BRAIN MAP

OSCILLATIONS

Class XI

Periodic Motion

A motion that repeats itself at regular interval of time is called periodic motion. The displacement is represented by a periodic function of time with time period T. *i.e.*, $f(t) = f(t+T) = f(t+2T) = ...$

Oscillatory Motion

If the body is given a small displacement from the position, a force comes into play which tries to bring the body back to the equilibrium point. Such motions are called oscillatory motion.

Simple Harmonic Motion

The motion arises when the force on the oscillating body is directly proportional to its displacement from mean position. Such motion is called simple harmonic motion.

System Excuting SHM | Dynamic of SHM | Charact-eristics of SHM

SHM IN SPRING

- Equation of motion

$$\frac{d^2y}{dt^2} = \frac{-ky}{m} = -\omega^2 y$$

- If the spring is not light but has a definite mass m_s then

$$T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$$

- Two bodies of masses m_1 and m_2 are attached through a light spring of spring constant k, the time period of oscillation

$$T = 2\pi\sqrt{\frac{\mu}{k}} \quad \text{where } \mu = \frac{m_1 m_2}{m_1 + m_2}$$

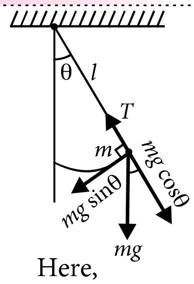
SIMPLE PENDULUM

- Time period

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{l}{g}}$$

- If the length of simple pendulum is very large,

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}}$$



Here, $\alpha = -\frac{mgl}{I}\theta$

where R is the radius of length of pendulum

FORCE LAW IN SHM

- The force acting on a particle of mass m in SHM is
  $\vec{F} = -m\omega^2\vec{x}$ or $\vec{F} = -k\vec{x}$
  where, $k = m\omega^2$ = force constant
- Linear SHM:
  - Angular velocity, $\omega = \sqrt{\frac{k}{m}}$
  - Time period, $T = 2\pi\sqrt{\frac{m}{k}}$
- Angular SHM:
  - Torque, $\tau = -k\theta$
  - Angular velocity, $\omega = \sqrt{k/I}$
  - Angular acceleration, $\alpha = -\frac{k\theta}{I}$
  - Time period, $T = 2\pi\sqrt{\frac{I}{k}}$
    where I = moment of inertia

ENERGY IN SHM

- Linear SHM:
  - Kinetic energy $(K) = \frac{1}{2}m\omega^2A^2\cos^2\omega t$
  - Potential energy $(U) = \frac{1}{2}m\omega^2A^2\sin^2\omega t$
  - Total energy $(E) = \frac{1}{2}m\omega^2A^2$
- Angular SHM:
  - Kinetic energy $(K) = \frac{1}{2}I\omega^2$
  - Potential energy $(U) = \frac{1}{2}k\theta^2 = \frac{1}{2}I\omega^2\theta^2$
  - Total energy $(E) = \frac{1}{2}I\omega^2\theta_0^2$

GENERAL EQUATIONS OF SHM

- Linear SHM:
  - Differential equation $\frac{d^2y}{dt^2} + \omega^2 y = 0$
  - Displacement $y = A\sin(\omega t + \phi)$
  - Velocity, $v = \omega\sqrt{A^2 - y^2}$
  - Acceleration, $a = -\omega^2 y$
- Angular SHM:
  - Differential equation
    $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$
  - Displacement $\theta = \theta_0\sin(\omega t + \delta)$

DAMPED AND FORCED OSCILLATIONS

- Damped oscillations
  - Angular frequency $(\omega') = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
  - Mechanical energy $E(t) = \frac{1}{2}kA^2e^{\frac{-bt}{m}}$
  - Amplitude $A' = Ae^{-bt/2m}$
    where b is damping constant.
- Forced oscillations
  - When driving frequency ω_d far from natural frequency ω:
    Amplitude $A' = \frac{F_0}{m(\omega^2 - \omega_d^2)}$
  - When driving frequency ω_d closed to natural frequency ω:
    Amplitude $A' = \frac{F_0}{\omega_d b}$

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