

MASTERJEE CLASSES ELECTROMAGNETIC INDUCTION

BRAIN MAP

CLASS XII

Magnetic Energy

- Energy stored in an inductor,
$$U_B = \frac{1}{2} LI^2$$
- Energy stored in the solenoid,
$$U_B = \frac{1}{2\mu_0} B^2 Al$$
- Magnetic energy density,
$$u_B = \frac{U_B}{V} = \frac{B^2}{2\mu_0}$$

Combination of Inductors

- Inductors in series, $L_S = L_1 + L_2 + 2M$
- Inductors in parallel, $L_P = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$
- If coils are far away, then $M = 0$.
So, $L_S = L_1 + L_2$ and $L_P = \frac{L_1 L_2}{L_1 + L_2}$

L-R Circuit

- Current growth in L-R circuit $I = I_0(1 - e^{-t/\tau_L})$
- Current decay in L-R circuit,
 $I = I_0(e^{-t/\tau_L})$
Here, τ_L = Time constant = $\frac{L}{R}$
$$I_0 = \frac{\mathcal{E}}{R}$$

Inductance

- Emf induced in the coil/conductor, $\mathcal{E} = -L \frac{dI}{dt}$
- Coefficient of self induction, $L = \frac{N}{I} \phi_B = \frac{-\mathcal{E}}{dI/dt}$
- Self inductance of a long solenoid, $L = \mu_0 \mu_r n^2 Al = \frac{\mu_0 \mu_r N^2 A}{l}$
- Mutual inductance, $M = \frac{N_2 \phi_2}{I_1} = \frac{-\mathcal{E}_2}{(dI_1/dt)} = \frac{-\mathcal{E}_1}{(dI_2/dt)}$
- Mutual inductance of two long coaxial solenoids,
$$M = \mu_0 \mu_r \pi r_1^2 n_1 n_2 l = \frac{\mu_0 \mu_r N_1 N_2 A_1}{l}$$
- Coefficient of coupling, $k = \frac{M}{\sqrt{L_1 L_2}}$
For perfect coupling, $k = 1$ so, $M = \sqrt{L_1 L_2}$

Induced Electric Field

- It is produced by change in magnetic field in a region. This is non-conservative in nature.
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt} = -A \frac{dB}{dt} \neq 0$$
- This is also known as integral form of Faraday's law.

Lenz's Law

- The direction of the induced current is such that it opposes the change that has produced it.
- If a current is induced by an increasing(decreasing) flux, it will weaken (strengthen) the original flux.
- It is a consequence of the law of conservation of energy.

Magnetic Flux and Faraday's Law

- Magnetic flux $\phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$
- Faraday's law : Whenever magnetic flux linked with a coil changes, an emf is induced in the coil.
 - Induced emf, $\mathcal{E} = -N \frac{d\phi_B}{dt}$
 - Induced current, $I = \frac{\mathcal{E}}{R} = N \frac{(-d\phi_B/dt)}{R}$
 - Induced charge flow, $\Delta Q = I \Delta t = -N \frac{\Delta \phi_B}{R}$

Motional emf

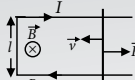
- On a straight conducting wire, $\mathcal{E} = Bvl$
- On a rotating wire about one end, $\mathcal{E} = \frac{B\omega l^2}{2}$
Here, \vec{B} , $\vec{v}(\omega r \hat{\phi})$ and \vec{l} are perpendicular to each other.

Electric Generator

- Mechanical energy is converted into electrical energy by virtue of electromagnetic induction.
- Induced emf,
 $\mathcal{E} = NAB\omega \sin \omega t = \mathcal{E}_0 \sin \omega t$
- Induced current,
$$I = \frac{NBA\omega}{R} \sin \omega t = I_0 \sin \omega t$$

Energy Consideration in Motional emf

- Emf in the wire, $\mathcal{E} = Bvl$
- Induced current, $I = \frac{\mathcal{E}}{R} = \frac{Bvl}{R}$
- Force exerted on the wire,
$$F = \frac{B^2 l^2 v}{R}$$


- Power required to move the wire, $P = \frac{B^2 l^2 v^2}{R}$
It is dissipated as Joule's heat.