BRAIN MAP

CLASS XI

MASTERJEE CLASSES
SYSTEM OF PARTICLES AND ROTATIONAL MOTION

Centre of Mass and Centre of Gravity

- The centre of gravity of a body coincides with its centre of mass only if the gravitational field does not vary from one point of the body to other.
- Mathematically,

$$\vec{R}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j} + z_{CM}\hat{k}$$

  - For discrete body, $x_{CM} = \frac{1}{M}\sum m_i x_i$,

$$y_{CM} = \frac{1}{M}\sum m_i y_i,\ z_{CM} = \frac{1}{M}\sum m_i z_i$$

  - For continuous body, $\vec{R}_{CM} = \frac{1}{M}\int \vec{r}\,dm$
- Centre of mass of symmetric body
  - Semi-circular ring, $y_{CM} = \frac{2R}{\pi}$
  - Semi-circular disc, $y_{CM} = \frac{4R}{3\pi}$

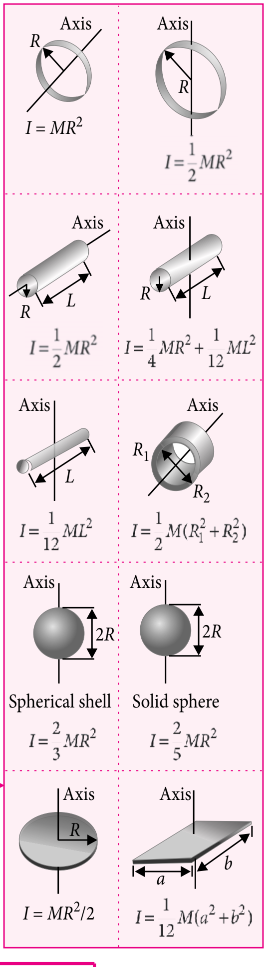
Rotational Motion

- Perpendicular distance of each particle remains constant from a fixed line or point and particle do not move parallel to the line.
- Angular displacement, $\theta = \frac{s}{r}$

  Angular velocity, $\omega = \frac{d\theta}{dt}$

  Angular acceleration, $\alpha = \frac{d\omega}{dt}$
- Equations of rotational motion
  - $\omega = \omega_0 + \alpha t$
  - $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
  - $\omega^2 = \omega_0^2 + 2\alpha\theta$
- Torque : Turning effect of the force about the axis of rotation.

  $\vec{\tau} = \vec{r}\times\vec{F}$; $\tau = rF\sin\theta$; $\tau = I\alpha$
- Angular momentum, $\vec{L} = \vec{r}\times\vec{p}$; $L = I\omega$
- Work done by torque, $W = \tau d\theta$
- Power, $P = \tau\omega$



Motion of Centre of Mass

- For a system of particles
  - Position, $\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...}{m_1 + m_2 + ...}$
  - Velocity, $\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ...}{m_1 + m_2 + ...}$
  - Acceleration, $\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$

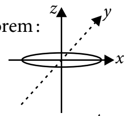
  $\vec{F}_{ext} = 0$, then $\vec{v}_{CM}$ = constant.

Moment of Inertia

- For a rigid body, $I = \sum_{i=1}^{n} m_i r_i^2$
- Perpendicular axes theorem :

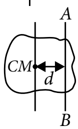
  $I_z = I_x + I_y$

  (Object is in x-y plane)



- Parallel axes theorem :

  $I_{AB} = I_{CM} + Md^2$



Conservation of Angular Momentum

- If the net external torque acting on a system is zero, the angular momentum $\vec{L}$ of the system remains constant, no matter what changes take place within the system.

  $\vec{L}$ = constant; $I_1\omega_1 = I_2\omega_2$

  (for isolated system)

Equilibrium of a Rigid Body

- A rigid body is said to be in mechanical equilibrium, if both of its linear momentum and angular momentum are not changing with time, *i.e.*, total force and total torque are zero.
- Linear momentum does not change implies the condition for the translational equilibrium of the body and angular momentum does not change implies the condition for the rotational equilibrium of the body.

Rolling Motion

- For a body rolling without slipping, velocity of centre of mass

  $v_{CM} = R\omega$

  Kinetic energy,

$$K = K_{translational} + K_{rotational}$$

$$= \frac{1}{2}mv_{CM}^2\left(1 + \frac{k^2}{R^2}\right)$$

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