

ELECTRIC CHARGES AND FIELDS

Basic Properties of Charges

Quantization of charge :
Total charge on a body is always an integral multiple of a basic unit of charge denoted by e and is given by $q = ne$.

Conservation of charge :
Total charge of an isolated system remains unchanged with time.

Additivity of charge : Total charge of a system is the algebraic sum (*i.e.* sum taking into account with proper signs) of all individual charges in the system.

Coulomb's Law

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^3} \hat{r}_{12} \text{ for like charges}$$

$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{21}^3} \hat{r}_{21} \text{ for unlike charges}$$

Field lines start from positive charges and end at negative charges.

using Superposition principle

The vector sum of forces would give us the total force.

Force between two charges is unaffected by the presence of the other charges.

Electric Field : Electric field intensity at a point distant r from a point charge q in air is $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$.

Basic characteristics

Electrostatic field lines do not form any closed loops.

Two field lines can never cross each other.

Electric field due to dipole

On Equatorial line
(Broad on position)

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

On Axial line
(End on position)

$$E = \frac{2p}{4\pi\epsilon_0 r^3}$$

Electric Dipole : Every dipole is associated with a dipole moment \vec{p} whose magnitude is equal to the product of the magnitude of either charge (q) and the distance $2a$ between the charges, *i.e.*, $\vec{p} = q \times (2\vec{a})$.

Dipole in an external field experiences

Torque

$$\vec{\tau} = \vec{p} \times \vec{E} \text{ or } \tau = pE \sin\theta.$$

Electric Flux : Electric flux over an area in an electric field represents the total number of electric field lines crossing this area.

Gauss's Theorem : Total normal electric flux over a closed surface S in vacuum is $1/\epsilon_0$ times the charge (Q) contained inside the surface. $\phi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}$.

Applications

volume charge density, $\rho = \frac{q}{V}$

Electric field due to a uniformly charged thin spherical shell

At a point outside the shell *i.e.*, $r > R$, $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{\rho R^3}{3\epsilon_0 r^2}$

On the surface $r = R$, $E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$

Inside the shell *i.e.*, $r < R$, $E = 0$

surface charge density, $\sigma = \frac{q}{A}$

Electric field due to a uniformly charged infinite thin plane sheet, $E = \frac{\sigma}{2\epsilon_0}$.

linear charge density, $\lambda = \frac{q}{l}$

Electric field due to an infinitely long thin uniformly charged straight wire, $E = \frac{\lambda}{2\pi\epsilon_0 r}$.

