

Thus, by (i) the gravitational field is

$$E = \frac{2\pi G\rho_0 a^3}{4a^2} = \frac{1}{2} \times G\rho_0 a.$$

Example 8: Two satellites of same mass are launched in the same orbit round the earth so as to rotate opposite to each other. They collide solidly and stick together as wreckage. Obtain the total energy of the system before and just after the collision. Describe the subsequent motion of the wreckage.

Sol: Both the satellites are moving in the same orbit so their orbital velocity will be same. As the masses of the satellites are equal, and they are moving in the opposite direction their total momentum before and after the collision is zero.

The two satellites round the earth are shown in figure

Potential energy of the satellite in its orbit = $-GMm/r$

Kinetic energy of satellite in its orbit is

$$K = GMm/2r$$

Where m is mass of satellite, M is the mass of the earth and r is the orbital radius.

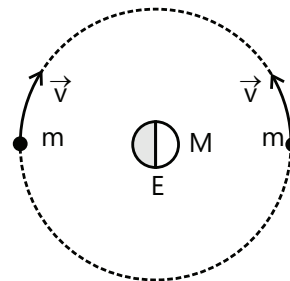
$$\text{Total energy} = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

When there are two satellites, the total energy would be

$$\left(-\frac{GMm}{2r}\right) + \left(-\frac{GMm}{2r}\right) = \left(-\frac{GMm}{r}\right)$$

Let after collision, v' be the velocity of wreckage by the law of conservation of momentum $mv - mv = (m + m)v'$

$$\therefore v' = 0$$



The wreckage of mass $(2m)$ has no kinetic energy, but it has only potential energy,

$$\text{So, energy after collision} = -\frac{GM(2m)}{r}$$

Now the combined mass has zero velocity just after collision and therefore, the wreckage stops rotating and falls down under gravity.

JEE Main/Boards

Exercise 1

Q.1 Why Newton's law of gravitation is called a universal law?

Q.2 On earth value of $G = 6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$. What is its value on moon, where g is nearly one-sixth than that of earth?

Q.3 An artificial satellite is revolving around the earth at a height 200 km from the earth's surface. If a packet is released from the satellite, what will happen to it? Will it reach the earth?

Q.4 A spring balance is suspended inside an artificial satellite revolving around the earth. If a boy of mass 2 kg is suspended from it, what would be its reading?

Q.5 The escape velocity from earth for a piece of 10 gram is 11.2kms^{-1} . What would it be for a piece of mass 100 gram?

Q.6 Where will the true weight of the body be zero?

Q.7 If the force of gravity acts on all bodies in proportion to their masses, why does not a heavy body fall correspondingly faster than a light body.

Q.8 The gravitational potential energy of a body at a distance from the center of earth is U . What is the weight of the body at that point?

Q.9 The distance of two planets from the sun are 10^{11} and 10^{10} meters respectively. What is the ratio of time periods of these two planets?

Q.10 For a satellite, escape speed is 11kms^{-1} . If the satellite is launched at an angle of 60° with the vertical, what will be the escape speed?

Q.11 Prove that the value of acceleration due to gravity at a point above the surface of the earth is inversely proportional to the square of the distance of that point from the center of the earth.

Q.12 Gravitational force between two bodies is 1 newton. If the distance between them is made twice, what will be the force?

Q.13 If a person goes to a height equal to radius of earth from its surface, what would be his weight relative to that on the earth?

Q.14 If the change in the value of g at a height h above the surface of the earth is the same as at a depth x below it, both x and h being much smaller than the radius of the earth, find the relation between x and h .

Q.15 The gravitational force acting on a rocket at a height h from the surface of earth is $1/3$ of the force acting on a body at sea level. What is the ratio of h and R (radius of earth)?

Q.16 Does the gravitational force of attraction of the earth becomes zero at some height above the surface of earth? Explain.

Q.17 What do you understand by gravity and acceleration due to gravity. Establish a relation between g and G .

Q.18 Explain how the knowledge of g helps us to find (i) mass of earth and (ii) mean density of earth?

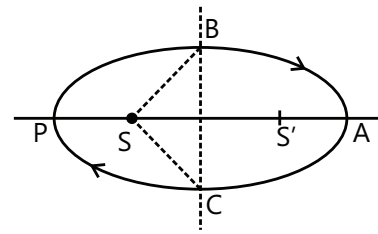
Q.19 What do you understand by 'Escape velocity'? Derive an expression for it in terms of parameters of given planet.

Q.20 What do you understand by Gravitational field, Intensity of gravitational field. Prove that gravitational intensity at a point is equal to the acceleration due to gravity at that point.

Q.21 Explain Kepler's laws of planetary motion and deduce Newton's law of gravitation from them.

Q.22 Explain Newton's law of gravitation. Define gravitational constant, and give its dimensional formula. Give the evidences in support of the Newton's law of gravitation.

Q.23 Let the speed of the planet at the perihelion P in the given figure be v_p and the sun-planet distance SP be r_p . Relate r_p , v_p to the corresponding quantities at the aphelion (r_A, v_A). Will the planet take equal times to traverse BAC and CPB ?



Q.24 Two satellites of a planet have period 32 days and 256 days. If the radius of the orbit of former is R , find the orbital radius of the latter.

Q.25 If the distance of the earth from the sun were half the present value, how many days will make one year? Given, 1 year = 365 days.

Q.26 Estimate the mass of the sun, assuming the orbit of earth round the sun to be a circle. The distance between the sun and the earth is $1.49 \times 10^{11} \text{m}$, and $G = 6.66 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$.

Q.27 If the mass of the sun is $2 \times 10^{30} \text{kg}$, the distance of the earth from sun is $1.5 \times 10^{11} \text{m}$ and period of revolution of the earth around sun is one year (=365.3 days), calculate the value of gravitational constant.

Q.28 Calculate the mass and mean density of earth from the following data:

Radius of earth = $6.37 \times 10^6 \text{m}$, acceleration due to gravity = 9.8ms^{-2} and

Gravitational constant = $6.6 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$

Q.29 If the radius of the earth shrinks by 2.5%, mass remaining constant, then how would the value of acceleration due to gravity change?

Q.30 At what altitude the acceleration due to gravity above the earth's surface would be half of its value on the surface of the earth? Radius of earth is 6400 km.

Q.31 The radius of earth is approximately 6000 km. What will be your weight at 600 km above the surface of earth? At 12000 km above? At 18000 km above? Your weight on earth is 80 kg wt.

Q.32 At what height from the surface of earth, the acceleration due to gravity is the same at a depth 160 km below the surface of earth. Radius of earth is 6400 km.

Q.33 What is the minimum energy required to launch a satellite of mass m from the surface of earth of mass M , radius R in a circular orbit at an altitude $2R$.

Q.34 A rocket is launched vertically from the surface of the earth with an initial velocity 10kms^{-1} . How far above the surface of the earth would it go? Radius of the earth = 6400km ; $g = 9.8\text{ms}^{-2}$.

Q.35 A remote sensing satellite of the earth revolves in a circular orbit at a height of 250km above the earth's surface. What is the (a) orbital speed, and (b) period of revolution of satellite? Radius of the earth = $6.38 \times 10^6\text{m}$, and acceleration due to gravity at the surface of earth = 9.8ms^{-2} .

Q.36 A satellite revolves round a planet in an orbit just above the surface of planet. Taking $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$ and the mean density of the planet = $5.51 \times 10^3\text{kgm}^{-3}$, find the period of satellite.

Q.37 Find the speed of escape at the moon given that its radius $1.7 \times 10^6\text{m}$ and the value of g at its surface is 1.63ms^{-2} .

Q.38 If the earth has a mass nine times and radius twice that of the planet Mars, calculate the maximum speed required by a rocket to pull out of the gravitational force of Mars. Given escape speed on the surface of earth is 11.2kms^{-1} .

Exercise 2

Single Correct Choice Type

Q. 1 At what altitude will the acceleration due to gravity be 25% of that at the earth's surface (given radius of earth is R)?

- (A) $R/4$ (B) R (C) $3R/8$ (D) $R/2$

Q.2 Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles. An object weighed at the equator gives the same reading as a reading taken at a depth d below earth's surface at a pole ($d < R$). The value of d is

- (A) $\frac{\omega^2 R^2}{g}$ (B) $\frac{\omega^2 R^2}{2g}$ (C) $\frac{2\omega^2 R^2}{g}$ (D) $\frac{\sqrt{Rg}}{g}$

Q.3 If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same?

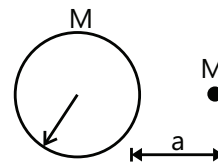
- (A) $1/25$ (B) $1/5$ (C) $1/\sqrt{5}$ (D) 5

Q.4 The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is a second's pendulum on earth?

- (A) $\sqrt{2}$ second (B) $2\sqrt{2}$ second

- (C) $\frac{1}{\sqrt{2}}$ second (D) $\frac{1}{2\sqrt{2}}$ second

Q.5 A particle of mass M is at a distance a from surface of a thin spherical shell of equal mass and having radius a .



- (A) Gravitational field and potential both are zero at center of the shell.
 (B) Gravitational field is zero not only inside the shell but at a point outside the shell also.
 (C) Inside the shell, gravitational field alone is zero.
 (D) Neither gravitational field nor gravitational potential is zero inside the shell.

Q.6 A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is v . Due to the rotation of planet about its axis the acceleration due to gravity g at equator is $\frac{1}{2}$ of g at poles. The escape velocity of a particle on the pole of planet in terms of V .

- (A) $v_e = 2v$ (B) $v_e = v$
 (C) $v_e = v\sqrt{2}$ (D) $v_e = \sqrt{3}v$

Q.7 Two planets A and B have the same material density. If the radius of A is twice that of B, then the ratio of the escape velocity $\frac{v_A}{v_B}$ is.

- (A) 2 (B) $\sqrt{2}$ (C) $1/\sqrt{2}$ (D) $1/2$

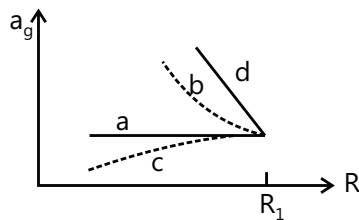
Q.8. The escape velocity for a planet is v_e . A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the center of the planet, its speed will be

- (A) v_e (B) $\frac{v_e}{\sqrt{2}}$ (C) $\frac{v_e}{2}$ (D) Zero

Q.9 A hollow spherical shell is compressed to half its radius. The gravitational potential at the center

- (A) Increases
 (B) Decreases
 (C) Remains same
 (D) During the compression increases then returns at the previous value.

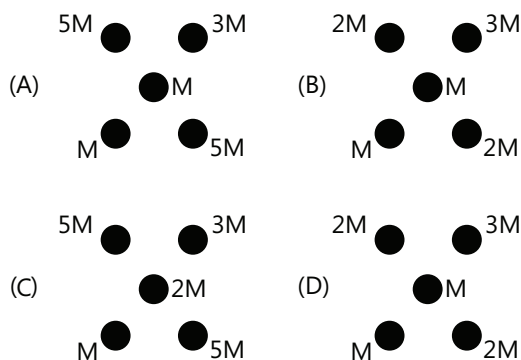
Q.10 A (nonrotating) star collapses onto itself from an initial radius R_1 with its mass remaining unchanged. Which curve in the following figure best gives the gravitational acceleration a_g on the surface of the star as function of the radius of the star during the collapse?



- (A) a (B) b (C) c (D) d

Q.11 A mass is at the center of a square, with four masses at the corners as shown

Rank the choices according to the magnitude of the gravitational force on the center mass.



- (A) $F_A = F_B < F_C = F_D$ (B) $F_A > F_B < F_D < F_C$
 (C) $F_A = F_B > F_C = F_D$ (D) None

Q.12 A satellite of the earth is revolving in circular orbit with a uniform velocity V . If the gravitational force suddenly disappears, the satellite will

- (A) Continue to move with the same velocity in the same orbit.
 (B) Move tangentially to the original orbit with velocity V .
 (C) Fall down with increasing velocity.
 (D) Come to a stop somewhere in its original orbit.

Q.13 A satellite revolves in the geostationary orbit but in a direction east to west. The time interval between its successive passing about a point on the equator is:

- (A) 48 hrs (B) 24 hrs
 (C) 12 hrs (D) Never

Q.14 Two point masses of mass $4m$ and m respectively separated by d distance are revolving under mutual force of attraction. Ratio of their kinetic energies will be:

- (A) 1:4 (B) 1:5 (C) 1:1 (D) 1:2

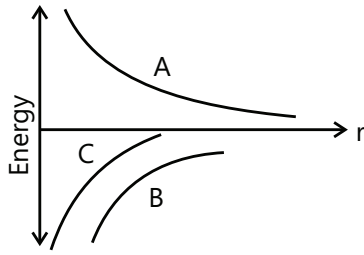
Q.15 Select the correct choice(s)

- (A) The gravitational field inside a spherical cavity, within a spherical planet must be non-zero and uniform.
 (B) When a body is projected horizontally at an appreciable large height above the earth, with a velocity less than for a circular orbit, it will fall to the earth along a parabolic path
 (C) A body of zero total mechanical energy placed in a gravitational field if it is travelling away from source of field will escape the field.
 (D) Earth's satellite must be in equatorial plane.

Q.16 A satellite of mass m , initially at rest on the earth, is launched into a circular orbit at a height equal to the radius of the earth. The minimum energy required is.

- (A) $\frac{\sqrt{3}}{4}mgR$ (B) $\frac{1}{2}mgR$
 (C) $\frac{1}{4}mgR$ (D) $\frac{3}{4}mgR$

Q.17 The following figure shows the variation of energy with the orbit radius of a circular planetary motion. Find the correct statement about the curves A, B and C



(A) A shows the kinetic energy, B the total energy and C the potential energy of the system.

(B) C shows the total energy, V the kinetic energy and A the potential energy of the system.

(C) C and A are kinetic and potential energies respectively and B is the total energy of the system.

(D) A and B are kinetic and potential energies and C is the total energy of the system.

Q.18 When a satellite moves around the earth in a certain orbit, the quantity which remains constant is:

- (A) Angular velocity (B) Kinetic energy
(C) Aerial velocity (D) Potential energy

Q.19 A satellite of mass $5M$ orbits the earth in a circular orbit. At one point in its orbit, the satellite explodes into two pieces, one of mass M and the other of mass $4M$. After the explosion the mass M ends up travelling in the same circular orbit, but in opposite direction. After explosion the mass $4M$ is.

- (A) In a circular orbit
(B) Unbound
(C) Elliptical orbit
(D) Data is insufficient to determine the nature of the orbit.

Q.20 A satellite can be in a geostationary orbit around earth at a distance r from the center. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the center is

- (A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$ (C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$

Q.21 A planet of mass m is in an elliptical orbit around the sun ($m \ll M_{\text{sun}}$) with an orbital period T . If A be the area of orbit, then its angular momentum would be:

- (A) $\frac{2mA}{T}$ (B) mAT (C) $\frac{mA}{2T}$ (D) $2mAT$

Q.22 Satellite A and B are orbiting around the in orbits of ratio R and $4R$ respectively. The ratio of their aerial velocities is:

- (A) 1:2 (B) 1:4 (C) 1:8 (D) 1:16

Q.23 In older times, people used to think that the Earth was flat. Imagine that the earth is indeed not a sphere of radius R , but an infinite plate of thickness H . What value of H is needed to allow the same gravitational acceleration to be experienced as on the surface of the actual Earth? (Assume that the Earth's density is uniform and equal in the two models.)

- (A) $\frac{2R}{3}$ (B) $\frac{4R}{3}$ (C) $\frac{8R}{3}$ (D) $\frac{R}{3}$

Q.24 A planet revolves about the sun in elliptical orbit.

The aerial velocity $\left(\frac{dA}{dt}\right)$ of the planet is $4.0 \times 10^{16} \text{ m}^2/\text{s}$.

The least distance between planet and the sun is $2 \times 10^{12} \text{ m}$. Then the maximum speed of the planet in km/s is:

- (A) 10 (B) 20 (C) 40 (D) None of these

Previous Years' Questions

Q.1 If the radius of x the earth were to shrink by one per cent, its mass remaining the same, the acceleration due to gravity on the earth's surface would **(1981)**

- (A) Decrease (B) Remain unchanged
(C) Increase (D) Be zero

Q.2 If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is **(1983)**

- (A) $\frac{1}{2}mgR$ (B) $2mgR$
(C) mgR (D) $\frac{1}{4}mgR$

Q.3 Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T . If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then **(1989)**

- (A) T^2 is proportional to R^2

- (B) T^2 is proportional to $R^{7/2}$
 (C) T^2 is proportional to $R^{3/2}$
 (D) T^2 is proportional to $R^{3.75}$

Q.4 If the distance between the earth and the sun were half its present value, the number of days in a year would have been **(1996)**

- (A) 64.5 (B) 129 (C) 182.5 (D) 730

Q.5 A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is **(2001)**

- (A) 1 (B) $\sqrt{2}$ (C) 4 (D) 2

Q.6 A geostationary satellite orbits around the earth in a circular orbit of radius 36,000km. Then, the time period of a spy satellite orbiting a few hundred km above the earth's surface ($R_e = 6400\text{km}$) will approximately be **(2002)**

- (A) 1/2h (B) 1 h (C) 2 h (D) 4 h

Q.7 A double star system consists of two stars A and B which have time periods T_A and T_B . Radius R_A and R_B and mass M_A and M_B . Choose the correct option. **(2006)**

(A) $T_A > T_B$ then $R_A > R_B$

(B) if $T_A > T_B$ then $M_A > M_B$

(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$

(D) $T_A = T_B$

Q.8 A satellite is moving with a constant speed v in a circular orbit about the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is **(2011)**

- (A) $\frac{1}{2}mv^2$ (B) mv^2 (C) $\frac{3}{2}mv^2$ (D) $2mv^2$

Q.9 A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 kms^{-1} , the escape velocity from the surface of the planet would be **(2008)**

- (A) 1.1 kms^{-1} (B) 11 kms^{-1}
 (C) 110 kms^{-1} (D) 0.11 kms^{-1}

Q. 10 Statement-I: For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is $4\pi GM$.

Statement-II: If the direction of a field due to a point source is radial and its dependence on the distance 'r' for the source is given as $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface **(2008)**

(A) Statement-I is false, statement-II is true.

(B) Statement-I is true, statement-II is true; statement-II is correct explanation for statement-I.

(C) Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I.

(D) Statement-I is true, statement-II is false.

Q.11 The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R , the radius of the earth is **(2009)**

- (A) $2R$ (B) $\frac{R}{\sqrt{2}}$ (C) $\frac{R}{2}$ (D) $\sqrt{2}R$

Q.12 Two bodies of masses m and $4m$ are placed at a distance r . The gravitational potential at a point on the line joining them where the gravitational field is zero is: **(2011)**

- (A) $-\frac{4Gm}{r}$ (B) $-\frac{6Gm}{r}$
 (C) $-\frac{9Gm}{r}$ (D) Zero

Q.13 The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400km respectively. The required energy for this work will be: **(2012)**

- (A) 6.4×10^{11} Joules (B) 6.4×10^8 Joules
 (C) 6.4×10^9 Joules (D) 6.4×10^{10} Joules

Q.14 What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$?

(2013)

(A) $\frac{5GmM}{6R}$

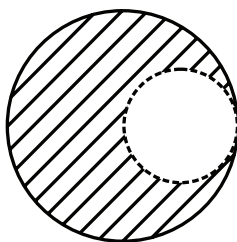
(B) $\frac{2GmM}{3R}$

(C) $\frac{GmM}{2R}$

(D) $\frac{GmM}{3R}$

Q.15 From a solid sphere of mass M and radius R , a spherical portion of radius $R/2$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is: ($G =$ gravitational constant)

(2015)



(A) $\frac{-GM}{2R}$

(B) $\frac{-GM}{R}$

(C) $\frac{-2GM}{3R}$

(D) $\frac{-2GM}{R}$

Q.16 A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R ; $h < R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to: (Neglect the effect of atmosphere.)

(2016)

(A) \sqrt{gR}

(B) $\sqrt{gR/2}$

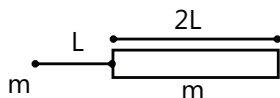
(C) $\sqrt{gR}(\sqrt{2}-1)$

(D) $\sqrt{2gR}$

JEE Advanced/Boards

Exercise 1

Q.1 A small mass and a thin uniform rod each of mass 'm' are positioned along the same straight line as shown. Find the force of gravitational attraction exerted by the rod on the small mass.



Q.2 A particle is forced vertically from the surface of the earth with a velocity kv_e where v_e is the escape velocity and $k < 1$. Neglecting air resistance and assuming earth's radius as R_e , calculate the height to which it will rise from the surface of the earth.

Q.3 A point P lies on the axis of a fixed ring of mass M and radius a , at a distance a from its center C. A small particle starts from P and reaches C under gravitational attraction only. Its speed at C will be _____.

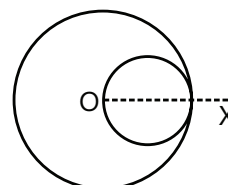
Q.4 Calculate the distance from the surface of the earth at which above and below the surface acceleration due to gravity is the same.

Q.5 An object is projected vertically upward from the surface of the earth of mass M with a velocity such that the maximum height reached is eight times the radius R of the earth. Calculate:

(i) The initial speed of projection

(ii) The speed at half the maximum height.

Q.6 A sphere of radius R has its center at the origin. It has a uniform mass density ρ_0 except that there is a spherical hole of radius $r = R/2$ whose center is at $x = R/2$ as in the given figure. (a) Find gravitational field at points on the axis for $x > R$

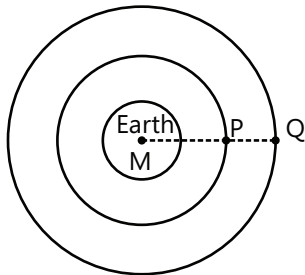


(b) Show that the gravitational field inside the hole is uniform. Find its magnitude and direction.

Q.7 A small body of mass is projected with a velocity just sufficient to make it reach from the surface of a planet (of radius $2R$ and mass $3M$) to the surface of another planet (of radius R and mass M). The distance between the centers of the two spherical planet is $6R$. The distance of the body from the center of bigger planet is 'x' at any moment. During the journey, find the distance x where the speed of the body is (a) maximum (b) minimum. Assume motion of body along the line joining centers of planets.

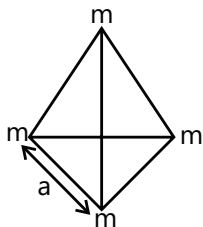
Q.8 A man can jump over $b=4\text{m}$ wide trench on earth. If mean density of an imaginary planet is twice that of the earth, calculate its maximum possible radius so that he may escape from it by jumping. Given radius of earth= 6400km .

Q.9 A satellite P is revolving around the earth at a height $h = \text{radius of earth } (R)$ above equator. Another satellite Q is at a height $2h$ revolving in opposite direction. At an instant the two are at same vertical line passing through center of sphere. Find the least time after which again they are in this situation.



Q.10 Two small dense stars rotate about their common center of mass as a binary system with the period 1 year for each. One star is of double the mass of the other and the mass of the lighter one is $1/3$ of the mass of the sun. Find the distance between the stars if distance between the earth & the sun is R .

Q.11 Four masses (each of m) are placed at the vertices of a regular pyramid (triangular base) of side 'a'. Find the work done by the system while taking them apart so that they form the pyramid of side '2a'



Q.12 A thin spherical shell of total mass M and radius R is held fixed. There is a small hole in the shell. A mass m is released from rest at a distance R from the hole along a line that passes through the hole and also through the center of the shell. This mass subsequently moves under the gravitational force of the shell. How long does the mass take to travel from the hole to the point diametrically opposite?

Q.13 A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time_____.

Q.14 A satellite is moving in a circular orbit around the earth. The total energy of the satellite is $E = -2 \times 10^5 \text{ J}$. The amount of energy to be imparted to the satellite to transfer it to a circular orbit where its potential energy is equal to_____.

Q.15 A satellite of mass m is orbiting the earth in a circular orbit radius r . It starts losing energy due to small air resistance at the rate of $C \text{ J/s}$. Then the time taken for the satellite to reach the earth is_____.

Q.16 A satellite is orbiting the Earth of mass M in equatorial plane in a circular orbit having radius $2R$ and same sense of rotation as that of the Earth. Find duration of time for which a man standing on the equator will be able to see the satellite continuously. Assume that the man can see the satellite when it is above horizon. Take Earth's angular velocity = ω

Q.17 A launching pad with a spaceship is moving along a circular orbit of the moon, whose radius R is triple that of moon R_m . The ship leaves the launching pad with a relative velocity equal to the launching pad's initial orbital velocity v_0 and the launching pad then falls to the moon. Determine the angle θ with the horizontal at which the launching pad crashes into the surface if its mass is twice that of the spaceship m .

Q.18 A body moving radially away from a planet of mass M , when at distance r from planet, explodes in such a way that two of its many fragments move in mutually perpendicular circular orbits around the planet. What will be

(a) Their velocity in circular orbits

(b) Maximum distance between the two fragments before collision and

(c) Magnitude of their relative velocity just before they collide.

Q.19 A cord of length 64 m is used to connect a 100 kg astronaut to spaceship whose mass is much larger than that of the astronaut. Estimate the value of the tension in the cord. Assume that the spaceship is orbiting near earth's surface. Assume that the spaceship and the astronaut fall on a straight line from the earth's center. The radius of the earth is 6400 km.

Q.20 Imagine a planet of mass M with a small moon of mass m and radius a orbiting it and keeping the same face toward it. If the moon now approaches the planet, there will be a critical distance from the planet's center at which loose material lying on the moon's surface will be lifted off. Show that this distance is given by $r_e = a(3M/m)^{1/3}$. This critical distance is called Roche's limit.

Q.21 A hypothetical planet of mass M has three moons each of equal mass ' m ' revolving in the same circular orbit of radius R . The masses are equally spaced and thus form an equilateral triangle. Find:

- The total P.E. of the system
- The orbital speed of each moon such that they maintain this configuration.

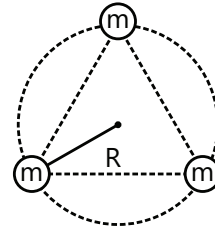
Q.22 A remote sensing satellite is revolving in an orbit of radius x over the equator of earth. Find the area on earth's surface in which satellite cannot send message.

Q.23 A pair of stars rotate about a common center of mass. One of the stars has a mass M which is twice as large as the mass m of the other. Their centers are a distance d apart, d being large compared to the size of either star.

- Derive an expression for the period of rotation of the star about their common center of mass in terms of d , m , G .
- Compare the angular momentum of the two stars about their common center of mass by calculating the ratio L_m/L_M .
- Compare the kinetic energies of the two stars by calculating the ratio K_m/K_M .

Q.24 Assume that a geosynchronous communications satellite is in orbit at the longitude of Mumbai. You are in Mumbai and want to pick up its signals. In what direction should you point the axis of your parabolic antenna? The latitude of Mumbai 30° N.

Q.25 The fastest possible rate of rotation of a planet such that for which the gravitational force on material at the equator barely provides the centripetal force needed for the rotation. Show that the corresponding shortest period of rotation is given by $T = \sqrt{\frac{3\pi}{G\rho}}$, where ρ is the density of the planet, assumed to be homogeneous.



Exercise 2

Multiple Correct Choice Type

Q.1 Assuming the earth to be a sphere of uniform density the acceleration due to gravity

- At a point outside the earth is inversely proportional to the square of its distance from the center.
- At a point outside the earth is inversely proportional to its distance from the center.
- At a point inside is zero.
- At a point inside is proportional to its distance from the center.

Q.2 Two masses m_1 and m_2 ($m_1 < m_2$) are released from rest from a finite distance. They start under their mutual gravitational attraction.

- Acceleration of m_1 is more than that of m_2 .
- Acceleration of m_1 is more than that of m_1 .
- Center of mass of system will remain at rest in all reference frames.
- Total energy of system remains constant.

Q.3 Inside a hollow isolated spherical shell

- Everywhere gravitational potential is zero.
- Everywhere gravitational field is zero.
- Everywhere gravitational potential is same.
- Everywhere gravitational field is same.

Q.4 When a satellite in a circular orbit around the earth centers the atmospheric region, it encounters small air resistance to its motion. Then

- (A) Its kinetic energy increases.
- (B) Its kinetic energy decreases.
- (C) Its angular momentum about the earth decreases.
- (D) Its period of revolution around the earth increases.

Q.5 A communications Earth satellite

- (A) Goes round the earth from east to west.
- (B) Can be in the equatorial plane only.
- (C) Can be vertically above any place on the earth.
- (D) Goes round the earth from west to east.

Q.6 An earth satellite is moved from one stable circular orbit to another larger and stable circular orbit. The following quantities increase for the satellite as a result of this change:-

- (A) Gravitational potential energy
- (B) Angular velocity
- (C) Linear orbital velocity
- (D) Centripetal acceleration

Q.7 A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth.

- (A) The acceleration of S is always directed towards the center of the earth.
- (B) The angular momentum of S about the center of the earth changes in direction, but its magnitude remains constant.
- (C) The total mechanical energy of S varies periodically with time.
- (D) The linear momentum of S remains constant in magnitude.

Q.8 If a satellite orbits as close to the earth's surface as possible,

- (A) Its speed is maximum
- (B) Time period of its rotation is minimum
- (C) The total energy of the 'earth plus satellite' system is minimum
- (D) The total energy of the 'earth plus satellite' system is maximum

Q.9 For a satellite to orbit around the earth, which of the following must be true?

- (A) It must be above the equator at some time.
- (B) Its cannot pass over the poles at any time
- (C) Its height above the surface cannot exceed 36,000 km
- (D) Its period of rotation must be $> 2\pi\sqrt{R/g}$ where R is radius of earth

Q.10 Two satellites s_1 & s_2 of equal masses revolve in the same sense around a heavy planet in coplanar circular orbit of radii R & 4R.

- (A) The ratio of period of revolution s_1 & s_2 is 1:8
- (B) Their velocities are in the ratio 2:1
- (C) Their angular momentum about the planet are in the ratio 2:1
- (D) The ratio of angular velocities of s_1 w.r.t. s_2 when all three are in same line is 9:5

Assertion Reasoning Type

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct the explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I false, statement-II is true.

Q.11 Statement-I: Moon revolving around earth does not come despite earth's gravitational attraction.

Statement-II: A radially outward force balances earth's force of attraction during revolution of moon.

Q.12 Statement-I: Time period of simple pendulum in an orbiting geostationary satellite is infinite.

Statement-II: Earth's gravitational field becomes negligible at large distance from it.

Q.13 Statement-I: Geostationary satellite may be setup in equatorial plane in orbits of any radius more than earth's radius.

Statement-II: Geostationary satellite have period of revolution of 24 hrs.

Q.14 Statement-I: For the calculation of gravitational force between any two uniform spherical shells, they

can always be replaced by particles of same mass placed at respective centers.

Statement-II: Gravitational field of a uniform spherical shell outside it is the same as that of particle of same mass placed at its center of mass.

Q.15 Statement-I: It takes more fuel for a spacecraft to travel from the earth to moon than for the return trip.

Statement-II: Potential energy of spacecraft at moon's surface is greater than that at earth surface.

Comprehension Type

Paragraph 1:

Two uniform spherical stars made of same material have radii R and $2R$. Mass of the smaller planet is m . They start moving from rest towards each other from a large distance under mutual force of gravity. The collision between the stars is inelastic with coefficient of restitution $\frac{1}{2}$.

Q.16 Kinetic energy of the system just after the collision is:

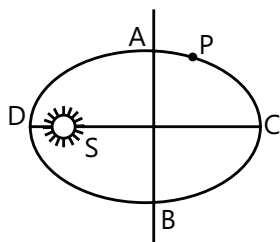
- (A) $\frac{8Gm^2}{3R}$ (B) $\frac{2Gm^2}{3R}$
 (C) $\frac{4Gm^2}{3R}$ (D) Cannot be determined

Q.17 The maximum separation between their centers after their first collision

- (A) $4R$ (B) $6R$ (C) $8R$ (D) $12R$

Paragraph 2:

The given figure shows the orbit of a planet P round the sun S , AB and CD are the minor and major axes of the ellipse.



Q.18 If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA , then

- (A) $t_1 = t_2$ (B) $t_1 > t_2$
 (C) $t_1 < t_2$ (D) Nothing can be concluded

Q.19 If U is the potential energy and K kinetic energy then $|U| > |K|$ at

- (A) Only D (B) Only C
 (C) Both D & C (D) Neither D nor C

Paragraph 3:

During the formation of stars from clouds of hydrogen gas in space, due to gravitational force of attraction, volume of gas decreases, which in turn heats the gas. Specific heat capacity of gas is S , universal gravitational constant is G and mass in a hydrogen cloud is M .

Q.20 If radius of gas cloud decreases from R to $R/2$, the increment in temperature of gas is (assume No loss of energy outside due to radiations, and clouds are spherical in shape)

- (A) $\frac{GM}{RS}$ (B) $\frac{3GM}{5RS}$ (C) $\frac{3GS}{5MR}$ (D) $-\frac{3GM}{RS}$

Q.21 Assume the initial temperature of gas is 0 K and thermonuclear reactions will start at $T_0\text{ K}$ temperature, the minimum mass of gas required so that thermonuclear reactions start when radius of cloud becomes half of initial radius (R). Assume uniform temperature in entire volume of gas.

- (A) $\frac{5 SRT_0}{3 G}$ (B) $\frac{3 SRT_0}{5 G}$
 (C) $\frac{SRT_0}{G}$ (D) None

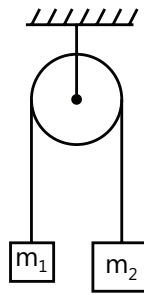
Paragraph 4:

In some parts of universe, it is found that acceleration produced in a body is inversely proportional to the square of its mass and directly proportional to the net

force (F) according to equation $a = c \frac{F}{m^2}$ where c is

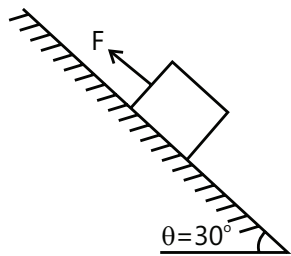
constant, whose magnitude is 1, if m is measured in kg , a is measured in m/s^2 and F is in N . Also action and reaction force are equal and opposite and on different interacting bodies.

Q.22 In the given figure shown, two blocks of mass $m_1 = 2\text{ kg}$ and $m_2 = 4\text{ kg}$ are attached via an ideal massless string over frictionless mass less pulley. If acceleration due to gravity $g = 5\text{ m/s}^2$. The tension in the string is



- (A) 6 Bosc
(C) 3 Bosc
- (B) 1.67 Bosc
(D) 32 Bosc

Q.23 In the given figure a block of mass $m=2$ kg is placed on smooth inclined plane. The minimum value of force F needed to support the block is ($g = 5m/s^2$)



- (A) Zero, Newton
(C) 20 Bosc
- (B) 10 Bosc
(D) 10 Newton

Previous Years' Questions

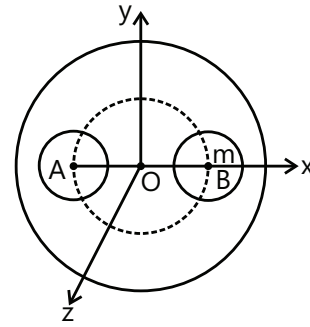
Q.1. Statement-I: An astronaut in an orbiting space station above the earth experiences weightlessness.

Statement-II: An object moving around the earth under the influence of earth's gravitational force is in a state of 'free-fall' **(2008)**

- (A) If statement-I is true, statement-II is true; statement-II is the correct explanation for statement-I
- (B) If statement-I is true; statement-II is true; statement-II is not a correct is true; statement-I
- (C) If statement-I is true; statement-II is false
- (D) If statement-I is false; statement-II is true

Q.2 A solid sphere of uniform density and radius 4 units is located with its center at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centers at $A(-2, 0, 0)$ and $B(2, 0, 0)$ respectively, are taken out of the solid leaving behind spherical cavities as shown in the given figure. **(1993)**

Then



- (A) the gravitational field due to this object at the origin is zero.
- (B) the gravitational field at the point $B(2, 0, 0)$ is zero
- (C) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$
- (D) the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$

Q.3 The magnitude of the gravitational field at distance r_1 and r_2 from the center of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then **(1994)**

- (A) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$
- (B) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$
- (C) $\frac{F_1}{F_2} = \frac{r_1^3}{r_2^3}$ if $r_1 < R$ and $r_2 < R$
- (D) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

Q.4 Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 h and 8 h respectively. The radius of the orbit of S_1 is 10^4 km. when S_2 is closed to S_1 . Find

- (a) the speed of S_2 relative to S_1 ,
- (b) the angular speed of S_2 as actually observed by an astronaut in S_1 **(1986)**

Q.5 Three particles, each of mass m , are situated at the vertices of an equilateral triangle of side length a . The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining the original mutual

separation a . Find the initial velocity that should be given to each particle and also the time period of circular motion **(1988)**

Q.6 An artificial satellite is moving in circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. **(1990)**

(a) Determine the height of the satellite above the earth's surface.

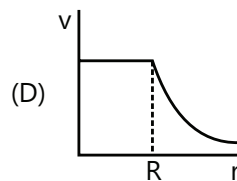
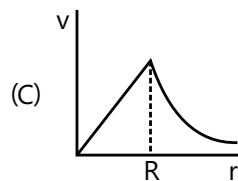
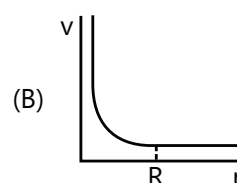
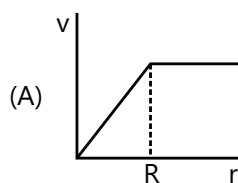
(b) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits surface of the earth.

Q.7 Distance between the centers of two stars is $10a$. The masses of these stars are M and $16M$ and their radii a and $2a$ respectively. A body of mass m is fired straight from the surface of the larger star towards the surface of the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G , M and a . **(1996)**

Q.8 There is a crater of depth $\frac{R}{100}$ on the surface of the moon (radius R). A projectile is fired vertically upward from the crater with velocity, which is equal to the escape velocity v from the surface of the moon. Find the maximum height attained by the projectile. **(2003)**

Q.9 Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 km/s the escape speed on the surface of the planet in km/s will be? **(2010)**

Q.10 A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$ where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r from the center of the system is represented by **(2008)**



Q.11 Two spherical planets P and Q have the same uniform density ρ , masses M_p and M_q , an surface areas A and $4A$, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_p + M_q)$. The escape velocities from the planets P, Q and R, are V_p , V_q and V_r respectively. Then **(2012)**

(A) $V_q > V_r > V_p$ (B) $V_r > V_q > V_p$

(C) $\frac{V_r}{V_p} = 3$ (D) $\frac{V_p}{V_q} = \frac{1}{2}$

Q.12 A planet of radius $R = \frac{1}{10}$ × (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density $10^{-3} \text{ kg m}^{-1}$ into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity of Earth is 10 ms^{-2}) **(2014)**

(A) 96 N (B) 108 N (C) 120 N (D) 150 N

Q.13 In an experiment to determine the acceleration due to gravity g , the formula used for the time period of a periodic motion is $T = 2\pi \sqrt{\frac{7(R-r)}{5g}}$. The values

of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is (are) true? **(2016)**

- (A) The error in the measurement of r is 10%
 (B) The error in the measurement of T is 3.57%
 (C) The error in the measurement of T is 2%
 (D) The error in the determined value of g is 11%

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.23 Q.31 Q.33
Q.35

Exercise 2

Q. 2 Q.13 Q.17
Q.20

JEE Advanced/Boards

Exercise 1

Q.1 Q.5 Q.6
Q.11 Q.16 Q.21

Exercise 2

Q.1 Q.2 Q.5
Q.6 Q.7 Q.8
Q.10 Q.11 Q.12
Q.16

Answer Key

JEE Main /Boards

Exercise 1

Q.3 No

Q.4 Zero

Q.9 $10\sqrt{10}$

Q.10 11 km/s

Q.13 One-Fourth

Q.14 $x=2h$

Q.15 0.732

Q.23 $v_A/v_P = r_P/r_A$; No, time for path BAC is greater than time for path CPB

Q.24 4 R

Q.25 129 days

Q.26 1.972×10^{30} kg

Q.27 $6.69 \times 10^{-11} \text{ m}^2 \text{ kg}^{-2}$

Q.28 6.025×10^{24} kg; $5.56 \times 10^3 \text{ kg/m}^3$

Q.29 Increase by 5%

Q.30 2649.6 km

Q.31 66.12 kg wt; 8.89 kg wt; 5 kg wt.

Q.32 80 km

Q.33 $\frac{5GmM}{6R}$

Q.34 2.56×10^4 km

Q.35 (a) 7756.6m/s; (b) 5373 s

Q.36 5064 s

Q.37 $2.354 \times 10^3 \text{ ms}^{-1}$

Q.38 5.28 kms^{-1}

Exercise 2

Single Correct Choice Type

Q.1 B	Q.2 A	Q.3 B	Q.4 B	Q.5 C	Q.6 A
Q.7 A	Q.8 B	Q.9 B	Q.10 B	Q.11 A	Q.12 B
Q.13 C	Q.14 A	Q.15 C	Q.16 D	Q.17 D	Q.18 C
Q.19 B	Q.20 C	Q.21 A	Q.22 A	Q.23 A	Q.24 C

Previous Years' Questions

Q.1 C	Q.2 A	Q.3 B	Q.4 B	Q.5 D	Q.6 C
Q.7 D	Q.8 B	Q.9 C	Q.10 B	Q.11 A	Q.12 C
Q.13 D	Q.14 A	Q.15 B	Q.16 C		

JEE Advanced/Boards

Exercise 1

$$\text{Q.1 } \frac{GM^2}{3L^2}$$

$$\text{Q.2 } \frac{R_e k^2}{1-k^2}$$

$$\text{Q.3 } \sqrt{\frac{2GM}{a} \left(1 - \frac{1}{\sqrt{2}}\right)}$$

$$\text{Q.4 } h = \frac{\sqrt{3}-1}{2}$$

$$\text{Q.5 (i) } \frac{4}{3} \sqrt{\frac{GM}{R}} \quad \text{(ii) } \frac{2}{3} \sqrt{\frac{2GM}{5R}}$$

$$\text{Q.6 (a) } E = \frac{\pi G \rho_0 R^3}{6} \left[\frac{1}{(x-(R/2))^2} - \frac{8}{x^2} \right], \quad \text{(b) } E = \frac{GM}{2R^2}$$

$$\text{Q.7 } 2R, 3R[3-\sqrt{3}]$$

$$\text{Q.8 } \sqrt{6.4} \text{ km}$$

$$\text{Q.9 } \frac{2\pi R^{3/2} (6\sqrt{6})}{\sqrt{GM} (2\sqrt{2} + 3\sqrt{3})}$$

$$\text{Q.10 R}$$

$$\text{Q.11 } \frac{6GM^2}{2a}$$

$$\text{Q.12 } 2 \times \sqrt{R^3/GM}$$

Q.13 1.6 hours if it is rotating from west to east, 24/17 hours if it is rotating from east to west

$$\text{Q.14 } 1 \times 10^5 \text{ J}$$

$$\text{Q.15 } t = \frac{GMm}{2C} \left(\frac{1}{R_e} - \frac{1}{r} \right)$$

$$\text{Q.16 } \frac{2\pi}{3 \left(\sqrt{\frac{Gm}{8R^3}} - \omega_e \right)}$$

$$\text{Q.17 } \cos \theta = \frac{3}{\sqrt{10}}$$

$$\text{Q.18 (a) } \sqrt{\frac{GM}{r}}; \text{ (b) } r\sqrt{2}; \text{ (c) } \sqrt{\frac{2GM}{R}}$$

$$\text{Q.19 } T = 3 \times 10^{-2} \text{ N}$$

$$\text{Q.20 } r_e = a(3M/m)^{1/3}$$

$$\text{Q.21 (i) } -\frac{3GM}{R} \left(\frac{m}{\sqrt{3}} + m \right), \text{ (ii) } \left(\sqrt{\frac{GM(2\sqrt{3}+R)}{R}} \right)$$

$$\text{Q.22 } 2\pi R^2 \left(1 + \frac{R}{x}\right)$$

$$\text{Q.23 (a) } T = \frac{2\pi d^{3/2}}{\sqrt{3GM}} \quad \text{(b) 2 (c) 2}$$

$$\text{Q.24 } \cot^{-1} \left(\sqrt{3} - \frac{32}{105} \right) \text{ to vertical}$$

$$\text{Q.25 } T = \sqrt{\frac{3\pi}{G\rho}}$$

Exercise 2

Multiple Correct Choice Type

Q.1 A, D

Q.2 A, D

Q.3 B, C, D

Q.4 A, C

Q.5 B, D

Q.6 A, C, D

Q.7 A, D

Q.8 A, B, C

Q.9 A, D

Q.10 A, B, D

Assertion Reasoning Type

Q.11 C

Q.12 B

Q.13 D

Q.14 D

Q.15 A

Comprehension Type

Paragraph 1: Q.16 B

Q.17 A

Paragraph 2: Q.18 B

Q.19 C

Paragraph 3: Q.20 B

Q.21 A

Paragraph 4: Q.22 D

Q.23 B

Previous Years' Questions

Q.1 A

Q.2 A

Q.3 B

Q.4 (a) $-\pi \times 10^4$ km/h (b) 3×10^{-4} rad/s

$$\text{Q.5 } v = \sqrt{\frac{Gm}{a}}, T = 2\pi \sqrt{\frac{a^3}{3Gm}}$$

Q.6 (a) 6400 km (b) 7.9 km/s

$$\text{Q.7 } \frac{3\sqrt{5}}{2} \sqrt{\frac{GM}{a}}$$

Q.8 99.5 R

Q.9 3

Q.10 C

Q.11 B, D

Q.12 B

Q.13 A

Solutions

JEE Main/Boards

Exercise 1

Sol 1: Newton's law of gravitation is called a universal law because it is applicable anywhere in the universe.

Sol 2: The value is same on moon. G is called universal gravitational constant, which is constant anywhere in the universe

Sol 3: The packet doesn't reach the earth (theoretically). Because in a satellite, centrifugal force balances the gravitational force on it. The same will happen with the packet, which has same initial velocity as that of satellite.

Sol 4: Its reading will be zero. A spring balance shows the net force the hanging body exerts on it net force by body = Mass \times Acceleration

\therefore The net acceleration is zero as the centrifugal and gravitational forces balance each other, it reads zero.

Sol 5: Escape velocity is always constant for a given celestial body. Escape velocity of earth is 11.2 km s^{-1} . It is same irrespective of mass.

Sol 6: Weight = Mass \times Net acceleration It will be zero, if net acceleration is zero like a satellite.

Sol 7: Gravitational force $(f) = \frac{GMm}{r^2}$

$\therefore f \propto m$

\therefore Acceleration due to gravity $g = \frac{f}{m}$

$$g = \frac{GM}{r^2}$$

It is independent of m .

Here both fall at same time

Sol 8: $U = \frac{GMm}{r}$

weight $W = mg$

$$= m \cdot \left(\frac{GM}{r^2} \right) = \frac{1}{r} \left(\frac{GMm}{r} \right)$$

$$W = \frac{U}{r}$$

$$\text{Hence weight} = \frac{U}{r}$$

Sol 9: $T \propto r^{3/2}$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2} \right)^{3/2} = \left(\frac{10^{11}}{10^{10}} \right)^{3/2} = 10\sqrt{10}$$

Sol 10: Escape speed is still 11 km s^{-1} because escape speed is irrespective of angle of launch (of course not towards ground). We calculate escape velocity by

$$\frac{1}{2} m v^2 = \frac{GMm}{r}$$

(Kinetic energy + Potential energy)

= it Irrespective of angle of each.

Sol 11: Acceleration due to gravity

$$g = \frac{GM}{r^2}$$

$$g \propto \frac{1}{r^2}$$

Hence it is inversely proportional to r^2

Sol 12: $f \propto \frac{1}{r^2}$

$$f_2 = f_1 \cdot \left(\frac{r_1}{r_2} \right)^2$$

$$\frac{f_1}{f_2} = \frac{1}{2}$$

$$\therefore f_2 = \frac{f_1}{4} = \frac{1}{4} \text{ N}$$

\therefore Gravitational force 0.25 N

Sol 13: If he goes to a height r , his distance from center is $2r$

$$\text{i.e., } \frac{r_1}{r_2} = \frac{1}{2}$$

$$w \propto \frac{1}{r^2} \quad (w = \text{Weight})$$

$$\frac{w_2}{w_1} = \left(\frac{r_1}{r_2}\right)^2$$

$$\Rightarrow \frac{w_2}{w_1} = \left(\frac{1}{2}\right)^2 = \frac{1}{4},$$

i.e., his weight quadrates

Sol 14: $g = \frac{GM}{r^2}$ for $r \geq r_0$ (r_0 is radius of earth)

$$\frac{dg}{dr} = -\frac{GM}{2r^3}$$

$$\Rightarrow \Delta g = -\frac{GM}{2r^3} \cdot \Delta r$$

$$\Rightarrow \Delta g = -\frac{GM}{2r_0^3} \cdot h$$

$$g = \frac{GM}{r_0^3} r_1 \quad r_1 \leq r_0$$

$$\Delta g = \frac{GM}{r_0^3} (r_1 - r_0)$$

$$r_1 - r_0 = -x$$

$$\Rightarrow \Delta g = \frac{GM}{r_0^3} \cdot (-x)$$

$$\Rightarrow \frac{GM}{r_0^3} (-x) = \frac{GM}{r_0^3} (-2h)$$

$$\Rightarrow x = 2h$$

$$\text{Sol 15: } \frac{f_1}{f_2} = \left(\frac{r_2}{r_1}\right)^2$$

$r_1 = r_0$, $r_2 =$ Distance of rocket from center of earth

$$\frac{f_2}{f_1} = \frac{1}{3}$$

$$\Rightarrow 3 = \left(\frac{r_2}{r_1}\right)^2 \Rightarrow \frac{r_2}{r_1} = \sqrt{3}$$

Height of rocket $n = r_2 - r_1$

$$\frac{r_2}{r_1} = \sqrt{3}$$

$$\frac{r_2}{r_1} - 1 = \sqrt{3} - 1$$

$$\Rightarrow \frac{r_2 - r_1}{r_1} = \sqrt{3} - 1$$

$$\Rightarrow \frac{h}{r_1} = 0.732$$

Sol 16: No. gravitational force $f = \frac{GMm}{r^2}$

$$\Rightarrow f \propto \frac{1}{r^2}$$

$\frac{1}{r^2}$ will never become zero.

So force doesn't become zero.

Sol 17: Gravity is the force with which a body pulls another body towards its center.

Acceleration due to gravity is the acceleration which it produces in the body due to force of gravity

$$g = \frac{GM}{r^2}$$

Sol 18: $g = \frac{GM}{r^2}$

$m =$ Mass of earth

$r =$ Radius of earth

$$m = \frac{r^2 g}{G}$$

= We can calculate g by physical means, r is known, hence M can be calculated

$$\text{Mean density of earth} = \frac{M}{\frac{4}{3}\pi r^3}$$

$\therefore M$ can be calculated, mean density can also be calculated.

Sol 19: Escape velocity is the velocity with which when a body is projected from the surface of a celestial body, it crosses its potential barrier and escapes into out space for bodies to escape total energy ≥ 0

i.e. $K.E + P.E \geq 0$

At escape velocity

$$K.E + P.E = 0$$

$$\Rightarrow \frac{1}{2}mv^2 - \frac{GMm}{r} = 0$$

$$\Rightarrow v = \sqrt{\frac{2GM}{r}}$$

Sol 20: Gravity field is a field in which a body produces a force on another body.

Intensity of gravitational force is the force which a body attracts a body of unit mass

$$\begin{aligned} \text{Intensity} &= \frac{GM}{r^2} \quad \dots (i) \\ &= \frac{GM}{r^2} = g \end{aligned}$$

Sol 21: Kepler's law $T^2 \propto r^3$

$$\begin{aligned} T &= \frac{2\pi r}{v} ; \Rightarrow \frac{r^2}{v^2} \propto r^3 \\ \Rightarrow v^2 &\propto \frac{1}{r} \quad \dots (i) \end{aligned}$$

For a planetary motion

$$PE + KE = 0$$

$$\Rightarrow \frac{1}{2} mv^2 + PE = 0$$

$$\Rightarrow PE = -\frac{1}{2} mv^2 ; \Rightarrow PE \propto -\frac{m}{r}$$

$$\text{Gravitational force} = \frac{d}{dr} PE$$

$$\Rightarrow F \propto \frac{d}{dr} \left(-\frac{m}{r} \right)$$

$$\Rightarrow f \propto \frac{m}{r^2} \Rightarrow f = \frac{km}{r^2} \text{ (k-some constant)}$$

Let two bodies m_1, m_2 exert gravity on each other.

$$f_{12} = \frac{k_2 m_2}{r^2} \Rightarrow f_{12} \propto m_1 \quad \dots (i)$$

$$f_{21} = \frac{k_1 m_1}{r^2} \Rightarrow f_{21} \propto m_2 \quad \dots (ii)$$

Where f_{12} is force on body 1 by body 2 similarly f_{21} defined

but $|f_{12}| = |f_{21}|$ newton's third law

$$\Rightarrow t_{12} \propto m_2$$

$$\Rightarrow f_{12} = k_3 \frac{m_1 m_2}{r^2} \text{ (k}_3\text{-some constant)}$$

$$\Rightarrow f \propto \frac{m_1 m_2}{r^2}$$

Hence newton's law of gravity is deduced.

Sol 22: Newton's law of universal gravitation states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of distance between them.

$$f \propto \frac{m_1 m_2}{r^2}$$

Gravitational constant (G) is a empirical physical constant involved in the calculation of gravitational

force between two bodies. In simple terms it can be sides the proportionality constant for Newton's law of gravitation

$$G = 6.67 \times 10^{-4} \text{ N} \left(\frac{\text{m}}{\text{kg}} \right)^2$$

$$\text{Dimensional formula [G]} = \frac{fL^2}{m^2}$$

$$f = \frac{GMm}{r^2}$$

$$[f] = \frac{[G][M][m]}{[r]^2} = \frac{FL^2}{m^2} \cdot \text{m.m.L}^{-2} = f$$

It supports Newton's law empirically

Sol 23: The distance travelled \overline{dx} in time dt is $d\overline{v}$

$$= \overline{v} \cdot dt$$

Area swept by radius vector

$$dA = \frac{1}{2} \vec{r} \times \vec{v} dt$$

$$dA_A = \frac{1}{2} r_A v_A dt \text{ and}$$

$$dA_P = \frac{1}{2} r_P v_P dt$$

$$\therefore \frac{dA}{dt} = \text{Constant, (Kepler's 2}^{\text{nd}} \text{ law)}$$

$$\Rightarrow r_A v_A = r_P v_P$$

The time taken is different as

area of SBAC \pm SCPB

Sol 24: $T^2 \propto r^3$

$$\Rightarrow \frac{r_1}{r_2} = \left(\frac{T_1}{T_2} \right)^{\frac{2}{3}} ; \Rightarrow r_1 = r_2 \left(\frac{T_1}{T_2} \right)^{\frac{3}{2}}$$

$$\text{gen } rL = R, T_1 = 32, T_2 = 256$$

$$\Rightarrow r_1 = 4R$$

$$\text{Sol 25: } \therefore T_1 = T_2 = \left(\frac{r_1}{r_2} \right)^{\frac{3}{2}} \text{ (kepler's 3}^{\text{rd}} \text{ law)}$$

$$\frac{r_1}{r_2} = \frac{1}{2}$$

$$\therefore T_1 = 365 \left(\frac{1}{2} \right)^{\frac{3}{2}} = 129 \text{ days}$$

1 year would have 129 days.

Sol 26: For planetary motion,

$$\frac{1}{2}mv^2 = \frac{GMm}{r}; \Rightarrow V = \sqrt{\frac{2GM}{r}}$$

$$V = \frac{2\pi r}{T}; \Rightarrow T = 2\pi \sqrt{\frac{\pi^3}{2GM}}$$

$$\Rightarrow M = \frac{(2\pi)^2 r^3}{2GT^2}$$

$$= \frac{4\pi^2 \cdot (1.49 \times 10^{11})^3}{2 \times 6.66 \times 10^{-11} \times (365 \times 24 \times 3600)^2}$$

$$= 1.972 \times 10^{30} \text{ kg}$$

Sol 27: $M = \frac{(2\pi)^2 r^3}{2GT^2}; \Rightarrow G = \frac{(2\pi)^2 r^3}{2MT^2}$

$$r = 1.5 \times 10^{11} \text{ m}$$

$$T = 365.3 \text{ days} = 365.3 \times 24 \times 3600 \text{ seconds}$$

$$M = 2 \times 10^{30} \text{ Kg}$$

$$\Rightarrow G = \frac{4(\pi)^2 \cdot (1.5 \times 10^{11})^3}{2 \times 2 \times 10^{30} \times (365.3 \times 24 \times 3600)^2}$$

$$= 6.69 \times 10^{-11} \text{ m}^2/\text{Kg}^{-2}$$

Note: For calculation purpose, you may take $\pi^2 = 10$

Sol 28: $g = \frac{GM}{r^2}$

$$M = \frac{gr^2}{G}$$

$$= \frac{9.8 \times (6.37 \times 10^6)^2}{(6.66 \times 10^{-11})} = 6.025 \times 10^{24} \text{ kg}$$

$$\text{Mean density} = \frac{M}{\frac{4}{3}\pi r^3} = \frac{gr^2}{G} \cdot \frac{1}{\frac{4}{3}\pi r^3}$$

$$= \frac{3}{4} \frac{g}{Gr} = \frac{3}{4} \times \frac{9.8}{6.66 \times 10^{-11} \times 6.37 \times 10^6}$$

$$\text{Mean density (e)} = 5.56 \times 10^3 \text{ kg/m}^3$$

Sol 29: $g = \frac{GM}{r^2}; \frac{dg}{dr} = -\frac{2GM}{r^3}$

$$\Rightarrow \Delta g = -\frac{2GM}{r^3} \Delta r; \frac{\Delta g}{g} = \frac{-2GM}{\frac{GM}{r^2}} \Delta r$$

$$\frac{\Delta g}{g} = -2 \left(\frac{\Delta r}{r} \right); \frac{\Delta g}{g} \times 100 = -2 \frac{\Delta r}{r} \times 100$$

$$\frac{\Delta r}{r} \times 100 = 2.5\%; \Rightarrow \frac{\Delta g}{g} \times 100 = -2(-2.5)$$

$$= 5\%$$

\therefore Acceleration due to gravity increases by 5%

Note :- Try focusing on the sign convention. If you get confused, use common sense which implies when body gets denser, its g increases like a black hole, etc.

Sol 30: $g \propto \frac{1}{r^2}$

$$\frac{g_1}{g_2} = \left(\frac{r_2}{r_1} \right)^2; \Rightarrow r_2 = r_1 \sqrt{\frac{g_1}{g_2}}$$

$$\frac{g_1}{g_2} = 2; \Rightarrow r_2 = r_1 \sqrt{2}$$

$$\text{Height} = r_2 - r_1 = r_1 (\sqrt{2} - 1)$$

$$= 2649.6 \text{ km}$$

Sol 31: $wt \propto \frac{1}{r^2}$

$$w_2 = w_1 \cdot \left(\frac{r_1}{r_2} \right)^2$$

$$r_1 = 6000 \text{ km}$$

$$r_2 = 6600, 18000, 2400 \text{ (} r_2 = r_1 + h \text{)}$$

$$w_1 = 80 \text{ kg wt.}$$

$$\text{for } r_2 = 6600$$

$$w_2 = 80 \left(\frac{6000}{6600} \right)^2 = 66.12 \text{ kg wt}$$

$$\text{for } r_2 = 18000$$

$$w_2 = 80 \left(\frac{6000}{18000} \right)^2 = 8.89 \text{ kg wt}$$

$$\text{for } r_2 = 24000$$

$$w_2 = 80 \left(\frac{6000}{24000} \right)^2 = 5 \text{ kg.wt}$$

Sol 32: g at a depth x , $g_x = \frac{GM}{r_0^3} (r_0 - x)$

$$g \text{ at a height } h, g_h = \frac{GM}{(r_0 + h)^2}$$

$$g_x = gh$$

By substituting we get solution

But for some intelligent manipulation

$$g_x = \frac{gm}{r_0^3}(r_0 - x) \Rightarrow \Delta g = \frac{-GMx}{r_0^3}$$

$$g = \frac{GM}{r_0^3}$$

$$\Delta g = -\frac{2GM}{x^2} \Delta r \text{ (Differentiation)}$$

$\therefore g$ is equal $\Rightarrow \Delta g$ is equal

$$\Rightarrow -\frac{2GM}{r_0^3} \Delta r = -\frac{GM}{r_0^3} x; \Rightarrow \Delta r = \frac{x}{2}$$

$$\Delta r = n$$

$$x = 160\text{km}; \Rightarrow h = \frac{160}{2} = 80\text{ km}$$

\Rightarrow It is same at a height 80 km

Note: $h < r_0$ is assumed hence we could apply this method of differentiation

Sol 33: Energy required = Total change in energy Initial

$$\text{energy} = -\frac{GMm}{R}$$

Find energy = P. E + K. E

$$= -\frac{GMm}{3R} + \frac{1}{2}mv^2$$

($r = 3R$ because altitude = $2R$)

For orbital motion, centrifugal force = Gravitational for

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$= \frac{GMm}{6R} \text{ (substitute } r = 3R)$$

$$\Rightarrow \text{Final energy} = \frac{GMm}{6R} - \frac{GMm}{3R} = -\frac{GMm}{6R}$$

$$\Rightarrow \text{Energy required} = -\frac{GMm}{6R} - \left(\frac{GMm}{R}\right)$$

$$= \frac{5GMm}{6R}$$

Sol 34: Kinetic energy = Change in potential energy

$$\Rightarrow \frac{1}{2}mv^2 = -\frac{GMm}{r} - \left(\frac{GMm}{R}\right)$$

$$\Rightarrow \frac{1}{r} = \frac{1}{R} - \frac{1}{2} \frac{v^2}{GM}$$

$$\frac{1}{r} = \frac{1}{R} - \frac{1}{2} \frac{v^2}{gR^2}$$

$$\frac{1}{r} = \frac{1}{6400} - \frac{1}{2} \times \frac{10^2}{9.8 \times 10^{-3} \times (6400)^2}$$

$$r = 2.56 \times 10^4 \text{ km}$$

Sol 35: Orbital velocity $v = \sqrt{\frac{GM}{r}}$

$$= \sqrt{\frac{GM}{R+H}} = \sqrt{\frac{gR^2}{R+H}}$$

$$g = 9.8 \text{ ms}^{-1}$$

$$R = 6.38 \times 10^6$$

$$H = 250 \text{ km} = 2.5 \times 10^5$$

$$v = 7756.6 \text{ ms}^{-1}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(R+H)}{v}$$

$$T = 5373 \text{ s}$$

Sol 36: Let orbital velocity = v

$$\Rightarrow v = \sqrt{\frac{GM}{R}} \text{ where } m = \text{Mass, } R = \text{Radius of point}$$

$$T = \frac{2\pi R}{v} = 2\pi \sqrt{\frac{R^3}{GM}} \quad \dots (i)$$

$$M = \frac{4}{3} \pi R^3 \rho$$

$$\Rightarrow T = 2\pi \sqrt{\frac{R^3}{G \cdot \frac{4}{3} \pi R^3 \rho}}$$

$$= 2\pi \sqrt{\frac{3}{4\pi G \rho}} = 5064 \text{ s}$$

Sol 37: For escape velocity

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$v_e = \sqrt{2gR}$$

$$= \sqrt{2 \times 1.63 \times 17 \times 10^6}$$

$$= 2.354 \times 10^3 \text{ ms}^{-1}$$

Sol 38: $v_e \propto \sqrt{\frac{M}{R}}$

$$\frac{v_m}{v_e} = \sqrt{\frac{M_m}{R_m} \times \frac{R_e}{M_e}}$$

$$v_m = v_e \sqrt{\left(\frac{M_m}{M_e}\right) \times \left(\frac{R_e}{R_m}\right)}$$

given = $\frac{M_m}{M_e} = \frac{1}{9}$

$$\frac{R_e}{R_m} = 2$$

$$\Rightarrow v_m = v_e \sqrt{\frac{2}{9}} = 2 \sqrt{\frac{2}{9}} = 5.28 \text{ kms}^{-1}$$

M_m = Mass of planet mass

R_m = Radius of planet mass

Exercise 2

Single Correct Choice Type

Sol 1: (B) $g \propto \frac{1}{r^2}$

$$\frac{g_1}{g_2} = \left(\frac{r_2}{r_1}\right)^2 ; \Rightarrow \frac{r_2}{r_1} = \sqrt{\frac{g_1}{g_2}} = \sqrt{4}$$

$$\Rightarrow r^2 = 2R ; \Rightarrow \text{altitude } h = r_2 - R$$

$$h = R$$

Sol 2: (A) Net acceleration at equator

$$g' = g - R\omega^2$$

($R\omega^2$ is radial acceleration)

$$\therefore \text{Weight at equator } mg' = mg - mR\omega^2$$

acceleration at a depth d

$$\Rightarrow g_d = \frac{g(R-d)}{R} = g - \frac{g}{R}d$$

given $mg_d = mg'$

$$\Rightarrow mg - mR\omega^2 = mg - \frac{mgd}{R} \Rightarrow d = \frac{R^2 \omega^2}{g}$$

Sol 3: (B) $g = \frac{GM}{r^2}$

$$m = \frac{4}{3} \pi r^3 \rho ; \Rightarrow g = \frac{4}{3} \pi r \rho$$

$rp = \text{constant}$

$$\therefore \frac{r_2}{r_1} = \frac{\rho_1}{\rho_2} = \frac{1}{5} \left(\frac{\rho_2}{\rho_1} = 5 \text{ given} \right)$$

\therefore Radius to be changed by a factor of $\frac{1}{5}$

Sol 4: (B) $T = 2\pi \sqrt{\frac{\ell}{g}}$

$$T \propto \frac{1}{\sqrt{g}} ; g = \frac{GM}{r^2}$$

$$\frac{1}{\sqrt{g}} = \frac{r}{\sqrt{M}} ; \frac{T_1}{T_2} = \frac{r_1}{r_2} \sqrt{\frac{M_2}{M_1}}$$

$$T_2 = T_1 \left(\frac{r_2}{r_1} \sqrt{\frac{M_1}{M_2}} \right)$$

$$\frac{r_2}{r_1} = 2 ; \frac{M_1}{M_2} = \frac{1}{2} ; T_1 = 2 \text{ seconds}$$

$$\therefore T_2 = (2 \times 2) \left(\frac{1}{\sqrt{2}} \right) = 2\sqrt{2} \text{ second}$$

Note: Time period of a seconds pendulum is 2 seconds.

Sol 5: (C) Inside the shell, the gravitational field due to sphere is zero, but there is gravity due to particle.

Sol 6: (A) $g - \frac{v^2}{R} = \frac{g}{2} ; \Rightarrow \frac{v^2}{R} = \frac{g}{2}$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{1}{2} \frac{GM}{R^2} (4R)}$$

$$= \sqrt{\left(\frac{g}{2}\right) (4R)} = \sqrt{\frac{v^2}{R} (4R)}$$

$$v_e = 2v$$

Sol 7: (A) $v \propto \sqrt{\frac{M}{R}}$

$$\frac{M}{R} = \frac{\frac{4}{3} \pi R^3 \rho}{R} \propto R^2 ; \Rightarrow v \propto R$$

$$\Rightarrow \frac{v_A}{v_B} = \frac{R_A}{R_B} = 2$$

$$\text{Sol 8: (B)} \quad v_e = \sqrt{\frac{2GM}{R}}$$

$$\text{P.E at surface} = \frac{GMm}{R}$$

$$\text{P.E at centre of earth} = \frac{3GMm}{2R}$$

$$\text{KE} = \Delta\text{PE} = \frac{1}{2} \frac{GMm}{R}$$

$$\frac{1}{2}mv^2 = \frac{1}{2} \frac{GMm}{R}; \Rightarrow v = \sqrt{\frac{GM}{R}}$$

$$= \frac{v_e}{\sqrt{2}}$$

Sol 9: (B) Potential at surface = Potential at center for hollow sphere

$$\text{Potential } P = -\frac{GM}{r}$$

r = radius

$$\text{Let } P_0 = -\frac{GM}{r_0}$$

$$\text{new potential } P = -\frac{GM}{\frac{r_0}{2}} = -\frac{2GM}{r_0}$$

$$P < P_0$$

\therefore Decreases

$$\text{Sol 10: (B)} \quad g \propto \frac{1}{r^2}$$

Sol 11: (A) In A, both $5M$ forces, cancel each other hence net force is proportional to $(3M - M)$ and M (at center)

The same is for B.

$$\therefore F_A = F_B$$

$$\text{Similarly } f_c \propto (3m - m)2m \therefore f_c > f_b$$

same is for F_D

$$\therefore F_c = f_D$$

$$\therefore F_A = F_B < F_c = F_D$$

Sol 12: (B) It moves tangentially as there is no centripetal force.

Sol 13: (C) Time period of a geo stationary satellite is 24 hrs but due to the given situation, it moves twice above same point in one day

$$\therefore \text{Time for successful interval} = \frac{24}{2} = 12 \text{ Hrs}$$

Sol 14: (A) Let centre of mass be at a distance ℓ from $4m$

$$\Rightarrow 4m\ell = m(d - \ell)$$

$$\Rightarrow \ell = \frac{d}{5}$$

$$\Rightarrow \text{Orbital radius of } 4m = \frac{d}{5}, m = \frac{4d}{5}$$

Both bodies have same angular velocities

$$\Rightarrow \frac{v}{r} = \text{constant}$$

$$\Rightarrow v = Kr \quad (K = \text{constant})$$

$$v_{4m} = \frac{kd}{5}, v_m = \frac{4kd}{5}$$

$$\frac{KE_{4m}}{KE_m} = \frac{\frac{1}{2}(4m)(v_{4m})^2}{\frac{1}{2}m(v_m)^2} = 4 \left(\frac{1}{4}\right)^2 = \frac{1}{4}$$

Sol 15: (C) C-options defines the information about the escape velocities

B-option it is elliptical path

Sol 16: (D) Change in potential energy

$$\Delta E = \frac{GMm}{R} - \frac{GMm}{2R} = \frac{GMm}{2R}$$

Final velocity or orbital velocity

$$v_0 = \sqrt{\frac{GM}{2R}}$$

Change in P.E = Change in K.E.

$$\frac{GMm}{2R} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2R} + \frac{1}{4}m \cdot \frac{GM}{R}$$

$$\frac{1}{2}mv^2 = \frac{3}{4} \frac{GMm}{R}$$

$$\frac{1}{2}mv^2 = \frac{3}{4}mgR$$

$$\therefore \text{Energy required is } \frac{3}{4}mgR$$

Sol 17: (D) $K.E \propto \frac{1}{r}$

\therefore A is K.E.

Total energy > Potential energy,

total energy, potential energy $\propto -\frac{1}{r}$

\therefore C is total energy

B is potential energy

Sol 18: (C) Kepler's 2nd law, areal velocity is constant.

Sol 19: (B) Let final velocity of 4 M be V_1

$$5MV = 4MV_1 - MV$$

(Conservation of linear momentum)

$$\Rightarrow V_1 = \frac{3}{2}V$$

$$\text{now } V = \sqrt{\frac{GM}{r}}$$

$$V_e = \sqrt{\frac{2GM}{r}} = \sqrt{2}V = V_1 > \sqrt{2}V$$

i.e., $V_1 > V_e$

\Rightarrow Body gets unbound

Sol 20: (C) $T \propto r^{3/2}$

$$\frac{T_2}{T_1} = \frac{1}{2}; \left(\frac{r_2}{r_1}\right)^{3/2} = \frac{1}{2}$$

$$\Rightarrow r_2 = r_1 \left(\frac{1}{2}\right)^{2/3} = \frac{r}{(4)^{2/3}}$$

Sol 21: (A) Consider the planet to be at one of the vertex.

Let its distance from sun be r , velocity be v .

Area covered in time dT

$$dA = \frac{1}{2}rv \cdot dT; \frac{2dA}{dT} = vr$$

$$mvr = \frac{2mdA}{dT}; \frac{dA}{dT} = \frac{A}{T}$$

$$\therefore \text{Angular momentum} = \frac{2mA}{T}$$

Sol 22: (A) $\frac{dA}{dt} \propto rv$

$$V \propto \frac{1}{\sqrt{r}}; \therefore \frac{dA}{dt} \propto \sqrt{r}$$

\therefore ratio of their area velocity

$$= \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{1}{4}} = 1 : 2$$

Sol 23: (A) Field due to gravity $E_1 = \frac{GM}{R^2}$

$$m = \frac{4}{3}\pi R^3; \therefore E_1 = \frac{4}{3}\pi G\rho R$$

Field due to infinite plate $E_2 = 2\pi\rho tG$

$$(t = H) = 2\pi\rho HG$$

$$E_1 = E_2$$

$$\therefore \frac{4}{3}\pi G\rho R = 2\pi\rho HG; \Rightarrow H = \frac{2R}{3}$$

Sol 24: (C) Maximum speed occurs at least distance

$$\frac{dA}{dt} = \frac{1}{2}r_{\min} v_{\max}$$

$$4 \times 10^{16} = \frac{1}{2} \times 2 \times 10^{12} \times v$$

$$v = 4 \times 10^4 \text{ ms}^{-1}$$

$$\therefore v = 40 \text{ kms}^{-1}$$

Previous Years' Questions

Sol 1: (C) $g = \frac{GM}{R^2}$

or $g \propto \frac{1}{R^2}$

g will increase if R decreases

Sol 2: (A) $\Delta U = \frac{mgh}{1 + \frac{h}{R}}$

Given, $h = R$

$$\Delta U = \frac{mgR}{1 + \frac{R}{R}} = \frac{1}{2}mgR$$

Sol 3: (B) $\frac{mv^2}{R} \propto R^{-5/2}; \therefore v \propto R^{-3/4}$

Now, $T = \frac{2\pi R}{v}$

$$\text{or } T^2 \propto \left(\frac{R}{v}\right)^2$$

$$\text{or } T^2 \propto \left(\frac{R}{R^{-3/4}}\right)^2$$

$$\text{or } T^2 \propto R^{7/2}$$

Sol 4: (B) From Kepler's third law

$$T^2 \propto r^3 \text{ or } T \propto (r)^{3/2}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2}$$

$$\text{or } T_2 = T_1 \left(\frac{r_2}{r_1}\right)^{3/2} = (365) \left(\frac{1}{2}\right)^{3/2}$$

$$T_2 \approx 129 \text{ days}$$

$$\text{Sol 5: (D)} \quad T \propto \frac{1}{\sqrt{g}} \text{ i.e., } \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}}$$

where g_1 = Acceleration due to gravity on Earth's surface
= g

g_2 = Acceleration due to gravity at a height $h = R$ from earth's surface = $g/4$

$$\left[\text{Using } g(h) = \frac{g}{\left(1 + \frac{h}{R}\right)^2} \right] \frac{T_2}{T_1} = \sqrt{\frac{g}{g/4}} = 2$$

Sol 6: (C) Time period of a satellite very close to earth's surface is 84.6 min. Time period increases as the distance of the satellite from the surface of earth increase. So, time period of spy satellite orbiting a few 100 km above the earth's surface should be slightly greater than 84.6 min. Therefore. The most appropriate option is (C) or 2 h.

Sol 7: (D) In case of binary star system angular velocity and hence the time period of both the stars are equal.

Sol 8: (B) In circular orbit of a satellite, potential energy

$$= -2 \times (\text{kinetic energy})$$

$$= -2 \times \frac{1}{2} mv^2 = -mv^2$$

Just to escape from the gravitational pull, its total mechanical energy should be zero therefore, its kinetic energy should be $+mv^2$

Sol 9: (C)

$$V_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G \times 10M}{R/10}} = 10 \times 11 = 110 \text{ km/s}$$

Sol 10: (B) $g = GM/r^2$

Sol 11: (A) $g' = \frac{GM}{(R+h)^2}$, acceleration due to gravity at height h

$$\Rightarrow \frac{g}{9} = \frac{GM}{R^2} \frac{R^2}{(R+h)^2} = g \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \frac{1}{9} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{3}$$

$$\Rightarrow 3R = R+h \Rightarrow 2R = h$$

Sol 12: (C) Position of the null point from mass m ,

$$x = \frac{r}{1 + \sqrt{\frac{4m}{m}}} = \frac{r}{3}$$

$$V = -Gm \left(\frac{3}{r} + \frac{12}{2r} \right) = -9 \frac{Gm}{r}$$

Sol 13: (D) To launch the spaceship out into free space, from energy conservation,

$$\frac{-GMm}{R} + E = 0$$

$$E = \frac{GMm}{R} = \left(\frac{GM}{R^2}\right) mR = mgR$$

$$= 6.4 \times 10^{10} \text{ J}$$

$$\text{Sol 14: (A)} \quad E_f = \frac{1}{2} mv_0^2 - \frac{GmM}{3R} = \frac{1}{2} m \frac{GM}{3R} - \frac{GmM}{3R}$$

$$= \frac{GmM}{3R} \left(\frac{1}{2} - 1 \right) = \frac{-GmM}{6R}$$

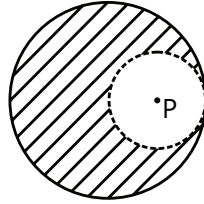
$$E_i = \frac{-GmM}{R} + K$$

$$E_i = E_f$$

$$K = \frac{5GmM}{6R}$$

Sol 15: (B) Potential at point P due to complete solid sphere

$$\begin{aligned}
 &= -\frac{GM}{2R^3} \left(3R^2 - \left(\frac{R}{2} \right)^2 \right) \\
 &= -\frac{GM}{2R^3} \left(3R^2 - \frac{R^2}{4} \right) \\
 &= -\frac{GM}{2R^3} \left(\frac{11R^2}{4} \right) = -\frac{11GM}{8R}
 \end{aligned}$$



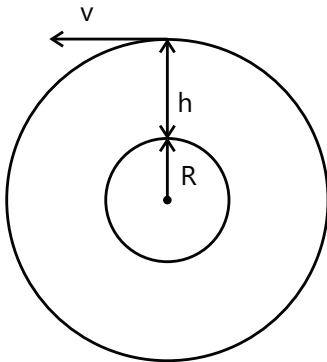
Potential at point P due to cavity part

$$= -\frac{3}{2} \frac{GM}{R} = -\frac{3GM}{8R}$$

So potential due to remaining part at point P

$$\begin{aligned}
 &= \frac{-11GM}{8R} - \left(\frac{-3GM}{8R} \right) \\
 &= \frac{-11GM + 3GM}{8R} = \frac{-8GM}{8R} = -\frac{GM}{R}
 \end{aligned}$$

Sol 16: (C) $\frac{GmM}{(R+h)^2} = \frac{GMm}{R}$



$$v = \sqrt{\frac{GM}{R}}$$

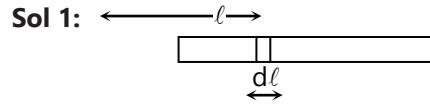
$$\frac{1}{2}mv_1^2 - \frac{GMm}{R} = 0$$

$$v_1 = \sqrt{\frac{2GM}{R}}$$

$$\Delta V = \sqrt{\frac{GM}{R}}(\sqrt{2} - 1) = \sqrt{gR}(\sqrt{2} - 1)$$

JEE Advanced/Boards

Exercise 1



consider a small strip of rod of length $d\ell$ at a distance ℓ from the small mass. Let the mass of strip be dm

$$df = \frac{GM}{\ell^2} dm \quad (df = \text{force of attraction between the strip and small mass})$$

and small mass)

$$dm = \sigma d\ell$$

where σ is linear density of rod

$$\sigma = \frac{M}{2L}$$

$$\Rightarrow df = \frac{GM}{\ell^2} \cdot \sigma d\ell$$

integrating from L to $3L$

$$\int_0^f df = \int_L^{3L} \frac{GM}{\ell^2} \sigma d\ell$$

$$f = GM\sigma \cdot \left. -\frac{1}{\ell} \right|_L^{3L}$$

$$f = GM\sigma \cdot \frac{2}{3L}$$

$$= GM \cdot \frac{M}{2L} \cdot \frac{2}{3L}$$

$$F = \frac{GM^2}{3L^2}$$

Note: Try understanding the boundary conditions. It is most important aspect of physics. Here it is integrated from L to $3L$ because the rod starts from distance L till distance $3L$ from the small mass.

Sol 2: $v_c = \sqrt{\frac{2GM}{R}}$

Kinetic energy = Change in potential energy

$$\frac{1}{2}mv^2 = -\frac{GMm}{r} - \left(-\frac{GMm}{R} \right)$$

$$\frac{1}{2}m \cdot (v_e)^2 = \frac{GMm}{R} - \frac{GMm}{r}$$

$$\Rightarrow \frac{1}{R} - \frac{1}{r} = \frac{1}{2} \cdot m \cdot k^2 \cdot 2 \frac{GM}{R} \cdot \frac{1}{GMm}$$

$$\Rightarrow \frac{1}{R} - \frac{1}{r} = \frac{k^2}{R}$$

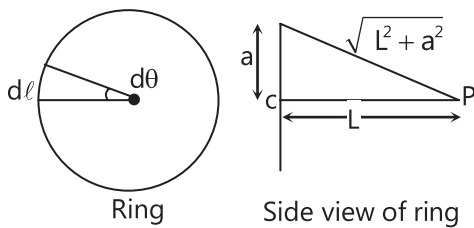
$$\Rightarrow r = \frac{R}{1-k^2}$$

height = $r - R$

$$= \frac{R}{1-k^2} - R = \frac{k^2 R}{1-k^2}$$

Hence it will rise to a height of $\frac{k^2 R}{1-k^2}$

Sol 3: Consider a small path on the ring of length $d\ell$, which subtends an angle $d\theta$ at the center. Let its mass be dM



$$d\ell = a d\theta$$

$$dM = \sigma d\ell$$

$$r = \text{Linear density of ring} = \frac{M}{2\pi a}$$

$$\Rightarrow dM = \frac{M}{2\pi a} \cdot a d\theta = \frac{M}{2\pi} d\theta$$

Let the particle be at a distance x along the axis from center.

Potential energy due to \rightarrow Mass patch

$$dE = \frac{GM}{r} \cdot dM \quad (m = \text{mass of particles})$$

$$= \frac{GM}{r} \cdot \frac{M}{2\pi} d\theta$$

$$r = \sqrt{L^2 + a^2}$$

$$\int_0^E dE = \int_0^{2\pi} \frac{GMm}{\sqrt{L^2 + a^2}} \cdot \frac{1}{2\pi} d\theta$$

$$E = \frac{GMm}{\sqrt{L^2 + a^2}} \times \frac{2\pi}{2\pi} = \frac{GMm}{L^2 + a^2}$$

Kinetic energy = Change in potential energy

$$\frac{1}{2} mv^2 = \frac{GMm}{a} - \frac{GMm}{\sqrt{a^2 + L^2}}$$

$$\frac{1}{2} mv^2 = \frac{GMm}{a\sqrt{a^2 + L^2}} (\sqrt{a^2 + L^2} - a)$$

$$v = \sqrt{\frac{2GM}{a\sqrt{a^2 + L^2}} (\sqrt{a^2 + L^2} - a)}$$

Here given $\ell = a$

$$\Rightarrow v = \sqrt{\frac{2GM(\sqrt{2}-1)}{a\sqrt{2}}}$$

$$v = \sqrt{\frac{2GM}{a} \left(1 - \frac{1}{\sqrt{2}}\right)}$$

Sol 4: Let the height be h

$$g = \frac{GM}{(R+h)^2} \quad (\text{above the surface})$$

g below the surface

$$g = \frac{GM(R-h)}{R^3}$$

$$\Rightarrow \frac{GM}{(R+h)^2} = \frac{GM(R-h)}{R^3}$$

$$\Rightarrow h^3 + h^2R - hR^2 = 0$$

$h = 0$ (which is an obvious solution)

$$h^2 + hR - R^2 = 0$$

$$h = -\frac{R \pm \sqrt{3R^2}}{2}$$

$$h = \frac{\sqrt{3}-1}{2} \quad (\because h > 0)$$

Sol 5: (i) Maximum height = $8R$

\Rightarrow Distance from center of earth (r)

$$= 8R + R$$

$$= 9R$$

Kinetic energy = Change in potential energy

$$\frac{1}{2} mv^2 = -\frac{GMm}{9R} - \left(-\frac{GMm}{R}\right)$$

$$\frac{1}{2} mv^2 = GMm \left(\frac{8}{9R}\right)$$

$$v = \sqrt{\frac{16GM}{9R}}; v = \frac{4}{3} \sqrt{\frac{GM}{R}}$$

(ii) Half minimum height = 4R

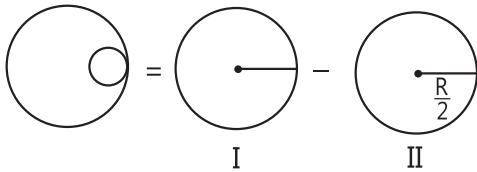
$$\Rightarrow r = 4R + R = 5R$$

$$\Rightarrow \frac{1}{2}mv^2 = -\frac{GMm}{9R} - \left(-\frac{GMm}{5R}\right)$$

$$\Rightarrow v = \sqrt{\frac{8GM}{45R}}; v = \frac{2}{3} \sqrt{\frac{2GM}{5R}}$$

Sol 6: We use principal of superposition gravitation field due to sphere I

$$E_1 = -\frac{GM}{x^2}; x > R,$$



$$= -\frac{GM}{R^3} \cdot x; x < R$$

Let M_2 mass of sphere II

Here the center is at $\frac{R}{2}$ hence distance function is $R - \frac{R}{2}$.

Assume sphere is uniform

$$\Rightarrow M = \frac{4}{3} \pi R^3$$

$$m_2 = \frac{4}{3} \pi \left(\frac{R}{2}\right)^3 = \frac{1}{8} \left(\frac{4}{3} \pi R^3\right) = \frac{M}{8}$$

$$E_2 = -\frac{GM_2}{\left(x - \frac{R}{2}\right)^2} \quad x < 0, x > R$$

$$-\frac{GM_2}{\left(\frac{R}{2}\right)^3} \left(x - \frac{R}{2}\right) \quad 0 < x < R$$

$$= -\frac{GM}{8\left(R - \frac{R}{2}\right)^2} \quad x < 0, x > R$$

$$-\frac{GM_2}{R^3} \left(x - \frac{R}{2}\right)$$

for $x > R$

$$\begin{aligned} E &= E_1 - E_2 \\ &= -\frac{GM}{x^2} + \frac{GM}{8\left(x - \frac{R}{2}\right)^2} \\ &= \frac{GM}{8} \left[\frac{1}{\left(x - \frac{R}{2}\right)^2} - \frac{8}{x^2} \right] \end{aligned}$$

$$M = \frac{4}{3} \pi R^3 \rho_0$$

$$\Rightarrow E = \frac{\pi G \rho_0 R^3}{6} \left[\frac{1}{\left(x - \frac{R}{2}\right)^2} - \frac{8}{x^2} \right]$$

for $x < R$

$$\begin{aligned} E &= E_1 - E_2 \\ &= -\frac{GM}{R^3} x + \frac{GM}{R^3} \left(x - \frac{R}{2}\right) \end{aligned}$$

$$E = \frac{GM}{2R^2}$$

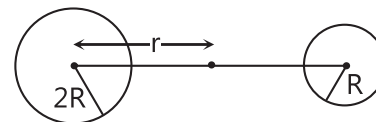
If is independent of x , hence uniform

Sol 7: Potential energy due to planet, at a distance r from its cents

$$P.E_1 = -\frac{G(3M)m}{r}; 2R < r < 5R$$

Potential due to plant 2

$$P.E_2 = -\frac{GMm}{(6R - r)}; R < r < 4R$$



$$\begin{aligned} P.E &= P.E_1 + P.E_2 \\ &= -\frac{3GMm}{r} - \frac{GMm}{6R - r} \end{aligned}$$

$$E(r) = -GMm \left(\frac{3}{r} + \frac{1}{6R - r} \right)$$

Differentiating

$$\frac{dE}{dr} = -GMm \left[-\frac{3}{r^2} - \frac{1}{(6R-r)}(-1) \right]$$

For maximum E , $\frac{dE}{dr} = 0$

$$\Rightarrow -\frac{3}{r^2} + \frac{1}{(6R-r)^2} = 0$$

$$\Rightarrow r = \frac{6\sqrt{3}R}{\sqrt{3}+1}$$

For particle to reach other side it is sufficient if its velocity is zero at

$$r_0 = \frac{6\sqrt{3}R}{\sqrt{3}+1}$$

i.e., K.E = 0 at this point

$$\begin{aligned} \therefore \text{Speed is minimum at } r_0 &= \frac{6\sqrt{3}R}{\sqrt{3}+1} \\ &= 3R(3 - \sqrt{3}) \end{aligned}$$

Potential energy at $x = 2R$

$$\begin{aligned} \therefore PE_1 + PE_2 \\ &= \frac{G \cdot 3M}{2R} - \frac{GMm}{6R-R} = -\frac{17GMm}{10R} \end{aligned}$$

Potential energy at $x = 5R$

$$\begin{aligned} \Rightarrow PE_1 + PE_2 \\ &= -\frac{G(3M)m}{5R} - \frac{GMm}{6R-R} \\ &= -\frac{8}{5} \frac{GMm}{R} \quad PE(2r) < PE(5R) \end{aligned}$$

$PE(r_0) - PE(2r) > PE(r_0) - PE(5R)$

Hence it has maximum speed at $x = 2R$

Sol 8: Maximum range = $\frac{v^2}{g}$

$$b = \frac{v^2}{g}$$

$$v = \sqrt{bg} = \sqrt{\frac{bGM}{r_e^2}}$$

$$M = \frac{4}{3} \pi r^3 \rho$$

$$\Rightarrow v = \sqrt{\frac{bG}{r_e^2} \cdot \frac{4}{3} \pi r_e^3 \rho}$$

$$v = \sqrt{\frac{4\pi bG r_e \rho}{3}}$$

Escape velocity $V_e = \sqrt{\frac{2GM}{r}}$

$$= \sqrt{\frac{2G}{r_p} \cdot \frac{4}{3} \pi r_p^3 \cdot 2\rho} = \sqrt{\frac{16\pi \rho r_p^2}{3}}$$

$$v = v_e$$

$$\Rightarrow \sqrt{\frac{4\pi bG r_e \rho}{3}} = \sqrt{\frac{16\pi \rho r_p^2}{3}}$$

$$\Rightarrow r_p = \sqrt{\frac{b r_e}{2}}$$

$$= \frac{\sqrt{4 \times 10^{-3} \times 6400}}{2}$$

$$r_p = \sqrt{6.4} \text{ km}$$

Maximum radius of planet is $\sqrt{6.4} \text{ km}$

Sol 9: $V = \sqrt{\frac{GM}{r}}$ where $V = \text{Orbital velocity}$

$$V_1 = \sqrt{\frac{GM}{2R}}, V_2 = \sqrt{\frac{GM}{3R}} \quad (r = R + \text{height})$$

$$\text{Angular velocity } \omega = \frac{V}{R}$$

$$\omega_1 = \frac{V}{2R} \sqrt{\frac{GM}{2R}}; \omega_2 = \frac{1}{3R} \sqrt{\frac{GM}{3R}}$$

Relative angular velocity $\omega_R = \omega_1 + \omega_2$

$$= \sqrt{\frac{GM}{R^3}} \left(\frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} \right)$$

$$t = \frac{2\pi}{\omega R} = \frac{2\pi}{\sqrt{\frac{GM}{R^3}} \frac{2\sqrt{2} + 3\sqrt{3}}{6\sqrt{6}}}$$

$$\therefore t = \frac{2\pi R^{\frac{3}{2}} (6\sqrt{6})}{\sqrt{GM} (2\sqrt{2} + 3\sqrt{3})}$$

Sol 10: Let d be distance between them. Distance of centre of mass from m

$$r_1 = \frac{Md}{m+M}; r_1 = \frac{2d}{3}$$

$$F = \frac{GMm}{d^2} = \frac{2GM^2}{d^2}$$

Gravitational force = Centrifugal force

$$\frac{2GM^2}{d^2} = \frac{mv_1^2}{2d}; \Rightarrow v_1 = \sqrt{\frac{4GM}{3d}}$$

$$T = \frac{2\pi}{v_1} r_1 = \frac{2\pi \frac{2d}{3}}{\sqrt{\frac{4GM}{3d}}} = \frac{2\pi d^{\frac{3}{2}}}{\sqrt{3GM}}$$

$$m = \frac{M_s}{3} \quad (M_s = \text{Mass of surfs})$$

$$\therefore T = \frac{2\pi d^{\frac{3}{2}}}{\sqrt{GM_s}}$$

$$\text{Time period of earth } T_e = \frac{2\pi d^{\frac{3}{2}}}{\sqrt{GM_s}}$$

given $T = T_e; \Rightarrow d = R$

$$\text{Sol 11: Total energy} = \sum_{i < j} \frac{-GM_i M_j}{r_{ij}}$$

$$= -G \left(\frac{M^2}{a} \right) \times (\Sigma 3) = -\frac{6GM^2}{a}$$

$$\text{Final energy} = -\frac{6GM^2}{2a}$$

$$\text{Change in energy} = -\frac{6GM^2}{2a} + \frac{6GM^2}{a}$$

$$= \frac{6GM^2}{2a}$$

$$\therefore \text{Work done is } \frac{6GM^2}{2a}$$

$$\text{Sol 12: Potential} = \frac{GM}{r}$$

$$\therefore \text{Change in potential} = GMm \left(\frac{1}{R} - \frac{1}{2R} \right)$$

$$= \frac{GMm}{2R}$$

K.E. = Change in P.E

$$\frac{1}{2}mv^2 = \frac{GMm}{2R}; v = \sqrt{\frac{GM}{R}}$$

Inside sphere v is constant

$$\therefore \text{Time} = \frac{2R}{v} = 2\sqrt{\frac{R^3}{GM}}$$

$$\text{Sol 13: } \omega_1 = \frac{2\pi}{T}$$

Angular velocity of earth

$$\omega_e = \frac{2\pi}{T_0} \quad (T_0 = 24 \text{ Hz})$$

$$\omega_r = \omega_1 + \omega_e \text{ or } \omega_1 - \omega_e$$

$$T = \frac{2\pi}{\omega_r} = \frac{2\pi}{\frac{2\pi}{T} \pm \frac{2\pi}{T_0}} = \left(\frac{T_0 \pm T}{T_0 T} \right)^{-1}$$

$$= \left(\frac{24 \pm 1.5}{24 \times 1.5} \right)^{-1} = \left(\frac{17}{24} \right)^{-1} \text{ hrs, } \left(\frac{5}{8} \right)^{-1} \text{ hrs}$$

Sol 14: For a satellite

$$|K.E| = \frac{1}{2}|P.E| = |\text{total energy}|$$

$$E_1 = -2 \times 10^5 \text{ J}$$

$$\Rightarrow U_1 = -4 \times 10^5, K_1 = 2 \times 10^5$$

$$U_2 = -2 \times 10^5$$

$$\Rightarrow E_1 = -1 \times 10^5 \text{ J}$$

$$\Delta E = 1 \times 10^5 \text{ J}$$

\therefore Energy required is 10^5 J

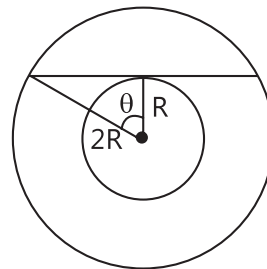
$$\text{Sol 15: Total energy} = -\frac{GMm}{2r}, r = \text{Radius}$$

$$\therefore \text{Change in energy} = \frac{GMm}{2} \left(\frac{1}{R_e} - \frac{1}{r} \right)$$

$t =$ Change in energy

$$\Rightarrow t = \frac{GMm}{2C} \left(\frac{1}{R_e} - \frac{1}{r} \right)$$

Sol 16:



Angle of view = 2θ

$$= 2\cos^{-1} \frac{R}{2R} = 2(60^\circ) = 120^\circ = \frac{2\pi}{3}$$

$$\text{Angular velocity of earth } \omega_e = \frac{2\pi}{T_0}$$

($T_0 = 24 \text{ Hrs}$)

Angular velocity of satellite $\omega = \sqrt{\frac{GM}{8R^3}}$

$$\left(\omega = \sqrt{\frac{GM}{r}} \right)$$

Relative angular velocity $\omega_r = \omega - \omega_e$

$$= \sqrt{\frac{GM}{8R^3}} - \omega_e; T = \frac{\theta}{\omega} = \frac{2\pi}{3\left(\sqrt{\frac{GM}{8R^3}} - \omega_e\right)}$$

Sol 17: Let final velocity of launch pad be x

$$\Rightarrow 3mv_0 = m(x + v_0) + 2m(x)$$

$$\Rightarrow x = \frac{2v_0}{3}$$

Angular momentum = mvr

$$= m \cdot \frac{2v_0}{3} \cdot (3R_m)$$

$$L = 2mv_0 R_m$$

Angular momentum is constant

$$\therefore mv_x \cdot R_m = L$$

$$\Rightarrow mv_x \cdot R_m = 2mv_0 R_m$$

$$v_x = 2v_0$$

$$\Delta KE = \Delta Pt$$

$$\Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = GMm \left(\frac{1}{R_m} - \frac{1}{3R_m} \right)$$

$$\Rightarrow v^2 = v_0^2 + 2GM \left(\frac{2}{3R_m} \right)$$

Now satellite equals

$$v = \sqrt{\frac{GM}{r}}; \Rightarrow v_0 = \sqrt{\frac{GM}{3R_m}}$$

$$\Rightarrow \frac{GM}{3R_m} = v_0^2; \Rightarrow v^2 = v_0^2 + 4v_0^2$$

$$x = \frac{2v_0}{3}; \Rightarrow v = \frac{2}{3}\sqrt{10}v_0$$

$$\cos\theta = \frac{v_x}{v}; \Rightarrow \theta = \cos^{-1} \frac{2v_0}{\frac{2\sqrt{10}}{3}v_0}$$

$$\Rightarrow \cos\theta = \frac{3}{\sqrt{10}} \Rightarrow \theta = \cos^{-1} \frac{3}{\sqrt{10}}$$

Sol 18: (a) Orbital velocity $V = \sqrt{\frac{GM}{R}}$

(b) At maximum distance, they are mutually perpendicular radially about the winter of planet

$$\therefore \text{Maximum distance} = \sqrt{2}r$$

$$(c) \text{ Their relative velocity} = \sqrt{2}V = \sqrt{\frac{2GM}{R}}$$

Sol 19: Gravitational force

$$F = \frac{GMm}{r^2}$$

$$\Rightarrow \Delta F = -\frac{GMm}{r^3} \Delta r$$

\therefore Net force at a height $r + \Delta r$

$$= \frac{GMm}{r^2} \left(1 - \frac{2\Delta r}{r} \right)$$

Centrifugal force $f = mrw^2$

$$\Delta f = mw^2 \Delta r$$

\therefore Net centrifugal force $-f + \Delta f$

$$= mrw^2 \left(1 + \frac{\Delta r}{r} \right)$$

$$= mrw^2 \left(1 + \frac{\Delta r}{r} \right) - \frac{GMm}{r^2} \left(1 - \frac{2\Delta r}{r} \right)$$

$$= mrw^2 \frac{\Delta r}{r} + \frac{GMm}{r^2} \frac{2\Delta r}{r}$$

$$mrw^2 = \frac{GMm}{r^2} = mg \quad (\because \text{satellite moon})$$

$$\therefore T = 3Mg \frac{\Delta r}{r} = 3 \times 100 \times 10 \times \frac{64 \times 10^{-3}}{6400}$$

$$T = 3 \times 10^{-2} \text{ N}$$

$$\text{Sol 21: (i) P.E} = \left(-\frac{GMm}{R} \right) 3 + 3 \left(-\frac{GMm}{\sqrt{3}R} \right)$$

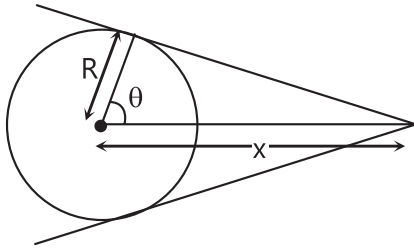
$$= -\frac{3GM}{R} \left(\frac{m}{\sqrt{3}} + m \right)$$

(ii) Centrifugal force = Force towards center

$$\frac{mv^2}{R} = \frac{GMm}{R^2} + \left(\frac{GMm}{(\sqrt{3}R)^2} \cdot \frac{\sqrt{3}}{2} \right) R$$

$$\Rightarrow v = \left(\sqrt{\frac{GM(2\sqrt{3} + R)}{R \cdot 2\sqrt{3}}} \right)$$

Sol 22: Surface area of earth $A = 4\pi R^2$



$$\cos \theta = \frac{R}{x}$$

Area covered by the satellite on surface of earth

$$A_1 = 2\pi R^2(1 - \cos \theta)$$

where θ is semi-vertical angle

$$\therefore \text{Area out of reach} = A - A_1$$

$$= 4\pi R^2 - 2\pi R^2(1 - \cos \theta)$$

$$= 2\pi R^2(1 + \cos \theta)$$

$$= 2\pi R^2 \left(1 + \frac{R}{x}\right)$$

Sol 23: (a) $M = 2m$

$$\text{Center of mass from } m (r_1) = \frac{md}{m+m}$$

$$= \frac{2d}{3}; \Rightarrow r^2 = \frac{d}{3}$$

$$\text{Force between them (t)} = \frac{GMm}{d^2}$$

$$= \frac{2GM^2}{d^2}$$

Let velocity of mass m be v_1

$$\frac{mv_1^2}{r_1} = (F); \Rightarrow \frac{mv_1^2}{\frac{2d}{3}} = \frac{2GM^2}{d^2}$$

$$\Rightarrow v_1 = \sqrt{\frac{4GM}{3d}}; T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$$

$$= \frac{2\pi \times \frac{2d}{3}}{\sqrt{\frac{4GM}{3d}}} = \frac{4\pi}{3} \sqrt{\frac{3d^3}{4GM}} = \frac{2\pi d^{3/2}}{\sqrt{3GM}}$$

$$(b) \frac{v_m}{r_m} = \frac{v_M}{r_M}$$

$$\frac{L_m}{L_M} = \frac{mv_m r_m}{2mv_M r_M} = \frac{1}{2} \left(\frac{r_m}{r_M}\right)^2 = \frac{1}{2}(2)^2$$

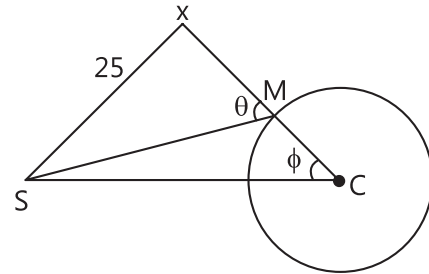
$$\frac{L_m}{L_M} = 2$$

$$(c) k = \frac{1}{2}mv^2$$

$$\frac{k_m}{k_M} = \frac{\frac{1}{2}mv_m^2}{\frac{1}{2}mv_M^2} = \frac{m}{2m} \cdot \left(\frac{r_m}{r_M}\right)^2 = \frac{1}{2}(2)^2$$

$$\frac{k_m}{k_M} = 2$$

Sol 24:



Let S be satellite, M Mumbai, C center of earth.

$$\phi = 30^\circ$$

$$\text{let } SC = r$$

$$MC = R$$

$$\Rightarrow r \sin \phi = (r \cos \phi - R) \tan \alpha$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\sin \phi}{\cos \phi - \frac{R}{r}} \right); r = \left(\frac{T\sqrt{GM}}{2\pi} \right)^{\frac{2}{3}}$$

$$T = 24 \text{ Hr}$$

$$\sqrt{GM} = R\sqrt{g}$$

$$R = \text{radius of earth}$$

Upon substitution we get

$$\theta = \cot^{-1} \left(\sqrt{3} - \frac{32}{105} \right)$$

Sol 25: Angular velocity be ω

$$mR\omega^2 = mg$$

$$\omega = \frac{2\pi}{T}; mR \cdot \frac{(2\pi)^2}{T^2} = mg$$

$$T = 2\pi \sqrt{\frac{R}{g}}$$

$$g = \frac{GM}{R^2}$$

$$= \frac{G}{R^2} \cdot \frac{4}{3} \pi R^3 \rho$$

$$\therefore T = 2\pi \sqrt{\frac{R}{GR \cdot \frac{4}{3} \pi \rho}}$$

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

Exercise 2

Multiple Correct Choice Type

Sol 1: (A, D)

$$g = \frac{GM}{r^2}; \quad r > R$$

$$\frac{GM}{R^2} r; \quad r < R$$

Sol 2: (A, D) $f_{12} = f_{21}$

$$\Rightarrow m_1 a_1 = m_2 a_2$$

$$\Rightarrow \text{if } m_1 < m_2$$

$$\therefore a_1 > a_2$$

Total energy is constant, by law of conservation of energy centre of mass is in motion in the reference from of the masses.

Sol 3: (B, C, D) Everywhere gravitational field is zero which is same everywhere inside the spherical shell.

$$\text{Everywhere potential is same as } -\frac{dE}{dx} = 0$$

i.e., gravitational field is same. Potential inside sphere is equal to that on surface.

Sol 4: (A, C) The satellite will always be in orbital motion at every instant

$$|u| = 2|k|$$

$$|\Delta u| = 2|\Delta k|$$

$$u = -2k$$

$$\Rightarrow \Delta u = -2\Delta k$$

$$\therefore \Delta u \text{ is } -ve$$

hence kinetic energy increases

$$L = mvr$$

$$\Delta L = mvr \left(\frac{\Delta v}{v} + \frac{\Delta r}{r} \right)$$

$$U = -\frac{GMm}{r}; \quad \Delta u = \frac{GMm}{r^2} \Delta r$$

$$k = \frac{1}{2} mv^2$$

$$\Delta k = mv \Delta u$$

$$\Delta u = -2\Delta k$$

$$\frac{GMm}{r^2} \Delta r = -2mv\Delta u$$

$$\frac{GMm}{r^2} \Delta r = 2 \left(\frac{1}{2} mv^2 \right)$$

$$\frac{\Delta r}{r} = -\frac{2\Delta v}{v}; \quad \Rightarrow \Delta L = mvr \left(\frac{\Delta r}{2r} \right)$$

$$\Delta r < 0$$

$$\Rightarrow \Delta L < 0$$

Sol 5: (B, D) Communication satellites are geo stationary.

Sol 6: (A, C, D) Only potential energy increase

$$PE = -\frac{GMm}{r}$$

$$v \text{ decrease; } v \propto \frac{1}{\sqrt{r}}$$

and hence angular velocity and centripetal acceleration decreases as r increases.

Sol 7: (A, D) Acceleration is always directed towards centre of earth. Centripetal force.

$$\text{Sol 8: (A, B, C)} \quad v \propto \frac{1}{r}, \quad T \propto r^{3/2}$$

\therefore Speed is maximum and time period is minimum

Potential energy is minimum

$$PE \propto \left(-\frac{1}{r} \right)$$

Sol 9: (A, D) Satellite has to be above equator at some time

$$T = 2\pi \sqrt{\left(\frac{r}{R} \right)^2 \cdot \frac{r}{g}}; \quad r = \text{radius of orbit}$$

$$> 2\pi\sqrt{\frac{R}{g}}$$

$$\therefore r > R, \left(\frac{r}{R}\right)^2 > 1$$

Sol 10: (A, B, D) $T \propto R^{3/2}$

$$\therefore S_1 : S_2 = 1^{3/2} : 4^{3/2} = 1 : 8$$

$$v \propto \frac{1}{\sqrt{r}} \Rightarrow v_1 : v_2 = 4^{1/2} : 1^{1/2} = 2 : 1$$

Angular momentum $L \propto r^{1/2}$

$$L_1 : L_2 = 1 : 4^{1/2} = 1 : 2$$

Let velocities be $2k, k$

Relative velocities are $3k, k$

$$\text{i.e. } v_1 : v_2 = 3 : 1$$

Relative radii = $4R + R, 4R - R$

$$= 5R, 3R$$

$$\text{i.e. } R_1 : R_2 = 4 : 3.$$

$$\omega = \frac{v}{R}$$

$$\therefore \omega_1 : \omega_2 = \frac{3}{5} : \frac{1}{3}$$

$$\omega_1 : \omega_2 = 9 : 5$$

Assertion Reasoning Type

Sol 11: (C) There is no such real radial force. It only appears in moon's frame of reference as centrifugal force.

Sol 12: (B) Statement-I is true because there is no net acceleration downward in it.

Sol 13: (D) Geostationary satellites have fixed orbital radius and do have 24 hours of time period of revolution.

Sol 14: (D) Statement-I is only true long distances between them.

Sol 15: (A) For travel, energy required

$$= \text{maximum P.E} - \text{P.E at surface}$$

Comprehension type

Paragraph 1:

Sol 16: (B) Let P.E at $\infty = 0$

$$\text{Final P.E} = \frac{GM_1M_2}{d}$$

final distance between center of masses

$$d = R + 2R = 3R$$

mass of small sphere is m

mass of smaller sphere

$$= m \left(\frac{R_2}{R}\right)^3 = m \left(\frac{2R}{R}\right)^3$$

$$= 8m$$

$$\text{Final energy} = \frac{GM \cdot 8m}{3R} = \frac{8GM^2}{3R}$$

Let initial velocities be v_1, v_2

Let final velocities be v_3, v_4

since centre of mass is at rest

$$\vec{v}_3 = -e \vec{v}_1$$

(e = Coefficient of restitution)

$$|V_3| = \frac{1}{2}|V_1|$$

$$\Rightarrow \frac{1}{2}mv_3^2 = \frac{1}{2}m\left(\frac{1}{2}v\right)^2$$

$$= \frac{1}{4}\left(\frac{1}{2}mv_1^2\right)$$

$$\text{Similarly } \frac{1}{2}mv_4^2 = \frac{1}{4}\left(\frac{1}{2}mv_2^2\right)$$

$$\therefore \text{Final energy} = \frac{1}{2}mv_3^2 + \frac{1}{2}mv_4^2$$

$$= \frac{1}{4}\left(\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2\right)$$

$$= \frac{1}{4}\left(\frac{8Gm^2}{3R}\right)$$

$$= \frac{2Gm^2}{3R}$$

Sol 17: (A) Change in P.E = kinetic energy

$$\frac{2GM^2}{3R} = \text{P.E} - \left(-\frac{8GM^2}{3R} \right)$$

$$\Rightarrow \text{PE} = \frac{6GM^2}{3R} = \frac{GM(8M)}{4R}$$

\therefore Maximum distance between them is $4R$

Note:- Try deriving the result

$\vec{V}_3 = -e\vec{V}_1$ used in the problem. Here centre of mass is at rest.

Paragraph 2:

Sol 18: (B) Area of ASBC > ASBD

$\therefore t_1 > t_2$

Kepler's 2nd law

Sol 19: (C) $|u| > |k|$ always

Because if $|k| \geq |u|$ body escapes from the sun's gravitational force

Paragraph 3:

Sol 20: (B) Self energy of a uniform sphere of radius R and mass M is given by

$$E = -\frac{3GM^2}{5R}$$

\therefore Change in energy

$$= -\frac{3GM^2}{5} \left(\frac{1}{R/2} - \frac{1}{R} \right)$$

$$= \frac{3GM^2}{5R}$$

Increase in temperature = $\frac{\text{energy}}{M.S}$

$$= \frac{3GM^2}{5R} \cdot \frac{1}{M.S}$$

$$= \frac{3GM}{5RS}$$

Sol 21: (A) $T_0 = \frac{3GM}{5RS}$

$$M = \frac{5SRT_0}{3G}$$

Note: Study the self-energy of objects here is a derivation.

Consider a sphere of density ρ and initial radius r initial

$$\text{mass } m = \frac{4}{3}\pi r^3\rho$$

Let additional mass added

$$dm = 4\pi r^2 \cdot dr\rho$$

Increase in energy $dE = \frac{GMdm}{r}$

$$dE = \frac{G \cdot \frac{4}{3}\pi r^3\rho \cdot 4\pi r^2 dr\rho}{r}$$

$$dE = \frac{(4\pi)^2}{3} \rho^2 \cdot G \cdot r^4 \cdot dr$$

$$E = \int_0^R dE = \frac{(4\pi)^2}{3} \rho^2 \cdot G \cdot \frac{r^5}{5}$$

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} \quad (M \text{ is final mass})$$

$$\Rightarrow E = \frac{m^2}{\left(\frac{4\pi}{3}\right)^2 R^6} \cdot \frac{R^5}{5} \times \frac{(4\pi)^2}{3}$$

$$E = \frac{3GM^2}{5R}$$

Paragraph 4:

Sol 22: (D) $1 \text{ Bose} = \frac{1}{c} \text{ Newton}$

Let T be tension in the string.

Let a_1, a_2 be acceleration of m_1, m_2 downward

$$a = \left(\frac{gm_1^2 - T}{m_1^2} \right) c$$

where $\frac{gm_1^2}{c}$ is downward gravitational force on m_1

$$\text{Similarly } a_2 = \left(\frac{gm_2^2 - T}{m_2} \right) c$$

$a_1 + a_2 = 0$ by constrain equation

$$\left(\frac{gm_1^2 - T}{m_1^2}\right)c + \left(\frac{gm_2^2 - T}{m_2^2}\right) = 0$$

$$\Rightarrow T = 2g \left(\frac{m_1^2 m_2^2}{m_1^2 + m_2^2}\right) \cdot \frac{1}{c}$$

$$= \frac{2 \times 5 \times 2^2 \times 4^2}{2^2 + 4^2} \cdot \frac{1}{c}$$

$$T = \frac{32}{c}$$

$$T = 32 \text{ Bose}$$

Note: If you do not know bosc, try guessing what it could be. 1 newton is the force which is produced when an object of mass 1 kg moves with an acceleration of 1 ms^{-2} . Similarly define bosc. This is the best assumption you can do with the given amount of information.

Sol 23: (B) Force due to gravity $F = \frac{gm^2}{c} = gm^2 \text{ bosc}$

force along slope $f_1 = f \sin \theta$

$$f = f_1$$

$$= f \sin \theta$$

$$= gm^2 \sin \theta \text{ bosc}$$

$$= 5 \times (2)^2 \cdot \frac{1}{2}$$

$$= 10 \text{ bosc}$$

Previous Years' Questions

Sol 1: (A) Force acting on astronaut is utilized in providing necessary centripetal force, thus he feels weightlessness, as he is in a state of free fall.

Sol 2: (A) The gravitational field is zero at the centre of a solid sphere. The small spheres can be considered as negative mass m located at A and B. The gravitational field due to these masses at O is equal and opposite. Hence, the resultant field at O is zero.

(c and d) \rightarrow are correct because plane of these circles is y - z , i.e., perpendicular to x -axis i.e., potential at any point on these two circles will be equal due to the positive mass M and negative masses $-m$ and $-m$.

Sol 3: (B) For $r \leq R$, $F = \frac{GM}{R^3} \cdot r$ or $F \propto r$

$$\frac{F_1}{F_2} = \frac{r_1}{r_2} \text{ for } r_1 < R \text{ and } r_2 < R$$

$$\text{And for } r \leq R, F = \frac{GM}{r^2} \text{ or } F \propto \frac{1}{r^2}$$

$$\text{i.e., } \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2} \text{ for } r_1 > R \text{ and } r_2 > R$$

Sol 4: $T \propto r^{3/2}$

or $r \propto T^{2/3}$

$$\frac{r_2}{r_1} = \left(\frac{T_2}{T_1}\right)^{2/3}$$

$$r_2 = \left(\frac{T_2}{T_1}\right)^{2/3} r_1 = \left(\frac{8}{1}\right)^{2/3} (10^4) = 4 \times 10^4 \text{ km}$$

$$\text{Now, } v_1 = \frac{2\pi r_1}{T_1} = \frac{(2\pi)(10^4)}{1} = 2\pi \times 10^4 \text{ km/h}$$

$$v_2 = \frac{2\pi r_2}{T_2} = \frac{(2\pi)(4 \times 10^4)}{8} = (\pi \times 10^4) \text{ km/h}$$

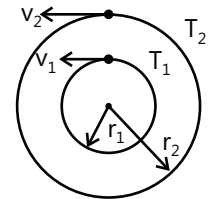
(a) Speed of S_2 relative to S_1

$$= v_2 - v_1 = -\pi \times 10^4 \text{ km/h}$$

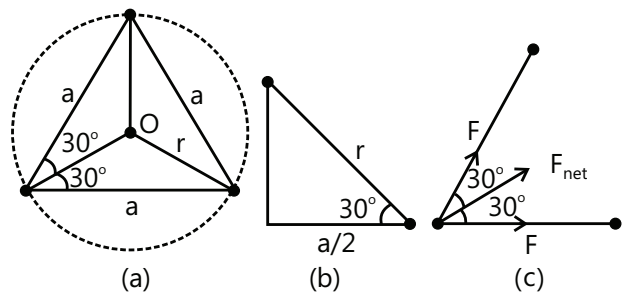
(b) Angular speed of S_2 as observed by S_1

$$\omega_r = \frac{|v_2 - v_1|}{|r_2 - r_1|} = \frac{\left(\pi \times 10^4 \times \frac{5}{18} \text{ m/s}\right)}{(3 \times 10^7 \text{ m})}$$

$$= 0.3 \times 10^{-3} \text{ rad/s} = 3 \times 10^{-4} \text{ rad/s}$$



Sol 5: Centre should be at O and radius r . We can calculate r from figure (b).



$$\frac{a/2}{r} = \cos 30^\circ = \frac{\sqrt{3}}{2} \therefore r = \frac{a}{\sqrt{3}}$$

Further net force on any particle towards centre

$$F_{\text{net}} = 2F \cos 30^\circ$$

$$= 2 \left(\frac{Gm^2}{a^2} \right) \left(\frac{\sqrt{3}}{2} \right) = \frac{\sqrt{3} Gm^2}{a^2}$$

This net force should be equal to $\frac{mv^2}{r}$

$$\therefore \frac{\sqrt{3} Gm^2}{a^2} = \frac{mv^2}{a/\sqrt{3}} \therefore v = \sqrt{\frac{Gm}{a}}$$

Time period of circular motion

$$T = \frac{2\pi r}{v} = \frac{2\pi (a/\sqrt{3})}{\sqrt{Gm/a}} = 2\pi \sqrt{\frac{a^3}{3Gm}}$$

Sol 6: (a) Orbital speed of a satellite at distance r from centre of earth,

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} \quad \dots(i)$$

$$\text{Given, } v_0 = \frac{v_e}{2} = \frac{\sqrt{2GM/R}}{2} = \sqrt{\frac{GM}{2R}} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$h = R = 6400 \text{ km}$$

(b) Decrease in potential energy = increase in kinetic energy

$$\text{or } \frac{1}{2} mv^2 = \Delta U \therefore v = \sqrt{\frac{2(\Delta U)}{m}}$$

$$= \sqrt{\frac{2 \left(\frac{mgh}{1+h/R} \right)}{m}} = \sqrt{gR}$$

$$(h=R) = \sqrt{9.8 \times 6400 \times 10^3} = 7919 \text{ m/s} = 7.9 \text{ km/s}$$

Sol 7: Let there are two stars 1 and 2 as shown below

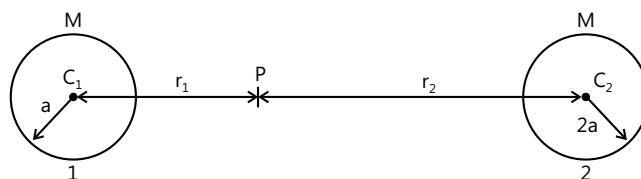
Let P is a point between C_1 and C_2 , where gravitational field strength is zero or at P field strength due to star 1 is equal and opposite to the field strength due to star 2 Hence,

$$\frac{GM}{r_1^2} = \frac{G(16M)}{r_2^2} \text{ or } \frac{r_2}{r_1} = 4$$

$$r_1 + r_2 = 10a \therefore r_2 = \left(\frac{4}{4+1} \right) (10a) = 8a$$

$$\text{and } r_1 = 2a$$

Now, the body of mass m is projected from the surface of larger star towards the smaller one. Between C_2 and P it is attracted towards 2 and between C_1 and P it will be attracted towards 1

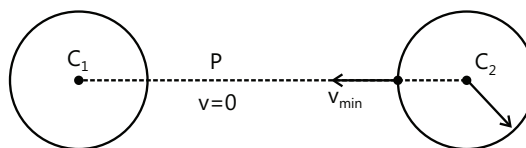


Therefore, the body should be projected to just cross point P because beyond that the particle is attracted towards the smaller star itself.

From conservation of mechanical energy $\frac{1}{2} mv_{\text{min}}^2$

= Potential energy of the body at P – Potential energy at the surface of larger star.

$$\therefore \frac{1}{2} mv_{\text{min}}^2 = \left[-\frac{GMm}{r_1} - \frac{16GMm}{r_2} \right] - \left[-\frac{GMm}{10a-2a} - \frac{16GMm}{2a} \right]$$

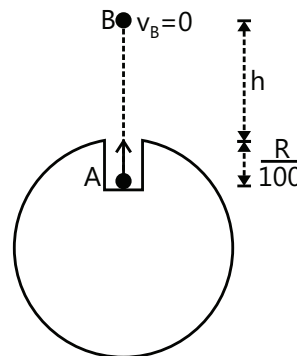


$$\text{Or } \frac{1}{2} mv_{\text{min}}^2 = \left(\frac{48}{8} \right) \frac{GMm}{a}$$

$$\therefore v_{\text{min}} = \frac{3\sqrt{5}}{2} \left(\sqrt{\frac{GM}{a}} \right)$$

Sol 8: Speed of particle at A, v_A = escape velocity on the surface of earth

$$= \sqrt{\frac{2GM}{R}}$$



At highest point B, $v_B = 0$

Applying conservation of mechanical energy, decrease in kinetic energy = increase in gravitational potential energy

$$\text{or } \frac{1}{2}mv_A^2 = U_B - U_A = m(v_B - v_A)$$

$$\text{or } \frac{v_A^2}{2} = v_B - v_A$$

$$\therefore \frac{GM}{R} = -\frac{GM}{R+h} - \left[-\frac{GM}{R^3} \left(1.5R^2 - 0.5 \left(R - \frac{R}{100} \right)^2 \right) \right]$$

$$\text{or } \frac{1}{R} = -\frac{1}{R+h} + \frac{3}{2R} - \left(\frac{1}{2} \right) \left(\frac{99}{100} \right)^2 \cdot \frac{1}{R}$$

Solving this equation, we get

$$h = 99.5 R$$

$$\text{Sol 9: } g = \frac{GM}{R^2} = \frac{G \left(\frac{4}{3} \pi R^3 \right) \rho}{R^2} \text{ or } g \propto \rho R$$

$$\text{or } R \propto \frac{g}{\rho}$$

Now escape velocity, $v_e = \sqrt{2gR}$ or $v_e \propto \sqrt{gR}$

$$\text{or } v_e \propto \sqrt{g \times \frac{g}{\rho}} \propto \sqrt{\frac{g^2}{\rho}}$$

$$\therefore (v_e)_{\text{planet}} = (11 \text{ km s}^{-1}) \sqrt{\frac{6}{121} \times \frac{3}{2}} = 3 \text{ km s}^{-1}$$

Sol 10: (C) For $r \leq R$

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \quad \dots (i)$$

$$\text{Here, } M = \left(\frac{4}{3} \pi r^3 \right) \rho_0$$

Substituting in Eq. (i), we get

$$v \propto r$$

i.e., v-r graph is a straight line passing through origin
For $r > R$

$$\frac{mv^2}{r} = \frac{Gm \left(\frac{4}{3} \pi R^3 \right) \rho_0}{r^2} \text{ or } v \propto \frac{1}{\sqrt{r}}$$

The corresponding v-r graph will be as shown in option (c)

$$\text{Sol 11: (B, D)} \quad V_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \cdot G \rho \frac{4}{3} \pi R^3}{R}} = \sqrt{\frac{4G\rho R}{3}}$$

$$V_{es} \propto R$$

$$\text{Surface area of P} = A = 4\pi R_P^2$$

$$\text{Surface area of Q} = 4A = 4\pi R_Q^2$$

$$\Rightarrow R_Q = 2R_P$$

$$\text{Mass R is } M_R = M_P + M_Q$$

$$\rho \frac{4}{3} \pi R_R^3 = \rho \frac{4}{3} \pi R_P^3 + \rho \frac{4}{3} \pi R_Q^3 \Rightarrow R_R^3 = R_P^3 + R_Q^3 = 9R_P^3$$

$$R_R = 9^{1/3} R_P \Rightarrow R_R > R_Q > R_P$$

Therefore $V_R > V_Q > V_P$

$$\frac{V_R}{V_P} = 9^{1/3} \text{ and } \frac{V_P}{V_Q} = \frac{1}{2}$$

Sol 12: (B) Inside planet

$$g_i = g_s \frac{r}{R} = \frac{4}{3} G \pi r \rho$$

Force to keep the wire at rest (F)

= Weight of wire

$$= \int_{4R/5}^R (\lambda dr) \left(\frac{4}{3} G \pi r \rho \right) = \left(\frac{4}{3} G \pi \rho \right) \left(\frac{9\lambda}{50} \right) R^2$$

$$\text{Here, } \rho = \text{density of earth} = \frac{M_e}{\frac{4}{3} \pi R_e^3}$$

$$\text{Also, } R = \frac{R_e}{10}; \text{ putting all values, } F = 108 N$$

Sol 13: (A) Measured value of $r = (10 \pm 1) \text{ mm}$

$$\Delta r = 1 \text{ mm}$$

$$\text{Relative error} = \frac{\Delta r}{r} = \frac{1}{10} = 10\%$$

Average value of

$$\bar{T} = \frac{\sum_{i=1}^{n=5} T_i}{n} = \frac{(0.52 + 0.56 + 0.57 + 0.54 + 0.59)}{5} \text{ s}$$

$$\Rightarrow \bar{T} = 0.556 \text{ s} \approx 0.56 \text{ s}$$

$$\text{Relative error in time period} \approx \frac{0.01}{0.56} = 1.79\%$$

Reported value of $(R - r) = (50 \pm 2) \text{ mm}$

$$\text{Relative error in } (R - r) = \frac{2}{50} = 4\%$$

$$T = 2\pi \sqrt{\frac{7(R-r)}{5g}} \Rightarrow \frac{\Delta g}{g} = 2 \left(\frac{\Delta T}{T} \right) + \frac{\Delta(R-r)}{(R-r)}$$

$$\Rightarrow \frac{\Delta g}{g} = 7.57\%$$