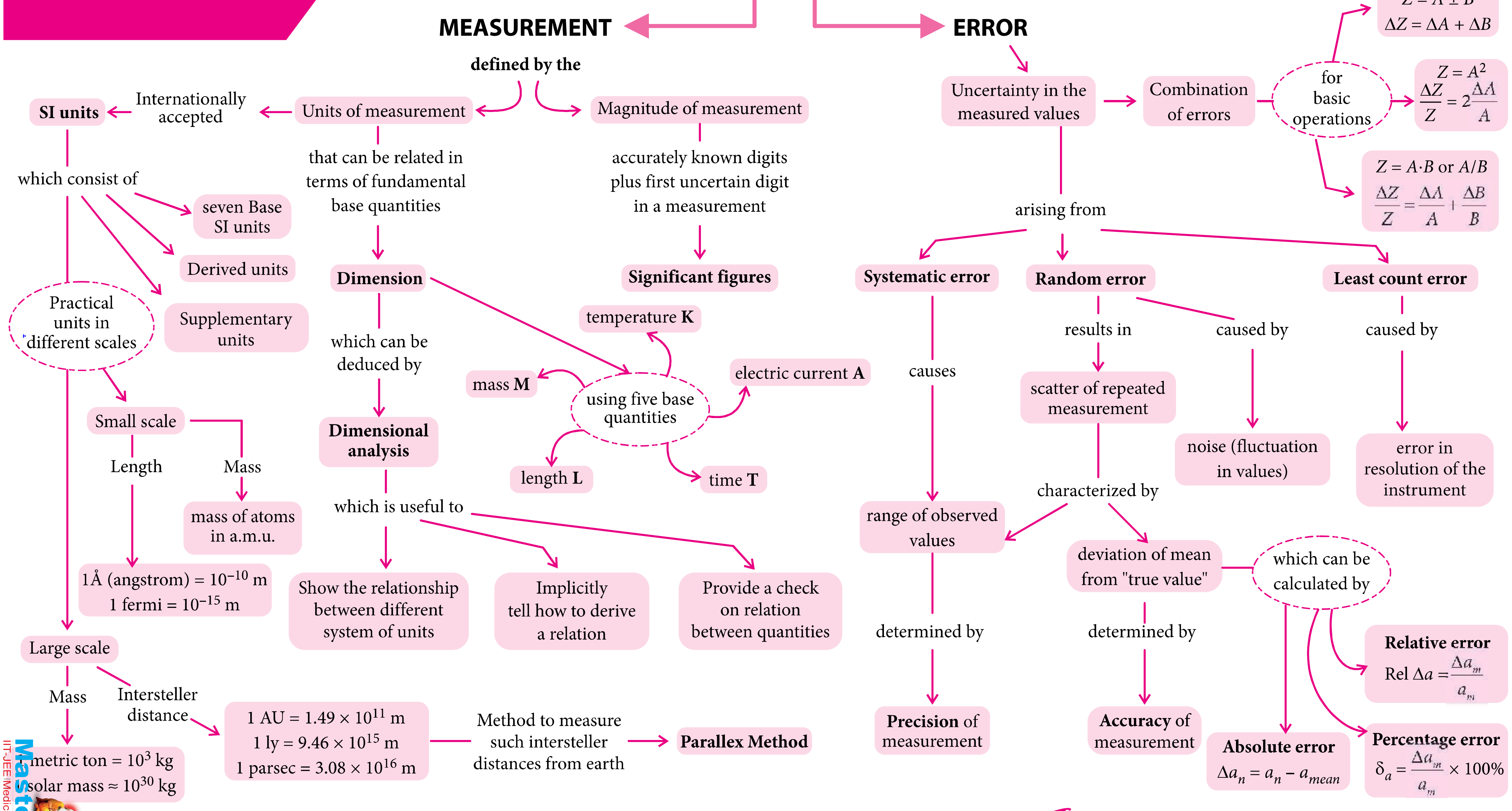
BRAIN MAP

UNITS AND MEASUREMENTS

CLASS XI



CLASS XII

BRAIN MAP

CLASS XI

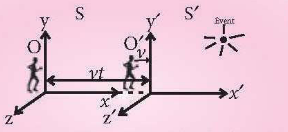
MOTION IN A STRAIGHT LINE

Motion

If a body changes its position as time passes w.r.t. frame of reference, it is said to be in motion.

Frame of Reference

A system consisting a set of coordinates and with reference to which observer describes any event.



The observers are moving in a particular direction x at a relative velocity of v and each observer has their own set of coordinates (x, y, z) and (x', y', z').

Distance

The actual path length covered by moving particle.

Displacement

The change in position vector.

Speed

The rate of distance covered with time is called speed,

$$v = \frac{\text{distance}}{\text{total time}} = \frac{d}{t}$$

Average Speed

$$v_{av} = \frac{\text{total distance}}{\text{total time}}$$

Instantaneous Speed

$$v = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta d}{\Delta t}$$

differentiating with respect to time

Velocity

The rate of change of position per unit time,

$$\vec{v} = \frac{\text{displacement}}{\text{time}} = \frac{\Delta \vec{x}}{\Delta t}$$

Average Velocity

$$\vec{v}_{av} = \frac{\Delta \vec{x}}{\Delta t}$$

Instantaneous Velocity

$$\vec{v} = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

differentiating with respect to time

Acceleration

The time rate of change of velocity, $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

Average Acceleration

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous Acceleration

$$\vec{a} = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Uniform Acceleration

Magnitude of velocity changes by equal amounts in equal intervals of time.

Non-uniform Acceleration

Acceleration changes with time.

Kinematic Equations

A mathematical treatment to describe the motion of a body in 1-dimension.

Constant Acceleration

For Uniformly Accelerated Motion

- $v = u + at$
- $s = ut + \frac{1}{2}at^2$
- $v^2 = u^2 + 2as$
- $S_n = u + \frac{a}{2}(2n-1)$

Acceleration changes with time

For Motion with Variable Acceleration

If $a = f(t) \longrightarrow$ a function of time

- $v = u + \int_0^t f(t)dt$
- $s = ut + \int_0^t (f(t)dt)dt$

For Motion Under Gravity

Vertically downward motion

(Free fall case) $u = 0, a = g$

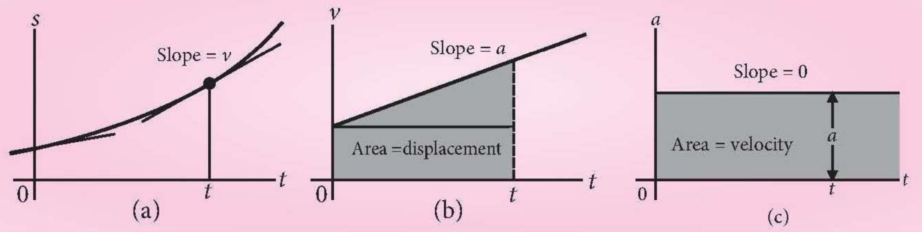
- $v = gt$
- $h = 1/2\, gt^2$
- $v^2 = 2gh$

Vertically upward motion

$v = 0$, acceleration $a = -g$

- $u = gt$
- $h = ut - 1/2\, gt^2$
- $u = \sqrt{2gh}$

Graphical Representation of Uniformly Accelerated Motion



(a) (b) (c)

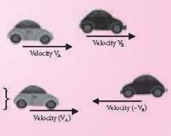
Relative Velocity

The velocity with which an object moves with respect to another object is called relative velocity

$V_{AB} = (V_A - V_B)$

$V_{AB} = \{V_A - (-V_B)\}$

$V_{AB} = (V_A + V_B)$



BRAIN MAP

MOTION IN A PLANE

CLASS XI

VECTOR

Physical quantities having both magnitude and direction

Addition of Vectors

Triangle Law

If two vectors are represented by two sides of a triangle in same order, the resultant will be the third side but in opposite order. $\vec{C} = \vec{A} + \vec{B}$

Parallelogram Law

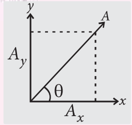
For two coinitial vectors represented by the two adjacent side of a parallelogram, the diagonal of the parallelogram so formed will be the resultant. $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Resolution of Vector

Horizontal component of $\vec{A}$, $A_x = A\cos\theta$

Vertical component of $\vec{A}$, $A_y = A\sin\theta$

$A = \sqrt{A_x^2 + A_y^2}$, $\tan\theta = \left(\dfrac{A_y}{A_x}\right)$



Multiplication by a Scalar Quantity

Multiplication of a vector by a scalar ϕ changes its magnitude by a factor of ϕ.

KINEMATICS IN TWO DIMENSIONS

Kinematic Equations in 2-Dimensions

- $\vec{v} = \vec{u} + \vec{a}t$
  - $v_x = u_x + a_x t$ (along x-axis)
  - $v_y = u_y + a_y t$ (along y-axis)
- $\vec{r} = \vec{r}_0 + \vec{u}_0 t + (1/2)\vec{a}t^2$
  - $x = x_0 + u_x t + (1/2)a_x t^2$ (along x-axis)
  - $y = y_0 + u_y t + (1/2)a_y t^2$ (along y-axis)

Relative Velocity

According to vector law of addition

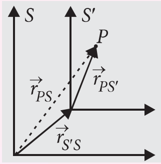
$\vec{r}_{PS} = \vec{r}_{PS'} + \vec{r}_{S'S}$

$\dfrac{d\vec{r}_{PS}}{dt} = \dfrac{d\vec{r}_{PS'}}{dt} + \dfrac{d\vec{r}_{S'S}}{dt}$

So, $\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$

i.e., $\vec{v}_{actual} = \vec{v}_{relative} + \vec{v}_{reference}$

$\vec{v}_{relative} = \vec{v}_{actual} - \vec{v}_{reference}$



Circular Motion

Uniform Circular Motion

A particle moves along a circular path with a constant speed.

Angular displacement, $\theta = \dfrac{s}{r}$

Angular velocity, $\omega = \dfrac{\theta}{t}$

Also, $\omega = \dfrac{2\pi}{T} = 2\pi\upsilon$

Linear velocity, $v = r\omega$

Centripetal acceleration, $a = \dfrac{v^2}{r} = r\omega^2$

Linear acceleration, $a = r\alpha$

Non Uniform Circular Motion

Circular motion with variable speed

Resultant acceleration of the particle

$a = \sqrt{a_t^2 + a_r^2}$

$a_r = \dfrac{v^2}{r}$, a_t = tangential acceleration

Projectile Motion

Projectile Fired Horizontally

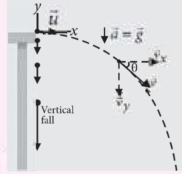
Equation of trajectory: $y = \dfrac{g}{2u^2}.x^2$

Velocity after time t: $v = \sqrt{u^2 + g^2t^2}$

$\beta = \tan^{-1}\dfrac{gt}{u}$

Time of flight: $T = \sqrt{\dfrac{2h}{g}}$

Horizontal range: $R = u\sqrt{\dfrac{2h}{g}}$



Projectile Fired at an Angle with Horizontal

Position after time t:

$x = (u\cos\theta)t, y = (u\sin\theta)t - \dfrac{1}{2}gt^2$

Equation of trajectory:

$y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}.x^2$

Maximum height: $H = \dfrac{u^2\sin^2\theta}{2g}$

Time of flight: $T = \dfrac{2u\sin\theta}{g}$

Horizontal range: $R = \dfrac{u^2\sin 2\theta}{g}$

BRAIN MAP

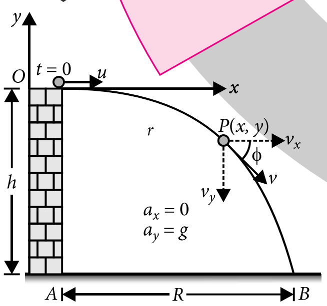
CLASS XI

PROJECTILE MOTION

PROJECTILE Motion

A body which is in flight through the atmosphere under the effect of gravity alone and is not being propelled by any fuel is called projectile and its motion is called projectile motion.

Horizontal Projectile Motion



- **Equation of Trajectory**

$$y = \frac{1}{2}\frac{gx^2}{u^2}$$

- **Time of Descent**

$$T = \sqrt{\frac{2h}{g}}$$

- **Horizontal Range**

$$R = u\sqrt{\frac{2h}{g}}$$

- **Instantaneous Velocity**

$$v = \sqrt{u^2 + 2gy} = \sqrt{u^2 + g^2t^2}$$

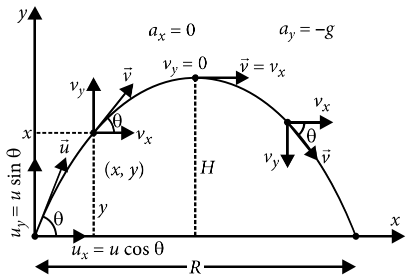
$$\tan\phi = \frac{v_y}{v_x} = \tan^{-1}\left(\frac{gt}{u}\right)$$

Oblique Projectile Motion

- **Equation of Trajectory**

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

This represents the parabolic path.



- **Maximum Height**

$$H = \frac{u^2\sin^2\theta}{2g}$$

- **Time of Flight**

$$T = \frac{2u\sin\theta}{g}$$

- **Horizontal Range**

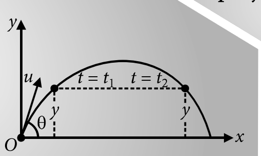
$$R = \frac{u^2\sin 2\theta}{g}$$

- Projectile passing through two different points on same height at time t_1 and t_2

$$y = \frac{gt_1t_2}{2}$$

$$t_2 = \frac{u\sin\theta}{g}\left[1 + \sqrt{1 - \left(\frac{2gy}{u\sin\theta}\right)^2}\right]$$

$$t_1 = \frac{u\sin\theta}{g}\left[1 - \sqrt{1 - \left(\frac{2gy}{u\sin\theta}\right)^2}\right]$$



- Ratio of time of ights for projectiles at complimentary angles θ and 90 – θ

$$\frac{T_\theta}{T_{90-\theta}} = \tan\theta$$

- Range R is n times the maximum height H

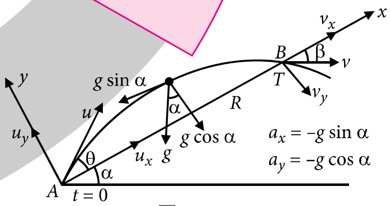
$R = nH; \theta = \tan^{-1}[4/n]$

- For complimentary angles θ and (90 – θ) range remains unchanged
- Relation between horizontal range and maximum height

$R = 4H\cot\theta$

- If $R = H$ then $\theta = \tan^{-1}(4)$ or $\theta = 76°$
- If $R = 4H$ then $\theta = \tan^{-1}(1)$ or $\theta = 45°$

Projectile Motion on an Inclined Plane



- **Time of Flight**

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

- **Maximum Height**

$$H = \frac{u^2\sin^2\theta}{2g\cos\alpha}$$

- **Horizontal Range**

$$R = \frac{2u^2}{g}\frac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$$

- Maximum range occurs when

$$\theta = \frac{\pi}{4} + \frac{\alpha}{2}$$

- Maximum range along the incline when projectile is thrown upwards

$$R_{max} = \frac{u^2}{g(1+\sin\alpha)}$$

- Maximum range along incline when the projectile thrown downwards

$$R_{max} = \frac{u^2}{g(1-\sin\alpha)}$$

BRAIN MAP

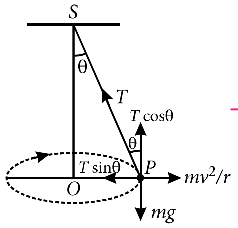
CLASS XI

CIRCULAR MOTION

Time period $T = 2\pi\sqrt{\dfrac{l\cos\theta}{g}}$

Linear velocity $v = \sqrt{gr\tan\theta}$

Angular velocity $\omega = \sqrt{\dfrac{g}{l\cos\theta}}$



Conical Pendulum : Tension in the string

$$T = mg\sqrt{1+\left(\frac{v^2}{rg}\right)^2}$$

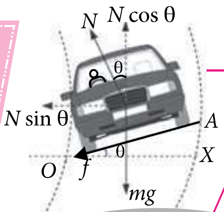
- As $F = \dfrac{mv^2}{r} \neq 0$, Particle is not in equilibrium
- $\tau_{net} = 0$ as central force is involved. *i.e.*, $p \neq$ constant but L = constant

Maximum safe speed on a banked frictional road

$$v = \sqrt{\frac{rg(\mu+\tan\theta)}{1-\mu\tan\theta}}$$

Banking of a road : Optimum speed of a vehicle on a banked road.

$$v = \sqrt{rg\tan\theta}$$



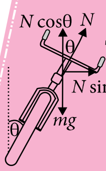
Forces

Centripetal force

$$F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$$

Tangential force, $F_t = ma_t$

Net force, $F_{net} = m\sqrt{a_c^2 + a_t^2}$



Bending of a Cyclist : The cyclist should bend through an angle

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Dynamics of Circular Motion

Tangential acceleration : $\vec{a}_t = \vec{\alpha} \times \vec{r}$

Centripetal acceleration : $\vec{a}_c = \vec{\omega} \times \vec{v}$

Uniform Circular Motion

A particle moves in a circle at a constant speed.

Kinematics of Uniform Circular Motion

CIRCULAR MOTION

Kinematics of Non-uniform Circular Motion

Non-uniform Circular Motion

Speed of the particle in a horizontal circular motion changes with respect to time.

- **Angular displacement :** It is denoted by θ. SI unit : rad or degree
- **Angular velocity (ω) :**

$$\omega = \lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Motion in a Vertical Circle

Body is under the influence of gravity of earth and total mechanical energy is conserved.

Condition of Leaving the Circle

Tension in the string becomes zero before reaching the highest point if $\sqrt{2gL} < u < \sqrt{5gL}$

Tension at any point on vertical loop :

$$T - mg\cos\theta = \frac{mv^2}{L}$$

Condition of Oscillations

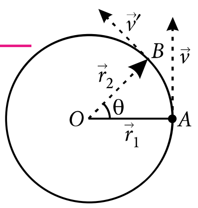
Particle will oscillate if velocity becomes zero the tension in the string is not zero.

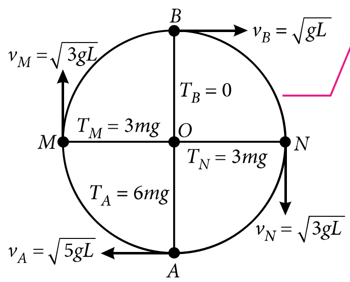
$$0 < u \leq \sqrt{2gL}$$

Condition of Looping the Loop

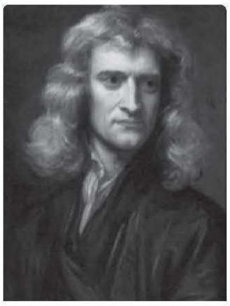
The particle will complete the circle if $u \geq \sqrt{5gL}$

Relations between :
- s and $\theta \Rightarrow s = r\,\theta$
- v and $\omega \Rightarrow \vec{v} = \vec{\omega} \times \vec{r}$





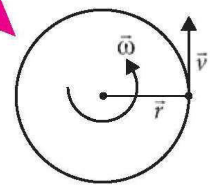
BRAIN MAP
CLASS XI
LAWS OF MOTION


SIR ISSAC NEWTON (1643-1727)

Inertia of Direction
The property due to which a body cannot change its direction of motion by itself.

Inertia of Motion
The tendency of a body to remain in a state of uniform motion in a straight line.

Non-uniform circular motion
Acceleration, $\vec{a} = \vec{a}_T + \vec{a}_C = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$
$a = \sqrt{a_T^2 + a_C^T}$



Inertia of Rest
The property of a body due to which it cannot change its state of rest by itself.

Newton's 1st Law
A body continues its state of rest or motion until unless an external force is acted on it.

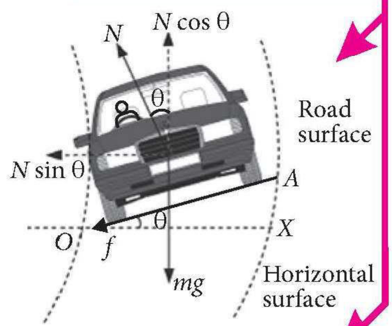
Circular Motion
A body moving in a circular path is called circular motion.
Centripetal force $F_e = \frac{mv^2}{r}$

Motion of a Car on Banked Road
- Maximum velocity on banked road to avoid skidding
$v_{max} = \sqrt{\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}}$
- Optimum speed on a smooth banked road
$v_0 = \sqrt{rg\tan\theta}$

Newton's 2nd Law
The rate of change of linear momentum of a body is directly proportional to the external force applied on the body in the direction of force.

LAWS OF MOTION AND THEIR CONSEQUENCES

CIRCULAR MOTION IN HORIZONTAL PLANE

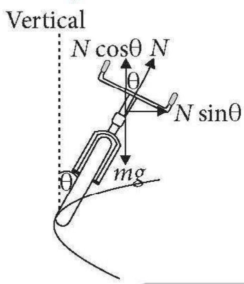


Introduces the Concept of Force

Force = mass × acceleration
$F = ma$, SI unit ≡ N

Newton's 3rd Law
To every action there is always an equal and opposite reaction
$\vec{F}_{AB} = -\vec{F}_{BA}$

Bending of a Cyclist on a circular turn
Angle of bending
$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$



Impulse
A large force acts for a very short duration of time producing a finite change in momentum.
$J = F \times t = \Delta p$

Conservation of Linear Momentum
Total momentum of an isolated system of interacting particles is conserved if there is no external force acting on it. $P_{initial} = P_{final}$

Motion of a Car on Level Road
Maximum velocity on a curved road to avoid skidding
$v_{max} = \sqrt{\mu_s rg}$

Friction
Resistance offered to the relative motion between the surface of the two bodies in contact.

- Acceleration of a body sliding down a rough inclined plane, $a = g(\sin\theta - \mu\cos\theta)$
- Angle of friction, $\theta = \tan^{-1}(\mu_s)$.
- Angle of repose, $\alpha = \tan^{-1}(\mu_s)$.
- Numerically, $\alpha = \theta$

Types of Friction
(a) Static friction : acts when a body is at rest on application of a force, $f_s = F_{ext}$
(b) Limiting friction : acts when a body is just at the verge of movement, $f_l = \mu_s N$
(c) Kinetic friction : acts when a body is actually sliding, $f_l = \mu_s N$

Problem Solving Techniques in Mechanics
- Make free body diagram of each element showing all external forces acting on it.
- Analyse the magnitude and direction of pseudo forces if there is any.
- For equilibrium of concurrent forces use
$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_n = 0$
$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$ (Lami's theorem)
- For horse cart type system
$a = \frac{F_x - f}{M_H + M_{cart}}$ F_x = horizontal component of reaction
f = frictional force

BRAIN MAP
CLASS XI

MASTERJEE CLASSES
NEWTON'S LAWS OF MOTION

Problem Solving Strategies
- *Identify the unknown forces and accelerations.*
- *Draw FBD of bodies in the system.*
- *Resolve forces into their components.*
- *Apply $\Sigma\vec{F} = M\vec{a}$ in the direction of motion.*
- *Apply $\Sigma\vec{F} = 0$ in the direction of equilibrium.*
- *Write constraint relation if exists.*
- *Solve equations $\Sigma\vec{F} = M\vec{a}$ and $\Sigma\vec{F} = 0$.*

LAWS OF MOTION AND THEIR CONSEQUENCES

Newton's 1st Law
A body continues its state of rest or motion until unless an external force is acted on it.

- Inertia of rest
- Inertia of motion
- Inertia of direction

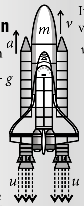
Newton's 2nd Law
The rate of change of linear momentum of a body is directly proportional to the external force applied on the body in the direction of force.

$$F = \frac{dp}{dt} = ma$$

Pseudo Force
For non-inertial frame of reference

$$\vec{F}_{ext} + \vec{F}_{Pseudo} = M\vec{a}$$

$$\vec{F}_{pseudo} = -M\vec{a}_{frame}$$

Rocket Propulsion


Acceleration $a = \frac{u}{m}\frac{dm}{dt} - g$

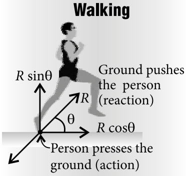
Instantaneous velocity $v = u\log_e\left(\frac{m_0}{m_r}\right) - gt$

Burn out speed $v_{max} = u\log_e\left(\frac{m_0}{m_r}\right)$

Thrust $F = -u\frac{dm}{dt}$

Newton's 3rd Law
To every action there is always an equal and opposite reaction.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Walking


Horse Cart Type System
For horse cart type system

$$a = \frac{F_x - f}{M_H + M_{cart}}$$

F_x = horizontal component of reaction force

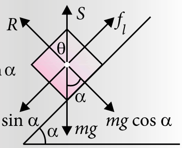
f = frictional force

FRICTION
The motion resisted by a bonding between the body and the surface in contact represented by single force called

Angle of Friction (θ) and Angle of Repose (α)
$S = \sqrt{R^2 + f_l^2}$

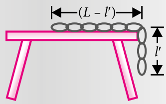
$\tan\theta = \frac{f_l}{R} = \mu_s = \tan\alpha$

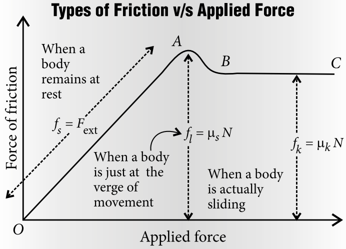
∴ Numerically, $\theta = \alpha$



Maximum Length of Hanging Chain
Length of a chain hanging in air

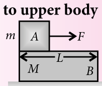
$$l' = \frac{\mu L}{1+\mu}$$



Types of Friction v/s Applied Force


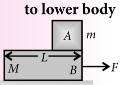
Motion of Two Bodies One Resting on the Other

Force F applied to upper body



- **When there is no friction**
  - $a_A = F/m;\ a_B = 0$
  - A will fall from B after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{F}}$
- **Friction present between A and B ($F < f_l$)**
  - Combined system will move together with $a = F/(M+m)$
- **Friction present between A and B ($F > f_l$)**
  - Relative acceleration $a = a_A - a_B = \frac{MF - \mu_k mg(m+M)}{mM}$
  - A will fall from B after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mML}{MF - \mu_k mg(m+M)}}$

Force F applied to lower body



- **When there is no friction**
  - $a_B = F/M$ and $a_A = 0$
  - A will fall from B (backward) after time t ∴ $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F}}$
- **Friction present between A and B ($F < f_l$)**
  - Both the bodies will move together $a = \frac{F}{M+m}$ and $f_l = \mu_s mg$
  - Pseudo force on the body A, $F' = ma = \frac{mF}{m+M}$
- **Friction present between A and B ($F > f_l$)**
  - Relative acceleration $a = a_A - a_B = -\left[\frac{F - \mu_k g(m+M)}{M}\right]$
  - A will fall from B (backward) after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F - \mu_k g(m+M)}}$

BRAIN MAP

CLASS XI

WORK, ENERGY AND POWER

WORK

Work is said to be done whenever a force acts on a body and the body moves through some distance.

$W = \vec{F}\cdot\vec{S} = FS\cos\theta$ (where θ is the angle between force applied $\vec{F}$ and displacement vector $\vec{S}$.)

The SI unit of work is joule (J).

Nature of Work Done

If $\theta = 0°$, $W = FS$ *i.e.*, work done is maximum.

If $\theta = 90°$, $W = 0$ *i.e.*, work done is zero.

Work Done by a Variable Force

The work done by a variable force in changing the displacement from S_1 to S_2 is $W = \int_{S_1}^{S_2} \vec{F}\cdot d\vec{S}$ = Area under the force-displacement graph

ENERGY

It is defined as the ability of a body to do work. It is measured by the amount of work that a body can do. The unit of energy used at the atomic level is electron volt (eV) and SI unit is J.

Kinetic Energy

It is the energy possessed by a body by virtue of its motion. The K.E. of a body of mass m moving with speed v is

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Potential Energy

It is the energy possessed by a body by virtue of its position (in a field) or configuration (shape or size). For a conservative force in one dimension, the potential energy function $U(x)$ may be defined as

$$F(x) = -\frac{dU(x)}{dx} \text{ or } \Delta U = U_f - U_i = -\int_{x_i}^{x_f} F(x)dx$$

Power

The rate of doing work is called power.

Average Power:

It is defined as the ratio of the small amount of work done W to the time taken t to perform the work.

$$P = \frac{W}{t}$$

The SI unit of power is watt (W).

Work Energy Theorem

The work done by the net force acting on a body is equal to the change in kinetic energy of the body.

W = Change in kinetic energy

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \Rightarrow W = \Delta \text{K.E.}$$

The work energy theorem may be regarded as the scalar form of Newton's second law of motion.

Potential Energy of a Spring

According to Hooke's law, when a spring is stretched through a distance x, the restoring force F is such that

$F \propto x$ (where k is the spring constant or $F = -kx$ and its unit is N m^{-1}.)

The work done is stored as potential energy U of the spring

$$W = \int_0^x kx\,dx = \frac{1}{2}kx^2 \Rightarrow U = \frac{1}{2}kx^2$$

COLLISION

A collision between two bodies is said to occur if either they physically collide against each other or the path of the motion of one body is influenced by the other.

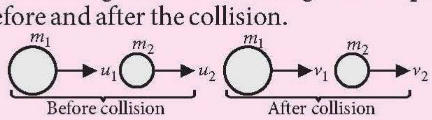
Types of Collision

Elastic collision : Both the momentum and kinetic energy of the system remain conserved.

Inelastic collision : Only the momentum of the system is conserved but kinetic energy is not conserved.

Head-on Collision or One-Dimensional Collision

It is a collision in which the colliding bodies move along the same straight line path before and after the collision.



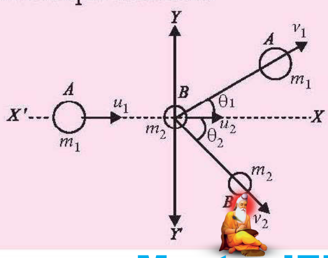
Velocity of approach = Velocity of separation or $u_1 - u_2 = v_2 - v_1$

Also, $v_1 = \frac{m_1 - m_2}{m_1 + m_2}\cdot u_1 + \frac{2m_2}{m_1 + m_2}\cdot u_2$ and

$$v_2 = \frac{2m_1}{m_1 + m_2}\cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2}\cdot u_2$$

Oblique Collision

If the two bodies do not move along the same straight line path before and after the collision, the collision is said to be oblique collision.



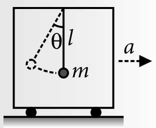
BRAIN MAP

CLASS XI

WORK AND ENERGY

Pendulum Suspended in an Accelerating Trolley

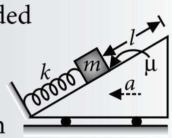
- For a pendulum suspended from the ceiling of a trolley moving with acceleration a, the maximum deflection θ of the pendulum from the vertical is $\theta = 2\tan^{-1}\left(\frac{a}{g}\right)$



Work Energy Theorem for Non-inertial Frames

For a block of mass m welded with light spring (relaxed) When the wedge fitted moves with an acceleration a, block slides through maximum distance l relative to wedge,

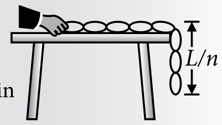
$$l = \frac{2m}{k}[a(\cos\theta - \mu\sin\theta) - g(\sin\theta + \mu\cos\theta)]$$



Work Done in Pulling the Chain

$$W = \frac{MgL}{2n^2}$$

{M = Mass of chain
L = Length
n = Fraction of chain hanged}



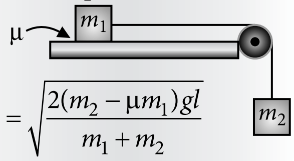
Nature of Work Done

- Positive work ($0° \le \theta < 90°$)
  Component of force is parallel to displacement
- Negative work ($90° < \theta \le 180°$)
  Component of force is opposite to displacement
- Zero work ($\theta = 90°$)
  Force is perpendicular to displacement

Different cases explained using work energy theorem

Motion of Blocks Connected with Pulley

- Two blocks connected by a string, as shown. If they are released from rest. After they have moved a distance l, their common speed is



$$v = \sqrt{\frac{2(m_2 - \mu m_1)gl}{m_1 + m_2}}$$

Work Energy Theorem

Work done by a force acting on a body is equal to the change in the kinetic energy of the body. It is valid for a system in presence of all types of forces.

$$W_{\text{total}} = \Delta K$$

Work

Work done by a force (F) is equal to the scalar product of the force and the displacement (S) of the body.

$W = \vec{F} \cdot \vec{S}$ (θ is the angle between F and S)

$W = FS\cos\theta$

Energy

The energy of a body is defined as its capacity for doing work. Energy is a scalar quantity.

Unit and dimensions for both energy and work are same

Dimensions : $[ML^2T^{-2}]$

S.I. Unit : joule (J)

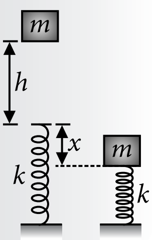
Work Depends on Frame of Reference

With change of the frame of reference (inertial), force does not change while displacement may change. So the work done by a force will vary in different frames.

An Application of Conservation of Energy

- A block of mass m, falling from height h, on a mass less spring of stiffness k.
  - The maximum compression in the spring will be

$$x = \frac{mg}{k}\left[1 + \sqrt{1 + \frac{2kh}{mg}}\right]$$



- If block is released slowly ($h = 0$), maximum compression, $x = \frac{2mg}{k}$
- Work done in bringing the block to stable equilibrium, $W_{ext} = -\frac{m^2g^2}{2k}$

Work Done by Friction

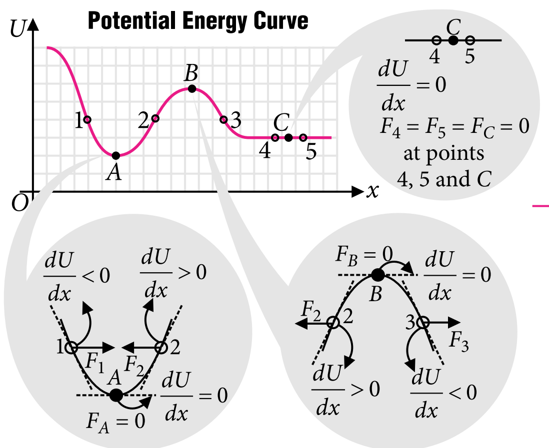
- Work done by static friction is always zero.
- Work done by kinetic friction on the system is always negative.

Work Done by a Spring Force

- Work done for a displacement from x_i to x_f

$$W_s = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

Potential Energy Curve



Potential Energy

It is the ability of doing work by a conservative force. It arises from the configuration of the system or position of the particles in the system.

Relation between Conservative Force and Potential Energy

Negative gradient of the potential energy gives force.

$$F = -\frac{dU}{dr}$$

BRAIN MAP

CLASS XI

COLLISION

CLASSIFICATION OF COLLISON

HEAD-ON COLLISION

The velocities of the particles are along the same line before and after the collision.

On the basis of line of impact

OBLIQUE COLLISION

The velocities of the particles are along different lines before and after the collision.

Head on Inelastic Collision

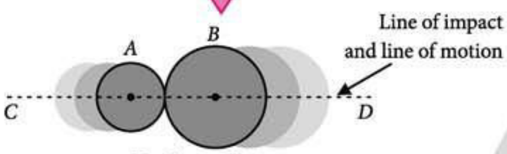
Velocity after collision :

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2$$

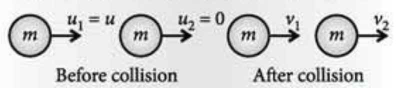
Loss in kinetic energy :

$$(\Delta K) = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(1-e^2)(u_1 - u_2)^2$$



Head-on collision

- **Velocities after inelastic collision :**



$$\therefore \frac{v_1}{v_2} = \frac{1-e}{1+e}$$

- **Coefficient of Restitution (e)**

$$e = \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

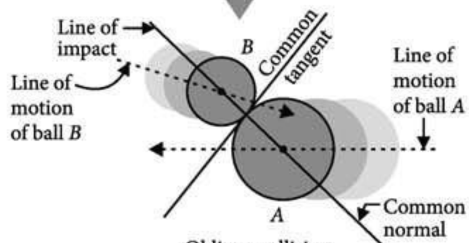
Rebounding of Ball After Collision

- **After first rebound**
  - **Speed :** $v_1 = ev_0 = e\sqrt{2gh_0}$
  - **Height :** $h_1 = e^2 h_0$
- **After n^{th} rebound :**
  - **Speed :** $v_n = e^n v_0$
  - **Height :** $h_n = e^{2n} h_0$
- **Total distance travelled by the ball before it stops bouncing :**
  $H = h_0[(1+e^2)/(1-e^2)]$

In Case of Smooth Surfaces

Common normal :
Force is exerted in common normal direction only. Momentum changes in common normal direction.

Common tangent :
$F = 0$
Neither momentum nor velocity changes in common tangent direction.



Oblique collision

On the basis of kinetic energy

INELASTIC COLLISION

If the kinetic energy after and before collision are not equal, the collision is said to be inelastic.

PERFECTLY INELASTIC COLLISION

If velocity of separation just after collision becomes zero, then the collision is perfectly inelastic.

ELASTIC COLLISION

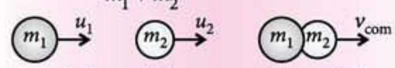
If the kinetic energy after and before collision is same, the collision is said to be perfectly elastic.

By substituting $e = 1$, we get $\Delta K = 0$

Perfectly Inelastic Collision

- **When the colliding bodies are moving in the same direction :**

$$v_{com} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$



Loss in kinetic energy

$$\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

- **When the colliding bodies are moving in the opposite direction :**

$$\therefore v_{com} = \frac{m_1 u_1 - m_2 u_2}{m_1 + m_2}$$

Loss in kinetic energy

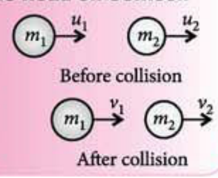
$$\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

Elastic or Perfectly Elastic Head on Collison

Velocity after collision :

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$$

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$$



Special Cases

If projectile and target are of same mass
For $m_1 = m_2 \Rightarrow v_1 = u_1$ and $v_2 = u_1$
i.e., their velocities get interchanged.

If massive projectile collides with a light target
For $m_1 >> m_2 \Rightarrow v_1 = u_1$ and $v_2 = 2u_1 - u_2$
Sub case : For $u_2 = 0$, *i.e.*, target is at rest
$v_1 = u_1$ and $v_2 = 2u_1$

If light projectile collides with a heavy target
For $m_1 << m_2 \Rightarrow v_1 = -u_1 + 2u_2$ and $v_2 = u_2$
Sub case : For $u_2 = 0$, *i.e.*, target is at rest
$v_1 = -u_1$ and $v_2 = 0$, the ball rebounds with same speed.

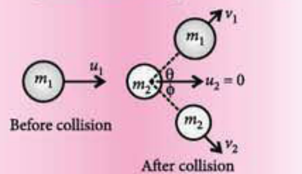
If $m_2 = nm_1$ and $u_2 = 0$
The fractional kinetic energy transferred by projectile

$$\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$$

Fractional kinetic energy retained by the projectile

$$\left(\frac{\Delta K}{K}\right)_{\text{Retained}} = 1 - \text{fractional kinetic energy transferred by projectile}$$

Perfectly Elastic Oblique Collison



After perfectly elastic oblique collision of two bodies of equal masses, the scattering angle $(\theta + \phi)$ would be 90°.

BRAIN MAP

CLASS XI

MASTERJEE CLASSES

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

Centre of Mass and Centre of Gravity

- The centre of gravity of a body coincides with its centre of mass only if the gravitational field does not vary from one point of the body to other.
- Mathematically,

$$\vec{R}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j} + z_{CM}\hat{k}$$

  - For discrete body, $x_{CM} = \frac{1}{M}\sum m_i x_i$,

$$y_{CM} = \frac{1}{M}\sum m_i y_i,\ z_{CM} = \frac{1}{M}\sum m_i z_i$$

  - For continuous body, $\vec{R}_{CM} = \frac{1}{M}\int \vec{r}\, dm$
- Centre of mass of symmetric body
  - Semi-circular ring, $y_{CM} = \frac{2R}{\pi}$
  - Semi-circular disc, $y_{CM} = \frac{4R}{3\pi}$

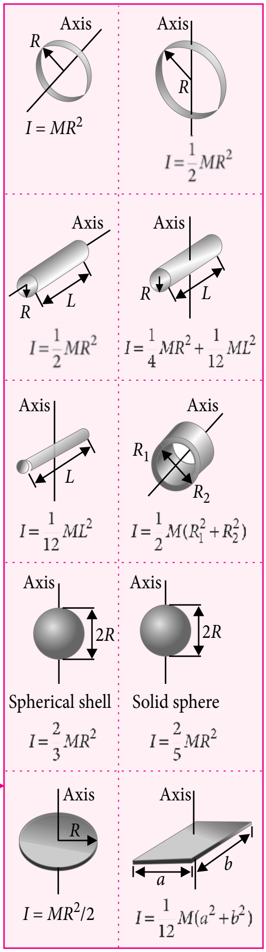
Rotational Motion

- Perpendicular distance of each particle remains constant from a fixed line or point and particle do not move parallel to the line.
- Angular displacement, $\theta = \frac{s}{r}$

  Angular velocity, $\omega = \frac{d\theta}{dt}$

  Angular acceleration, $\alpha = \frac{d\omega}{dt}$
- Equations of rotational motion
  - $\omega = \omega_0 + \alpha t$
  - $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
  - $\omega^2 = \omega_0^2 + 2\alpha\theta$
- Torque : Turning effect of the force about the axis of rotation.

  $\vec{\tau} = \vec{r} \times \vec{F}$; $\tau = rF\sin\theta$; $\tau = I\alpha$
- Angular momentum, $\vec{L} = \vec{r} \times \vec{p}$; $L = I\omega$
- Work done by torque, $W = \tau d\theta$
- Power, $P = \tau\omega$



Motion of Centre of Mass

- For a system of particles
  - Position, $\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...}{m_1 + m_2 + ...}$
  - Velocity, $\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ...}{m_1 + m_2 + ...}$
  - Acceleration, $\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$

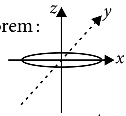
  $\vec{F}_{ext} = 0$, then $\vec{v}_{CM} =$ constant.

Moment of Inertia

- For a rigid body, $I = \sum_{i=1}^{n} m_i r_i^2$
- Perpendicular axes theorem :

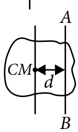
  $I_z = I_x + I_y$

  (Object is in x-y plane)



- Parallel axes theorem :

  $I_{AB} = I_{CM} + Md^2$



Conservation of Angular Momentum

- If the net external torque acting on a system is zero, the angular momentum $\vec{L}$ of the system remains constant, no matter what changes take place within the system.

  $\vec{L}$ = constant; $I_1\omega_1 = I_2\omega_2$

  (for isolated system)

Equilibrium of a Rigid Body

- A rigid body is said to be in mechanical equilibrium, if both of its linear momentum and angular momentum are not changing with time, *i.e.*, total force and total torque are zero.
- Linear momentum does not change implies the condition for the translational equilibrium of the body and angular momentum does not change implies the condition for the rotational equilibrium of the body.

Rolling Motion

- For a body rolling without slipping, velocity of centre of mass

  $v_{CM} = R\omega$

  Kinetic energy,

$$K = K_{translational} + K_{rotational}$$

$$= \frac{1}{2}mv_{CM}^2\left(1 + \frac{k^2}{\ }\right)$$

MOTION OF A RIGID BODY

RIGID BODY MOTION

PURE TRANSLATIONAL MOTION

- During such motion, all the particles have same displacement (s), velocity (v) and acceleration (a) during any interval and at any instant.

- For a system of particles
  - Position, $\vec{r}_{CM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...}{m_1 + m_2 + ...}$
  - Velocity, $\vec{v}_{CM} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2 + ...}{m_1 + m_2 + ...}$
  - Acceleration, $\vec{a}_{CM} = \dfrac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$
  - If $\vec{F}_{ext.} = 0$, then $\vec{v}_{CM}$ = constant.

PURE ROTATIONAL MOTION

- Every point of the body moves in a circle whose centre lies on the axis of rotation and every point moves through same angle.

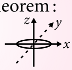
Kinematics of Rotational Motion

- Angular displacement, $\theta = \dfrac{s}{r}$
- Angular velocity, $\omega = \dfrac{d\theta}{dt}$
- Angular acceleration, $\alpha = \dfrac{d\omega}{dt}$
- Equations of rotational motion
  - $\omega = \omega_0 + \alpha t$
  - $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$
  - $\omega^2 = \omega_0^2 + 2\alpha\theta$

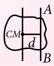
Rotational Analogue of Mass

Moment of Inertia

- For a rigid body, $I = \sum_{i=1}^{n} m_i r_i^2$
- Perpendicular axes theorem: $I_z = I_x + I_y$ (Object is in x-y plane)



- Parallel axes theorem: $I_{AB} = I_{CM} + Md^2$



Dynamics of Rotational Motion

Angular Momentum

- Of a particle about a point $\vec{L} = \vec{r} \times \vec{p}$; $L = r\ p \sin\theta$
- Of a rigid body rotating about a fixed axis $\vec{L} = \sum_i m_i(\vec{r}_i \times \vec{v}_i)$; $L = I\omega$

- From Newton's 2nd law $\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$
- Torque about the axis of rotation $\vec{\tau} = \vec{r} \times \vec{F}$; $\tau = rF\sin\theta = I\alpha$
- Work done by torque, $W = \tau d\theta$
- Power, $P = \tau\omega$

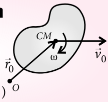
Conservation of Angular Momentum

If $\tau_{net} = 0$, then $\dfrac{dL}{dt} = 0$, so that $L = I\omega$ = constant

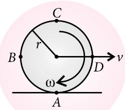
COMBINED TRANSLATIONAL AND ROTATIONAL MOTION

- If all points in the body rotates about an axis of rotation and the axis of rotation moves with respect to the ground.

Angular momentum of a rigid body in combined motion



$\vec{L} = \vec{L}_{CM} + M(\vec{r}_0 \times \vec{v}_0)$



- Velocity of any point of the rigid body in combined motion is the vector sum of $\vec{v}$ and $\vec{r} \times \vec{\omega}$.

- The speed of a point on the circumference at any instant t is $2r\omega \sin(\omega t/2)$
- x and y coordinates of the bottommost point at any time t, $(x, y) \equiv (vt - r\sin\omega t, r - r\cos\omega t)$

PURE ROLLING MOTION

- For a body rolling without slipping, velocity of centre of mass, $v_{CM} = R\omega$

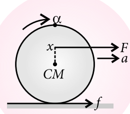
  Kinetic energy,

  $K = K_{trans.} + K_{rot.}$

  $= \dfrac{1}{2}mv_{CM}^2(1 + C)$

When a force acts on a body

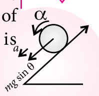
- A force F is applied at a distance x above the centre of a rigid body of radius R, mass M and moment of inertia CMR^2 about an axis passing through the centre of mass



$a = \dfrac{F(R+x)}{MR(C+1)}, f = \dfrac{F(x - RC)}{R(C+1)}$ $\left\{ f \text{ must be } \leq \mu_s mg \right.$

On an Inclined Plane

- A rigid body of radius R, mass M and moment of inertia, $I = CMR^2$ is released at rest.



$a = \dfrac{g\sin\theta}{1 + C}$ $\left\{ C\ (= k^2/R^2) \text{ is a constant varies for different bodies} \right.$

Snapshot of Rolling Motion

For rigid bodies : solid cylinder, hollow cylinder, solid sphere and hollow sphere,

- *Order of acceleration*

  $a_{\substack{solid\\sphere}} > a_{\substack{solid\\cylinder}} > a_{\substack{hollow\\sphere}} > a_{\substack{hollow\\cylinder}}$

- *Order of required friction force for pure rolling*

  $f_{\substack{hollow\\cylinder}} > f_{\substack{hollow\\sphere}} > f_{\substack{solid\\cylinder}} > f_{\substack{solid\\sphere}}$

- *Order of required minimum friction coefficient for pure rolling*

  $\mu_{\substack{hollow\\cylinder}} > \mu_{\substack{hollow\\sphere}} > \mu_{\substack{solid\\cylinder}} > \mu_{\substack{solid\\sphere}}$

BRAIN MAP
CLASS XI

GRAVITATION

Newton's Law of Gravitation

Gravitational force (F) between two bodies is directly proportional to product of masses and inversely proportional to square of the distance between them.

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\cdot\hat{r}$$

Kepler's Laws of Planetary Motion

Law of orbits : Every planet revolves around the sun in an elliptical orbit and the sun is situated at one of its foci.

Law of areas : The areal velocity of the planet around the sun is constant i.e., $\frac{dA}{dt} = \text{a constant}$

Law of periods : The square of the time period of revolution of a planet is directly proportional to the cube of semi major axis of the elliptical orbit.

$$T^2 \propto a^3$$

Acceleration due to gravity

- For a body falling freely under gravity, the acceleration in the body is called acceleration due to gravity.
- Relationship between g and G

$$g = \frac{GM_e}{R_e^2} = \frac{4}{3}\pi G R_e \rho$$

where G = gravitational constant
ρ = density of earth
M_e and R_e be the mass and radius of earth

Characteristics of gravitational force

- It is always attractive.
- It is independent of the medium.
- It is a conservative and central force.
- It holds good over a wide range of distance.

Gravitational Potential Energy

Work done in bringing the given body from infinity to a point in the gravitational field.

$$U = -GMm/r$$

Gravitational potential

Work done in bringing a unit mass from infinity to a point in the gravitational field.

$$V = \frac{-GM}{r}$$

Escape speed

The minimum speed of projection of a body from surface of earth so that it just crosses the gravitational field of earth.

$$v_e = \sqrt{\frac{2GM}{R}}$$

Variation of acceleration due to gravity (g)

Due to altitude (h)

$$g_h = g\left(1 - \frac{2h}{R_e}\right)$$

The value of g goes on decreasing with height.

Due to depth (d)

$$g_d = g\left(1 - \frac{d}{R_e}\right)$$

The value of g decreases with depth.

Due to rotation of earth

$g_\lambda = g - R_e\omega^2\cos^2\lambda$
At equator, $\lambda = 0°$
$g_{\lambda\min} = g - R_e\omega^2$
At poles, $\lambda = 90°$
$g_{\lambda\max} = g_p = g$

Earth's Satellite

Types of Satellite

Polar satellilte

- Time period : 100 min
- Revolves in polar orbit around the earth.
- Height : 500-800 km.
- Uses : Weather forecasting, military spying

Geostationary satellite

- Time period : 24 hours
- Same angular speed in same direction with earth.
- Height : 36000 km.
- Uses : GPS, satellite communication (TV)

Orbital speed of satellite

The minimum speed required to put the satellite into a given orbit.

$$v_0 = R_e\sqrt{\frac{g}{R_e + h}}$$

For satellite orbiting close to the earth's surface

$$v_0 = \sqrt{gR_e}$$

Time period of satellite

$$T = \frac{2\pi}{R_e}\sqrt{\frac{(R_e+h)^3}{g}}$$

For satellite orbiting close to the earth's surface

$$T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6 \text{ min}$$

Energy of satellite

- Kinetic energy $K = \frac{GM_e m}{2(R_e + h)}$
- Potential energy $U = \frac{-GM_e m}{R_e + h}$
- Total energy

$$E = K + U = -\frac{GM_e m}{2(R_e + h)}$$

BRAIN MAP

CLASS XI

GRAVITATIONAL FIELD AND POTENTIAL

Gravitational Field

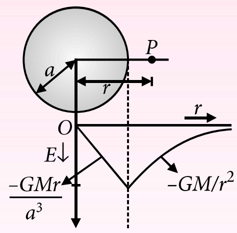
The space surrounding the body within which its gravitation force of attraction is experienced by other bodies is called gravitational field.

Gravitational Field Intensity (E)

The gravitational force experienced by a unit mass placed at a point.

$\vec{E} = \dfrac{\vec{F}}{m}$ SI unit : N kg^{-1}. $[E] = [M^0LT^{-2}]$

Due to uniform solid sphere at point P



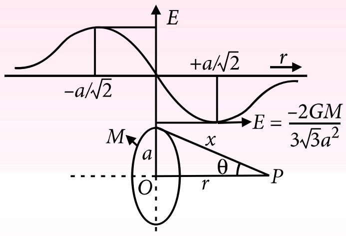
- Inside $r < a$

$$E = -\frac{GMr}{a^3}$$

- Outside $r \geq a$

$$E = -\frac{GM}{r^2}$$

Due to a uniform ring at a distance r on the axis from centre

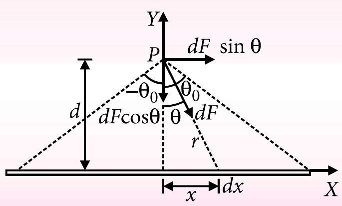


$$E = \frac{-GMr}{(a^2+r^2)^{3/2}}\hat{r}$$

towards the center of ring

Due to a linear mass of finite length on its axis

$$E = \frac{-GM}{d\sqrt{L^2+d^2}}$$



Due to infinite uniform linear mass distribution

Here $\theta_0 = \pi/2$

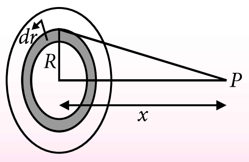
linear mass density λ

Field intensity, $E = \dfrac{2G\lambda}{d}$

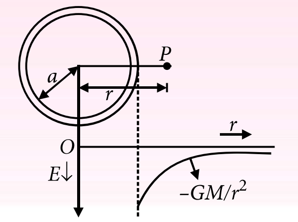
Due to a uniform disc of mass M

At a distance x on the axis from centre

$$E = -\frac{2GMx}{R^2}\left[\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right]$$



Due to uniform spherical shell



- For $r \geq a$

$$E = -\frac{GM}{r^2}$$

- For point inside the shell

$E = 0,\ r < a$

Relation between gravitational field and potential

$$E = -\frac{dV}{dr}$$

Relation between E and R

For two planets A and B of masses m_A, m_B and radius R_A and R_B having

- *equal mass* $\dfrac{E_A}{E_B} = \dfrac{R_B^2}{R_A^2}$
- *equal density* $\dfrac{E_A}{E_B} = \dfrac{R_A}{R_B}$

Gravitational Potential (V)

The amount of work done by an external agent in bringing a body of unit mass from infinity to that point in the gravitational field.

$V = -\dfrac{GM}{r}$ SI unit is J kg^{-1}. $[V] = [M^0L^2T^{-2}]$

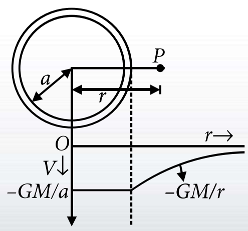
Due to uniform thin spherical shell

- Inside $r \leq a$

$$V = \frac{-GM}{a}$$

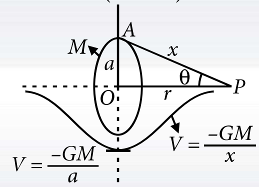
- Outside $r > a$

$$V = \frac{-GM}{r}$$

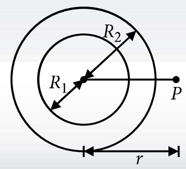


Due to a uniform ring at a distance r on the axis from centre

$$V = \frac{-GM}{(a^2+r^2)^{1/2}}$$



Uniform Thick Spherical Shell



- Outside $V = -G\dfrac{M}{r}$
- Inside $V = -\dfrac{3}{2}GM\left(\dfrac{R_2+R_1}{R_2^2+R_1R_2+R_1^2}\right)$

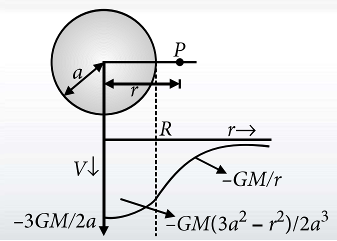
Due to uniform solid sphere at point P

- Inside $r \leq a$,

$$V = -\frac{GM}{2a^3}(3a^2 - r^2)$$

- Outside $r \geq a$

$$V = -\frac{GM}{r}$$

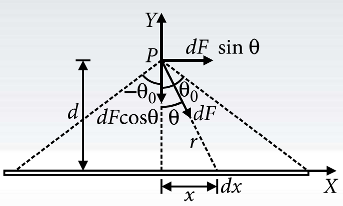


Due to infinite uniform linear mass distribution (λ)

Potential difference between two points at distance d_1 and d_2

$$V_{12} = 2G\lambda \ln\frac{d_2}{d_1}$$

Due to a linear mass of finite length on its axis



$$V = -\frac{GM}{L}\ln\left\{\frac{L+\sqrt{L^2+d^2}}{d}\right\}$$

BRAIN MAP

CLASS XI

MECHANICAL PROPERTIES OF SOLIDS AND FLUIDS

PROPERTIES OF SOLIDS

Young's modulus

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

Bulk modulus,

$$B = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

Compressibility, $k = 1/B$

HOOKE'S LAW

Stress $\propto$ Strain

or Stress $= E \times$ Strain,

(E = modulus of elasticity)

Modulus of rigidity

$$G = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

Poisson's ratio

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

RELATION BETWEEN Y, B, G AND σ

- $Y = 3B(1 - 2\sigma)$
- $Y = 2G(1 + \sigma)$
- $\sigma = \frac{3B - 2G}{2G + 6B}$
- $\frac{9}{Y} = \frac{1}{B} + \frac{3}{G}$

ELASTIC POTENTIAL ENERGY

$$U = \frac{1}{2} F \times \Delta L = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

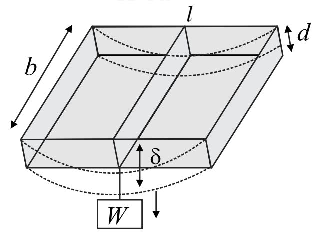
P.E. stored per unit volume of stretched wire,

$$u = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$$

APPLICATION OF ELASTICITY

Designing beams for bridges

The depression in rectangular beam, $\delta = \frac{Wl^3}{4Ybd^3}$



STRESS AND STRAIN

- Stress $= \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$
- Strain $= \frac{\text{Change in configuration}}{\text{Original configuration}}$

ELASTICITY AND PLASTICITY

Elasticity : Ability of a body to regain its original shape, on removing deforming force.

Plasticity : The inability of a body to regain its original size and shape on the removal of the deforming forces.

PROPERTIES OF FLUIDS

FLUIDS IN MOTION

FLUIDS AT REST

VISCOSITY

Coefficient of viscosity:

$$\eta = \frac{F}{A\left(-\frac{dv}{dx}\right)}$$ where $-\frac{dv}{dx}$ is the velocity gradient between two layers of liquid.

SURFACE TENSION

Surface tension: The property by which the free surface of liquid at rest tends to have minimum surface area.

Surface energy: Work done against the force of surface tension in forming the liquid surface.

BERNOULLI'S THEOREM

Bernoulli's theorem : For the streamline flow of an ideal liquid, the total energy per unit volume remains constant

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

PRESSURE

Pascal's law

The pressure is same at all points inside the liquid lying at the same depth in a horizontal plane.

Gauge pressure $= P - P_0 = h\rho g$.

CAPILLARITY

The phenomenon of rise or fall of liquid in a capillary tube is called capillarity.

Height of the liquid within capillary tube

$$h = \frac{2S\cos\theta}{a\rho g}$$ Where, θ = angle of contact, ρ = density of liquid, a = radius of tube

Basic results on viscosity

Stoke's law : Backward dragging force on a spherical body, $F = 6\pi\eta rv$.

Poiseuille's formula

$$Q = \frac{\pi}{8}\frac{Pr^4}{\eta l}$$

Reynold's number : Determines nature of fluid flow $R = \frac{\rho vd}{\eta}$

ARCHIMEDE'S PRINCIPLE

When a body is immersed fully or partly in a liquid at rest, it loses some of its weight, which is equal to the weight of the liquid displaced by the immersed part of the body.

Apparent weight $= mg\left(1 - \frac{\rho'}{\rho}\right)$

(For fully immersed body)

Excess Pressure

In an air bubble

$$\Delta P = \frac{2S}{R}$$

Inside a soap bubble

$$\Delta P = \frac{4S}{R}$$

Inside a liquid drop

$$\Delta P = \frac{2S}{R}$$

BRAIN MAP

CLASS XI

FLUID IN MOTION

Flow of Fluid

- **Streamline flow :** The flow in which path taken by a fluid particle under a steady flow is a streamline in direction of the fluid velocity at that point.
- **Laminar flow :** The liquid is flowing with a steady flow and moves in the form of layers of different velocities and do not mix with each other, is called laminar flow.
- **Turbulent flow :** The flow in which velocity is greater than its critical velocity and the motion of particles becomes irregular is called turbulent flow.
- **Critical velocity :** The velocity of liquid flow upto which the flow is streamlined and above which it becomes turbulent is called critical velocity.
- In compressible flow, the density of fluid varies from point to point, whereas in incompressible flow, the density of the fluid remains constant throughout. Liquids are generally incompressible while gases are compressible.
- Rotational flow is the flow in which the fluid particles while flowing along path-lines also rotate about their own axis. In irrotational flow, particles do not rotate about their axis.

Reynold's Number

- Reynold's number $= \dfrac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}}$

or $N_R = \dfrac{v\rho d}{\eta}$

Where v = velocity of liquid, ρ = density of liquid, d = diameter of tube, η = coefficient of viscosity of liquid.

▸ On the basis of Reynold's number, we have,

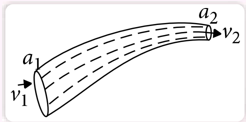
$0 < N_R < 2000 \rightarrow$ streamline flow.

$2000 < N_R < 3000 \rightarrow$ streamline to turbulent flow.

$3000 < N_R \rightarrow$ purely turbulent flow.

Equation of Continuity

- According to conservation of mass, mass of liquid entering per second at wider end = mass of liquid leaving per second at narrower end

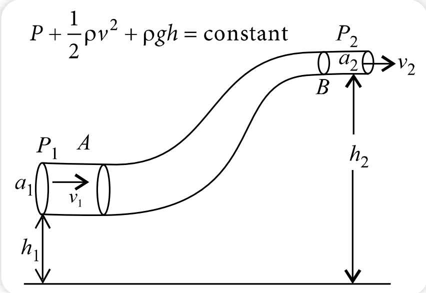


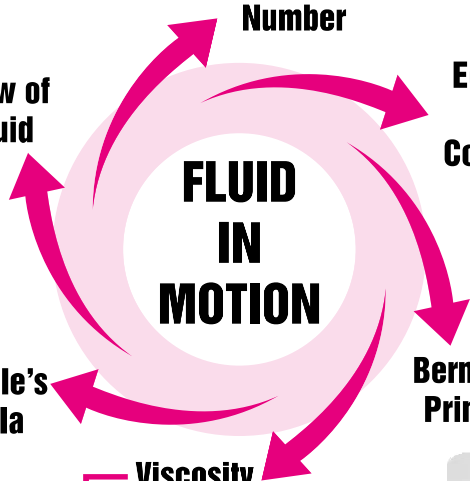
$a_1v_1\rho_1 = a_2v_2\rho_2$

$a_1v_1 = a_2v_2$

(If liquid is incompressible, $\rho_1 = \rho_2 = \rho$)

or av = constant





Bernoulli's Principle

- It states that for a steady flow of an incompressible and non-viscous liquid the sum of the pressure (P), kinetic energy per unit volume (K) and potential energy per unit volume (U) remains constant throughout the flow.

Poiseuille's Formula

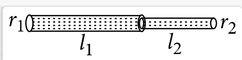
- The rate of volume of fluid coming out of a narrow tube is

$$\frac{V}{t} = \frac{\pi P r^4}{8\eta l}$$

where P = pressure difference, l = length of tube, r = radius of cross-section of the tube.

▸ Liquid resistance, $R = \dfrac{8\eta l}{\pi r^4}$

▸ Series combination of tubes ($V_1 = V_2$)

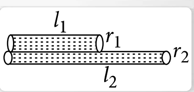


$$\frac{V}{t} = \frac{P}{\left[\dfrac{8\eta l_1}{\pi r_1^4} + \dfrac{8\eta l_2}{\pi r_2^4}\right]}$$

Here, $P = P_1 + P_2$

P_1 and P_2 are the pressure difference across the first and second tubes.

▸ Parallel combination of tubes ($P_1 = P_2$)

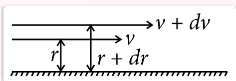


Here, $V = V_1 + V_2$

$$\frac{V}{t} = P\left[\frac{\pi r_1^4}{8\eta l_1} + \frac{\pi r_2^4}{8\eta l_2}\right]$$

Viscosity

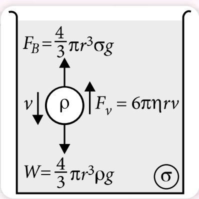
- The property of fluid due to which it opposes the relative motion between its different layers in a steady flow is called viscosity.



▸ Tangential force between the layers, $F = -\eta A(dv/dr)$, where η = a constant called coefficient of viscosity.

▸ SI unit of η is = N s m^{-2} or Poiseuille (Pl), Dimensions of $[\eta] = [ML^{-1}T^{-1}]$

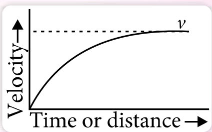
- **Stokes' law :** The viscous drag opposing the motion is $F_v = 6\pi\eta r v$



▸ Terminal velocity : $v = (2/9)[r^2(\rho - \sigma)g/\eta]$

where ρ = density of sphere, σ = density of fluid medium, r = radius of sphere.

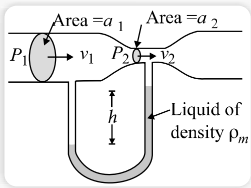
- The variation of velocity with time (or distance)



- **Venturi-meter :** It is a device to measure the speed of flow of incompressible fluid.

▸ Volume of the fluid flowing out per second

$$Q = a_1v_1 = a_1a_2\sqrt{\frac{2h\rho_m g}{\rho(a_1^2 - a_2^2)}}$$



▸ $v_1 = \sqrt{\dfrac{2h\rho_m g}{\rho} \times \dfrac{a_2^2}{a_1^2 - a_2^2}}$

- **Torricelli's law :**

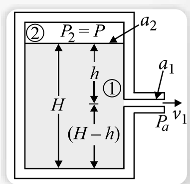
▸ If the container is open at the top to the atmosphere then speed of efflux $v_1 = \sqrt{2gh}$.

▸ Horizontal range, $R = v_1 \times t$

$$= \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$$

R will be maximum if $h = \dfrac{H}{2}$, i.e., $R_{max} = H$

▸ In general as shown in figure, speed of outflow,



$$v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}$$

BRAIN MAP

CLASS XI

MASTERJEE CLASSES

THERMAL PROPERTIES OF MATTER

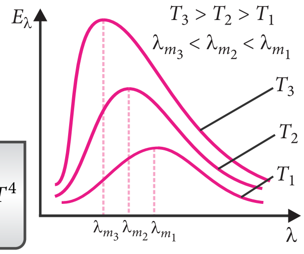
Latent Heat of Fusion (L_f)
$\text{Solid} \xrightarrow{Q = mL_f} \text{Liquid}$

Latent Heat of Vapourisation(L_v)
$\text{Liquid} \xrightarrow{Q = mL_v} \text{Gas}$

Latent Heat of Sublimation (L_s)
$\text{Solid} \xrightarrow{Q = mL_s} \text{Gas}$

$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$

Wien's Displacement Law
$\lambda_m T = b = \text{constant}$
$b = 2.89 \times 10^{-3}$ m K



Linear Expansion
$L = L_0[1 + \alpha\Delta T]$

Relation between α, β and γ

Superficial Expansion
$A = A_0(1 + \beta\Delta T)$

Volume Expansion
$V = V_0(1 + \gamma\Delta T)$

Stefan's Boltzmann Law
Emissive power, $E \propto T^4 \Rightarrow E = \sigma T^4$
For ordinary body, $E = e\sigma T^4$

CHANGE OF STATE
During the change of phase, temperature remains constant but heat is being supplied to the body.

THERMAL EXPANSION
Increase in configuration of a solid with increasing temperature.

Radiation
Heat is transferred without heating the intervening medium.

Kirchhoff's Law
Ratio of emissive power to absorptive power at temperature, T is
$\frac{E_1}{A_1} = \frac{E_2}{A_2} = \ldots \left(\frac{E}{A}\right)_{\text{Perfectly black body}}$

HEAT
Heat is the form of energy transferred between two or more systems or a system and its surroundings by virtue of temperature difference.

TEMPERATURE
It is defined as the thermal state of the body, which would determine the direction of flow of heat when this body is placed in contact with another body.

HEAT TRANSFER
Transfer of heat from one system to another system arises due to temperature difference.

Convection
Heat is transferred due to actual motion of heated particles.

Applications of Convection
- Formation of trade winds
- Causes monsoons
- Land and sea breezes etc.

Conduction
Heat is transferred without any actual motion of particles of the substance.

Rate of Heat Flow
$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{d}; (T_1 > T_2)$
(K = Coefficient of thermal conductivity)

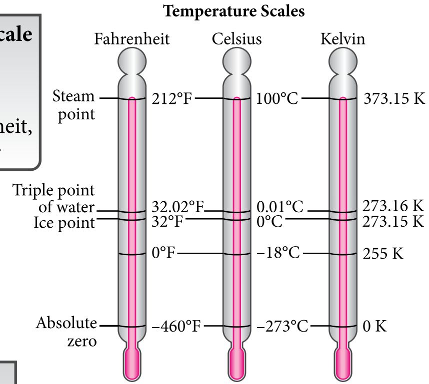
PRINCIPLE OF CALORIMETRY

It is based on the law of conservation of energy.
Heat lost by one body = Heat gained by other body

THERMOMETRY
A branch of physics which deals with the measurement of temperature, is known as the thermometry.

Conversion of Temperature Scale
$\frac{C}{5} = \frac{F - 32}{9} = \frac{K - 273}{5} = \frac{R}{4}$
where, C = Celsius, F = Fahrenheit, K = Kelvin, R= Reaumur

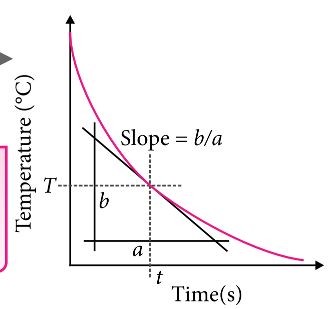
Temperature Scales



Triple Point of Water
Temperature and pressure at which all the three phases of water exists in equilibrium. $T_3 = 273.16$ K

NEWTON'S LAW OF COOLING
The rate of loss of heat by a body is directly proportional to the excess of temperature of the body over surroundings, provided excess is small.

Newton's Law of Cooling
Rate of cooling of a body is proportional to the excess temperature of the body over the surroundings
$\frac{dT}{dt} = \frac{-k}{ms}(T_2 - T_1)$



HEAT CAPACITY
The amount of heat absorbed by a given amount of substance to change the temperature by unity.

Molar Specific Heat
The amount of heat required to raise the temperature of one mole of a substance by unity. $C = \frac{\Delta Q}{n\Delta T}$

Specific Heat Capacity
The amount of heat required to raise the temperature of 1g of substance by 1 °C. $s = \frac{\Delta Q}{m\Delta T}$

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BRAIN MAP
CLASS XI
THERMODYNAMICS

Thermal Equilibrium
The macroscopic variables such as pressure, temperature, volume, mass, composition, etc., which characterize a system, do not change with time.

Zeroth Law
Two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other.

First Law
Heat supplied to a gas may
(i) raise its internal energy
(ii) enable it to expand and thereby do external work. $dQ = dU + dW$

Second Law
There is no heat engine can have efficiency η equal to 1 or no refrigerator can have coefficient of performance is equal to infinity.

Specific Heat Capacity
$\Delta Q = ms\Delta T = ms(T_f - T_i)$ or $s = \dfrac{\Delta Q}{m\Delta T}$

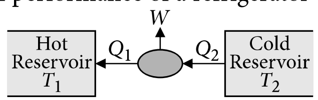
Molar specific heat capacity, $C = \dfrac{1}{\mu}\left(\dfrac{\Delta Q}{\Delta T}\right)$

State Variables and Equation of State
The relation between the state variables (*P, V, T*) of the system is called equation of state.
For μ moles of an ideal gas, equation of state is $PV = \mu RT$ and for 1 mole of an ideal gas it is $PV = RT$.

THERMODYNAMICS
Laws of Thermodynamics
Thermodynamic Processes

Refrigerator
The coefficient of performance of a refrigerator is ;

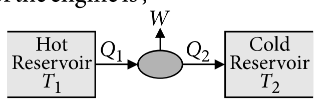
$\alpha = \dfrac{Q_2}{W} = \dfrac{Q_2}{Q_1 - Q_2}$



here, $(Q_1 < Q_2)$

Heat Engine
The efficiency η of the engine is ;

$\eta = \dfrac{W}{Q_1} = 1 - \dfrac{Q_2}{Q_1}$



here $(Q_1 > Q_2)$

Carnot's Cycle
Isothermal expansion:

$$W_1 = \mu RT_1 \log_e \frac{V_2}{V_1}$$

Adiabatic expansion:

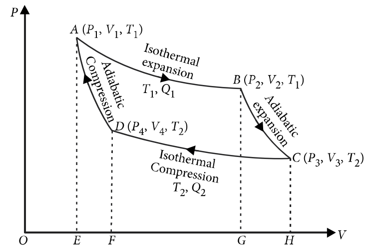
$$W_2 = \mu \frac{R(T_1 - T_2)}{\gamma - 1}$$

Isothermal compression:

$$W_3 = \mu RT_2 \log_e \frac{V_3}{V_4}$$

Adiabatic compression:

$$W_4 = \mu \frac{R(T_1 - T_2)}{\gamma - 1}$$



Net work done during the complete cycle,

$W = W_1 + W_2 + (-W_3) + (-W_4)$
$= W_1 - W_3 = \text{Area ABCD}$
(As $W_2 = W_4$)

Efficiency, $\eta = \dfrac{\text{Work done}}{\text{Heat input}} = \dfrac{W}{Q_1}$

$\eta = 1 - \dfrac{Q_2}{Q_1} = \dfrac{W}{Q_1}$

Relation between Coefficient of Performance and Efficiency of Refrigerator
$$\alpha = \frac{1 - \eta}{\eta}$$

Isothermal Process
Isothermal process : A thermodynamic process in which the temperature remains constant.
- Equation of isothermal process, $PV = \text{constant}$.
- Work done during isothermal process,

$W = \mu RT \ln\left(\dfrac{V_f}{V_i}\right)$; $W = \mu RT \ln\left(\dfrac{P_i}{P_f}\right)$

- The slope of isothermal curve on a *P-V* diagram at any point on the curve is given by

$\dfrac{dP}{dV} = -\dfrac{P}{V}$

Adiabatic Process
Adiabatic Process : A thermodynamic process in which no heat flows between the system and the surroundings.
- Equation of adiabatic process,
$PV^\gamma = \text{constant}$, where $\gamma = C_P/C_V$.
- Work done during adiabatic process,

$W = \dfrac{(P_iV_i - P_fV_f)}{(\gamma - 1)}$; $W = \dfrac{\mu R(T_i - T_f)}{\gamma - 1}$

- The slope of adiabatic curve on a *P-V* diagram at any point on the curve is given by

$\dfrac{dP}{dV} = -\gamma\left(\dfrac{P}{V}\right)$

Isochoric Process
Isochoric (isometric) process : A thermodynamic process in which volume remains constant.
- Equation of isochoric process :
$\dfrac{P}{T} = \text{constant}$.
- No work is done by the gas in an isochoric process.
- The slope of the isochoric curve on a *P-V* diagram is infinite.

Isobaric Process
Isobaric process : A thermodynamic process in which pressure remains constant.
- Equation of isobaric process : $\dfrac{V}{T} = \text{constant}$.
- Work done during isobaric process,
$W = P(V_f - V_i) = \mu R(T_f - T_i)$.
- The slope of the isobaric curve on a *P-V* diagram is zero.

MASTERJEE CLASSES

BRAIN MAP

KINETIC THEORY

Class XI

KINETIC THEORY

Kinetic Theory of Ideal Gases

Relation between v_{rms}, v_{av} and v_{mp}

$$v_{rms} : v_{av} : v_{mp}$$

$$= \sqrt{\frac{3RT}{M}} : \sqrt{\frac{8RT}{\pi M}} : \sqrt{\frac{2RT}{M}}$$

$$= \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2} \; ; (v_{rms} > v_{av} > v_{mp})$$

Maxwell's Law of Distribution of Velocities

The distribution of molecules at different speed is given as,

$$dN = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv$$

Mean Free Path

The average distance travelled between successive collisions of molecules of a gas is called mean free path (λ).

$\lambda = \frac{1}{\sqrt{2}n\pi d^2}$; where n is the number density and d is the diameter of the molecule.

Kinetic Interpretation of Temperature

$$KE_{avg} = E = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$

$$KE / \text{mole} = \left(\frac{3}{2}kT\right)N_A = \frac{3}{2}RT$$

Pressure Exerted by a Gas

$$P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2 = \frac{2}{3}E'$$

E' = Average KE per unit volume

Specific Heat Capacity

Specific Heat of a Gas

At constant pressure (C_P) :

$$C_P = \frac{(\Delta Q)_P}{n\Delta T} \text{ or } C_P = \left(1 + \frac{f}{2}\right)R$$

At constant volume (C_V) :

$$C_V = \frac{(\Delta Q)_V}{n\Delta T} \text{ or } C_V = \frac{1}{2}fR$$

Mayer's relation : $C_P - C_V = R$

(f = degree of freedom)

Monoatomic Gas ($f = 3$)

$$U = \frac{3}{2}RT, C_V = \frac{3}{2}R, C_P = \frac{5}{2}R, \gamma = \frac{5}{3}$$

Diatomic Gas ($f = 5$)

$$U = \frac{5}{2}RT, C_V = \frac{5}{2}R, C_P = \frac{7}{2}R, \gamma = \frac{7}{5}$$

Polyatomic Gas

$U = (3 + f')\,RT$

$C_V = (3 + f')\,R$

$C_P = (4 + f')\,R$

$\gamma = (4 + f')/(3 + f')$

f' = a certain number of vibrational mode

Law of Equipartition of Energy

For any system in thermal equilibrium, the total energy is equally distributed among its various degrees of freedom and each degree of freedom is associated with energy $\frac{1}{2}kT$.

Behaviour of Gases

Vander Waal's Equation

For n moles of a gas,

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

$$\begin{bmatrix} [a] = [ML^5T^{-2}] \\ [b] = [L^3] \end{bmatrix}$$

- **Critical Temperature :** $T_c = \frac{8a}{27Rb}$
- **Critical Pressure :** $P_c = \frac{a}{27b^2}$
- **Critical Volume :** $V_c = 3b$

Graham's Law of Diffusion

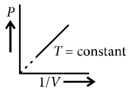
For given temperature and pressure, the rate of diffusion of gas is inversely proportional to the square root of the density of the gas. $r \propto \frac{1}{\sqrt{\rho}} \propto \frac{1}{\sqrt{M}}$

Gas Laws

Boyle's Laws

At constant temperature, volume of a fixed mass of a gas is inversely proportional to its pressure.

$P \propto \frac{1}{V}$ or PV = constant

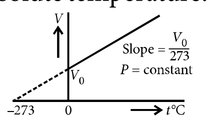


Charle's Laws

The volume of the gas is directly proportional to its absolute temperature.

$V \propto T$ (at constant P)

$$V_t = V_0\left(1 + \frac{t}{273}\right)$$



Gay-Lussac's Law

Pressure of the gas varies directly with the temperature at constant volume.

$P \propto T$ (at constant volume)

$$P_t = P_0\left[1 + \frac{t}{273}\right]$$

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OSCILLATIONS

Periodic Motion

A motion that repeats itself at regular interval of time is called periodic motion. The displacement is represented by a periodic function of time with time period T. *i.e.*, $f(t) = f(t+T) = f(t+2T) = ...$

Oscillatory Motion

If the body is given a small displacement from the position, a force comes into play which tries to bring the body back to the equilibrium point. Such motions are called oscillatory motion.

Simple Harmonic Motion

The motion arises when the force on the oscillating body is directly proportional to its displacement from mean position. Such motion is called simple harmonic motion.

System Excuting SHM

Dynamic of SHM

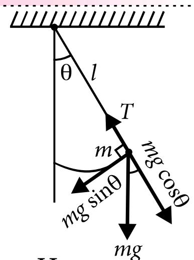
Characteristics of SHM

SHM IN SPRING

- Equation of motion
  $$\frac{d^2y}{dt^2} = \frac{-ky}{m} = -\omega^2 y$$
- If the spring is not light but has a definite mass m_s then
  $$T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$$
- Two bodies of masses m_1 and m_2 are attached through a light spring of spring constant k, the time period of oscillation
  $$T = 2\pi\sqrt{\frac{\mu}{k}} \quad \text{where } \mu = \frac{m_1 m_2}{m_1 + m_2}$$

SIMPLE PENDULUM

- Time period
  $$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{l}{g}}$$
- If the length of simple pendulum is very large,
  $$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}}$$



Here, $\alpha = -\frac{mgl}{I}\theta$

where R is the radius of length of pendulum

FORCE LAW IN SHM

- The force acting on a particle of mass m in SHM is
  $\vec{F} = -m\omega^2\vec{x}$ or $\vec{F} = -k\vec{x}$
  where, $k = m\omega^2$ = force constant
- Linear SHM:
  - Angular velocity, $\omega = \sqrt{\frac{k}{m}}$
  - Time period, $T = 2\pi\sqrt{\frac{m}{k}}$
- Angular SHM:
  - Torque, $\tau = -k\theta$
  - Angular velocity, $\omega = \sqrt{k/I}$
  - Angular acceleration, $\alpha = -\frac{k\theta}{I}$
  - Time period, $T = 2\pi\sqrt{\frac{I}{k}}$
    where I = moment of inertia

ENERGY IN SHM

- Linear SHM:
  - Kinetic energy $(K) = \frac{1}{2}m\omega^2A^2\cos^2\omega t$
  - Potential energy $(U) = \frac{1}{2}m\omega^2A^2\sin^2\omega t$
  - Total energy $(E) = \frac{1}{2}m\omega^2A^2$
- Angular SHM:
  - Kinetic energy $(K) = \frac{1}{2}I\omega^2$
  - Potential energy $(U) = \frac{1}{2}k\theta^2 = \frac{1}{2}I\omega^2\theta^2$
  - Total energy $(E) = \frac{1}{2}I\omega^2\theta_0^2$

GENERAL EQUATIONS OF SHM

- Linear SHM:
  - Differential equation $\frac{d^2y}{dt^2} + \omega^2 y = 0$
  - Displacement $y = A\sin(\omega t + \phi)$
  - Velocity, $v = \omega\sqrt{A^2 - y^2}$
  - Acceleration, $a = -\omega^2 y$
- Angular SHM:
  - Differential equation
    $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$
  - Displacement $\theta = \theta_0\sin(\omega t + \delta)$

DAMPED AND FORCED OSCILLATIONS

- Damped oscillations
  - Angular frequency $(\omega') = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
  - Mechanical energy $E(t) = \frac{1}{2}kA^2e^{\frac{-bt}{m}}$
  - Amplitude $A' = Ae^{-bt/2m}$
    where b is damping constant.
- Forced oscillations
  - When driving frequency ω_d far from natural frequency ω:
    Amplitude $A' = \frac{F_0}{m(\omega^2 - \omega_d^2)}$
  - When driving frequency ω_d closed to natural frequency ω:
    Amplitude $A' = \frac{F_0}{\omega_d b}$

BRAIN MAP

WAVES

Class XI

TYPES OF WAVES

Electromagnetic Waves

Waves propagating in form of oscillating electric and magnetic fields.
Do not require medium for propagation.

Matter Waves

Waves associative with microscopic particles such as electrons, protons etc. in motion are called matter waves.

Mechanical Waves

Waves which require a material medium for their propagation are called mechanical waves.

Transverse Waves

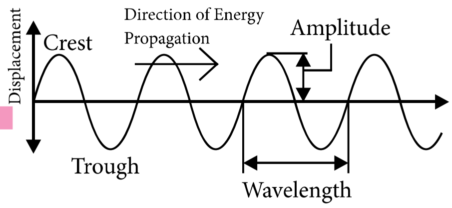
The individual particles of the medium oscillate perpendicular to the direction of wave propagation.

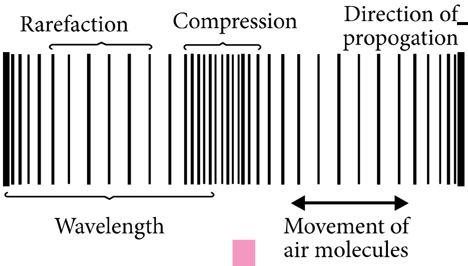
Longitudinal Waves

The individual particles of medium oscillate along the direction of wave propagation.

Velocity of Transverse Wave in Solids and Strings

- In solids, $v=\sqrt{\dfrac{\eta}{\rho}}$
  where η is modulus of rigidity and ρ is density of solids.
- In stretched string, $v=\sqrt{\dfrac{T}{m}}$
  here, T is tension in string and m is mass per unit length of string.





Velocity of Longitudinal Waves

- In a solid of bulk modulus κ, modulus of rigidity η and density ρ is
  $$v=\sqrt{\frac{\kappa+\frac{4}{3}\eta}{\rho}}$$
- In a fluid of bulk modulus κ and density ρ is
  $$v=\sqrt{\frac{\kappa}{\rho}}$$
- Newton's formula for the velocity of sound in a gas is
  $$v=\sqrt{\frac{\kappa_{iso}}{\rho}}=\sqrt{\frac{P}{\rho}}\ (P=\text{pressure of the gas})$$

WAVE MOTION

Progressive Waves

- **Displacement,** $y=A\sin(\omega t-kx+\phi_0)$
  $$y=A\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)=A\sin\frac{2\pi}{\lambda}(vt-x)$$
- **Phase,** $\phi=2\pi\left(\dfrac{t}{T}-\dfrac{x}{\lambda}\right)+\phi_0$
  where ϕ_0 is the initial phase.
- **Phase change:**
  (a) with time $\Delta\phi=\dfrac{2\pi}{T}\Delta t$
  (b) with position $\Delta\phi=\dfrac{2\pi}{\lambda}\Delta x.$

Doppler's Effect in Sound

- If v, v_0, v_s and v_m are the velocities of sound, observer, source and medium respectively, then the apparent frequency,
  $$\upsilon'=\frac{v+v_m-v_0}{v+v_m-v_s}\times\upsilon$$
- If the medium is at rest, $(v_m=0)$ then
  $$\upsilon'=\frac{v-v_0}{v-v_s}\times\upsilon$$

Superposition of Waves

Identical waves of same speed superposes in opposite direction

Waves with same speed and different frequency superposes in same direction

Stationary Waves

- Wave formed by the superposition of incident wave and reflected wave is given by $y=\pm 2a\sin\dfrac{2\pi x}{\lambda}\cos\dfrac{2\pi t}{T}$
- Position of antinodes: $x=0,\dfrac{\lambda}{2},\lambda,\dfrac{3\lambda}{2}$
- Position of nodes: $x=\dfrac{\lambda}{4},\dfrac{3\lambda}{4},\dfrac{5\lambda}{4}...$
- Frequency of vibration of a string fixed at both ends, $\upsilon=\dfrac{nv}{2L}=\dfrac{n}{2L}\sqrt{\dfrac{T}{m}}$
  L = length of string, n = mode of vibration

- **Open organ pipe:**
  Fundamental mode,
  $\upsilon_1=v/2L=\upsilon$ (1^{st} harmonic)
  n^{th} mode, $\upsilon_n=nv/2L$ (n^{th} harmonic and $(n-1)^{th}$ overtone)
- **Closed organ pipe:**
  Fundamental mode,
  $\upsilon_1=v/4L=\upsilon$ (1^{st} harmonic)
  n^{th} mode, $\upsilon_n=(2n-1)\upsilon$
  [$(2n-1)^{th}$ harmonic or $(n-1)^{th}$ overtone]

Beats Formation

- **Beat frequency** = Number of beats sec^{-1}
  = Difference in frequencies of two sources.
  $\upsilon_{beat}=(\upsilon_1-\upsilon_2)$ or $(\upsilon_2-\upsilon_1)$
  $\therefore\ \upsilon_2=\upsilon_1+\upsilon_{beat}$
- If prongs of tuning fork is filed υ increases.
- If prongs is loaded with a wax υ decreases.
- **Uses:**
  - For tuning musical instruments
  - For detection of marsh gas in mines
  - For using as a low frequency oscillator.

ELECTRIC CHARGES AND FIELDS

Basic Properties of Charges

Quantization of charge : Total charge on a body is always an integral multiple of a basic unit of charge denoted by e and is given by $q = ne$.

Conservation of charge : Total charge of an isolated system remains unchanged with time.

Additivity of charge : Total charge of a system is the algebraic sum (*i.e.* sum taking into account with proper signs) of all individual charges in the system.

Coulomb's Law

$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r}_{12}$ for like charges

$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^3}\vec{r}_{21}$ for unlike charges

using Superposition principle

- The vector sum of forces would give us the total force.
- Force between two charges is unaffected by the presence of the other charges.

Electric Field

Electric Field : Electric field intensity at a point distant r from a point charge q in air is $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$.

Basic characteristics

- Field lines start from positive charges and end at negative charges.
- Electrostatic field lines do not form any closed loops.
- Two field lines can never cross each other.

Electric Dipole

Electric Dipole : Every dipole is associated with a dipole moment $\vec{p}$ whose magnitude is equal to the product of the magnitude of either charge (q) and the distance $2\vec{a}$ between the charges, *i.e.*, $\vec{p} = q \times (2\vec{a})$.

Dipole in an external field experiences

Torque
$\vec{\tau} = \vec{p} \times \vec{E}$ or $\tau = pE \sin\theta$.

Electric field due to dipole

- On **Equatorial line** (Broad on position) $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$
- On **Axial line** (End on position) $E = \frac{2p}{4\pi\varepsilon_0 r^3}$

Electric Flux

Electric Flux : Electric flux over an area in an electric field represents the total number of electric field lines crossing this area.

Gauss's Theorem : Total normal electric flux over a closed surface S in vacuum is $1/\varepsilon_0$ times the charge (Q) contained inside the surface. $\phi_E = \oint_S \vec{E}.d\vec{S} = \frac{Q}{\varepsilon_0}$.

Applications

volume charge density, $\rho = \frac{q}{V}$

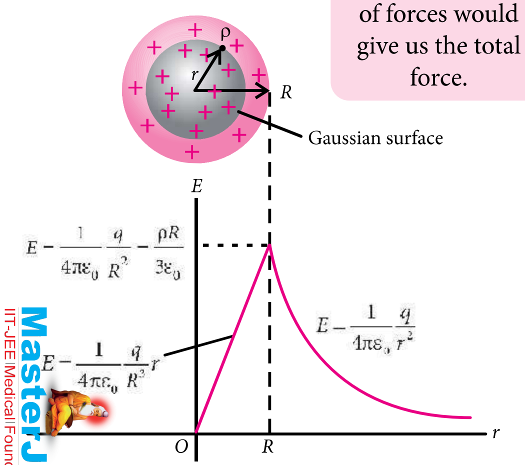
Electric field due to **a uniformly charged thin spherical shell**
At a point outside the shell *i.e.*, $r > R$, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}$
On the surface $r = R$, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$
Inside the shell *i.e.*, $r < R$, $E = 0$

surface charge density, $\sigma = \frac{q}{A}$

Electric field due to **a uniformly charged infinite thin plane sheet,** $E = \frac{\sigma}{2\varepsilon_0}$.

linear charge density, $\lambda = \frac{q}{l}$

Electric field due to **an infinitely long thin uniformly charged straight wire,** $E = \frac{\lambda}{2\pi\varepsilon_0 r}$.



For uniformly charged solid sphere

ELECTRIC FLUX AND GAUSS'S LAW

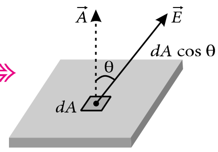
BRAIN MAP

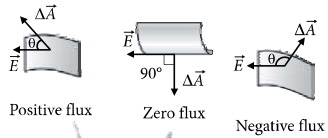
CLASS XII

ELECTRIC FLUX

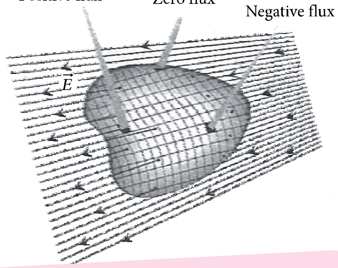
Electric flux is a measure of flow of electric field through a surface. It is equal to the dot product of an area vector and electric field

- Flux of electric field E through any area A: $\phi = EA\cos\theta$ or $\phi = \vec{E}\cdot\vec{A}$
- For variable electric field or curved area $\phi = \int \vec{E}\cdot d\vec{A}$





- For a closed surface outward flux is taken to be positive while inward flux is taken to be negative.



To get a direct connection between the electric flux through closed surface and the total charge inside it.

Problem Solving Strategies

- *Select a symmetric gaussian surface as per the charge distribution.*
- *Calculate total electric charge inside the gaussian surface.*
- *For uniform charge density, simply multiply it by length, area and volume of surface.*
- *For non uniform charge density integrate it over the region enclosed the surface.*
- *Calculate electric field on the gaussians surface as per the given uniform charge distribution.*

GAUSS'S LAW

The total flux linked with a closed Gaussian surface is $(1/\varepsilon_0)$ times the charge enclosed by the closed surface *i.e.*,

$$\phi = \int_s \vec{E}\cdot\vec{dA} = \frac{1}{\varepsilon_0}(Q_{enc})$$

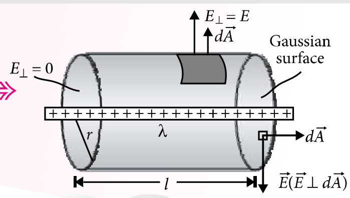
APPLICATIONS OF GAUSS'S LAW

Flux across some definite symmetric closed surfaces

- **Hemispherical body**
  In uniform electric field $\phi_{circular} = -\phi_{curved}; |\phi_{circular}| = \pi R^2 E$
  In non uniform electric field $\phi_{circular} = -\phi_{curved}; |\phi_{curved}| = 2\pi R^2 E$
- **Gaussian Cube**
  Charge kept at corner $\phi_{cube} = \frac{Q}{8\varepsilon_0}$ $\phi_{face} = \frac{Q}{24\varepsilon_0}$ $\phi_{total} = \frac{Q}{\varepsilon_0}$
  Charge kept at centre of face $\phi_{cube} = \frac{Q}{2\varepsilon_0}(5\,faces)$ $\phi_{total} = \frac{Q}{\varepsilon_0}$

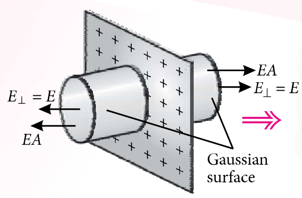
Field of a line charge

$$E_{net} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$



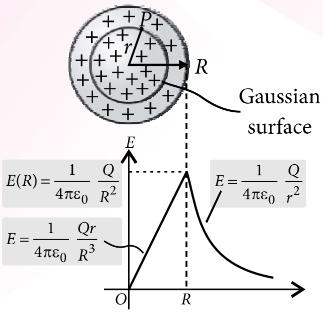
Field of an infinite plane sheet of charge

$$E = \frac{\sigma}{2\varepsilon_0}$$

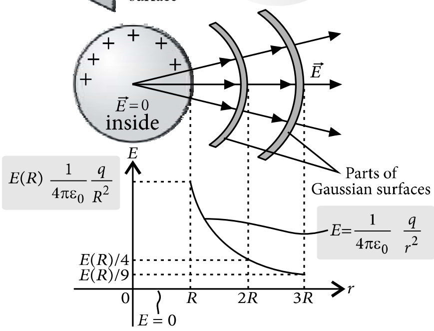


Result in field due to a sheet depends only on total charge of the sheet and independent of distribution of charge.

Field of a uniformly charged solid sphere and charged conducting sphere

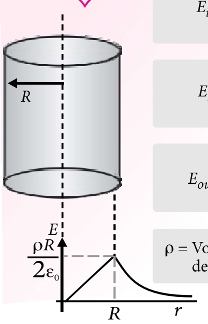


Uniformly charged sphere



Charged conducting sphere

Field due to a long uniformly charged solid cylinder



$$E_{in} = \frac{\rho r}{2\varepsilon_0}$$

$$E_s = \frac{\rho R}{2\varepsilon_0}$$

$$E_{out} = \frac{\rho R^2}{2\varepsilon_0 r}$$

ρ = Volume charge density

ELECTROSTATIC POTENTIAL AND CAPACITANCE

BRAIN MAP

CLASS XII

Electrostatic Potential

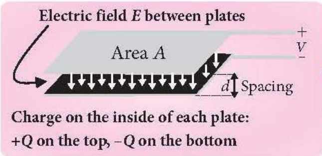
Work done per unit positive test charge by an external force in bringing a unit positive charge from infinity to a point in the presence of another point charge.

$$V = -\frac{W}{q_0} = \frac{q}{4\pi\varepsilon_0 r}$$

Electric Potential Energy

For a system of two charges

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$



Capacitor and Capacitance

Capacitor is used to store electrical energy. Capacitance is defined as the ratio of the charge stored to the potential between the plates.

$$C = \frac{Q}{V}$$

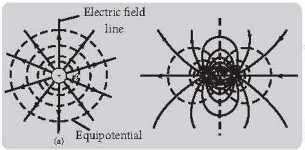
Relation between $\vec{E}$ and V

$$\vec{E} = -\vec{\nabla} V$$

$$E = -\frac{dV}{dr}$$

Equipotential Surface

Surface having same electrostatic potential at every point.



Properties

- Do not intersects each other
- At every point $\vec{E} \perp$ surface
- Work done in moving a charge is zero $W_{net} = 0$
- Closely spaced in the region of strong field and vice-versa.

Electrostatic Potential Due to an Electric Dipole

At any arbitrary point; $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

At axial point $V = \frac{p}{4\pi\varepsilon_0 r^2}$

At equatorial point $V = 0$

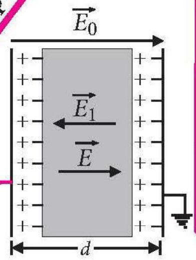
Potential energy of a dipole in external field

$U(\theta) = pE(\cos\theta_0 - \cos\theta)$

$\rightarrow$ When initially at $\theta_0 = 90°$

$\Rightarrow U = -\vec{p}\cdot\vec{E}$

Capacitances of different types of capacitors



Parallel plate capacitor with dielectric slab of thickness t

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

Parallel plate capacitor filled with dielectric

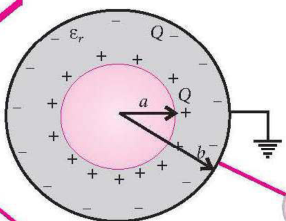
$$C = \frac{K\varepsilon_0 A}{d}$$

Air filled parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

Parallel plate capacitor with metallic conductor inserted in it

$$C = \frac{\varepsilon_0 A}{(d-t)}$$



Spherical capacitor

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

Combination of Capacitor

Series combination

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

Parallel combination

$$C_p = C_1 + C_2$$

Electrostatic Shielding

To shield an electronic circuit from external field by surrounding it with conducting walls.

Lightning Conductor

Lightning conductors fitted above the highest part of a building to protect a tall building from being struck by lightning.

Electric Potential due to Uniformly Charged Spherical Shell

Outside the shell

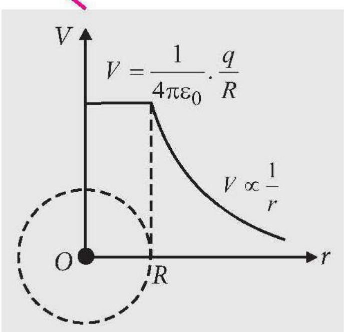
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}; r > R$$

On the shell

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}; r = R$$

Inside the shell

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$



Energy Stored in a Capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

Energy Density

$$u = \frac{U}{V} = \frac{1}{2}\varepsilon_0 E^2$$

Electric Potential Due to a Non-Conducting Solid Sphere

outside the sphere

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}; r > R$$

on the sphere i.e.,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}; r = R$$

inside the sphere

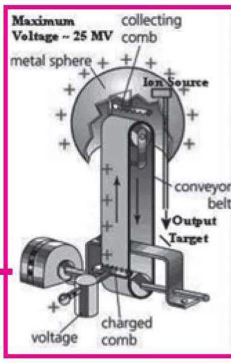
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q(3R^2 - r^2)}{2R^3}; r < R$$

Van de Graaff Generator

An electrostatic generator design to produce high voltage of the order of 10 million volt, used to accelerate charged particles.

Principle

- If an electric charge is imparted to the inside of a spherical conductor, it is distributed entirely on its outer surface.
- Pointed ends cannot retain charge due to high charge density on them.



MASTERJEE CLASSES

CAPACITOR AND CAPACITANCE

BRAIN MAP

CLASS XII

$$K = \frac{\varepsilon\,(\text{Permitivity of medium})}{\varepsilon_0(\text{Permitivity of free space})}$$

Dielectric Constant

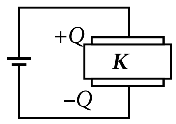
Polar dielectric
Permanent dipole moment exist in absence of electric field also.

Non-polar dielectric
Each molecule has zero dipole moment in its normal state.

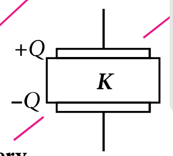
Types of Dielectric

For linear isotropic dielectric
$\vec{P} = \chi_e \vec{E}$ (χ_e = electrical susceptibility)

- $C' = KC$
- $U = KU_0$
- $Q = KQ_0$
- $E = E_0$
- $V = V_0$



Battery Connected



Battery Disconnected

- $C' = KC$
- $Q = Q_0$
- $V = \frac{V_0}{K}$
- $U = \frac{U_0}{K}$
- $E = \frac{E_0}{K}$

Dielectric is an insulator which transmits electric effect without conducting. If field exceeds the dielectric strength, the dielectric begins to conduct.

Energy Stored

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$$

Energy Density

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Parallel Plate Capacitor

Energy Stored

$$U = \frac{1}{2}QV = \frac{Q^2}{8\pi\varepsilon_0 a}$$

Energy Density

$$u = \frac{Q^2}{32\pi^2\varepsilon_0 r^4}$$

Spherical Conductor of Radius *a*

Dielectric and its Properties

Energy Storage in a Capacitor

Capacitors with Dielectrics

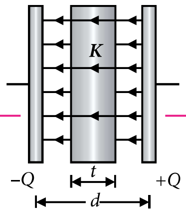
CAPACITOR

Capacitor is a passive electronic component that stores energy in the form of an electrostatic field. The ratio of charge and the potential it raised is called **capacitance**.

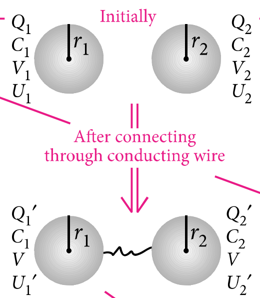
Combination of Capacitors

Charge Sharing Between Conductors

Types of Capacitors and their Capacitances



- **Partially filled dielectric**
$$C' = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

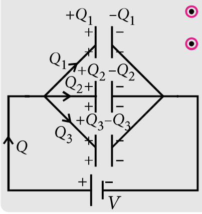


- **New charge**
$$Q_1' = Q\left[\frac{r_1}{r_1 + r_2}\right]$$
$$Q_2' = Q\left[\frac{r_2}{r_1 + r_2}\right]$$

- **Common potential**
$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

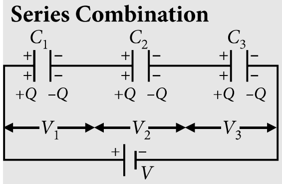
- **Energy loss**
$$\Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

Parallel Combination



- $Q = Q_1 + Q_2 + Q_3$
- $C_{eq} = C_1 + C_2 + C_3$
- For n identical capacitor $C_{eq} = nC_1$ and $Q' = Q/n$
- Used when high capacity is required at low potential

Series Combination



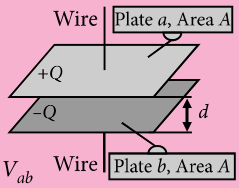
- $V = V_1 + V_2 + V_3$
- $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$
- For n identical capacitor $C_{eq} = C/n$ and $V' = V/n$

Parallel Plate Capacitor

It consists of two large plates placed parallel to each other with a separation d.

Capacitance:

$$C = \frac{\varepsilon_0 A}{d}$$



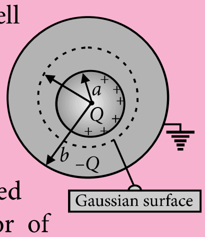
Potential difference = V_{ab}

Spherical Capacitor

It consists of two concentric spherical conducting shells of radii a and b, say $b > a$. The outer shell is earthed.

Capacitance:

$$C = \frac{4\pi\varepsilon_0 ab}{b - a}$$



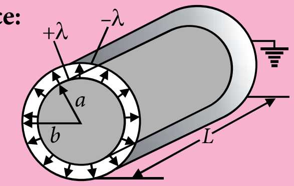
For a single isolated spherical conductor of radius R, $C = 4\pi\varepsilon_0 R$

Cylindrical Capacitor

It consists of two coaxial cylinders of radii a and b, say $b > a$ and length L. The outer one is earthed.

Capacitance:

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$



CURRENT ELECTRICITY

CLASS XII

Current Electricity

Electric Current

- $I = \frac{q}{t} = \frac{ne}{t}$
- In case of an electron revolving in a circle of radius r with speed v, period of revolution is $T = \frac{2\pi r}{v}$
- Frequency of revolution $\upsilon = \frac{v}{2\pi r}$
- Current at any point of the orbit is $I = \frac{e}{T} = \frac{ev}{2\pi r}$

Ohm's Law, Resistance and Resistivity

- $V = IR$
- Resistance of uniform conductor of length l and cross sectional area A, $R = \frac{\rho l}{A}$
- Resistivity or specific resistance, $\rho = \frac{RA}{l}$
- Effective specific resistance in series combination is
  $\frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2}$ (A is same).
- Effective specific resistance in parallel combination is
  $\frac{(A_1 + A_2)\rho_1\rho_2}{A_1\rho_2 + A_2\rho_1}$ (l is same).

Current Density, Conductance and Conductivity

- Conductance, $G = \frac{1}{R}$
- Conductivity, $\sigma = \frac{1}{\rho} = \frac{l}{RA}$
- Current density, $J = \frac{I}{A} = \sigma E = nev_d$

Electric Power

$P = VI = I^2R = \frac{V^2}{R}$

Kirchhoff's Laws

- Law of conservation of charge applied at a junction, *i.e.*, $\Sigma I = 0$
- Law of conservation of energy applied in closed loop,
  i.e., $\Sigma\varepsilon = \Sigma IR$

Drift Velocity and Mobility of Charge

- Drift speed, $v_d = \frac{eE}{m}\tau$
- Mobility, $\mu_e = \frac{v_d}{E}$
- Current in terms of drift velocity,
  $I = neAv_d = \frac{ne^2 A\tau E}{m} = neA\mu_e E = neA\mu_e \frac{V}{l}$
- In terms of relaxation time τ,
  $R = \frac{ml}{ne^2\tau A}$ and $\rho = \frac{m}{ne^2\tau}$

Variation of Resistance with Temperature

- Temperature coefficient of resistance, $\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$

If $T_1 = 0°C$ and $T_2 = T\,°C$ then $\alpha = \frac{R_T - R_0}{R_0 \times T}$ or $R_T = R_0(1 + \alpha T)$

Combination of Resistances

- In series, equivalent resistance, $R_s = R_1 + R_2 + R_3 + ...$
- In parallel, equivalent resistance, $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$
- For two resistances in parallel current through the two resistances will be, $I_1 = \frac{R_2 I}{R_1 + R_2}, I_2 = \frac{R_1 I}{R_1 + R_2}$
- When resistances are connected in series, the current through each resistance is same. In parallel combination voltage is same.

Emf, Internal Resistance, Current in case of Grouping of Cells

- Emf of a cell, $\varepsilon = \frac{W}{q}$
- Terminal potential difference where current is being drawn from the cell, $V = \varepsilon - Ir$
- Terminal potential difference when the cell is being charged $V = \varepsilon + Ir$
- Internal resistance of a cell, $r = R\left[\frac{\varepsilon - V}{V}\right]$
- Grouping of identical cells :
  - Cells in series, $I = \frac{n\varepsilon}{R + nr}$ (n cells)
  - Cells in parallel, $I = \frac{m\varepsilon}{mR + r}$ (m cells)
  - Cells in mixed grouping, $I = \frac{mn\varepsilon}{mR + nr}$

OHM'S LAW AND KIRCHHOFF'S RULE

BRAIN MAP

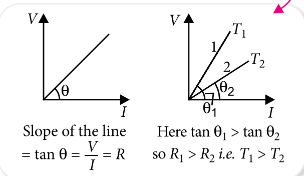
CLASS XII

OHM'S LAW

If the physical conditions of the conductor (length, temperature, mechanical strain etc.) remain same, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends *i.e.*, $I \propto V \Rightarrow V = IR$

Basic Features of Ohm's Law

- Vector form of Ohm's law, $\vec{J} = \sigma \vec{E}$
  where conductivity $\sigma = \frac{1}{\rho}$ and $\vec{J}$ is the current density.
- Graph between V and I for a metallic conductor

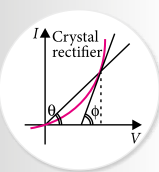


Slope of the line $= \tan\theta = \frac{V}{I} = R$

Here $\tan\theta_1 > \tan\theta_2$ so $R_1 > R_2$ *i.e.* $T_1 > T_2$

V-I curve for non-ohmic substance is not linear

- Static resistance
  $$R_{st} = \frac{V}{I} = \frac{1}{\tan\theta}$$
- Dynamic resistance
  $$R_{dyn} = \frac{\Delta V}{\Delta I} = \frac{1}{\tan\phi}$$

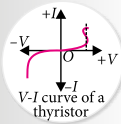


Limitations of Ohm's Law

It is not a universal law that applies everywhere under all conditions. Ohm's law is obeyed by metallic conductors, that too at about normal working temperatures.

Ohm's law is not followed in the following cases

- Materials : Crystal rectifiers, thermistors, thyristors, semi-conductors.



V-I curve of a thyristor

- Conditions :
  (i) At very high temperatures
  (ii) At very low temperatures
  (iii) At very high potential differences.

R is a proportionality constant, known as

Resistance

The property of a substance by virtue of which it opposes the flow of current through it.

$$R = \rho\frac{l}{A} = \frac{m}{ne^2\tau}\cdot\frac{l}{A}$$

ρ is specific resistance of the material of conductor

Resistivity

It is numerically equal to the resistance of a substance having unit area of cross-section and unit length.

Temperature Dependence

For a conductor

then $R_t = R_0(1 + \alpha t + \beta t^2)$

$R_t = R_0(1 + \alpha t)$ $(\beta \approx 0)$

also for resistivity, $\rho_t = \rho_0(1 + \alpha t)$

R_0 = resistance at 0°C
R_t = resistance at t°C
α, β = temperature co-efficients

KIRCHHOFF'S RULE

For circuit containing multiple batteries

Guidelines to applying Kirchhoff's rule

Junction Rule

At any junction of circuit, the sum of currents entering and leaving must be zero.

$\Sigma I = 0.$

It is based on conservation of charge.

Loop Rule

The algebraic sum of changes in potential around any closed loop must be zero.

$\Sigma\varepsilon - \Sigma IR = 0$

It is based on conservation of energy.

Grouping of Batteries

Series grouping

For n identical batteries

$$I = \frac{n\varepsilon}{nr + R} \quad \begin{cases}\varepsilon = \text{emf} \\ r = \text{Internal resistance}\end{cases}$$

If polarity of m batteries is reversed

$I = (n - 2m)\varepsilon/(nr + R)$

Parallel grouping

- With identical batteries :
  $$I = \frac{\varepsilon_{net}}{R_{net}},\ \varepsilon_{net} = \varepsilon,\ R_{net} = \frac{r}{n} + R$$
- With unidentical batteries :
  $$\varepsilon_{net} = \frac{\Sigma(\varepsilon/r)}{\Sigma(1/r)},\ I = \frac{\varepsilon_{net}}{R_{net}}$$

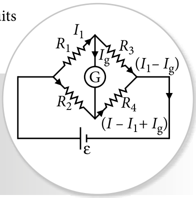
Mixed grouping

- For n rows of identical batteries with m cells in each row. Then,
  $$\varepsilon_{net} = m\varepsilon,\ R_{net} = \frac{mr}{n} + R,\ I = \frac{\varepsilon_{net}}{R_{net}}$$

An important application for few circuits

Wheatstone Bridge

- In balanced condition,
  If $\frac{R_1}{R_2} = \frac{R_3}{R_4}$
  then $I_g = 0$.



Problem Solving Strategies

- *Distribute current at various junctions in the circuit starting from positive terminal.*
- *Pick a point and begin to walk around a closed loop.*
- *Write down the voltage change for that element according to the sign convention.*
- *By applying KVL, select the required number of loops as many as unknowns are available and apply KVL across each loop.*
- *Solve the set of simultaneous equation to find the unknowns.*

HANS CHRISTIAN OERSTED (1777-1851)

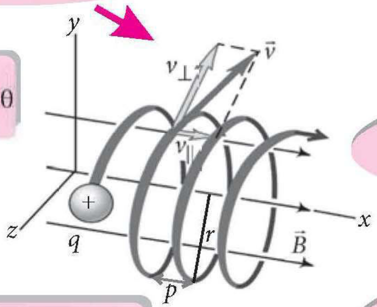
MOVING CHARGES AND MAGNETISM

BRAIN MAP CLASS XII

Magnetic Force on a charge particle in a uniform magnetic field
$\vec{F} = q(\vec{v} \times \vec{B})$, $F = qvB\sin\theta$

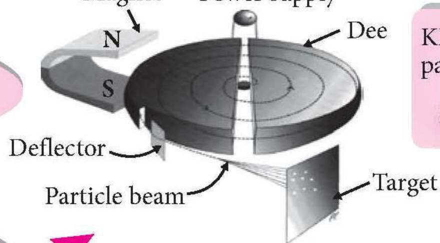
For arbitrary angle θ ($< 90°$) $F = qvB\sin\theta$ and charge will attain helical path

Pitch $(p) = \dfrac{2\pi mv}{qB}\cos\theta$



For $v \parallel B$, $\theta = 0°$ ($F = 0$) no force is experienced

For $v \perp B$, $\theta = 90°$, $F_{max} = qvB$ charge will attain circular path



KE_{max} of charge particle
$K = \dfrac{q^2B^2R^2}{2m}$

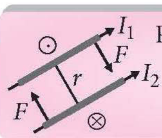
Cyclotron
A device use to accelerate positively charged particles.

- Radius of circular path
  $R = \dfrac{mv}{Bq} = \dfrac{\sqrt{2mK}}{qB}$
- Time period of revolution
  $T = \dfrac{2\pi R}{v} = \dfrac{2\pi m}{qB}$
- Cyclotron frequency
  $\upsilon = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$

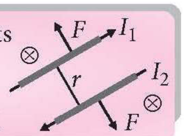
Magnetic field at the centre of a circular coil
$B = \dfrac{\mu_0 I}{2a}$

Magnetic field at a point on the axis of the circular current carrying coil
$B = \dfrac{\mu_0}{4\pi}\dfrac{2\pi NIa^2}{(a^2+x^2)^{3/2}}$

Biot Savart's Law
Magnetic field varies directly with current and length element and inversely with square of the distance.
$d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\theta}{r^2}$



Parallel currents attract while antiparallel currents repel



The force of attraction or repulsion acting on each conductor of length l due to currents in two parallel conductor is $F = \dfrac{\mu_0}{4\pi}\dfrac{2I_1I_2}{r}l$

Force on a current carrying conductor in a uniform magnetic field
$\vec{F} = I(\vec{l} \times \vec{B})$, $F = IlB\sin\theta$

MAGNETIC EFFECT OF CURRENT

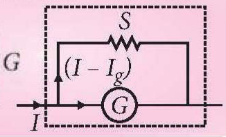
Ampere's Circuital Law
The line integral of magnetic field is equal to μ_0 times the current passing through area bounded by closed path.
$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

Magnetic field due to an infinitely long straight wire of radius a, carrying current I at a point
$$B = \begin{cases} \dfrac{\mu_0 Ir}{2\pi a^2} & ; \; r < a \\ \dfrac{\mu_0 I}{2\pi a} & ; \; r = a \\ \dfrac{\mu_0 I}{2\pi r} & ; \; r > a \end{cases}$$

Magnetic field due to a current carrying solenoid and toroid
$B_S = \mu_0 nI = (\mu_0 NI)/l$
$B_T = \mu_0 nI = \dfrac{\mu_0 NI}{2\pi R_m}$

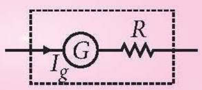
Torque on a current carrying coil placed in a uniform magnetic field
$\tau = NIAB\sin\theta = MB\sin\theta$

Galvanometer into Ammeter
$S = \left(\dfrac{I_g}{I - I_g}\right)G$



Conversion

Galvanometer into Voltmeter
$R = \dfrac{V}{I_g} - G$



Moving Coil Galvanometer
Current I passing through the galvanometer is directly proportional to its deflection (θ). $I \propto \theta$ or $I = G\theta$.
where $G = \dfrac{k}{NAB}$ = galvanometer constant

Current sensitivity : $I_s = \dfrac{\theta}{I} = \dfrac{NAB}{k}$

Voltage sensitivity : $V_s = \dfrac{\theta}{IR} = \dfrac{NAB}{kR}$

MAGNETISM AND MATTER

BRAIN MAP

CLASS XII

THE BAR MAGNET

Bar Magnet as an Equivalent Solenoid

For solenoid of length $2l$ and radius a consisting n turns per unit length, the magnetic field is given by

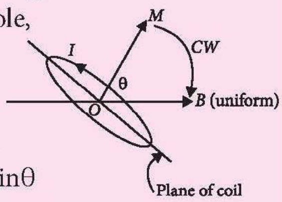
$B = \dfrac{\mu_0 2m}{4\pi r^3}$ (where m = magnetic moment of solenoid $= n\,(2l)\,I\,(\pi a^2)$)

Magnetic Dipole in Magnetic Field

Torque on magnetic dipole,

$\tau = MB\sin\theta$

Torque on coil or loop,



$\vec{\tau} = \vec{M}\times\vec{B}$, here $\vec{M} = NI\vec{A}$,

$\vec{\tau} = NI(\vec{A}\times\vec{B})$, $\tau = BINA\sin\theta$

$\theta = 90^\circ \quad \Rightarrow \tau_{max} = BINA$

$\theta = 0^\circ$ or $180^\circ \quad \Rightarrow \tau_{min} = 0$

Potential energy of magnetic dipole,

$U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$

Magnetisation and Magnetic Intensity

Relation between B, χ_m and H

$\vec{B} = \vec{B}_0 + \mu_0\vec{M}$

$\vec{B} = \mu_0(\vec{H} + \vec{M}) \qquad (\because \vec{B}_0 = \mu_0\vec{H})$

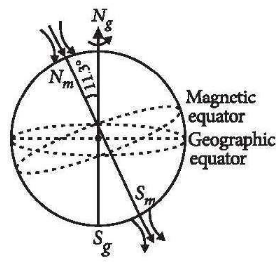
$\vec{B} = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H} \qquad (\because \vec{M} = \chi_m\vec{H})$

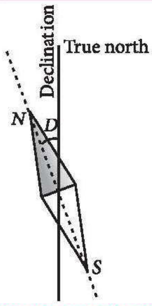
$\mu = \mu_0\mu_r = \mu_0(1 + \chi_m) \Rightarrow \mu_r = 1 + \chi_m$

THE EARTH'S MAGNETISM

Cause of Earth's Magnetism

The magnetic field of earth arises due to electrical current produced by convective motion of metallic fluids in the outer core of the earth. This is known as the dynamo effect.

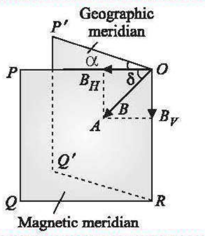




The Earth's Magnetism

Magnetic declination (α): The angle between the geographic meridian and magnetic meridian.

Magnetic dip (δ): The angle made by the earth's magnetic field with the horizontal in the magnetic meridian.



Relation between angle of dip(δ) and components of earth's magnetic field.

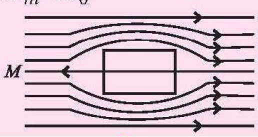
$\tan\delta = \dfrac{B_V}{B_H}$, $B = \sqrt{B_H^2 + B_V^2}$

CLASSIFICATION OF MAGNETIC MATERIALS

Diamagnetic

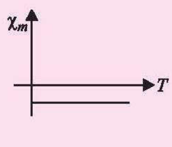
Poor magnetisation in opposite direction.

Here $B_m < B_0$



$\chi_m \rightarrow$ Small, negative and temperature independent

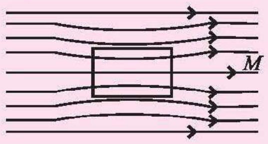
$\chi_m \propto T_0$



Paramagnetic

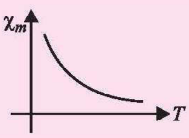
Poor magnetisation in same direction.

Here $B_m > B_0$



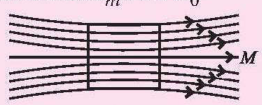
$\chi_m \rightarrow$ Small, positive and varies inversely with temperature

$\chi_m \propto \dfrac{1}{T}$ (Curie's law)



Ferromagnetic

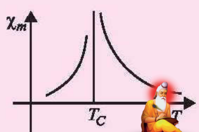
Strong magnetisation in same direction. Here $B_m >>> B_0$



$\chi_m \rightarrow$ Very large, positive and temperature dependent

$\chi_m \propto \dfrac{1}{T - T_C}$ (Curie-Weiss law)

(for $T > T_C$)



MASTERJEE CLASSES

BRAIN MAP

ELECTROMAGNETIC INDUCTION

CLASS XII

Combination of Inductors

- Inductors in series, $L_S = L_1 + L_2 \pm 2M$
- Inductors in parallel, $L_P = \dfrac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$
- If coils are far away, then $M = 0$.

  So, $L_S = L_1 + L_2$ and $L_P = \dfrac{L_1L_2}{L_1 + L_2}$

Magnetic Energy

- Energy stored in an inductor, $U_B = \dfrac{1}{2}LI^2$
- Energy stored in the solenoid, $U_B = \dfrac{1}{2\mu_0}B^2Al$
- Magnetic energy density, $u_B = \dfrac{U_B}{V} = \dfrac{B^2}{2\mu_0}$

L–R Circuit

- Current growth in *L–R* circuit $I = I_0(1 - e^{-t/\tau_L})$
- Current decay in *L–R* circuit,

  $I = I_0(e^{-t/\tau_L})$

  Here, τ_L = Time constant $= \dfrac{L}{R}$

  $I_0 = \dfrac{\varepsilon}{R}$

Inductance

- Emf induced in the coil/conductor, $\varepsilon = -L\dfrac{dI}{dt}$
- Coefficient of self induction, $L = \dfrac{N}{I}\phi_B = \dfrac{-\varepsilon}{dI/dt}$
- Self inductance of a long solenoid, $L = \mu_0\mu_r n^2 Al = \dfrac{\mu_0\mu_r N^2 A}{l}$
- Mutual inductance, $M = \dfrac{N_2\phi_2}{I_1} = \dfrac{-\varepsilon_2}{(dI_1/dt)} = \dfrac{-\varepsilon_1}{(dI_2/dt)}$
- Mutual inductance of two long coaxial solenoids,

  $$M = \mu_0\mu_r \pi r_1^2 n_1 n_2 l = \frac{\mu_0\mu_r N_1N_2A_1}{l}$$

- Coefficient of coupling, $k = \dfrac{M}{\sqrt{L_1L_2}}$

  For perfect coupling, $k = 1$ so, $M = \sqrt{L_1L_2}$

Lenz's Law

- The direction of the induced current is such that it opposes the change that has produced it.
- If a current is induced by an increasing(decreasing) flux, it will weaken (strengthen) the original flux.
- It is a consequence of the law of conservation of energy.

Induced Electric Field

- It is produced by change in magnetic field in a region. This is non-conservative in nature.

  $$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt} = -A\frac{dB}{dt} \neq 0$$

- This is also known as integral form of Faraday's law.

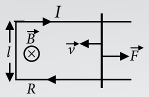
Magnetic Flux and Faraday's Law

- Magnetic flux $\phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$
- Faraday's law : Whenever magnetic flux linked with a coil changes, an emf is induced in the coil.
  - Induced emf, $\varepsilon = -N\dfrac{d\phi_B}{dt}$
  - Induced current, $I = \dfrac{\varepsilon}{R} = N\dfrac{(-d\phi_B/dt)}{R}$
  - Induced charge flow, $\Delta Q = I\Delta t = -N\dfrac{\Delta\phi_B}{R}$

Energy Consideration in Motional emf

- Emf in the wire, $\varepsilon = Bvl$
- Induced current, $I = \dfrac{\varepsilon}{R} = \dfrac{Bvl}{R}$
- Force exerted on the wire,

  $F = \dfrac{B^2l^2v}{R}$



- Power required to move the wire, $P = \dfrac{B^2l^2v^2}{R}$

  It is dissipated as Joule's heat.

Motional emf

- On a straight conducting wire, $\varepsilon = Bvl$
- On a rotating conducting wire about one end, $\varepsilon = \dfrac{B\omega l^2}{2}$

  Here, $\vec{B}, \vec{v}(=\omega r\hat{v})$ and $\vec{l}$ are perpendicular to each other.

Electric Generator

- Mechanical energy is converted into electrical energy by virtue of electromagnetic induction.
- Induced emf,

  $\varepsilon = NAB\omega\sin\omega t = \varepsilon_0\sin\omega t$
- Induced current,

  $I = \dfrac{NBA\omega}{R}\sin\omega t = I_0\sin\omega t$

MasterJEE

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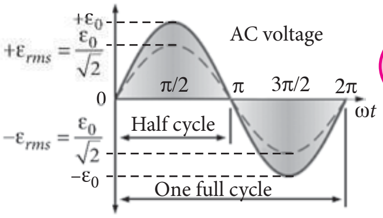
ALTERNATING CURRENT
ELECTROMAGNETIC WAVES



Alternating Current
Current which changes continuously in magnitude and periodically in direction.

Transformer
- Transformer ratios
$$\frac{\varepsilon_S}{\varepsilon_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = k$$
- Efficiency of a transformer,
$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\varepsilon_S I_S}{\varepsilon_P I_P}.$$

Step-up transformer, $\varepsilon_S > \varepsilon_P, I_S < I_P$ and $N_S > N_P$.

Step-down transformer, $\varepsilon_S < \varepsilon_P, I_S > I_P$ and $N_S < N_P$.

Alternating voltage
$\varepsilon = \varepsilon_0 \sin \omega t$

Applied across capacitor

Purely capacitive circuit
Current leads the voltage by a phase angle of $\pi/2$.
$I = I_0 \sin(\omega t + \pi/2)$; $I_0 = \frac{\varepsilon_0}{X_C} = \omega C \varepsilon_0$
where $X_C = 1/\omega C$

Applied across resistor

Purely resistive circuit
Alternating voltage is in phase with current.
$I = \varepsilon/R = I_0 \sin\omega t$

Applied across inductor

Purely inductive circuit
Current lags behind the voltage by a phase angle of $\pi/2$.
$I = I_0 \sin(\omega t - \pi/2)$; $I_0 = \varepsilon_0/X_L = \varepsilon_0/\omega L$
where $X_L = \omega L$

Combining LCR in series

Series *LCR* circuit
- $\varepsilon = \varepsilon_0 \sin\omega t$, $I = I_0 \sin(\omega t - \phi)$
  Impedance of the circuit: $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- Phase difference between current and voltage is ϕ
$$\tan\phi = \frac{X_L - X_C}{R}$$
- For $X_L > X_C$, ϕ is +ve. (Predominantly inductive)
- For $X_L < X_C$, ϕ is –ve. (Predominantly capacitive)

Power in ac circuit
Average power (P_{av})
$$P_{av} = \varepsilon_{rms} I_{rms} \cos\phi = \frac{\varepsilon_0 I_0}{2}\cos\phi$$

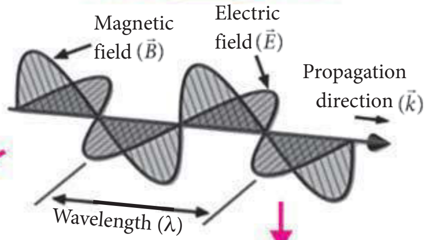
Power factor
- **Power factor:** $\cos\phi = \frac{R}{Z}$
- In pure resistive circuit, $\phi = 0°$; $\cos\phi = 1$
- In purely inductive or capacitive circuit
$$\phi = \pm\frac{\pi}{2}; \cos\phi = 0$$
- In series *LCR* circuit,
  At resonance, $X_L = X_C$
  $\therefore$ $Z = R$ and $\phi = 0°, \cos\phi = 1$

Resonant series *LCR* circuit
When $X_L = X_C$, $Z = R$, current becomes maximum.
Resonant frequency $\omega_r = \frac{1}{\sqrt{LC}}$

Quality factor
It is a measure of sharpness of resonance.
$\therefore$ $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Electromagnetic Waves
Waves having sinusoidal variation of electric and magnetic field at right angles to each other and perpendicular to direction of waves propagation.

Displacement current
Displacement current arises wherever the electric flux is changing with time.
$I_D = \varepsilon_0 \, d\phi_E/dt$



Energy density of electromagnetic waves
Average energy density
$$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$
Intensity of electromagnetic wave $= \frac{1}{2}\varepsilon_0 E_0^2 c$

Production of electromagnetic waves
- Through accelerating charge
- By harmonically oscillating electric charges.
- Through oscillating electric dipoles.

Maxwell's equations
$\oint \vec{E}\cdot d\vec{S} = \frac{q}{\varepsilon_0}$ (Gauss's law for electrostatics)

$\oint \vec{B}\cdot d\vec{S} = 0$ (Gauss's law for magnetism)

$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt}$ (Faraday's law of electro-magnetic induction)

$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$ (Maxwell-Ampere's circuital law)

AC CIRCUITS

Series Resonance Circuit

- At resonance : $X_L = X_C \Rightarrow Z_{min} = R$
- Phase difference : $\phi = 0° \Rightarrow \cos\phi = 1$
- Resonant frequency : $\upsilon_0 = \dfrac{1}{2\pi\sqrt{LC}}$

Quality Factor (Q-factor)

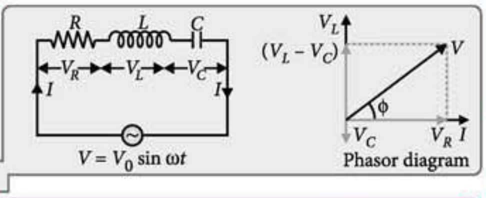
$$Q\text{-factor} = \frac{\text{Resonant frequency}}{\text{Band width}} = \frac{\omega_0}{2\Delta\omega}$$

Parallel Resonance Circuit

- At resonance : $I_C = I_L; Z_{max} = R$
- Phase difference : $\phi = 0° \Rightarrow \cos\phi = 1$
- Resonant frequency : $\upsilon_0 = \dfrac{1}{2\pi\sqrt{LC}}$

Q-factor of Series Resonant Circuit

$$Q\text{-factor} = \frac{V_L}{V_R} \text{ or } \frac{V_C}{V_R} = \frac{\omega_0 L}{R} \text{ or } \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$



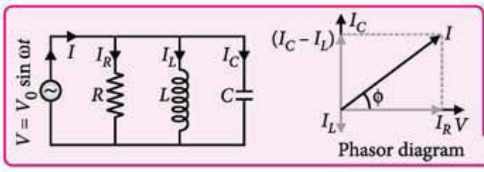
Q-factor of Parallel Resonant Circuit

$$Q\text{-factor} = R\sqrt{\frac{C}{L}}$$

Series *RLC*-Circuit

- Voltage : $V = \sqrt{V_R^2 + (V_L - V_C)^2}$
- Impedance : $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$
- Phase difference :

$$\phi = \tan^{-1}\frac{V_L - V_C}{V_R} = \tan^{-1}\frac{X_L - X_C}{R}$$

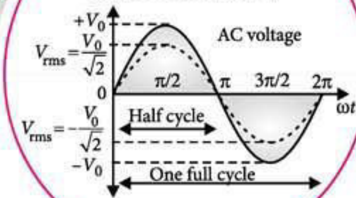


Parallel *RLC* Circuits

- Current : $I = \sqrt{I_R^2 + (I_C - I_L)^2}$
- Phase difference : $\phi = \tan^{-1}\dfrac{(I_C - I_L)}{I_R}$
- Impedance : $Z = 1\Big/\sqrt{\left(\dfrac{1}{R}\right)^2 + \left(\dfrac{1}{X_L} - \dfrac{1}{X_C}\right)^2}$

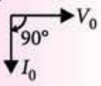
AC VOLTAGE APPLIED TO COMBINED LCR CIRCUITS

ALTERNATING CURRENT AND VOLTAGE

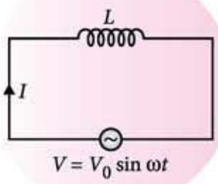


Purely Inductive Circuit

- Voltage : $V = V_0 \sin \omega t$
- Current : $I = I_0 \sin(\omega t - \pi/2)$
- Phase difference : $+(\pi/2)$
- Impedance : $X_L = \omega L$
- Peak current : $I_0 = V_0/X_L$

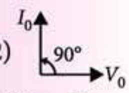


Phasor diagram (Voltage leads current by $\pi/2$)

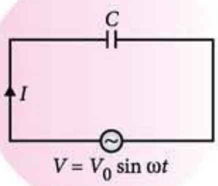


Purely Capacitive Circuit

- Voltage : $V = V_0 \sin \omega t$
- Current : $I = I_0 \sin(\omega t + \pi/2)$
- Phase difference : $-(\pi/2)$
- Impedance : $X_C = 1/\omega C$
- Peak current : $I_0 = V_0/X_C$



Phasor diagram (Current leads voltage by $\pi/2$)

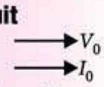


Combined *RL* circuit

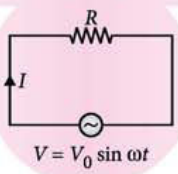
Combined *RC* circuit

Purely Resistive Circuit

- Voltage : $V = V_0 \sin \omega t$
- Current : $I = I_0 \sin \omega t$
- Phase difference : zero
- Impedance : R
- Peak current : $I_0 = V_0/R$

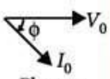


Phasor diagram (Both current and voltage are in same phase)

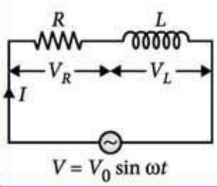


(Series *RL*-Circuit)

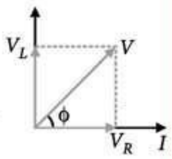
- Applied voltage : $V = \sqrt{V_R^2 + V_L^2}$
- Impedance : $Z = \sqrt{R^2 + 4\pi^2\upsilon^2 L^2}$
- Current : $I = I_0 \sin(\omega t - \phi)$
- Phase difference : $\phi = \tan^{-1}\dfrac{\omega L}{R}$
- Power factor : $\cos\phi = \dfrac{R}{\sqrt{R^2 + X_L^2}}$



Phasor diagram



$V_R = IR,$
$V_L = IX_L$

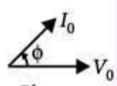


Power in AC Circuit

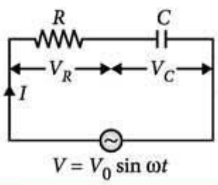
- Power factor : It may be defined as cosine of the angle of lag or lead (*i.e.*, $\cos\phi$)
- Average power (P_{av}) :
  $P_{av} = V_{rms} I_{rms} \cos\phi = (V_0 I_0/2)\cos\phi$

(Series *RC*-Circuit)

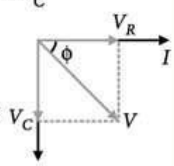
- Applied voltage : $V = \sqrt{V_R^2 + V_C^2}$
- Impedance : $Z = \sqrt{R^2 + (1/\omega C)^2}$
- Current : $I = I_0 \sin(\omega t + \phi)$
- Phase difference : $\phi = \tan^{-1}(1/\omega CR)$
- Power factor : $\cos\phi = \dfrac{R}{\sqrt{R^2 + X_C^2}}$



Phasor diagram



$V_R = IR,$
$V_C = IX_C$



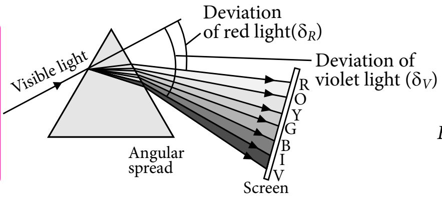
RAY OPTICS AND OPTICAL INSTRUMENTS

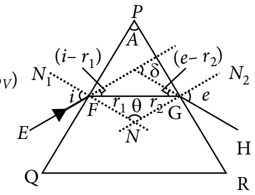
BRAIN MAP

CLASS XII

APPLICATIONS OF TIR

- Fiber optics communication
- Medical endoscopy
- Periscope (Using prism)
- Sparkling of diamond





POWER OF LENSES

Power of lens: $P=\dfrac{1}{f\text{ (in m)}}$

Combination of lenses:

Power: $P=P_1+P_2-dP_1P_2$

(d = small separation between the lenses)

For $d=0$ (lenses in contact)

Power: $P=P_1+P_2+P_3+\ldots$

TOTAL INTERNAL REFLECTION

TIR conditions

- Light must travel from denser to rarer.
- Incident angle $i>$ critical angle i_c

Relation between μ and i_c: $\mu=\dfrac{1}{\sin i_c}$

REFRACTION THROUGH PRISM

Relation between μ and δ_m

$$\mu=\frac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}}$$

where, δ_m = angle of minimum deviation

A = angle of prism

or $\delta=(\mu-1)A$ (Prism of small angle)

Angular dispersion

$$=\delta_V-\delta_R=(\mu_V-\mu_R)A$$

Dispersive power,

$$\omega=\frac{\delta_V-\delta_R}{\delta}=\frac{\mu_V-\mu_R}{\mu-1}$$

Mean deviation, $\delta=\dfrac{\delta_V+\delta_R}{2}$

THIN SPHERICAL LENS

Thin lens formula: $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$

Magnification: $m=\dfrac{v}{u}=\dfrac{h_i}{h_o}$

REFRACTION OF LIGHT

Snell's law: When light travels from medium a to medium b, ${}^a\mu_b=\dfrac{\mu_b}{\mu_a}=\dfrac{\sin i}{\sin r}$

Refractive index,

$$\mu=\frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}}=\frac{c}{v}$$

Real and apparent depth

$$\mu=\frac{\text{real depth}(x)}{\text{apparent depth }(y)}$$

REFRACTION BY SPHERICAL SURFACE

Relation between object distance (u), image distance (v) and refractive index (μ)

$$\frac{\mu_{\text{denser}}}{v}-\frac{\mu_{\text{rarer}}}{u}=\frac{\mu_{\text{denser}}-\mu_{\text{rarer}}}{R}$$

(Holds for any curved spherical surface.)

Lens maker's formula

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

REFLECTION OF LIGHT

According to the laws of reflection, $\angle i=\angle r$

If a plane mirror is rotated by an angle θ, the reflected rays rotates by an angle 2θ.

RAY OPTICS

OPTICAL INSTRUMENTS

REFLECTION BY SPHERICAL MIRRORS

Mirror formula, $\dfrac{1}{u}+\dfrac{1}{v}=\dfrac{1}{f}=\dfrac{2}{R}$

Magnification, $m=-\dfrac{v}{u}=\dfrac{h_i}{h_o}$

SIMPLE MICROSCOPE

Magnifying power

For final image is formed at D (least distance) $M=1+\dfrac{D}{f}$

For final image formed at infinity

$$M=\frac{D}{f}$$

COMPOUND MICROSCOPE

Magnifying power, $M=m_o\times m_e$

For final image formed at D (least distance) $M=\dfrac{L}{f_o}\left(1+\dfrac{D}{f_e}\right)$

For final image formed at infinity

$$M=\frac{L}{f_o}\cdot\frac{D}{f_e}$$

TELESCOPE

Astronomical telescope

For final image formed at D (least distance) $M=\dfrac{f_o}{f_e}\left(1+\dfrac{f_e}{D}\right)$

In normal adjustment, image formed at infinity $M=f_o/f_e$

REFLECTING TELESCOPE

Magnifying power

$$M=\frac{f_o}{f_e}=\frac{R/2}{f_e}$$

TERRESTRIAL TELESCOPE

For normal adjustment $M=\dfrac{f_o}{f_e}$

Distance between objective and eyepiece $d=f_o+4f+f_e$

BRAIN MAP

CLASS XII

INTERFERENCE OF LIGHT

Interference of Light

When two light waves having same frequency and to nearly equal amplitude are moving in the same direction, superimpose each other at some point, then intensity of light is maximum at some point and it is minimum at some another point.

Conditions for Sustained Interference

1. *The two sources of light should be coherent.*
2. *Interfering waves must be in same state of polarisation.*
3. *Sources should be monochromatic otherwise fringes of different colour will overlap.*
4. *Distance between two coherent sources must be small.*

Addition of Coherent Waves

Intensity $\propto$ (Amplitude)^2

$I = KA^2$

Resultant intensity

$I = I_1 + I_2 + \underbrace{2\sqrt{I_1}\sqrt{I_2}\cos\phi}$

Resultant amplitude

$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

If intensity of both sources is same then

$I_1 = I_2 = I_0 ; I = 4I_0\cos^2\phi/2$

Intensity $\propto$ Width of slit

$$\left[\frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{a_1^2}{a_2^2}\right]$$

Interference Term
Depending upon $\cos\phi$

Phase difference $\phi = (2\pi/\lambda)\,\Delta$

Path difference $\Delta = (\lambda/2\pi)\,\phi$

$\frac{\phi}{2\pi} = \frac{\Delta}{\lambda} = \frac{\text{Time difference}}{T}$

Constructive Interference

Phase difference $\phi = 0, 2\pi, 4\pi, 6\pi$ ...

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$A_{max} = a_1 + a_2$

For $I_1 = I_2 = I_0 ; I_{max} = 4I_0$

Destructive Interference

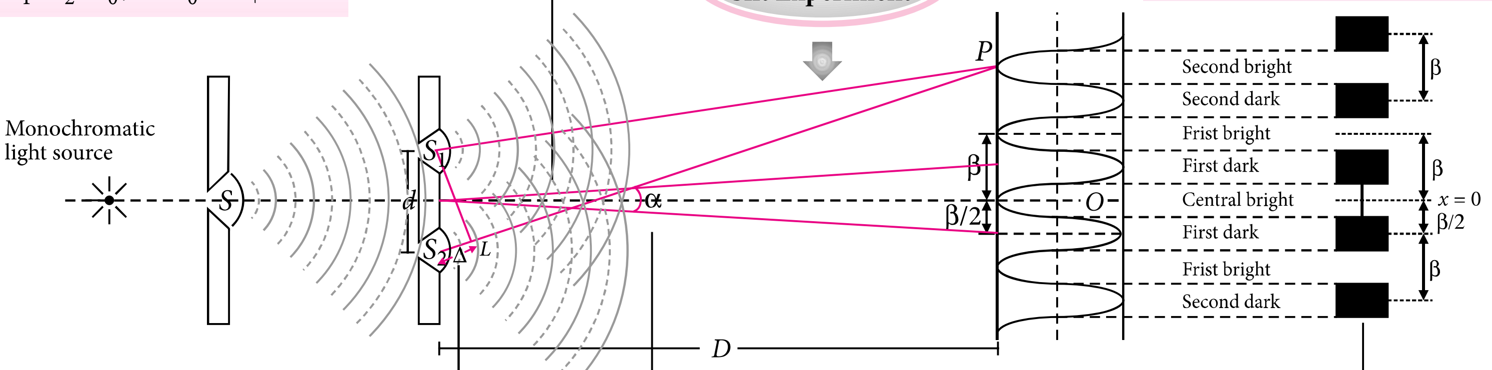
Phase difference $\phi = \pi, 3\pi, 5\pi$ ...

Resultant amplitude, $A_{min} = a_1 - a_2$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

For $I_1 = I_2 = I_0 ; I_{min} = 0$

Young's Double Slit Experiment



Path Difference

$\Delta = PS_2 - PS_1 = S_2L$

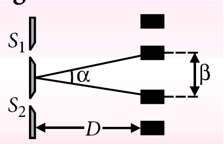
Path difference, $\Delta = xd/D$

$xd/D = n\lambda$ (Bright fringe)

$xd/D = (2n + 1)\,\lambda/2$ (Dark fringe)

Angular Fringe Width

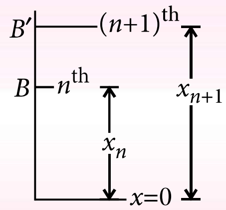
$\alpha = \frac{\beta}{D} = \frac{\lambda}{d}$



Fringe Width

The distance between two successive bright or dark fringe is known as fringe width.

$\beta = x_{n+1} - x_n = \frac{D\lambda}{d}$



Special Cases

- If two glass plates of R.I. μ_1 and μ_2 of same thickness t is placed in front of S_1 and S_2 then
  - Extra path difference $\Delta = (\mu_1 - \mu_2)t$
  - Shifting distance of central fringe $x = \beta(\mu_1 - \mu_2)t/\lambda$
- If a glass plate of thickness t and R.I. μ is placed in front of the slit then the central fringe shift towards that side in which glass plate is placed, because extra path difference is introduced by the glass plate.
  - Extra path difference $\Delta = (\mu - 1)t$
  - Shifting distance of central fringe $x = \beta(\mu - 1)t/\lambda$

INTERFERENCE IN THIN FILMS

- **For reflected Light**

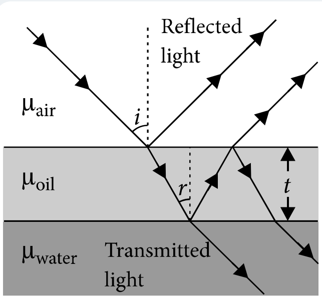
  Maxima $\rightarrow 2\mu t\cos r = (2n + 1)\frac{\lambda}{2}$

  Minima $\rightarrow 2\mu t\cos r = n\lambda$

- **For transmitted light**

  Maxima $\rightarrow 2\mu t\cos r = n\lambda$

  Minima $\rightarrow 2\mu t\cos r = (2n + 1)\frac{\lambda}{2}$



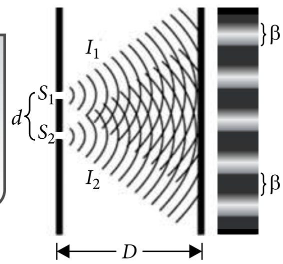
WAVE OPTICS

BRAIN MAP

CLASS XII

REFLECTION AND REFRACTION

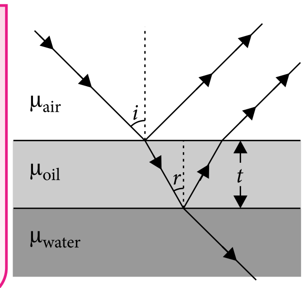
Law of reflection $\angle i = \angle r$ & law of refraction $\frac{\sin i}{\sin r} = \mu$ can be explained by Huygens wave theory.



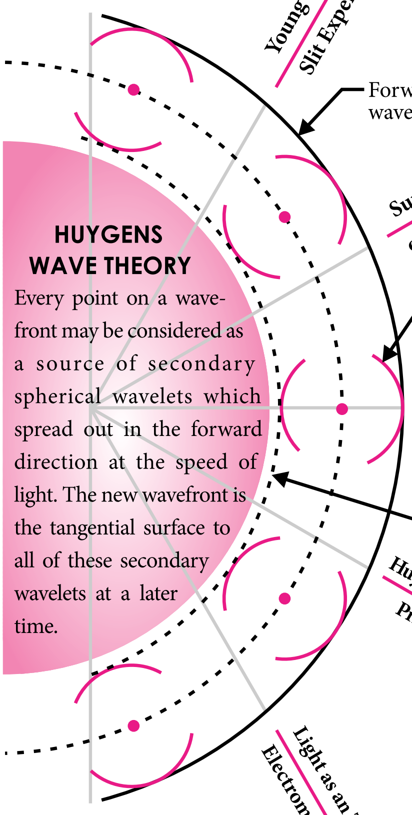
INTERFERENCE OF LIGHT

The superposition of two coherent waves resulting in a pattern of alternating dark and bright fringes of equal width.

- Position of bright fringes $x_n = \frac{n\lambda D}{d}$
- Position of dark fringes $x'_n = \frac{(2n-1)\lambda D}{2d}$
- Fringe width $\beta = \frac{\lambda D}{d}$
- Ratio of slit width with intensity : $\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$



Young's Double Slit Experiment



HUYGENS WAVE THEORY

Every point on a wave-front may be considered as a source of secondary spherical wavelets which spread out in the forward direction at the speed of light. The new wavefront is the tangential surface to all of these secondary wavelets at a later time.

ADDITION OF COHERENT WAVE

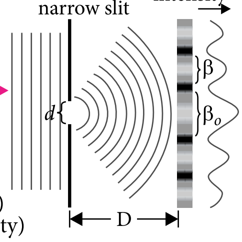
- Resultant intensity
  $I_R = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$
  for bright fringes,
  $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$ at $\phi = 0°, 2\pi, 4\pi...$
  for dark fringes,
  $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$ at $\phi = \pi, 3\pi, 5\pi...$
  for $I_1 = I_2 = I_0$; $I_R = 4I_0\cos^2\frac{\phi}{2}$

INTERFERENCE IN THIN FILM

- For reflected Light
  Maxima $\rightarrow 2\mu t\cos r = (2n+1)\frac{\lambda}{2}$
  Minima $\rightarrow 2\mu t\cos r = n\lambda$
- For transmitted light
  Maxima $\rightarrow 2\mu t\cos r = n\lambda$
  Minima $\rightarrow 2\mu t\cos r = (2n+1)\frac{\lambda}{2}$
  Shift in fringe pattern
  $\Delta x = \frac{\beta}{\lambda}(\mu-1)t = \frac{D}{d}(\mu-1)t$
  (t = thickness of film, μ = R.I. of the film)

Superposition of Light Waves

Single Slit Diffraction



DIFFRACTION

- Single slit experiment
  - Angular position of n^{th} minima, $\theta_n = \frac{n\lambda}{d}$
  - Angular position of n^{th} maxima, $\theta'_n = \frac{(2n+1)\lambda}{2d}$
  - Width of central maximum $\beta_o = 2\beta = \frac{2D\lambda}{d}$

Huygen Fresnel Principle

FRESNEL'S DISTANCE

Ray optics as a limiting case of wave optics

- Diffraction at circular aperture
  Linear spread, $x = D\theta$
  Areal spread, $x^2 = (D\theta)^2$ $\left\{\theta = \frac{1.22\lambda}{d}\right\}$
- Fresnel's distance : Distance at which diffraction spread is equal to the size of aperture, $D_F = \frac{d^2}{\lambda}$
- Size of Fresnel zone, $d_F = \sqrt{\lambda D}$

RESOLVING POWER (R.P.)

The ability to resolve the images of two nearby point objects distinctly.

R.P. = $\frac{1}{\text{Limit of resolution}}$

R.P. of a microscope $= \frac{1}{d} = \frac{2\mu\sin\theta}{\lambda}$

θ = Semi vertical angle subtended at objective.

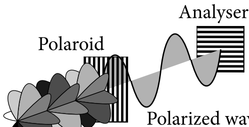
R.P. of a telescope $= \frac{1}{d\theta} = \frac{D}{1.22\lambda}$

D = Diameter of objective lens of telescope.

Light as an Transverse Electromagnetic Wave

DOPPLER'S EFFECT

- Apparent frequency received during relative motion of source and observer
  $\upsilon' = \upsilon\left(1 - \frac{v}{c}\right)$; (red shift)
  $\upsilon' = \upsilon\left(1 + \frac{v}{c}\right)$; (blue shift)
  Doppler shift : $\Delta\upsilon = \pm\frac{v}{c}\times\upsilon$
  $\Delta\lambda = \pm\frac{v}{c}\times\lambda \Rightarrow \lambda' - \lambda = \pm\frac{v}{c}\lambda$



POLARISATION OF LIGHT

Malus Law: The intensity of transmitted light passed through an analyser is

$I = I_0\cos^2\theta$

(θ = angle between transmission directions of polariser and analyser)

POLARISATION BY REFLECTION

- **Brewster's Law:** The tangent of polarising angle of incidence at which reflected light becomes completely plane polarised is numerically equal to refractive index of the medium
  $\mu = \tan i_p$; i_p = Brewster's angle.
  and $i_p + r_p = 90°$

MASTERJEE CLASSES

GEOMETRICAL OPTICS

BRAIN MAP

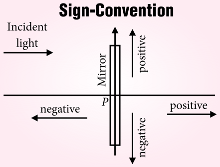
CLASS XII

Velocity of the Image of a Moving Object

Object is approaching the focus of a concave mirror from infinite with speed v_{obj},

$$v_{image} = \frac{dv}{dt} = -\frac{f^2}{(u-f)^2}\frac{du}{dt} = -m^2 v_{obj}$$

Sign-Convention

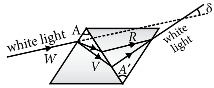


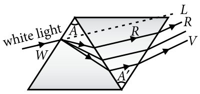
Combination of Prism

- Deviation without dispersion

$(\theta = 0)\ A' = -\frac{(\mu_V - \mu_R)A}{\mu'_V - \mu'_R}$

$\delta_{net} = (\mu - 1)A + (\mu' - 1)A'$





- Dispersion without deviation

$(\delta = 0)\ A' = -\frac{(\mu - 1)A}{\mu' - 1}$

$\theta_{net} = (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A'$

Angular dispersion, $\theta = (\mu_V - \mu_R)A$

Dispersive power, $\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1}$

Newton's Formula

If object distance (x_1) and image distance (x_2) are measured from focus,

$$f^2 = x_1 x_2$$

Through Spherical Mirrors

- **Mirror formula,** $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R}$
- **Magnification,** $m = -v/u$
  - Longitudinal magnification: $m_L = -\frac{dv}{du} = \left[\frac{v}{u}\right]^2 = m^2$
  - Superficial magnification: $m_s = \frac{\text{area of image}}{\text{area of object}} = m^2$

REFLECTION OF LIGHT

The bouncing back of a light ray to other side of normal in a same medium. According to the law of reflection, $\angle i = \angle r$

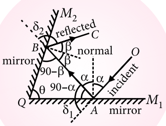
Relation between μ and δ_m

$$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$

where, δ_m = angle of minimum deviation

A = angle of prism

or $\delta_m = (\mu - 1)A$ (Prism of small angle)



Deviation produced by the combination of two plane mirrors,

$\delta = 360 - 2(\alpha + \beta)$

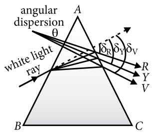
$\delta = 360 - 2\theta$

Through Plane Mirrors

GEOMETRICAL OPTICS

Deals with light propagation in the form of rays.

Through Prism



For two plane mirrors inclined at an angle θ, the number of images of a point object formed are

- $n = 360/\theta - 1$ [If $360/\theta$ is even]
- $n = 360/\theta$ [If $360/\theta$ is odd]

Minimum length (L_m) of a mirror to see complete Image of

- A person in the mirror $L_m = 1/2 \times$ (height of person)
- A wall behind a person in the mirror $L_m = 1/3 \times$ (height of wall)

REFRACTION OF LIGHT

Snell's law : When light travels from medium a to medium b, ${}^a\mu_b = \frac{\mu_b}{\mu_a} = \frac{\sin i}{\sin r}$

Refractive index, $\mu = \frac{c}{v}$

Through Spherical Lenses

General relation for spherical surfaces

$$\frac{\mu_{denser}}{v} - \frac{\mu_{rarer}}{u} = \frac{\mu_{denser} - \mu_{rarer}}{R}$$

Lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Thin Spherical Lens

Thin lens formula : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Magnification : $m = \frac{v}{u} = \frac{h_i}{h_0}$

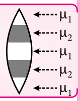
Through Different Medium

Apparent Depth (d_{ap}) and Normal Shift (x)

- **Object in denser medium is observed from rarer :** $d_{ap} = \frac{d_{ac}}{\mu}$; $x = d_{ac}\left[1 - \frac{1}{\mu}\right]$
- **Object in rarer medium is observed from denser :** $\frac{d_{ac}}{d_{ap}} = \frac{1}{\mu} (< 1)$; $x = [\mu - 1]d_{ac}$
- **Lateral shift :** $d = \frac{t}{\cos r}\sin(i - r)$

Special Cases

- If half portion of lens is covered by black paper then only intensity of image will be reduced.
- If a lens is made of number of layers of different R.I. for a given λ number of images = number of R.I.



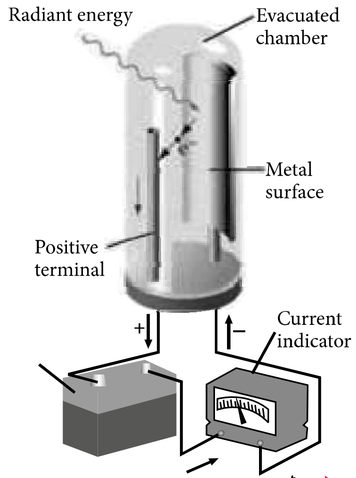
- If lens is cut into two equal parts by a vertical plane, focal length of each part $f' = 2 \times$ focal length of original lens (f)

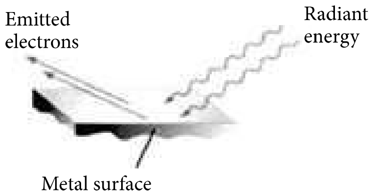
QUANTUM THEORY OF LIGHT

MASTERJEE CLASSES

BRAIN MAP

CLASS XII

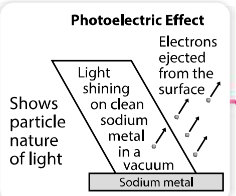




Photoelectric Cell

- An electrical device which converts light energy into electrical energy, is called as photocell or photoelectric cell.
- It works on the principle of photoelectric emission of electrons.

Application of Photoelectric Effect



Photoelectric Effect

- The phenomenon of emission of electrons from a metal surface when an electromagnetic wave of suitable frequency is incident on it is called photoelectric effect.

PARTICLE NATURE OF RADIATION

Basic Quantum Theory of Light

- According to Planck, the energy of a photon, $E \propto \upsilon$;

$$E = h\upsilon = \frac{hc}{\lambda} = \frac{1240}{\lambda(\text{in nm})}\text{ eV}$$

- Momentum of photon, $p = \frac{E}{c} = \frac{h\upsilon}{c} = \frac{h}{\lambda}$
- If source is 100% efficient, then the number of photons emitted per second by the source can be given by

$$n = \frac{\text{Power of source}}{\text{Energy of photon}} = \frac{P}{E} = \frac{P}{h\upsilon} = \frac{P\lambda}{hc}$$

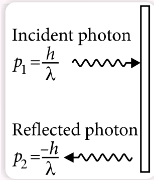
- The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave.

$I = E/At = P/A$

- Force exerted on perfectly reflecting surface

$$F = \frac{\Delta p}{t} = \frac{2Nh}{t\lambda} = n\left(\frac{2h}{\lambda}\right) = \frac{2P}{c}$$

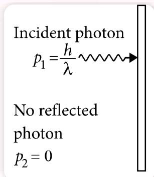
$$\text{Pressure} = \frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c}$$



- Force exerted on perfectly absorbing surface

$$F = \frac{\Delta p}{t} = \frac{Nh}{t\lambda} = n\left(\frac{h}{\lambda}\right) = \frac{P}{c}$$

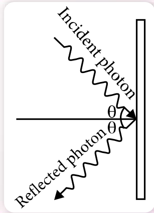
$$\text{Pressure} = \frac{F}{A} = \frac{P}{cA} = \frac{I}{c}$$



- When a beam of light is incident at an angle θ on perfectly reflector surface then force exerted on the surface,

$$F = \frac{2P}{c}\cos\theta = \frac{2IA\cos\theta}{c}$$

$$\text{Pressure} = \frac{2I\cos\theta}{c}$$



Photoelectric Equation

- $E = K_{max} + \phi_0$

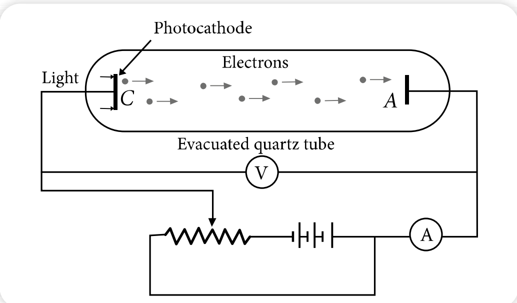
where ϕ_0 = work function of metal,

E = energy of incident light,

K_{max} = maximum kinetic energy of electrons

- $\frac{1}{2}mv_{max}^2 = h(\upsilon - \upsilon_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

where, $\lambda_0 = \frac{hc}{\phi_0}$ = threshold wavelength



Conclusions of Experimental Study of Photoelectric Effect

- Photo-current is directly proportional to the intensity of incident light, *i.e.*, $i_p \propto I$.

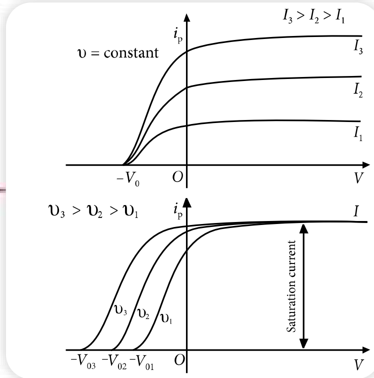
(At constant frequency υ and potential V)

- At constant frequency and intensity, the minimum negative potential at which the photocurrent becomes zero is called stopping potential (V_0).
- At stopping potential $V_0, K_{max} = eV_0$
- For a given frequency of the incident radiation, the stopping potential is independent of its intensity.
- The stopping potential varies linearly with the frequency of incident radiation but saturation current value remains constant for a fixed intensity of incident radiation.

- Quantum efficiency $= \frac{n_e}{n_{ph}}$

n_e = number of electron emitted per second

n_{ph} = total number of photon incident per second

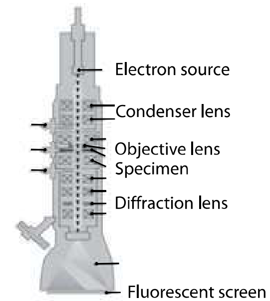


BRAIN MAP

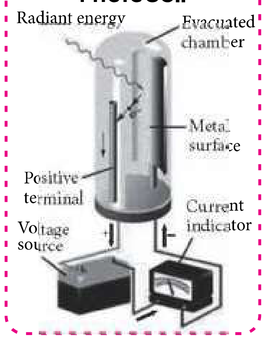
CLASS XII

DUAL NATURE OF RADIATION AND MATTER

Transmission electron microscope



Photocell



Photoelectric Cell

- An electrical device which converts light energy into electrical energy, is called as photocell or photoelectric cell.
- It works on the principle of photoelectric emission of electrons.

Electron Microscope

- Electron microscope is a device designed to study very minute objects.
- Based on principle of de Broglie wave and the fast moving electrons can be focussed by E or B field in a same way as beam of light is focussed by glass lenses.

de Broglie Wavelength

$\lambda = \frac{h}{p}$

- For electron having K.E. (K) is $\lambda = \frac{h}{\sqrt{2mK}}$, here $p = \sqrt{2mK}$
- For a charged particle accelerated by potential V is $\lambda = \frac{h}{\sqrt{2qmV}}$, here $p = \sqrt{2qmV}$

Application of Photoelectric Effect

Application of de Broglie Waves

Particle Nature of Radiation

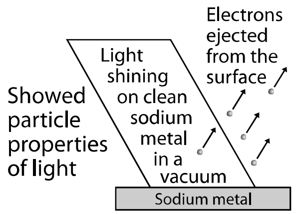
DUAL NATURE OF RADIATION AND MATTER
Nature's Love with symmetry arises the matter-wave duality

Wave Nature of Matter

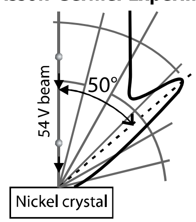
Photoelectric Effect

- The phenomenon of emission of electrons from a metal surface when an electromagnetic wave of suitable frequency is incident on it is called photoelectric effect.

Photoelectric Effect



Davisson-Germer Experiment

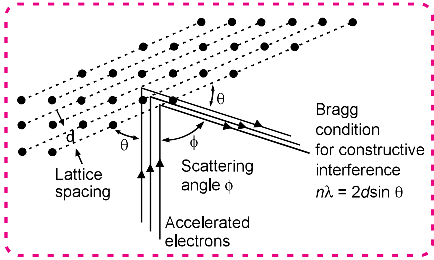


de Broglie Hypothesis

- Due to symmetry in nature, the particle in motion also possesess wave-like properties. And these waves are called matter waves.

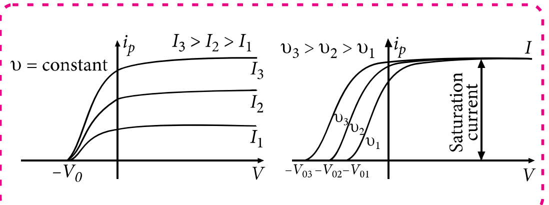
Photoelectric Equation

- $E = K_{max} + \phi_0$, where ϕ_0 = work function,
  E = energy of incident light, K_{max} = maximum K.E. of e^-
  $\Rightarrow h\upsilon = \frac{1}{2}mv_{max}^2 + h\upsilon_0 \Rightarrow \frac{1}{2}mv_{max}^2 = h(\upsilon - \upsilon_0)$



Experimental Study and Conclusion of Photoelectric Effect

- At constant frequency υ and potential V
  (Photo-current) $i_p \propto I$ (intensity)
- At constant frequency and intensity, the minimum negative potential at which the photocurrent becomes zero is called **stopping potential (V_0)**.
- At stopping potential V_0, K_{max} of $e^- = eV_0$
- For a given frequency of the incident radiation, the V_0 is independent of I.
- The V_0 varies linearly with υ.



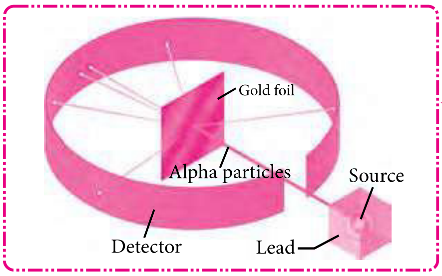
Davisson and Germer Experiment

- Study of wave nature of electron.
- At a suitable potential V, the fine beam of electrons from electron gun is allowed to strike on the nickel crystal. The electrons are scattered in all directions and following assumptions were made :
  - ➢ Intensity of scattered electrons depends over scattering angle ϕ.
  - ➢ A kink occurs in curve at $\phi = 50°$ for 54 eV beam.
  - ➢ The intensity is maximum at accelerating voltage 54 V. After this voltage, intensity starts decreasing.
  - ➢ Here, $\theta = \frac{1}{2}(180° - \phi) \Rightarrow \theta = 65°$ at $\phi = 50°$
    From Bragg's law (particle nature), $\lambda = 2d\sin\theta \Rightarrow \lambda = 1.65$ Å.
    Also, from wave nature at $V = 54$ volt, $\lambda = \frac{12.27}{\sqrt{54}} = 1.65$ Å

Class XII

ATOMS AND NUCLEI

BRAIN MAP



Line Spectra of Hydrogen

- While transition between different atomic levels, light radiated in various discrete frequencies are called spectral series of hydrogen atom.
- Rydberg formula :

Wave number $\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right]$

$R =$ Rydberg's constant
$= 1.097 \times 10^7\ \text{m}^{-1}$

Radioactivity

- **Law of radioactive decay**
  $\frac{dN}{dt} = -\lambda N(t)$ or $N(t) = N_0 e^{-\lambda t}$
- **Half-life**
  $T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$
- **Mean life or Average life**
  $\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44\,T_{1/2}$
- **Fraction of nuclei left undecayed after n half lives is**
  $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T_{1/2}}$, where $t = nT_{1/2}$

Decay Schemes

- α-Decay :
  $^{A}_{Z}X \xrightarrow{\alpha-\text{decay}} {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{He} + Q$
  (Energy released)
- β-Decay :
  $^{A}_{Z}X \xrightarrow{\beta^+} {}^{A}_{Z-1}Y + {}^{0}_{+1}e + \upsilon$
  $^{A}_{Z}X \xrightarrow{\beta^-} {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\upsilon}$
- γ-Decay :
  $^{A}_{Z}X^* \xrightarrow{\gamma-\text{decay}} {}^{A}_{Z}X + {}^{0}_{0}\gamma$ + Energy
  (Excited state) (Ground state)

Rutherford's Model of Atom

- K.E. of α−particles, $K = \frac{1}{2}mv^2$
- Distance of closest approach,
  $$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{K} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4Ze^2}{mv^2}$$
- Impact parameter,
  $$b = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{K} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{\frac{1}{2}mv^2}$$
- **Conclusion :** An atom consists of a small and massive central core in which entire positive charge and whole mass of atom is concentrated.
- **Drawback :** The revolving electron continuously loses its energy due to centripetal acceleration and finally it should collapse into the nucleus.

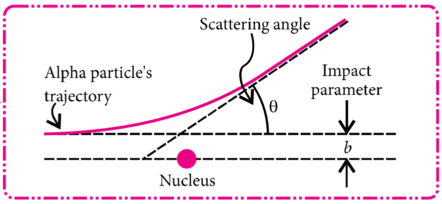
ATOMS & NUCLEI

Composition and Size of Nucleus

- Nucleus of an atom consists of protons and neutrons collectively called nucleons.
- Radius of a nucleus is proportional to its mass number as $R = R_0 A^{(1/3)}$. ($R_0 = 1.2$ fm)

Concept of Binding Energy

- The binding energy is defined as the surplus energy which the nucleons give up by virtue of their attractions when they bound together to form a nucleus.
  $\Delta E_b = [Zm_p + (A-Z)m_n - M_N]c^2$
- **Binding energy per nucleon :** $\therefore\ E_{bn} = \frac{E_b}{A}$



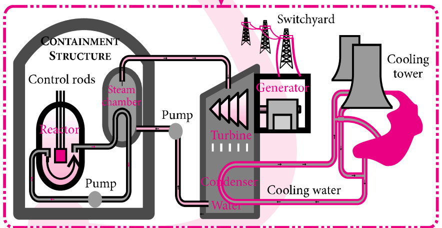
Bohr's Atomic Model

Electron orbits and their energy

- Radius of permitted n^{th} orbits,
  $r_n = \frac{n^2h^2}{4\pi^2mkZe^2} \Rightarrow r_n \propto n^2$
- Velocity of electron in n^{th} orbit,
  $v_n = \frac{2\pi kZe^2}{nh} \Rightarrow v_n \propto \frac{1}{n}$
- Energy of electron in n^{th} orbit
  $E_n = \frac{-2\pi^2mk^2Z^2e^4}{n^2h^2} \Rightarrow E_n \propto \frac{1}{n^2}$
  where the symbols have their usual meanings.

Nuclear Reactions

- **Nuclear fission :** It is the phenomenon of splitting a heavy nucleus into two or more smaller nuclei of nearly comparable masses.
- **Nuclear fusion :** It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.



Application of Nuclear Reactions

A. Fission

- Uncontrolled chain reaction: Principle of atomic bombs.
- Controlled chain reaction: Principle of nuclear reactors.

B. Fusion

Nuclear fusion is the source of energy in the Sun and stars.

Class XII

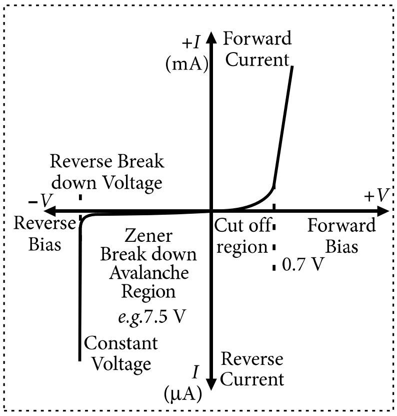
SEMICONDUCTOR ELECTRONICS

BRAIN MAP

TYPES OF SEMICONDUCTORS

INTRINSIC SEMICONDUCTORS

The pure semiconductors have thermally generated current carriers. Here, $n_e = n_h = n_i$



EXTRINSIC SEMICONDUCTORS

The semiconductor whose conductivity is mainly due to doping of impurity.

***p*-type semiconductor**

- Doped with trivalent atom.
- Here, $n_h >> n_e$

***n*-type semiconductor**

- Doped with pentavalent atom.
- Here, $n_e >> n_h$

SEMICONDUCTOR DIODE

***p–n* junction diode :** A *p*-type semiconductor is brought into contact with an *n*-type semiconductor such that structure remains continuous at boundary.

BIASING CHARACTERSTICS

Forward bias characteristic

- Width of depletion layer decreases
- Effective barrier potential decreases
- Low resistance at junction
- High current flow of the order of mA.

Reverse bias characteristic

- Width of depletion layer increases
- Effective barrier potential increases
- High resistance at the junction
- Low current flow of the order of μA.
- Reverse break down occurs at a high reverse bias voltage.

APPLICATIONS OF DIODE

- **Diode as a rectifier**
  - Half wave rectifier
  - Full wave rectifier
- **Zener diode** as a voltage regulator.
- **Photo diode** for detecting light signals.
- **LED:** light emitting diode.
- **Solar cells:** Generates emf from solar radiations.

JUNCTION TRANSISTOR

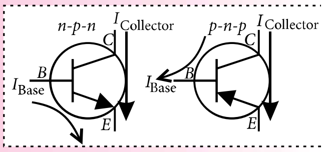
A semiconductor device possessing fundamental action of transfer resistor.

Junction transistors are of two types

- ***n-p-n* transistor:** A thin layer of *p*-type semiconductor is sandwiched between two *n*-type semiconductors.
- ***p-n-p* transistor:** A thin layer of *n*-type semiconductor is sandwiched between two *p*-type semiconductors.

There are three configurations of transistors

- CB (Common Base)
- CE (Common Emitter)
- CC (Common Collector).



Transistor characteristics

- Input resistance $(r_i)_{(CE)} = \left(\dfrac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$
- Output resistance $(r_o)_{(CE)} = \left(\dfrac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B=\text{constant}}$
- Current amplification factor

$$\beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}=\text{constant}} \qquad \alpha_{ac} = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{V_{CB}=\text{constant}}$$

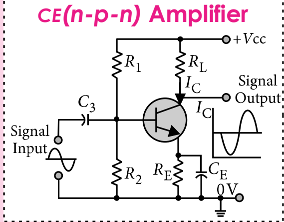
APPLICATIONS OF TRANSISTOR

- **Transistor as an Amplifier**
  - Its operating voltage is fix in active region.
  - Voltage gain,

$$A_v = \frac{V_o}{V_i} = -\beta_{ac}\frac{R_{out}}{R_{in}}$$

  - Power gain, $A_p = A_v \times \beta_{ac}$
- **Transistor as a Switch**
- **Transistor as an Oscillator**

CE(n-p-n) Amplifier

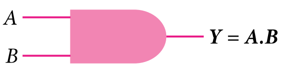


DIGITAL ELECTRONICS AND LOGIC GATES

VARIOUS TYPES OF LOGIC GATE

AND Gate

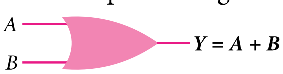
Output is high only when both inputs are high.



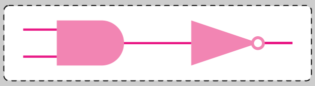
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate

Output is high if any one or both inputs are high.

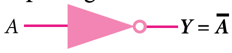


A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

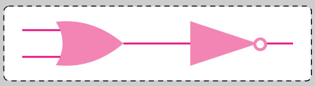


NOT Gate

It just inverts the input signal.

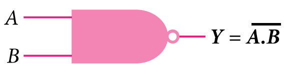


A	Y
0	1
1	0



NAND Gate

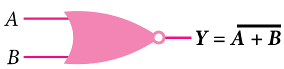
An AND Gate followed by a NOT Gate.



A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate

An OR Gate followed by a NOT Gate.



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Class XII

COMMUNICATION SYSTEMS

BRAIN MAP

MODES OF COMMUNICATION

POINT TO POINT COMMUNICATION
Communication takes place over a link between a single transmitter and a receiver.

BROADCAST MODE COMMUNICATION
Large number of receivers corresponding to a single transmitter.

BASIC COMMUNICATION TECHNIQUE

Transmitter
Converts the message signal suitable for transmission.

Communication Channel
A medium that connects a transmitter to a receiver.

Receiver
Retrieves the message signal into original form.

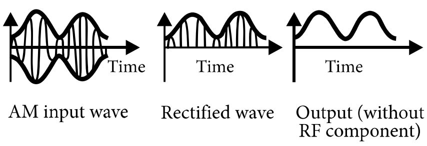
Modulation
Process of variation of some characteristic of a high frequency wave in accordance with the message signal.

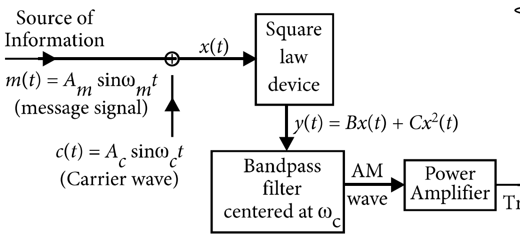
- **Phase Modulation**
- **Frequency Modulation**

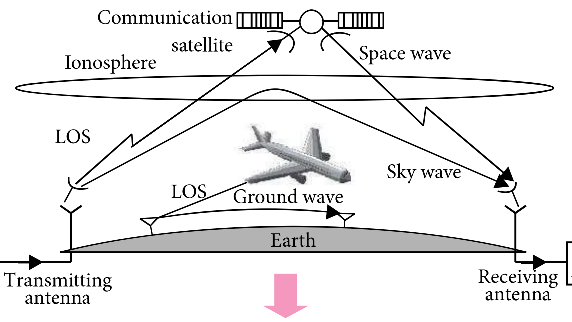
Amplitude Modulation
Amplitude of the high frequency carrier wave changes in accordance with modulating signal.

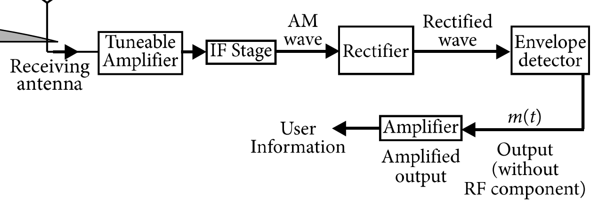
Necessity of Modulation
- *To reduce the size of antenna, need a high frequency carrier wave.*
- *We need high power transmission as $P \propto (1/\lambda)^2$*
- *To avoid the mixing up of signals a band of frequency is allotted to each user for different radio channels.*

Demodulation
Process of recovering the audio signal from the modulated wave is called demodulation or detection.







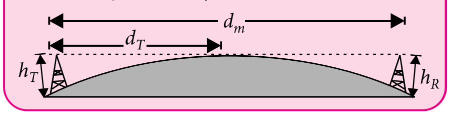


SPACE COMMUNICATION

Space Wave Propagation
A radio wave transmitted from an antenna, directly reaches the receiving antenna by LOS propagation.

- Maximum LOS distance

$$d_m = \sqrt{2h_T R} + \sqrt{2h_R R}$$



Range and Application:
- VHF : 30 MHz - 300 MHz
  TV, FM radio, metrology devices
- UHF : 300 MHz - 3 GHz
  TV, aircraft landing systems

Ground Wave Propagation
Here EM wave glides over the earth surface along its curvature from transmitter to receiver placed close to the surface of earth.

Range and Application:
- LF : 30 kHz - 300 kHz
  Long wave radio communication
- MF : 300 kHz - 3 MHz
  AM radio broadcast for local areas

Sky Wave Propagation
The radio wave directed towards the sky and reflected by the ionosphere towards the desired location on the earth.

- Critical frequency $\upsilon_c = 9(N_{max})^{1/2}$

Range and Application:
- HF : 3 MHz - 30 MHz
  Short wave radio communication, CB radio