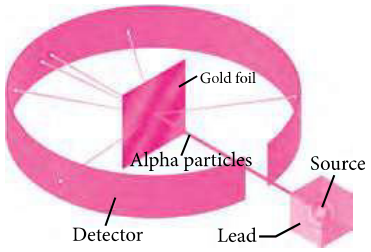
ATOMS AND NUCLEI



Line Spectra of Hydrogen

- While transition between different atomic levels, light radiated in various discrete frequencies are called spectral series of hydrogen atom.
- Rydberg formula :

$$\text{Wave number } \bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

R = Rydberg's constant $= 1.097 \times 10^7\ \text{m}^{-1}$

Radioactivity

- **Law of radioactive decay**

$$\frac{dN}{dt} = -\lambda N(t) \text{ or } N(t) = N_0 e^{-\lambda t}$$

- **Half-life**

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

- **Mean life or Average life**

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44\, T_{1/2}$$

- **Fraction of nuclei left undecayed after *n* half lives is**

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T_{1/2}}, \quad \text{where } t = nT_{1/2}$$

Decay Schemes

- **α-Decay :**

$${}^{A}_{Z}X \xrightarrow{\alpha\text{-decay}} {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{He} + Q$$

(Energy released)

- **β-Decay :**

$${}^{A}_{Z}X \xrightarrow{\beta^+} {}^{A}_{Z-1}Y + {}^{0}_{+1}e + \upsilon$$

$${}^{A}_{Z}X \xrightarrow{\beta^-} {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\upsilon}$$

- **γ-Decay :**

$${}^{A}_{Z}X^{*} \xrightarrow{\gamma\text{-decay}} {}^{A}_{Z}X + {}^{0}_{0}\gamma$$

(Excited state) (Ground state) + Energy

Rutherford's Model of Atom

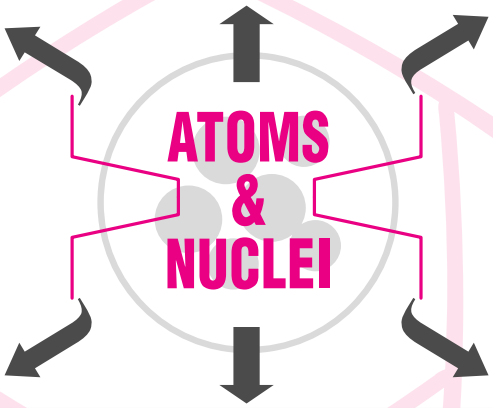
- K.E. of α–particles, $K = \frac{1}{2}mv^2$
- Distance of closest approach,

$$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{K} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4Ze^2}{mv^2}$$

- Impact parameter,

$$b = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{K} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{\frac{1}{2}mv^2}$$

- **Conclusion :** An atom consists of a small and massive central core in which entire positive charge and whole mass of atom is concentrated.
- **Drawback :** The revolving electron continuously loses its energy due to centripetal acceleration and finally it should collapse into the nucleus.



Composition and Size of Nucleus

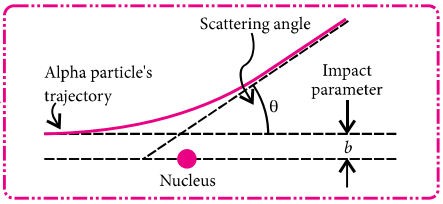
- Nucleus of an atom consists of protons and neutrons collectively called nucleons.
- Radius of a nucleus is proportional to its mass number as $R = R_0 A^{(1/3)}$. ($R_0 = 1.2$ fm)

Concept of Binding Energy

- The binding energy is defined as the surplus energy which the nucleons give up by virtue of their attractions when they bound together to form a nucleus.

$$\Delta E_b = [Zm_p + (A-Z)m_n - M_N]c^2$$

- **Binding energy per nucleon :** $\therefore\ E_{bn} = \frac{E_b}{A}$



Bohr's Atomic Model

Electron orbits and their energy

- Radius of permitted n^{th} orbits,

$$r_n = \frac{n^2h^2}{4\pi^2 mkZe^2} \Rightarrow r_n \propto n^2$$

- Velocity of electron in n^{th} orbit,

$$v_n = \frac{2\pi kZe^2}{nh} \Rightarrow v_n \propto \frac{1}{n}$$

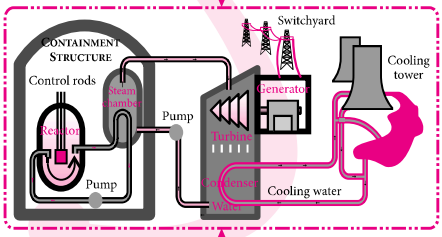
- Energy of electron in n^{th} orbit

$$E_n = \frac{-2\pi^2 mk^2Z^2e^4}{n^2h^2} \Rightarrow E_n \propto \frac{1}{n^2}$$

where the symbols have their usual meanings.

Nuclear Reactions

- **Nuclear fission :** It is the phenomenon of splitting a heavy nucleus into two or more smaller nuclei of nearly comparable masses.
- **Nuclear fusion :** It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.



Application of Nuclear Reactions

A. Fission

- Uncontrolled chain reaction: Principle of atomic bombs.
- Controlled chain reaction: Principle of nuclear reactors.

B. Fusion

Nuclear fusion is the source of energy in the Sun and stars.