1. UNITS, DIMENSIONS AND ERRORS

1. INTRODUCTION

Physics is an experimental science and experiments require measurement of physical quantities. Measuring a physical quantity involves comparing the quantity with a reference standard called the unit of the quantity. Some physical quantities are taken as base quantities and other quantities are expressed in terms of the base quantities called derived quantities. This forms a system of base quantities and their units. Without performing proper measurements we cannot describe the physical phenomena quantitatively.

2. UNITS

To measure a physical quantity we need some standard unit of that quantity. The measurement of the quantity is mentioned in two parts, the first part gives magnitude and the second part gives the name of the unit. Thus, suppose I say that length of this wire is 5 meters. The numeric part 5 says that it is 5 times of the unit of length and the second part meter says that unit chosen here is meter.

2.1 Fundamental and Derived Quantities

The basic physical quantities, which are independent of other quantities, are known as the fundamental quantities. For example, mass, length and time are considered to be the fundamental quantities. In the same manner, the units which can be derived from the fundamental units are known as derived units. In mechanics, virtually all quantities can be expressed in terms of mass, length and time. The main systems of units are given as follows:

(a) CGS or Centimetre, Gram, Second System

(b) FPS or Foot, Pound, Second System

(c) MKS or Metre, Kilogram, Second System

(d) SI system: Totally, there are seven basic or fundamental quantities in the international system of units called the SI system which can express all physical quantities including heat, optics and electricity and magnetism. We now provide these basic seven quantities with their units and symbols:

S. No.	Physical Quantity	SI Unit	Symbol
1	Mass	Kilogram	kg
2	Length	Metre	m
3	Time	Second	s
4	Temperature	Kelvin	K
5	Luminous intensity	Candela	cd

S. No.	Physical Quantity	SI Unit	Symbol
6	Electric current	Ampere	A
7	Amount of substance	Mole	mol.

There are also two supplementary units used as radian (rad) for plane angle and steradian (sr) for solid angle.

The above mentioned International System of Units (SI) is now extensively used in scientific measurements.

However, the following practical units of length are also conveniently used and are expressed in terms of SI system of units.

(a) **Micron** is a small unit for measurement of length. 1 micron $= 1\ \mu m = 10^{-6} m$

(b) **Angstrom** is a unit of length in which the size of an atom is measured and is used in atomic physics.

1 Angstrom = 1Å = 10^{-10} m.

(c) **Light year** is a unit of distance travelled by light in 1 year free space and is used in astrophysics.

1 Light year = $3 \times 10^8 m/s \times 365 \times 24 \times 60 \times 60 = 9.5 \times 10^{15}$ meters

(d) **Fermi** is a unit of distance in which the size of a nucleus is measured. 1 Fermi $= 10^{-15}$ m

(e) **Atomic mass unit:** It is a unit of mass equal to 1/12th of mass of carbon-12 atom.

1 atomic mass unit $\cong 1.67 \times 10^{-27}$ kg

Note: There are only seven fundamental units. Apart from these, there are two supplementary units—plane angle (radian) and solid angle (steradian). By using these units, all other units can be derived. However, we need to know the fact that both radian and steradian have no dimensions.

3. DIMENSIONS

All the physical quantities of interest can be derived from the base quantities. Thus, when a quantity is expressed in terms of the base quantities, it is written as a product of different powers of the base quantities. Further, the exponent of a base quantity that enters into the expression is called the dimension of the quantity in that base. To make it clear, consider the physical quantity "force." As we shall learn later, force is equal to mass times acceleration. We know that acceleration is change in velocity divided by time interval but velocity is length divided by time interval. Thus,

$$\text{Force} = \text{Mass} \times \text{Acceleration} = \text{Mass} \times \frac{\text{Velocity}}{\text{Time}} = \text{Mass} \times \frac{\text{Length / Time}}{\text{Time}} = \text{Mass} \times \text{Length} \times (\text{Time})^{-2}$$

Thus, the dimensions of force are 1 in mass, 1 in length and –2 in time. The dimensions in all other base quantities are zero. Note, however, that in this type of calculation, the magnitudes are not considered. This is because only equality of the type of quantity is what that matters. Thus, change in velocity, average velocity, or final velocity all are equivalent in this discussion, as each one is expressed in terms of length/time.

Illustration 1: Validate the relation $s = ut + \frac{1}{2}at^2$, where u is the initial velocity, a is the acceleration, t is the time and s is the displacement. **(JEE MAIN)**

Sol: The above relation is having units of displacement. To validate above relation dimensionally correct, we need to match the dimensions of each quantity to the right of equality with the dimensions of displacement.

By writing the dimensions of either side of the equation, we obtain

LHS = s = displacement = $[M^0LT^0]$; RHS = ut = velocity$\times$time $= [M^0LT^{-1}][T] = [M^0LT^0]$

Further, $\frac{1}{2}at^2 = (\text{acceleration}) \times (\text{time})^2 = [M^0LT^{-2}][T]^2 = [M^0LT^0]$

As LHS = RHS, the formula is dimensionally correct.

Table 1.1: SI units and dimensions of commonly used quantities

S. No.	Quantity	SI Units	Dimensional Formula
1.	Area	m^2	$[L^2]$
2.	Density	$kg\ m^{-3}$	$[ML^{-3}]$
3.	Velocity	ms^{-1}	$[LT^{-1}]$
4.	Acceleration	ms^{-2}	$[LT^{-2}]$
5.	Angular velocity	rad s^{-1}	$[T^{-1}]$
6.	Frequency	s^{-1} or hertz(Hz)	$[T^{-1}]$
7.	Momentum	$kg\ ms^{-1}$	$[MLT^{-1}]$
8.	Force	$kg\ ms^{-2}$ or newton (N)	$[MLT^{-2}]$
9.	Work, energy	$kg\ m^2\ s^{-2}$ or joule(J)	$[ML^2T^{-2}]$
10.	Power	$kg\ m^2\ s^{-3}$ or Js^{-1} orWatt	$[ML^2T^{-3}]$
11.	Pressure, stress	Nm^{-2} or pascal (Pa)	$[ML^{-1}T^{-2}]$
12.	Coefficient of elasticity	Nm^{-2}	$[ML^{-1}T^{-2}]$
13.	Moment of inertia	$kg\ m^2$	$[ML^2]$
14.	Torque	Nm	$[ML^2T^{-2}]$
15.	Angular momentum	$kg\ m^2\ s^{-1}$	$[ML^2T^{-1}]$
16.	Impulse	Ns	$[MLT^{-1}]$
17.	Universal gravitational constant	$Nm^2\ kg^{-2}$	$[M^{-1}L^3T^{-2}]$
18.	Latent heat	Jkg^{-1}	$[L^2T^{-2}]$
19.	Specific heat	$Jkg^{-1}\ K^{-1}$	$[L^2T^{-2}K^{-1}]$

S. No.	Quantity	SI Units	Dimensional Formula
20.	Thermal conductivity	$Jm^{-1}\ s^{-1}\ K^{-1}$	$\left[MLT^{-3}K^{-1}\right]$
21.	Electric charge	Coulomb(C)	$\left[AT\right]$
22.	Electric potential	JC^{-1} or volt (V)	$\left[ML^2T^{-3}A^{-1}\right]$
23.	Electric resistance	VA^{-1} or ohm (Ω)	$\left[ML^2T^{-3}A^{-2}\right]$
24.	Electric resistivity	(Ω) m	$\left[ML^3T^{-3}A^{-2}\right]$
25.	Capacitance	CV^{-1} or farad (F)	$\left[ML^{-1}T^{-2}\ T^4\ A^2\right]$
26.	Inductance	VsA^{-1} or henry (H)	$\left[ML^2T^{-2}\ A^{-2}\right]$
27.	Electric field	NC^{-1} or Vm^{-1}	$\left[ML^2T^{-3}\ A^{-1}\right]$
28.	Magnetic induction	$NA^{-1}\ m^{-1}$ or tesla(T)	$\left[MT^{-2}A^{-1}\right]$
29.	Magnetic flux	Tm^2 or weber (Wb)	$\left[ML^2T^{-2}A^{-1}\right]$
30.	Permittivity	$C^2\ N^{-1}\ m^{-2}$	$\left[M^{-1}L^{-3}T^4A^2\right]$
31.	Permeability	$Tm\ A^{-1}$ or $Wb\ A^{-1}\ m^{-1}$	$\left[MLT^{-2}A^{-2}\right]$
32.	Plank's constant	Js	$\left[ML^2T^{-1}\right]$
33.	Boltzmann constant	JK^{-1}	$\left[ML^2T^{-2}K^{-1}\right]$

Table 1.2: Table of physical quantity having same dimensional formula

S. No.	Dimensional Formula	Physical Quantities
1.	$\left[M^0\ L^0\ T^{-1}\right]$	(a) Frequency (b) Angular frequency (c) Angular velocity (d) Velocity gradient
2.	$\left[M^0\ L^2\ T^{-2}\right]$	(a) Square of velocity (b) Gravitational potential (c) Latent heat

S. No.	Dimensional Formula	Physical Quantities
3.	$\left[ML^2 T^{-2}\right]$	(a) Work (b) Energy (c) Torque (d) Heat
4.	$\left[MLT^{-2}\right]$	(a) Force (b) Weight (c) Thrust (d) Energy gradient
5.	$\left[ML^{-1}T^{-2}\right]$	(a) Pressure (b) Stress (c) Modulii of elasticity (d) Energy density

4. USES OF DIMENSIONS

The major uses of dimensions are listed hereunder:

(a) Conversion from one system of units to another.

(b) To test and validate the correctness of a physical equation or formula.

(c) To derive a relationship between different physical quantities in any physical phenomenon.

(d) **Conversion from one system of units to another:** If we consider n_1 as numerical value of a physical quantity with dimensions a, b and c for units of mass, length and time as M_1, L_1, and T_1, then the numerical value of the same quantity, n_2 can be calculated for different units of mass, length and time as M_2, L_2 and T_2, respectively.

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

(e) **To test and validate the correctness of a physical equation or formula:** The principle of homogeneity requires that the dimensions of all the terms on both sides of physical equation or formula should be equal if the physical equation of any derived formula is correct.

(f) **To derive a relationship between different physical quantities in any physical phenomenon:** Suppose that if a physical quantity depends upon a number of parameters whose dimensions are not known, then the principle of homogeneity of dimensions can be used. As we know that the dimensions of a correct dimensional equation are equal on both sides, it can be used to find the unknown dimensions of these parameters on which the physical quantity depends. Further, it can be used to derive the relationships between any physical quantity and its dependent parameters.

Derivation: $S_n = u + \frac{a}{2}(2n-1)$

$$S_n = S_n + S_{n-1} = \left(un + \frac{1}{2}an^2\right) - (n(n-1) + \frac{1}{2}a(n-1)^2)$$

$$S_n = u(1) + \frac{1}{2}a(1)(2n-1) = \left[u + \frac{a}{2}(2n-1)\right](1)$$ (We ignore '1' in formula but it carries dimension of time.)

Where, n – dimension of time; u – dimension of velocity; s – dimension of displacement; and a – dimension of acceleration.

MASTERJEE CONCEPTS

The formula for displacement in nth second by a moving body is wrong using dimensional analysis.

NO! Actually, if we go back deeper in derivation we would very easily find that although the equation looks dimensionally incorrect but it is precise and accurate.

Vaibhav Gupta (JEE 2009, AIR 54)

Illustration 2: A calorie is a unit of heat or energy and it equals about 4.2 J. Suppose that we employ a system of unit in which the unit of mass equals α kg, the unit of length equals β metre, and the unit of time is γ second. Then, show that a calorie has a magnitude $4.2\ \alpha^{-1}\beta^{-2}\gamma^{2}$ in terms of the new units. **(JEE MAIN)**

Sol: Here the system is expressed in one set of units. When we want to convert the units in order of magnitude only, the conversion factor is obtained by dividing the original units by new set of units.

1 cal = 4.2 kg m^2 s^{-2}

SI	New system
$N_1 = 4.2$	$N_2 = ?$
$M_1 = 1$ kg	$M_2 = \alpha$ kg
$L_1 = 1$ m	$L_2 = \beta$ m
$T_1 = 1$ s	$T_2 = \gamma$ s

Dimensional formula of energy is $\left[ML^2T^{-2}\right]$

Comparing with $\left[M^aL^bT^c\right]$, we find that a = 1, b = 2, and c = –2

$$\text{Now, } N_2 = N_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 4.2\left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1\left[\frac{1\text{ m}}{\beta\text{ m}}\right]^2\left[\frac{1\text{ s}}{\gamma\text{ s}}\right]^{-2} = 4.2\ \alpha^{-1}\beta^{-2}\gamma^{2}$$

Illustration 3: The centripetal force F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v), and radius (r) of the circle. Derive the formula for F using the method of dimensions. **(JEE MAIN)**

Sol: To obtain the relation between force F, mass M, velocity V and radius r, we use the dimensional analysis. The power of base units of each quantity on the right of the equality are matched with the power of same unit on left of the equality.

$$\text{Let } F = K(m)^x(v)^y(r)^z \quad \text{... (i)}$$

Here, k is a dimensionless constant of proportionality. By writing the dimensions of RHS and LHS in Eq. (i), we have

$$\left[MLT^{-2}\right] = [M]^x\left[LT^{-1}\right]^y[L]^z = \left[M^xL^{y+z}T^{-y}\right]$$

By equating the powers of M, L, and T of both sides, we have

$$x = 1, y = 2 \text{ and } y + z = 1 \text{ or } z = 1 - y = -1$$

By substituting the values in Eq. (i), we obtain $F = kmv^2r^{-1} = k\frac{mv^2}{r}$; $F = \frac{mv^2}{r}$

(where k = 1). [The value of K cannot be calculated wring dimensional analysis].

MASTERJEE CONCEPTS

A dimensionally correct equation may or may not be an exact equation but an exact equation must be dimensionally correct.

Example: F = ma and F = 0.5ma, both are dimensionally correct but only one is correct w.r.t the physical relation.

Vaibhav Krishnan (JEE 2009, AIR 22)

5. LIMITATIONS OF DIMENSIONS

(a) From a dimensionless equation, the nature of physical quantities cannot be decided, i.e., whether a given quantity is scalar or vector.

(b) The value of proportionality constant also cannot be determined.

(c) The relationship among physical quantities having exponential, logarithmic, and trigonometric functions cannot be established.

6. ORDER OF MAGNITUDE

In physics, we often learn quantities which vary over a wide rage. For example, we discuss regarding the size of a mountain and the size of the tip of a pin. In the same way, we also discuss regarding the mass of our galaxy and the mass of a hydrogen atom. Sometimes, we also discuss regarding the age of universe and the time taken by an electron to complete a circle around the proton in a hydrogen atom. However, we observe that it is quite difficult to get a feel of largeness or smallness of such quantities. Therefore, to express such drastically varying numbers, we use the power of ten method.

In this method, each number is expressed as $a \times 10^b$ where $1 \le a \le 10$ and b is an integer. Thus, we represent the diameter of the sun as 1.39×10^9 m and diameter of a hydrogen atom as 1.06×10^{-10} m. However, to have an approximate idea of the number, we may round the number 'a' to 1 if it is less than or equal to 5 and 10 if it is greater than 5. Thereafter, the number can be expressed approximately as 10^b. Further, we then obtain the order of magnitude of that number. Thus, now we can more clearly state that the diameter of the sun is of the order of 10^9m and that of a hydrogen atom is of the order of 10^{-10}m. More precisely, we say that the exponent of 10 in such a representation is called the order of magnitude of that quantity. Thus, now we can say that the diameter of the sun is 19 orders of magnitude larger than the diameter of a hydrogen atom. This is due to the fact that the order of magnitude of 10^9 is 9 and of 10^{-10} is –10. The difference is 9 –(–10) = 19.

Table 1.3: Table of SI prefixes

Power of 10	Prefix	Symbol
18	exa	E
15	peta	P
12	tera	T
9	giga	g
6	mega	M
3	kilo	k
2	hecto	h
1	deka	da

–1	deci	d
–2	centi	c
–3	milli	m
–6	micro	m
–9	nano	n
–12	pico	p
–15	femto	f
–18	atto	a

Tip: The best way to remember is by memorizing from milli to atto, kilo to exa and thereafter to go the power of 3.

For example, if one asks for giga since we have already memorized from kilo to exa, then we need to go like kilo mega giga and since it is 3 in the order shown, giga would be assigned a value of 3 * 3 = 9, i.e., 10^9.

7. SIGNIFICANT FIGURES

Significant figures in the measured value of a physical quantity provide information regarding the number of digits in which we have confidence. Thus, the larger the number of significant figures obtained in a measurement, the greater is the precision of the measurement.

“All accurately known digits in a measurement plus the first uncertain digit together form significant figures.”

7.1 Rules for Counting Significant Figures

For counting significant figures, we make use of the rules listed hereunder:

(a) All non-zero digits are significant. For example, x = 2567 has clearly four significant figures.

(b) The zeroes appearing between two non-zero digits are counted in significant figures. For example, 6.028 has 4 significant figures.

(c) The zeroes located to the left of the last non-zero digit are not significant. For example, 0.0042 has two significant figures.

(d) In a number without decimal, zeroes located to the right of the non-zero digit are not significant. However, when some value is assigned on the basis of actual measurement, then the zeroes to the right non-zero digit become significant. For example, L = 20 m has two significant figures but x = 200 has only one significant figure.

(e) In a number with decimal, zeroes located to the right of last non-zero digit are significant. For example, x = 1.400 has four significant figures.

(f) The power of ten is not counted as significant digit(s). For example, 1.4×10^{-7} has only two significant figures, i.e., 1 and 4.

(g) Change in the units of measurement of a quantity, however, does not change the number of significant figures. For example, suppose the distance between two stations is 4067 m. It has four significant figures. The same distance can be expressed as 4.067 km or 4.067×10^5 cm. In all these expressions, however, the number of significant figures continues to be four.

Table 1.4: Significant figures

Measured value	Number of significant figures	Rule
12376	5	1
6024.7	5	2

0.071	2	3
410 m	3	4
720	2	4
2.40	3	5
1.6×10^{14}	2	6

7.2 Rounding Off a Digit

The rules for rounding off a measurement are listed hereunder:

(a) If the number lying to the right of cut off digit is less than 5, then the cut off digit is retained as such. However, if it is more than 5, then the cut off digit is increased by 1.

For example, x = 6.24 is rounded off to 6.2 (two significant digits) and x = 5.328 is rounded off to 5.33 (three significant digits).

(b) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is increased by 1. For example, x = 14.252 is rounded off to x = 14.3 to three significant digits.

(c) If the digit to be dropped is simply 5 or 5 followed by zeroes, then the preceding digit is left unchanged if it is an even number. For example, x = 6.250 or x = 6.25 becomes x = 6.2 after rounding off to two significant digits.

(d) If the digit to be dropped is 5 or 5 followed by zeroes, then the preceding digit is raised by one if it is an odd number.

For example, x = 6.350 or 6.35 becomes x = 6.4 after rounding off to two significant digits.

Table 1.5: Significant digits

Measured value	After rounding off to three significant digits	Rules
7.364	7.36	1
7.367	7.37	1
8.3251	8.33	2
9.445	9.44	3
9.4450	9.44	3
15.75	15.8	4
15.7500	15.8	4

7.3 Algebraic Operations with Significant Figures

(a) **Addition and subtraction:** Suppose in the measured values to be added or subtracted, the least number of significant digits after the decimal is n. Then, in the sum or difference also, the number of significant digits after the decimal should be n.

Example: Suppose that we have to find the sum of number 420.42 m, 420.4m and 0.402m by arithmetic addition

$$\begin{array}{r} 420.42 \\ 420.4 \\ 0.402 \\ \hline 441.222 \end{array}$$

But the least precise measurement of 420.4 m is correct to only one decimal place. Therefore, the final answer will be 441.2 m.

(b) Multiplication or division: Suppose in the measured values to be multiplied or divided, the least number of significant digits is n; then, in the product or quotient, the number of significant digits should also be n.

Example: $1.2 \times 36.72 = 44.064 \approx 44$

In the example shown, the least number of significant digits in the measured values is two. Hence, the result when rounded off to two significant digits becomes 44. Therefore, the answer is 44.

Example: $\dfrac{1100\,ms^{-1}}{10.2\,ms^{-1}} = 107.8431373 \approx 108$

MASTERJEE CONCEPTS

Tip: In algebraic operations with significant figures, the result shall have significant figures corresponding to their number in the least accurate variable involved.

Nivvedan (JEE 2009, AIR 113)

Illustration 4: Round off the following number to three significant digits: (a) 15462, (b) 14.745, (c) 14.750 and (d) 14.650×10^{12}. **(JEE MAIN)**

Sol: The values above when rounded off to the three significant figures, if the fourth digit of the number is greater than or equal to 5, we increase the third digit by 1 and discard the digits after third digit. If the fourth digit is not greater than or equal to 5, we discard the digits from fourth onwards and write the number up to third significant figure. The power of 10 is not considered as the significant number.

(a) The third significant digit is 4. Now, this digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore, be increased by 1. The digits to be dropped should be replaced by zeroes because they appear to the left of the decimal point. Thus, 15462 becomes 15500 on rounding to three significant digits.

(b) The third significant digits in 14.745 is 7. The number next to it is less than 5. Therefore, 14.745 becomes 14.7 on rounding to three significant digits.

(c) 14.750 will become 14.8 because the digit to be rounded is odd and the digit next to it is 5.

(d) 14.650×10^{12} will become 14.6×10^{12} because the digit to be rounded is even and the digit next to it is 5.

Illustration 5: Evaluate $\dfrac{25.2 \times 1374}{33.3}$. All the digits in this expression are significant. **(JEE MAIN)**

Sol: The result of the above fraction is rounded off to the same number of significant figure as is contained by the least precise term used in calculation, like 25.2 and 33.3.

We have $\dfrac{25.2 \times 1374}{33.3} = 1039.7838.$

Out of the three numbers given in the expression, both 25.0 and 33.3 have 3 significant digits, whereas 1374 has four. The answer, therefore, should have three significant digits. Rounding 1039.7838 to three significant digits, it hence becomes 1040. Thus, we write $\dfrac{25.2 \times 1374}{33.3} = 1040.$

8. ERROR ANALYSIS

We define the uncertainty in a measurement as an 'error'. By this we mean the difference between the measured and the true values of a physical quantity under investigation. There are three possible ways of calculating an error as listed hereunder:

(i) Absolute error (ii) Relative error (iii) Percentage error

Let us consider a physical quantity measured by taking repeated number of observations say $x_1, x_2, x_3, x_4, ...$ if <x> or $\bar{x}$ be the average value of the measurement, then the error in the respective measurement is

$$\Delta x_1 = x_1 - \bar{x}\ ;\ \Delta x_2 = x_2 - \bar{x}\ ....;\ \Delta x = \left| x_{\text{experimental value}} - x_{\text{true value}} \right|$$

However, if we take the arithmetic mean of all absolute errors, then we obtain the final absolute error Δx_{mean}. When arithmetic mean alone is considered, then only the magnitudes of the absolute errors are taken into account.

$$\Delta x_{mean} = \frac{|\Delta x_1| + |\Delta x_2| + ... + |\Delta x_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta x_i|$$

It then follows clearly from the above discussion that any single measurement of x has to be such that

$$x_{mean} - \Delta x_{mean} \le x \le x_{mean} + \Delta x_{mean}$$

Relative error= $\dfrac{\Delta x_{mean}}{x_{mean}}$; percentage error= $\dfrac{\Delta x_{mean}}{x_{mean}} \times 100$

9. PROPAGATION OF ERRORS

9.1 Addition and Subtraction

If $x = A \pm B$; then $\Delta x = \Delta A + \Delta B$

i.e., for both addition and subtraction, the absolute errors are to be added up. The percentage error, then, in the value of x is

Percentage error in the value of $x = \left(\dfrac{\Delta A + \Delta B}{A \pm B}\right) \times 100\%$

9.2 Multiplication and Division

If y = AB or y = $\dfrac{A}{B}$ then, $\dfrac{\Delta y}{y} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B} \Rightarrow \dfrac{\Delta y}{y} \times 100\% = \dfrac{\Delta A}{A} \times 100\% + \dfrac{\Delta B}{B} \times 100\%$

$\Rightarrow$ Percentage error in Value of y = percentage error in value of A + percentage error in value of B

Illustration 6: Two resistors R_1 = 100 $\pm$ 3 Ω and R_2 = 200 $\pm$ 4 Ω are connected in series. Find the equivalent resistance. **(JEE MAIN)**

Sol: When resistance are added in the series, the error in the resultant combination is given by $\Delta R_{eq} = \Delta R_1 + \Delta R_2$ where ΔR_1 = 3 Ω and ΔR_2 = 4 Ω.

The equivalent resistance $R = R_1 + R_2 = (100 \pm 3)\ \Omega + (200 \pm 4)\Omega = 300 \pm 7\Omega$

Illustration 7: A capacitor of capacitance C = 2.0 $\pm$ 0.1 μF is charged to a voltage V = 20 $\pm$ 0.2 volt. What will be the charge Q on the capacitor? Use Q = CV. **(JEE MAIN)**

Sol: The relative error of result of the above product is given by $\dfrac{\Delta Q}{Q} = \pm\left(\dfrac{\Delta C}{C} + \dfrac{\Delta V}{V}\right)$ where $\dfrac{\Delta C}{C}$ and $\dfrac{\Delta V}{V}$ is the relative error in determination in C and V respectively.

If we omit all errors, then $Q = CV = 2.0 \times 10^{-6} \times 20\ C = 40 \times 10^{-6}\ C$

Error in C= 0.1 part in 2 = 1 part in 20 =5%

Error in V 0.2 part in 20 = 2 part in 200 = 1 part in 100 =1%; error $Q = 5\% + 1\% = 6\%$

$\therefore$ Charge, $Q = 40 \times 10^{-6} \pm 6\%\ C = 40 \pm 2.4 \times 10^{-6}\ C$

9.3 Power Functions

If $y = k\dfrac{A^{\ell}B^{m}}{C^{n}}$ then, $\dfrac{\Delta y}{y} = \ell\left(\dfrac{\Delta A}{A}\right) + m\left(\dfrac{\Delta B}{B}\right) + n\left(\dfrac{\Delta C}{C}\right)$

$$\begin{pmatrix}\text{Percentage error}\\ \text{in value of y}\end{pmatrix} = \ell\begin{pmatrix}\text{Percentage error}\\ \text{in value of A}\end{pmatrix} + m\begin{pmatrix}\text{Percentage error}\\ \text{in value of B}\end{pmatrix} + n\begin{pmatrix}\text{Percentage error}\\ \text{in value of C}\end{pmatrix}$$

MASTERJEE CONCEPTS

- The error in a measurement is always equal to the least count of the measuring instrument.
- Errors never propagate particularly in case of constants.

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 8: A physical quantity P is related to four observables a, b, c and d as follows: $P = \dfrac{a^3b^2}{\sqrt{cd}}$. The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P? **(JEE MAIN)**

Sol: The relative error of ratio of $P = \dfrac{a^3b^2}{\sqrt{cd}}$ is calculated as $\dfrac{\Delta P}{P} = \pm\left(\dfrac{\Delta N}{N} + \dfrac{\Delta D}{D}\right)$ where $N = a^3b^2$ and $D = \sqrt{c}\,d$ and $\dfrac{\Delta N}{N}$ and $\dfrac{\Delta D}{D}$ are the relative error in the N and D.

$$P = \frac{a^3b^2}{\sqrt{cd}};\ \frac{\Delta P}{P} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}$$

But $\dfrac{\Delta a}{a} = \dfrac{1}{100}, \dfrac{\Delta b}{b} = \dfrac{3}{100}, \dfrac{\Delta c}{c} = \dfrac{4}{100}, \dfrac{\Delta d}{d} = \dfrac{2}{100}$ $\therefore$ $\dfrac{\Delta P}{P} = 3\times\dfrac{1}{100} + 2\times\dfrac{3}{100} + \dfrac{1}{2}\times\dfrac{4}{100} + \dfrac{2}{100}$

% error in P = 3% + 6% + 2% + 2% = 13%.

10. LENGTH-MEASURING INSTRUMENTS

We know that length is an elementary physical quantity. The device generally used in everyday life for measurement of length is a metre scale. This scale can be used for measurement of length with accuracy to the extent of 1 mm. Therefore, the least count of a metre scale is 1 mm. Further, to measure length accurately up to $(1/10)$th or $\left(\dfrac{1}{100}\right)$th of a millimetre, we use the following instruments.

(1) Vernier calipers (2) Micrometer (3) Screw gauge

10.1 Vernier Calipers

This instrument has three parts.

(i) Main scale: It consists of a strip M, graduated in cm and mm at one of its edge. Also, it carries two fixed jaws A and C as shown in the Fig. 1.1.

(ii) Vernier scale: Vernier scale V slides on metallic strip M. This scale can be fixed in any position

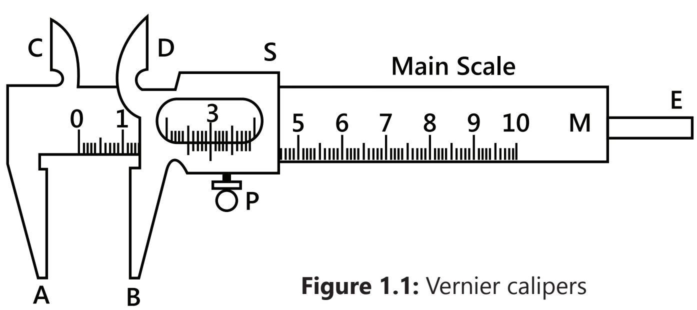

Figure 1.1: Vernier calipers

using the screw S. The side of the Vernier scale which slides over the mm sides has 10 divisions over a length of 9 mm. Further, B and D are two movable jaws that are fixed with it. When the Vernier scale is pushed toward A and C, then B touches A and straight side of C will touch straight side of D. In this position, however, if the instrument is free from error, zeroes of Vernier scale will coincide with zeroes of the main scales. Further, to measure the external diameter of an object, it is held between the jaws A and B, while the straight edges of C and D are used for measuring the internal diameter of a hollow object.

(iii) Metallic strip: There is a thin metallic strip E attached to the back side of M and connected with Vernier scale. When jaws A and B touch each other, the edge of E touches the edge of M. When the jaws A and B are separated, the E moves outward. This strip E is used for measuring the depth of a vessel.

10.1.1 Principle (Theory)

In the common form, the divisions on the Vernier scale V are smaller in size than the smallest division on the main scale M; however, in some special cases the size of the Vernier division may be larger than the main scale division.

Let n Vernier scale divisions (VSD) coincide with (n–1) main scale divisions (MSD). Then,

$$n\text{V.S.D.} = (n-1)\ \text{M.S.D.};\ \ 1\text{V.S.D.} = \left(\frac{n-1}{n}\right)\ \text{M.S.D.};\ 1\ \text{M.S.D.} - 1\ \text{V.S.D.}$$

$$= 1\ \text{M.S.D.} - \left(\frac{n-1}{n}\right)\text{M.S.D.} = \frac{1}{n}\text{M.S.D.}$$

The difference between the values of one main scale division and one Vernier scale division is known as Vernier constant (VC) or the **least count** (LC). This is precisely the smallest distance that can be accurately measured with the Vernier scale. Thus,

$$\text{V.C} = \text{L.C} = 1\text{M.S.D.} - 1\text{V.S.D.} = \left(\frac{1}{n}\right)\text{M.S.D.} = \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$$

In the ordinary Vernier caliper, one main scale division is 1 mm and 10 Vernier scale divisions coincide with 9 main scale divisions.

$$1\text{V.S.D.} = \frac{9}{10}\text{M.S.D.} = 0.9\text{mm};\ \text{V.C.} = 1\text{M.S.D.} - 1\text{V.S.D.} = 1\text{mm} - 0.9\text{mm} = 0.1\text{mm} = 0.01\text{cm}$$

10.1.2 Reading a Vernier Caliper

If we have to measure a length AB, then the end of A is coincided with the zero of the main scale. Now, suppose that the end B lies between 1.0 cm and 1.1 cm on the main scale. Then, let the 5th division of Vernier scale coincides with 1.5 cm of the main scale.

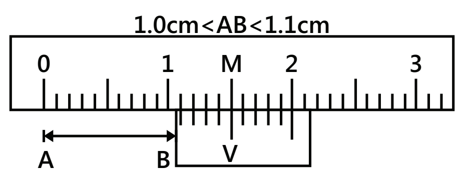

Figure 1.2: Reading main scale and vernier scale

Then, AB=1.0+5 × V.C = (1.0 +5 × 0.001) cm = 1.05 cm

Thus, we can make use of the following formula, i.e.,

Total reading = N+n × V.C

Here, N = main scale reading just before on the left of the zero of the Vernier scale.

n = number of Vernier division which just coincides with any of the main scale divisions.

MASTERJEE CONCEPTS

The main scale reading with which the Vernier scale division coincides has no connection with reading.

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10.1.3 Zero Error and Zero Correction

If the zero of the Vernier scale does not coincide with the zero of the main scale when jaw B touches jaw A and the straight edge of D touches the straight edge of C, then the instrument has an error called zero error. This zero error is always algebraically subtracted from the measured length.

Zero correction has a magnitude that is equal to zero error but its sign is opposite to that of the zero error. Zero correction is, as a rule of the thumb, always algebraically added to the measured length.

Zero error $\rightarrow$ algebraically subtracted

Zero correction $\rightarrow$ algebraically added

10.1.4 Positive and Negative Zero Error

If the zero of Vernier scale lies to the right of the main scale, then the zero error is positive and if it lies to the left of the main scale then the zero error is negative (when jaws A and B are in contact)

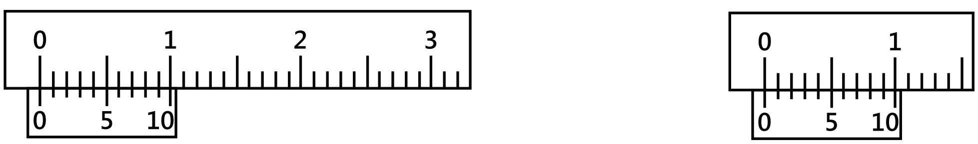
Positive zero error=$\left(\mathrm{N}+\mathrm{x}\times\mathrm{V.C.}\right)$

Here, N = main scale reading on the left of zero of Vernier scale.

X = Vernier scale division which coincides with any main scale division.

When the Vernier zero lies before the main scale zero, then the error is said to be negative zero error. If the 5th Vernier scale division coincides with the main scale division, then

$$\text{Negative zero error}=-\left[0.00\mathrm{cm}+5\times\mathrm{V.C.}\right]=-\left[0.00\mathrm{cm}+5\times0.01\mathrm{cm}\right]=-0.05\mathrm{cm}$$



(A) Positive zero error (B) Negative zero error

Figure 1.3: Types of error of vernier scale

Illustration 9: N-divisions on the main scale of a Vernier caliper coincides with N + 1 divisions on the Vernier scale. If each division on the main scale is of 'a' units, then determine the least count of the instrument. **(IIT JEE 2003)**

Sol: Least count of Vernier caliper is given by L. C. =1 M.S.D – 1 V.S.D.

(N+1) divisions on the Vernier scale = N divisions on the main scale

$\therefore$ 1 division on vernier scale=$\frac{\mathrm{N}}{\mathrm{N}+1}$divisions on main scale

If each division on the main scale is of 'a' units

$\therefore$ 1 division on vernier scale=$\left(\frac{\mathrm{N}}{\mathrm{N}+1}\right)\times$a unit=a'(say)

Least count = 1 main scale division – 1 Vernier scale division = $a-a'=a-\left(\frac{N}{N+1}\right)a=\frac{a}{N+1}$

Illustration 10: In the diagram provided, find the magnitude and the nature of zero error. **(JEE MAIN)**

Sol: Here as the zero division of the main scale is to the left of the zero division of the Vernier scale, thus Vernier caliper is said to have positive error. Thus the measured length of any object will be greater than actual length. This error is to be subtracted from measured length to get actual length.

Here, zero of Vernier scale lies to the right of zero of the main scale; hence, it has positive zero error.

Further, N = 0, x = 5,

L.C. of, V.C. = 0.01 cm.

Hence, zero error $= N + x \times 0.01 = 0.05\text{cm}$ Zero correction = –0.05cm

$\therefore$The actual length will be 0.05 cm less than the measured length.

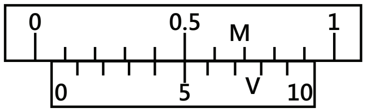


Figure 1.4

10.2 Micrometer Screw

10.2.1 Principle of a Micrometer Screw

The least count of Vernier calipers ordinary available in the laboratory is 0.01 cm. However, when lengths are to be measured with greater accuracy, say up to 0.001 cm, then screw gauge and spherometre are used which are based on the principle of micrometer screw as discussed here under.

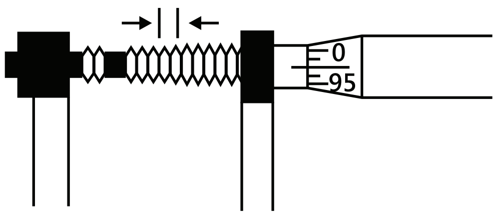


Figure 1.5: Micrometer screw gauge

If an accurately cut single threaded screw is rotated in a closely fitted nut, then in addition to the circular motion of the screw there is also a linear motion of the screw head in the forward or backward direction, along the axis of the screw. The linear distance moved by the screw, when it is given one complete rotation, is called the pitch (p) of the screw. The pitch is basically equal to the distance between two consecutive threads as measured along the axis of the screw. In most of the cases, it is either 1 mm or 0.5 mm. The screw moves forward or backward by $\frac{1}{100}\left(\text{or}\frac{1}{50}\right)$ of the pitch, if the circular scale (we will discuss later about the circular scale) is rotated through one circular division. This is exactly the minimum distance which can be accurately measured and hence called the least count (LC) of the screw.

Thus, $\text{Least count} = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$. If pitch is 1 mm and there are 100 divisions on the circular scale, then

$$\text{L.C.} = \frac{1\text{mm}}{100} = 0.01\text{mm} = 0.001\text{cm} = 10\mu\text{m}$$

Since, LC is of the order of $10\mu\text{m}$, the screw is called a micrometer screw.

10.3 Screw Gauge

Screw gauge functions based on the principle of a micrometer screw. It consists of a U-shaped metal frame M. At one end of it is fixed a small metal piece A. It is called stud and has a plane face. The other end N of M carries a cylindrical hum, H. It is graduated in millimeters or half millimeters based on the pitch of the screw. This scale is called a linear scale or a pitch scale.

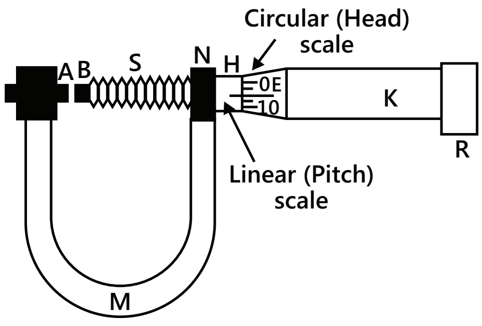


Figure 1.6: Main scale and circular scale of screw gauge

A nut is threaded through both the hub and the frame N. Through the nut moves a screw S. The front face B of the screw, however, facing the plane face A, is also plane in nature. There is a hollow cylindrical cap L that is capable of rotating over the hub when screw is rotated. As the cap is rotated, the screw either moves in or out. The surface E of the cap K is divided into 50 or 100 equal parts. It is called the circular scale or the head scale. In an accurately adjusted instrument when the faces A and B are just touching each other, then the zero of the circular scale should coincide with the zero of the linear scale.

10.3.1 To measure diameter of a given wire using a screw gauge

If with the wire between plane faces A and B, the edge of the cap lies ahead of Nth division of the linear scale, and nth division of circular scale lies over reference line then, Total reading = N + n × L.C

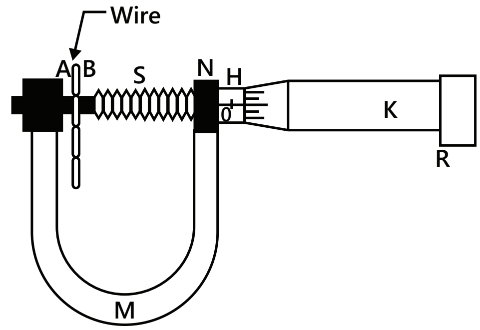


Figure 1.7: Determination of thickness of wire

10.3.2 Zero error and zero correction

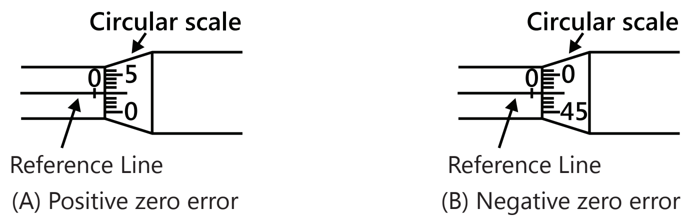


Figure 1.8: Types of errors in reading of screw gauge

If the zero mark of the circular scale does not coincide with the zero of the pitch scale when the faces A and B are just touching each other, then the instrument is said to possess zero error. However, if the zero of the circular scale advances beyond the reference line then the zero error is negative and zero correction is positive. Further, if it is left behind the reference line the zero error is positive and zero correction is negative. For example, if zero of the circular scale advances beyond the reference line by 5 divisions, then zero correction $= +5 \times (\text{L.C.})$ and if the zero of the circular scale is left behind the reference line by 4 divisions, then zero correction $= -4 \times (\text{L.C.})$.

10.3.3 Backlash error

When the direction of rotation of the screw is suddenly changed, then the screw head may rotate, but the screw itself may not move forward or backward. Consequently, the scale reading may not change even by the actual movement of the screw. This is what we meant by backlash error. This error is basically due to loose fitting of the screw. This arises mainly due to wear and tear of the threading due to prolonged use of the screw. To reduce this error, we recommend that the screw must always be rotated in the same direction for a particular set of observations.